

StatsI — Exercise 5

1. Let X_1, \dots, X_n be a sample from $\text{Bernoulli}(p)$, and Y_1, \dots, Y_m be a sample from $\text{Bernoulli}(q)$, and the two samples are independent with each other.
 - (a) Find a reasonable estimator for $p - q$ and its standard error.
 - (b) Find an approximate 95% confidence interval for $p - q$ when both n and m are large.
2. 100 people are given a standard antibiotic to treat an infection and another 100 are given a new antibiotic. In the first group, 90 people recover; in the second group, 85 people recover. Let p_1 be the probability of recovery with the standard antibiotic and p_2 be the probability of recovery with the new antibiotic. We are interested in estimating $\theta = p_1 - p_2$. Provide an estimate, standard error, an 80% confidence interval, and a 95% confidence interval for θ .
3. Let X_1, \dots, X_n be a sample from a distribution F and let \hat{F}_n be the empirical distribution. Let x and y be two distinct points, and $x < y$.
 - (a) Compute $E\{\hat{F}(x)\}$ and $\text{Var}\{\hat{F}(x)\}$.
 - (b) Compute $\text{Cov}\{\hat{F}(x), \hat{F}(y)\}$.
 - (c) Based on the CLT, find the limiting distribution of $\hat{F}(x)$. Furthermore, find an approximate $1 - \alpha$ confidence interval for $F(x)$.
 - (d) Suppose we are interested in estimating $\theta \equiv F(y) - F(x)$, and we use $\hat{\theta} \equiv \hat{F}(y) - \hat{F}(x)$ as our estimate. Find the standard deviation and the standard error of $\hat{\theta}$.