MATH453 Elementary Number Theory1Strings, Enumeration, and PermuationReila ZhengKevin Gao

**Q**: How many outcomes ar ethere if you roll a die and toss a coin?

**A**: # of outcomes for a die: 6; # of outcomes for a coin: 2; Hence, $6 \times 2 = 12$

**Q**: How many outcomes are there if you either roll a die or toss a coin?

**A**: $6 + 2 = 8$

**Q**: How many ways to get from home to Bahen without backtracking?

Strings

[Binary String] A ***binary string*** of length $n$ is an ordered list of length $n$ of elements from $\{0, 1\}$.

10011 is a binary string of length 5. $\emptyset = \{\}$ is a binary string of length 0.

Similarly, a ***ternary string*** of length $n$ is an ordered list of length $n$ of elements from $\{0, 1, 2\}$.

[X-String] Let $X = \{a_1, \ldots, a_n\}$ be the alphabet. An $X$-string of length $k$ is an ordered list of elements from the set $X$

In the previous example, the path from home to Bahen without backtracking can be thought of as an $\{N, E\}$-strings.

**Q**: How to relate the set of paths from home to Bahen without backtracking with some set of $\{N, E\}$-strings?

**A**: Note that not every $\{N, E\}$-strings can represent a path. In our specific examples, we need exactly walk 5 blocks east ar

**Q**: How many $\{N, E\}$-strings of length 9 are there?

**A**: Let $\mathcal{S} = \{all\{N, E\}-stringsoflength9\}$. For each $S \in \mathcal{S}$, $S = s_1 s_2 \ldots s_9$. For each position, there are two choices, N or

For $X = \{a_1, \ldots, a_n\}$, the number of $X$-strings of length $k$ is $n^k$, where $n, k \geq 1$ are natural numbers.

Denote $S = s_1 \ldots s_k \in \mathcal{S}$. For $s_i$, there are $n$ choices for $i \in \{1, \ldots, k\}$. So there are

k$times = n^k suchstrings.$

[Array] An ***array*** of length $n$ is an ordered list where the elements in position $i$ comes from some alphabet $X_i$.

Ontario health cards have the format: 10 digits of $\{0, 1, \ldots, 9\}$ followed by two letters from $\{\text{A}, \ldots, \text{Z}\}$. There are $(10)^1$

Permutations

Let $X = \{a_1, \ldots, a_n\}$ be a set of distinct object.

[Permutation] A permutation of length $k$ is an $X$-string of length $k$ such that there is no repetition.

Suppose $X = \{a, b, c, d\}$. $abc$ is a permutation of length 3. $\emptyset$ is a permutation of length 0. However, $bab$ is not a permut

Because a permutation requires there be no repetition, there is no permutation of length $k$ for $X$ of size $n$ if $k > n$.

**Q**: How many permutations of length 4 are there of $X = \{a, b, c, d\}$?

**A**: Let $S = s_1 s_2 s_3 s_4$ be a permutation. There are 4 choices for $s_1$. After choosing $s_1$, there are 3 choices left for $s_2$. After c

[Factorial] For any $n \in$ such that $n \geq 0$, $n$ ***factorial*** is defined as

In particular, we define $0! = 1$.