Nash Bargaining Solutions: Groupon or Standalong?

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Outline

- Preliminary
- Nash Bargaining Problem
- Bargaining Solutions
- Future Work and Conclusion

Preliminary

Game Theory

Non-Cooperative		Cooperative
Individual user	player	Group of users (Coalition)
Specific details	strategy	Abstract
Specific details	payoff	Abstract
Normal / Extensive	representation	Characteristic Function
Self-enforcing	interacting	Self-enforcing / Axiomatic
Equilibrium	outcome	Solution

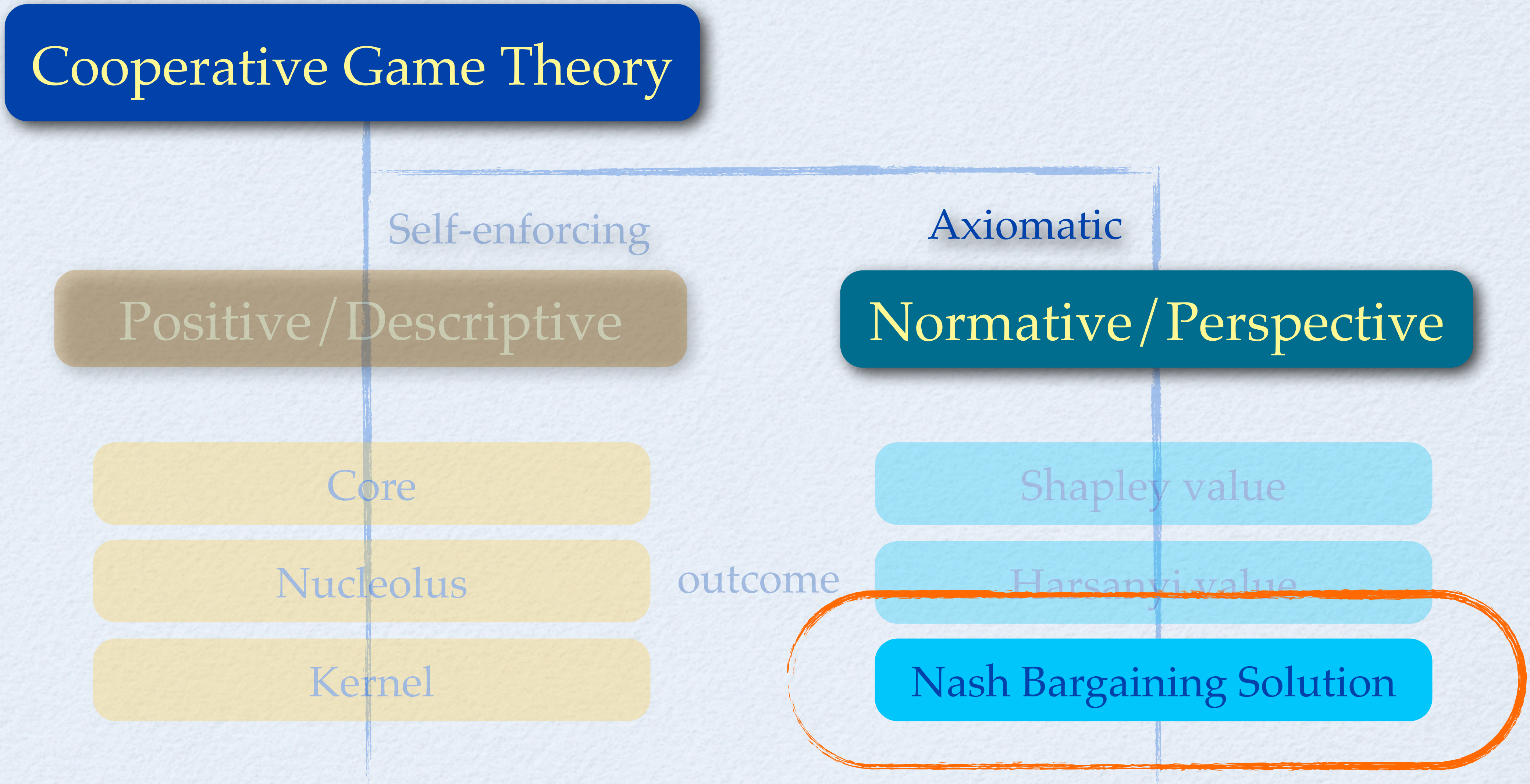
Preliminary

Cooperative Game Theory

- Self-enforcing
  - **Positive / Descriptive**
    - Core
    - Nucleolus
    - Kernel
- Axiomatic
  - **Normative / Perspective**
    - Shapley value
    - Harsanyi value
    - Nash Bargaining Solution

outcome

Preliminary



Preliminary

Nash Bargaining Model

Transferable Utility

1. The interaction of players generates certain social welfare;

2. The social welfare can be distributed among the players in an arbitrary way.

(Welfare space is a convex set)

Non-transferable Utility

1. The interaction of players generates certain social welfare;

2. The social welfare can be distributed among the players in some specific way.

Preliminary

- Example 1: TU and NTU
  - Tom and Jerry, bargaining for sth exchanging;
  - Tom: a book, \$10 for himself, and \$50 for Jerry;
  - Jerry: a pen, \$20 for Tom, and \$40 for himself.

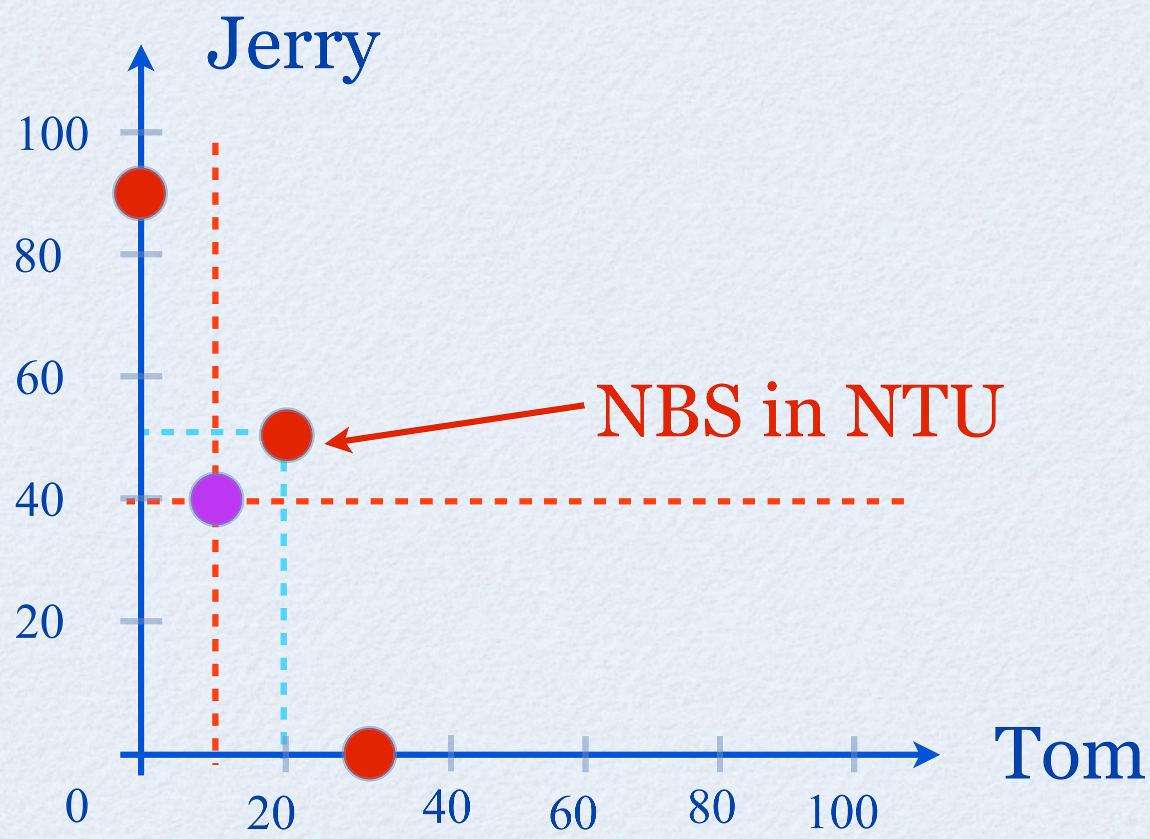
  -- Nash Bargaining Problem

$$\max_{\{v_1\, v_2\}} \quad (v_1 - c_1)(v_2 - c_2) \qquad \text{s.t., } v_i \geq c_{i},\ {}_{i=1,2}$$

Preliminary

- Example 1: TU and NTU

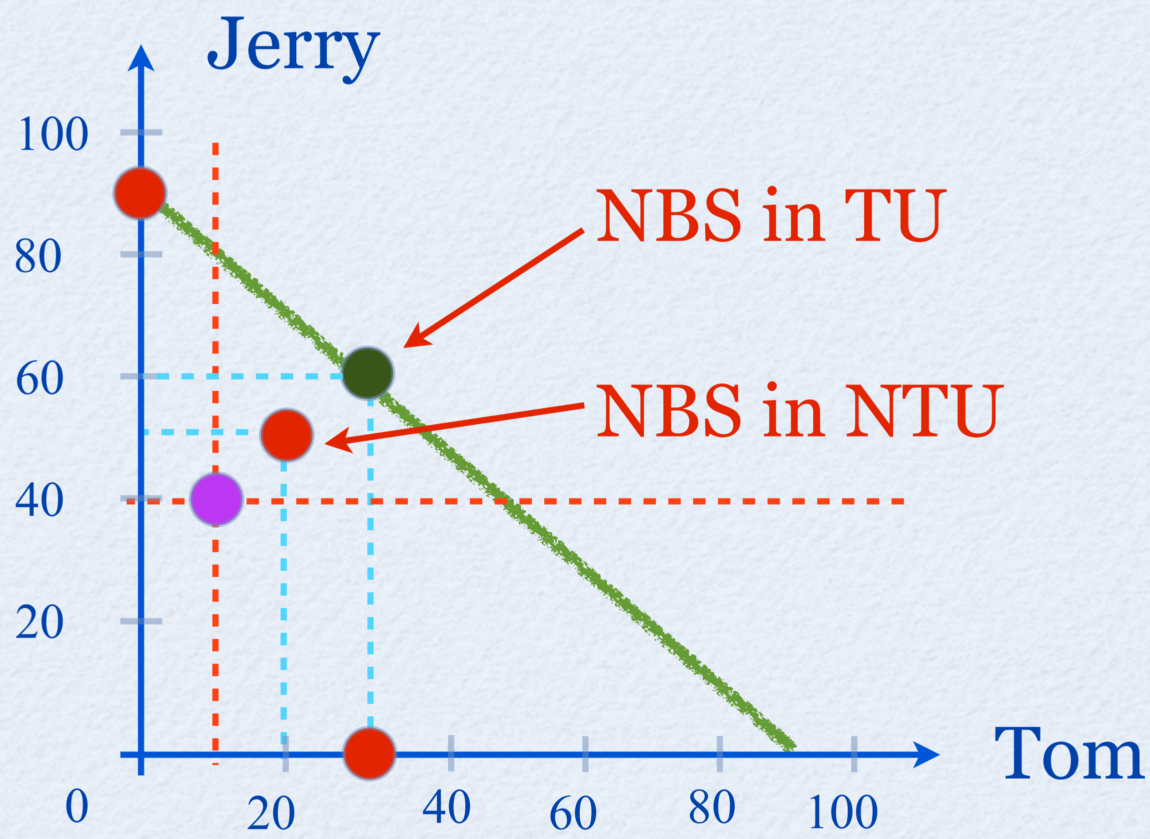
  -- NBS: (20, 50) in NTU, and (30, 60) in TU.



Preliminary

- Example 1: TU and NTU

  -- NBS: (20, 50) in NTU, and (30, 60) in TU.



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Nash Bargaining Problem

- Problem: How should N players divide the total surplus generated by their cooperation?
- Nash's Axioms:

  (1) Pareto optimality,

  (2) Invariant to affine transformations,

  (3) Independence of irrelevant alternatives,

  (4) Symmetry.

Nash Bargaining Problem

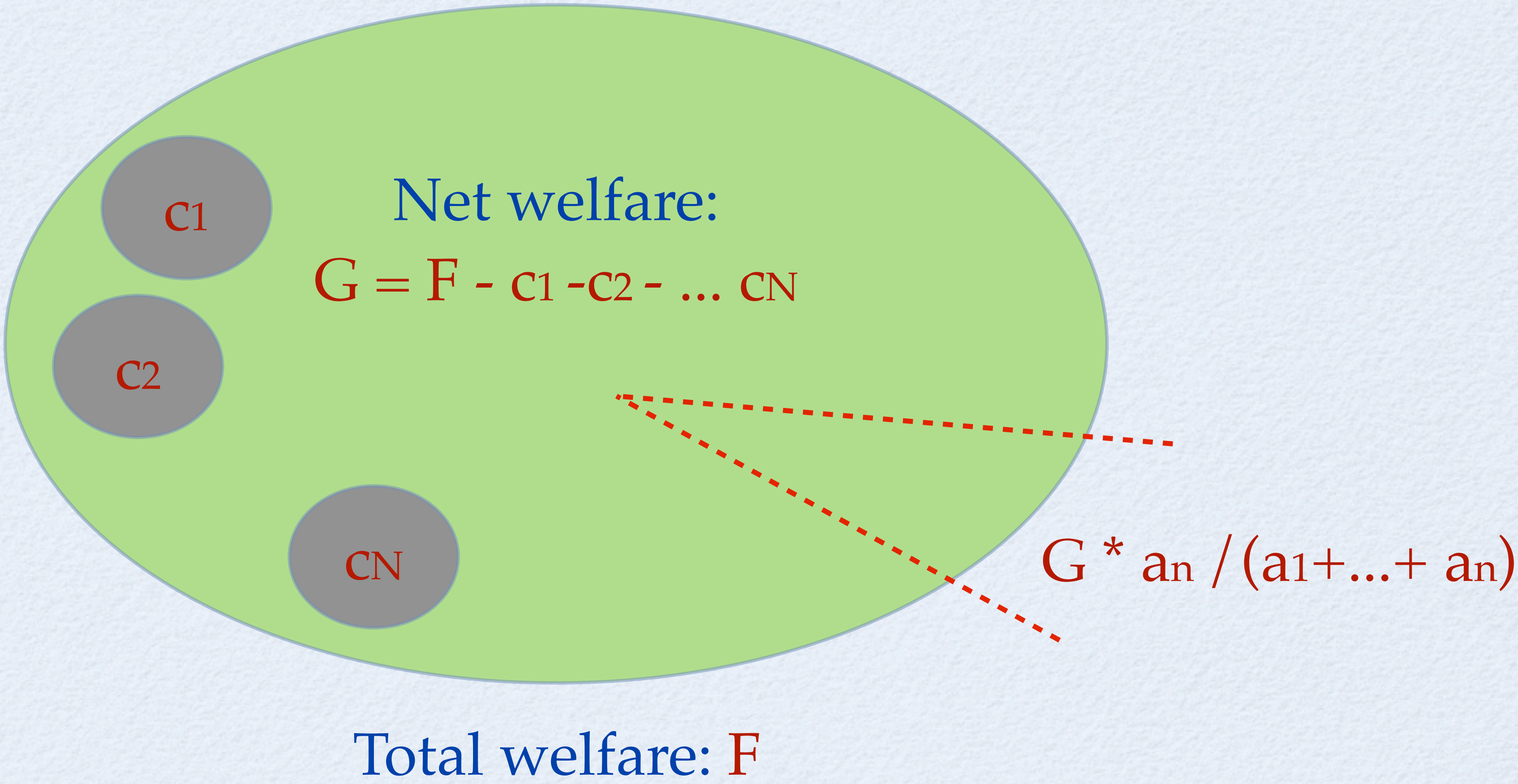
- Nash Bargaining Solution: the only solution satisfying above 4 axioms is given by:

$$\boxed{\max_{\{v_1 \ldots v_{\cancel{2}N}\}} (v_1 - c_1)^{a_1} (v_2 - c_2)^{a_2} \ldots (v_N - c_N)^{a_N} \quad \text{s.t., } v_i \geq c_{i,\ i=1,2,\ldots,N}}$$

$$v_{n^*} = c_n + \frac{(F - c_1 - \ldots - c_N)\, a_n}{a_1 + \ldots + a_N}$$

Nash Bargaining Problem

- Geometrical Illustration for NBS



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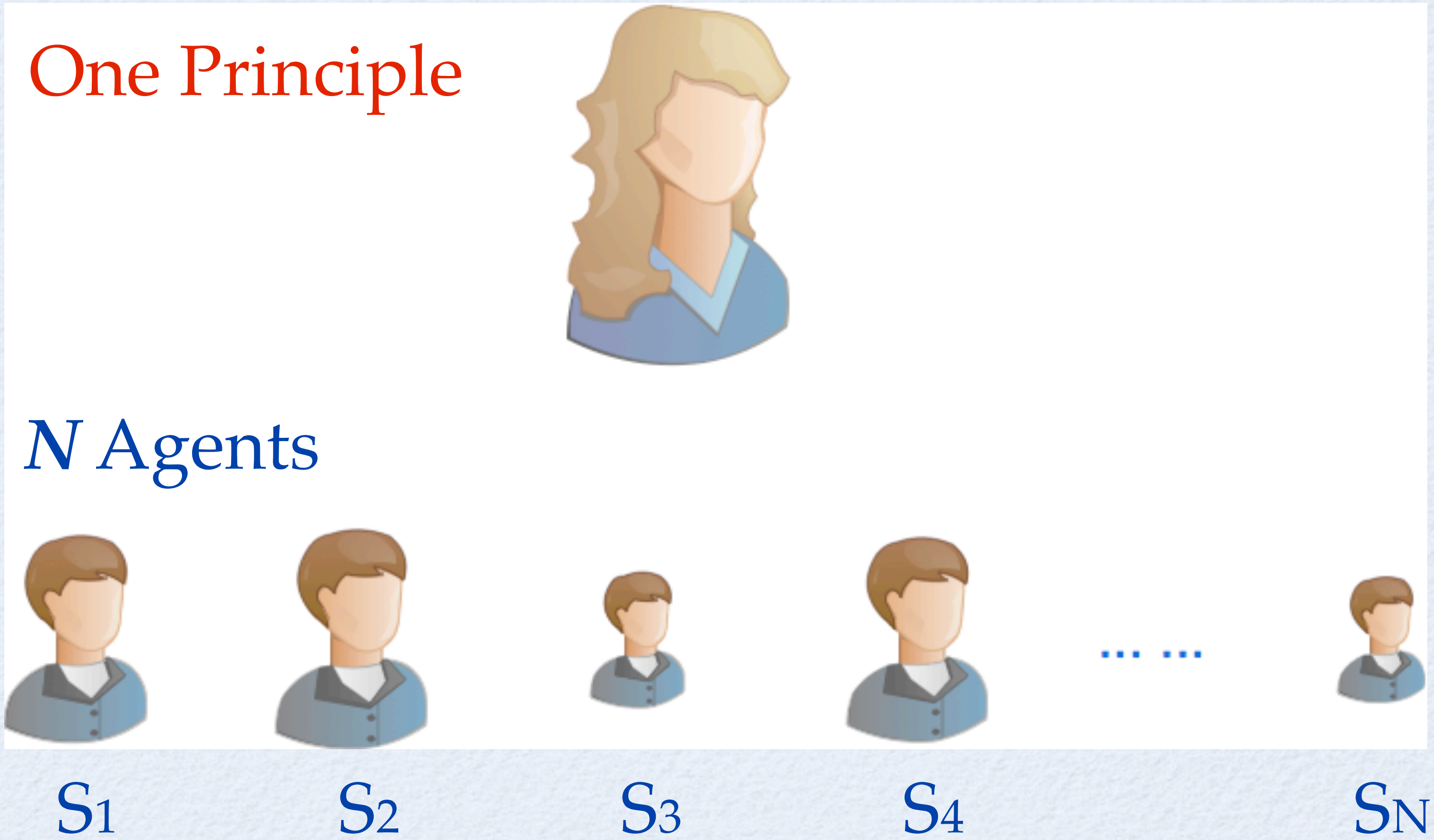
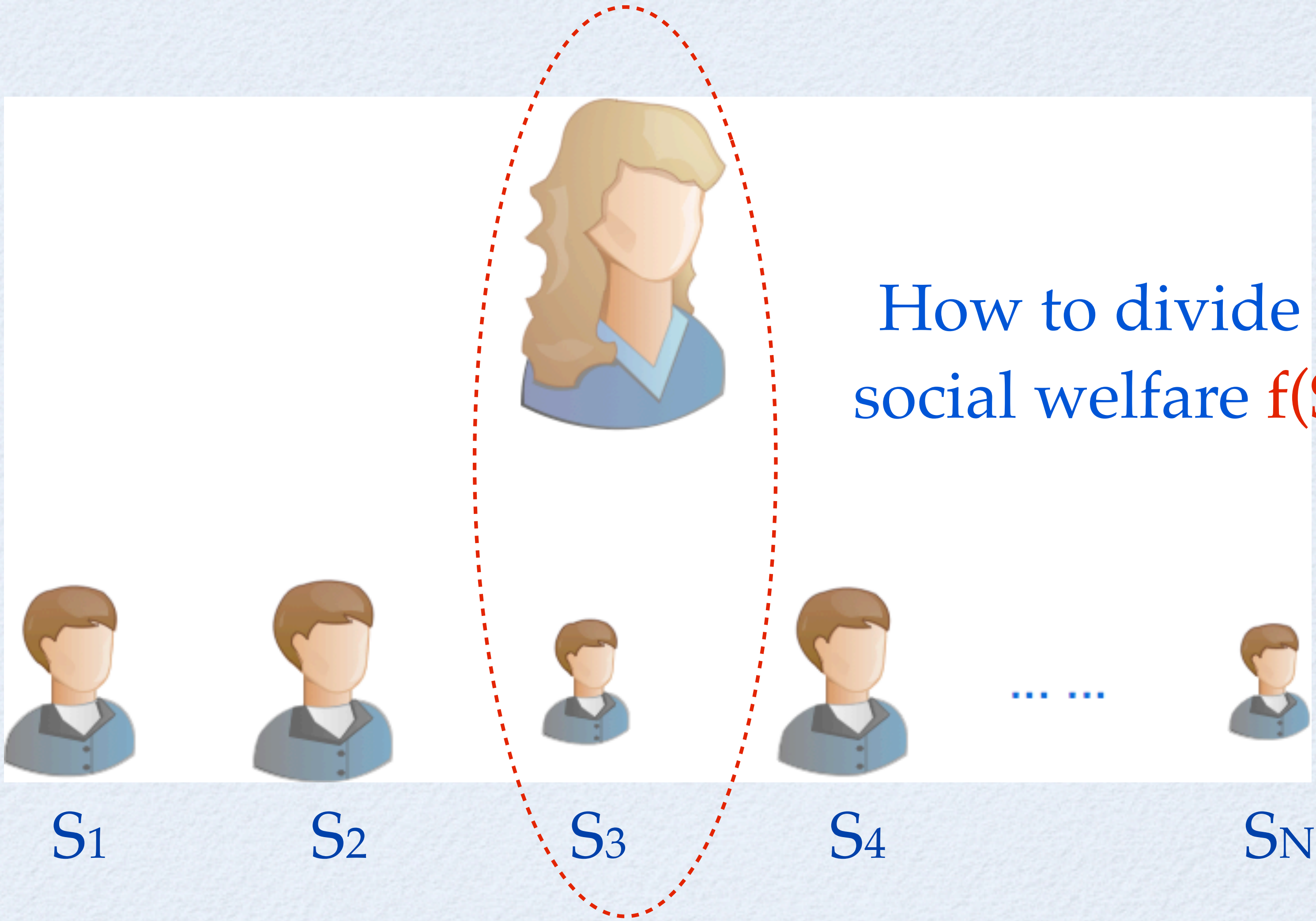
General Model

- Two types of players: Principle and Agent

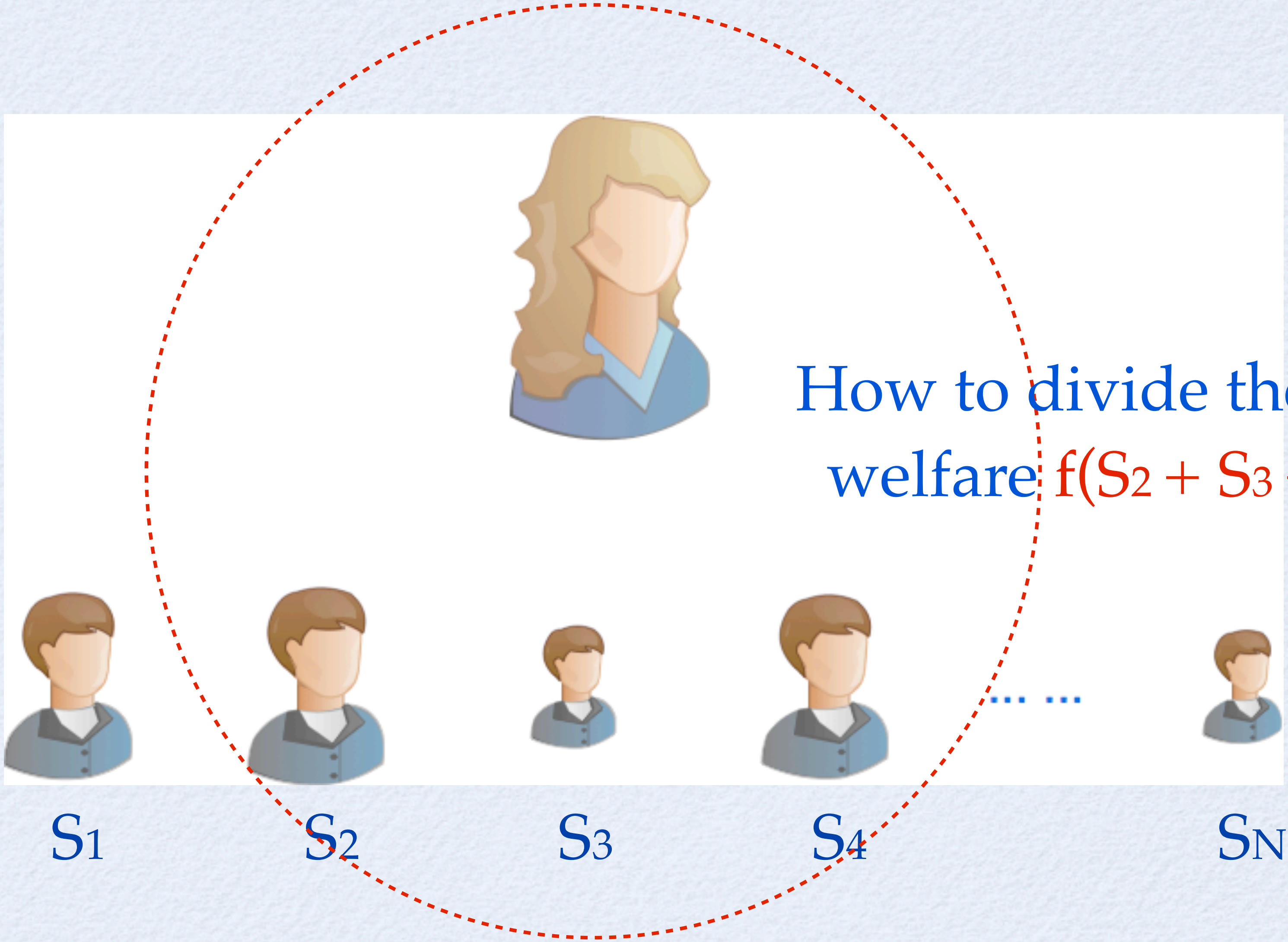


General Model



How to divide the social welfare $f(S_3)$?

General Model



How to divide the social welfare $f(S_2 + S_3 + S_4)$?

General Model

- Generalization for practical problem
  - In wireless networks, Operator vs Mobile users;
  - In cognitive networks, PU vs SUs;
  - In coop. communications, Source vs Relays;
  - In cloud systems, Server vs End users;
  - In social networks, Host vs Guests;
  - … …

Bargaining Solutions

- Three Bargaining Scenarios:

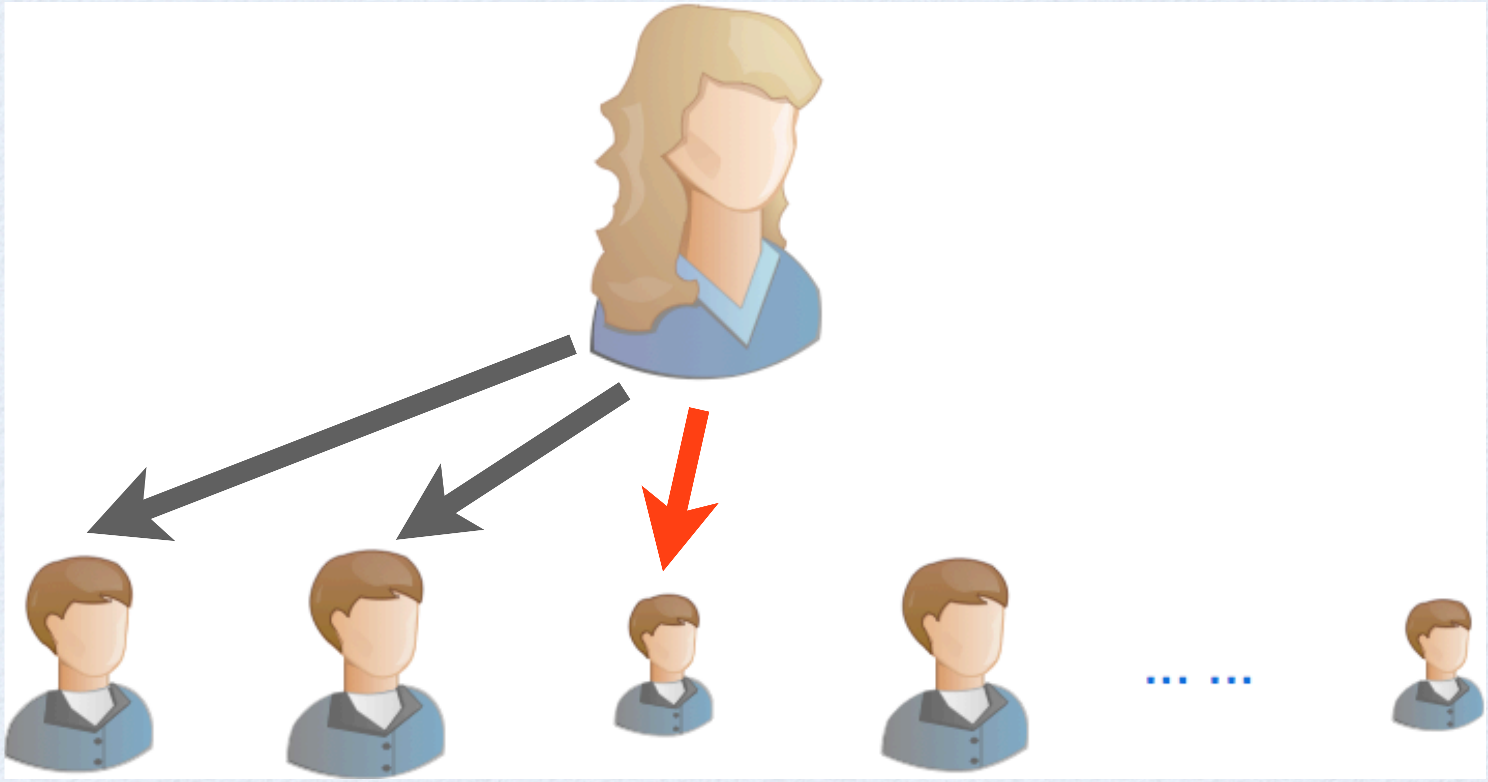
  (1) Sequential Bargaining

  (2) Independent Bargaining

  (3) Group Bargaining

Bargaining Solutions

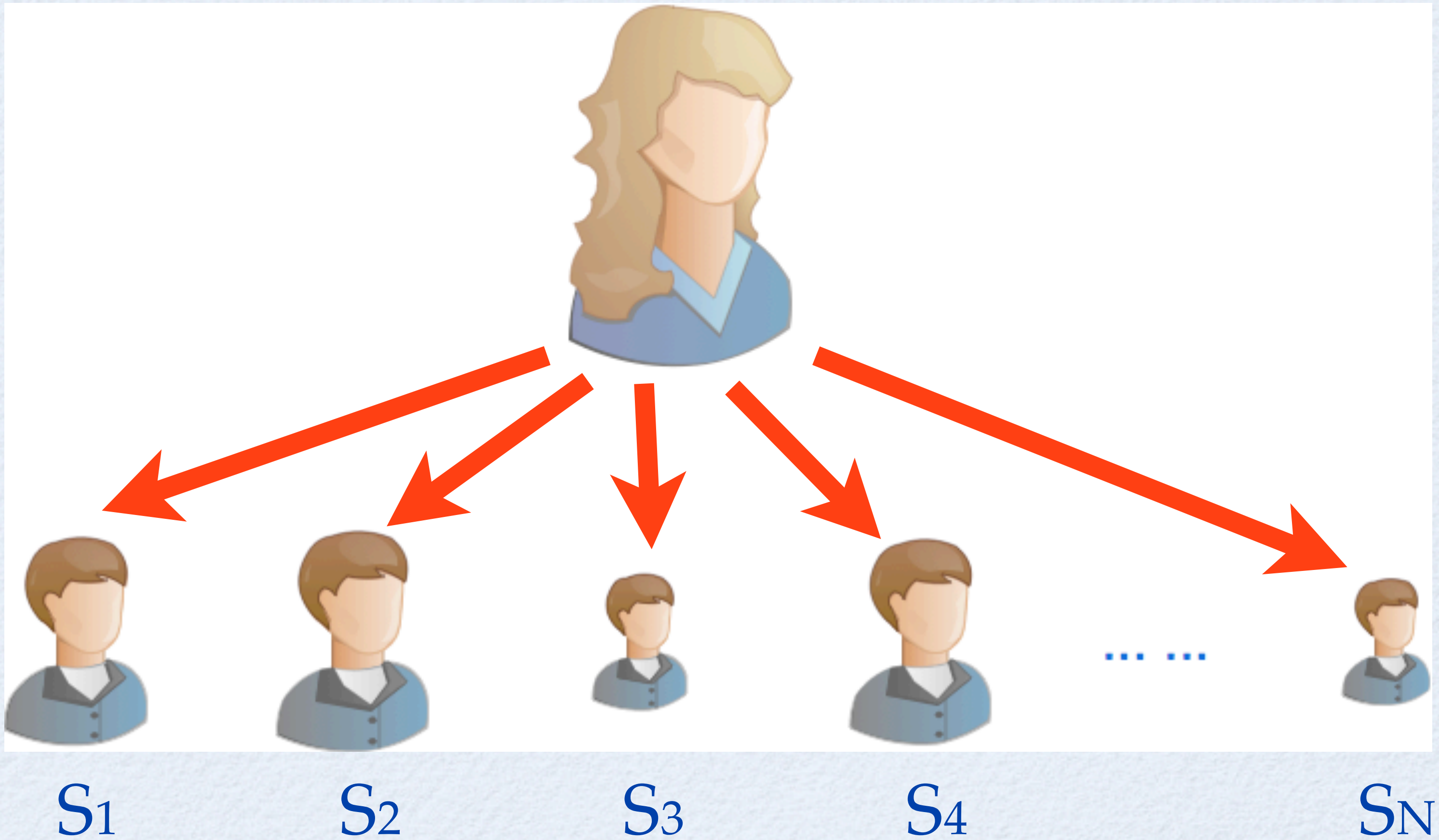
- Sequential Bargaining



S_1 S_2 S_3 S_4 S_N

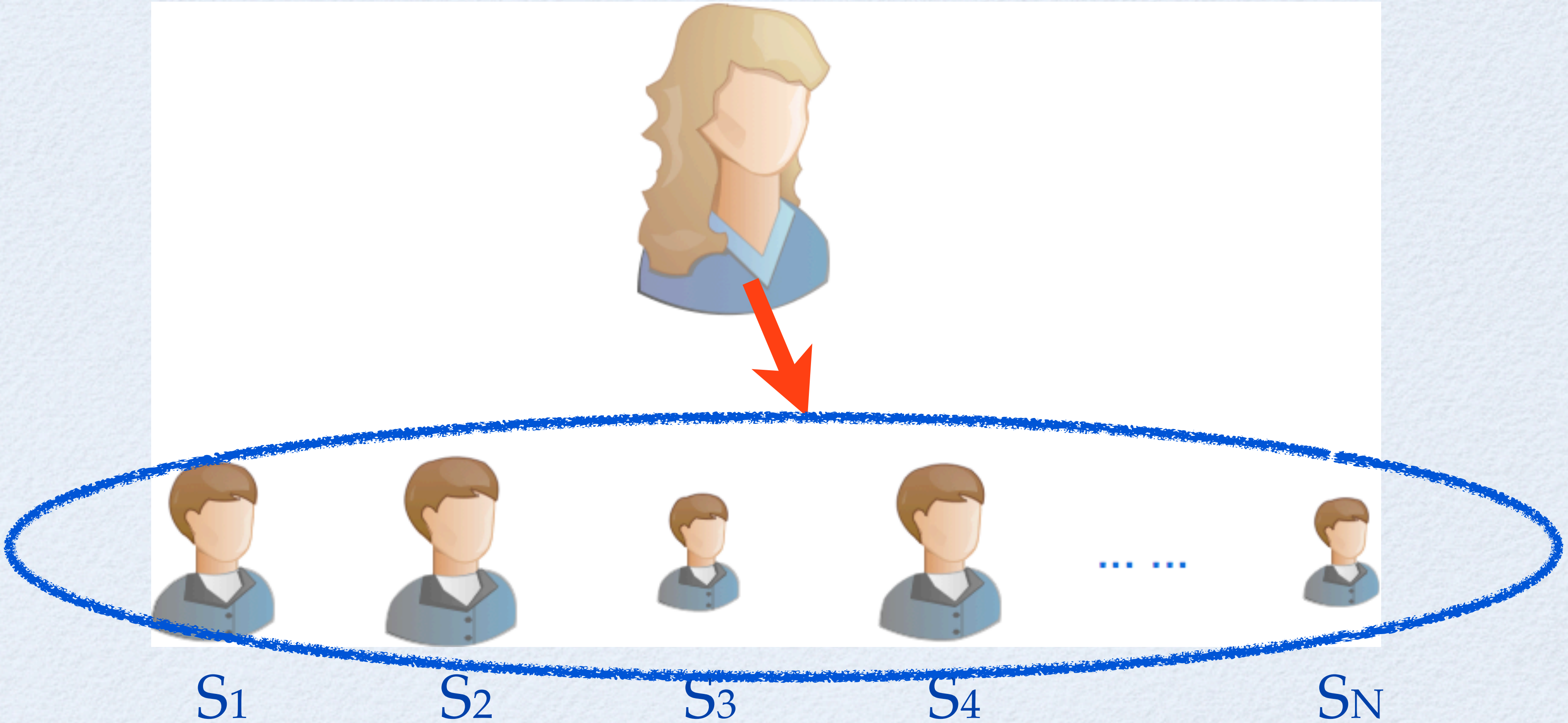
Bargaining Solutions

- Independent Bargaining



Bargaining Solutions

- Group Bargaining



Bargaining Solutions

- Sequential Bargaining Solution
- Step 1:

$$\max_{v,u_1} \ (v - c_0) \cdot (u_1 - c_1)$$
$$s.t. \ v + u_1 \leq f(S_1),$$
$$v \geq c_0, \ u_1 \geq c_1,$$

$$v^* = v_1^* = \frac{G_1}{2} + c_0, \ u_1^* = \frac{G_1}{2} + c_1.$$

$G_1 \triangleq f(S_1) - c_0 - c_1$ Marginal Welfare by Agent 1

Bargaining Solutions

- Sequential Bargaining Solution
- Step n:

$$\max_{v, u_n} \ (v - v_{n-1}^*) \cdot (u_n - c_n)$$
$$s.t. \ v + u_n + u_1^* + \ldots + u_{n-1}^* \leq f(S_1 + \ldots + S_n),$$
$$v \geq v_{n-1}^*, \ u_n \geq c_n.$$

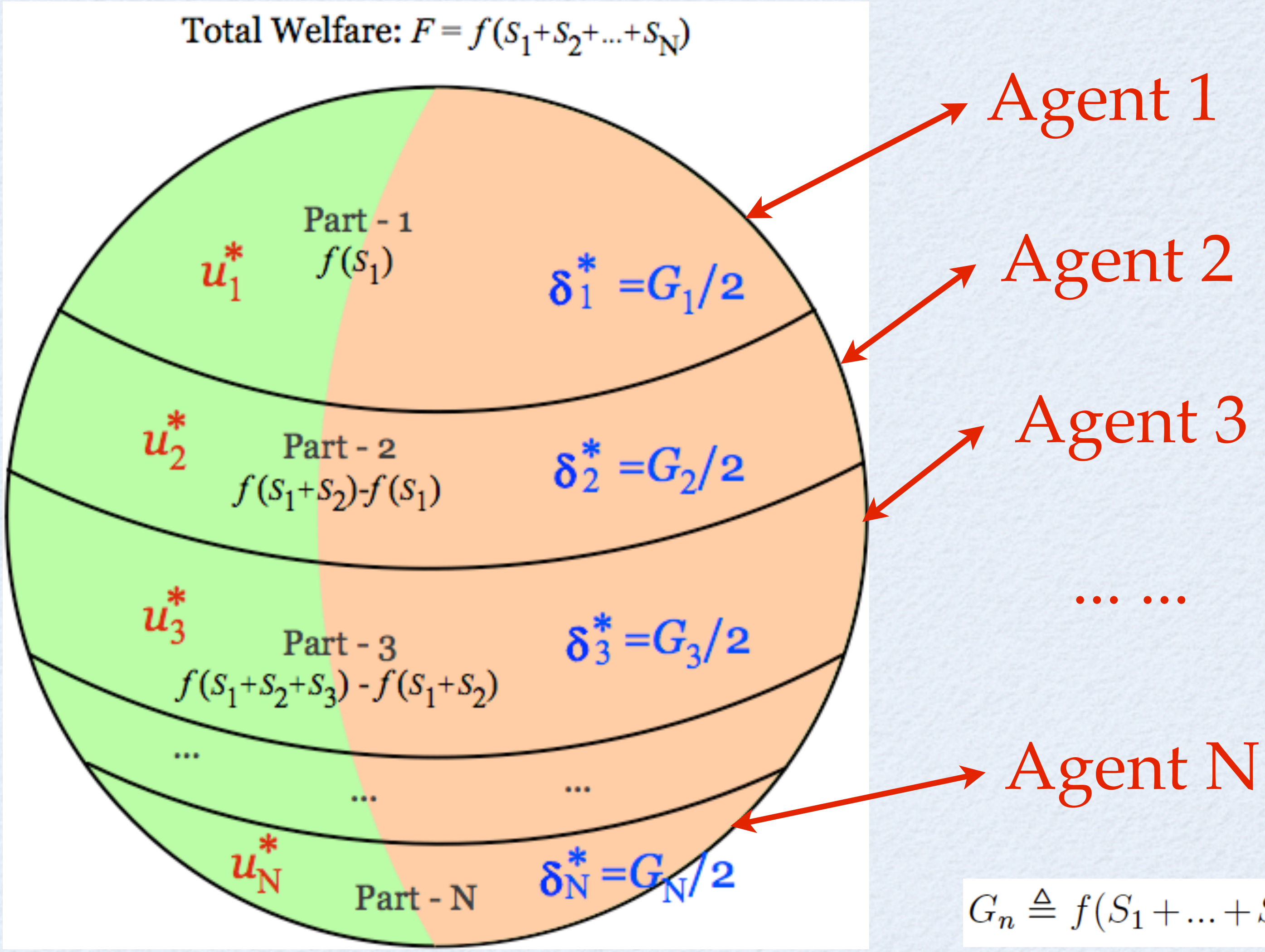
$$v^* = v_n^* = \frac{G_n}{2} + v_{n-1}^*, \ u_n^* = \frac{G_n}{2} + c_n.$$

$$G_n \triangleq f(S_1 + \ldots + S_n) - f(S_1 + \ldots + S_{n-1}) - c_n$$

Marginal Welfare by Agent n

Bargaining Solutions

- Sequential Bargaining Solution



$$G_n \triangleq f(S_1 + ... + S_n) - f(S_1 + ... + S_{n-1}) - c_n$$

Bargaining Solutions

- Independent Bargaining Solution
- Agent 1:

$$\max_{v_1, u_1} (v_1 - V_{-1}) \cdot (u_1 - c_1)$$
$$s.t.\ v_1 + u_1 + U_{-1} \leq F,$$
$$v_1 \geq V_{-1},\ u_1 \geq c_1,$$

$$V_{-1} = F_{-1} - U_{-1} \geq c_0,$$
$$F_{-1} \triangleq f(S_2 + \ldots + S_N),$$
$$U_{-1} = u_2^* + \ldots + u_N^*.$$

$$v^* = v_1^* = \frac{G_1}{2} + V_{-1},\ u_1^* = \frac{G_1}{2} + c_1,$$

$$G_1 \triangleq F - F_{-1} - c_1$$

Marginal Welfare by Agent 1 provided all other Agents is in the active set.

Bargaining Solutions

- Independent Bargaining Solution
- Agent n:

$$\max_{v_N, u_N} \ (v_N - V_{-N}) \cdot (u_N - c_N)$$
$$s.t. \ v_N + u_N + U_{-N} \leq F,$$
$$v_N \geq V_{-N}, \ u_N \geq c_N,$$

$$V_{-N} = F_{-N} - U_{-N},$$

$$F_{-N} \triangleq f(S_1 + ... + S_{N-1})$$

$$U_{-N} \triangleq u_1^* + ... + u_{N-1}^*$$

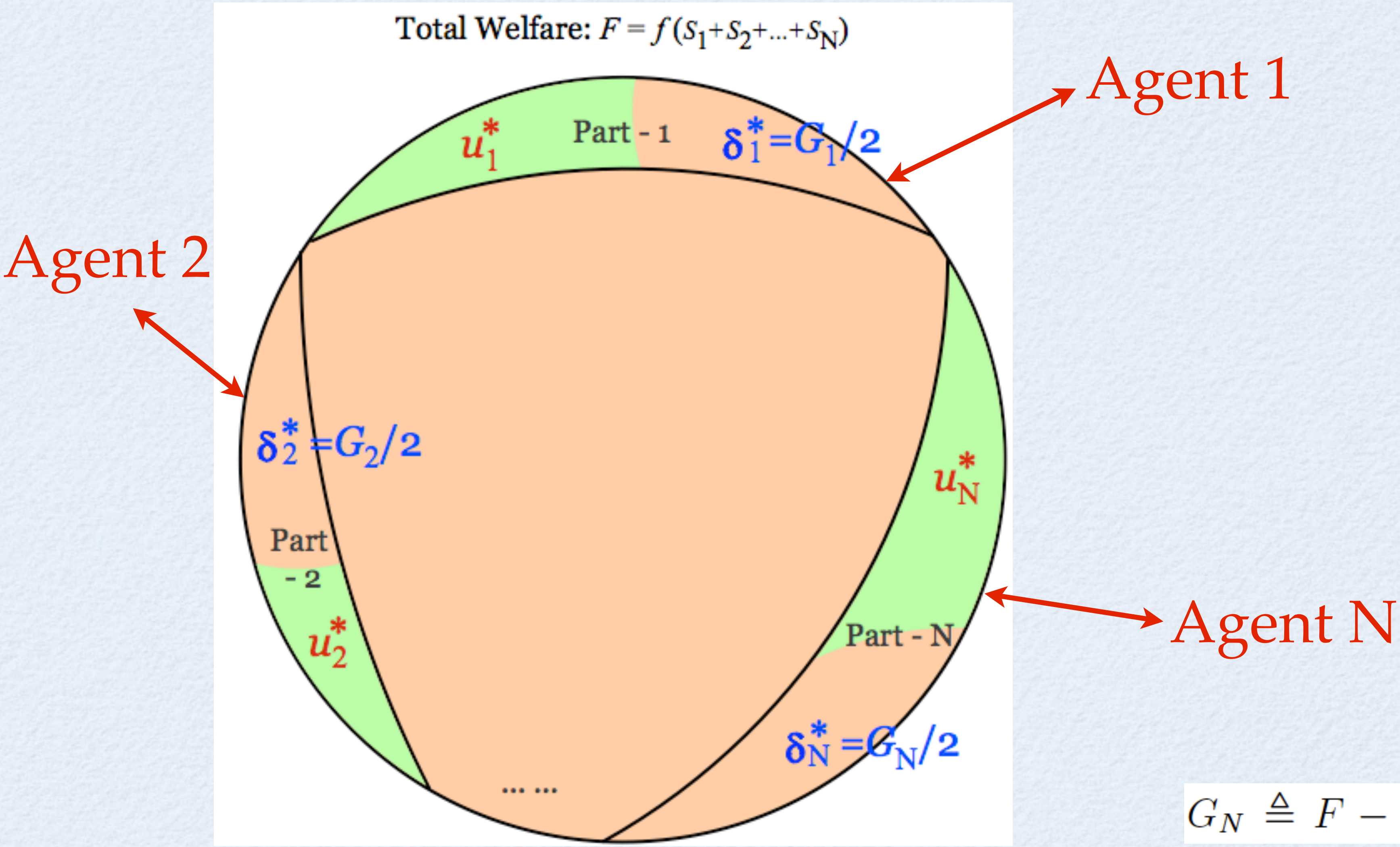
$$v^* = v_N^* = \frac{G_N}{2} + V_{-N}, \ u_N^* = \frac{G_N}{2} + c_N,$$

$$G_N \triangleq F - F_{-N} - c_N$$

Marginal Welfare by Agent n provided all other Agents is in the active set.

Bargaining Solutions

- Independent Bargaining Solution



$$G_N \triangleq F - F_{-N} - c_N$$

Bargaining Solutions

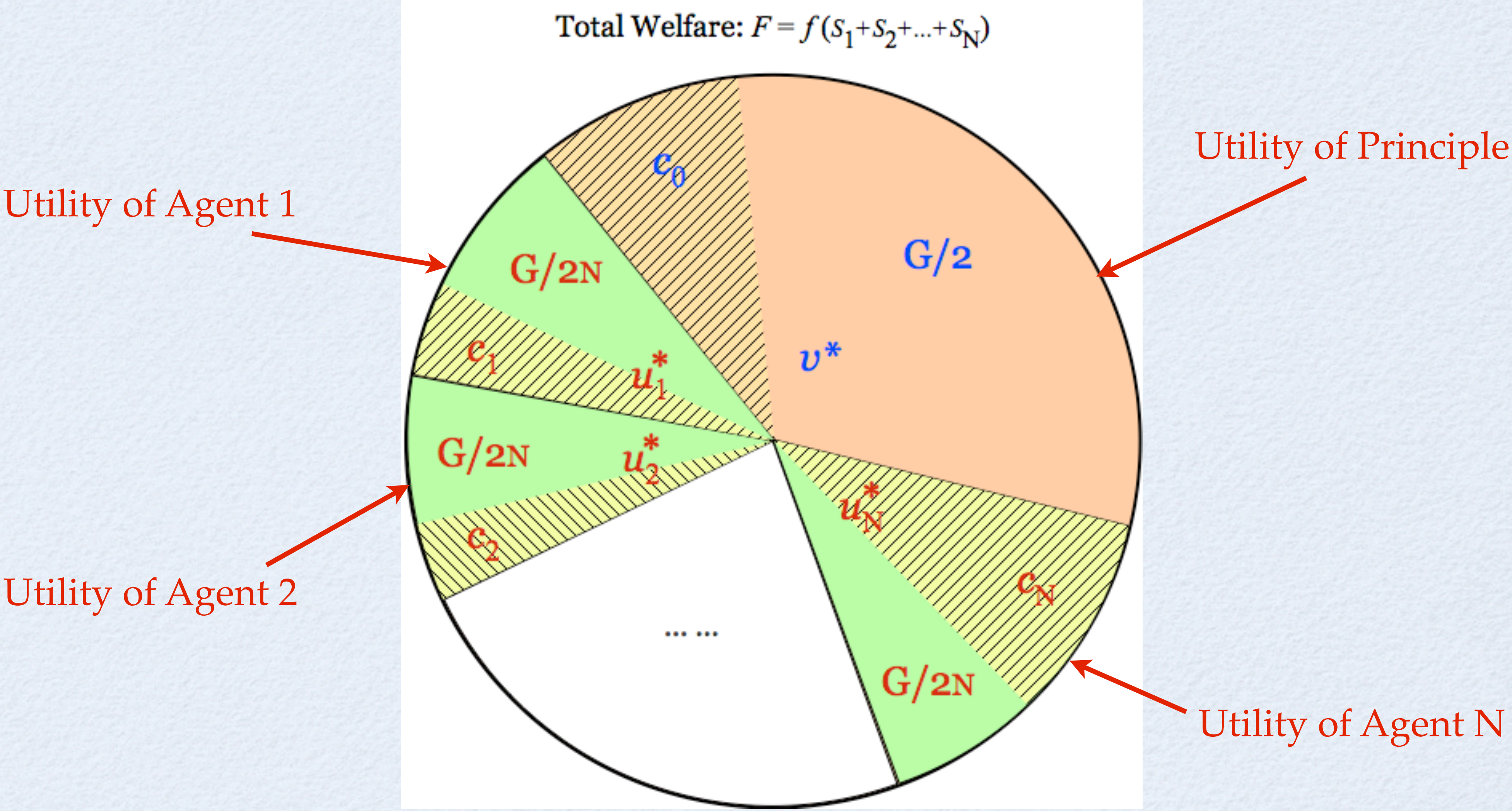
- Group Bargaining Solution

$$\max_{v,u_1,\ldots,u_N} (v - c_0) \cdot (u_1 - c_1)^{1/N} \cdot \ldots \cdot (u_N - c_N)^{1/N}$$
$$s.t.\ v + u_1 + \ldots + u_N \leq F,$$
$$v \geq c_0,\ u_n \geq c_n, \forall n = 1, \ldots, N.$$

$$\begin{cases} v^* = \dfrac{F - c_0 - c_1 - \ldots - c_N}{2} + c_0, \\ u_n^* = \dfrac{F - c_0 - c_1 - \ldots - c_N}{2N} + c_n, \forall n = 1, \ldots, N. \end{cases}$$

Bargaining Solutions

- Group Bargaining Solution



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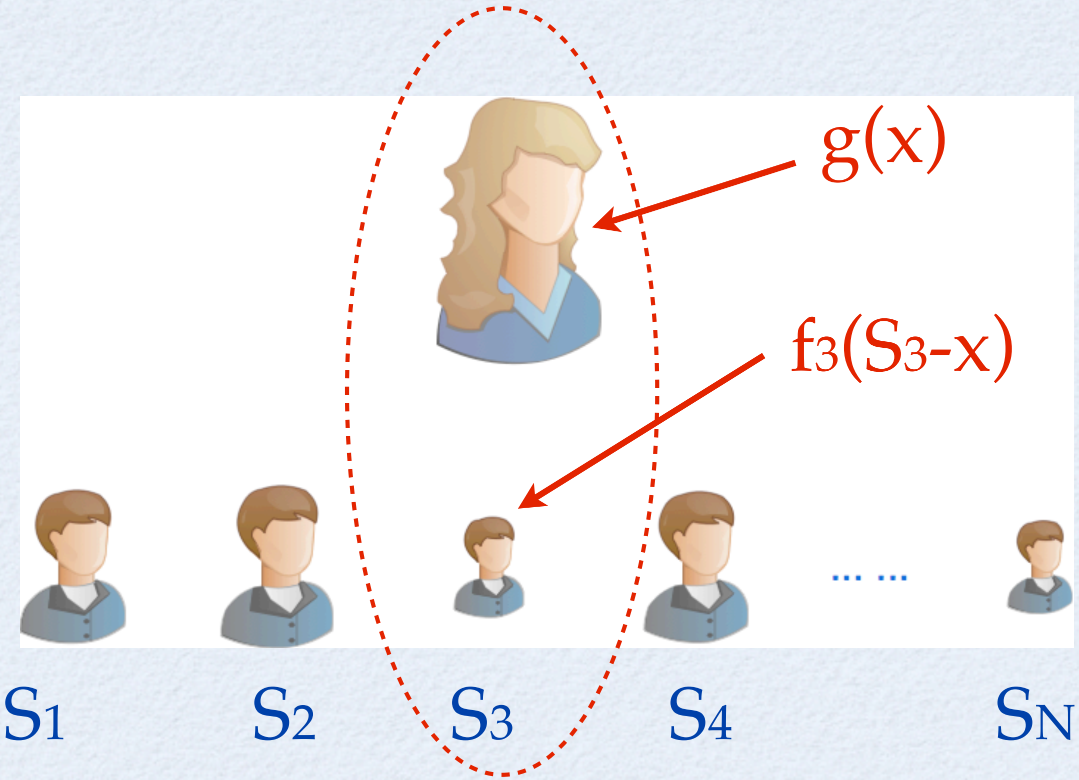
Future Work

- Non-Transferable Utility Model
  - How to extend the solution to NTU model?



How to divide resource S_3 among the Principle and Agent 3 ?

Conclusion

- We consider the Nash bargaining problem between one Principle and multiple Agents in Transferrable Utility;
- We propose three different bargaining schemes, and derive the respective bargaining solutions.

Q & A