StatsI — Exercise 1

1. Write down the sample space of the following two experiments:

   (a) Toss a fair six-sided die two times.

   (b) Toss two fair six-sided dies together for once.

   Identify the probability for each element in the sample space in the above two cases. Find the probabilities of the following events.

   i). For experiment (a), let A_1 = {sum of two tosses is even}, A_2 = {1st toss is even, 2nd toss is odd}.

   ii). For experiment (b), let B_1 = {sum of two dies is even}, B_2 = {one even and one odd}.

2. (*Monty Hall Problem*) A prize is placed at random behind one of three doors. You pick a door. To be concrete, let's suppose you always pick door 1. Now Monty Hall chooses one of the other two doors, opens it and shows you that it is empty. He then gives you the opportunity to keep your door or switch to the other unopened door. Should you stay or switch? Intuition suggests it doesn't matter. The correct answer is that you should switch. Prove it by calculating the probabilities of the following two events:

   (a) winning by staying at door 1,

   (b) winning by switching to the other unopened door.

   (You may gain some hands-on experience by playing this game a large number of times by using the R code available from the course website.)

3. Suppose that a fair coin is tossed repeatedly until both a head and tail have appeared at least once.

   (a) Describe the sample space.

   (b) What is the probability that three tosses will be required?

4. The probability that a child has blue eyes is 0.25. Assume independence between children. Consider a family with 3 children.

   (a) If it is known that at least one child has blue eyes, what is the probability that at least two children have blue eyes?

   (b) If it is known that the youngest child has blue eyes, what is the probability that at least two children have blue eyes?

5. Suppose that 30% of computer owners use a Macintosh, 50% use Windows, and 20% use Linux. Suppose that 65% of the Mac users have succumbed to a computer virus, 82% of the windows users get the virus, and 30% of the Linux users get the virus. We select a person at random. What is the probability that

   (a) her computer has infected by the virus, or

   (b) she is a Windows user, given the condition that her system has already infected by the virus.

6. There are three cards: one is green on both sides, one is red on both sides, and one is green on one side and red on the other side. We choose a card at random and we see one side (also chosen at random). If the side we see is green, what is the probability that the other side is also green? Many people intuitively answer 1/2. Show that the correct answer is 2/3.