1 Simple drifter dynamics

In absence of ambient flow, a submerged drogue is forced by wave-induced velocities relative to the drogue motion. For a monochromatic wave of mild slope and small amplitude a, water surface has a shape $\eta = acos(kx - \omega t)$, and associated velocities:

$$u(x, z, t) = a\omega e^{kz} cos(kx - \omega t) \tag{1}$$

$$w(x, z, t) = a\omega e^{kz} sin(kx - \omega t) \tag{2}$$

ω is angular frequency; k is wavenumber. The above is true in deep water. For water of arbitrary depth d, e^{kz} is replaced with $cosh(k(z+d))/sinh(kd)$.

The acceleration of water particles induced by the pressure gradient due to the surface slope is then:

$$a_x(x, z, t) = \frac{du}{dt} = a\omega^2 e^{kz} sin(kx - \omega t) \tag{3}$$

$$a_z(x, z, t) = \frac{dw}{dt} = -a\omega^2 e^{kz} cos(kx - \omega t) \tag{4}$$

The drag force induced by the flow on a submerged object such as drifter drogue can be appoximated as:

$$F_d = \frac{1}{2}\rho C_D \mathbf{u}_{rel}^2 A \tag{5}$$

where ρ is water density; C_D is drag coefficient of the drogue; $\mathbf{u}_{rel}$ is the velocity of the flow relative to the drogue and it can be of either sign; A is the suface area perpendicular to $\mathbf{u}_{rel}$.

Because the wave-induced velocity field is strongly sheared near the surface where the drifter is, a vertical integral of the velocity is considered to be acting on the drogue:

$$u = \frac{1}{L}\int_{z_D-0.5L}^{z_D+0.5L} u\ dz = \frac{a\omega}{kL}\left(e^{k(z_D+0.5L)} - e^{k(z_D-0.5L)}\right) cos(kx - \omega t) \tag{6}$$

where L is the drogue height and z_D is the depth of of the drogue center.

If the drogue of surface area A has some tilt ϕ, which corresponds to the deflection of its vertical axis away from z-axis, then the forces acting on the drogue in x and z are:

$$F_x = sgn(\mathbf{u}_{rel})\frac{1}{2}\rho C_D \mathbf{u}_{rel}^2 A cos\phi \tag{7}$$

$$F_z = sgn(\mathbf{w}_{rel})\frac{1}{2}\rho C_D \mathbf{w}_{rel}^2 A sin\phi \tag{8}$$

The tilt ϕ is positive when the drogue is tilted in the forward direction of the wave, so F_z vanishes if the drogue is perfectly vertical.

A tilted drogue also projects some of the horizontal force into the vertical and vice versa, so in fact the force experienced by the drogue is:

$$F_{dx} = F_x cos\phi - F_z sin\phi \tag{9}$$

$$F_{dz} = -F_x sin\phi + F_z cos\phi \tag{10}$$

Combine equations (7)-(10) to obtain:

$$F_{dx} = \frac{1}{2}\rho C_D A \left[sgn(u_{rel})\mathbf{u}_{rel}^2 cos^2\phi - sgn(w_{rel})\mathbf{w}_{rel}^2 sin^2\phi \right] \tag{11}$$

$$F_{dz} = \frac{1}{2}\rho C_D A \left[-sgn(u_{rel})\mathbf{u}_{rel}^2 + sgn(w_{rel})\mathbf{w}_{rel}^2 \right] sin\phi cos\phi \tag{12}$$

Assumptions made:

1. Linear wave theory holds;
2. Force on an object can be approximated using drag coefficient;
3. Drifter drogue does not disturb the flow;