StatsI — Exercise 7

1. Let $N(0,1)$, χ_k^2 and t_k denote, respectively, the standard normal, χ_k^2-distributed and t_k-distributed random variables. Find the unknown constants C and α in the equations below, either using the relevant tables in `StatisticalTables.pdf` or using R, and make a table itemizing these values.

$$P\{N(0,\ 1) > C\} = 0.975, \qquad P\{N(0,\ 1) < -2.3\} = \alpha, \qquad P\{-1.3 < N(0,\ 1) < 1.5\} = \alpha,$$

$$P\{\chi_{10}^2 > C\} = 0.975, \qquad P\{\chi_{14}^2 < C\} = 0.025, \qquad P\{13.5 < \chi_{17}^2 < 35.7\} = \alpha,$$

$$P\{t_{10} > C\} = 0.975, \qquad P\{t_{15} < -2.6\} = \alpha, \qquad P\{|t_{20}| < C\} = 0.95.$$

2. A random sample $X_1, \cdots, X_n$ of size n is selected from a normal distribution with known mean μ and unknown variance σ^2. Two possible confidence intervals for σ^2 are shown below, where a_1, a_2, b_1 and b_2 are constants.

$$(a_1^{-1}\sum_{i=1}^{n}(X_i - \bar{X})^2,\ a_2^{-1}\sum_{i=1}^{n}(X_i - \bar{X})^2), \qquad (b_1^{-1}\sum_{i=1}^{n}(X_i - \mu)^2,\ b_2^{-1}\sum_{i=1}^{n}(X_i - \mu)^2).$$

For the case $n = 10$, find values of these constants which give intervals with confidence level 0.90. Compare the expected lengths of these intervals. Comment on your findings.

3. Let $X_1, \cdots, X_n$ be a random sample from the uniform distribution on the interval $[0, \theta]$ ($\theta > 0$). Find a confidence interval for θ.

4. There is a theory that people can postpone their death until after an important event. To test this theory, Phillips and King (1988, *Lancet*, pp.728–) collected data on deaths around the Jewish holiday Passover. Of 1919 deaths, 922 died the week before the holiday and 997 died the week after. Think of this as a binomial and test the null hypothesis that $\theta = 1/2$, where θ is the probability that a death occurs after the holiday. Also construct a confidence interval for θ.

5. A sample of 11 observations from population $N(\mu,\ \sigma^2)$ yields the sample mean $\bar{X} = 8.68$ and the sample variance $S^2 = 1.21$. At 5% significance level, test the following hypotheses.

   (a) $H_0 :\ \mu = 8$ against $H_1 :\ \mu > 8$
   (b) $H_0 :\ \mu = 8$ against $H_1 :\ \mu < 8$
   (c) $H_0 :\ \mu = 8$ against $H_1 :\ \mu \neq 8$

   Repeat the above exercise with the additional assumption $\sigma^2 = 1.21$. Compare the results with those derived without this assumption and comment.

6. (a) Two independent random samples, of n_1 and n_2 observations, are drawn from normal distributions with the same variance σ^2. Let S_1^2 and S_2^2 be the sample variances of the first and the second sample, respectively. Show that $\widehat{\sigma}^2 = \frac{1}{n_1+n_2-2}\{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2\}$ is an unbiased estimator for σ^2.

   (b) Two makes of car safety belts, A and B have breaking strengths which are normally distributed with the same variance. A sample of 140 belts of make A and a sample of 220 belts of make B were tested, the sample means, and the sums of squares about the means (i.e. $\sum_i (X_i - \bar{X})^2$), of the breaking strengths (in lbf units) were (2685, 19000) for make A, and (2680, 34000) for make B. Is there any significant evidence to support the hypothesis that belts of make A are stronger than belts of make B?