Solutions to Quizzes

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Quiz 11:

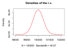
If X_n is $\mathrm{AN}(n, 2n)$, then so is $\frac{n-1}{n}X_n$, but NOT $\frac{n^{1/2}-1}{n^{1/2}}X_n$, why? and find a r.v. X_n s.t. $X_n \sim \mathrm{AN}(n, 2n)$

Solution to Quiz 11

Proof:

According to Lemma 23, if X_n is $\mathrm{AN}(n, 2n)$, then also $a_n X_n + b_n$ is $\mathrm{AN}(n, 2n) \iff a_n \to 1, \frac{\mu_n(a_n - 1) + b_n}{\sigma_n} \to 0$.

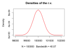
- For $\frac{n-1}{n} X_n$:
  $a_n = \frac{n-1}{n}, b_n = 0$, we have obviously
  $a_n = \frac{n-1}{n} \to 1, \frac{\mu_n(a_n - 1) + b_n}{\sigma_n} = \frac{n(\frac{n-1}{n} - 1) + 0}{\sqrt{2n}} = \frac{-1}{\sqrt{2n}} \to 0$
  By Lemma 23, $\frac{n-1}{n} X_n$ is also $\mathrm{AN}(n, 2n)$.



Solution to Quiz 11

Proof:

- For $\frac{n^{1/2}-1}{n^{1/2}} X_n$:
  $a_n = \frac{n^{1/2}-1}{n^{1/2}}$, $b_n = 0$, we have obviously
  $a_n = \frac{n^{1/2}-1}{n^{1/2}} \to 1$, $\frac{\mu_n(a_n-1)+b_n}{\sigma_n} = \frac{n(\frac{n^{1/2}-1}{n^{1/2}}-1)+0}{\sqrt{2n}} = -\sqrt{2}/2 \nrightarrow 0$
  By Lemma 23, $\frac{n^{1/2}-1}{n^{1/2}}$ is not $AN(n, 2n)$.



Solution to Quiz 11

Proof:

- For $\frac{n^\alpha - 1}{n^\alpha} X_n$ to follow $\mathrm{AN}(n, 2n)$, it must satisfy the following two conditions:

  1. $a_n = \frac{n^\alpha - 1}{n^\alpha} \to 1$
  2. $\frac{n\left(\frac{n^\alpha - 1}{n^\alpha} - 1\right) + 0}{\sqrt{2n}} \to 0$

  From 1, we have $\alpha > 0$;
  From 2, we have $\frac{-\sqrt{n}}{\sqrt{2}n^\alpha} \to 0$, thus $\alpha > 1/2$.
  Combining these 2 rsults, $\frac{n^\alpha - 1}{n^\alpha} X_n$ follows $\mathrm{AN}(n, 2n)$ when $\alpha > 1/2$.