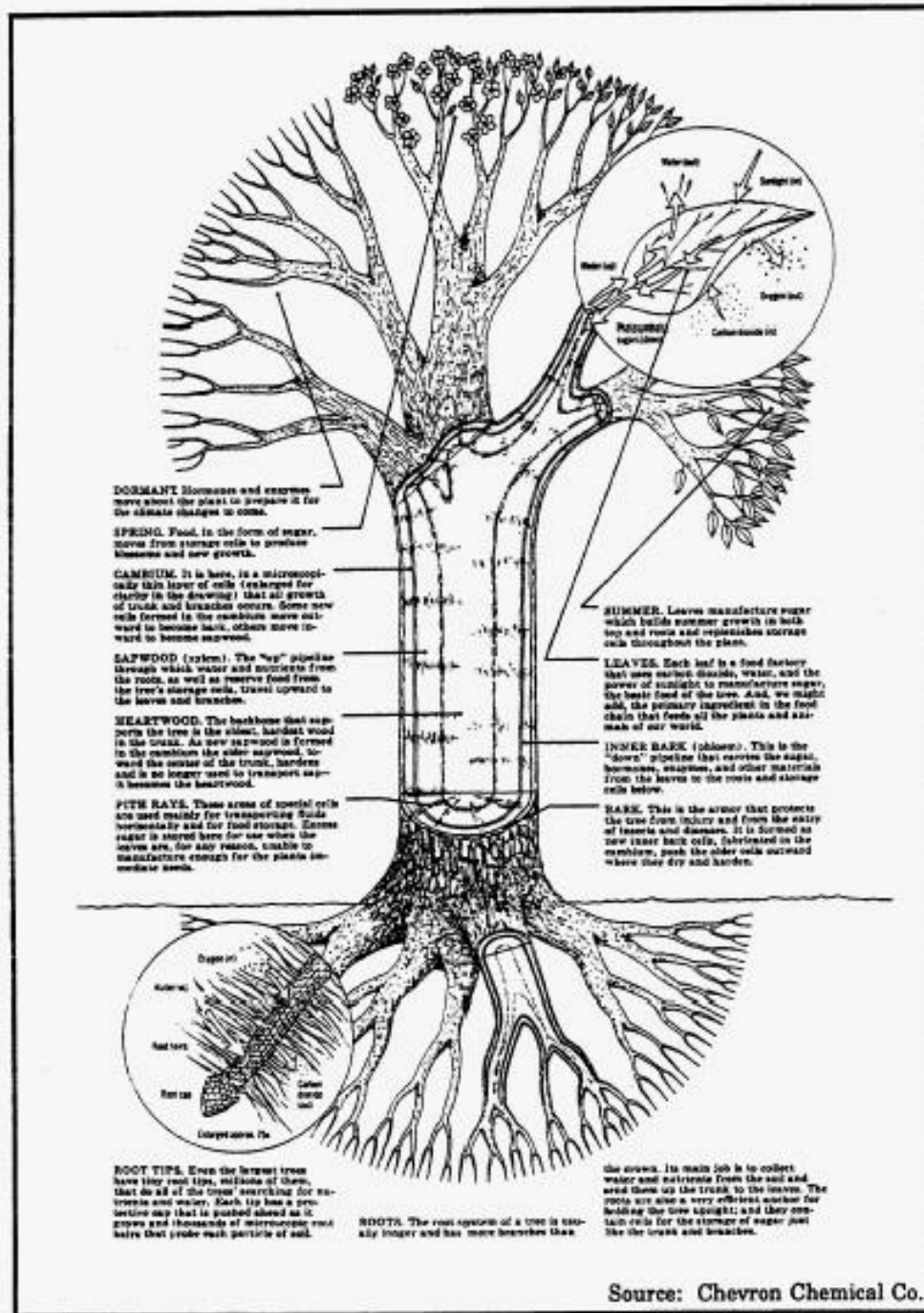
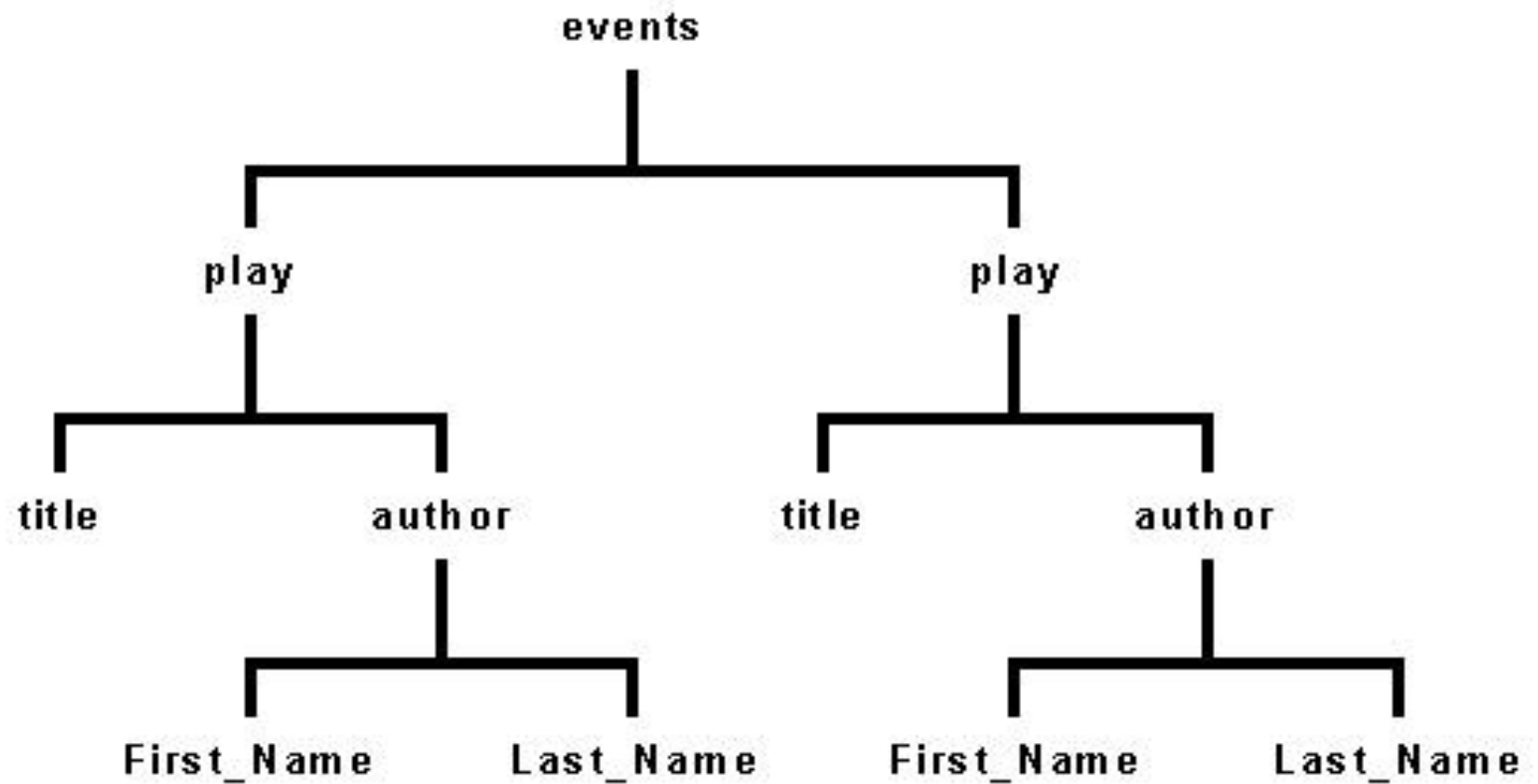


Tree

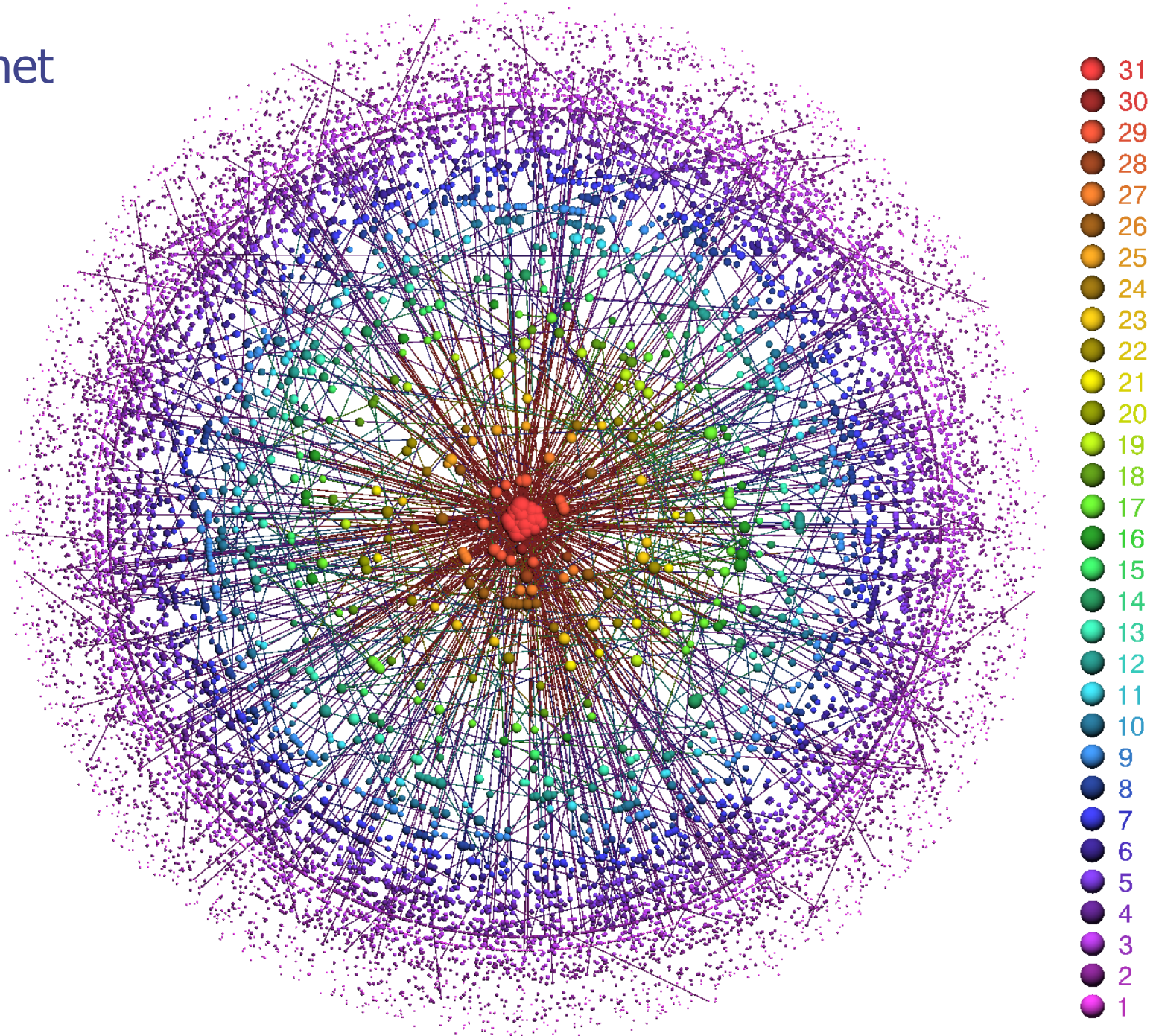


Source: Chevron Chemical Co.

Tree



Internet



Organizational chart

Drawn from Official Register of the United States (1925); Administrative History: Expansion of the National Park Service in the 1930's (1983)

NATIONAL PARK SERVICE
ORGANIZATION IN: 1925

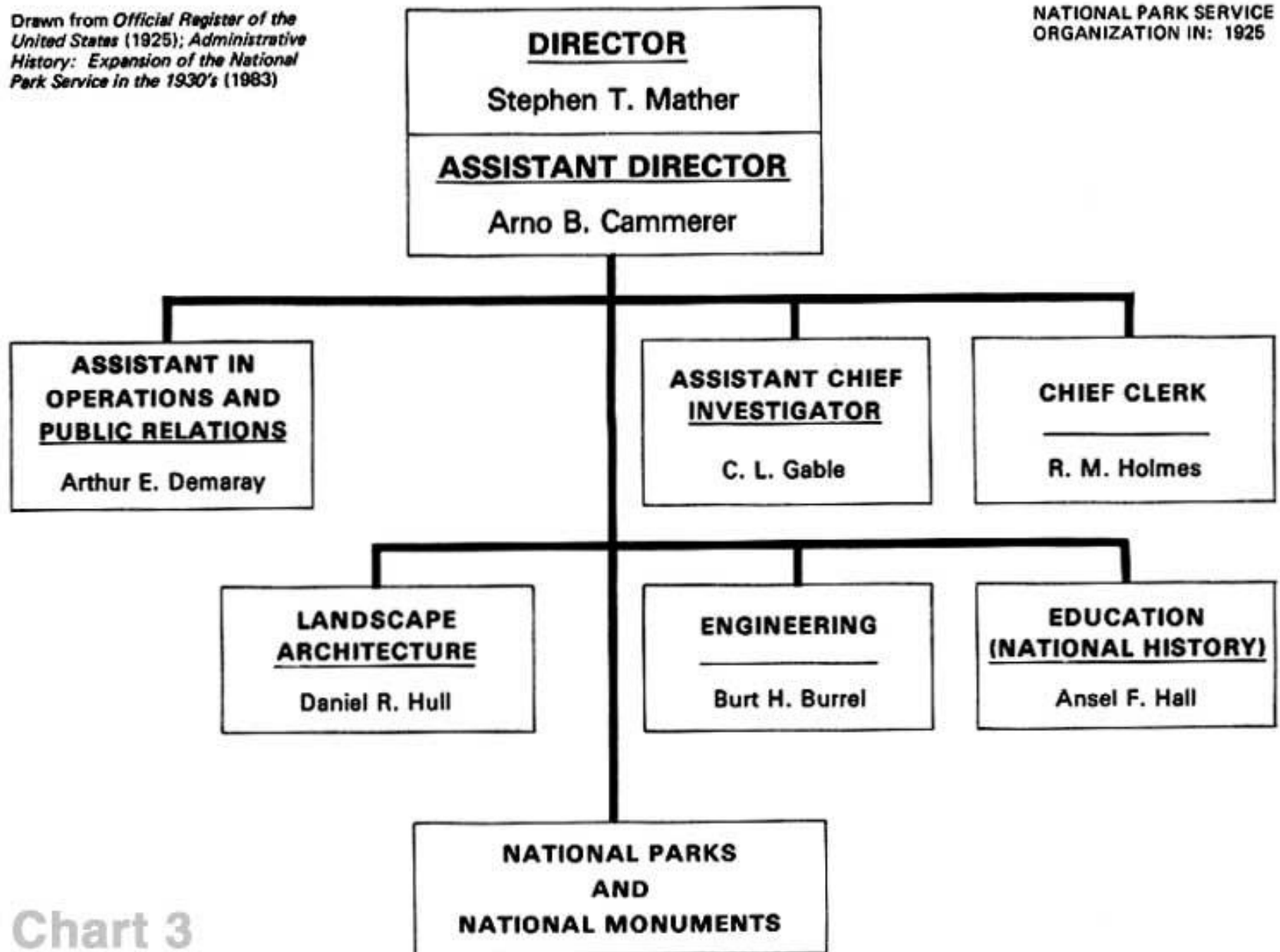
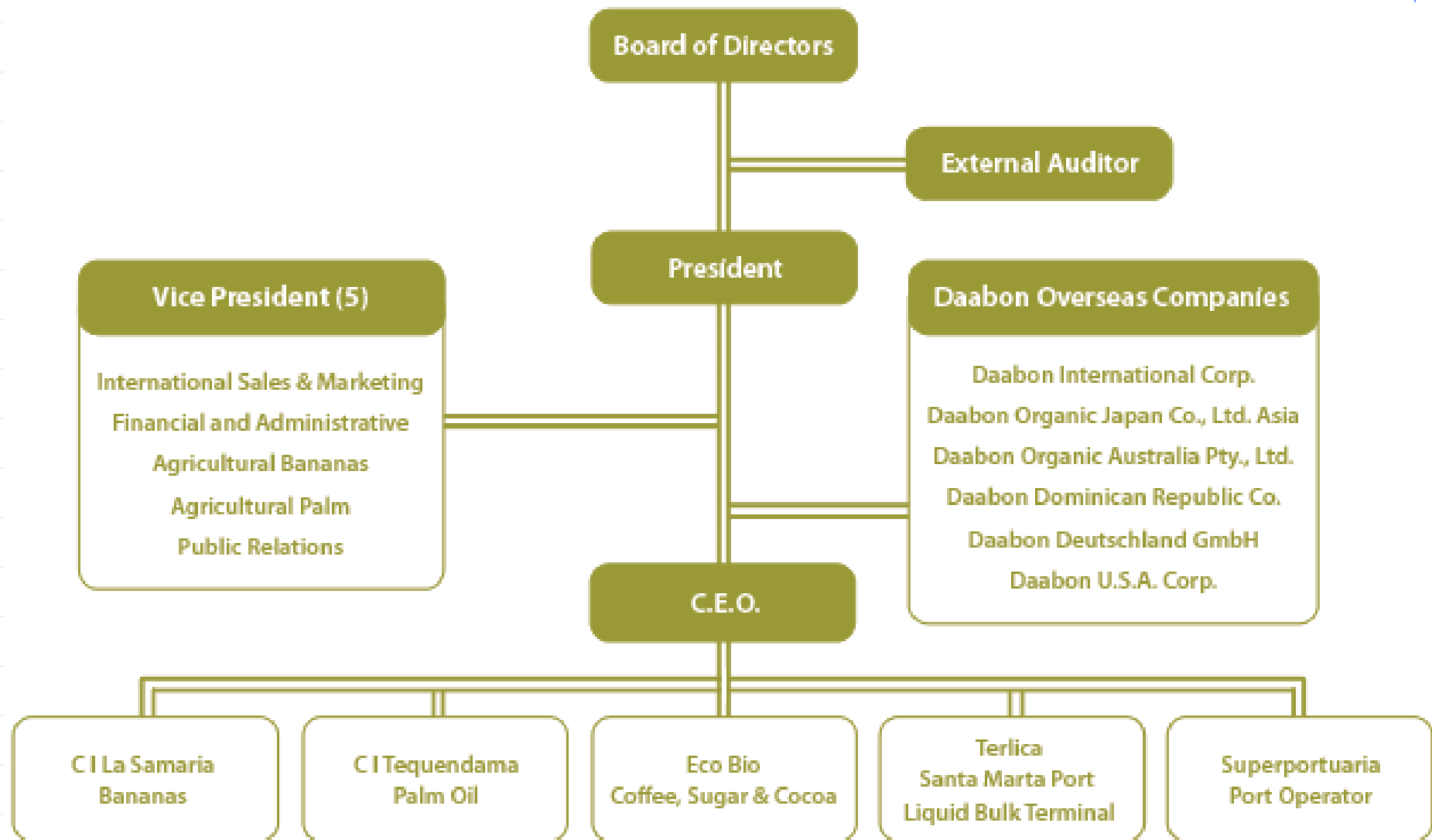
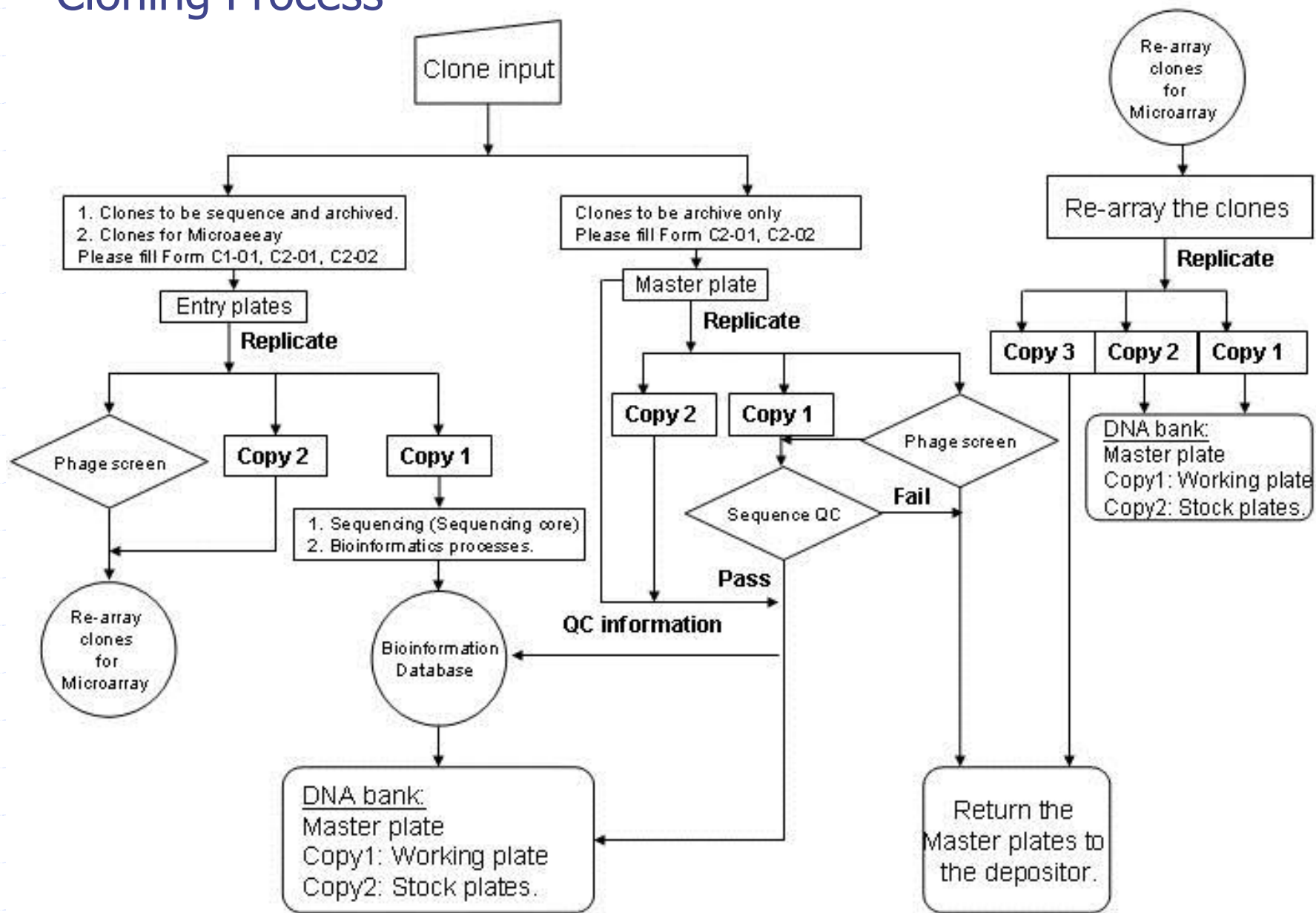


Chart 3

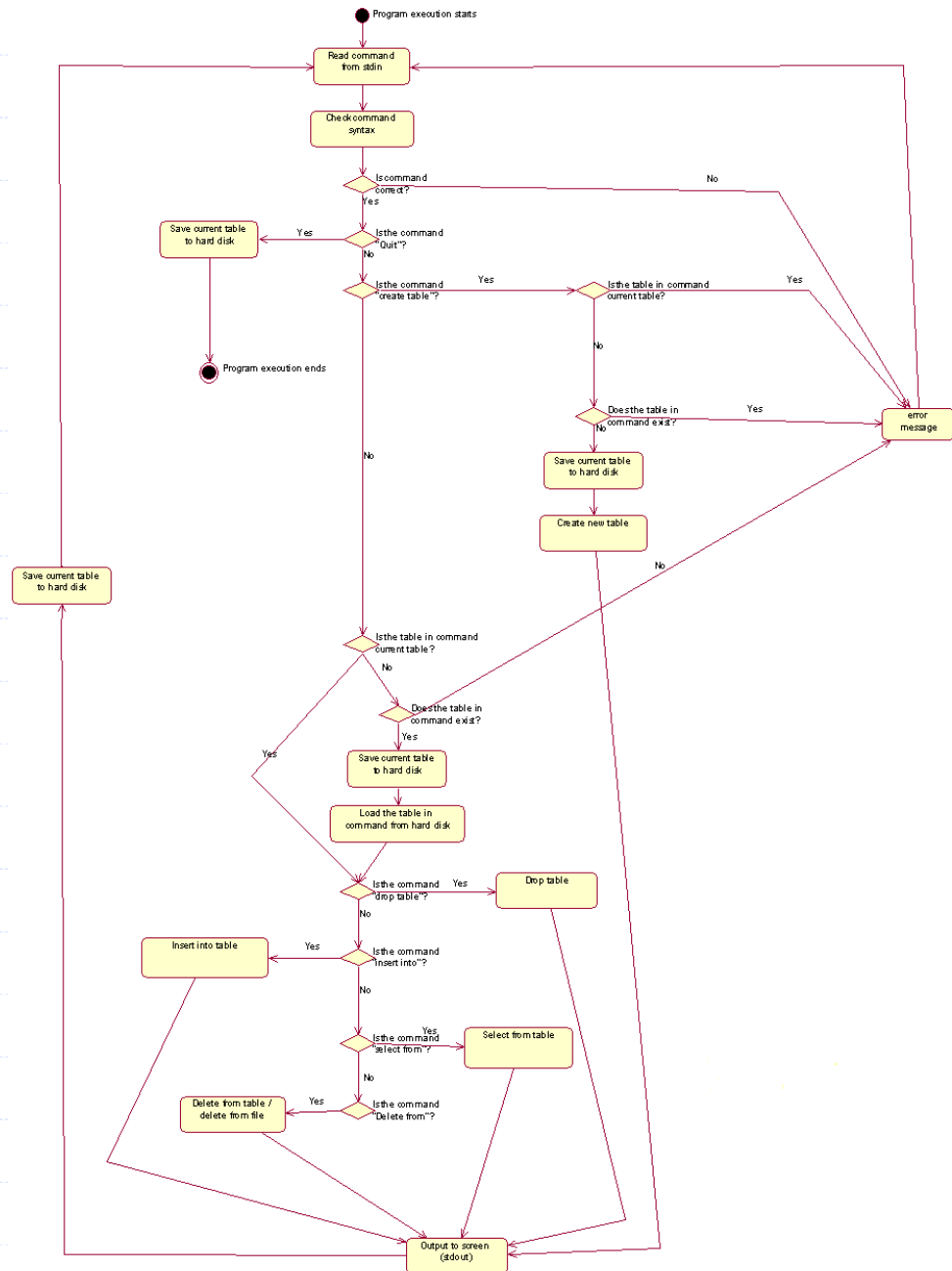
Organizational Chart



Cloning Process



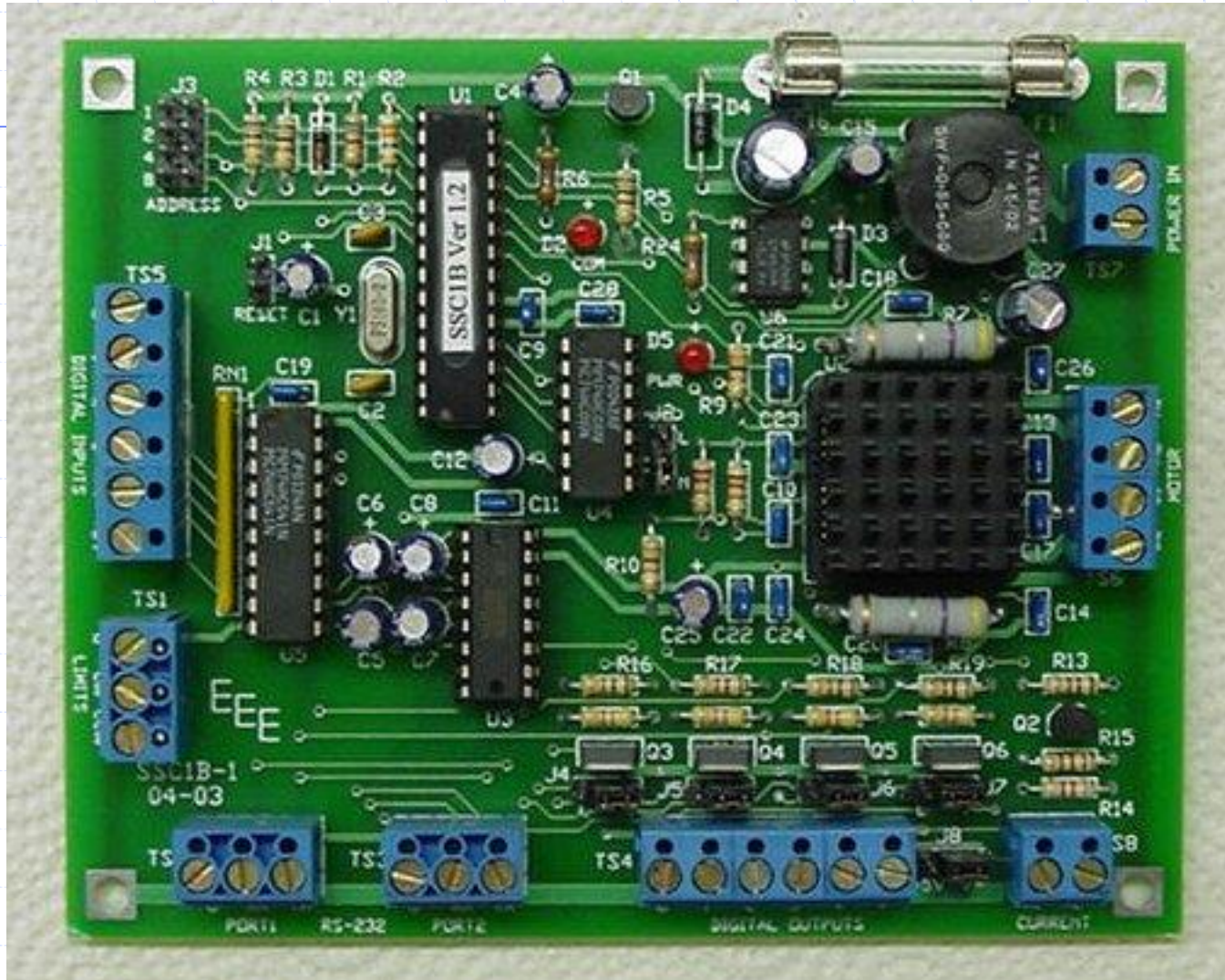
Program Flow Chart



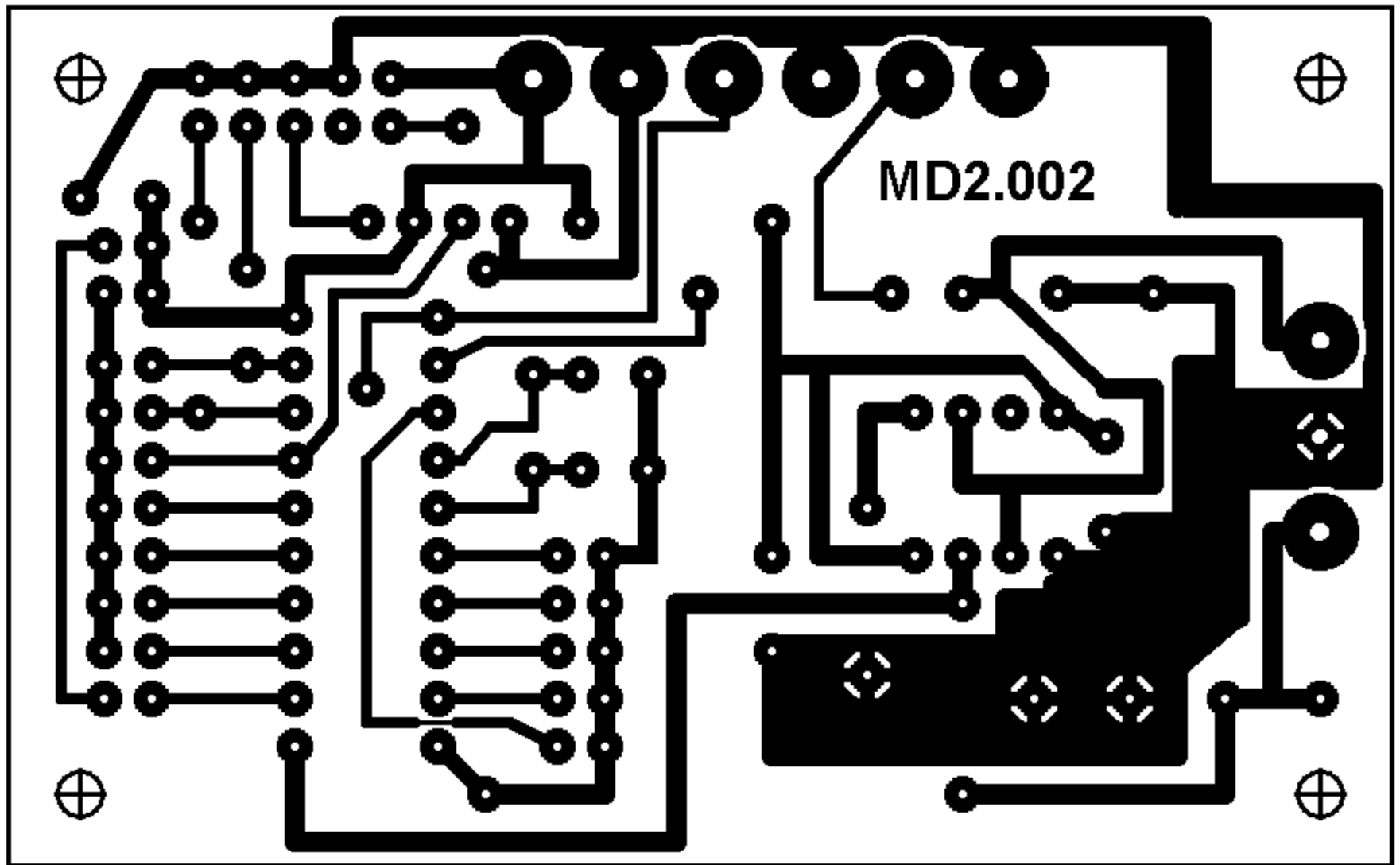
Flight Routes



Electronic Circuit Board



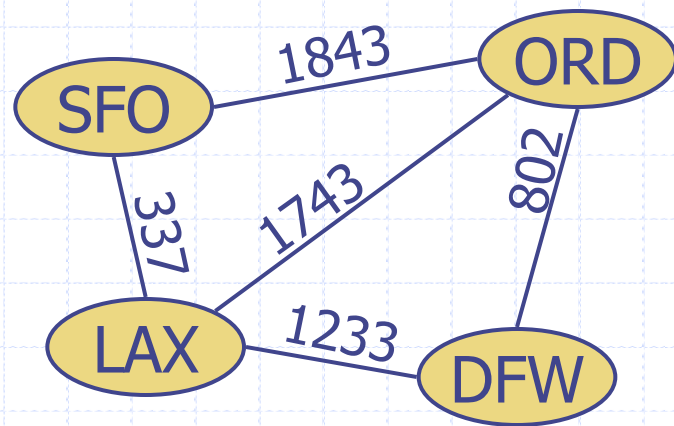
Electronic Circuit Board



What do these have in common?

◆ They are graphs...

Graphs



Outline and Reading

◆ Graphs

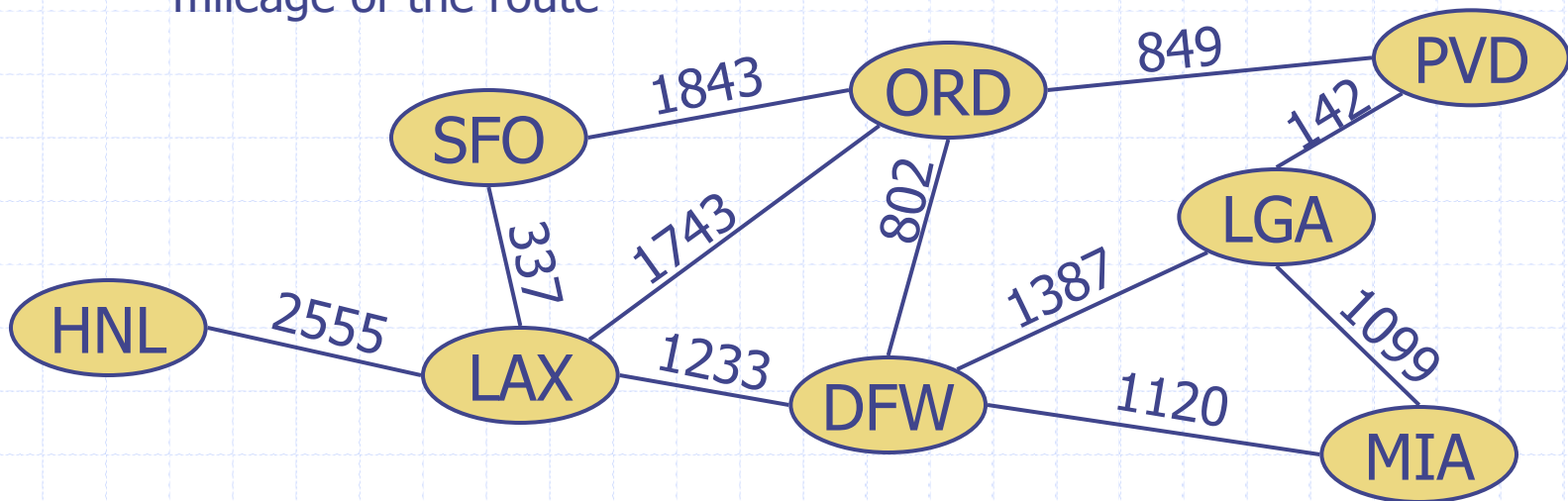
- Definition
- Applications
- Terminology
- Properties
- ADT

◆ Data structures for graphs

- Edge list structure
- Adjacency list structure
- Adjacency matrix structure

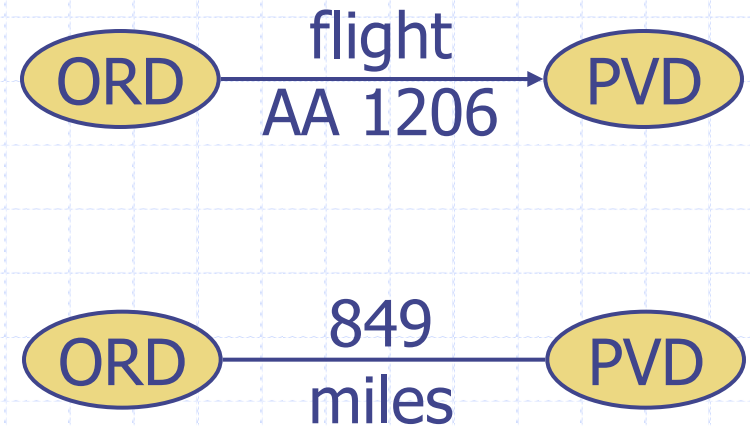
Graph

- ◆ A graph is a pair (V, E) , where
 - V is a set of nodes, called **vertices**
 - E is a collection of pairs of vertices, called **edges**
 - Vertices and edges are positions and store elements
- ◆ Example:
 - A vertex represents an airport and stores the three-letter airport code
 - An edge represents a flight route between two airports and stores the mileage of the route



Edge Types

- ◆ Directed edge
 - ordered pair of vertices (u,v)
 - first vertex u is the origin
 - second vertex v is the destination
 - e.g., a flight
- ◆ Undirected edge
 - unordered pair of vertices (u,v)
 - e.g., a flight route
- ◆ Directed graph
 - all the edges are directed
 - e.g., route network
- ◆ Undirected graph
 - all the edges are undirected
 - e.g., flight network



Applications

◆ Computer networks

- Local area network
- Internet
- Web

◆ Electronic circuits

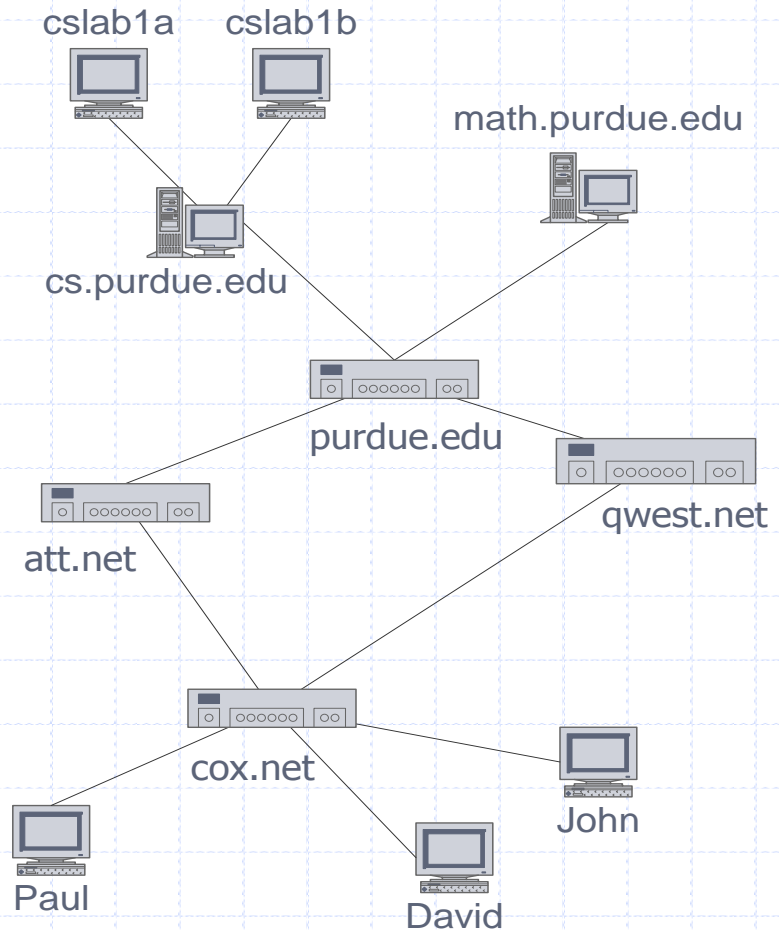
- Printed circuit board
- Integrated circuit

◆ Transportation networks

- Highway network
- Flight network

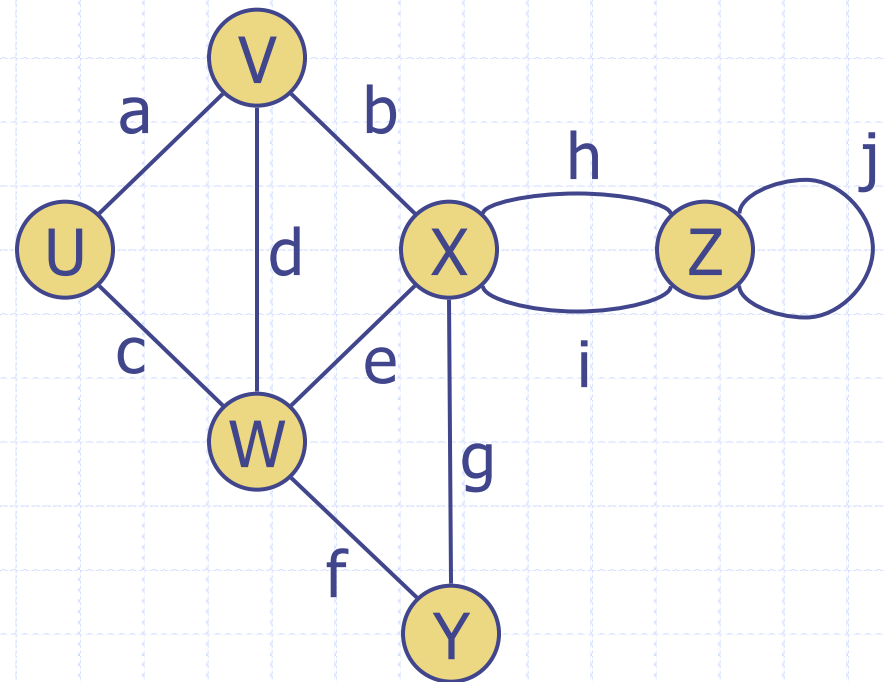
◆ Databases

- Entity-relationship diagram



Terminology

- ◆ End vertices (or endpoints) of an edge
 - U and V are the *endpoints* of an edge
- ◆ Edges incident on a vertex
 - a, d, and b are *incident* on V
- ◆ Adjacent vertices
 - U and V are *adjacent*
- ◆ Degree of a vertex
 - X has *degree* 5
- ◆ Parallel edges
 - h and i are *parallel edges*
- ◆ Self-loop
 - j is a *self-loop*



Terminology (cont.)

◆ Path

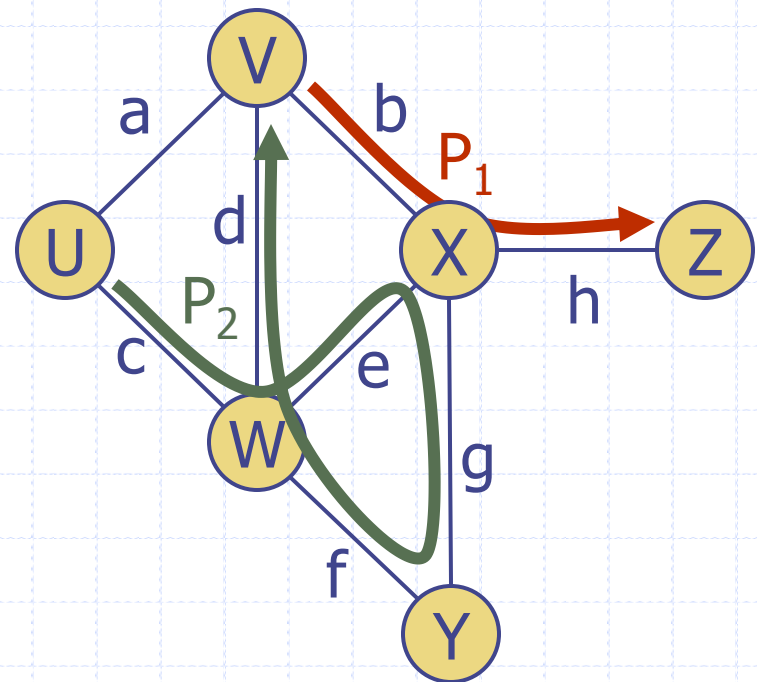
- sequence of alternating vertices and edges
- begins with a vertex
- ends with a vertex
- each edge is preceded and followed by its endpoints

◆ Simple path

- path such that all its vertices and edges are distinct

◆ Examples

- $P_1 = (V, b, X, h, Z)$ is a simple path
- $P_2 = (U, c, W, e, X, g, Y, f, W, d, V)$ is a path that is not simple



Terminology (cont.)

◆ Cycle

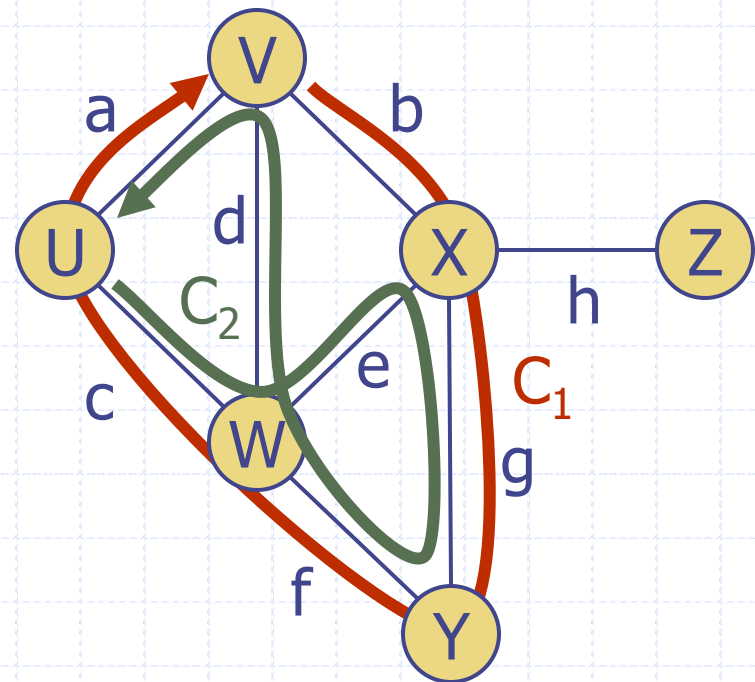
- circular sequence of alternating vertices and edges
- each edge is preceded and followed by its endpoints

◆ Simple cycle

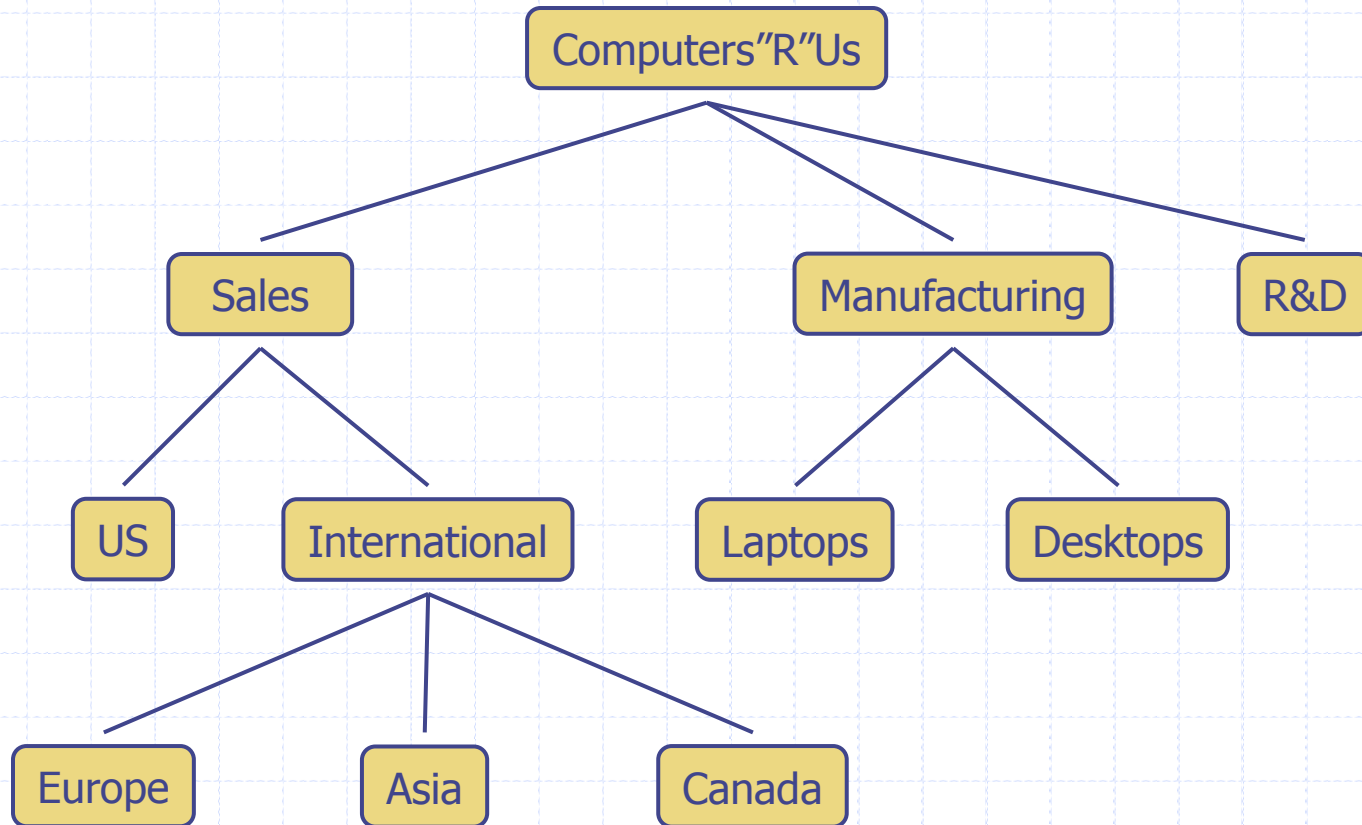
- cycle such that all its vertices and edges are distinct

◆ Examples

- $C_1 = (V, b, X, g, Y, f, W, c, U, a, \downarrow)$ is a simple cycle
- $C_2 = (U, c, W, e, X, g, Y, f, W, d, V, a, \downarrow)$ is a cycle that is not simple



Trees: Revisited



Trees: Revisited

- ◆ A tree is a graph
- ◆ Using tree terminology,
 - What are the vertices of a tree called?
 - ◆ Nodes
 - What are the edges of a tree called?
 - ◆ Edges (or pointers?)
 - Is a tree “directed” or “undirected”
 - ◆ You could say:
 - **Directed:** have parent->child pointers
 - **Undirected:** have parent->child and child->parent pointers

Trees: Revisited

◆ Using tree terminology,

- What is the degree of the vertices of a binary tree?
 - ◆ 3
- What is an example path through a tree?
 - ◆ A pre/in/post-order traversal
- What is an example cycle through a tree?
 - ◆ Trick question:
 - if you consider the tree a “directed graph”, then no cycles
 - Otherwise, a pre/in/post-order traversal yields a cycle

Properties

Property 1: Degree sum

$$\sum_n \deg(v_n) = 2m \quad \text{Why?}$$

Proof: each edge is counted twice

Property 2: Edge count bound

In an undirected graph with no self-loops and no multiple edges

$$m \leq n(n-1)/2$$

Proof: each vertex has degree at most $(n-1)$

Notation

n

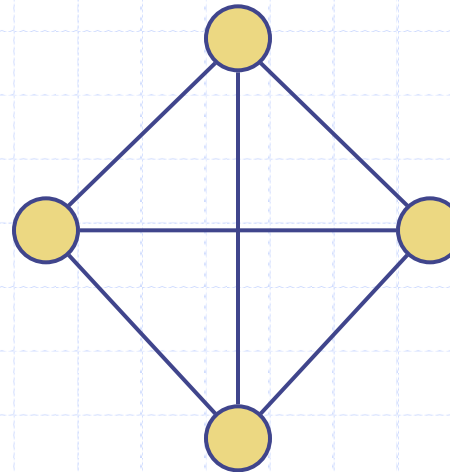
number of vertices

m

number of edges

$\deg(v)$

degree of vertex v



Example

- $n = 4$
- $m = 6$
- $\deg(v) = 3$

Main Methods of the Graph ADT

◆ Vertices and edges

- are positions
- store elements

◆ Accessor methods

- `aVertex()`
- `incidentEdges(v)`
- `endVertices(e)`
- `isDirected(e)`
- `origin(e)`
- `destination(e)`
- `opposite(v, e)`
- `areAdjacent(v, w)`

◆ Update methods

- `insertVertex(o)`
- `insertEdge(v, w, o)`
- `insertDirectedEdge(v, w, o)`
- `removeVertex(v)`
- `removeEdge(e)`

◆ Generic methods

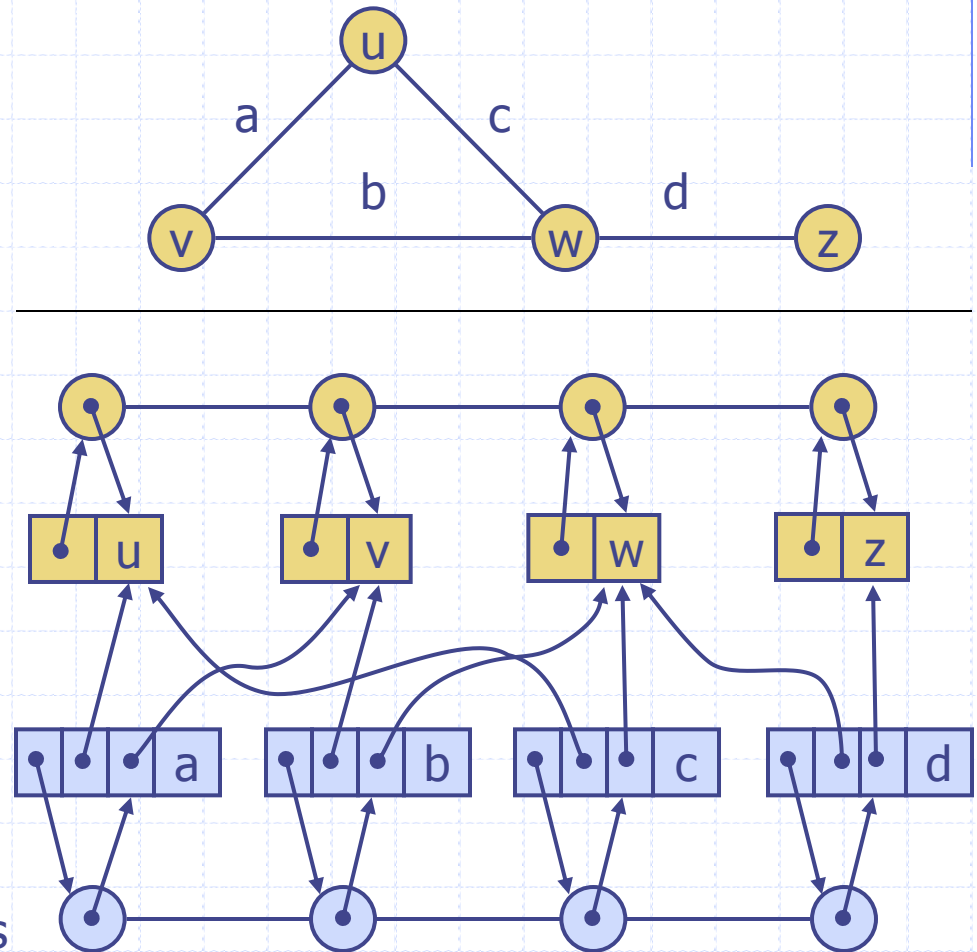
- `numVertices()`
- `numEdges()`
- `vertices()`
- `edges()`

Data Representation

- ◆ How do you store a graph's vertices, edges and connectivity?

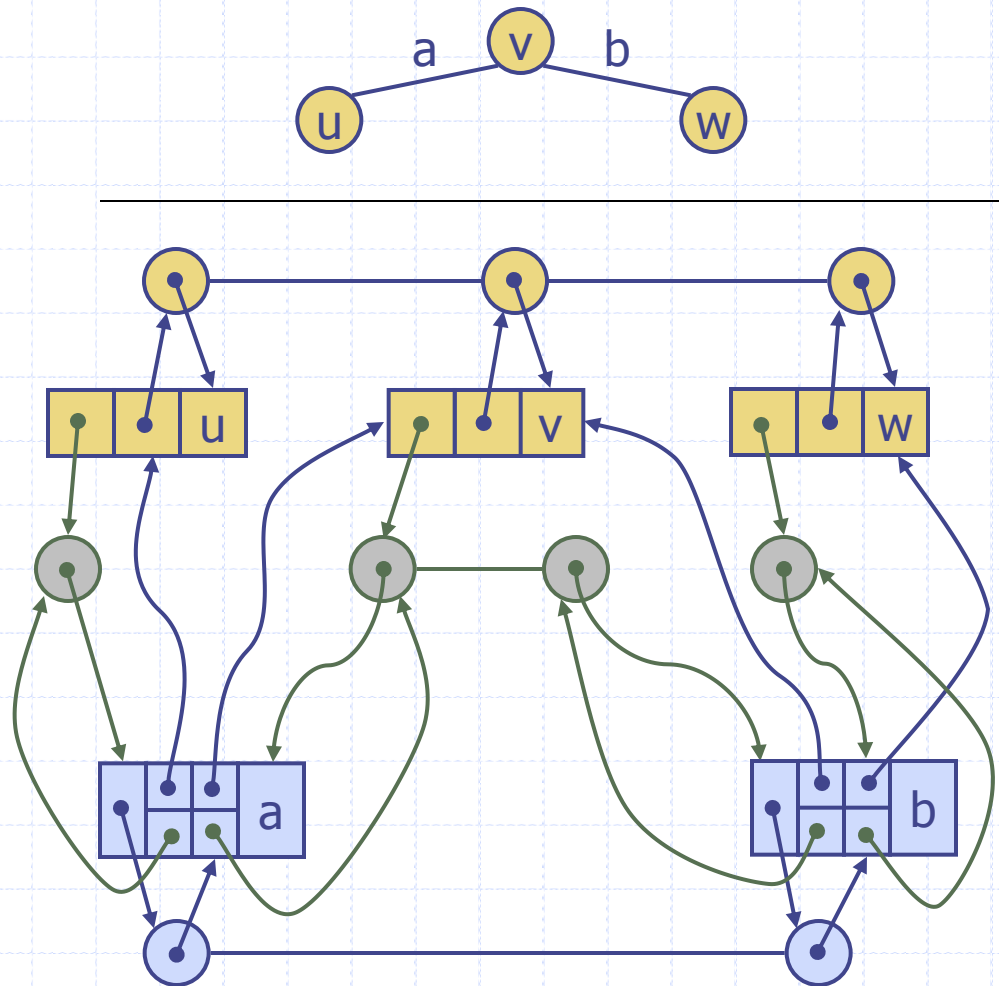
Edge List Structure

- ◆ Vertex object
 - element
 - reference to position in vertex sequence
- ◆ Edge object
 - element
 - origin vertex object
 - destination vertex object
 - reference to position in edge sequence
- ◆ Vertex sequence
 - sequence of vertex objects
- ◆ Edge sequence
 - sequence of edge objects



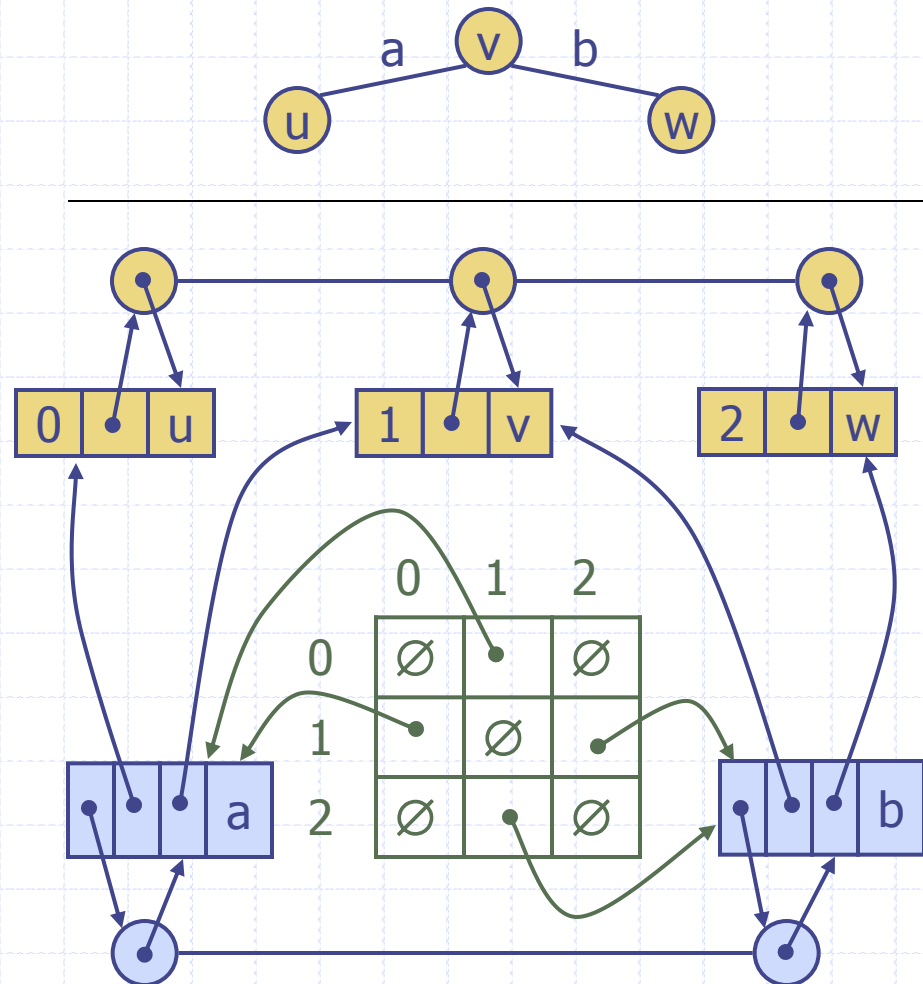
Or, Adjacency List Structure

- ◆ Edge list structure
- ◆ Incidence sequence for each vertex
 - sequence of references to edge objects of incident edges
- ◆ Augmented edge objects
 - references to associated positions in incidence sequences of end vertices



Or, Adjacency Matrix Structure

- ◆ Edge list structure
- ◆ Augmented vertex objects
 - Integer key (index) associated with vertex
- ◆ 2D-array adjacency array
 - Reference to edge object for adjacent vertices
 - Null for non adjacent vertices
- ◆ The “old fashioned” version just has 0 for no edge and 1 for edge



Asymptotic Performance

| | |
|--|-----------|
| ◆ n vertices, m edges ◆ no parallel edges ◆ no self-loops ◆ Bounds are “big-Oh” | Edge List |
| Space | $n + m$ |
| incidentEdges(v) | m |
| areAdjacent (v, w) | m |
| insertVertex(o) | 1 |
| insertEdge(v, w, o) | 1 |
| removeVertex(v) | m |
| removeEdge(e) | 1 |

Asymptotic Performance

| | | |
|--|-----------|--------------------------|
| <ul style="list-style-type: none"> ◆ n vertices, m edges ◆ no parallel edges ◆ no self-loops ◆ Bounds are “big-Oh” | Edge List | Adjacency List |
| Space | $n + m$ | $n + m$ |
| incidentEdges(v) | m | $\deg(v)$ |
| areAdjacent (v, w) | m | $\min(\deg(v), \deg(w))$ |
| insertVertex(o) | 1 | 1 |
| insertEdge(v, w, o) | 1 | 1 |
| removeVertex(v) | m | $\deg(v)$ |
| removeEdge(e) | 1 | 1 |

Asymptotic Performance

| | | | |
|--|-----------|--------------------------|------------------|
| <ul style="list-style-type: none"> ◆ n vertices, m edges ◆ no parallel edges ◆ no self-loops ◆ Bounds are “big-Oh” | Edge List | Adjacency List | Adjacency Matrix |
| Space | $n + m$ | $n + m$ | n^2 |
| incidentEdges (v) | m | $\deg(v)$ | n |
| areAdjacent (v, w) | m | $\min(\deg(v), \deg(w))$ | 1 |
| insertVertex (o) | 1 | 1 | n^2 |
| insertEdge (v, w, o) | 1 | 1 | 1 |
| removeVertex (v) | m | $\deg(v)$ | n^2 |
| removeEdge (e) | 1 | 1 | 1 |