EE 599 Spring 2020

Homework1

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https://github.com/gaochaousc/EE-599_-gaoc-_-4458723430-

1 Odd-even transposition sort [40 Points]

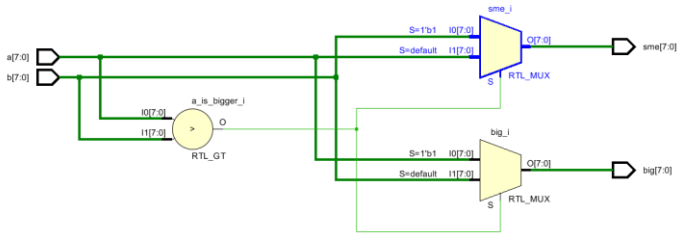
Odd-even transposition sort algorithm is a parallel sorting algorithm. It sorts n elements in n clocks (n is even), each of which requires $n/2$ compare-exchange operations. This algorithm alternates between two phases, called the odd and even phases. Let $< a_1, a_2, ..., a_n >$ be the sequence to be sorted. During the odd phase, elements with odd indices are compared with their right neighbors, and if they are out of sequence they are exchanged; thus, the pairs $(a_1, a_2), (a_3, a_4), ..., (a_{n-1}, a_n)$ are compare-exchanged (assuming n is even). Similarly, during the even phase, elements with even indices are compared with their right neighbors, and if they are out of sequence they are exchanged; thus, the pairs (a_2, a_3), (a_4, a_5), ..., (a_{n-2}, a_{n-1}) are compare-exchanged. After n phases of odd-even exchanges, the sequence is sorted. An example sorting instance is shown in Figure 1 and the sequential algorithm is shown in figure 2.

1.1 Implementation : Using Xilinx Vivado Software (Use Zynq-7000 xc7z007sclg225-2 FPGA)

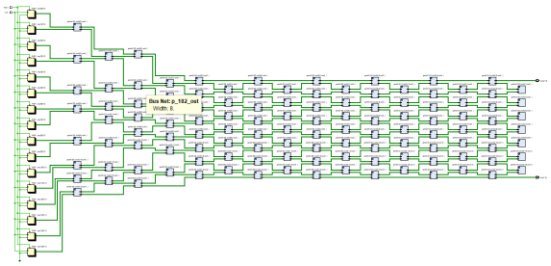
Consider only 8-bit arithmetic. All the values are already stored in the BRAM.

1. Using Verilog, implement **odd-even transposition circuit**, which takes n, 8 bit inputs and sort them.
2. For a 16 elements write a test bench and verify the waveforms.
3. Elaborate the design and include all the schematics' screenshots of the modules in the report.
4. Synthesis the design and include the screenshots.
5. Generate Resource and timing estimations and include them in the report.
6. Redo part 3, 4, 5 for 32, 64, 128.

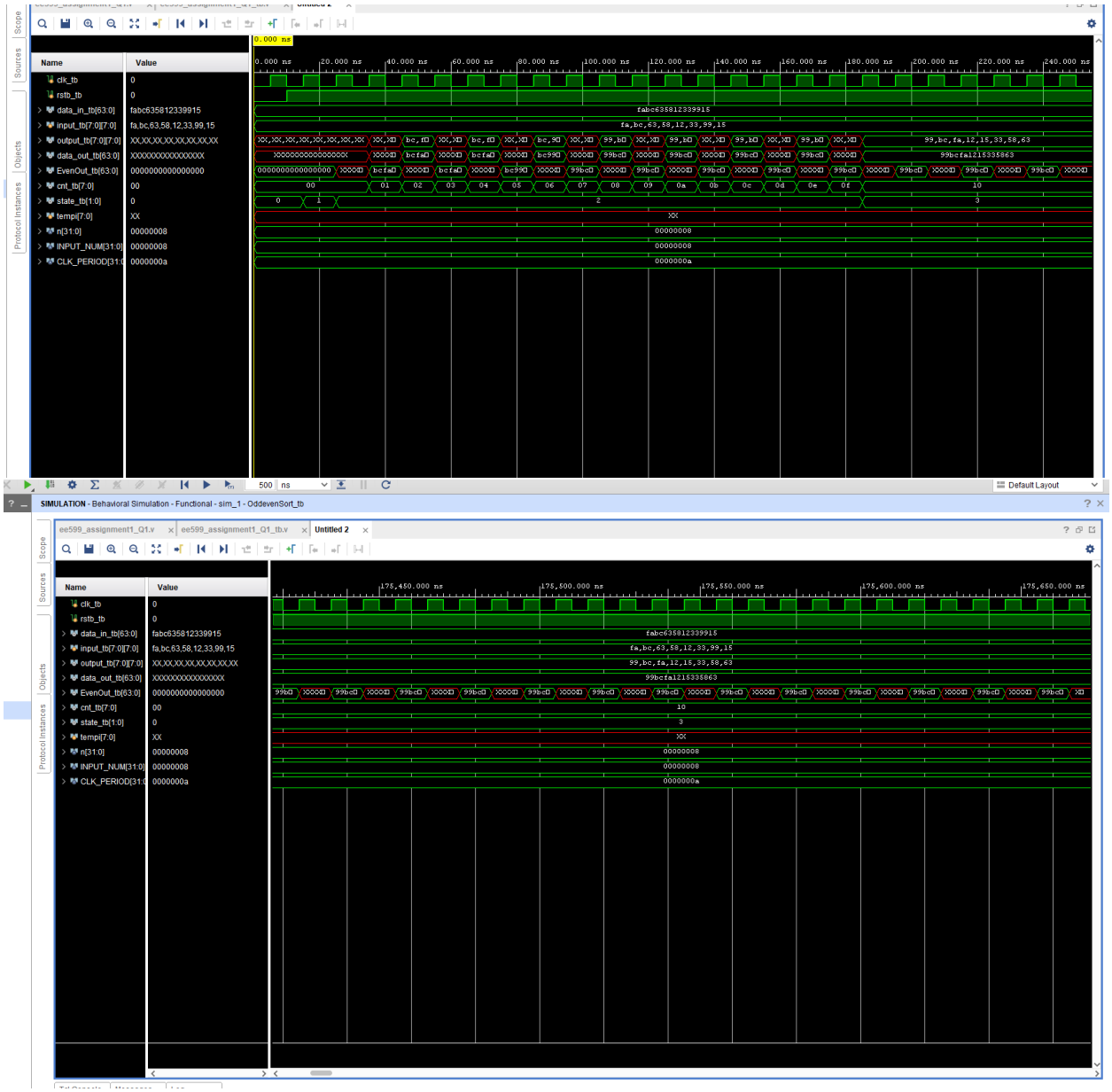
1. Schematic's screenshots
   a. CaS model: this is the model that compare two 8-bit number A and B, and sort them in the order of magnitude.



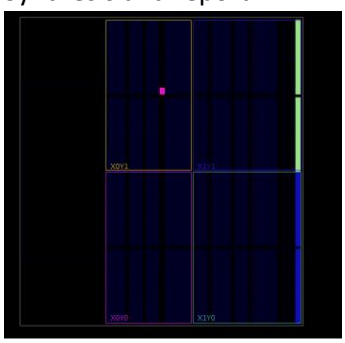
b. OES is the top module of the design (the schematic of 32,64,128 will be included in the later part of the report)



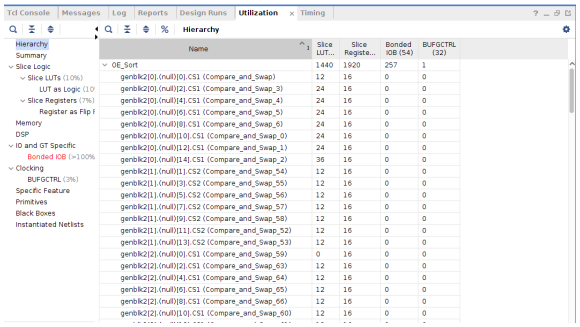
c. Function waveform
In this simulation, we can get the output in order of big and small



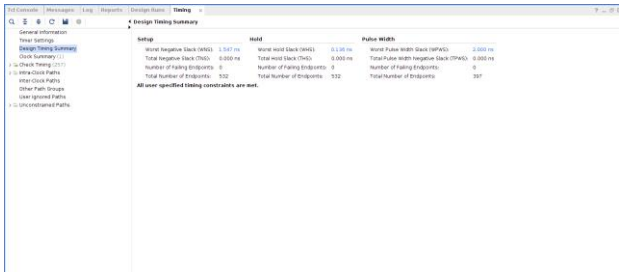
d. Simulation Report for 16 elements:
Synthesis and report:



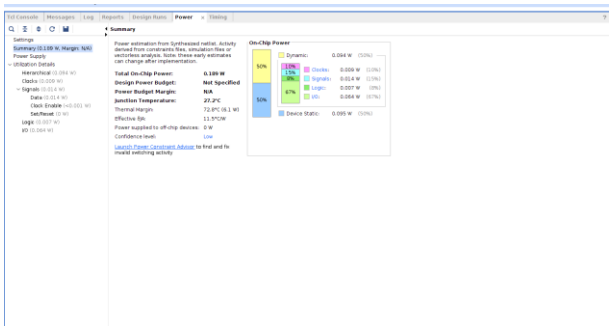
1. Resource estimation



2. Timing estimation

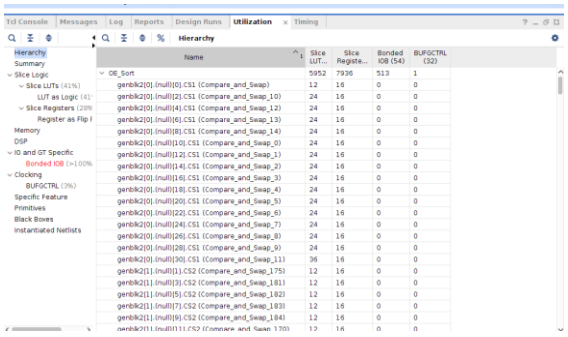


3. Power estimation

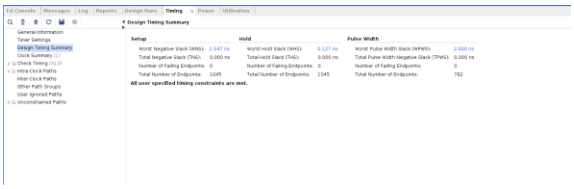


e. Simulation for 32 elements

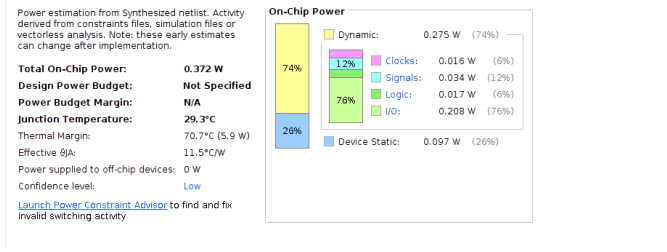
1. Resource estimation



2. Timing estimation

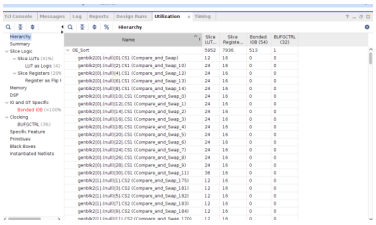


3. Power estimation

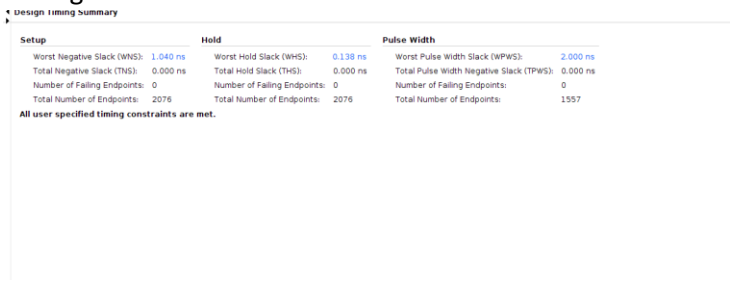


f. Simulation for 64 elements

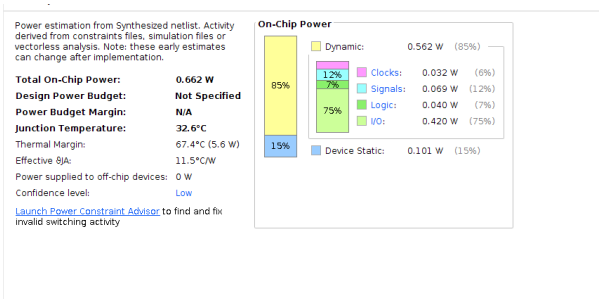
1. Resource estimation



2. Timing estimation

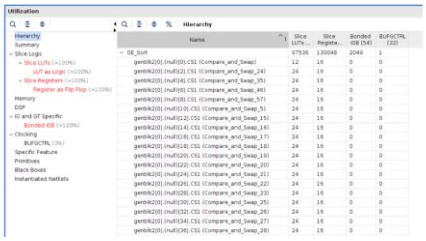


3. Power estimation



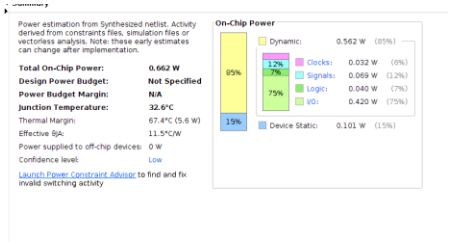
g. Simulation for 128 elements

1. Resource estimation



2. Timing estimation

3. Power estimation



2 Dense Matrix-Matrix Multiplication [60 Points]

2.1 Scalable Multiply and adder tree

If A is an $m \times n$ matrix and B is an $n \times p$ matrix, the matrix product $C = AB$ (denoted without multiplication signs or dots) is defined to be the $m \times p$ matrix such that,

$$c_{i,j} = a_{i,1}b_{1,j} + a_{i,2}b_{2,j} + ... + a_{i,n}b_{n,j} = \sum_{k=1}^{n} a_{i,k}b_{k,j};$$

where for i = 1, ..., m and j = 1, ..., p

Consider two matrices A and B, each having the size of $n \times n$ where $n = 2^r$.

Figure 3 shows an example design of Multiply and Adder Tree. The adder tree consists of a Multiplication Step following Adder Steps. Given the size of matrices is $n \times n$, there are n multipliers in the first stage. Assume that matrix A saved in row order, and matrix B saved in column order in the memory. In the beginning, the first row of A and the first column of B loaded and multiplied together. Then in each Adder Step, partial sums are added together until it produces the final result corresponding to an element in the output matrix. Adder steps consist of 2 element adders as shown in Figure 2. Notice that Multiply and adder tree is a pipeline process. Notice that in each step, after corresponding rows and columns of A and B going through the pipe, it produces one element of the output matrix.

2.1.1 Design Problems

Consider simple Multiply and Adder Tree design with n element multiplication,

1. How many Multiplier units needed for the entire design?

n

2. Consider an adder stage r, how many adder modules needed for the stage?

$$2^{logn-r}$$

3. If all the inputs to the design represented using k bits, how many bits are needed to represent the final result of the Multiply and Adder Tree?

$$2k\text{-}1+logn$$

4. How may Adder modules need for the entire multiply and adder tree design?

n-1

5. How many clock cycles need to produce the first output element in the adder tree?

$$logn + 1$$

6. How many clock cycles need to multiply two n× n matrices?

$$n^2 + logn$$

2.1.2 Implementation

1. Write a testbench which provides two 4X4 matrices to the design and takes output final 4X4 resultant matrix.
2. Simulate the design using the simulator and include the waveform in the report (Clearly indicate the locations where final outputs produced)
3. Elaborate the design and include all the schematics' screenshots of the modules (MulandAddTree, adder, and multiply) in the report.
4. Synthesis the design and include the screenshots like part 3.
5. Generate Resource and timing estimations and include them in the report.

Sche

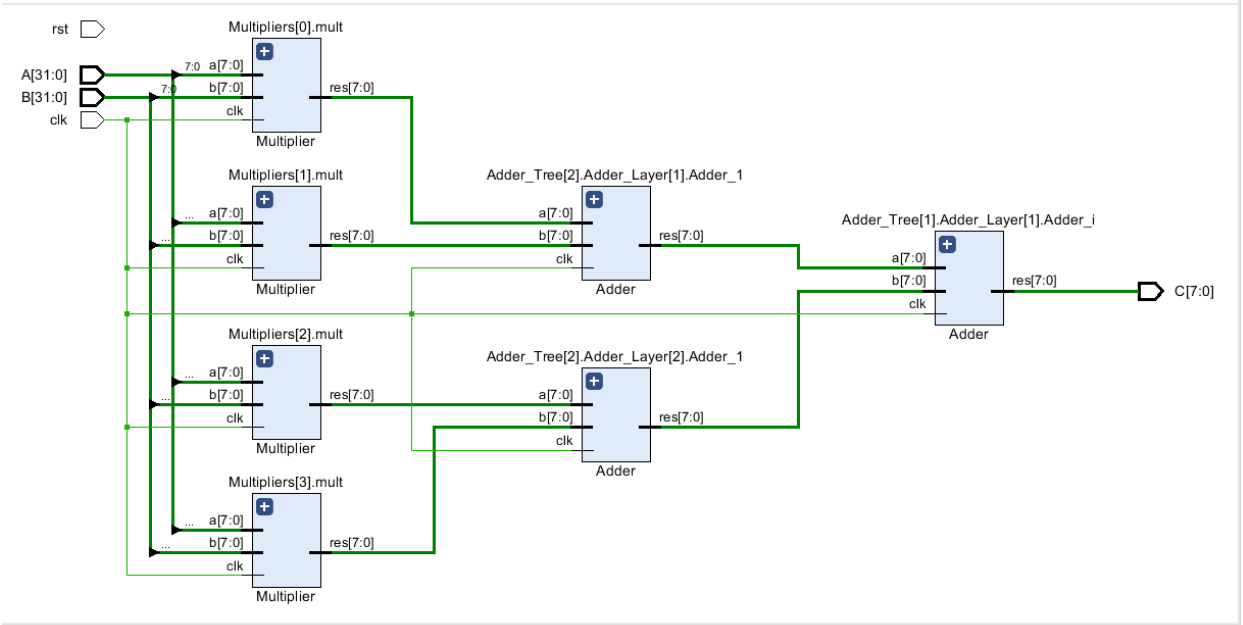
6. Generate power estimation reports and include them in the report.
7. How many of parallel *MulandAddTrees* can be implemented in this FPGA (Provide resource utilization reports with parallel *MulandAddTres*)?
8. Redo part 4,5,6 for 8x8, 16x16 and 32x32 matrices.

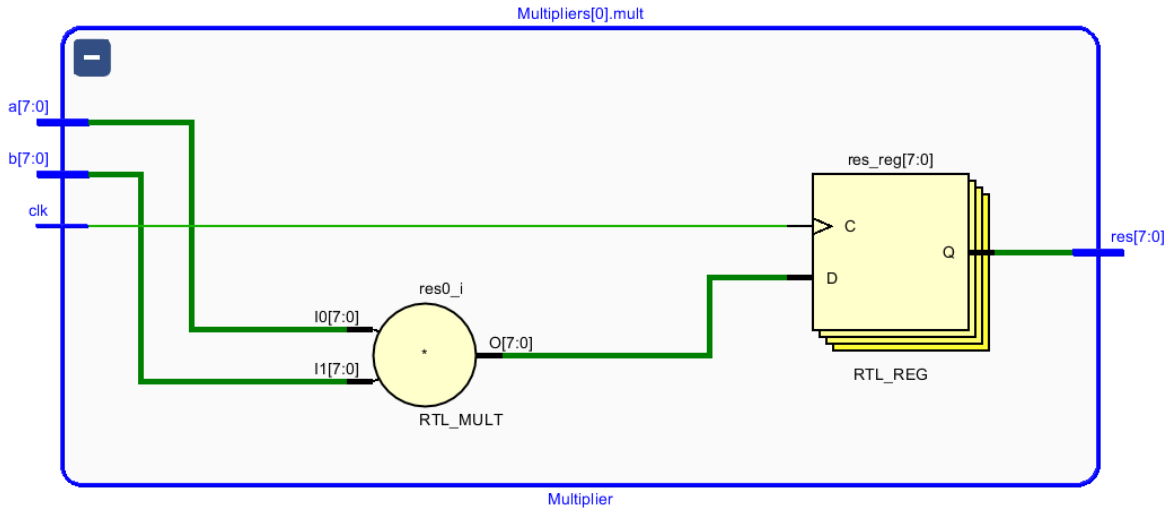
Schematic's screenshots are shown below:

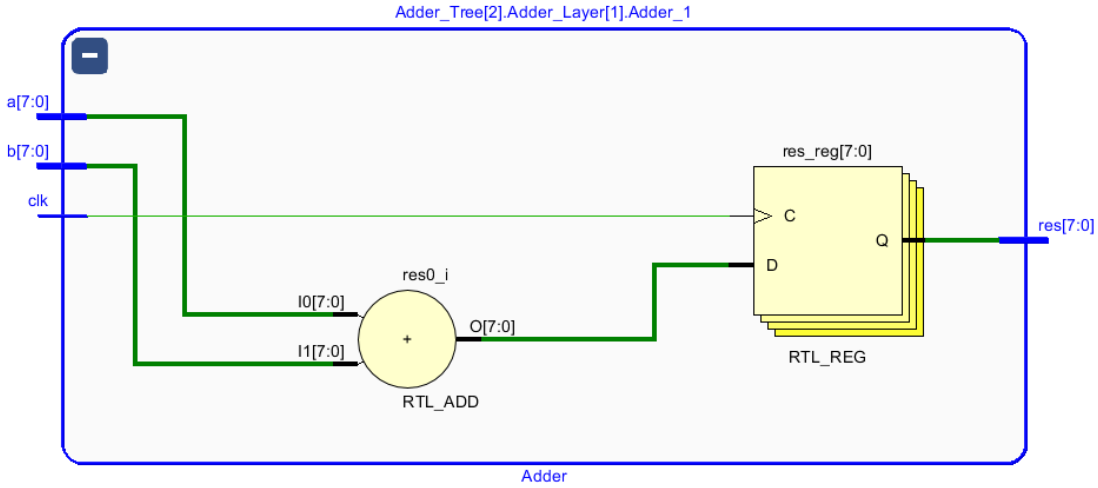
a. Multiply Unit is designed to get the product of two 8-bit number A and B.

b. And according to the number of elements of matrix, combine all the multiply units into Multiply Combination.

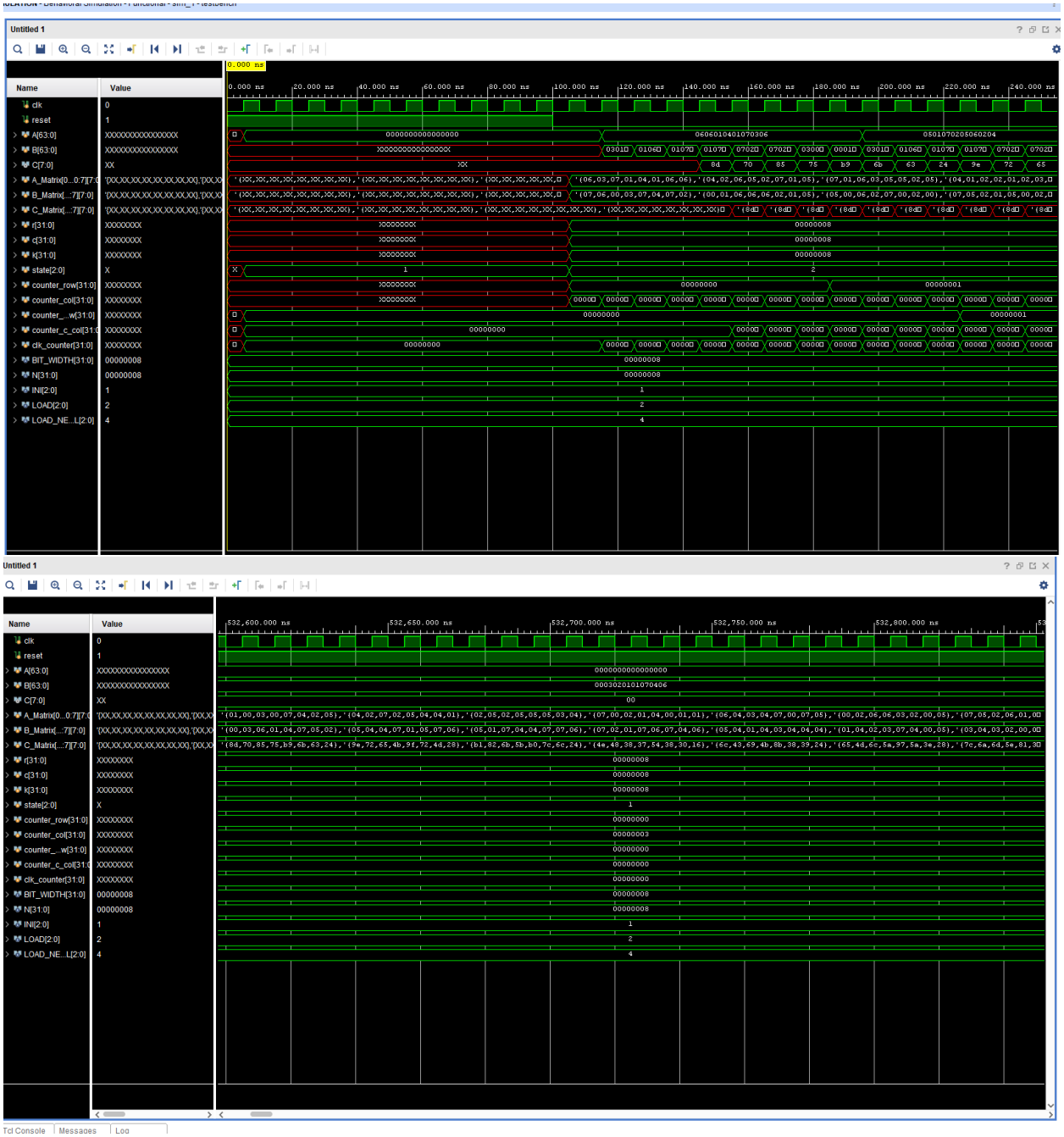
c. For adder, it computes the 4 A and B

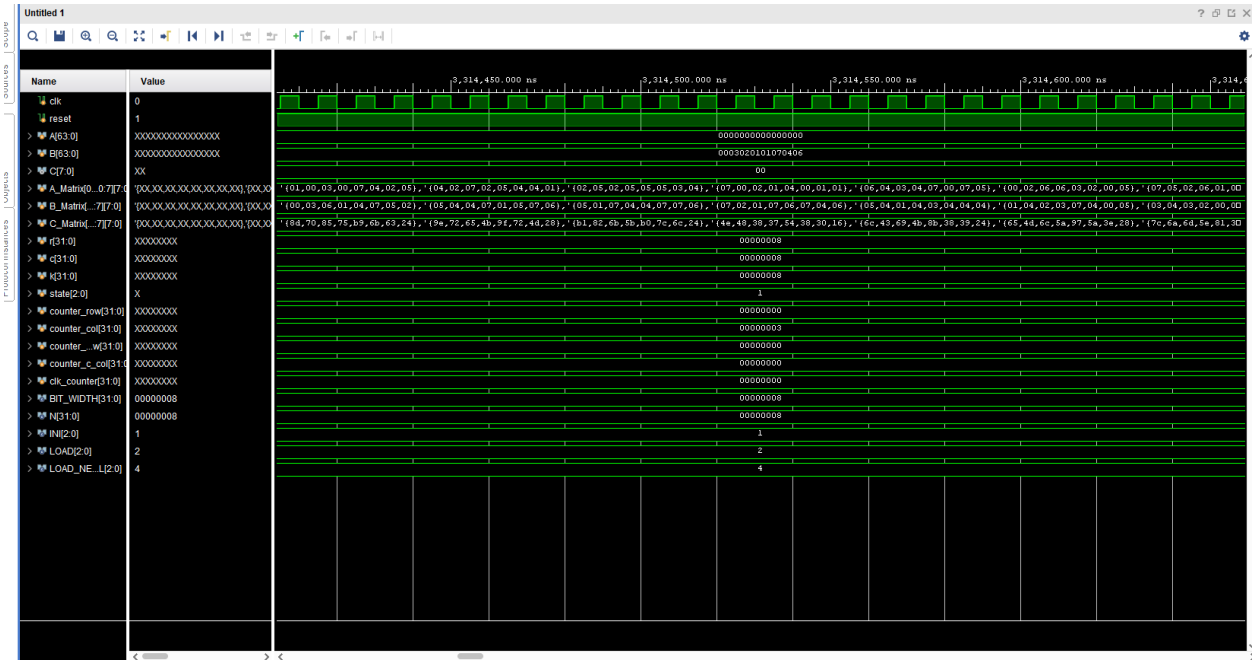

rst
A[31:0]
B[31:0]
clk
Multipliers[0].mult
7:0
a[7:0]
b[7:0]
clk
res[7:0]
Multiplier
Multipliers[1].mult
Multiplier
Multipliers[2].mult
Multiplier
Multipliers[3].mult
Multiplier
Adder_Tree[2].Adder_Layer[1].Adder_1
Adder
Adder_Tree[2].Adder_Layer[2].Adder_1
Adder
Adder_Tree[1].Adder_Layer[1].Adder_i
Adder
C[7:0]





d. Function waveform


Untitled 1
Name
Value
clk
reset
A[63:0]
B[63:0]
C[7:0]
A_Matrix[0...7][7:0]
B_Matrix[...7][7:0]
C_Matrix[...7][7:0]
r[31:0]
c[31:0]
k[31:0]
state[2:0]
counter_row[31:0]
counter_col[31:0]
counter_...w[31:0]
counter_c_col[31:0]
clk_counter[31:0]
BIT_WIDTH[31:0]
N[31:0]
INI[2:0]
LOAD[2:0]
LOAD_NE...L[2:0]
0.000 ns
20.000 ns
40.000 ns
60.000 ns
80.000 ns
100.000 ns
120.000 ns
140.000 ns
160.000 ns
180.000 ns
200.000 ns
220.000 ns
240.000 ns
Untitled 1
532,600.000 ns
532,650.000 ns
532,700.000 ns
532,750.000 ns
532,800.000 ns
Tcl Console
Messages
Log



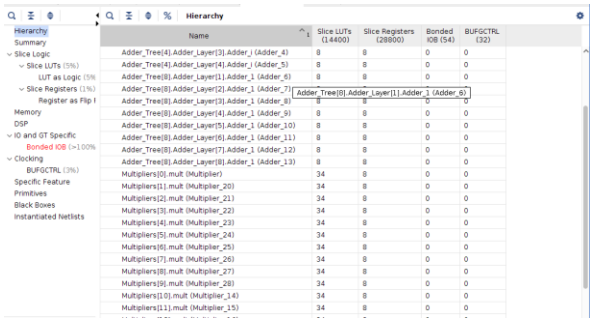
e. 8-bit timing analysis

Design Timing Summary

Setup		Hold		Pulse Width	
Worst Negative Slack (WNS):	3.161 ns	Worst Hold Slack (WHS):	0.170 ns	Worst Pulse Width Slack (WPWS):	2.000 ns
Total Negative Slack (TNS):	0.000 ns	Total Hold Slack (THS):	0.000 ns	Total Pulse Width Negative Slack (TPWS):	0.000 ns
Number of Failing Endpoints:	0	Number of Failing Endpoints:	0	Number of Failing Endpoints:	0
Total Number of Endpoints:	120	Total Number of Endpoints:	120	Total Number of Endpoints:	249

All user specified timing constraints are met.

f. 8-bit resources analysis



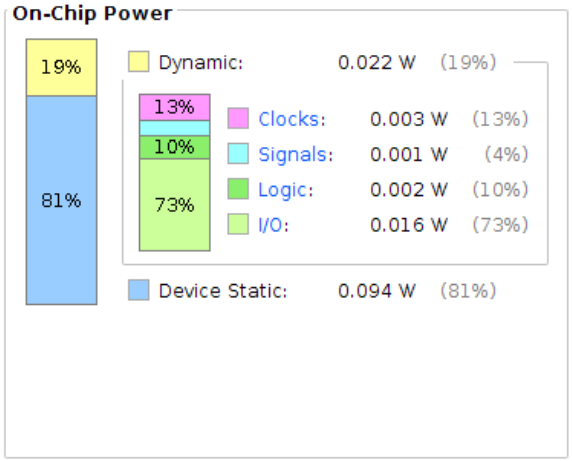
g. 8-bit power analysis

Summary

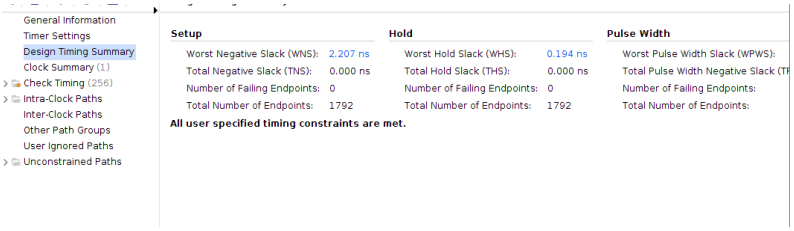
Power estimation from Synthesized netlist. Activity derived from constraints files, simulation files or vectorless analysis. Note: these early estimates can change after implementation.

Total On-Chip Power:	**0.116 W**
Design Power Budget:	**Not Specified**
Power Budget Margin:	**N/A**
Junction Temperature:	**26.3°C**
Thermal Margin:	73.7°C (6.2 W)
Effective θJA:	11.5°C/W
Power supplied to off-chip devices:	0 W
Confidence level:	Low

Launch Power Constraint Advisor to find and fix invalid switching activity



h. 16 inputs timing analysis



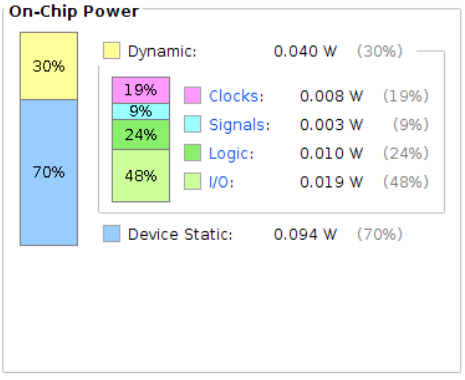
i. 16 inputs power analysis

Summary

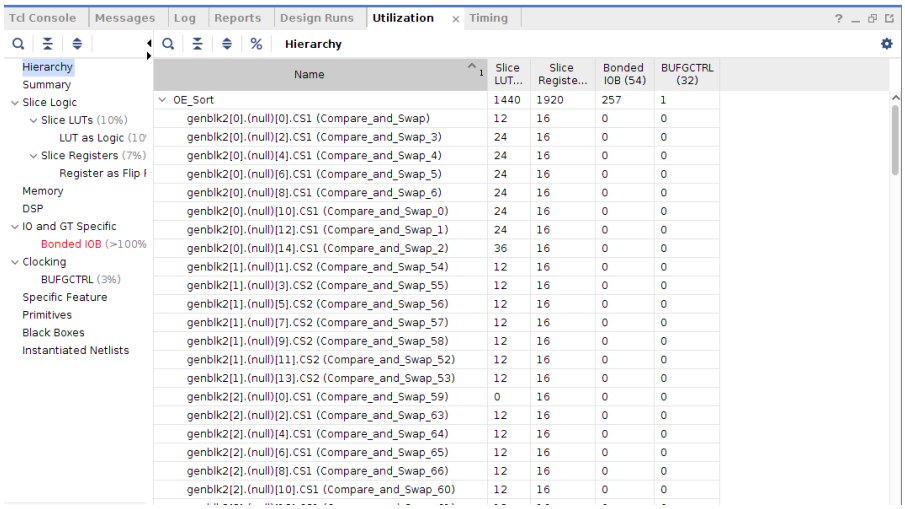
Power estimation from Synthesized netlist. Activity derived from constraints files, simulation files or vectorless analysis. Note: these early estimates can change after implementation.

Total On-Chip Power:	**0.134 W**
Design Power Budget:	**Not Specified**
Power Budget Margin:	**N/A**
Junction Temperature:	**26.5°C**
Thermal Margin:	73.5°C (6.2 W)
Effective θJA:	11.5°C/W
Power supplied to off-chip devices:	0 W
Confidence level:	Low

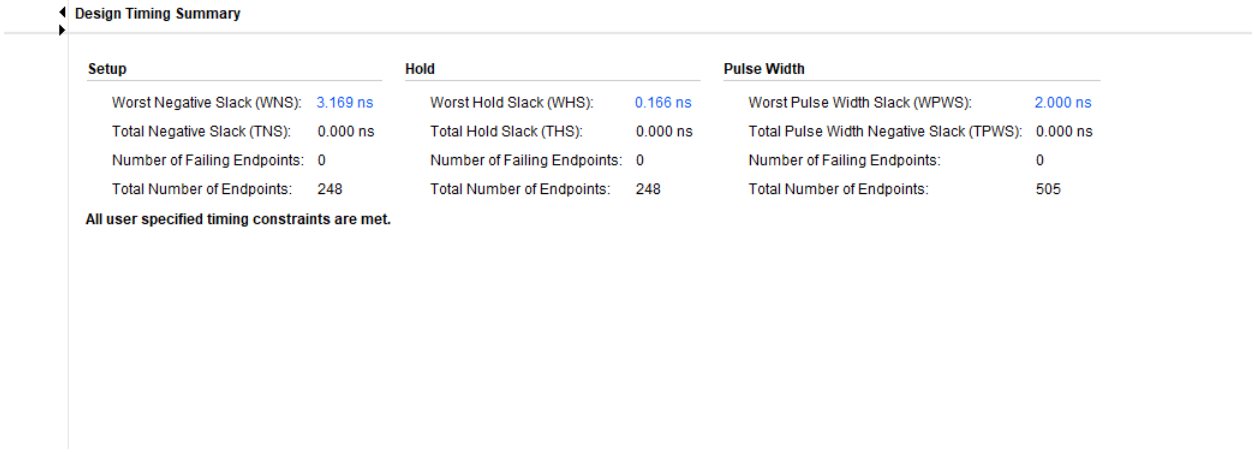
Launch Power Constraint Advisor to find and fix invalid switching activity



j. 16 inputs resources analysis



32inputs timing analysis



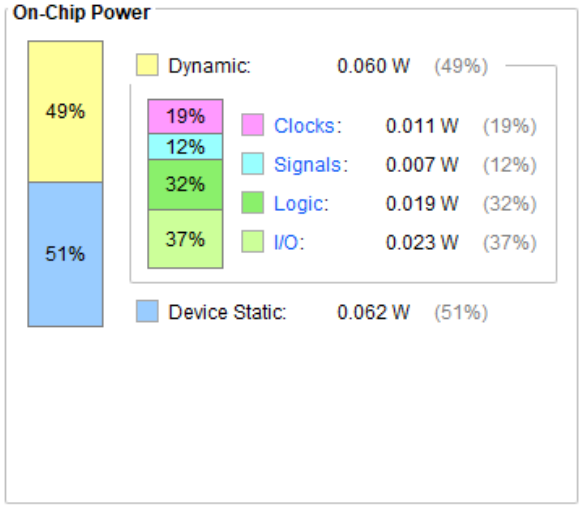
32 inputs power analysis

Summary

Power estimation from Synthesized netlist. Activity derived from constraints files, simulation files or vectorless analysis. Note: these early estimates can change after implementation.

Total On-Chip Power:	**0.123 W**
Design Power Budget:	**Not Specified**
Power Budget Margin:	**N/A**
Junction Temperature:	**25.8°C**
Thermal Margin:	74.2°C (12.0 W)
Effective ϑJA:	6.2°C/W
Power supplied to off-chip devices:	0 W
Confidence level:	Low

Launch Power Constraint Advisor to find and fix invalid switching activity



32 inputs resource analysis

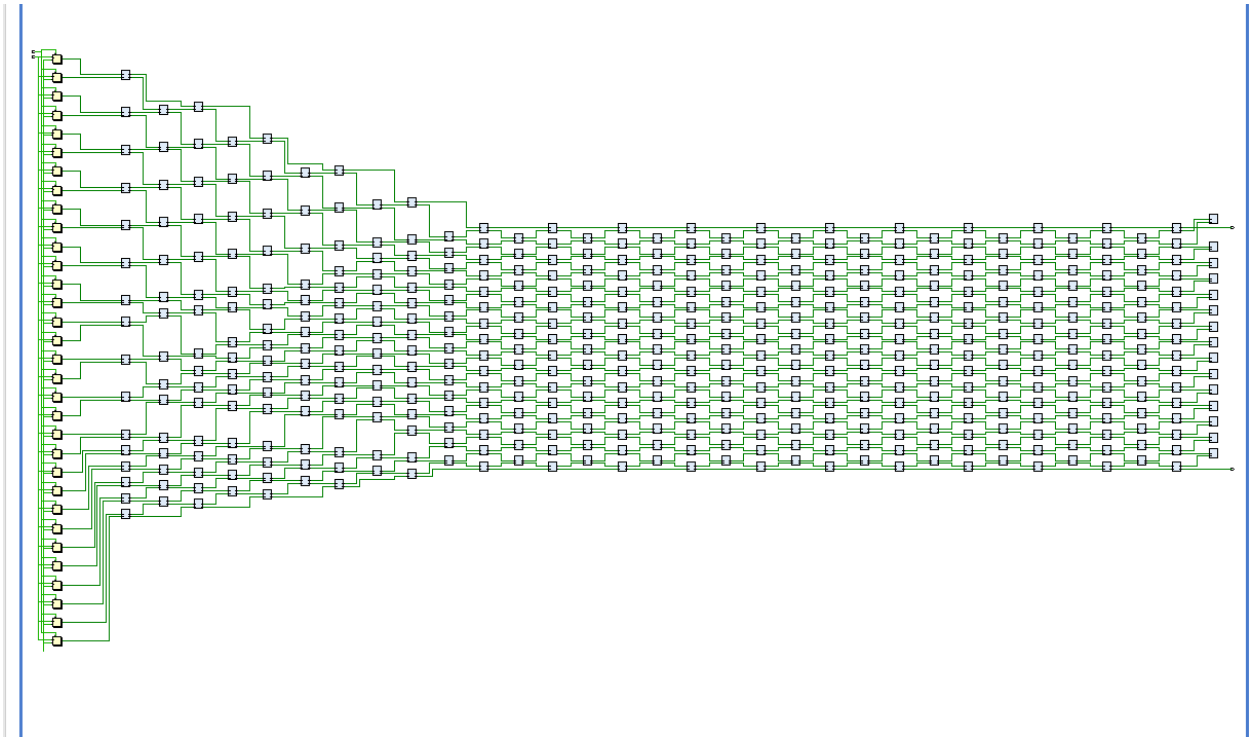
Hierarchy

Name	Slice LUTs (8000)	Slice Registers (16000)	Bonded IOB (112)	BUFGCTRL (32)
MulandAddTree	1336	504	521	1
Adder_Tree[1].Adder_Layer[1].Adder_i (Adder_15)	8	8	0	0
Adder_Tree[2].Adder_Layer[1].Adder_i (Adder_16)	8	8	0	0
Adder_Tree[2].Adder_Layer[2].Adder_i (Adder_17)	8	8	0	0
Adder_Tree[4].Adder_Layer[1].Adder_i (Adder_18)	8	8	0	0
Adder_Tree[4].Adder_Layer[2].Adder_i (Adder_19)	8	8	0	0
Adder_Tree[4].Adder_Layer[3].Adder_i (Adder_20)	8	8	0	0
Adder_Tree[4].Adder_Layer[4].Adder_i (Adder_21)	8	8	0	0
Adder_Tree[8].Adder_Layer[1].Adder_i (Adder_22)	8	8	0	0
Adder_Tree[8].Adder_Layer[2].Adder_i (Adder_23)	8	8	0	0
Adder_Tree[8].Adder_Layer[3].Adder_i (Adder_24)	8	8	0	0
Adder_Tree[8].Adder_Layer[4].Adder_i (Adder_25)	8	8	0	0
Adder_Tree[8].Adder_Layer[5].Adder_i (Adder_26)	8	8	0	0
Adder_Tree[8].Adder_Layer[6].Adder_i (Adder_27)	8	8	0	0
Adder_Tree[8].Adder_Layer[7].Adder_i (Adder_28)	8	8	0	0
Adder_Tree[8].Adder_Layer[8].Adder_i (Adder_29)	8	8	0	0
Adder_Tree[16].Adder_Layer[1].Adder_1 (Adder_6)	8	8	0	0
Adder_Tree[16].Adder_Layer[2].Adder_1 (Adder_7)	8	8	0	0
Adder_Tree[16].Adder_Layer[3].Adder_1 (Adder_8)	8	8	0	0
Adder_Tree[16].Adder_Layer[4].Adder_1 (Adder_9)	8	8	0	0
Adder_Tree[16].Adder_Layer[5].Adder_1 (Adder_10)	8	8	0	0
Adder_Tree[16].Adder_Layer[6].Adder_1 (Adder_11)	8	8	0	0
Adder_Tree[16].Adder_Layer[7].Adder_1 (Adder_12)	8	8	0	0
Adder_Tree[16].Adder_Layer[8].Adder_1 (Adder_13)	8	8	0	0
Adder_Tree[16].Adder_Layer[9].Adder_1 (Adder_14)	8	8	0	0
Adder_Tree[16].Adder_Layer[10].Adder_1 (Adder)	8	8	0	0
Adder_Tree[16].Adder_Layer[11].Adder_1 (Adder_0)	8	8	0	0
Adder_Tree[16].Adder_Layer[12].Adder_1 (Adder_1)	8	8	0	0
Adder_Tree[16].Adder_Layer[13].Adder_1 (Adder_2)	8	8	0	0
Adder_Tree[16].Adder_Layer[14].Adder_1 (Adder_3)	8	8	0	0
Adder_Tree[16].Adder_Layer[15].Adder_1 (Adder_4)	8	8	0	0
Adder_Tree[16].Adder_Layer[16].Adder_1 (Adder_5)	8	8	0	0

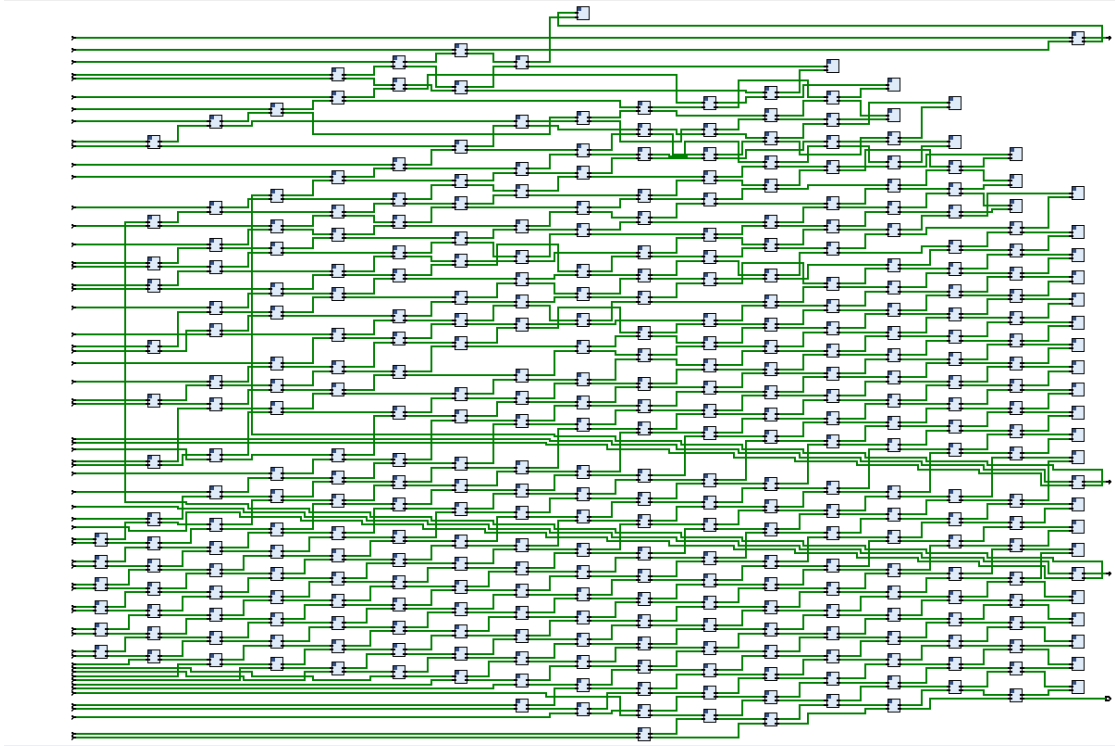
Multipliers[0].mult (Multiplier)	34	8	0	0
Multipliers[1].mult (Multiplier_40)	34	8	0	0
Multipliers[2].mult (Multiplier_51)	34	8	0	0
Multipliers[3].mult (Multiplier_54)	34	8	0	0
Multipliers[4].mult (Multiplier_55)	34	8	0	0
Multipliers[5].mult (Multiplier_56)	34	8	0	0
Multipliers[6].mult (Multiplier_57)	34	8	0	0
Multipliers[7].mult (Multiplier_58)	34	8	0	0
Multipliers[8].mult (Multiplier_59)	34	8	0	0
Multipliers[9].mult (Multiplier_60)	34	8	0	0
Multipliers[10].mult (Multiplier_30)	34	8	0	0
Multipliers[11].mult (Multiplier_31)	34	8	0	0
Multipliers[12].mult (Multiplier_32)	34	8	0	0
Multipliers[13].mult (Multiplier_33)	34	8	0	0
Multipliers[14].mult (Multiplier_34)	34	8	0	0
Multipliers[15].mult (Multiplier_35)	34	8	0	0
Multipliers[16].mult (Multiplier_36)	34	8	0	0
Multipliers[17].mult (Multiplier_37)	34	8	0	0
Multipliers[18].mult (Multiplier_38)	34	8	0	0
Multipliers[19].mult (Multiplier_39)	34	8	0	0
Multipliers[20].mult (Multiplier_41)	34	8	0	0
Multipliers[21].mult (Multiplier_42)	34	8	0	0
Multipliers[22].mult (Multiplier_43)	34	8	0	0
Multipliers[23].mult (Multiplier_44)	34	8	0	0
Multipliers[24].mult (Multiplier_45)	34	8	0	0
Multipliers[25].mult (Multiplier_46)	34	8	0	0
Multipliers[26].mult (Multiplier_47)	34	8	0	0
Multipliers[27].mult (Multiplier_48)	34	8	0	0
Multipliers[28].mult (Multiplier_49)	34	8	0	0
Multipliers[29].mult (Multiplier_50)	34	8	0	0
Multipliers[30].mult (Multiplier_52)	34	8	0	0
Multipliers[31].mult (Multiplier_53)	34	8	0	0

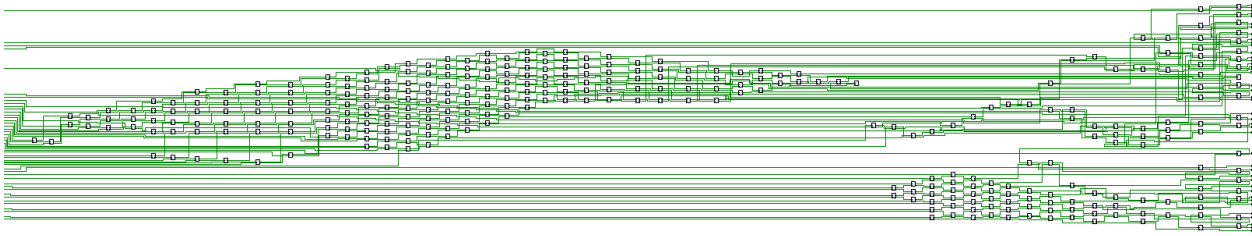
Add for more information

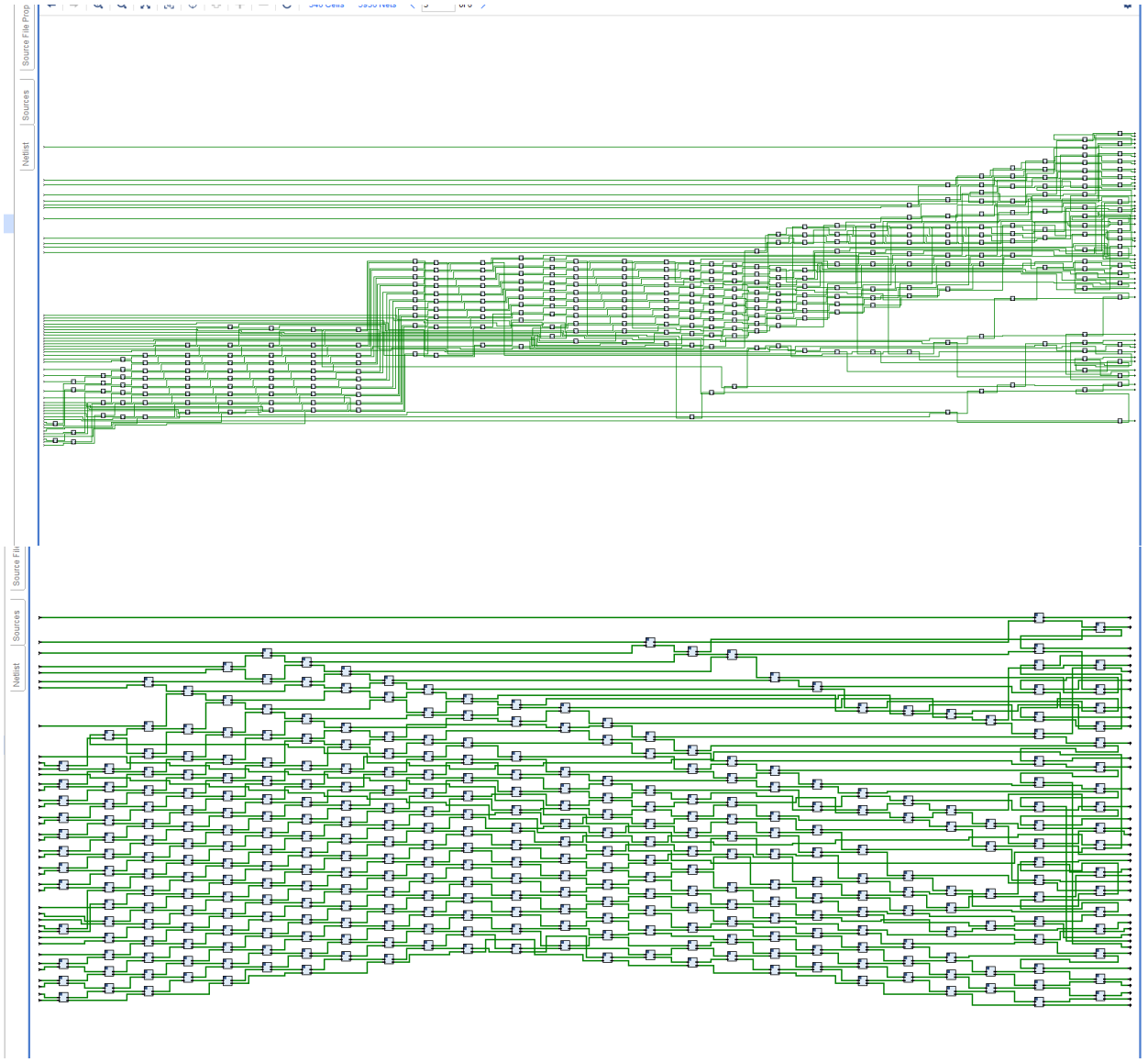
1. 32 odd-even sorting

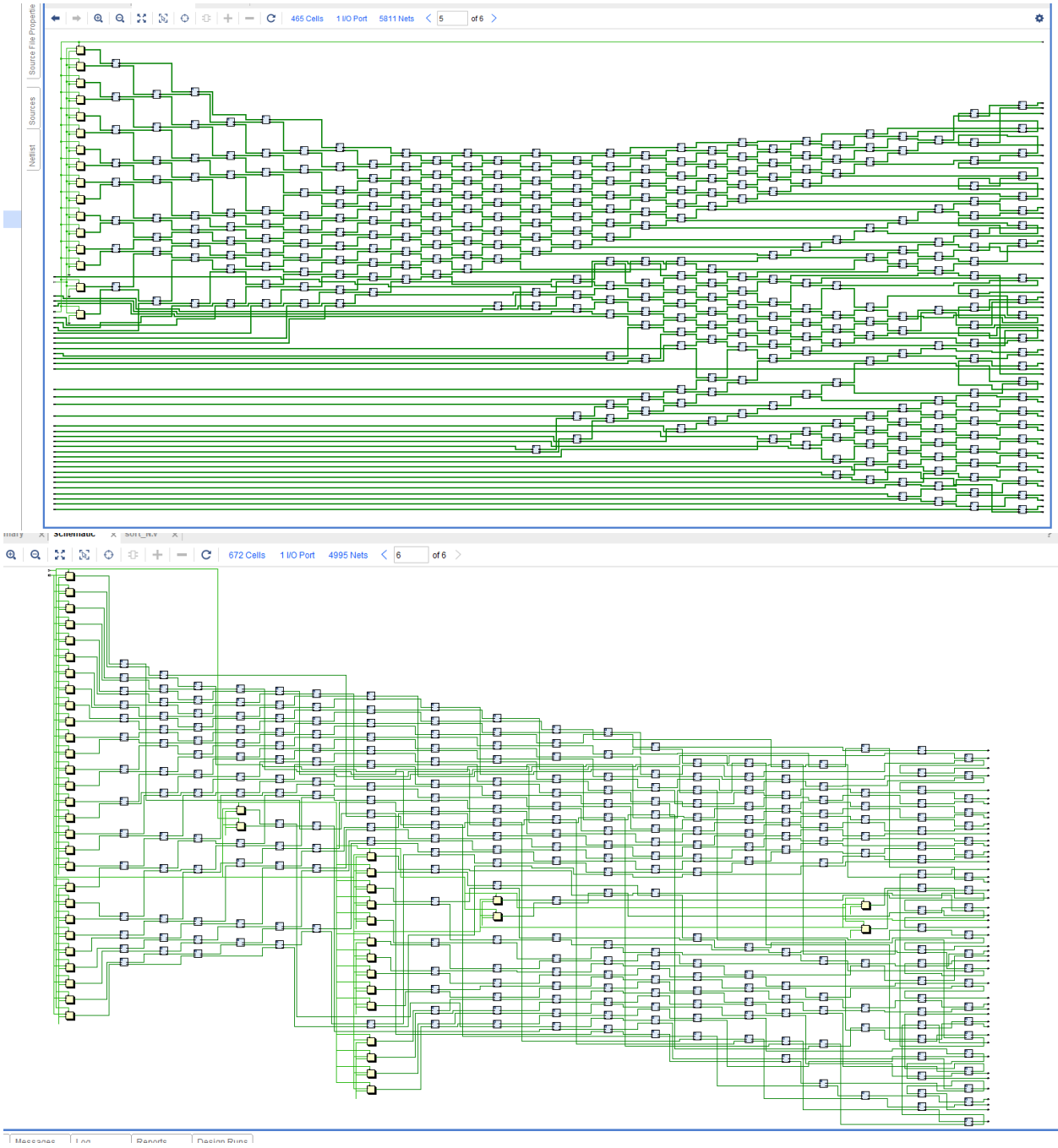


2. 64 odd-even sorting

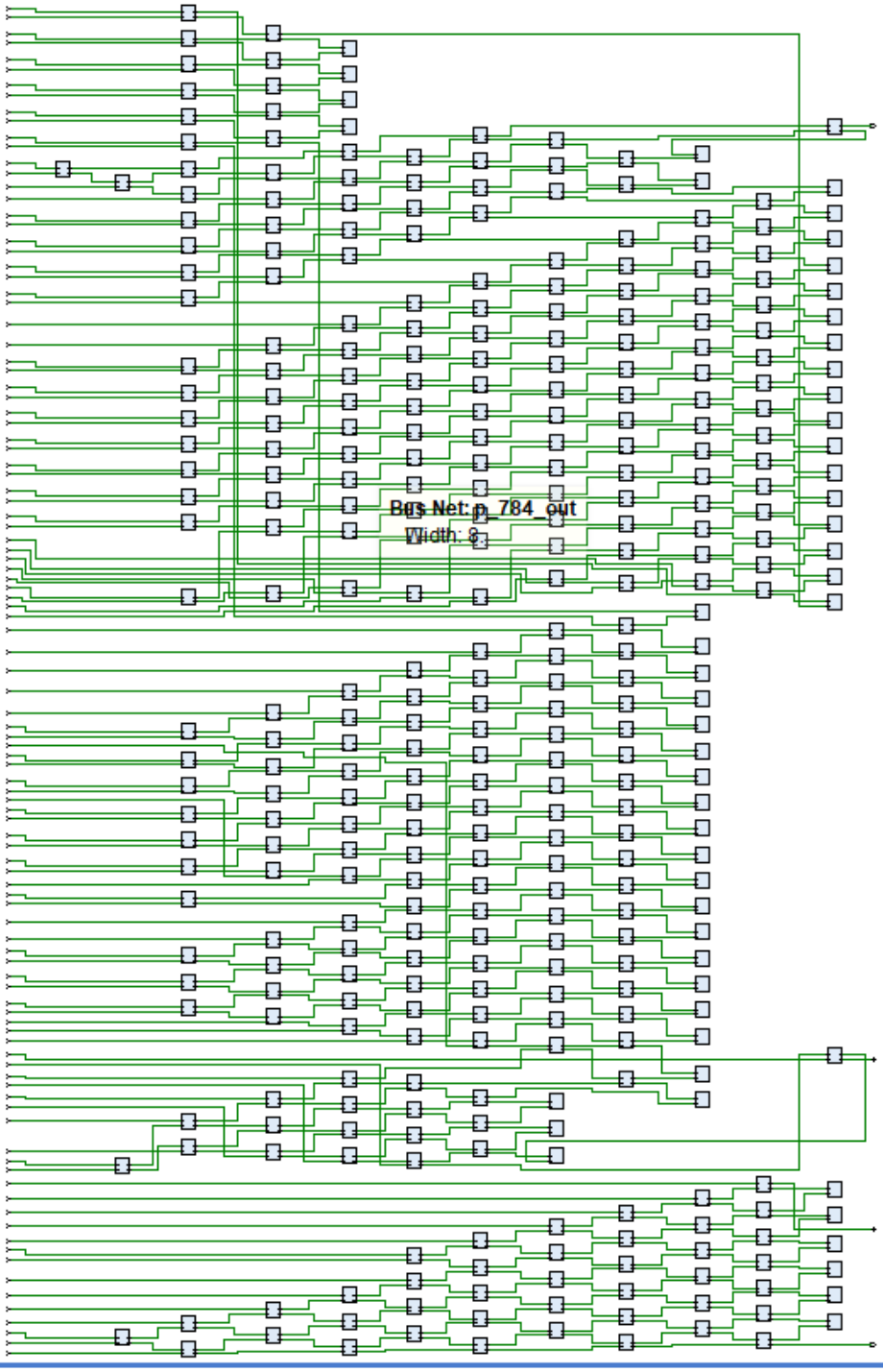





Source File Prop
Sources
Netlist
Source File
Sources
Netlist



3. 128 odd-even sorting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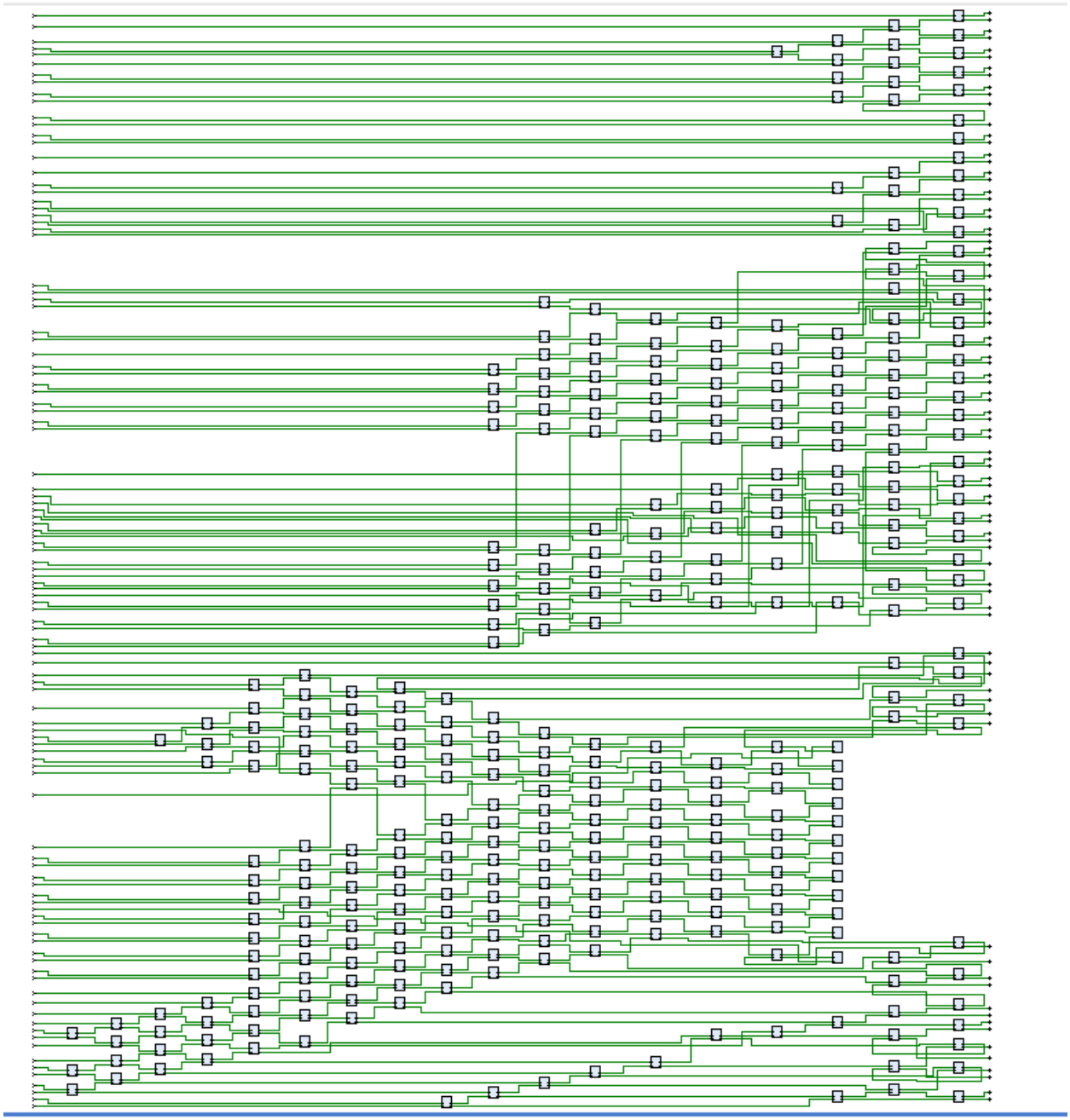

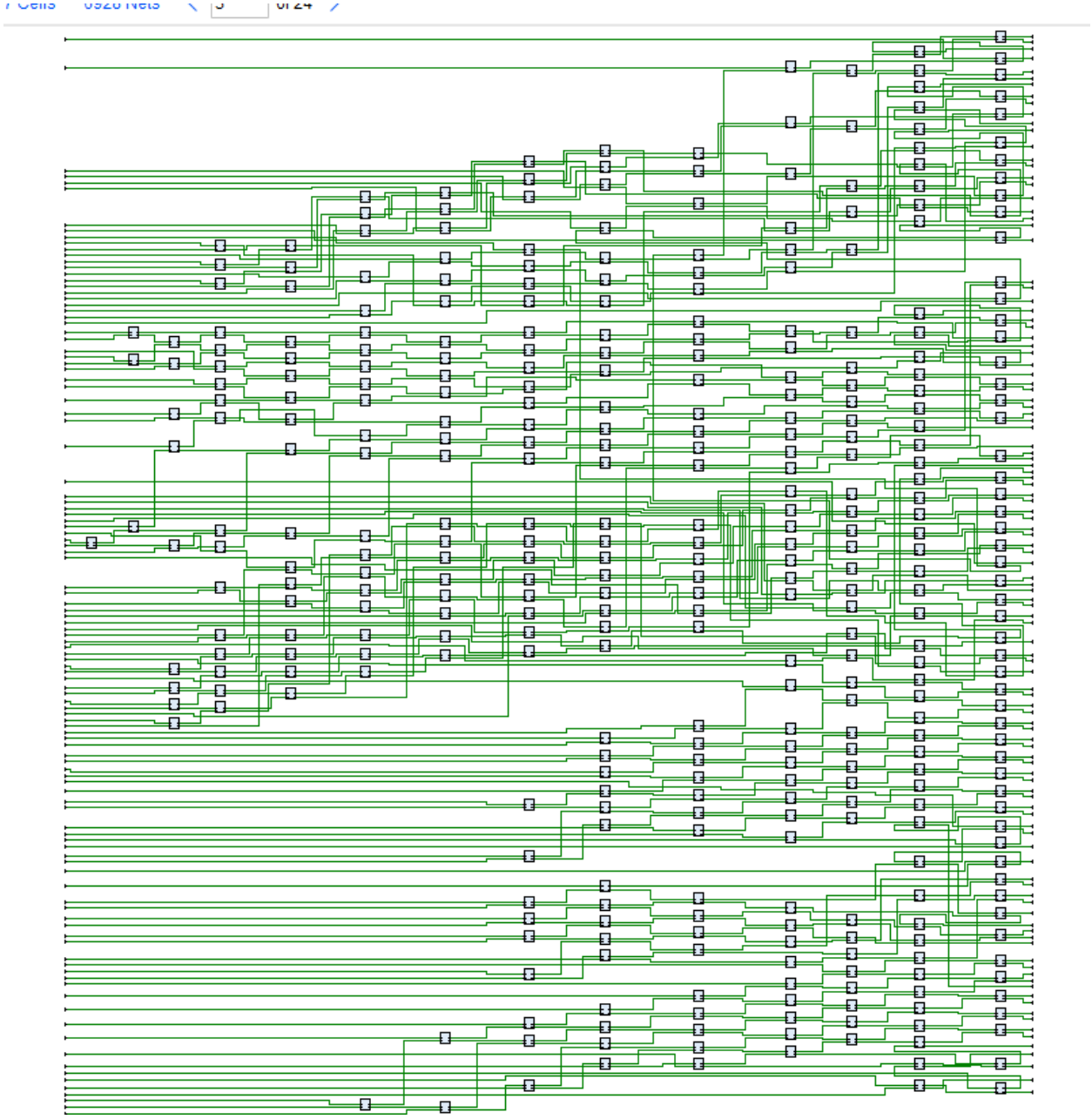

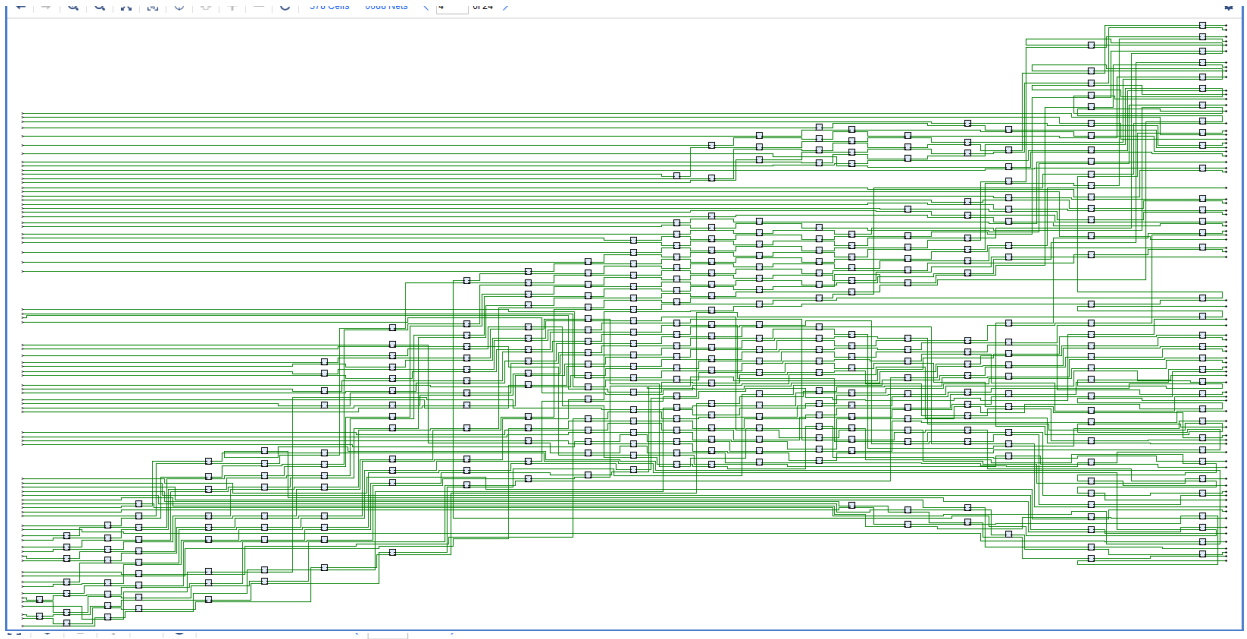

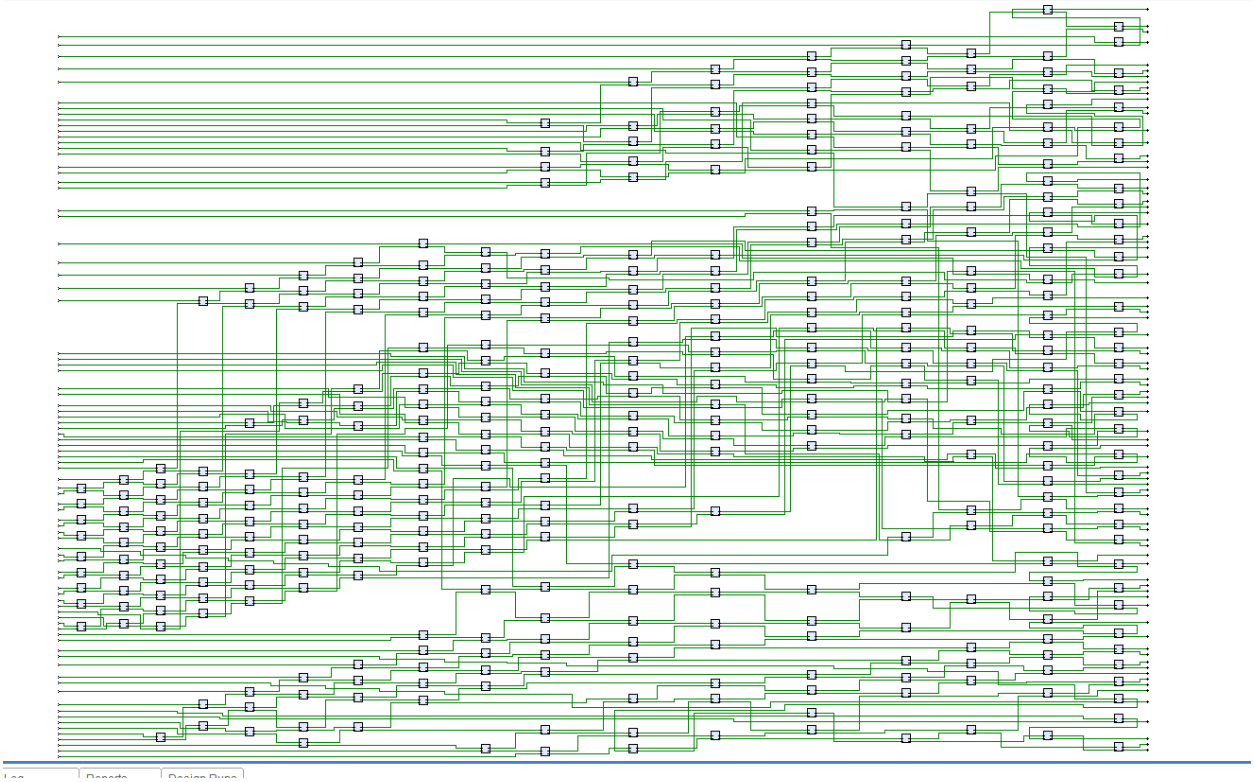

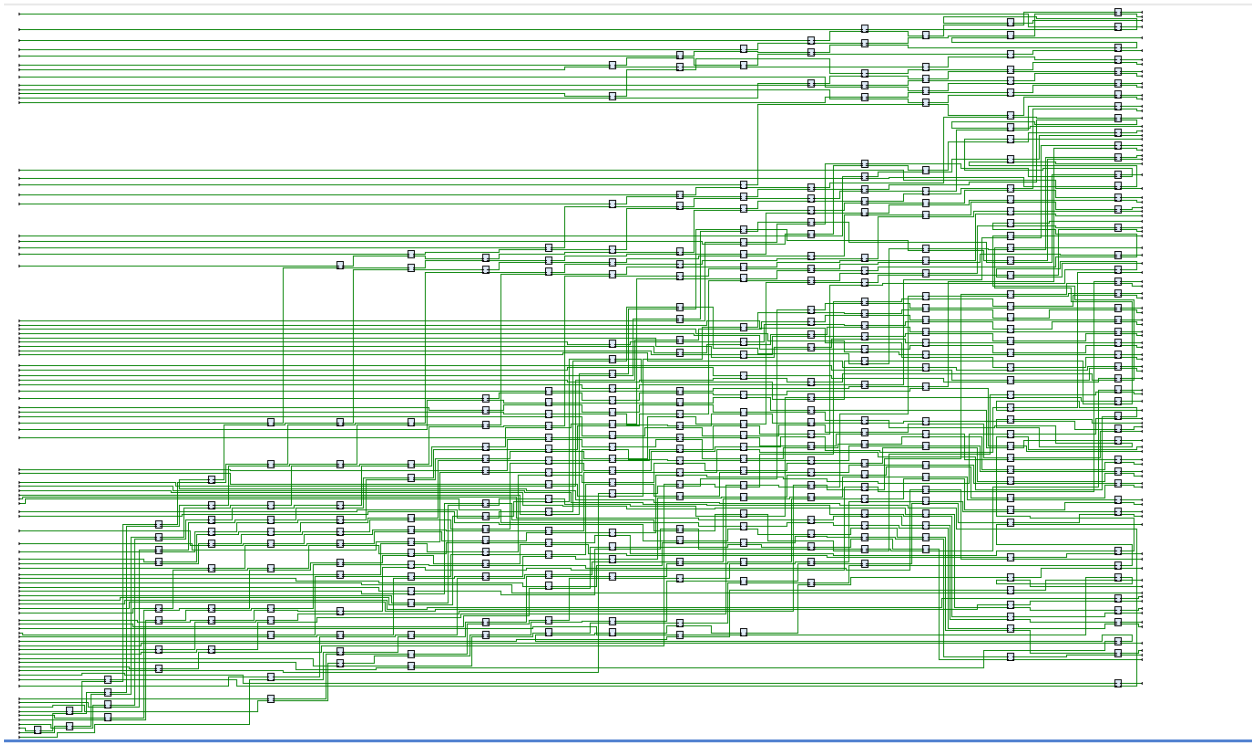

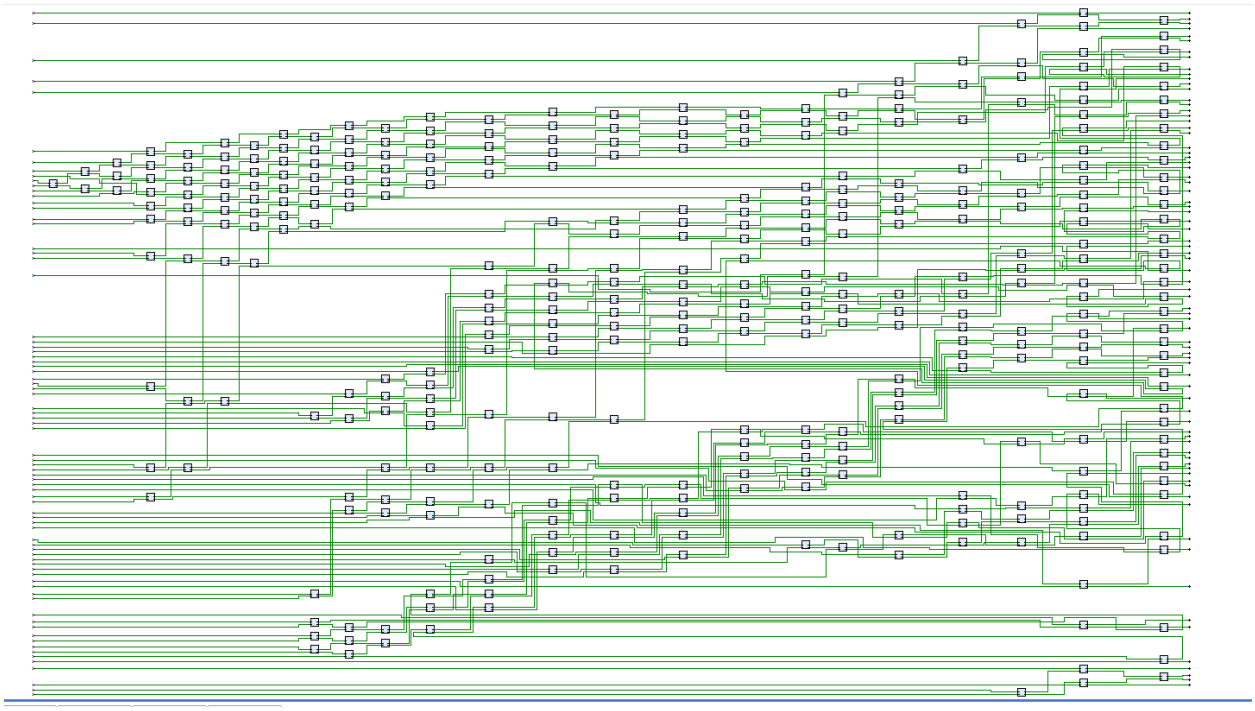

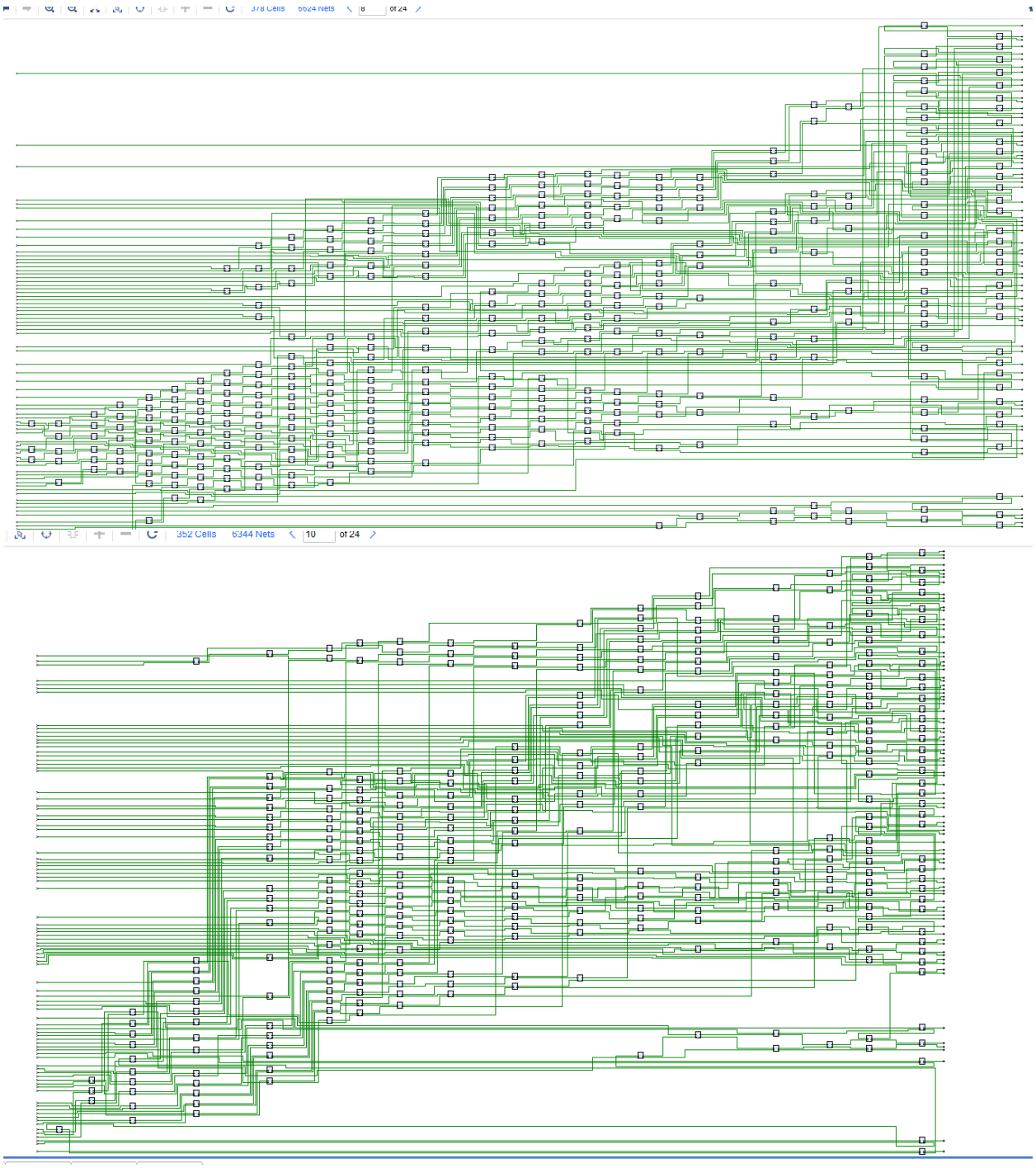
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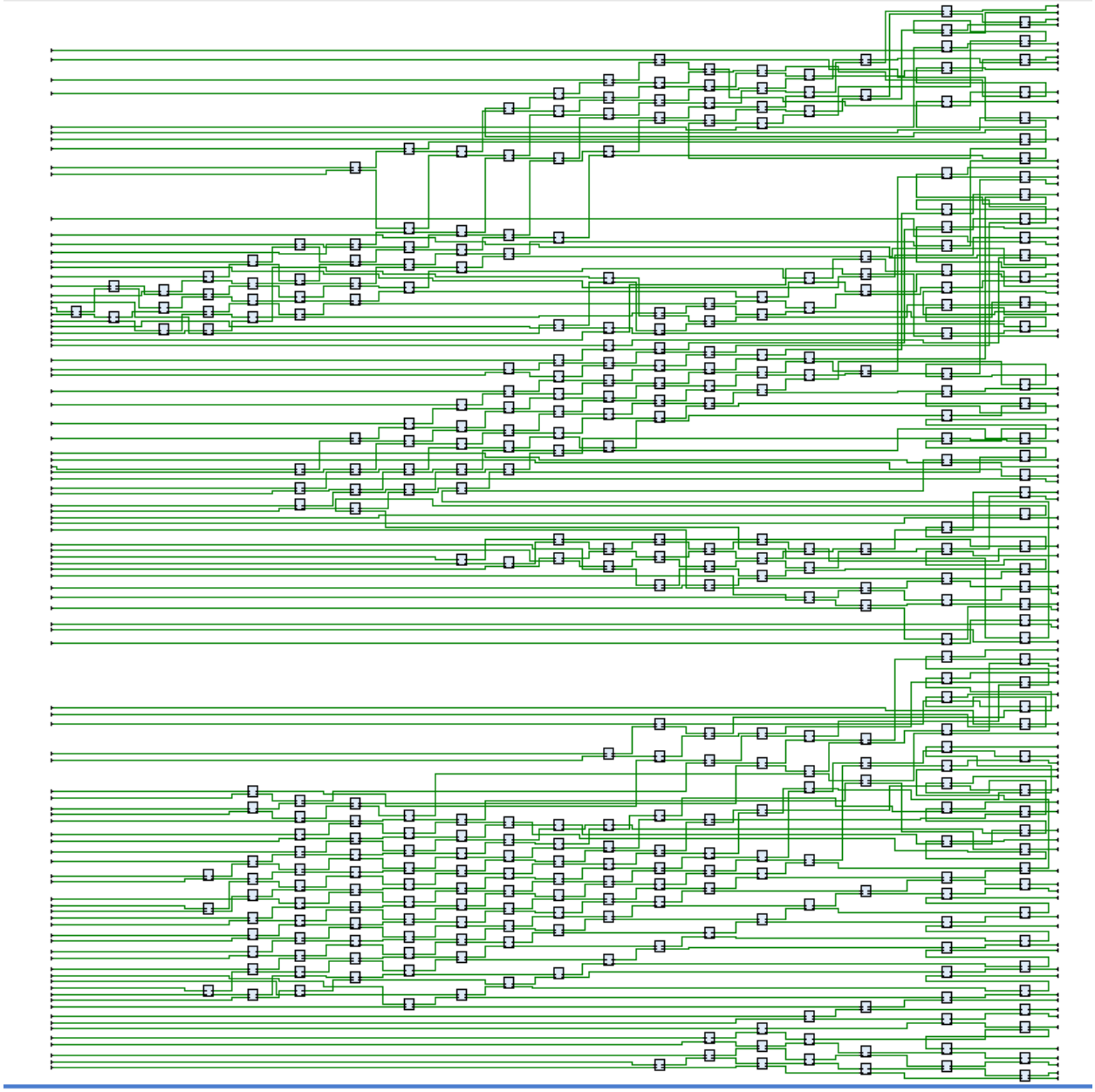
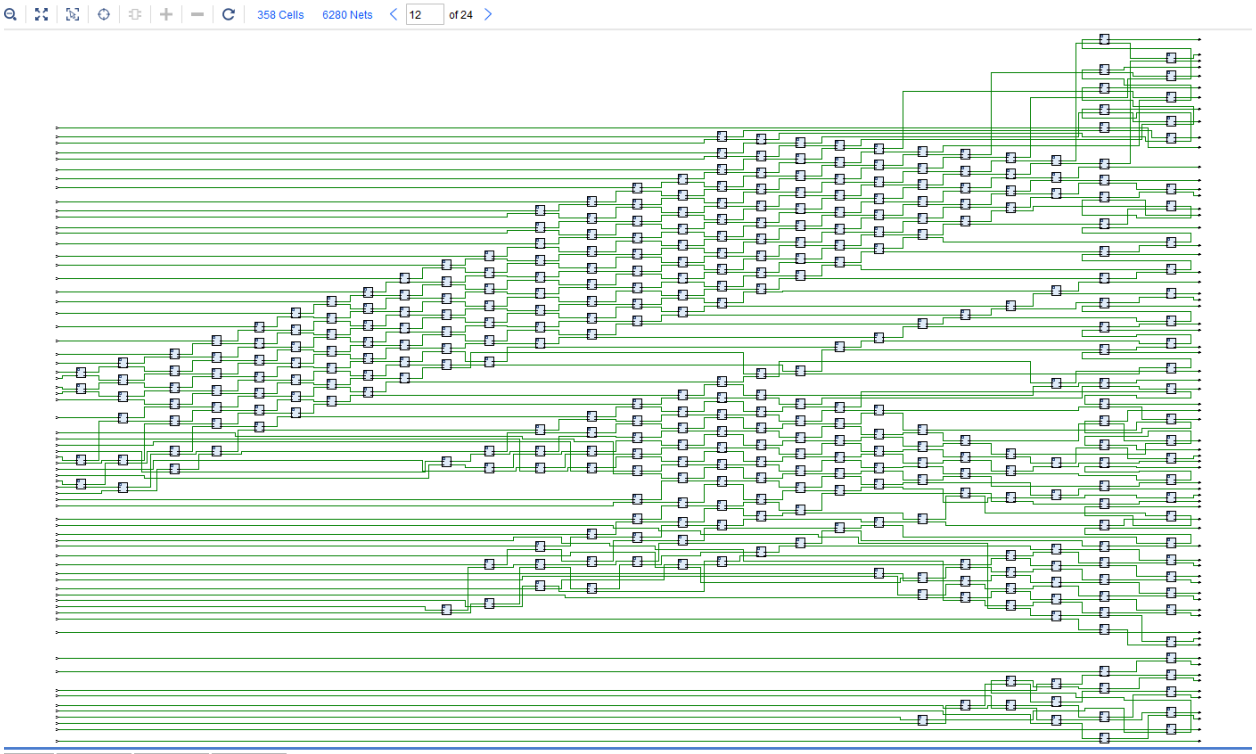
8
of 24
352 Cells
6344 Nets
10
of 24

57 Cells 6496 Nets < 11 of 24 >




358 Cells
6280 Nets
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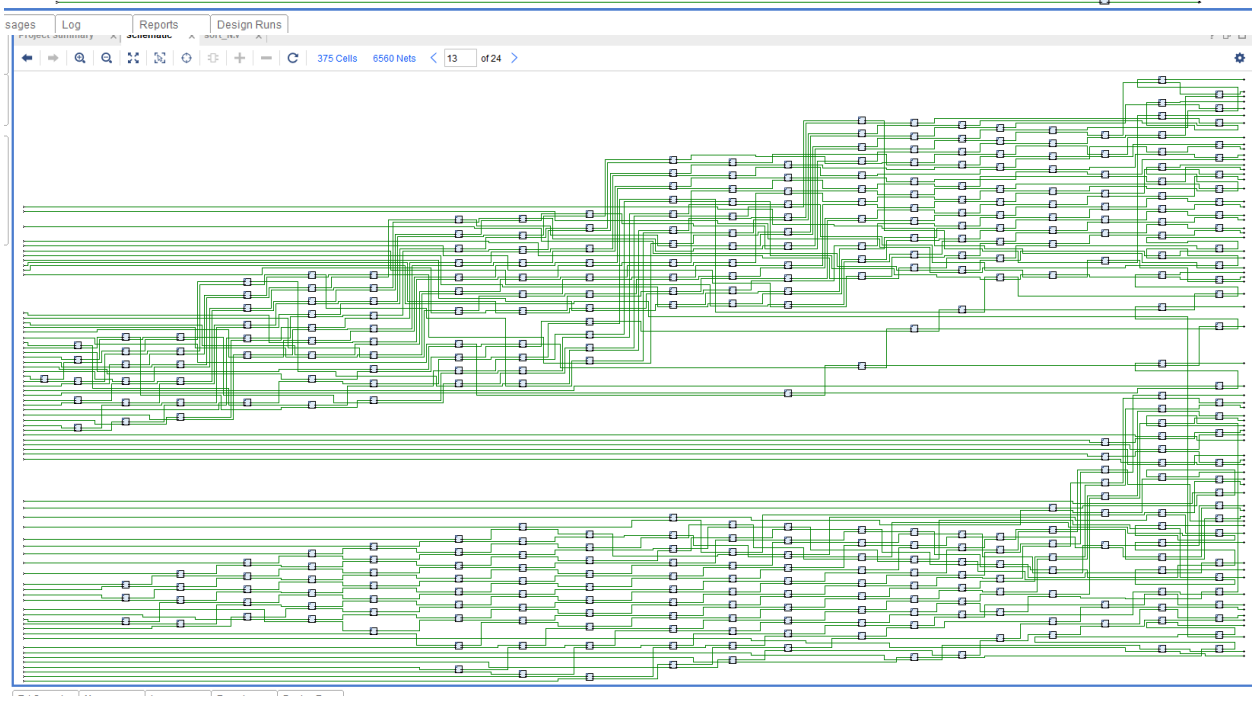
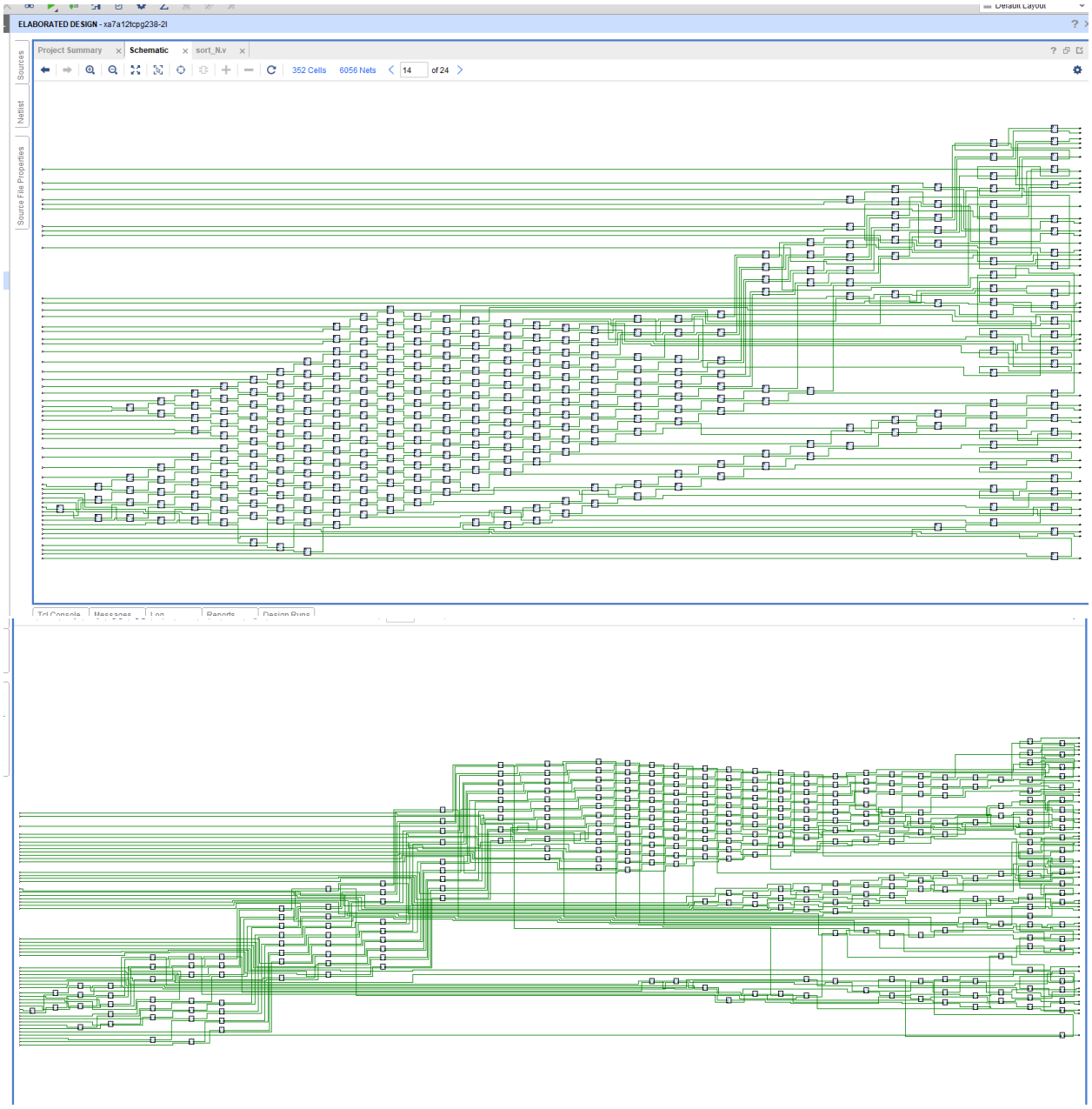
of 24


sages
Log
Reports
Design Runs
375 Cells
6560 Nets
13
of 24


ELABORATED DESIGN - xa7a12tcpg238-2I
Project Summary
Schematic
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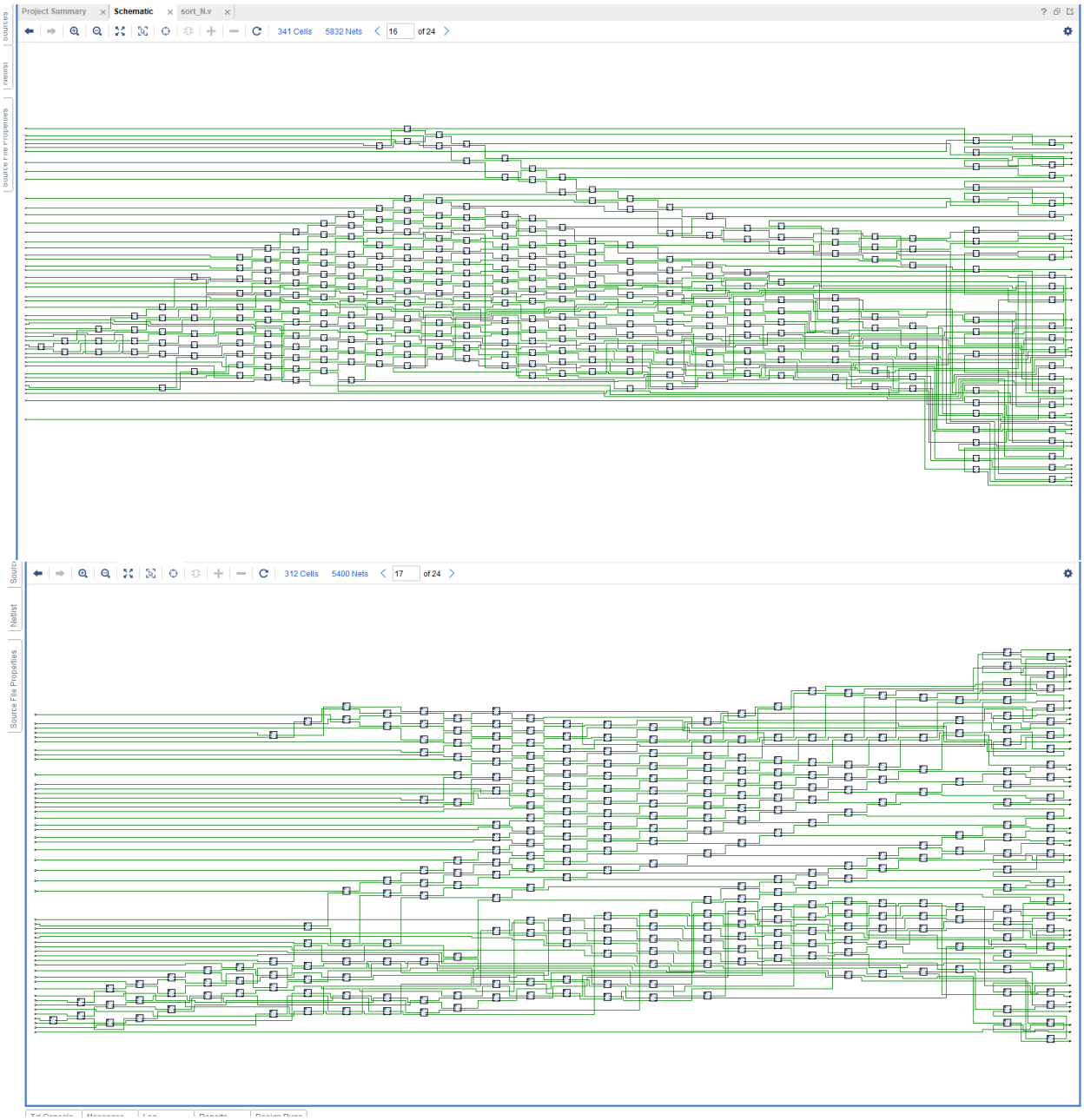
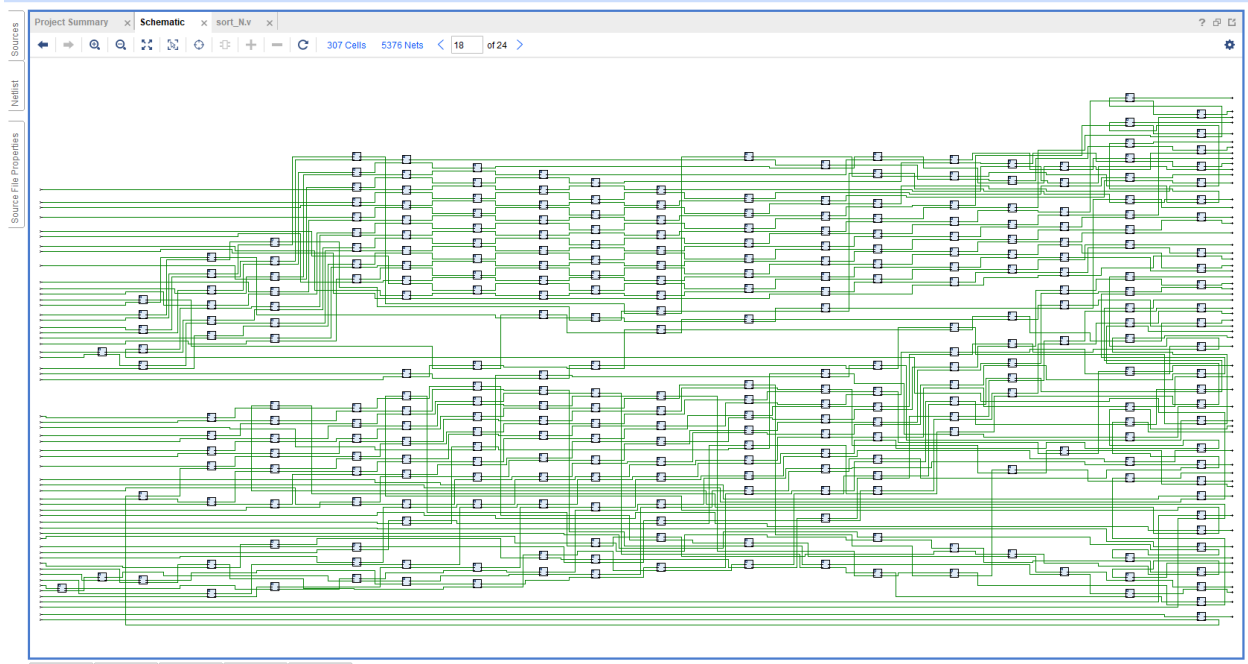
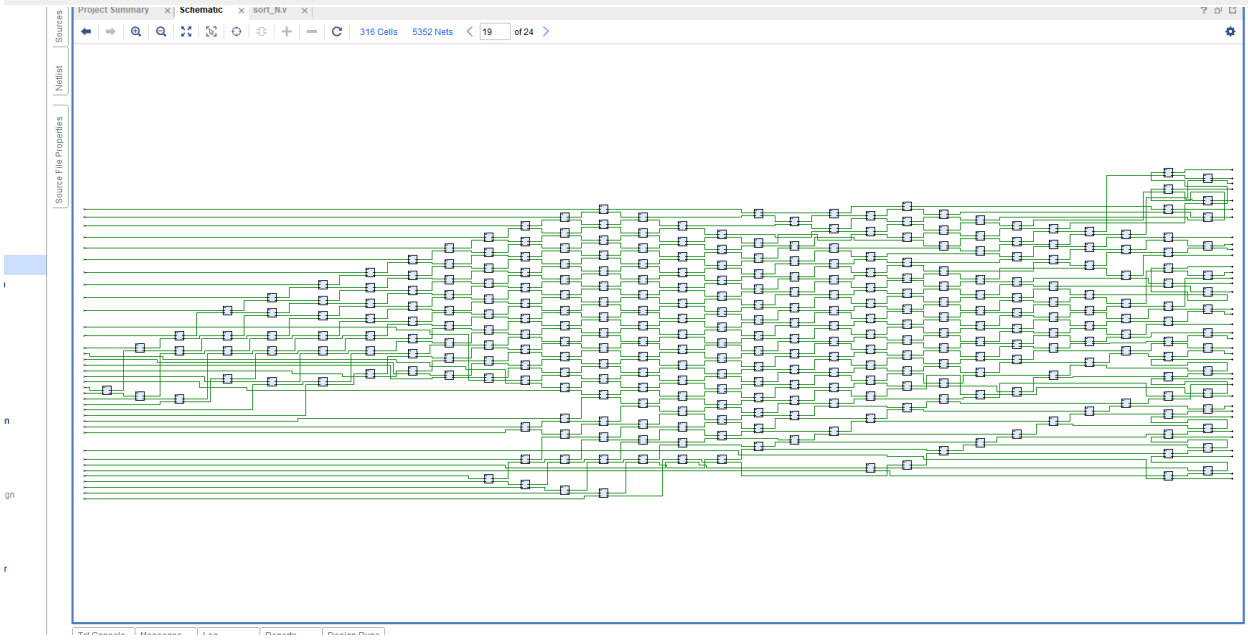
352 Cells
6056 Nets
14
of 24
Sources
Netlist
Source File Properties


Project Summary
Schematic
sort_N.v
341 Cells
5832 Nets
16
of 24
Sources
Netlist
Source File Properties
312 Cells
5400 Nets
17
of 24
Netlist
Source File Properties


Project Summary
Schematic
sort_N.v
307 Cells
5376 Nets
18
of 24
Sources
Netlist
Source File Properties
Tcl Console
Messages
Log
Reports
Design Runs


Project Summary
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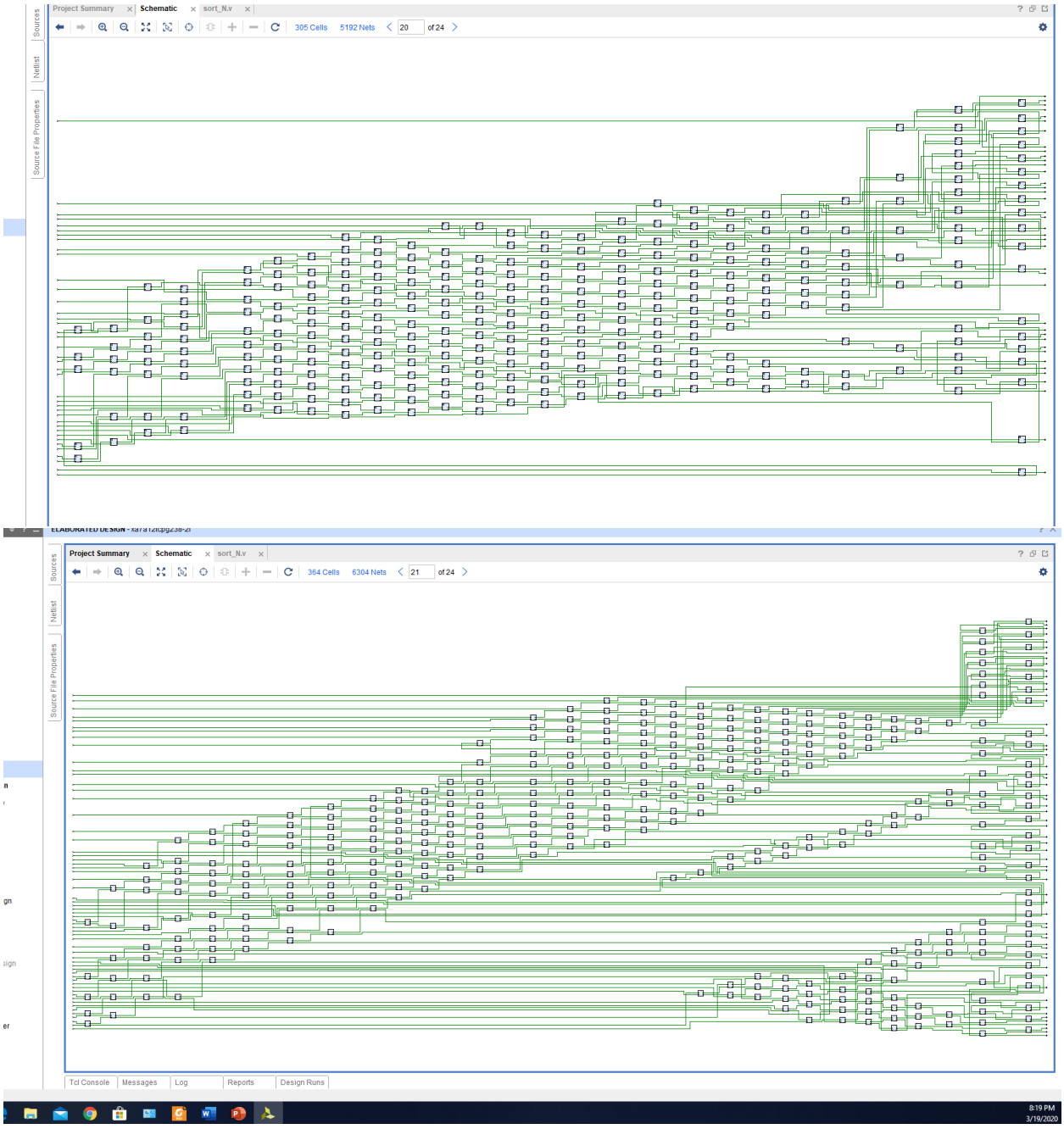
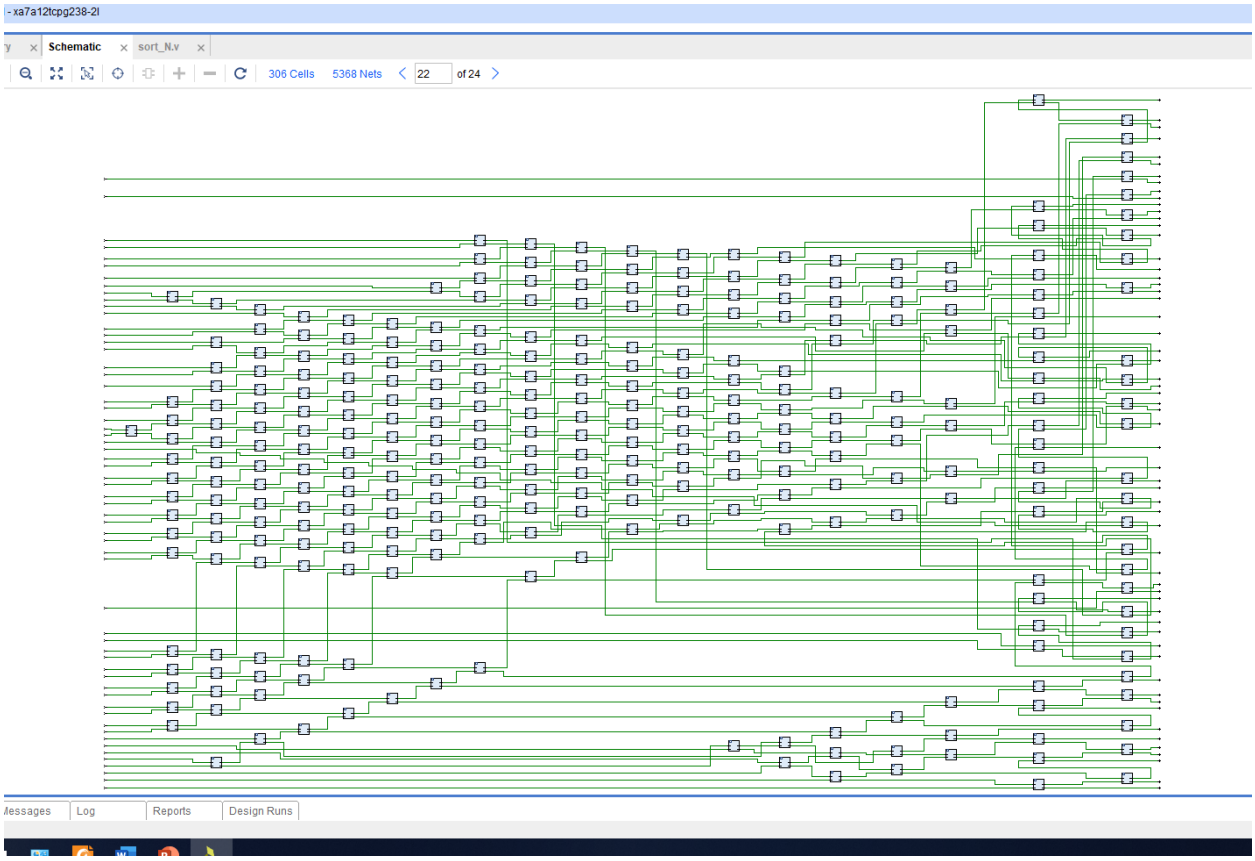
19
of 24
Sources
Netlist
Source File Properties


Project Summary
Schematic
sort_N.v
305 Cells
5192 Nets
20
of 24
Sources
Netlist
Source File Properties
Project Summary
Schematic
sort_N.v
364 Cells
6304 Nets
21
of 24
Sources
Netlist
Source File Properties
Tcl Console
Messages
Log
Reports
Design Runs
8:19 PM
3/19/2020


- xa7a12tcpg238-2I
Schematic
sort_N.v
306 Cells
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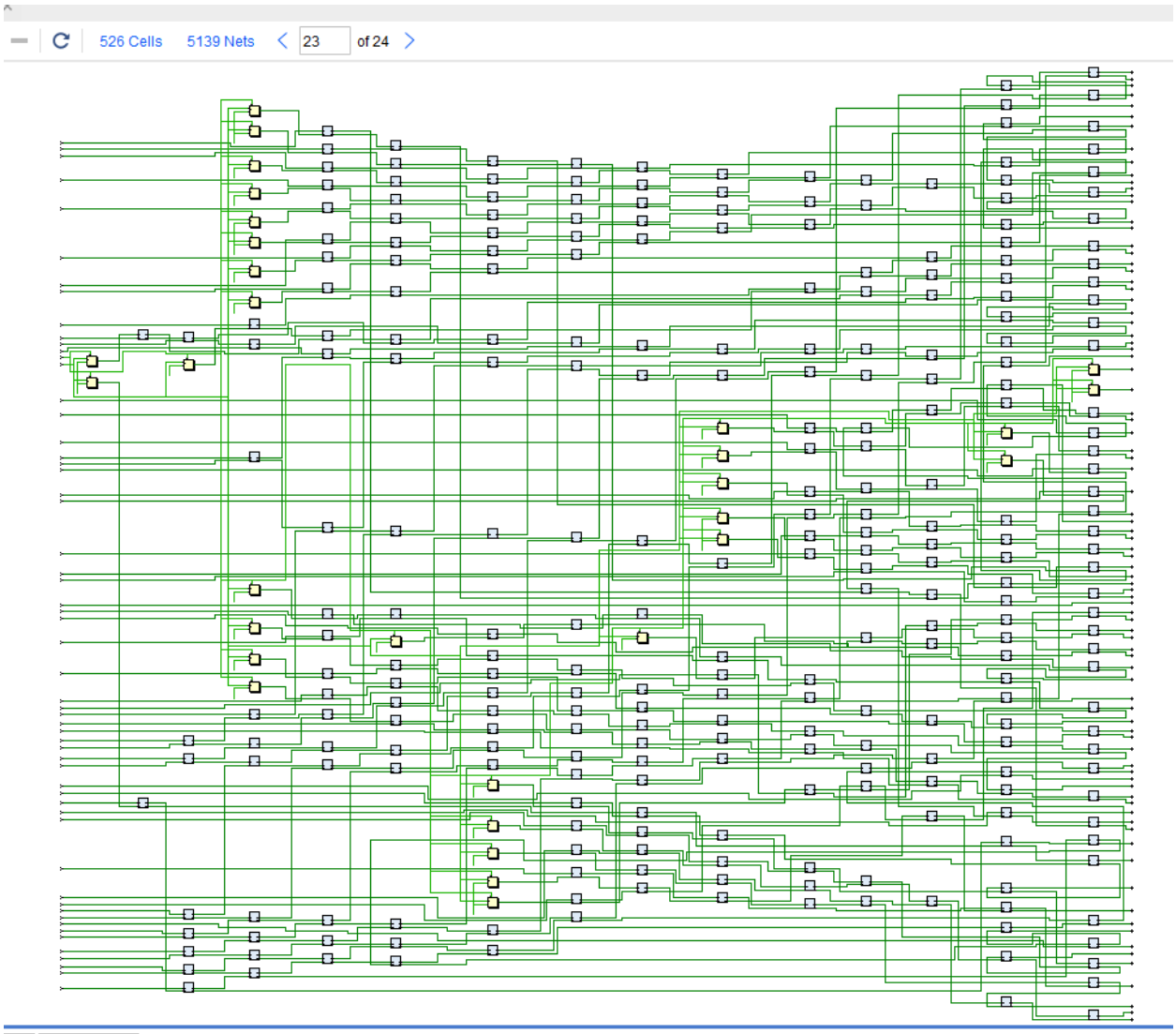
22
of 24
Messages
Log
Reports
Design Runs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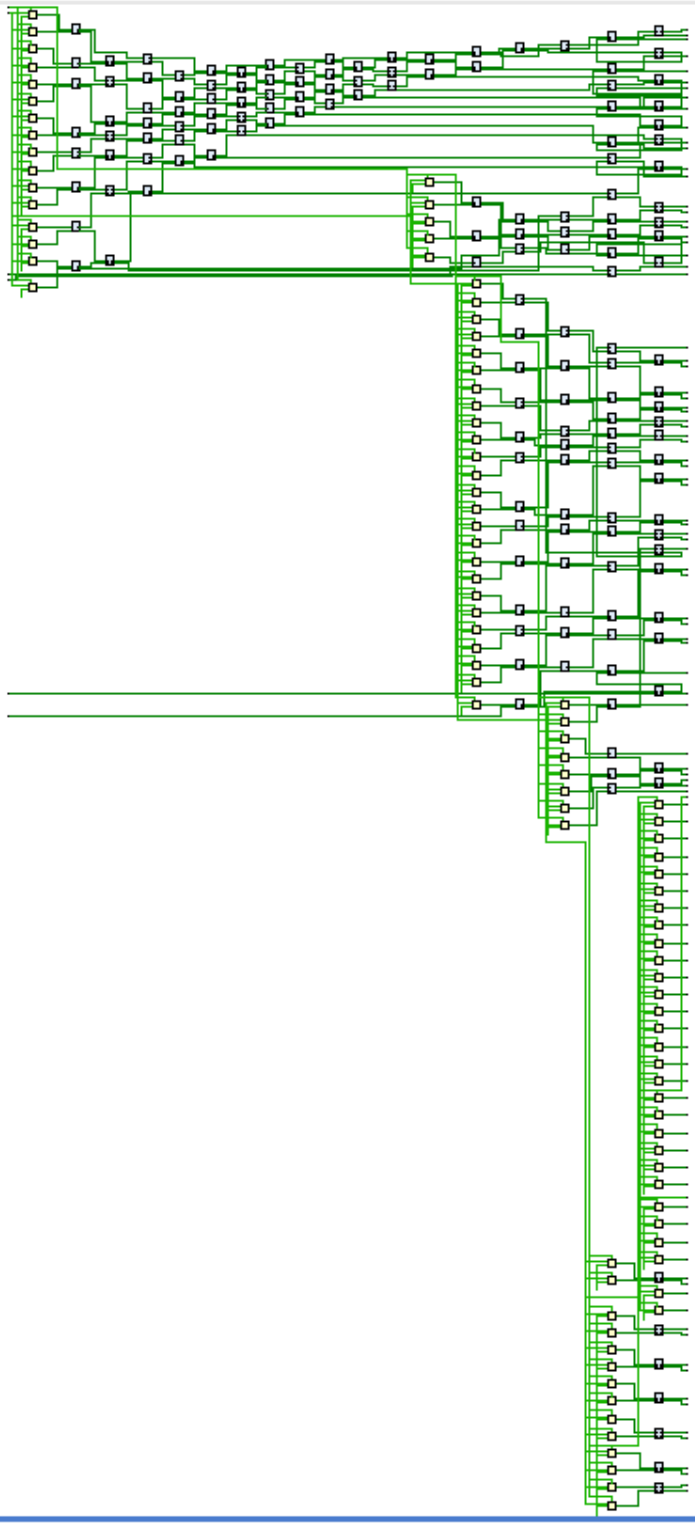
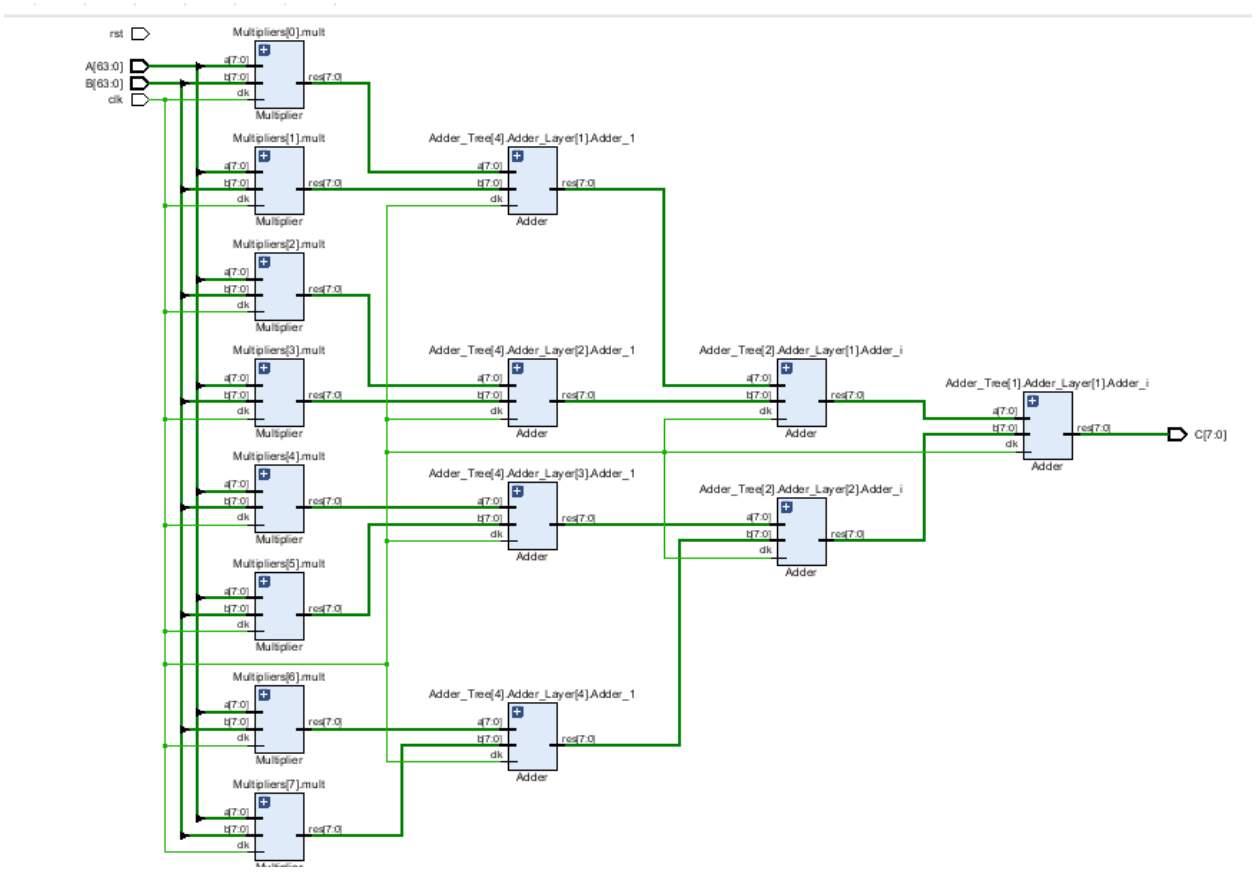
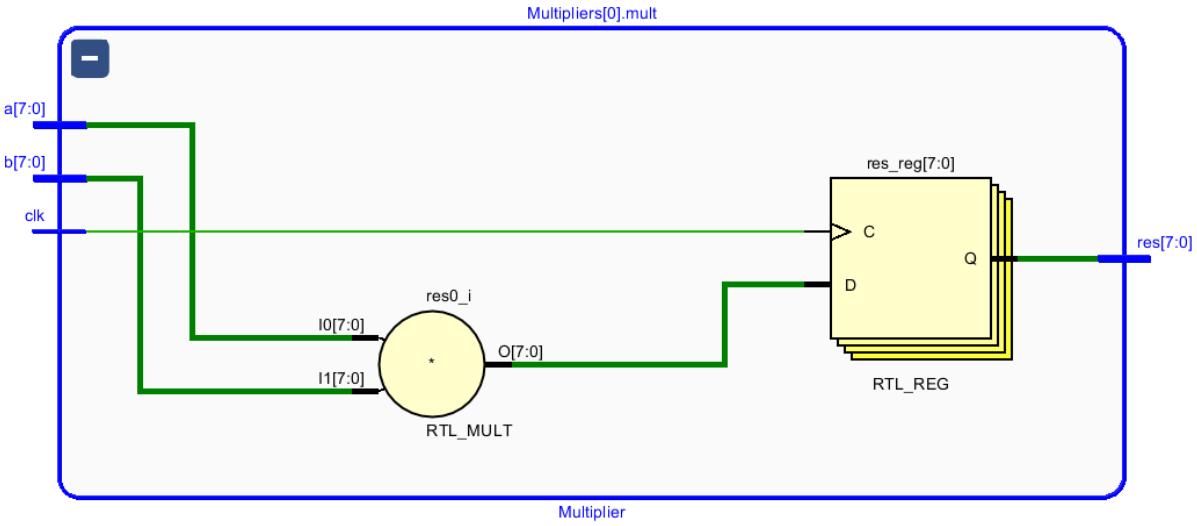
23
of 24

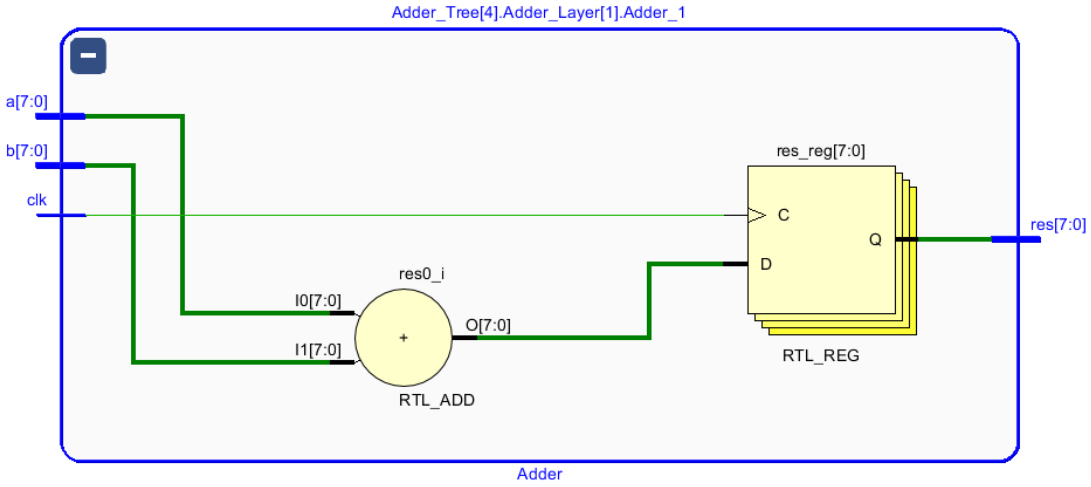
3371 Nets 24 of 24



8-bit Matrix multiplier schematic




Multipliers[0].mult
a[7:0]
b[7:0]
clk
res_reg[7:0]
C
Q
D
res[7:0]
res0_i
I0[7:0]
I1[7:0]
*
O[7:0]
RTL_REG
RTL_MULT
Multiplier


Adder_Tree[4].Adder_Layer[1].Adder_1
a[7:0]
b[7:0]
clk
res_reg[7:0]
C
Q
D
res[7:0]
res0_i
I0[7:0]
I1[7:0]
+
O[7:0]
RTL_REG
RTL_ADD
Adder