Distance Optimal Target Assignment for Networked Robots with Communication and Target-Sensing Limitations*

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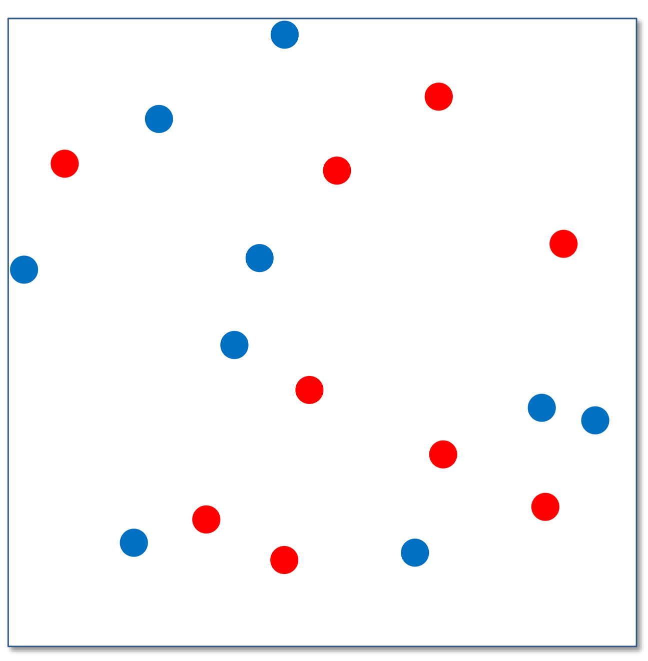
Stochastic Target Assignment Problem

Stochasticity in Target Assignment Problems



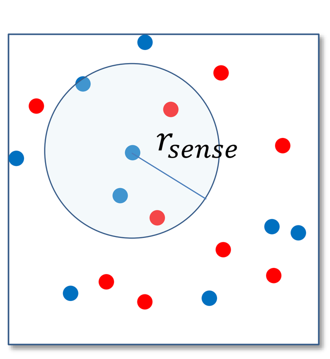
Unit square workspace: $Q = [0,1] \times [0,1]$

Robots: $X = \{x_1, \ldots, x_n\}$
Targets: $Y = \{y_1, \ldots, y_n\}$ } $i.i.d.$ and uniform

Control law: $\dot{x}_i = u_i, ||u_i|| = 1$

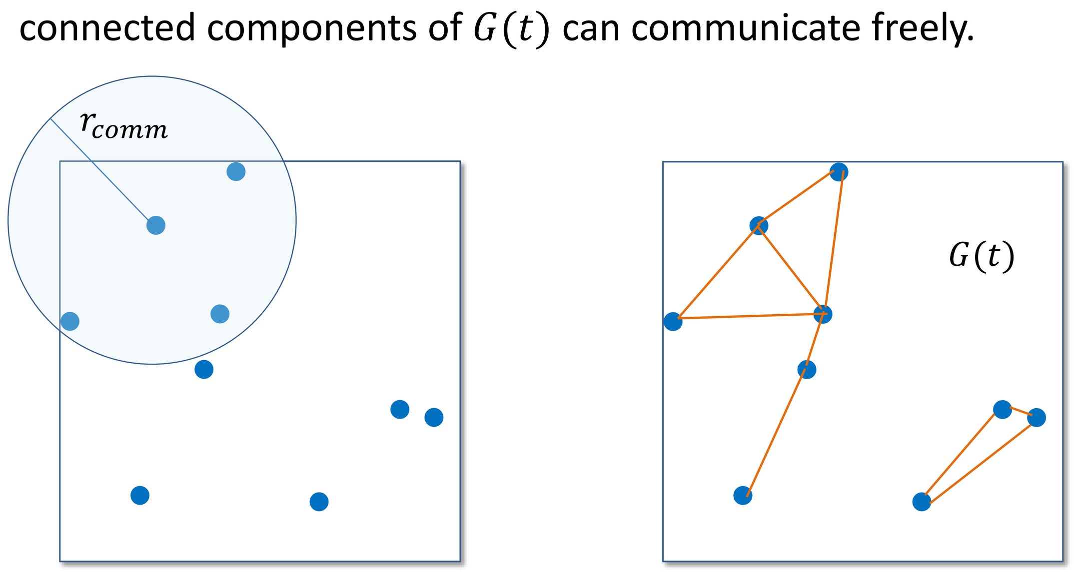
Objective: $\underset{\{u_i\}}{\text{minimize}} \quad D_n = \sum_i \int |\dot{x}_i(t)| dt$

Sensing and Communication Models



A robot can detect the presence of targets within some sensing radius r_{sense}.

A robot can communicate with any robot within a radius of r_{comm} (see the left figure below). Moreover, a robot can communicate with additional robots through relaying information. This gives us a $communication\ graph\ G(t)$ (se the right figure below). Any two robots within the same connected components of $G(t)$ can communicate freely.

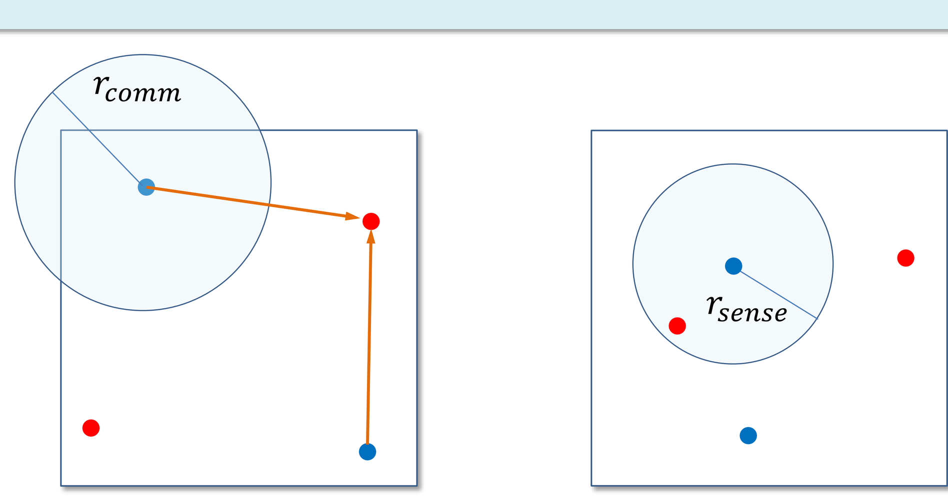


Guaranteeing Distance Optimality

Necessary and Sufficient Conditions

Theorem 1 (Necessary and Sufficient Conditions): In a unit square, under sensing and communication constraints (i.e., $r_{comm}, r_{sense} < \sqrt{2}$), distance optimality can be achieved with probability one if and only if at t = 0:

i. the communication graph is connected, and
ii. every target is within a distance of r_{sense} to some robot.



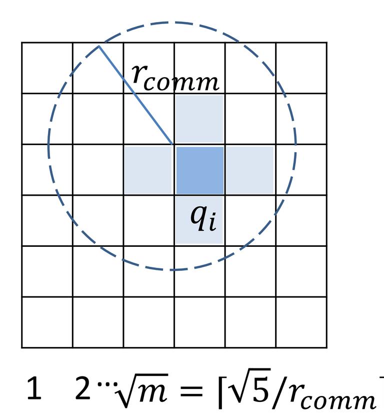
The two pictures above show the necessity of the conditions for two robots. Sufficiency is straightforward to verify.

Non-Asymptotic Probabilistic Guarantees

Theorem 2 (Probabilistic Optimality Guarantee): Suppose that n robots and n targets are uniformly randomly distributed in the unit square. Fixing $0 < \epsilon < 1$, at $t = 0$, the communication graph is connected and every target is observable by some robot with probability at least $1 - \epsilon$ if

$$n \geq \left\lceil \frac{\sqrt{10}}{\theta} \right\rceil^2 \log \left(\frac{1}{\epsilon} \left\lceil \frac{\sqrt{10}}{\theta} \right\rceil^2 \right),$$

in which $\theta \coloneqq \min\{\sqrt{5} r_{sense}, \sqrt{2} r_{comm}\}$.



Proof sketch: We partition the unit square into $\lceil \sqrt{5}/r_{comm} \rceil^2$ small squares. If each small square (*e.g.*, q_i) contains at least one robot, then the communication graph is connected. This allows us to compute n via the union bound. A similar bound can be obtained over r_{sense}.

Our bounds are **non-asymptotic** (does not require r_{comm}, r_{sense} to be small to hold well). Furthermore, we can show that the bounds from the theorem are also asymptotically tight if high probability guarantee is required.

Decentralized Hierarchical Strategies

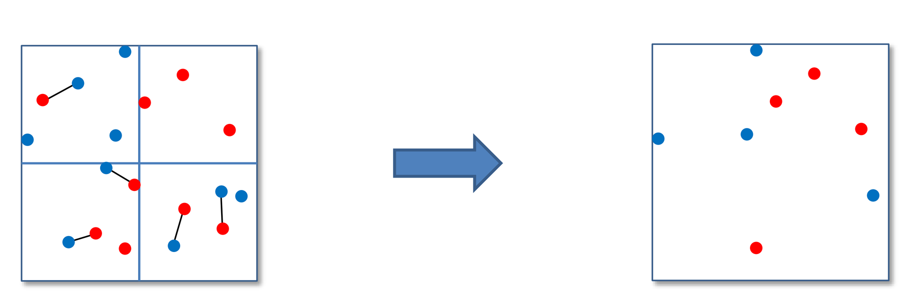
An Ideal Hierarchical Divide-and-Conquer Strategy

Assume temporarily that we are not constrained by sensing and communication limitations. We partition the unit square into m small squares and perform assignments hierarchically. That is, local assignments are performed first (*e.g.*, the example below).

We can prove the following for the local assignments

Lemma 1: The minimum total distance cost from the local assignments is bounded by $O(\sqrt{n \log n})$ in expectation.

After the local assignments, the leftover robots and targets at the higher hierarchy can then be matched. The number of the leftover robots can be easily bounded.



Lemma 2: The total number of robots that are left unmatched is no more than $\sqrt{n(m-1)/2}$ in expectation.

We can then bound the expected performance of a general class of hierarchical divide-and-conquer strategies.

Theorem 3 (Performance of Ideal Hierarchical Strategies): Suppose that n robots and n targets are uniformly randomly distributed in the unit square and the unit square is divided into m_i equal-sized small squares at hierarchy i with a total of $h \geq 2$ hierarchies. For all applicable $i \geq 1$, assume that $m_{i+1} \geq m_i$ and any square at hierarchy $i + 1$ falls within a single square at hierarchy i. Then a hierarchical divide-and-conquer strategy yields

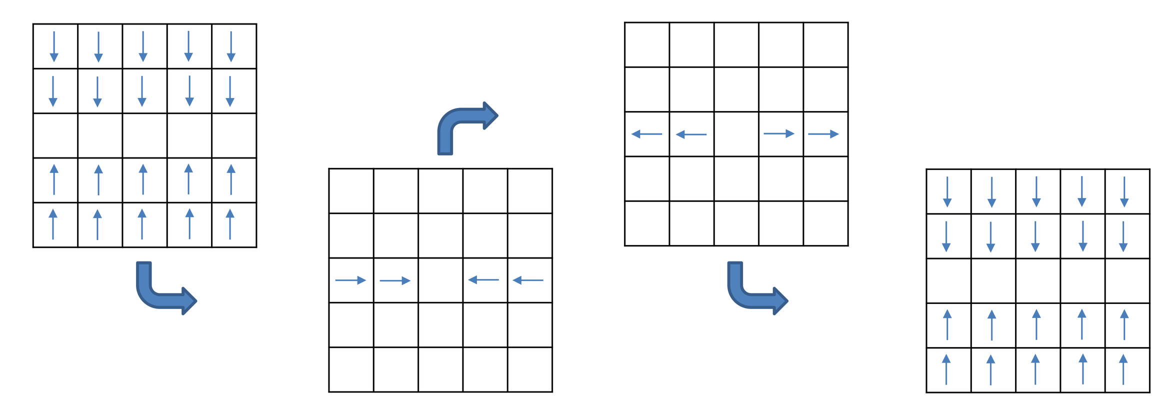
$$\mathbf{E}[D_n] \leq C\sqrt{n \log n} + \sum_{i=1}^{h-1} \sqrt{\frac{n m_{i+1}}{m_i}}.$$

We can further show that these strategies are asymptotically $O(1)$ optimal.

Corollary 1: For fixed h and $m_1, \ldots, m_h$ that do not depend on n, as $n \to \infty$, the hierarchical divide-and-conquer strategy yields targets assignments with $O(1)$ distance optimality.

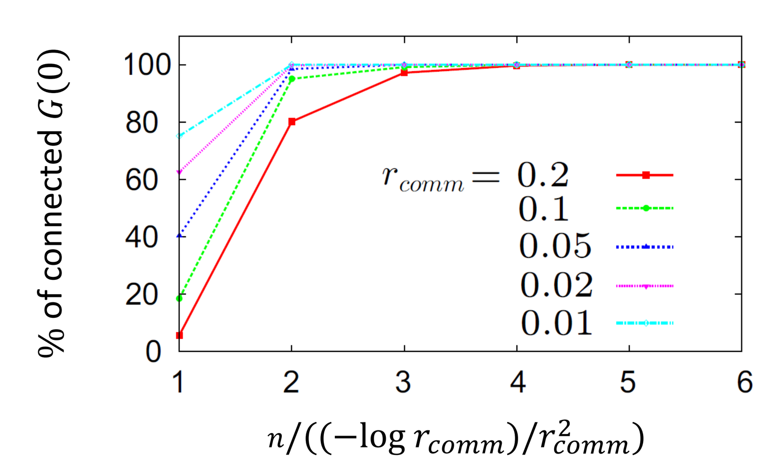
Decentralized Hierarchical Strategies

To handle the communication and sensing constraints (*i.e.*, $r_{comm}, r_{sense} < \sqrt{2}$) , we combine an ideal hierarchical strategy with a rendezvous-like strategy that simulates a connected communication graph and global sensing. For example, for communication, we may let a subset of robots move around to pass along information. One way to do this is follow the movement pattern given in the figure below.

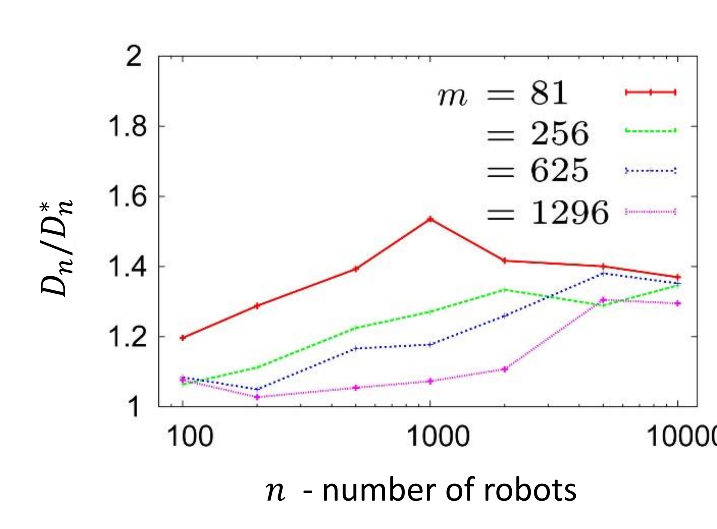


It turns out that the cost from simulating a connected communication graph and global sensing capability (which can be done similarly) is asymptotically insignificant when compared with the cost of assignment. Thus, we have obtained decentralized hierarchical strategies that give us constant approximations on distance optimality asymptotically, in expectation.

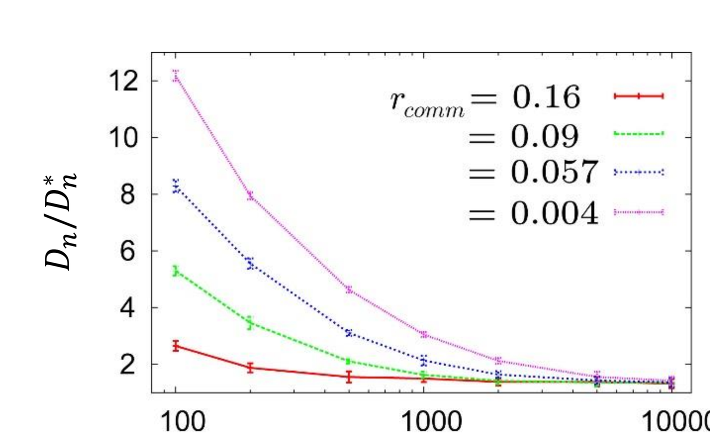
Simulation Study



We use the formula from Theorem 2 to compute for a range of r_{comm} −probability pairs the required number of robots for guaranteeing a connected $G(0)$ with the desired probability. We then plot (in the figure above) the percentage of connected $G(0)$ with respect to $n/((-\log r_{comm})/r_{comm}^2)$. Each data point is collected over 1000 simulations. As we can see, the threshold is fairly sharp.



We test the (2-level) ideal hierarchical strategies over various n-r_{comm} combinations. The average optimality ratios are plotted in the figure above. We observe that an optimality ratio of less than two is achieved consistently and the ratio can be very close to one. The performance of the corresponding decentralized strategy is plotted in the figure below, which shows asymptotic near $O(1)$ optimality.



Key takeaway: For a fairly general class of assignment problems, local optimality implies near-global optimality.

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