CS 251 Homework 1: Algorithms Analysis (100 points)

Out: January 11th, 2019 (8:00 pm)
Due: January 18th, 2019 (8:00 pm)

Important: Each question has only one correct answer. Additionally, you must provide an **explanation** for your answer (2-3 short sentences is sufficient). Answers without proper explanations, even though it is correct, will be graded with 0 points.

1) Select the **tightest big-O** expression for the following pseudo codes (30 points)

1.1)

```
int sum = 0;
for(int i = 0; i < n; i++)
        for(int j = 0; j < n * n; j++)
                sum ++;
```

A. *O(n)*
B. *O(n^2)*
C. *O(n^3)*
D. *O(n^4)*

1.2)

```
int sum = 0;
for(int i = 0; i < n; i++)
        for(int j = 0; j < i; j++)
                sum ++;
```

A. *O(n)*
B. *O(n logn)*
C. *O(n^2)*
D. *O(n^2 logn)*

1.3)

```
int prod = 3;
for(int i = 1; i <= n; i = i * 2)
        for(int j = 0; j < n; j++)
                prod = prod * prod;
```

A. *O(n)*
B. *O(n logn)*
C. *O(n^2)*
D. *O(n^2 logn)*

1.4)

```
int sum(n) {
        if(n == 1) {
                return 1;
        }
        return n + sum(n - 1);
}
```

A. *O(n)*
B. *O(n logn)*
C. $O(n^2)$
D. $O(n^2 \log n)$

1.5)

```
count1=0; count2=0; count3=0;
for(i=0; i<n; i++)
        for(j=1; j<n; j*=2)  {
                count1++;
                count2++;
                count3++;
        }
for(i=0; i<n; i++) {
        count1++;
        count2++;
        count3++;
}
```

A. *O(n)*
B. *O(n logn)*
C. $O(n^2)$
D. $O(n^2 \log n)$

2) Answer the following questions on the below expressions (15 points)

2.1) Choose **tightest Big-O for:**

$$n^3 + n! + 3^n + 2147483647 * \log^3 n$$

A. $O(n^3)$
B. $O(n!)$
C. $O(3^n)$
D. $O(2147483647 * \log^3 n)$

B

2.2) Choose **tightest Big-Ω for:**

$$\sum_{i=0}^{2n} 5i + i^2$$

A. $\Omega(n^3)$
B. $\Omega(n^2)$
C. $\Omega(3^n)$
D. $\Omega(2147483647 * log^3 n)$

3) What is the **worst-case** amount of work to find a particular card in a deck of cards of size (n)? Note that cards are unique and are **not** sorted. (10 points)

n

4) Order the following functions by growth rate. Indicate which functions grow at the same rate (15 points)

N, N^2, log N, N log N, $log(N^2)$, log^2 N, N log^2 N, 2, 2^N, 37, N^2 log N, 5logN, N^3, 10N log N^2

2^N -> N^3 ->N^2logN >N^2 > Nlog2N > 10NlogN^2 > NlogN > N > log^2N > 5logN > logN^2 > logN > 37=2

5) In each of the following problems, you are given two algorithms that do the same job. You are required to indicate which of the two algorithms is "**better**" and why (30 points)

a. The following two algorithms count the number of values that are above the average of a given set of values.

AboveAvg1 is better. O(n) vs O(n^2).

Algorithms: AboveAvg1 & AboveAvg2
Input → L: List of integer values, N: size of the List
Output → C: Number of values above the average of all values

AboveAvg1 (L, N)	AboveAvg2 (L, N)

1. Set Avg = 0, count = 0, C = 0
2. For I = 0 to N - 1
  1- set count = count + L[I]
3. Set Avg = count/N
4. For I = 0 to N - 1
  1- IF (L[I] > Avg)
  1- Set C = C + 1
5. Return C
	1. Set Avg = 0, C = 0
	2. For J = 0 to N - 1
	  1- set count = 0
	  2- For I 0 to N - 1
	  1- set count = count + L[I]
	  3- Set Avg = count/N
	  4- IF (L[J] > Avg)
	  1- Set C = C + 1
	3. Return C

b. The following two algorithms take a list L and output another list M such that M contains the even numbers of L first then L's odd numbers.

EvensFirst2 is better. Omega(2n) vs Omega(n).

Algorithms: EvensFirst1 & EvensFirst2
Input → L: List of integer values, n: size of List L
Output → M: List of integers such that even numbers are first then odd numbers

EvensFirst1 (L, n, M)	EvensFirst2 (L, n, M)

1. J = 0
2. For I = 0 to n - 1
  1- IF (L[I] % 2 == 0) // Even
  a. M[J] = L[I]
  b. J = J+1
3. For I = 0 to n - 1
  2- IF (L[I] % 2 != 0) // Odd
  a. M[J] = L[I]
  b. J = J+1
4. Return M
	1. J = 0
	2. K = Size - 1
	3. For I = 0 to n - 1
	  1- IF (L[I] % 2 == 0) // Even
	  a. M[J] = L[I]
	  b. J = J+1
	  2- ELSE // Odd
	  a. M[K] = L[I]
	  b. K = K - 1
	4. Return M

Submit Instructions:

The homework must be turned in by the due date and time through GradeScope.

- It is preferred that you type in your answers and provide them as one PDF file for submission. Hand-written solutions are acceptable if they are very clear and easy to read. However, you will still need to scan and upload them as a single PDF file.
- On GradeScope you will be requested to assign each of the questions to pages on your PDF file. It is preferable that you solve each question in a separate page.
- GradeScope will open for submission 2-3 days before the deadline. Please watch for an announcement on Piazza.