61A Lecture 4

Wednesday, January 28

Announcements

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- Project 1 due Thursday 2/5 at 11:59pm

Iteration Example

The Fibonacci Sequence

The Fibonacci Sequence

The Fibonacci Sequence

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987

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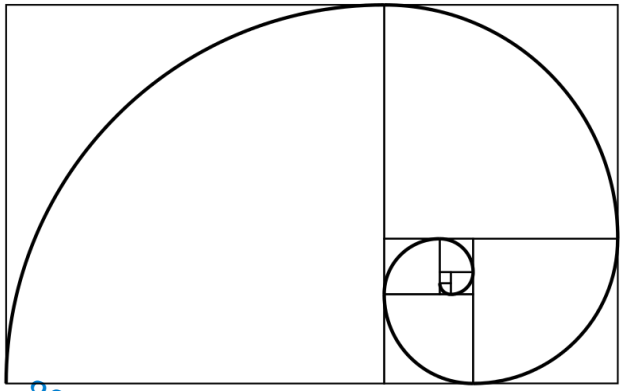

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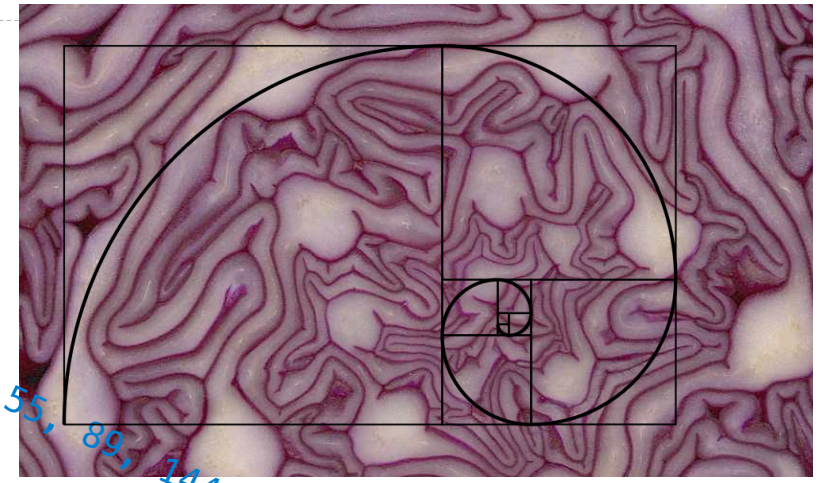

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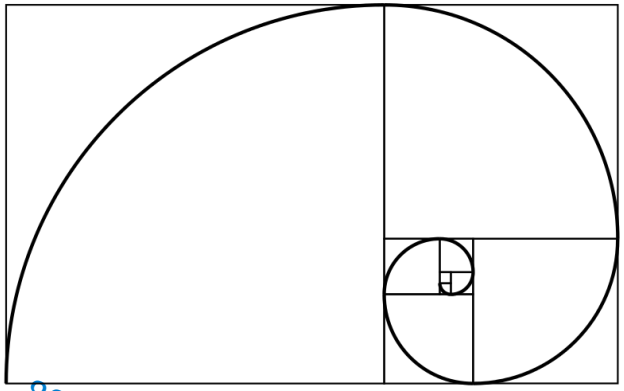

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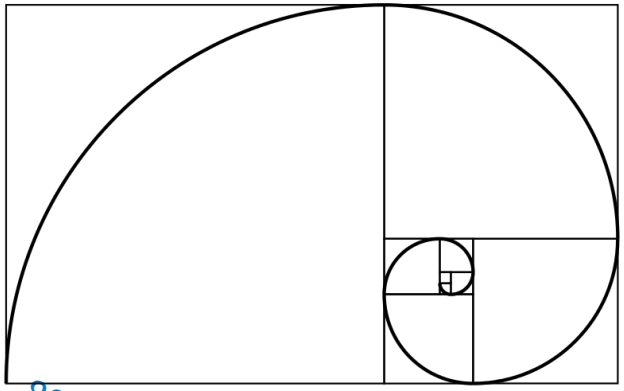

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    """Compute the nth Fibonacci number, for N >= 1."""
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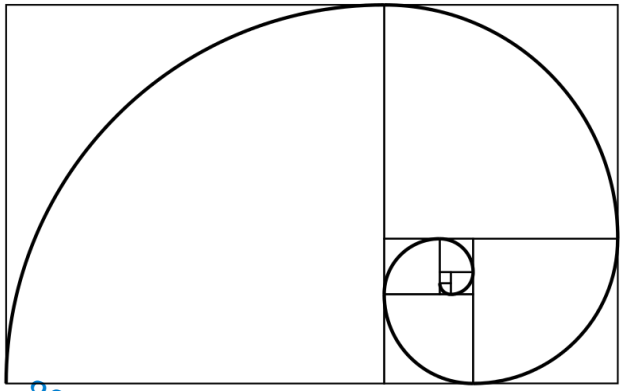

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The next Fibonacci number is the sum of the current one and its predecessor

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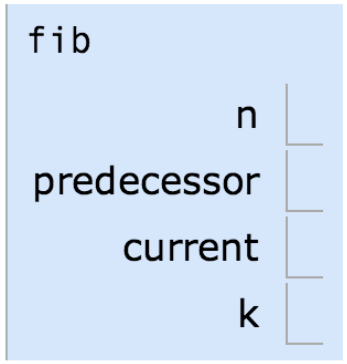

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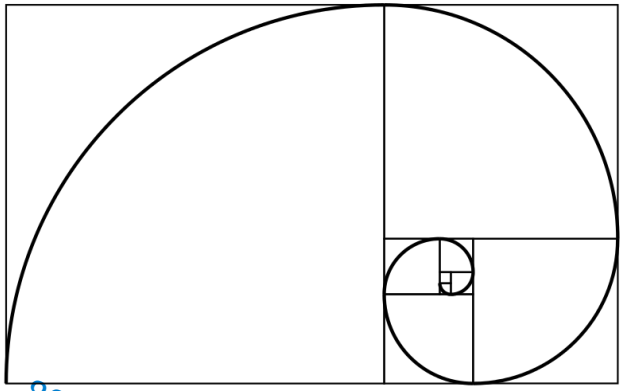

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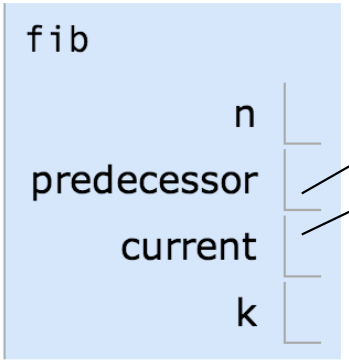

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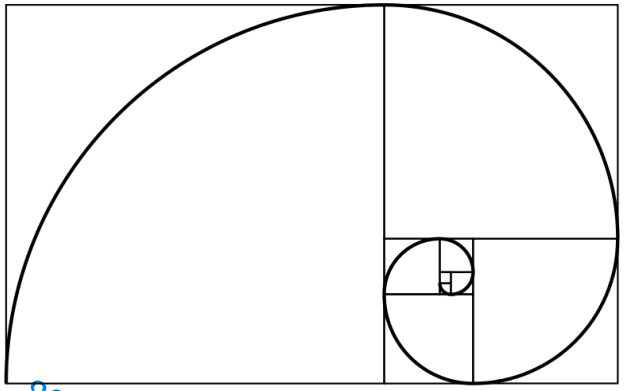

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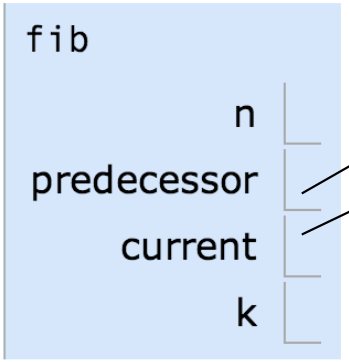

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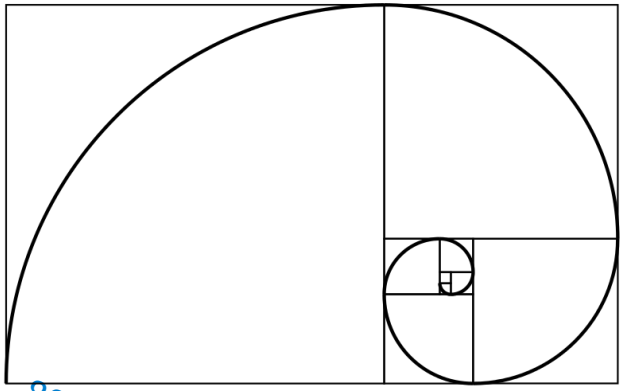

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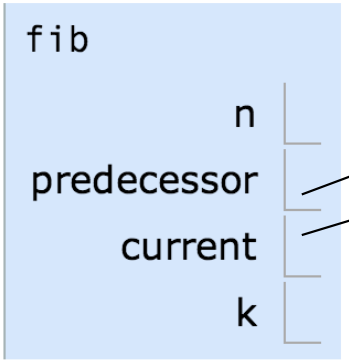

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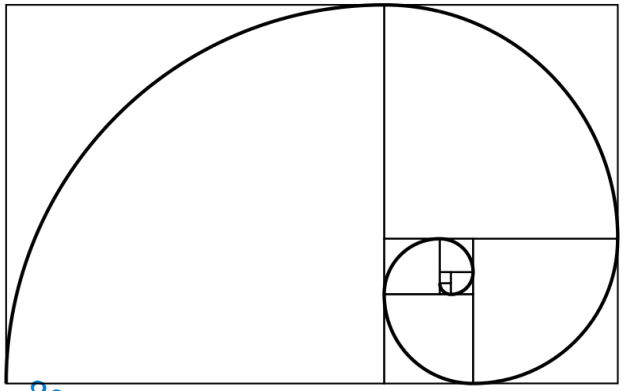

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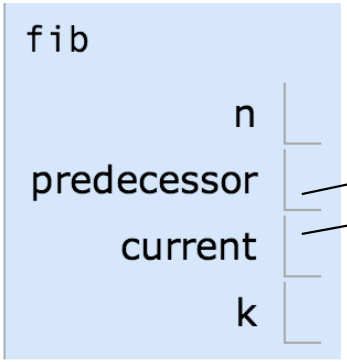

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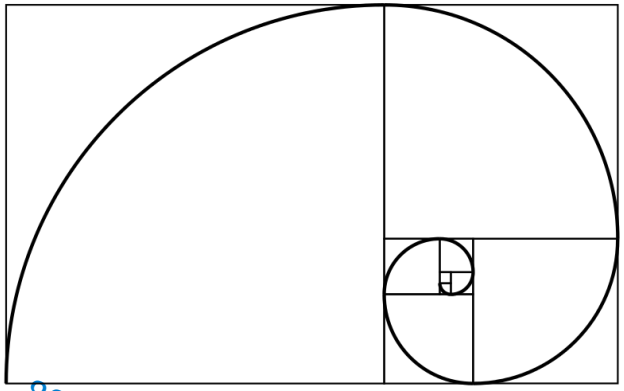

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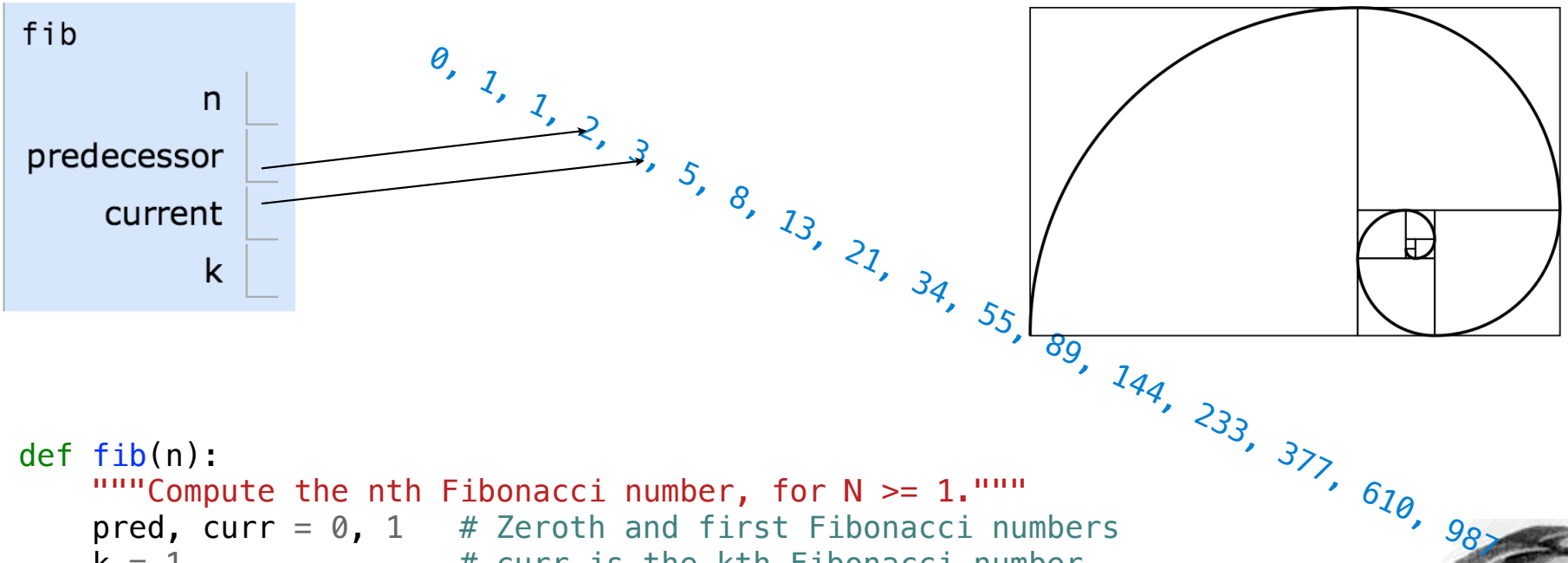


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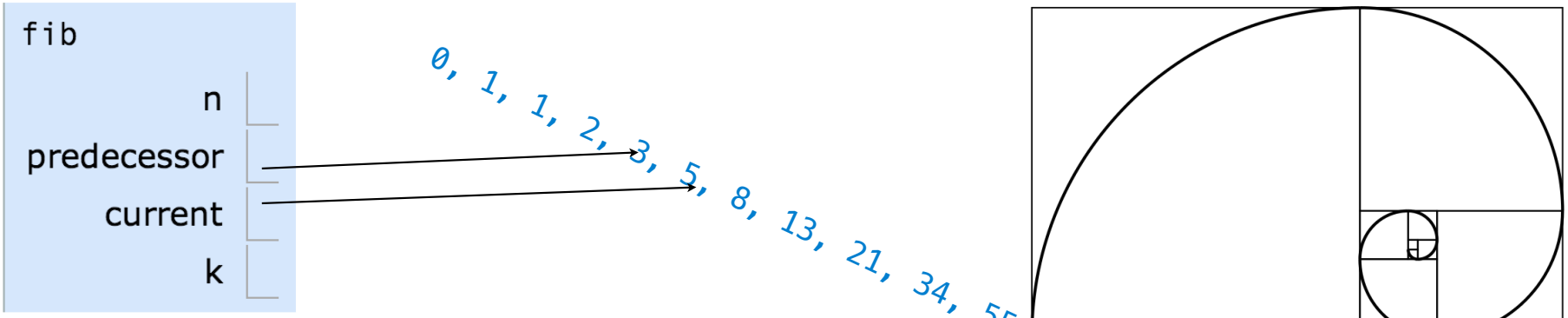


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a **b**

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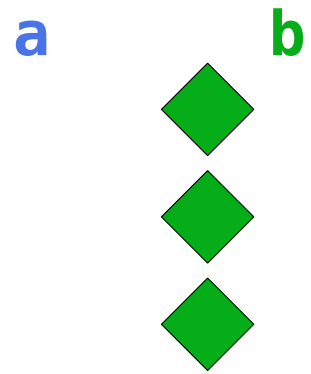

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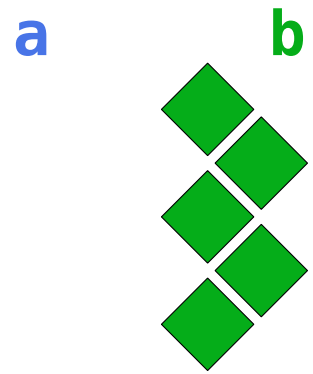

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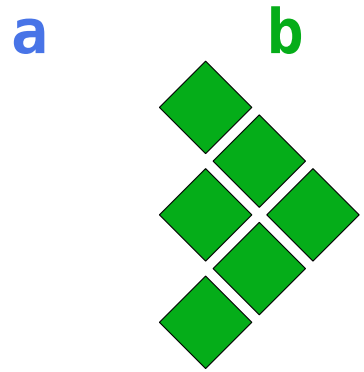

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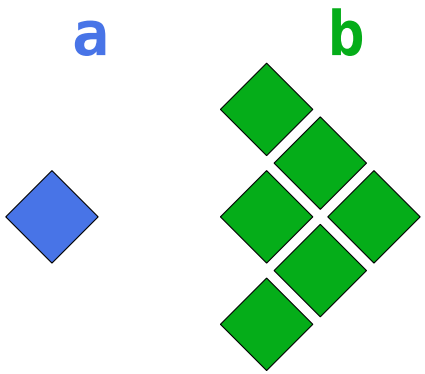

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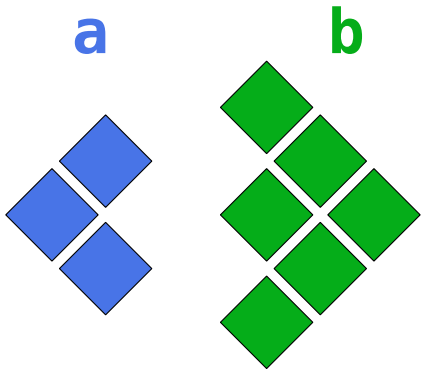

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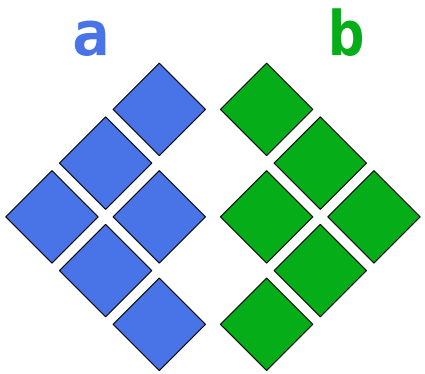

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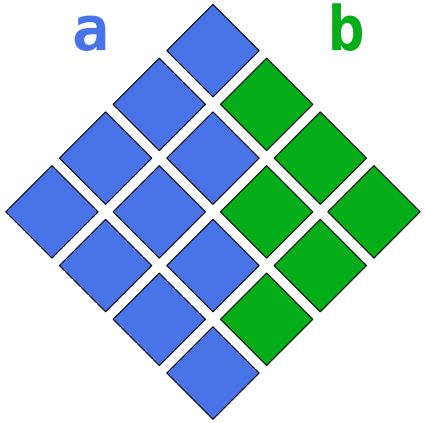

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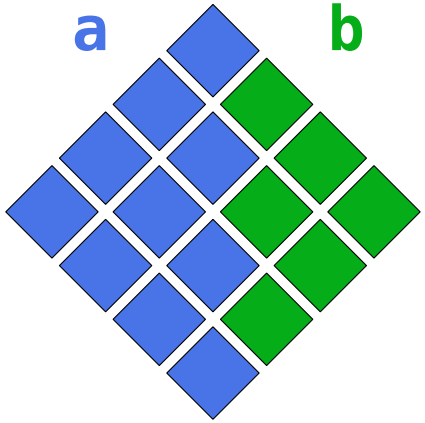

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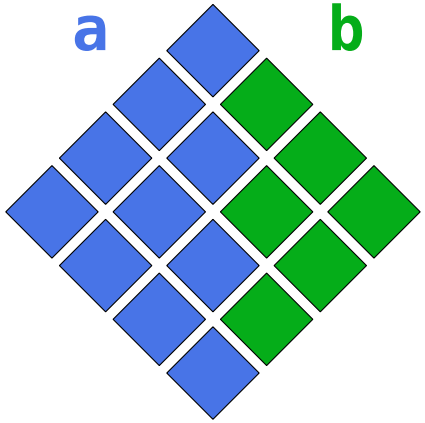

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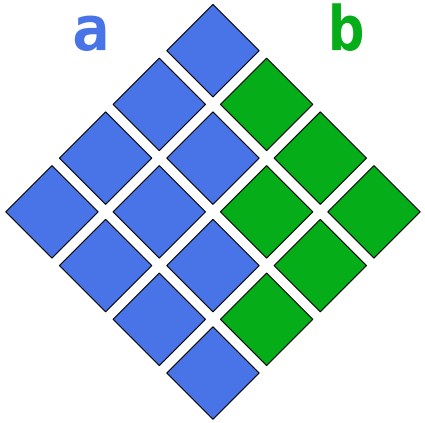

Designing Functions

Characteristics of Functions

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A pure function's behavior is the relationship it creates between input and output.

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def square(x):
    """Return X * X."""
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def square(x):
    """Return X * X."""
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A function's domain is the set of all inputs it might possibly take as arguments.

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A function's range is the set of output values it might possibly return.

returns a non-negative real number

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def square(x):
    """Return X * X."""
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```

A function's domain is the set of all inputs it might possibly take as arguments.

x is a real number

A function's range is the set of output values it might possibly return.

returns a non-negative real number

A pure function's behavior is the relationship it creates between input and output.

return value is the square of the input

Characteristics of Functions

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A Guide to Designing Function

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Generalization

Generalizing Patterns with Arguments

Generalizing Patterns with Arguments

```
Regular geometric shapes relate length and area.
```

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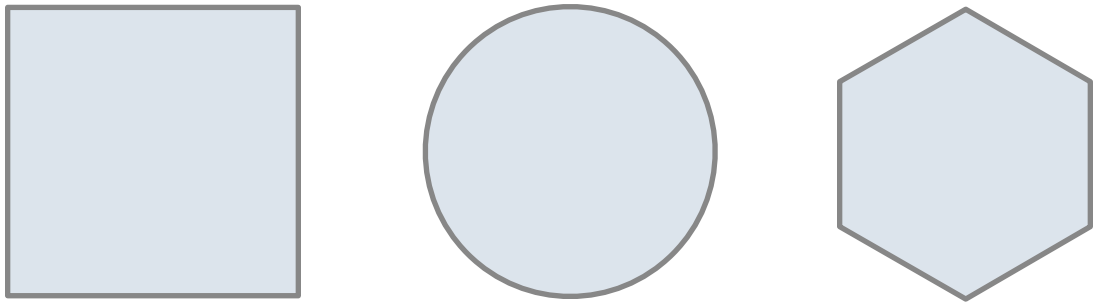
Shape:



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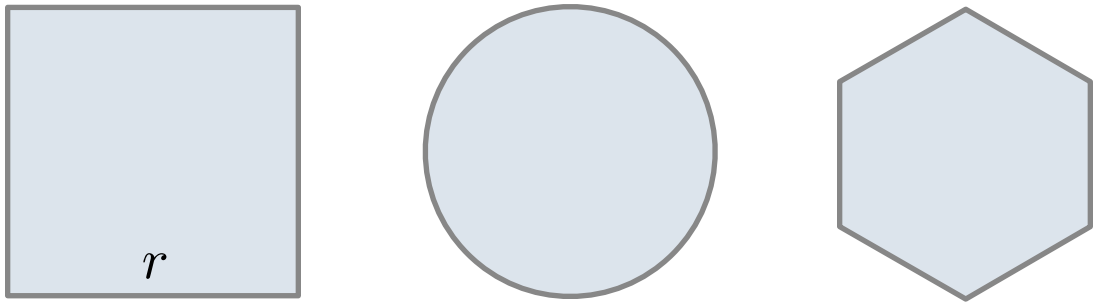

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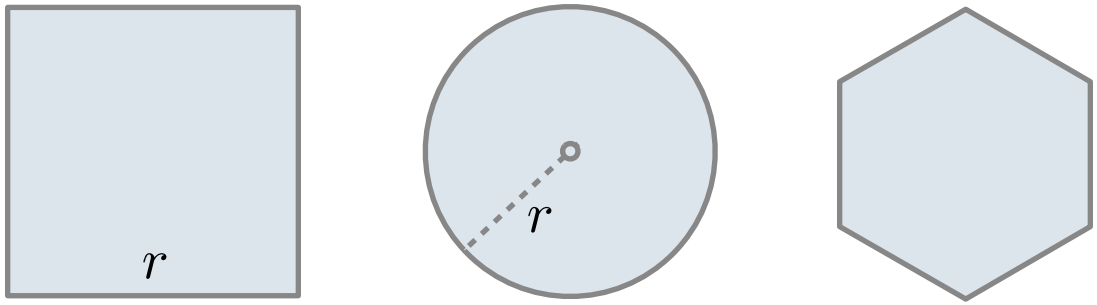

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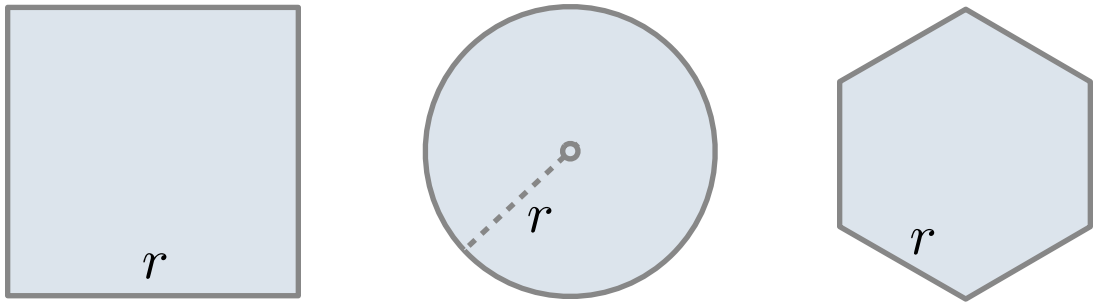

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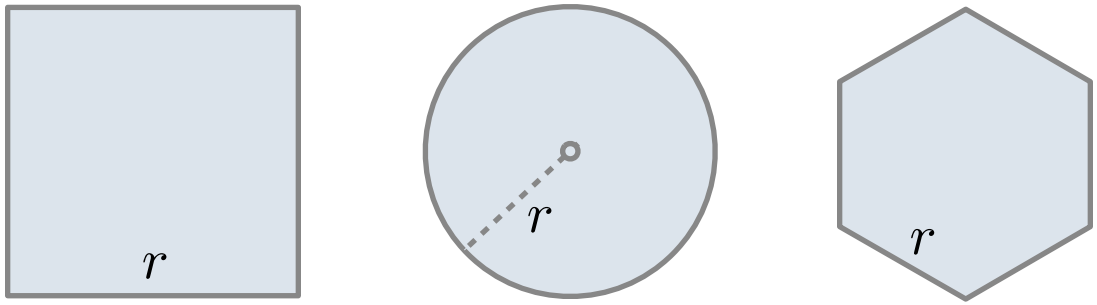

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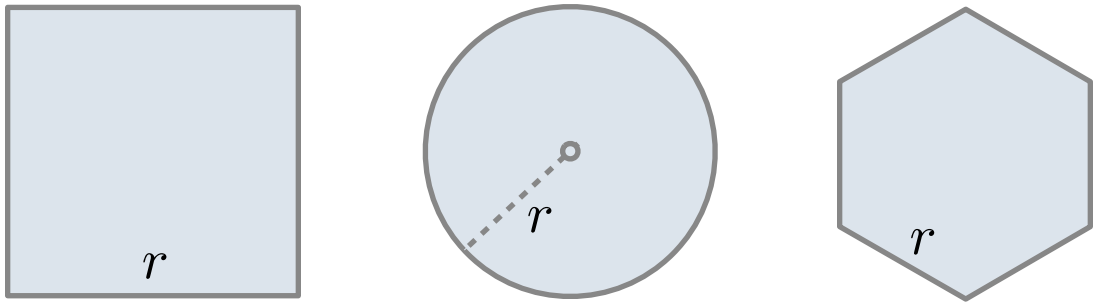

Area: r^2

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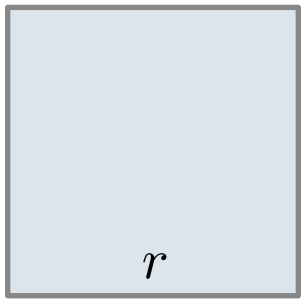
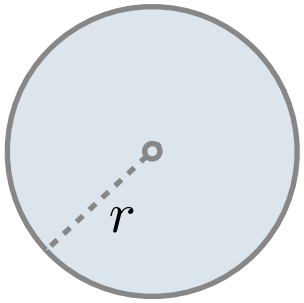
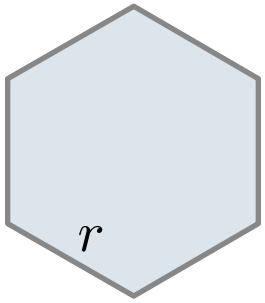
Shape:			
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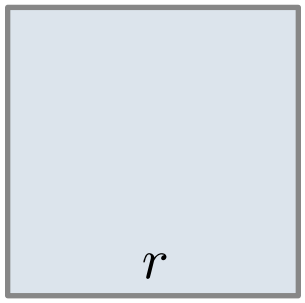
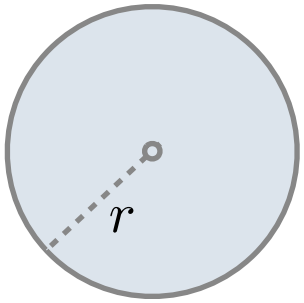
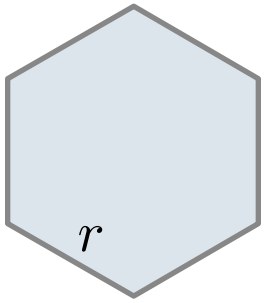
Shape:			
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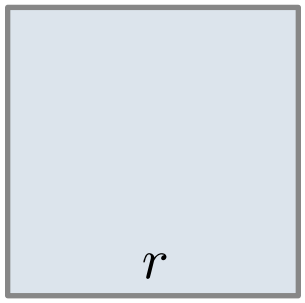
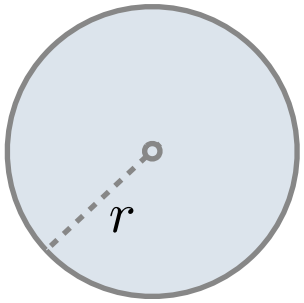
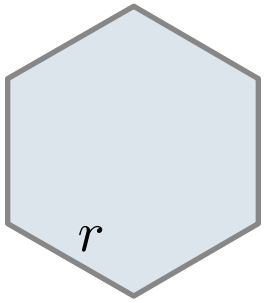
Shape:			
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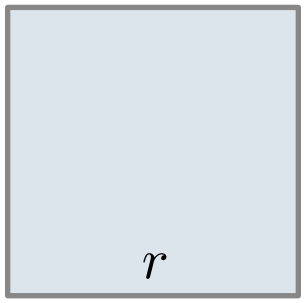
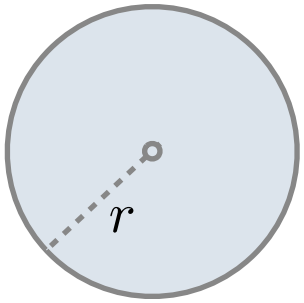
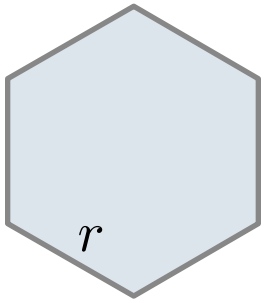
Shape:			
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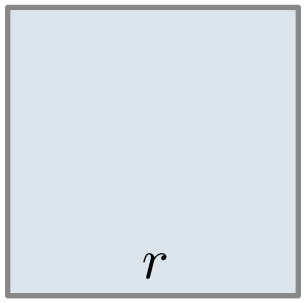
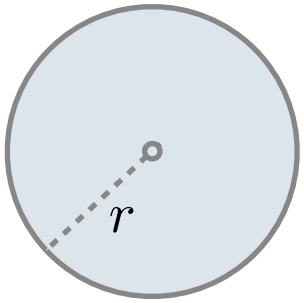
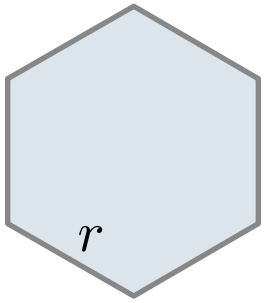
Shape:			
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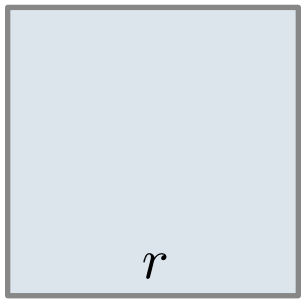

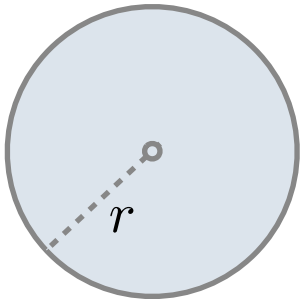

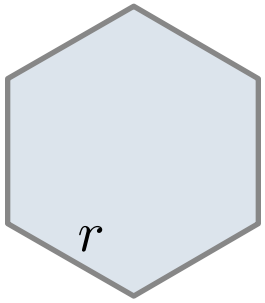

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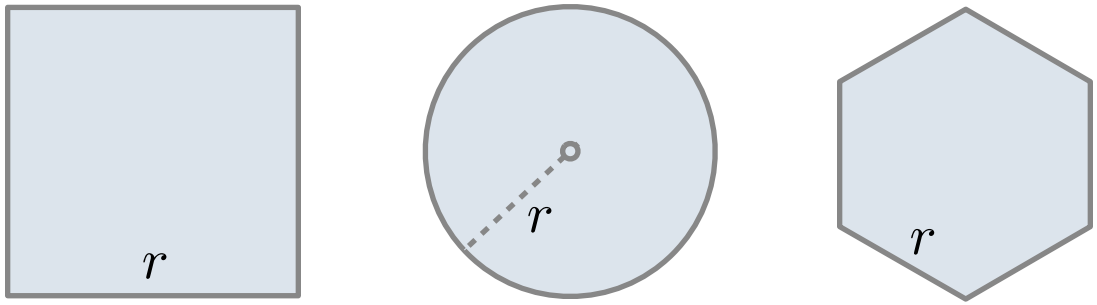

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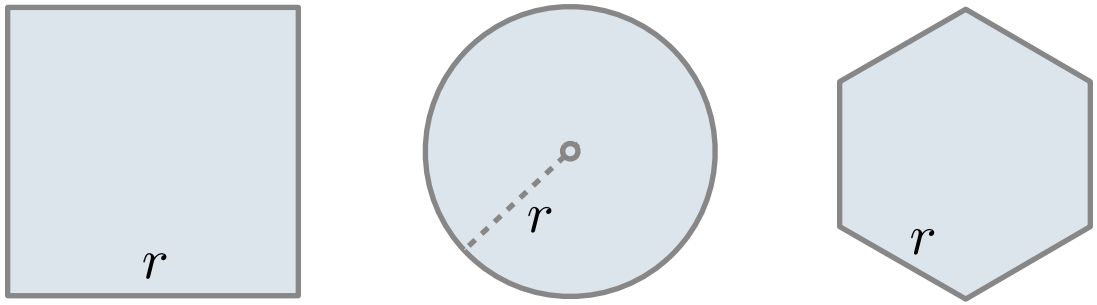

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Finding common structure allows for shared implementation

(Demo)

Higher-Order Functions

Generalizing Over Computational Processes

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(Demo)

Summation Example

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def cube(k):
    return pow(k, 3)

def summation(n, term):
    """Sum the first n terms of a sequence.

    >>> summation(5, cube)
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    """
    total, k = 0, 1
    while k <= n:
        total, k = total + term(k), k + 1
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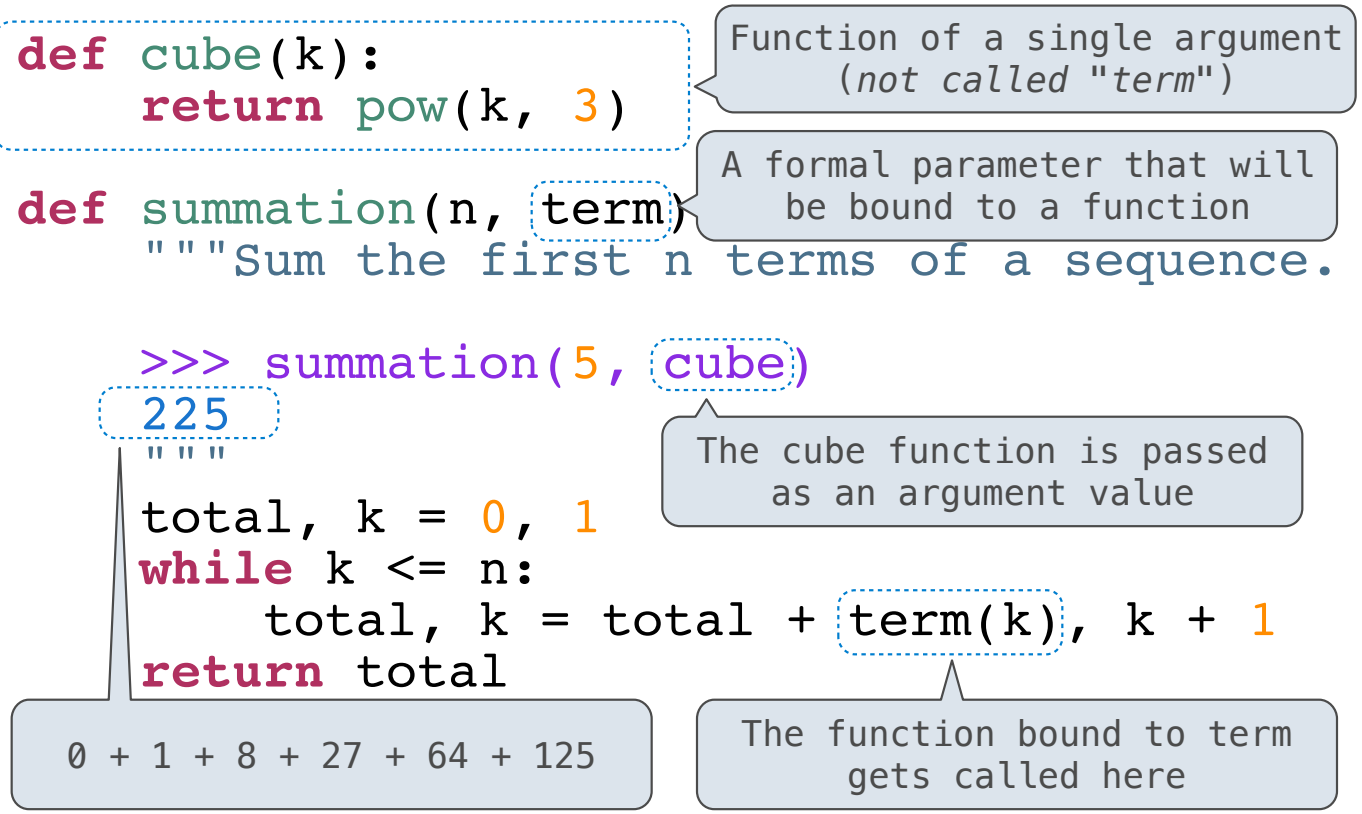

Functions as Return Values

(Demo)

Locally Defined Functions

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def make_adder(n):
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A def statement within another def statement

Can refer to names in the enclosing function

Call Expressions as Operator Expressions

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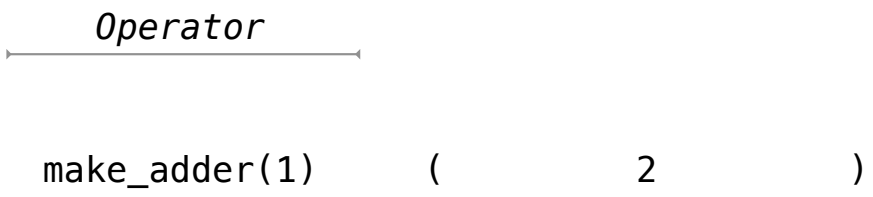
```
make_adder(1)     (         2         )
```

Call Expressions as Operator Expressions



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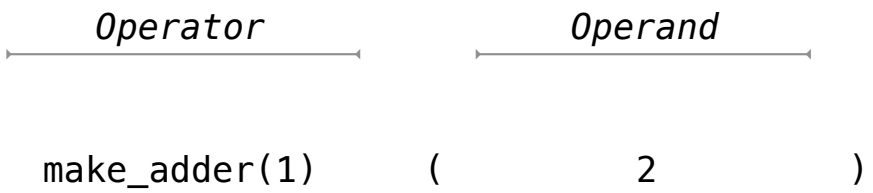

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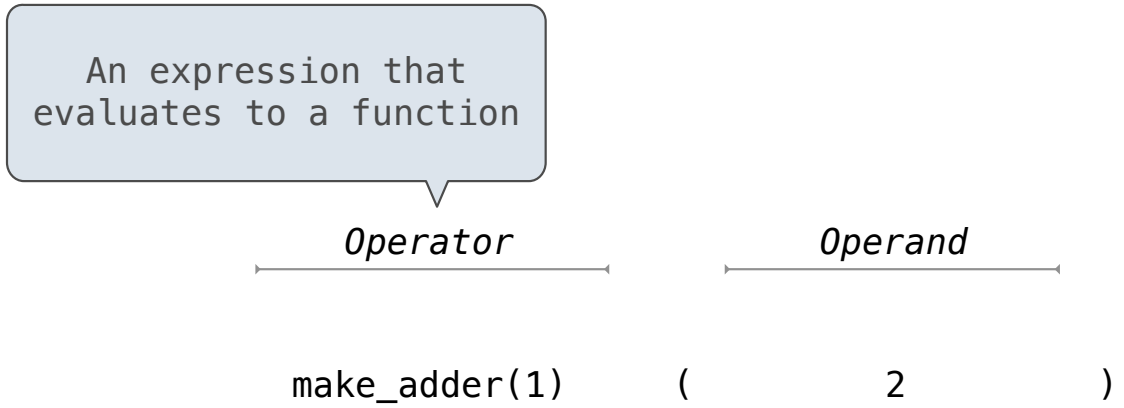

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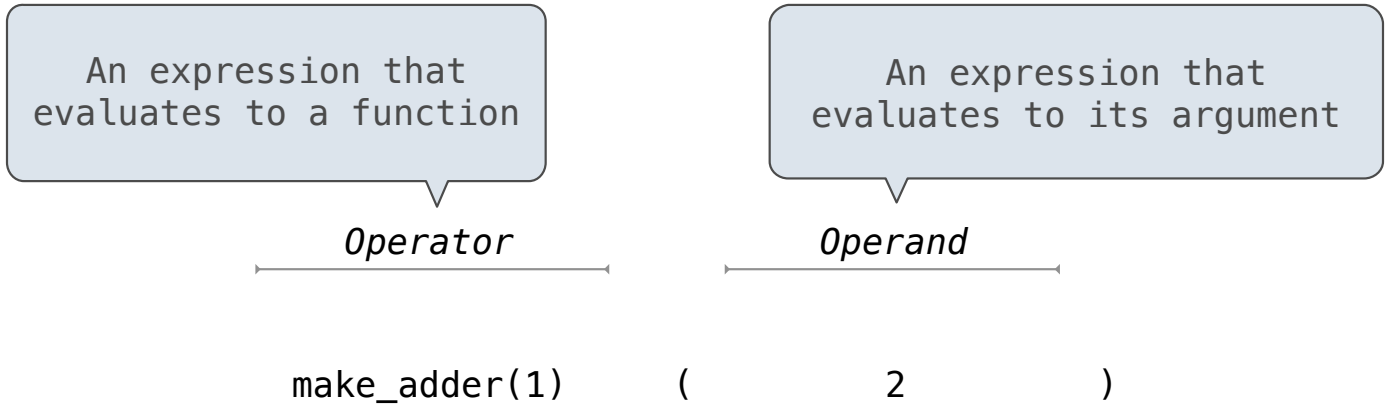

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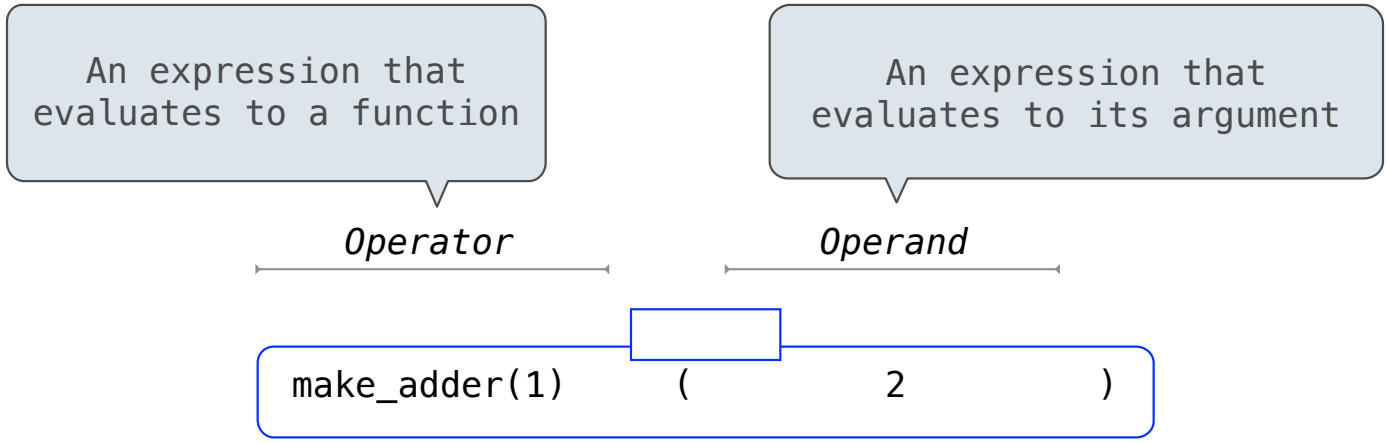

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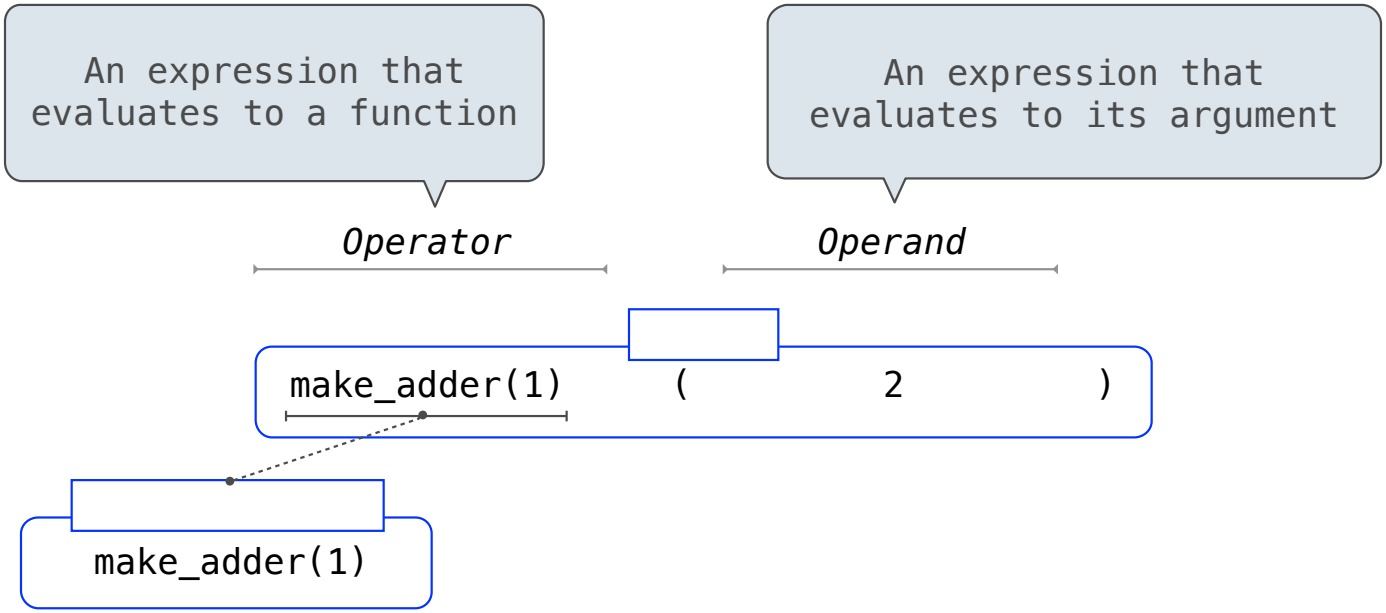

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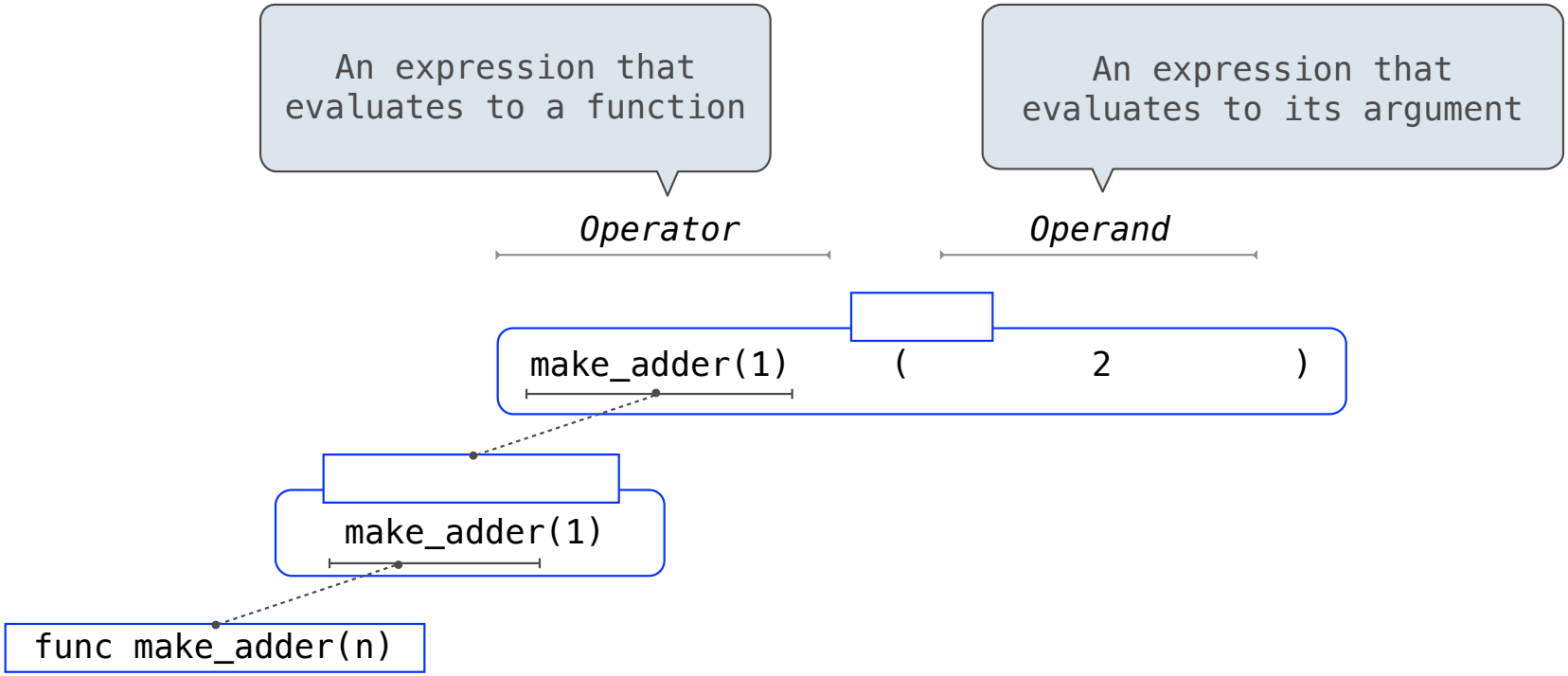

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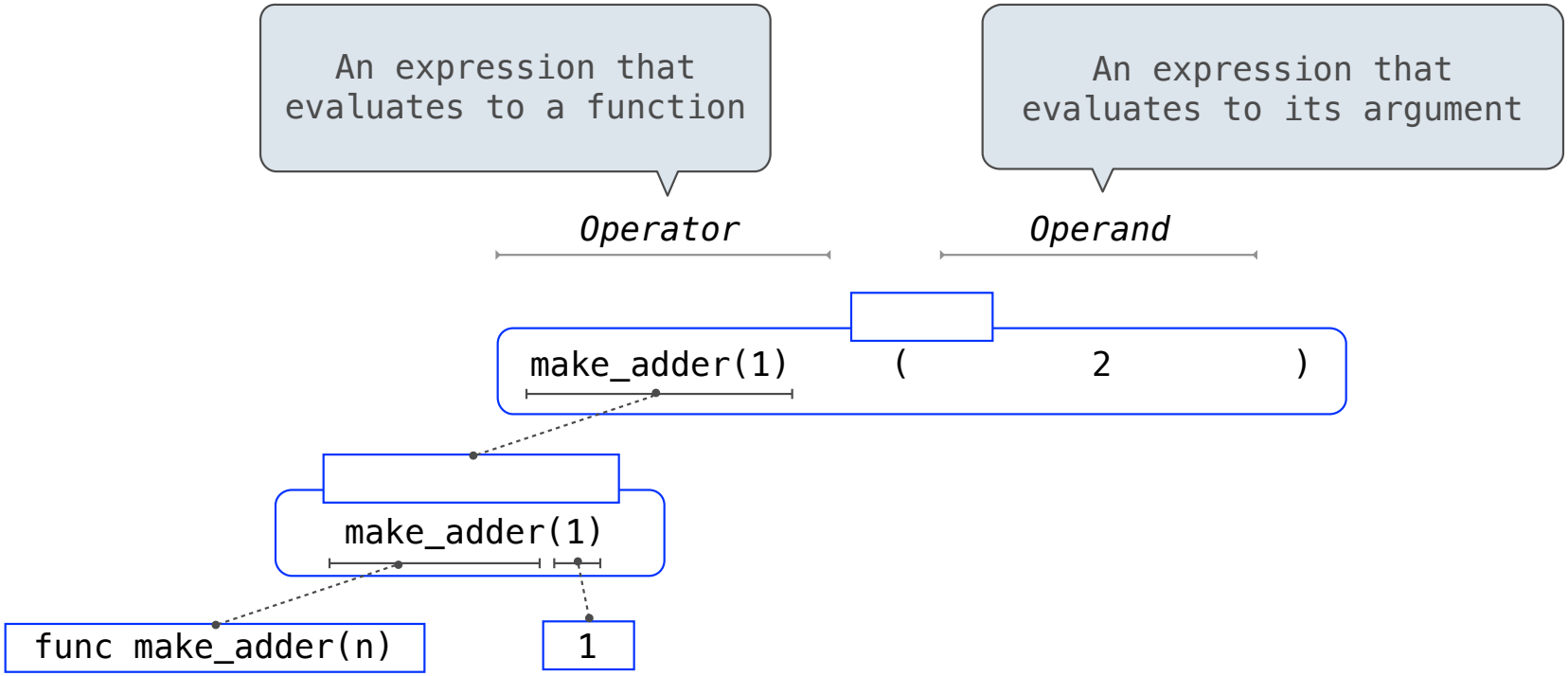

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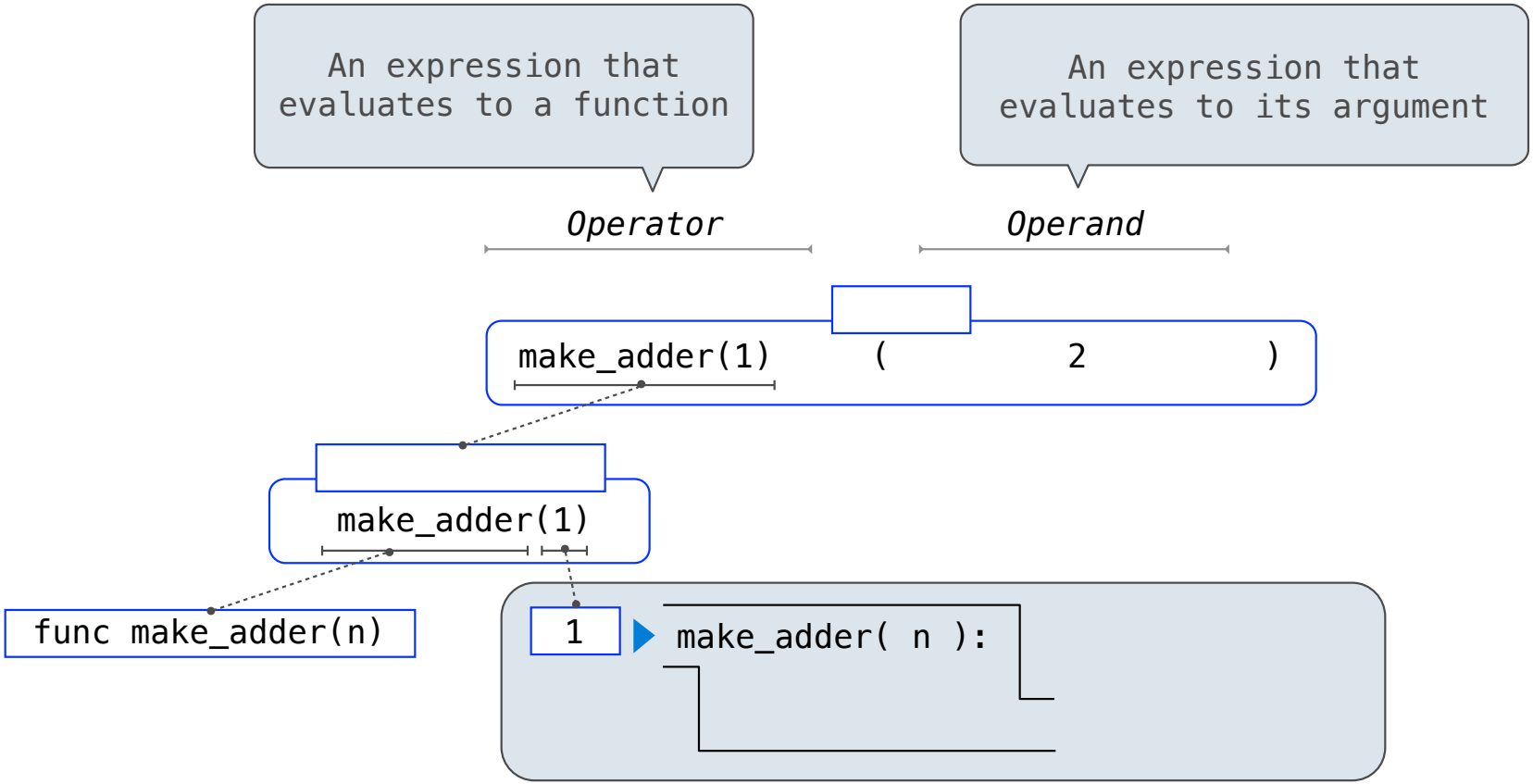

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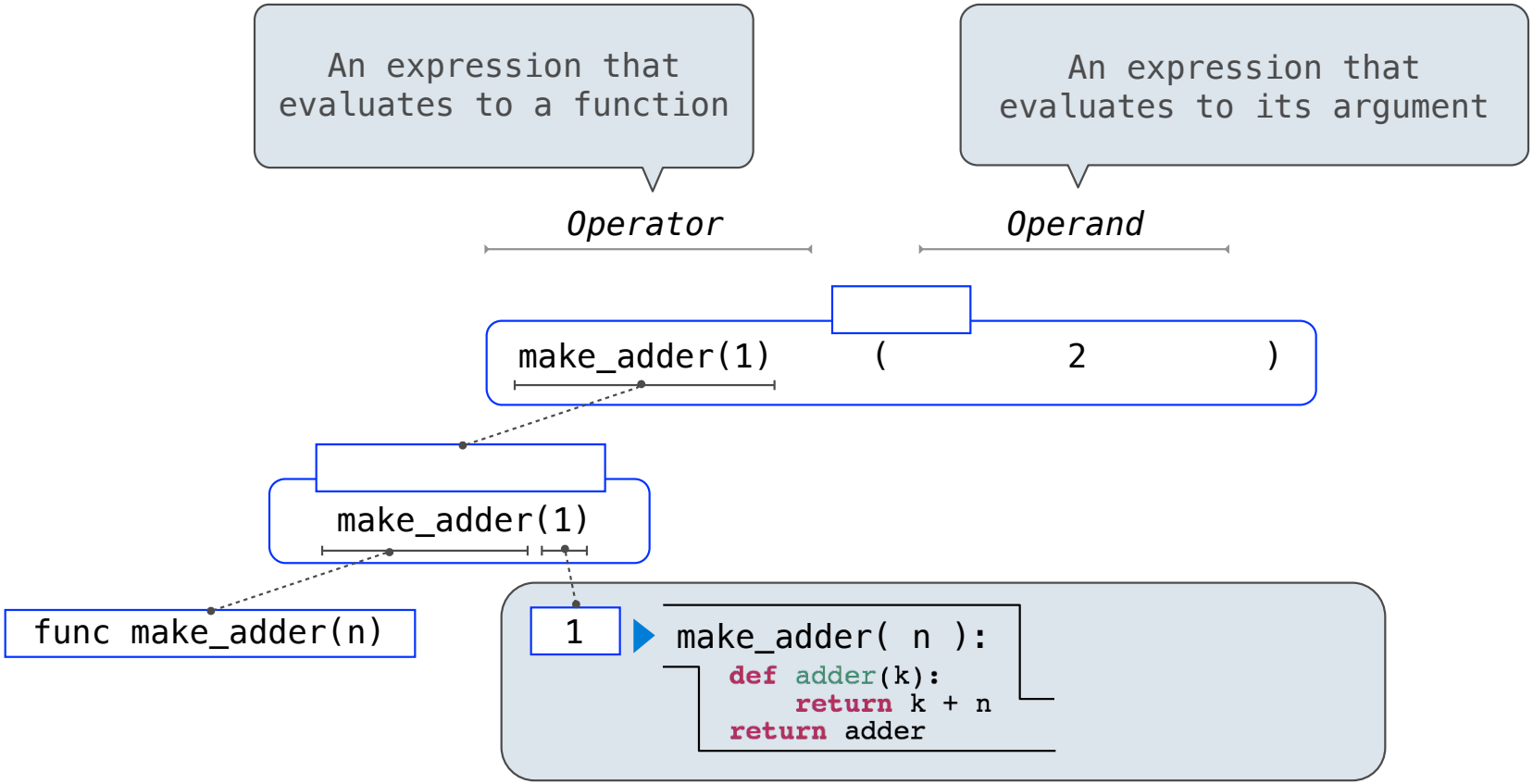

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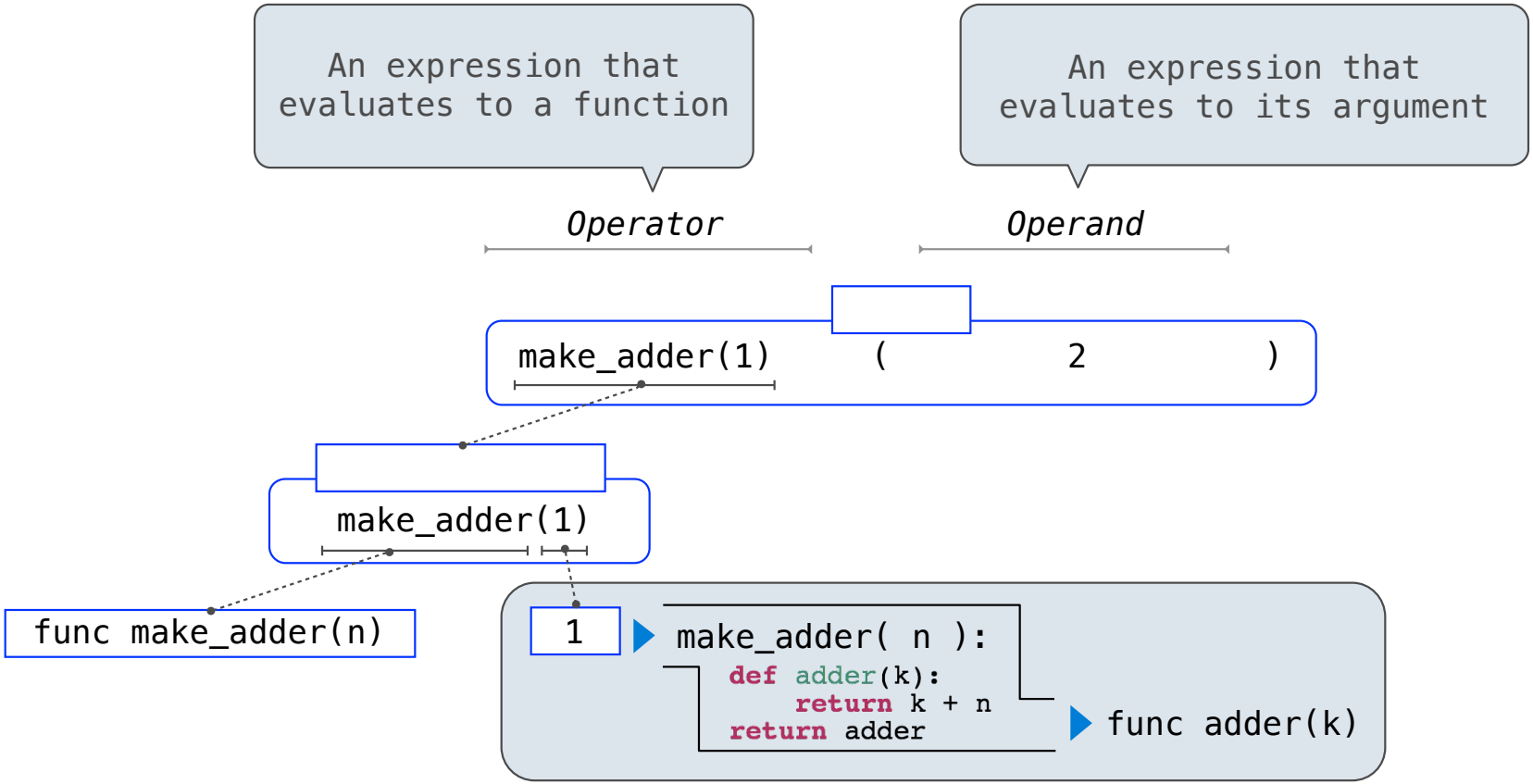

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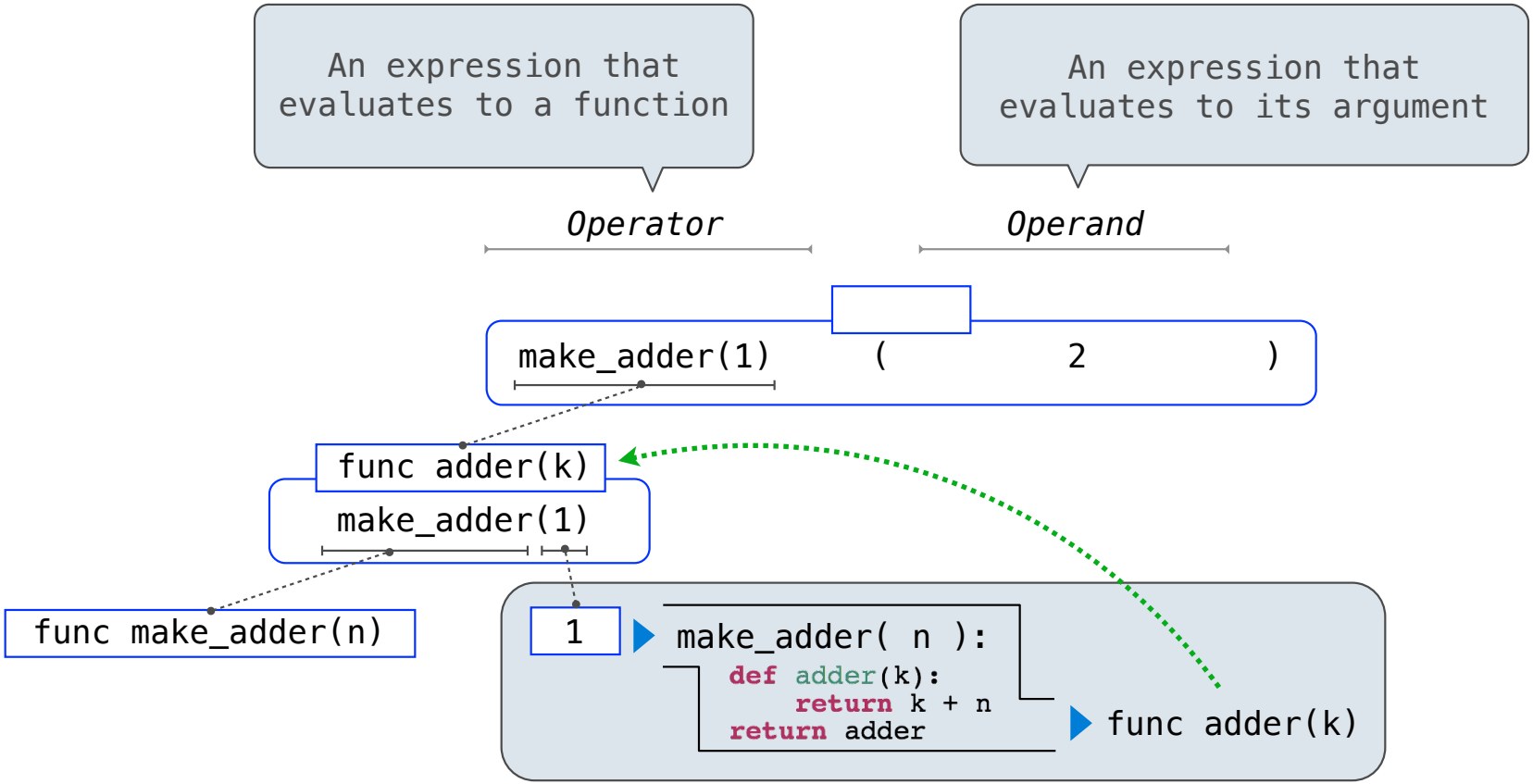

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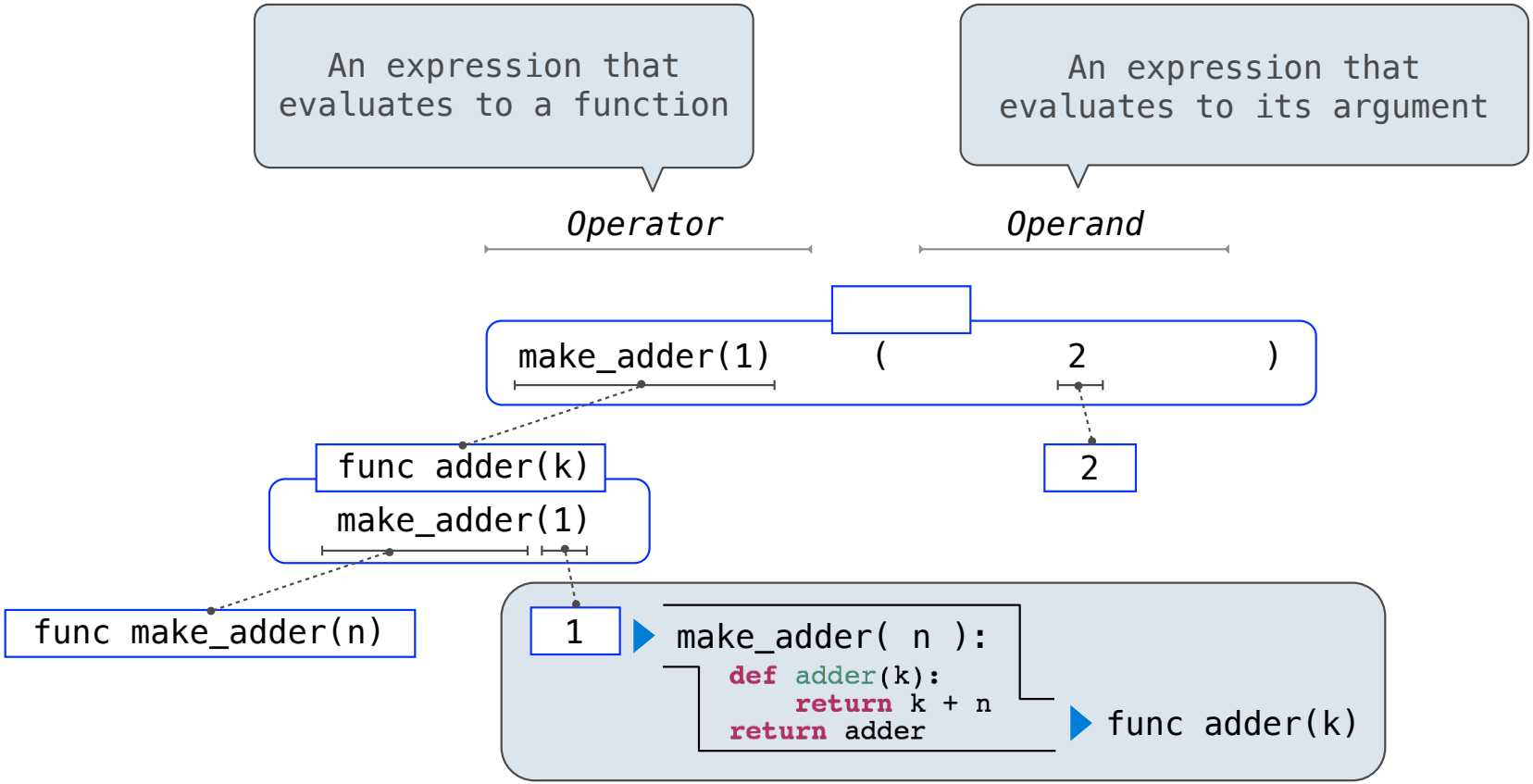

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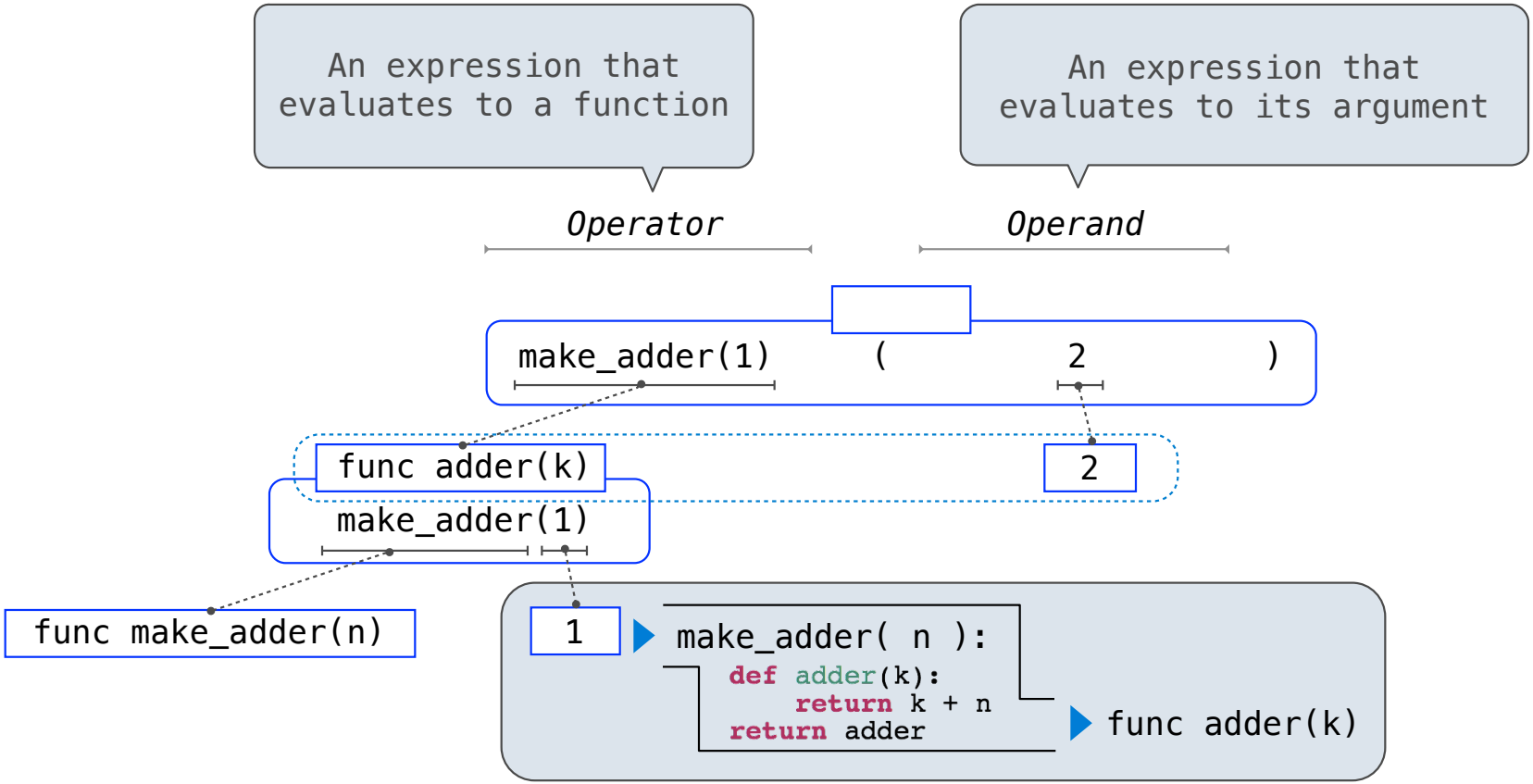

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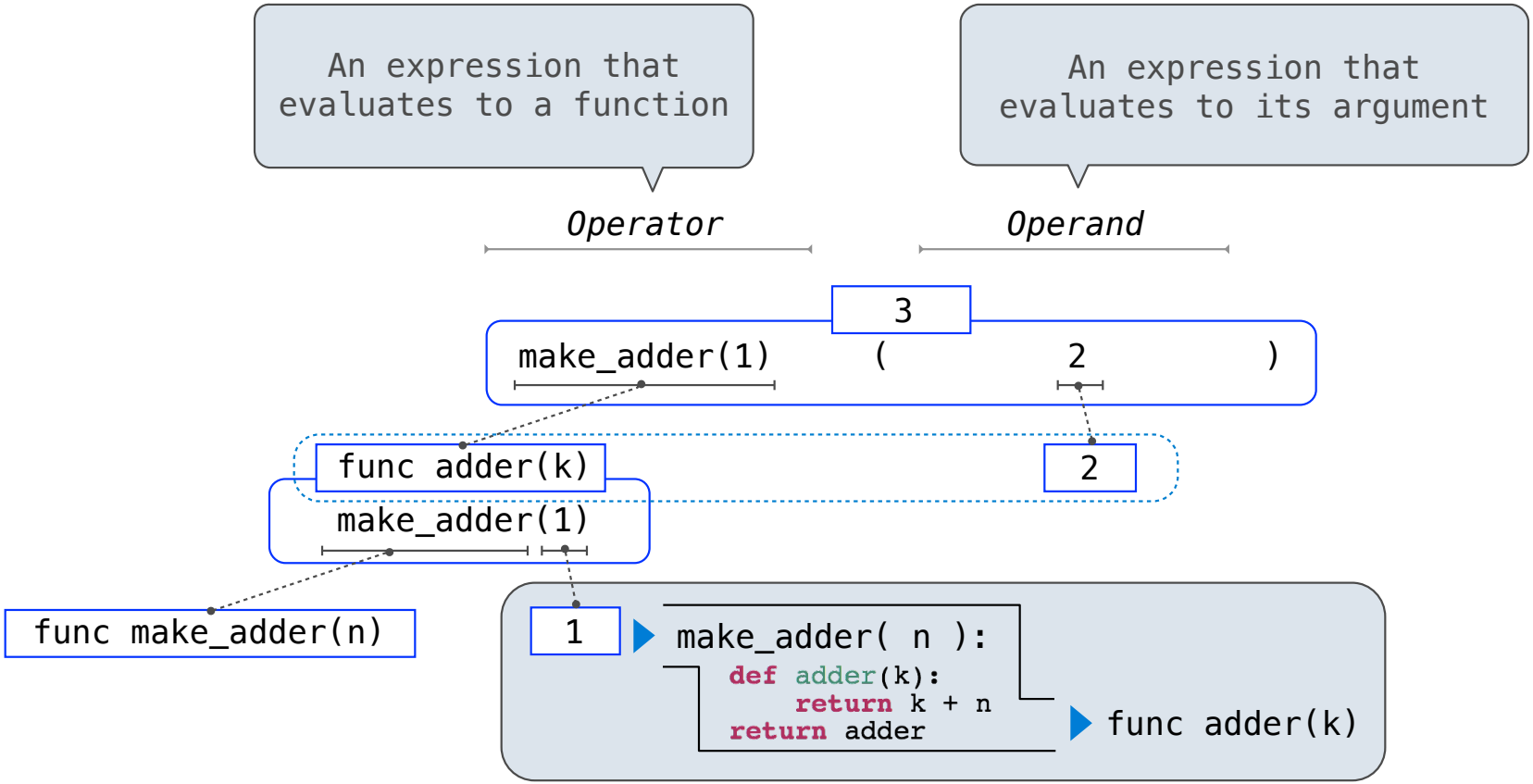
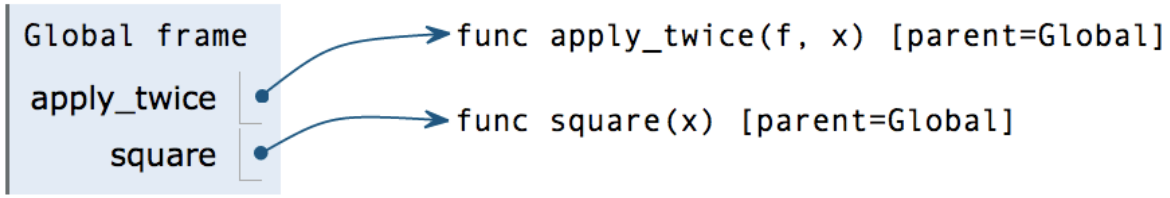


Environments for Higher-Order Functions

Names can be Bound to Functional Arguments

```
1  def apply_twice(f, x):
2      return f(f(x))
3
4  def square(x):
5      return x * x
6
7  result = apply_twice(square, 2)
```



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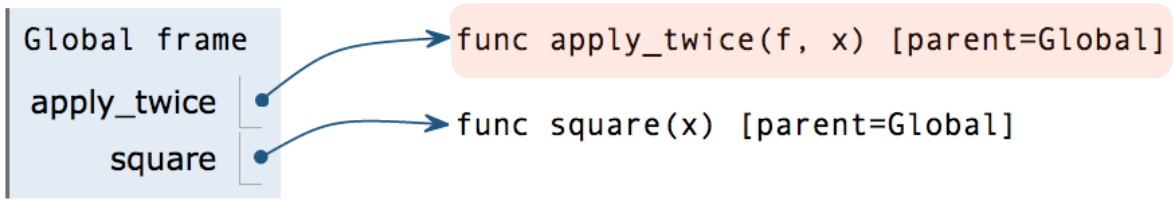

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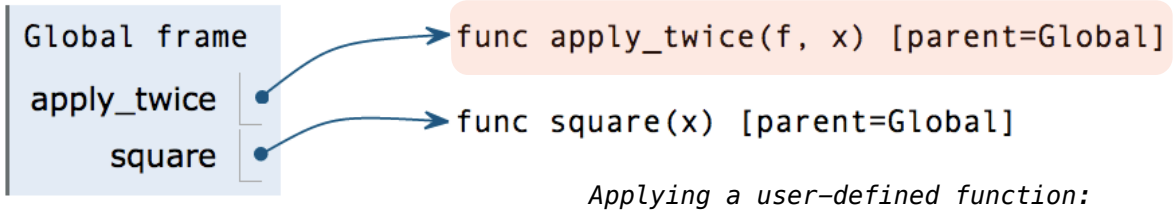

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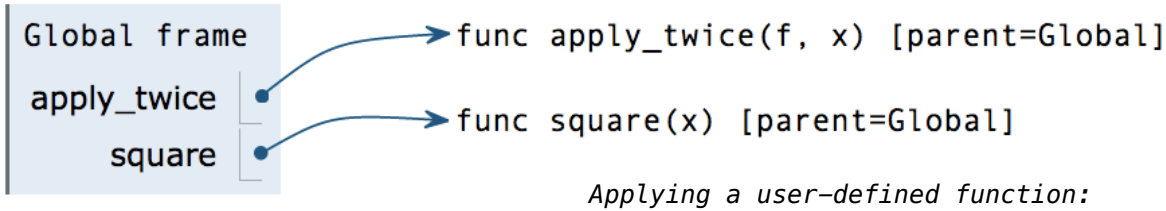

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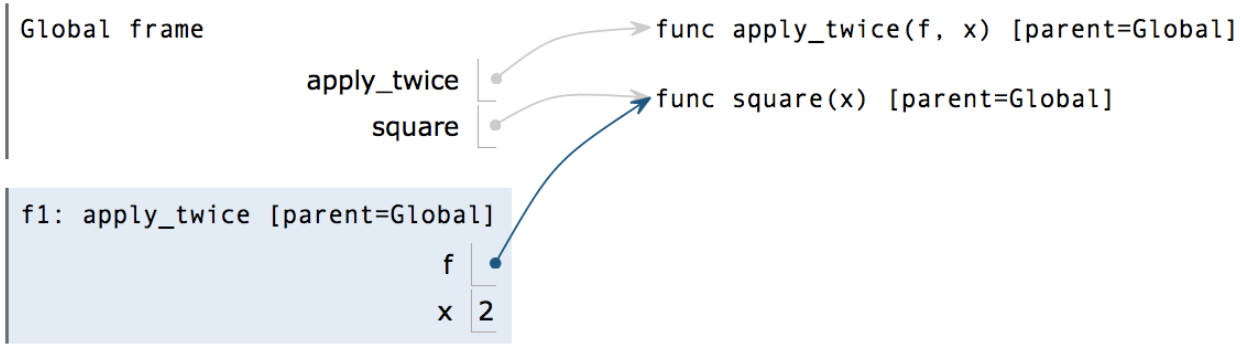

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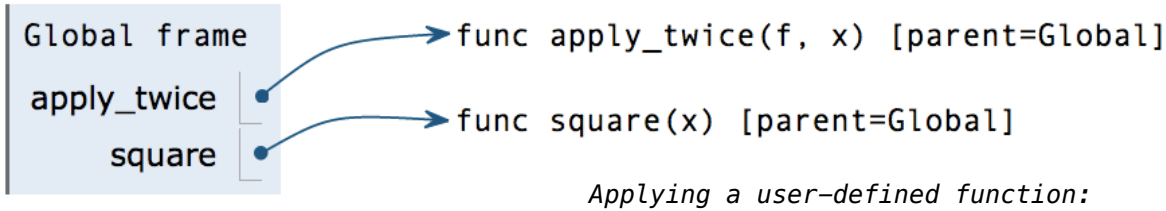

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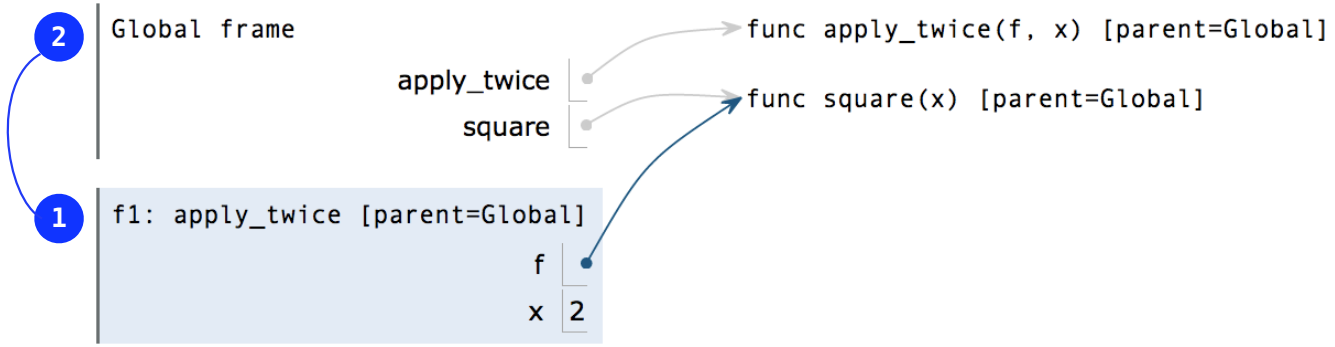

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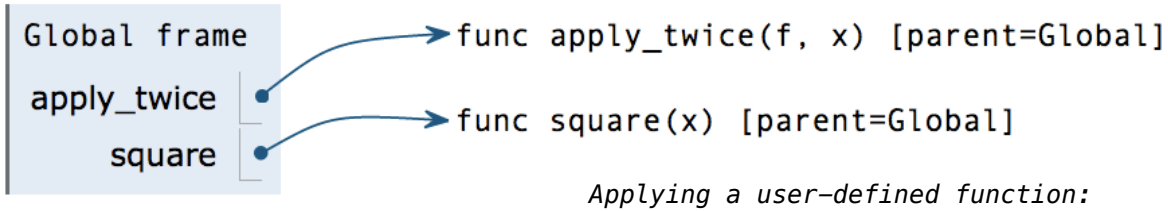

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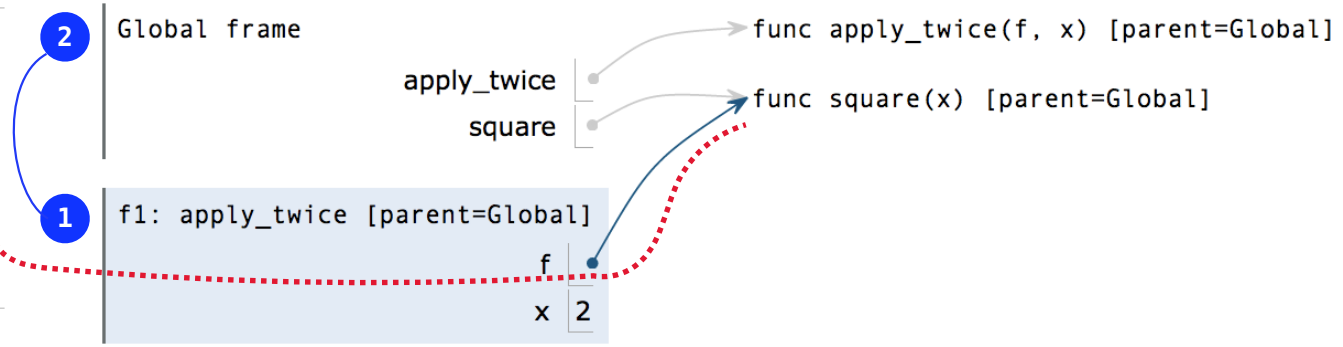

Interactive Diagram

Discussion Question

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def g(y):
    return (y + 5) // 3

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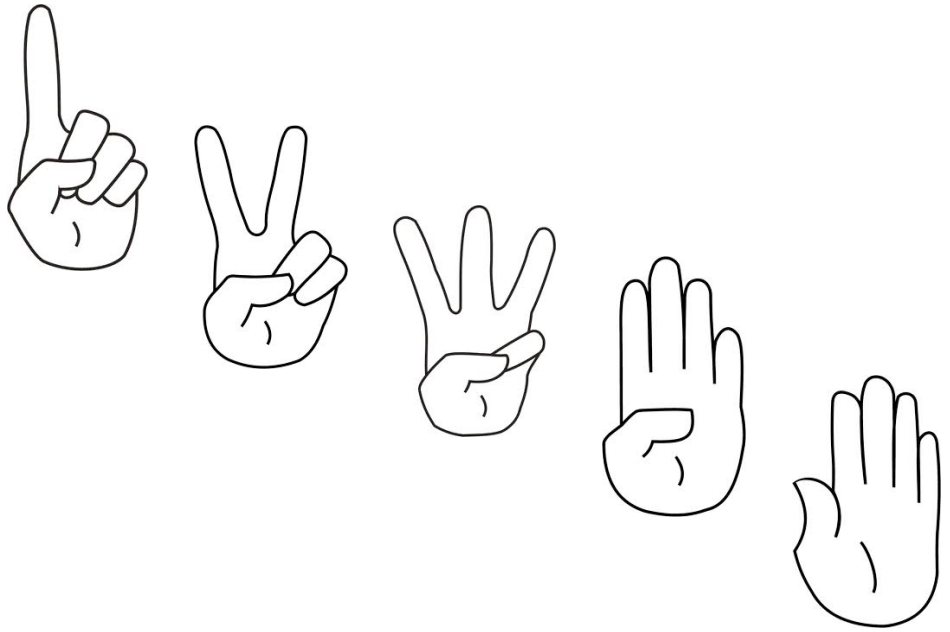

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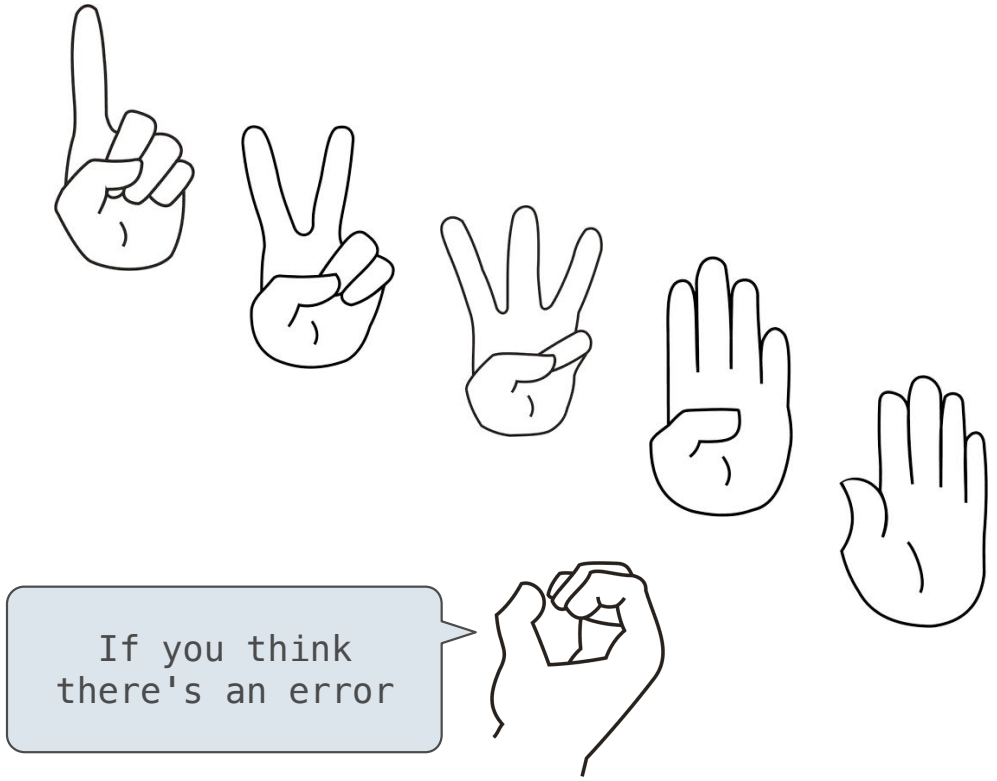

Interactive Diagram