Lidar-Lidar Extrinsic Matrix Manual Calibration User Guide

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1 Notation

Consider a point in left lidar's frame $P_l = (x_l, y_l, z_l)^T$, its corresponding 3D point in top-center lidar's frame is $P_t = (x_t, y_t, z_t)^T$. The extrinsic matrix from left lidar to top-center lidar's frame is $\boldsymbol{E}_{l2t}$. Therefore, we have:

$$P_t = \boldsymbol{E}_{l2t} P_l \tag{1}$$

Consider a point in right lidar's frame $P_r = (x_r, y_r, z_r)^T$, its corresponding 3D point in top-center lidar's frame is $P_t = (x_t, y_t, z_t)^T$. The extrinsic matrix from right lidar to top-center lidar's frame is $\boldsymbol{E}_{r2t}$. Therefore, we have:

$$P_t = \boldsymbol{E}_{r2t} P_r \tag{2}$$

$\boldsymbol{E}_{l2t}$ and $\boldsymbol{E}_{r2t}$ could be formualted as:

$$\boldsymbol{E}_{l2t} = \begin{bmatrix} \boldsymbol{R}_{l2t}^{(3\times3)} & \boldsymbol{T}_{l2t}^{(3\times1)} \\ \boldsymbol{0}_{1\times3} & \boldsymbol{1}_{1\times1} \end{bmatrix}, \boldsymbol{E}_{r2t} = \begin{bmatrix} \boldsymbol{R}_{r2t}^{(3\times3)} & \boldsymbol{T}_{r2t}^{(3\times1)} \\ \boldsymbol{0}_{1\times3} & \boldsymbol{1}_{1\times1} \end{bmatrix} \tag{3}$$

$\boldsymbol{T}_{l2t}^{(3\times1)}$ is the coordinate of the left lidar's origin in top-center lidar's frame.
$\boldsymbol{T}_{r2t}^{(3\times1)}$ is the coordinate of the right lidar's origin in top-center lidar's frame.

2 Notation

Consider a pixel point of lane lines in image plane $P_i = (u, v, 1)^T$, its corresponding 3D point in camera frame is noted as $P_c = (x_c, y_c, z_c)^T$ and its representation in ego frame is $P_w = (x_w, z_w, z_w)^T$.

The rotation matrix from world frame to camera frame is represented as $\boldsymbol{R}_{3\times3}$.
The position of camera frame's origin point in world frame is represented as $\boldsymbol{O}_{3\times1}$, we have extrinsic translation matrix $\boldsymbol{T}_{3\times1} = \boldsymbol{R}_{3\times3}\boldsymbol{O}_{3\times3}$.
The camera intrinsic matrix is represented as $\boldsymbol{I}_{3\times3}$.

3 Formualation

Follow the camera extrinisc and intrinsic model, the relationship between P_w and P_c, P_c and P_i are formulated as:

$$P_c = \boldsymbol{R}_{3\times3} P_w - \boldsymbol{T}_{3\times1} \tag{4}$$

$$P_i = \frac{1}{z_c} \boldsymbol{I}_{3\times3} P_c \tag{5}$$

respectively. Combine Equation 1 and 2, we can obtain:

$$z_c \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \boldsymbol{I}_{3\times3}\boldsymbol{R}_{3\times3} \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix} - \boldsymbol{I}_{3\times3}\boldsymbol{T}_{3\times1} \tag{6}$$

Therefore,

$$\boldsymbol{R}_{3\times3}^{-1}\boldsymbol{I}_{3\times3}^{-1} \left(z_c \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} + \boldsymbol{I}_{3\times3}\boldsymbol{T}_{3\times1} \right) = \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix} \tag{7}$$

In the equation above, it is worth mentioning that the extrinsic matrix $\boldsymbol{R}_{3\times3}$, $\boldsymbol{T}_{3\times1}$ and the camera intrisic matrix $\boldsymbol{I}_{3\times3}$ are calibirated in advance and already known. However, the depth z_c is unknown and it is crital for calculating P_w.

We introduce three assumptions:
1. the ground plane is extremely flat;
2. the ego frame's $XY - plane$ is parallel with ground;
3. the origin point of ego frame is with height H_t with respect to ground.

Then, we could config z_w in Equation 4 to $-H_t$, we represent $\boldsymbol{R}_{3\times3}^{-1}\boldsymbol{I}_{3\times3}^{-1}$ and $\boldsymbol{I}_{3\times3}\boldsymbol{T}_{3\times1}$ as below:

$$\boldsymbol{R}_{3\times3}^{-1}\boldsymbol{I}_{3\times3}^{-1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \boldsymbol{I}_{3\times3}\boldsymbol{T}_{3\times1} = \begin{bmatrix} b1 \\ b2 \\ b3 \end{bmatrix} \tag{8}$$

Combine Equation 5 and Equation 4 with condition $z_w = -H_t$, we can obtain:

$$z_c(a_{31}u + a_{32}v + a_{33}) = -H_t - (a_{31}b_1 + a_{32}b2 + a_{33}b3) \tag{9}$$

i.e, the depth z_c is:

$$z_c = \frac{-H_t - (a_{31}b_1 + a_{32}b2 + a_{33}b3)}{a_{31}u + a_{32}v + a_{33}} \tag{10}$$

After z_c is calculated, then the corresponding point P_w could be easily obtained with Equation 4.