## CS 251 Homework 1: Algorithms Analysis (100 points)

**Out:** January 11th, 2019 (8:00 pm) **Due:** January 18th, 2019 (8:00 pm)

**Important:** Each question has only one correct answer. Additionally, you must provide an **explanation** for your answer (2-3 short sentences is sufficient). Answers without proper explanations, even though it is correct, will be graded with 0 points.

1) Select the **tightest big-O** expression for the following pseudo codes (30 points)

```
1.1)
int sum = 0;
for(int i = 0; i < n; i++)
        for(int j = 0; j < n * n; j++)
                sum ++;
    A. O(n)
    B. O(n^2)
    C. O(n^3)
   D. O(n<sup>4</sup>)
1.2)
int sum = 0;
for(int i = 0; i < n; i++)
        for(int j = 0; j < i; j++)
                sum ++;
    A. O(n)
    B. O(n logn)
    C. O(n^2)
   D. O(n^2 \log n)
1.3)
int prod = 3;
for(int i = 1; i \le n; i = i * 2)
        for(int j = 0; j < n; j++)
                prod = prod * prod;
   A. O(n)
   B. O(n logn)
    C. O(n^2)
    D. O(n^2 \log n)
```

```
1.4)
int sum(n) {
       if(n == 1) {
              return 1;
       return n + sum(n - 1);
}
   A. O(n)
   B. O(n logn)
   C. O(n^2)
   D. O(n^2 \log n)
1.5)
count1=0; count2=0; count3=0;
for(i=0; i<n; i++)
       for(j=1; j<n; j*=2) {
              count1++;
              count2++;
              count3++;
       }
for(i=0; i<n; i++) {
       count1++;
       count2++;
       count3++;
}
   A. O(n)
   B. O(n logn)
   C. O(n^2)
   D. O(n^2 \log n)
```

- 2) Answer the following questions on the below expressions (15 points)
- 2.1) Choose tightest Big-O for:

$$n^3 + n! + 3^n + 2147483647 * log^3 n$$

В

- A.  $O(n^3)$
- B. *O(n!)*

C.  $O(3^n)$ 

D.  $O(2147483647 * log^3 n)$ 

## **2.2)** Choose **tightest Big-\Omega for**:

$$\sum_{i=0}^{2n} 5i + i^2$$

- A.  $\Omega(n^3)$
- B.  $\Omega(n^2)$
- C.  $\Omega(3^n)$
- D.  $\Omega(2147483647 * log^3 n)$
- 3) What is the **worst-case** amount of work to find a particular card in a deck of cards of size (n)? Note that cards are unique and are **not** sorted. (10 points)
- **4**) Order the following functions by growth rate. Indicate which functions grow at the same rate (15 points)
- N, N<sup>2</sup>, log N, N log N, log(N<sup>2</sup>), log<sup>2</sup> N, N log<sup>2</sup> N, 2,  $2^N$ , 37, N<sup>2</sup> log N, 5log N, N log N<sup>2</sup>

 $2^N - N^3 - N^2 \log N > N^2 > N \log 2N > 10N \log N^2 > N \log N > N > \log^2 N > \log N^2 > \log N^2 > \log N > 37=2$ 

- **5)** In each of the following problems, you are given two algorithms that do the same job. You are required to indicate which of the two algorithms is "**better**" and why (30 points)
- a. The following two algorithms count the number of values that are above the average of a given set of values.

AboveAvg1 is better. O(n) vs O(n^2).

Algorithms: AboveAvg1 & AboveAvg2

**Input** → L: List of integer values, N: size of the List

Output → C: Number of values above the average of all values

AboveAvg1 (L, N)	AboveAvg2 (L, N)
<pre>1. Set Avg = 0, count = 0, C = 0 2. For I = 0 to N - 1</pre>	<pre>1. Set Avg = 0, C = 0 2. For J = 0 to N - 1</pre>

b. The following two algorithms take a list L and output another list M such that M contains the even numbers of L first then L's odd numbers.

**EvensFirst2** is better. Omega(2n) vs Omega(n).

Algorithms: EvensFirst1 & EvensFirst2

**Input** → L: List of integer values, n: size of List L

Output → M: List of integers such that even numbers are first then odd numbers

a. $M[J] = L[I]$ 1- 1	t2 (L, n, M)
b. J = J+1 3. For I = 0 to n - 1 2- IF ( L[I] % 2 != 0 ) // Odd a. M[J] = L[I] b. J = J+1 4. Return M 4. Return	= 0 to n - 1 F ( L[I] % 2 == 0 ) // Even a. M[J] = L[I] b. J = J+1 LSE // Odd a. M[K] = L[I] b. K = K - 1

## **Submit Instructions:**

The homework must be turned in by the due date and time through GradeScope.

- It is preferred that you type in your answers and provide them as one PDF file for submission. Hand-written solutions are acceptable if they are very clear and easy to read. However, you will still need to scan and upload them as a single PDF file.
- On GradeScope you will be requested to assign each of the questions to pages on your PDF file. It is preferable that you solve each question in a separate page.
- GradeScope will open for submission 2-3 days before the deadline. Please watch for an announcement on Piazza.