Quiz 13 - Derive the above m.f. for $X \sim \mathrm{N}(\mu, \sigma^2)$ and $X \sim \mathrm{B}(1, p)$

- For continuous $r.v.X \in \mathbb{R}$, moment generating function is $M_X(t) = \mathrm{E}[e^{tX}] = \int_{-\infty}^{\infty} e^{tx} f(x) dx$:
  Therefore for $X \sim \mathrm{N}(\mu, \sigma^2)$, we have

$$
\begin{aligned}
M_X(t) = \mathrm{E}[e^{tX}] &= \int_{-\infty}^{\infty} e^{tx} \cdot \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx \\
&= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{[x-(\mu+t\sigma^2)]^2}{2\sigma^2}} \cdot e^{(t\mu + t^2\sigma^2/2)} dx \\
&= 1 \cdot exp(t\mu + t^2\sigma^2/2) = exp(t\mu + t^2\sigma^2/2)
\end{aligned}
$$

Quiz 13 - Derive the above m.f. for $X \sim \mathrm{N}(\mu, \sigma^2)$ and $X \sim \mathrm{B}(1, p)$

- For discrete $r.v. X \in \mathbb{Z}$, moment generating function is
  $M_X(t) = \sum_i e^{tx_i} \cdot p_i$
  Therefore for $X \sim \mathrm{B}(1, p)$, we have

$$M_X(t) = e^{t \cdot 0} \cdot (1 - p) + e^{t \cdot 1} \cdot p = 1 - p + pe^t$$