Homework to Week 3

Statistics: Principle, Methods and R (II)

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Week 3, 13 March 2017

The homework is due on Monday, 20 March 2017. Please hand in the solutions to the teaching assistant He Siyuan at the beginning of the lecture.

1. In each of the following models, find the Bayes risk of the MLE and the Bayes rule, using squared error loss.
   (a) $X \sim \text{Binomial}(n, p)$, $p \sim \text{Beta}(\alpha, \beta)$. (Please note here the latter means the prior for p is $\text{Beta}(\alpha, \beta)$.)
   (b) $X \sim \text{Poisson}(\lambda)$, $\lambda \sim \text{Gamma}(\alpha, \beta)$.
   (c) $X \sim N(\theta, \sigma^2)$ where σ^2 is known and $\theta \sim N(a, b^2)$.

2. Let $\Theta = \{\theta_1, \ldots, \theta_k\}$ be a finite parameter space. Prove that the posterior mode is the Bayes rule under zero-one loss.

3. Consider the **regression through the origin** model:

$$Y_i = \beta X_i + \varepsilon_i.$$

   Find the least square estimate for β. Assume further $\mathbb{E}[\varepsilon_i | X_i] = 0$ and $\text{Var}(\varepsilon_i | X_i) = \sigma^2$. Given all the covariates X_i's, find the standard error of the estimate.

4. Assuming only the simple linear regression model (without the normal assumption), prove the following formula mentioned in the lecture with elementary calculations. You should regard the X_i's as fixed constants.

$$\text{Var}(\hat{\beta} | X^{(n)}) = \frac{\sigma^2}{n s_X^2} \begin{pmatrix} \sum_{i=1}^n X_i^2 / n & -\overline{X}_n \\ -\overline{X}_n & 1 \end{pmatrix}$$

   where $s_X^2 = \sum_{i=1}^n (X_i - \overline{X}_n)^2$.

5. Get the passenger car mileage data from
   `http://lib.stat.cmu.edu/DASL/Datafiles/carmpgdat.html`

(a) Fit a simple linear regression model to predict MPG (miles per gallon) from HP (horsepower). Summarize your analysis including a plot of the data with the fitted line.

(b) Repeat the analysis but use log(MPG) as the response. Compare the analyses.