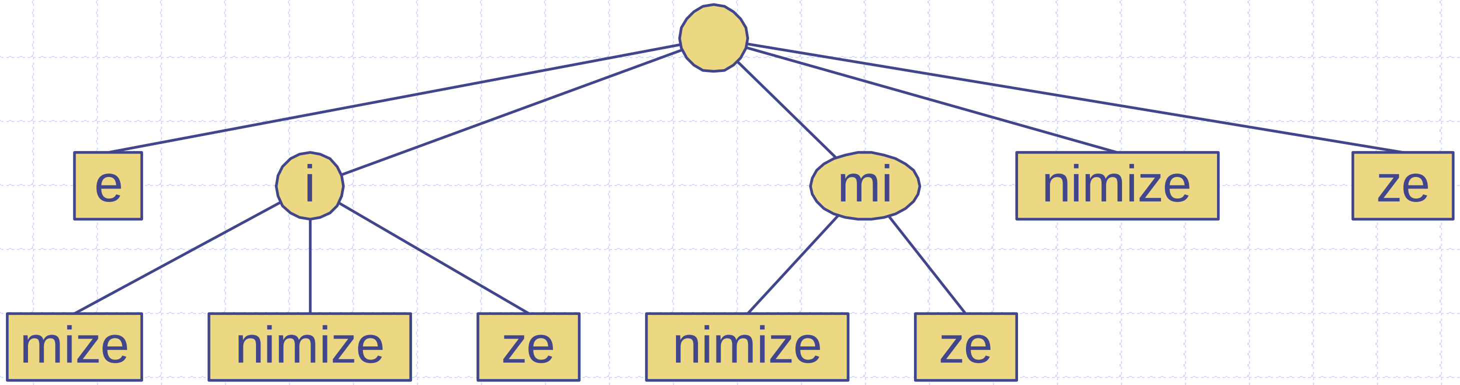
Tries



Outline

- Standard tries
- Compressed tries
- Suffix tries
- Huffman encoding tries

Where does "trie" come from?

- From the word re**trie**val
- Introduced in the 1960's

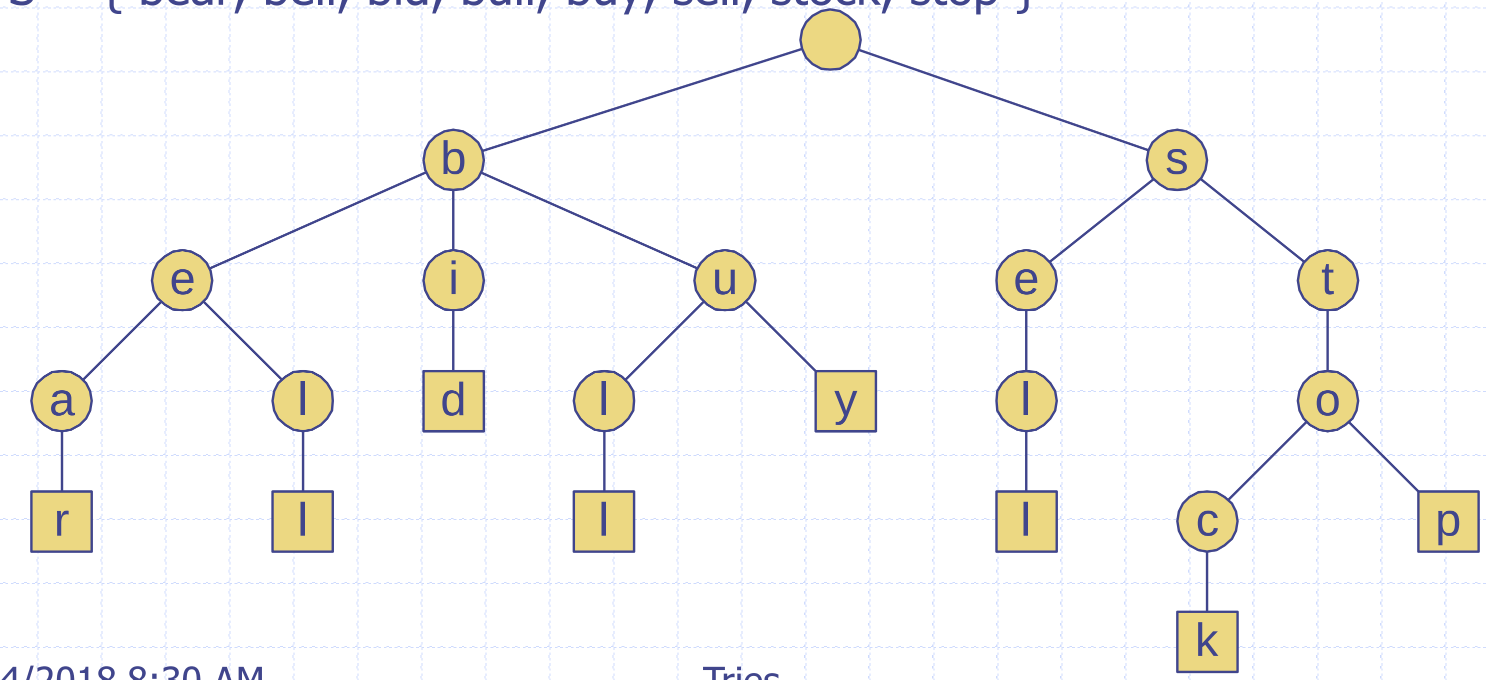
Preprocessing Strings

- Preprocessing the pattern speeds up pattern matching queries
  - After preprocessing the pattern, KMP's algorithm performs pattern matching in time proportional to the text size
- If the text is large, immutable and searched for often (e.g., works by Shakespeare), we may want to preprocess the text instead of the pattern
  - Thus do <u>better</u> than O(n+m) for text of size n and pattern of size m
- A trie is a compact data structure for representing a set of strings, such as all the words in a text
  - A tries supports pattern matching queries in time proportional to the <u>pattern size</u> (~O(m))!

Standard Trie

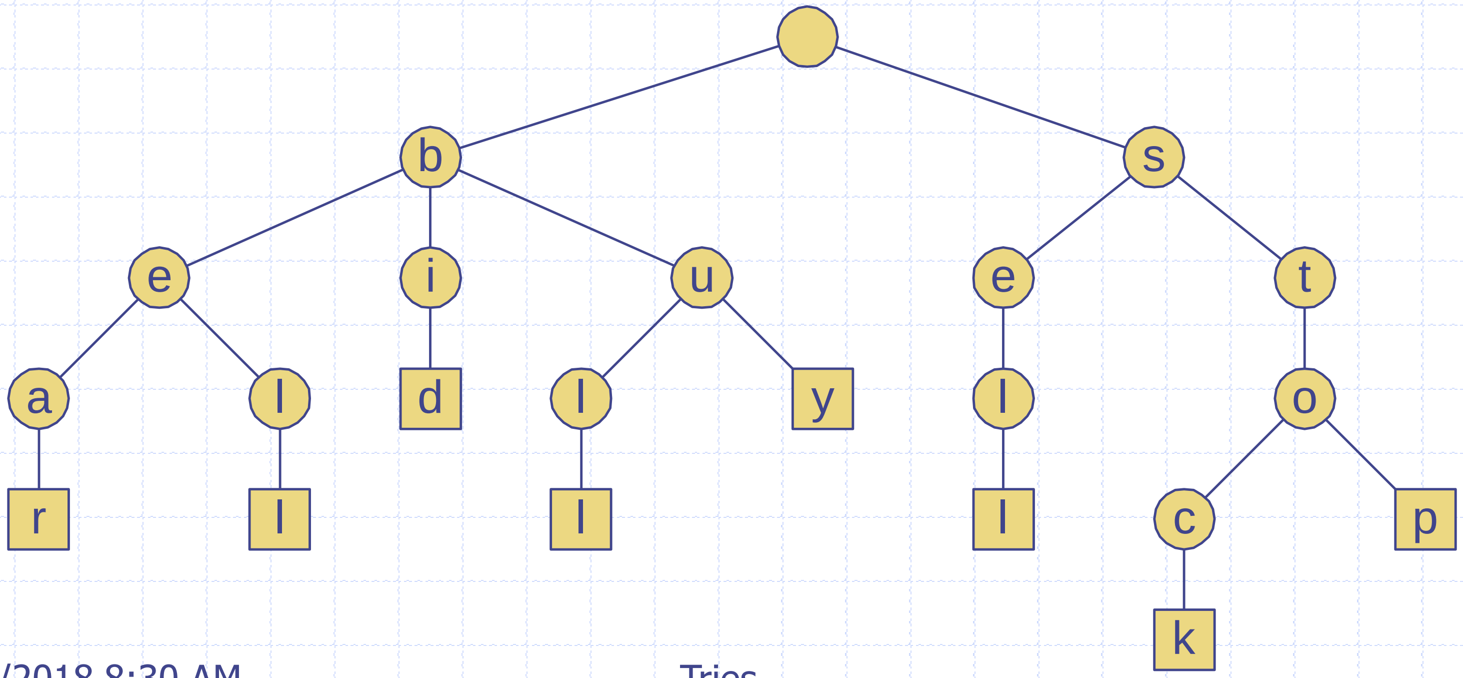
- The standard trie for a set of strings S is an ordered tree such that:
  - Each node but the root is labeled with a character
  - The children of a node are alphabetically ordered
  - The paths from the external nodes to the root yield the strings of S
- Example: standard trie for the set of strings

  S = { bear, bell, bid, bull, buy, sell, stock, stop }



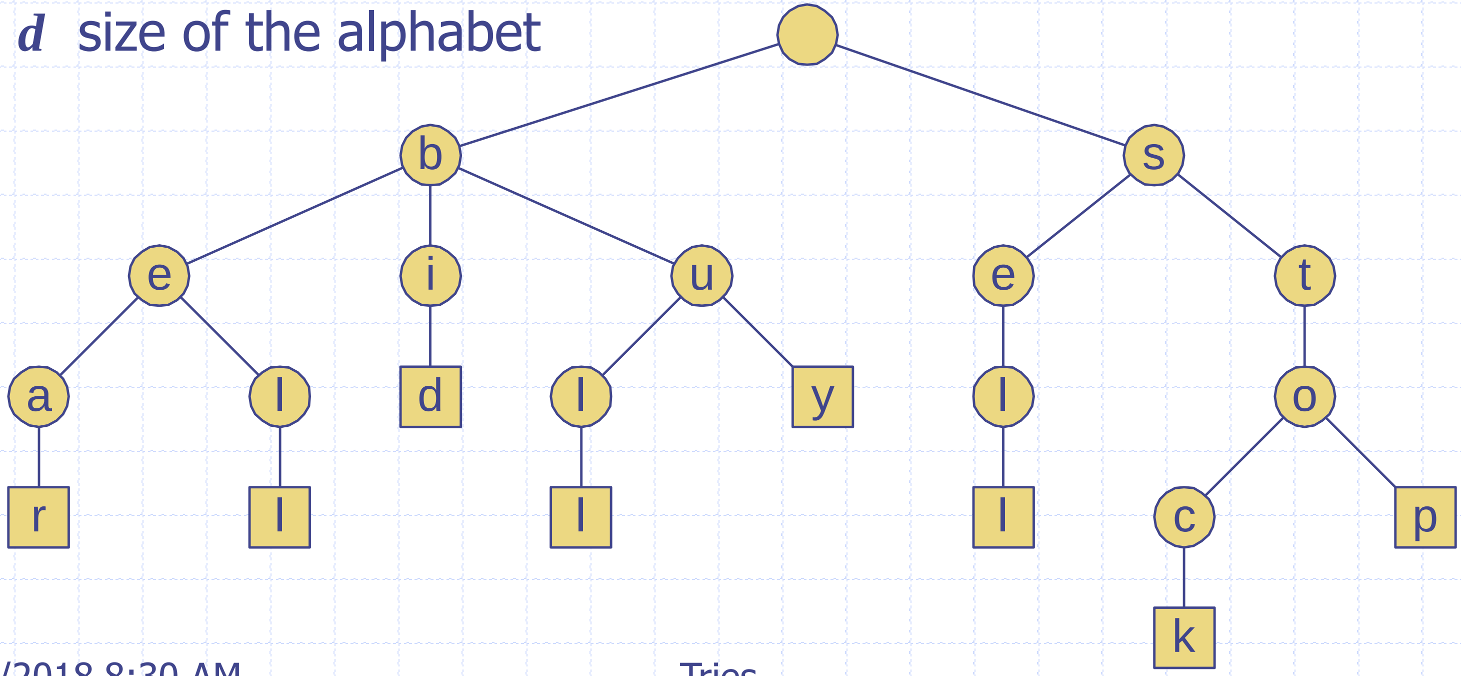
Standard Trie

- What space does the trie use?
- What is the maximum height of the tree?



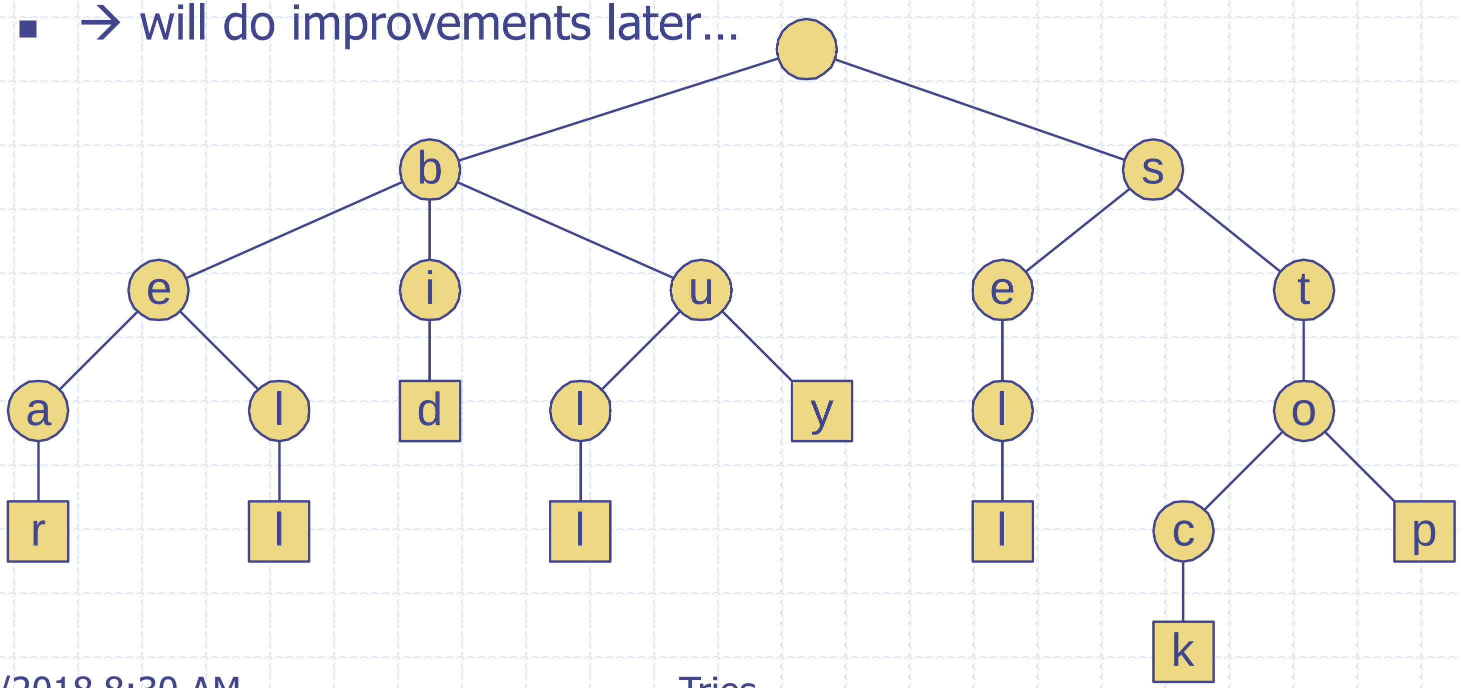
Standard Trie

- A standard trie uses $O(n)$ space and supports searches, insertions and deletions in time $O(dm)$, where:
  - n total size of the strings in S
  - m size of the (maximum) string parameter of the operation
  - d size of the alphabet



Standard Trie

- When is n, and thus space, maximum?
  - When S consists of mutually unique words with no letters in common
- What type of word(s) produces the largest search time?
  - Short word? Long word?
  - Answer: long words, especially those whose prefix is very common
  - → will do improvements later…



Word Matching with a Trie

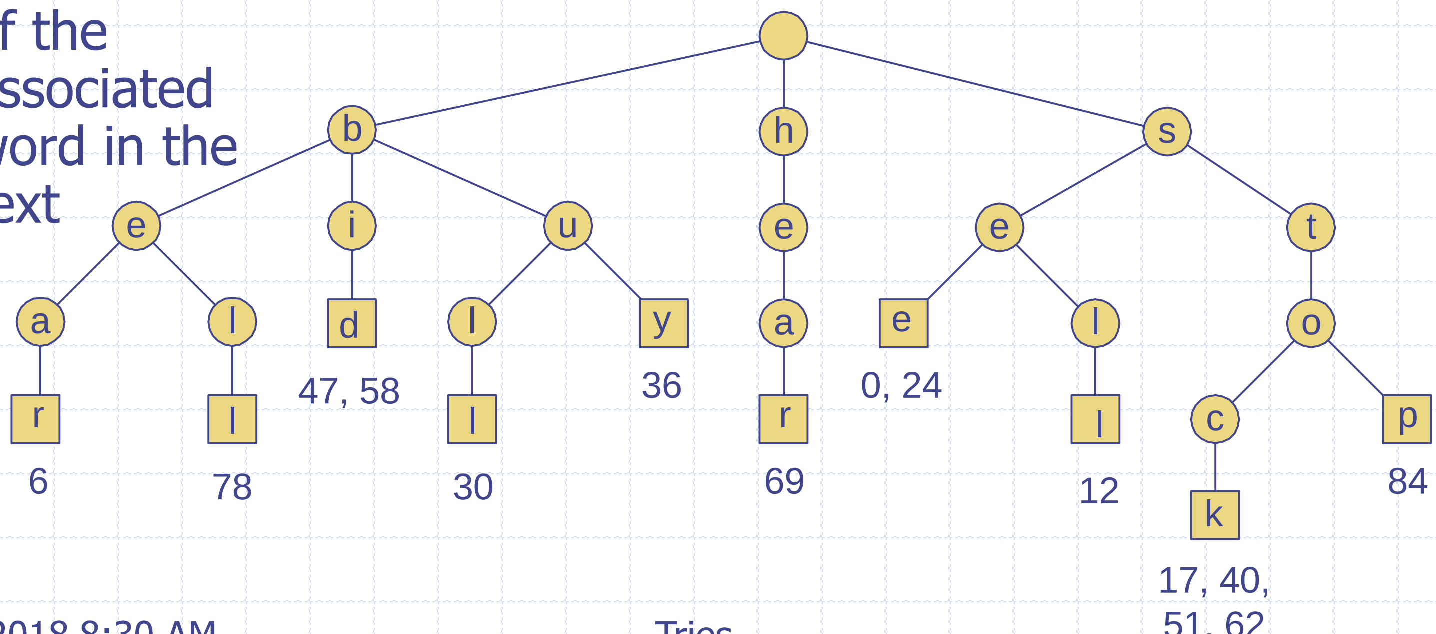
- We insert the words of the text into a trie
- Each leaf stores the occurrences of the associated word in the text

s	e	e		a		b	e	a	r	?		s	e	l	l		s	t	o	c	k	!	
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23

s	e	e		a		b	u	l	l	?		b	u	y		s	t	o	c	k	!	
24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46

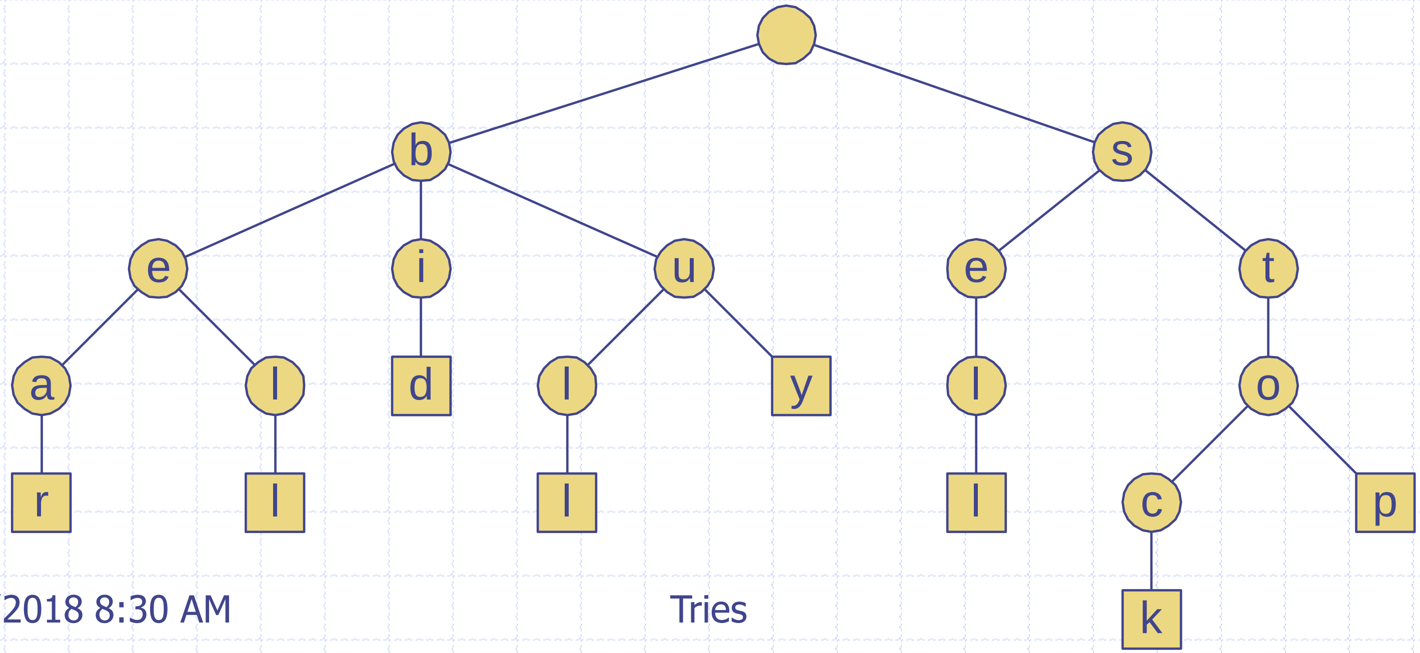
b	i	d		s	t	o	c	k	!		b	i	d		s	t	o	c	k	!	
47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68

h	e	a	r		t	h	e		b	e	l	l	?		s	t	o	p	!
69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88



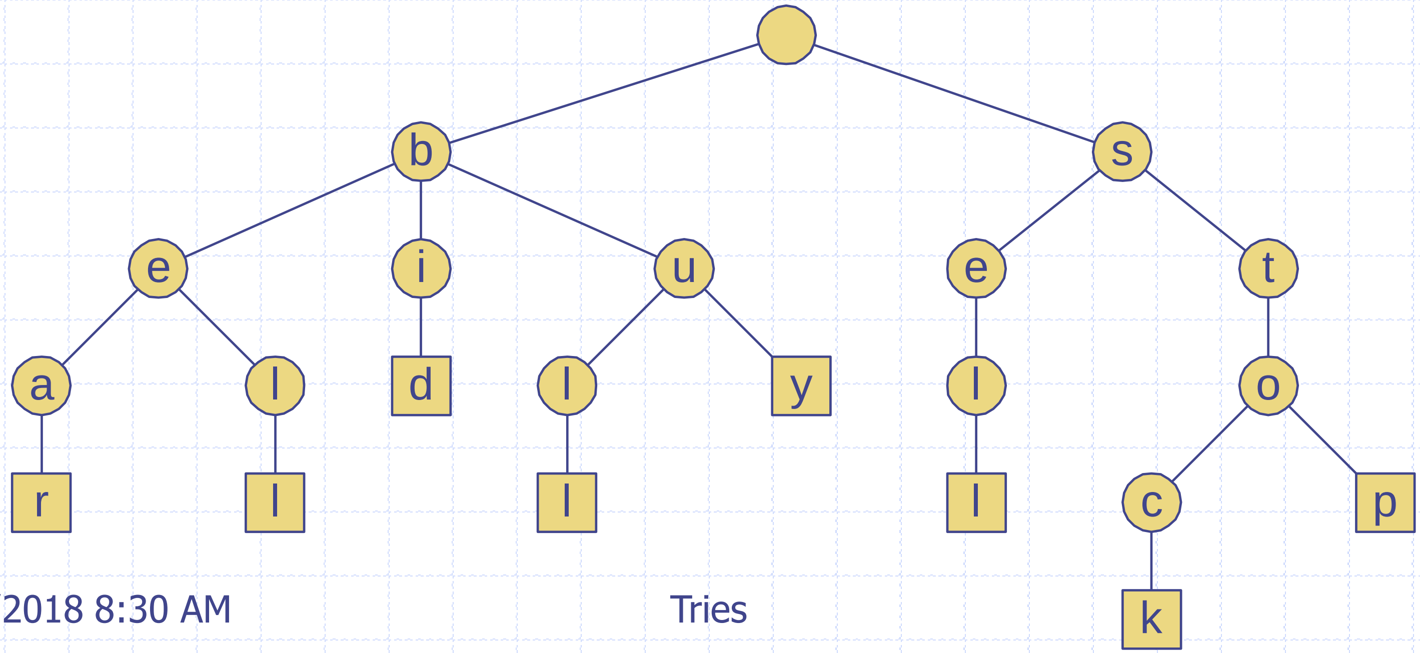
Standard Trie Construction

- How do you build it?



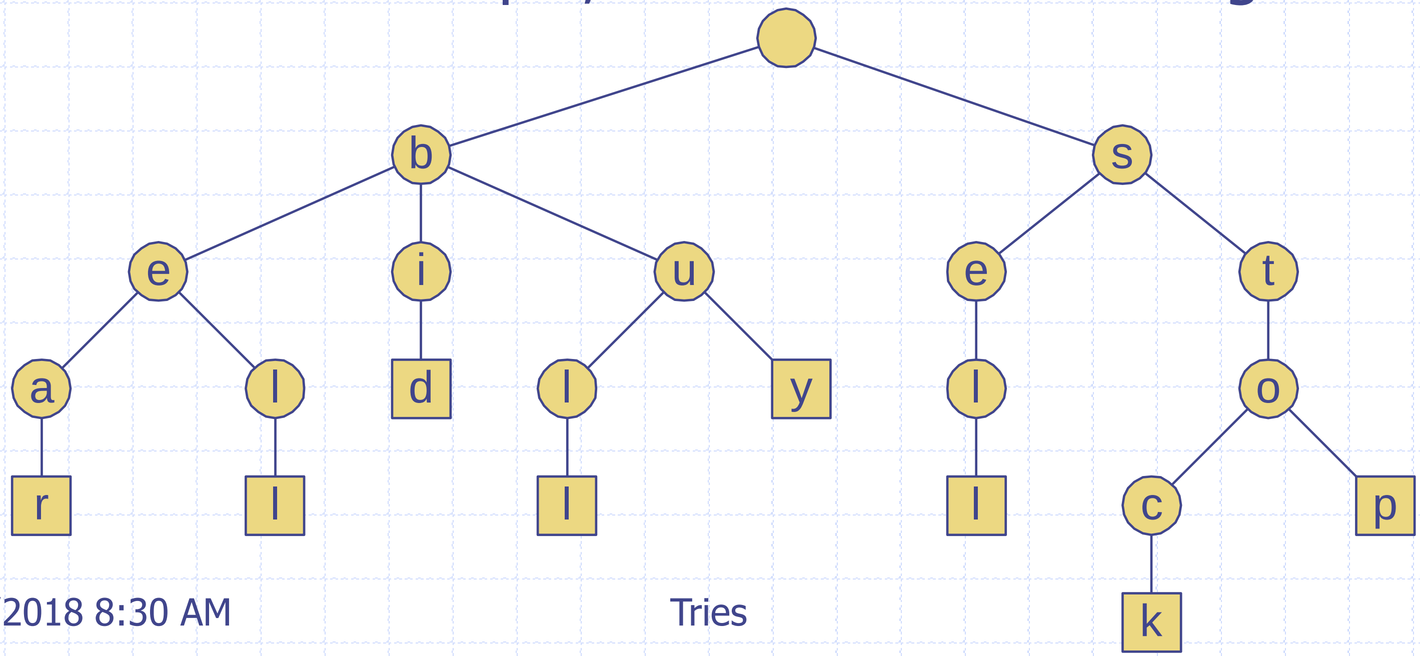
Standard Trie Construction

- Assuming the input strings are words in the English language, how many children does the root node have?



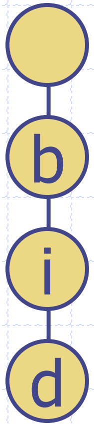
Standard Trie Construction

- The number of children of the root node equals to the maximum number of distinct first letters all the words in the input string
  - 2 in this example, maximum of 26 in English



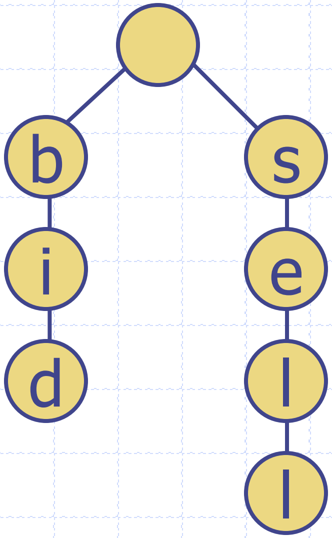
Standard Trie Construction

- What does the tree for “bid” look like?



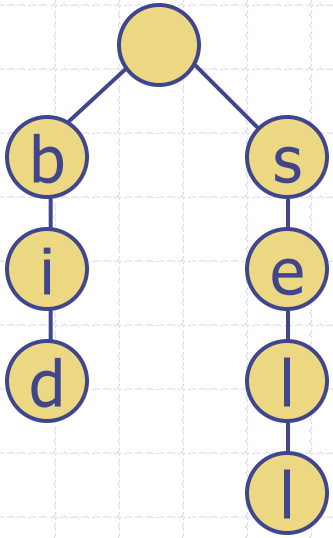
Standard Trie Construction

- What does the tree for "bid" and "sell" look like?



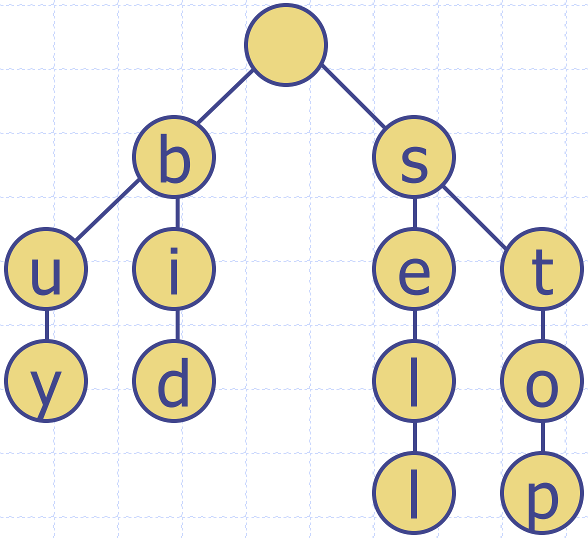
Standard Trie Construction

- What is the tree after adding “buy” and “stop”?



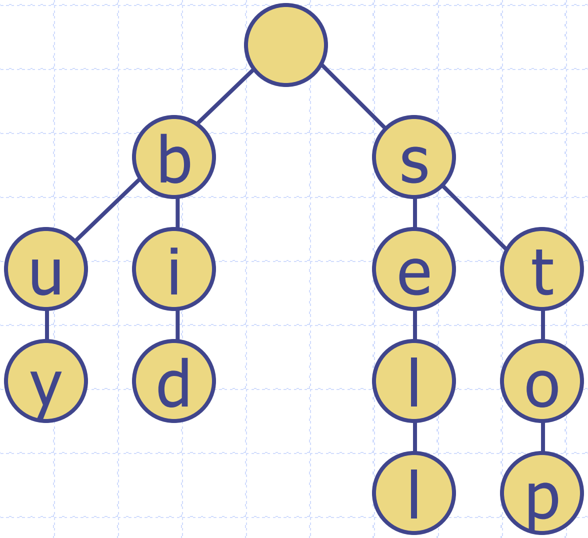
Standard Trie Construction

- What is the tree after adding “buy” and “stop”?



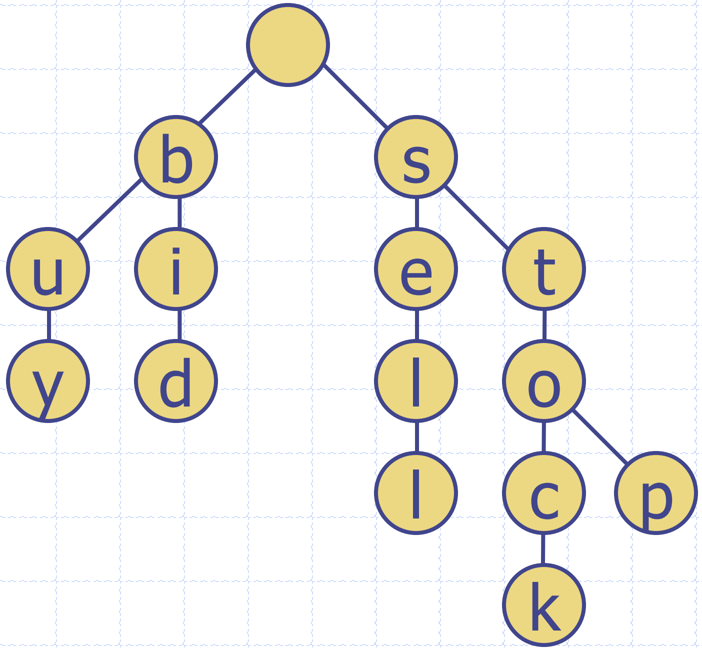
Standard Trie Construction

- What is the tree after adding “stock”?



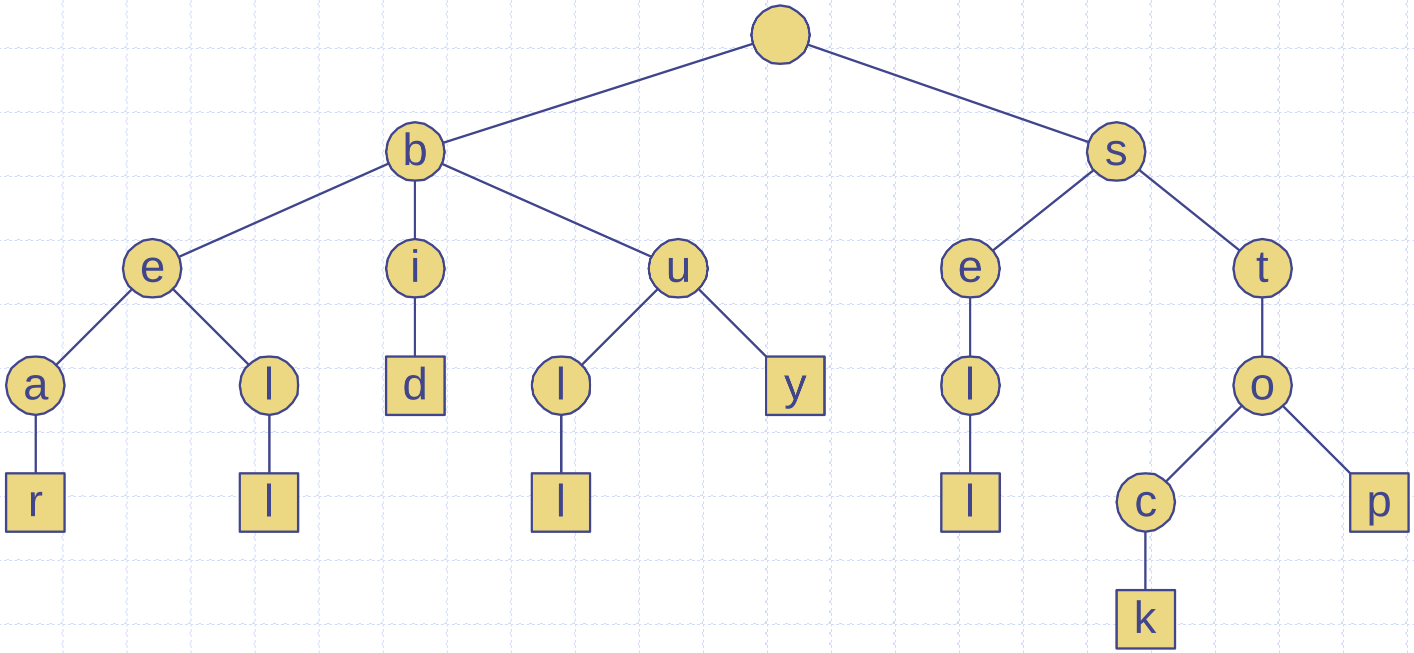
Standard Trie Construction

- What is the tree after adding “stock”?



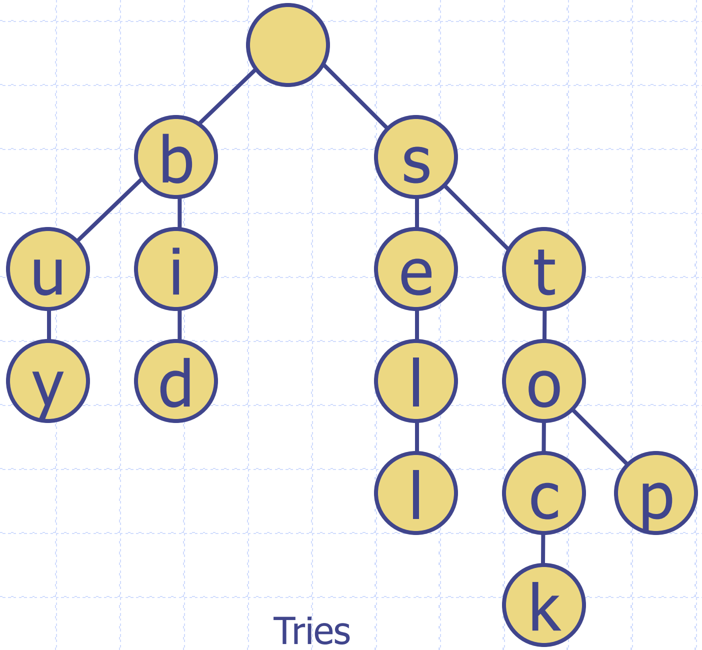
Standard Trie Construction

- After "bear, bell, bid, bull, buy, sell, stock, stop"...



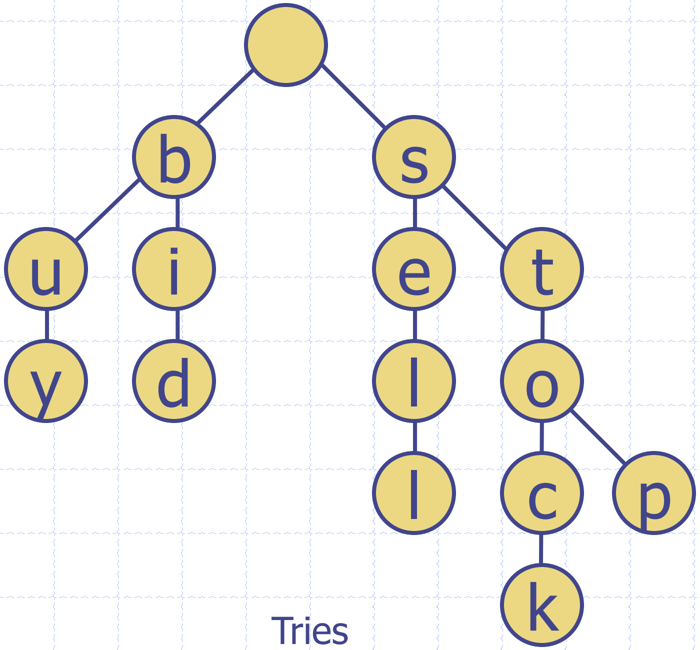
Improvements

- What comes to mind for tries?
  - e.g., is this entire tree really necessary?



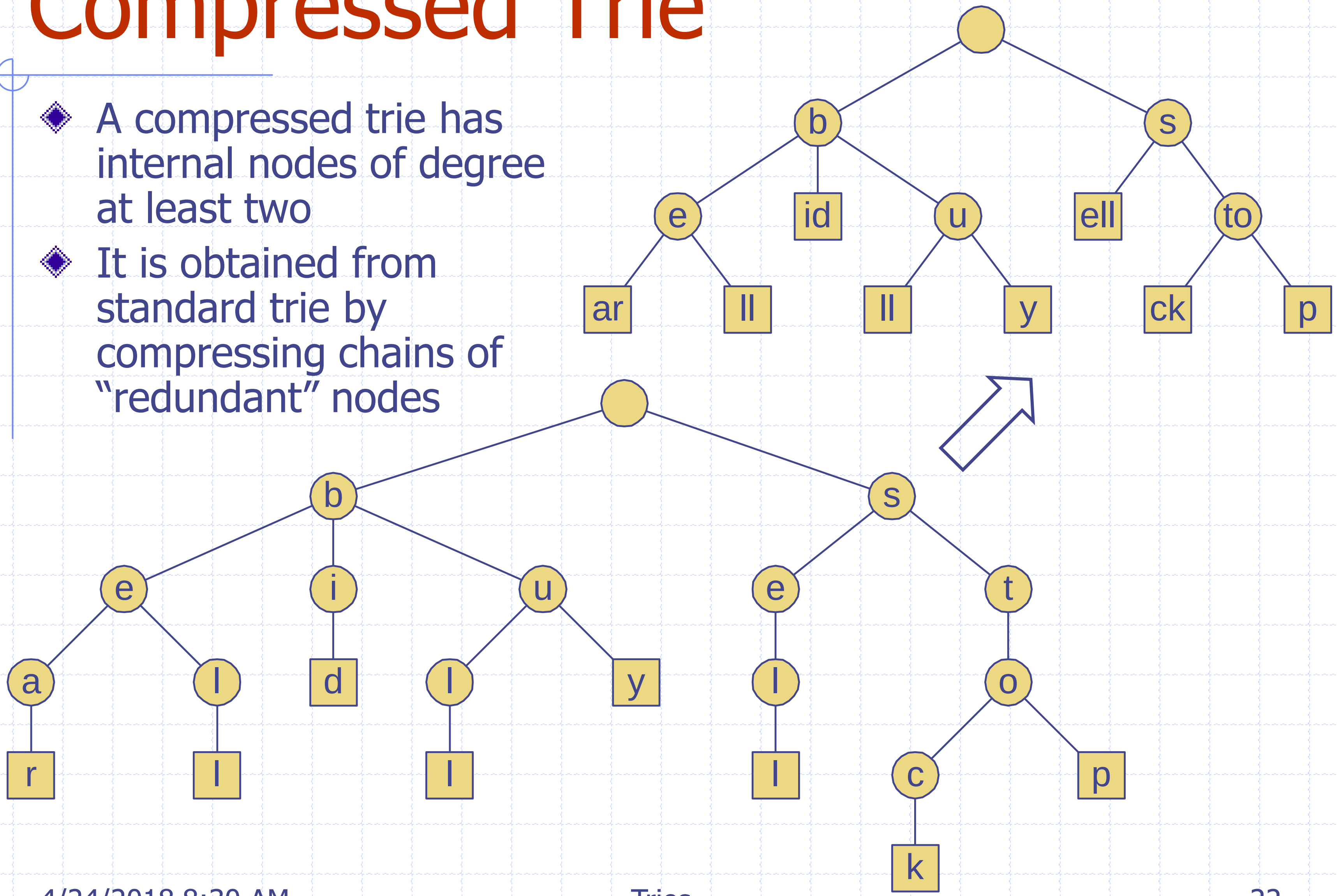
Improvements

- Two types of compression
  - Compress internal single-children node sequences
    - Also called "PATRICIA Tries" – Why?
    - = Practical AlgoriThm to Retrieve Information Coded In Alphanumeric (also called a "radix tree")
  - Compress external single-children leaf-node sequences



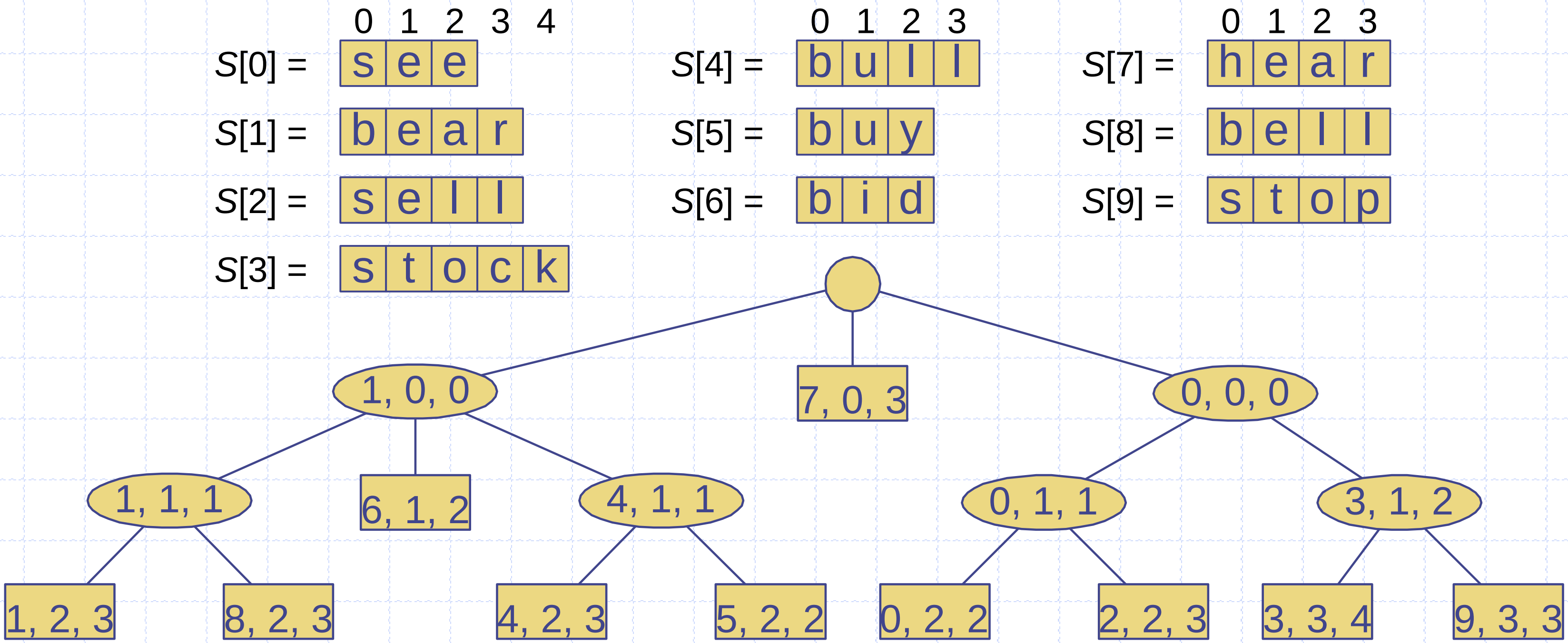
Compressed Trie

- A compressed trie has internal nodes of degree at least two
- It is obtained from standard trie by compressing chains of "redundant" nodes



Compact Representation

- Compact representation of a compressed trie for an array of strings:
  - Stores at the nodes ranges of indices instead of substrings
  - Serves as an auxiliary index structure



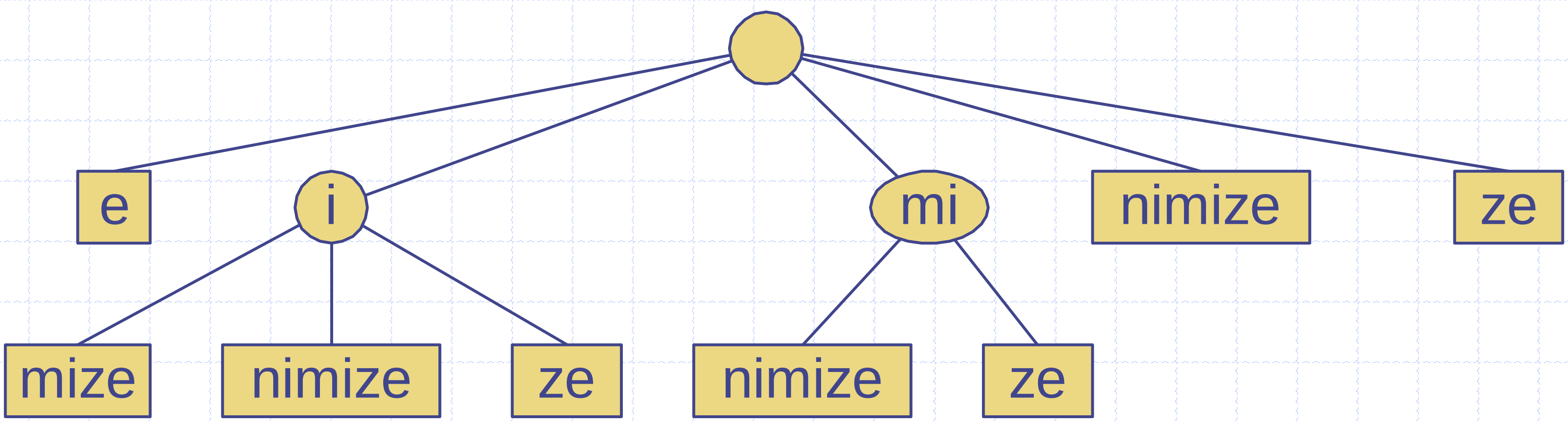
More Improvements

- We can build a standard trie and then compress it
- But, can we build some sort of compressed trie directly?
- Ideas?

Suffix Trie

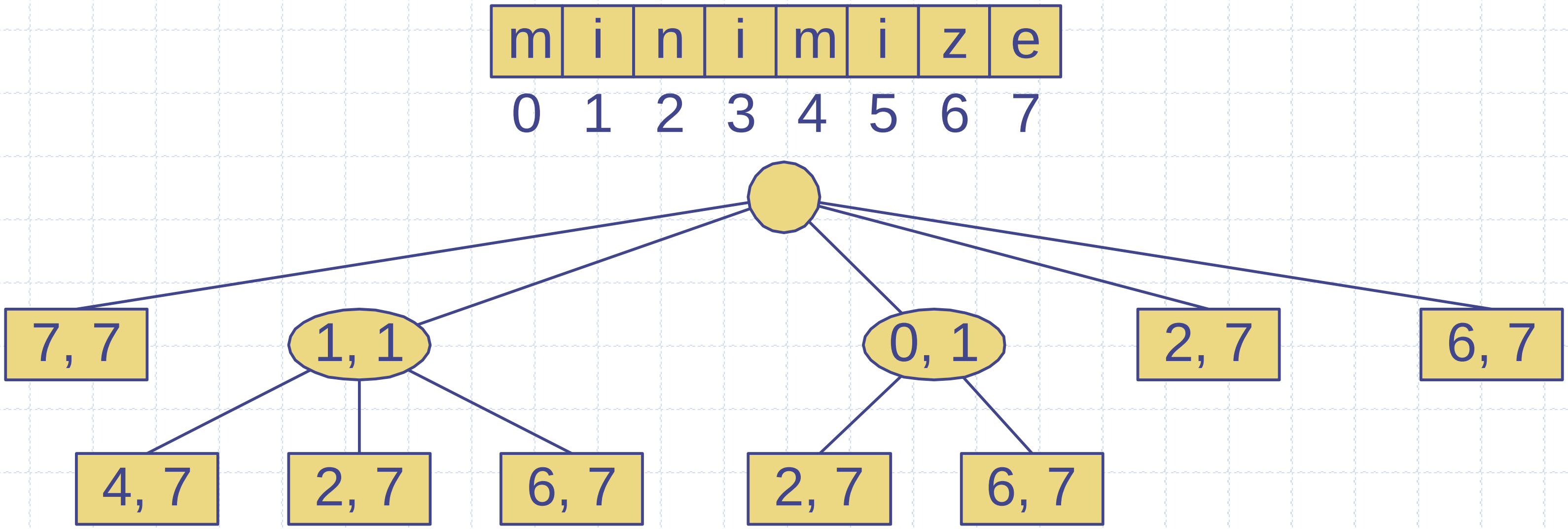
- The suffix trie of a string X is the compressed trie of all the suffixes of X

m	i	n	i	m	i	z	e
0	1	2	3	4	5	6	7



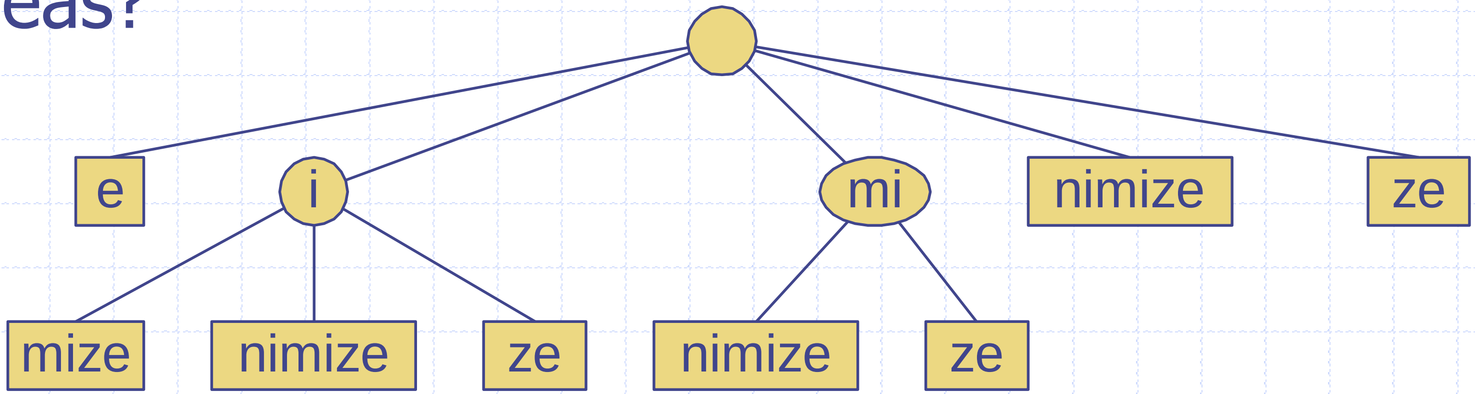
Suffix Trie

- Compact representation of the suffix trie for a string X of size n from an alphabet of size d
  - Uses $O(n)$ space
  - Supports arbitrary pattern matching queries in X in $O(dm)$ time, where m is the size of the pattern
  - Repetitive words not stored repetitively



More Improvements

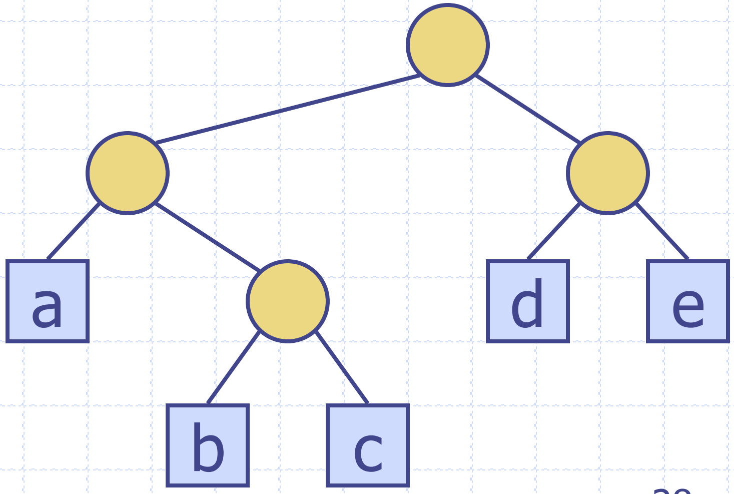
- There is still some repetition in this tree
  - e.g., "mize" appears several times
- How can we further compress the trie, thus reducing space and improving query time?
- Ideas?



Encoding Trie

- A code is a mapping of each character of an alphabet to a binary code-word
- A prefix code is a binary code such that no code-word is the prefix of another code-word
- An encoding trie represents a prefix code
  - Each leaf stores a character
  - The code word of a character is given by the path from the root to the leaf storing the character (0 for a left child and 1 for a right child)

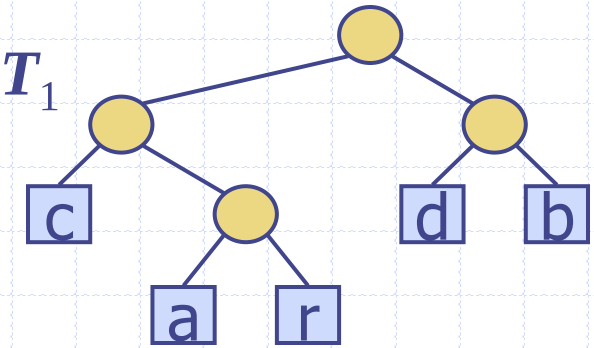
00	010	011	10	11
a	b	c	d	e



Encoding Trie

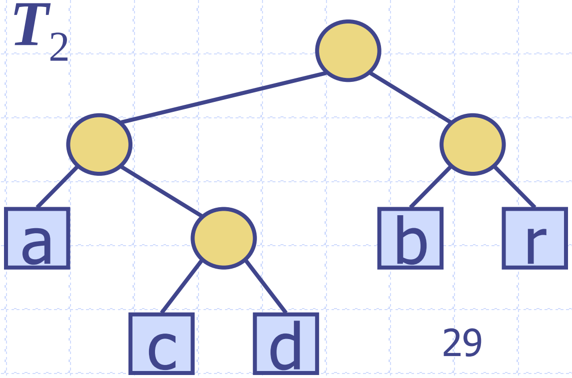
- Given a text string X, we want to find a prefix code for the characters of X that yields a small encoding for X
  - Frequent characters should have short code-words
  - Rare characters should have long code-words
  - Why?
- Example
  - X = abracadabra
  - T_1 encodes X into 29 bits
    - 29=3+2+3+3+2+3+2+3+2+3+3
  - T_2 encodes X into how many bits?
    - 24=2+2+2+2+3+2+3+2+2+2+2

T_1



How can we build a good encoding trie?

T_2



Huffman's Algorithm

- Given a string X, Huffman's algorithm constructs a prefix code that minimizes the size of the encoding of X
- It runs in time $O(n + d \log d)$, where n is the size of X and d is the number of distinct characters of X
- A heap-based priority queue is used as an auxiliary structure

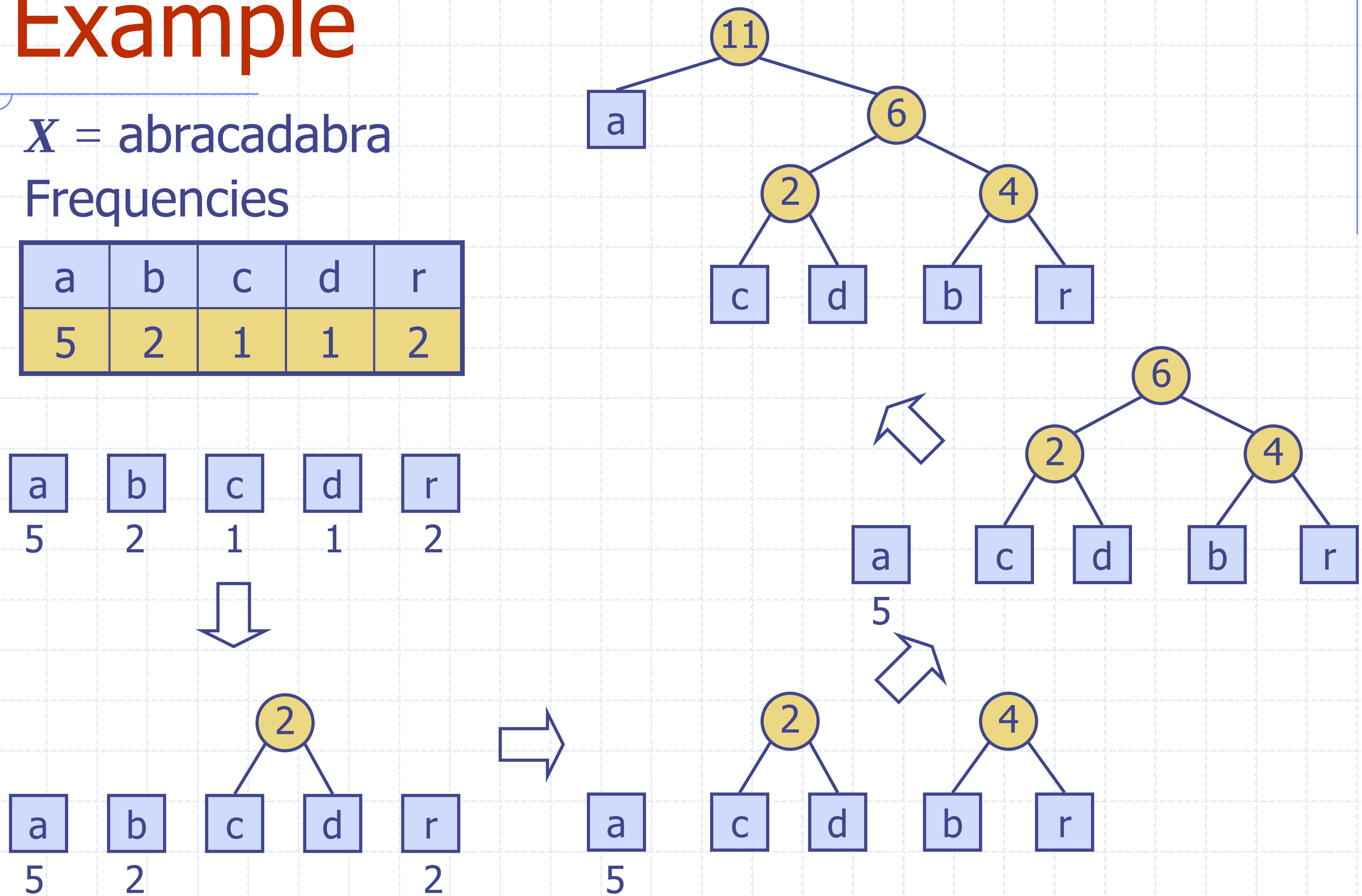
Algorithm *HuffmanEncoding*(X)
  Input string X of size n
  Output optimal encoding trie for X
  $C \leftarrow distinctCharacters(X)$
  $computeFrequencies(C, X)$
  $Q \leftarrow$ new empty heap
  for all $c \in C$
    $T \leftarrow$ new single-node tree storing c
    $Q.insert(getFrequency(c), T)$
  while $Q.size() > 1$
    $f_1 \leftarrow Q.minKey()$
    $T_1 \leftarrow Q.removeMin()$
    $f_2 \leftarrow Q.minKey()$
    $T_2 \leftarrow Q.removeMin()$
    $T \leftarrow join(T_1, T_2)$
    $Q.insert(f_1 + f_2, T)$
  return $Q.removeMin()$

Example

X = abracadabra

Frequencies

a	b	c	d	r
5	2	1	1	2



Summary of Pattern Matching

Algorithm	Search Time	Notes
Brute force	$O(nm)$	◈ simple, no preprocessing ◈ slow (good for small inputs)
Boyer-Moore	$O(nm+s)$	◈ O(m) preprocessing ◈ significantly faster than previous in practice
KMP	$O(n+m)$	◈O(m+s) preprocessing ◈ more complex, but ideal very fast
Standard Trie	$O(dm)$	◈O(n) preprocessing, d = size of alphabet ◈ fast
Suffix Trie	$O(dm)$	◈ O(n) preprocessing ◈ faster in practice because "compressed"
Huffman-Encoding Trie	$O(dm)$	◈ O(n+dlogd) preprocessing ◈ fastest and smallest in practice ◈ leads to lossless compression: ZIP