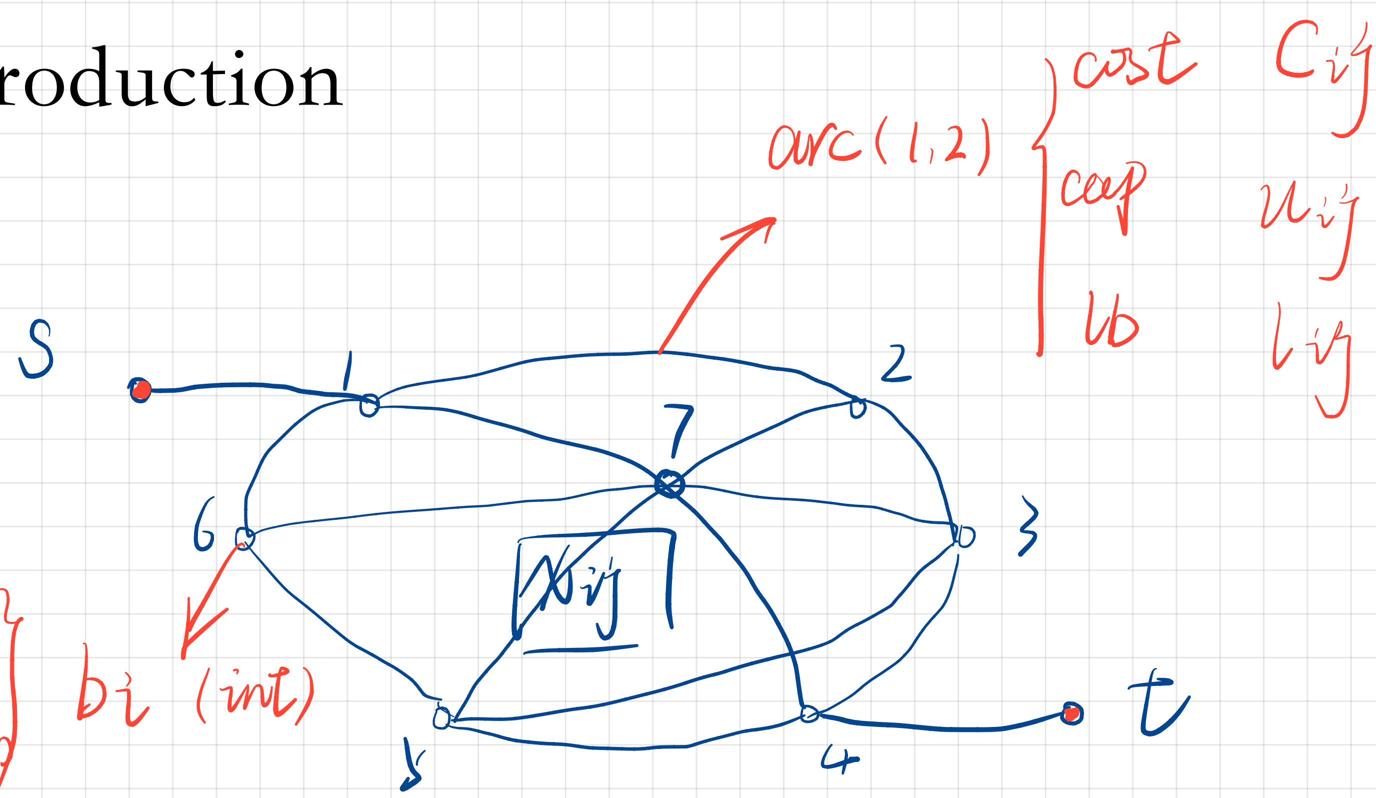
Chapter I Introduction

最小费用流问题

$G = (N, A)$

Network   nodes   Arcs

> 0 , supply
< 0 , demand
$= 0$ transship



Model: $\min \sum c_{ij} x_{ij}$

s.t. $\underset{\text{out}}{\sum x_{ij}} - \underset{\text{in}}{\sum x_{ji}} = b_i$ (对所有节点)

$lb \le x_{ij} \le ub$ (对所有弧)

$\left(\sum_{i=1}^{n} b_i = 0 \right)$

① **最短路问题**

let: $b_s = 1$, $b_t = -1$ (s到t的最短)

$b_s = n-1$, $b_i = -1$ (s到其余所有的一)

无 u_{ij} 限制 , $\min \sum x_{ij} c_{ij}$

② **最大流问题**

$$\max \sum_{u:(s,u)\in E} f_{su}$$

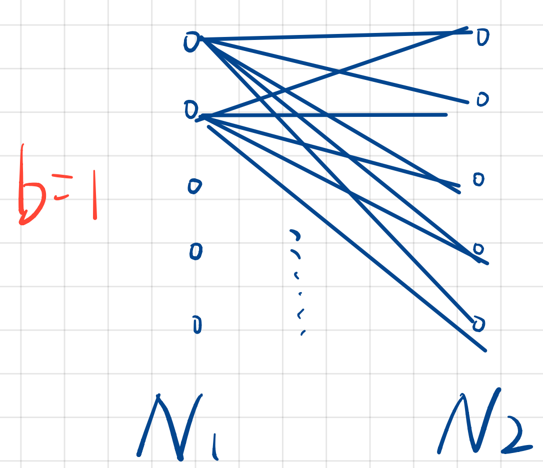
$$\text{s.t.} \quad 0 \le f_{uv} \le c_{uv} \qquad \forall (u,v) \in E$$

$$\sum_{w:(w,u)\in E} f_{wu} - \sum_{v:(u,v)\in E} f_{uv} = 0 \qquad \forall u \in V \setminus \{s,t\}$$

All $b_i = 0$, $c_{ij} = 1$

$\max \sum c_{ij} x_{ij}$

③ 指派问题

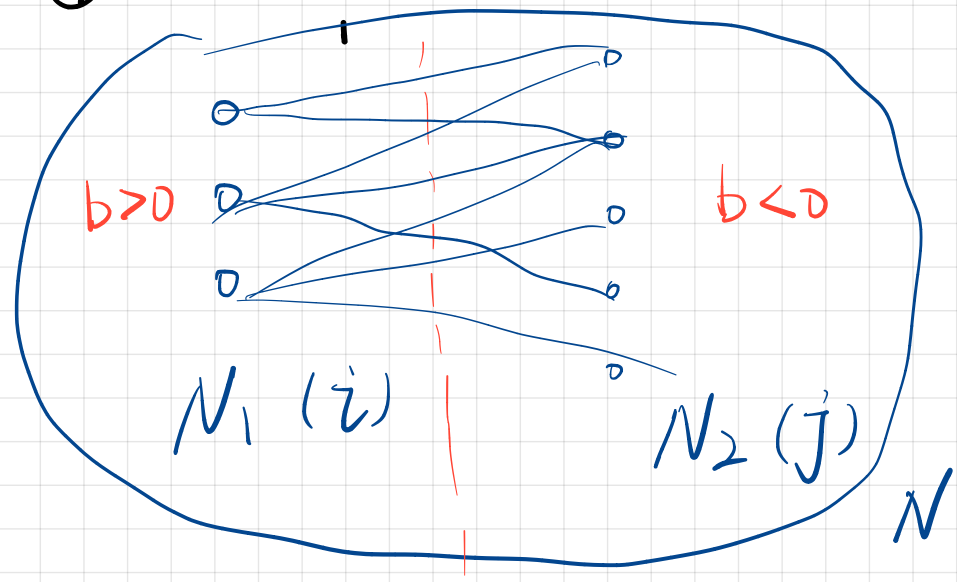


$u_{ij}=1$

$b=-1$

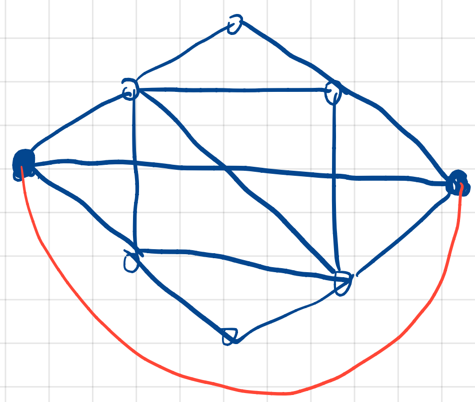
$\min \sum x_{ij} c_{ij}$

④ 运输问题



$\min \sum x_{ij} c_{ij}$

⑤ 循环问题



all $b_i = 0$

$x_{ij} \in [lb, ub]$

$\min \sum x_{ij} c_{ij}$

变式问题：

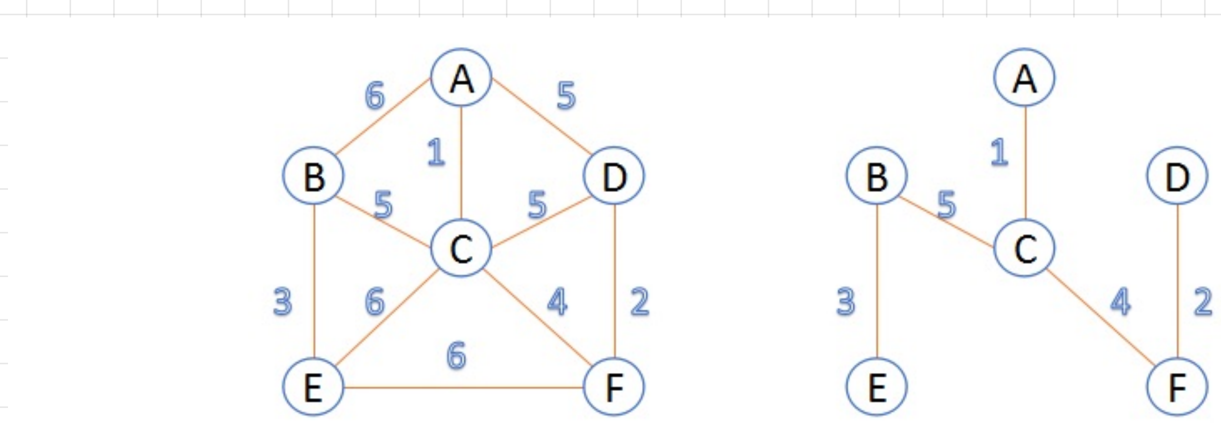
① 凸成本流问题 （费用非线性）

② 一般流问题 （arc 可消耗/产生 flow）

③ 多商品流问题 （arc 容量约束多个商品流下，寻找总体最小花费）

其他模型：

① 最小生成树



连通网G　　　　最小生成树

② 匹配问题

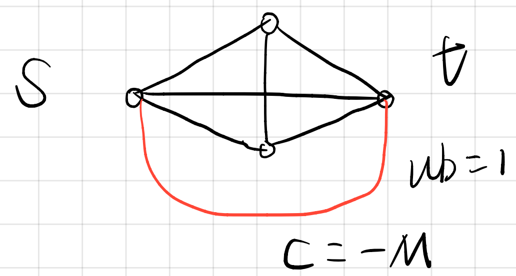
- 二分匹配
- 非二分匹配

- 基数匹配
- 质量匹配

1.1. Formulate the following problems as circulation problems: (1) the shortest path problem; (2) the assignment problem; and (3) the transportation problem.

(1) 最短路

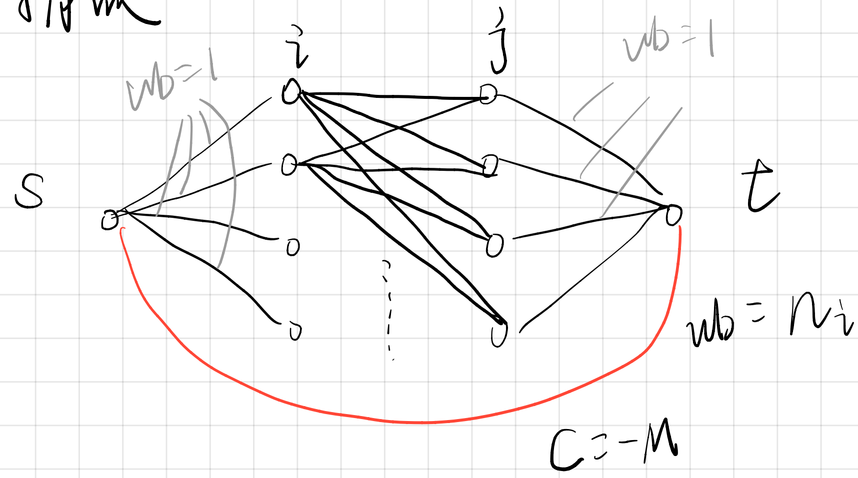
转换为循环问题的关键：给flow施加动力



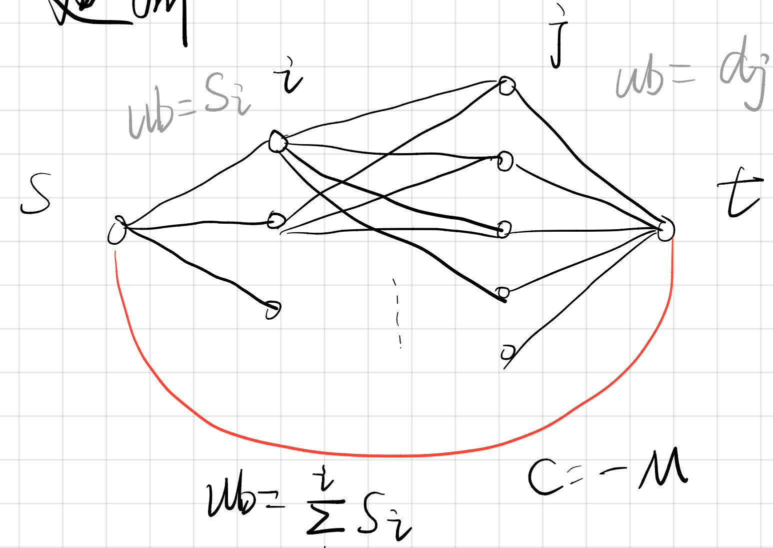
$$\min \sum c_{ij} x_{ij}$$

$$b_i = 0$$

(2) 指派



(3) 运输



1.2. Consider a variant of the transportation problem for which (1) the sum of demands exceeds the sum of supplies, and (2) we incur a penalty p_j for every unit of unfulfilled demand at demand node j. Formulate this problem as a standard transportation problem with total supply equal to total demand.

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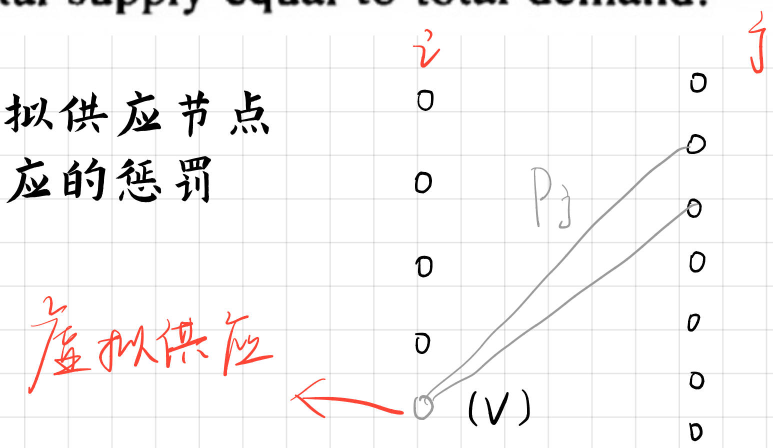

$$\min \sum x_{ij} c_{ij} + x_{vj} p_j$$

s.t.

$$\sum x_{ij} = b_i$$

$$\sum x_{ji} = -b_j$$

1.3. In this exercise we examine a generalization of Application 1.2, concerning assortment of structural steel beams. In the discussion of that application, we assumed that if we must cut a beam of length 5 units to a length of 2 units, we obtain a single beam of length 2 units; the remaining 3 units have no value. However, in practice, from a beam of length 5 we can cut two beams of length 2; the remaining length of 1 unit will have some scrap value. Explain how we might incorporate the possibility of cutting multiple beam lengths (of the same length) from a single piece and assigning some salvage value to the scrap. Assume that the scrap has a value of β per unit length.

$$C_{ij} = k_j + C_j \cdot \sum_{k=i+1}^{j} D_k - \beta \sum_{k=i+1}^{j} D_k (L_j - L_k)$$

?