Quiz 11: If X_n is $\mathrm{AN}(\mu_n, \sigma_n^2)$, then so is $\frac{n-1}{n}X_n$, but NOT $\frac{n^{1/2}-1}{n^{1/2}}X_n$, why? and find a $r.v. X_n s.t. X_n \sim \mathrm{AN}(n, 2n)$

Proof:

According to Lemma 23, if X_n is $\mathrm{AN}(n, 2n)$, then also $a_n X_n + b_n$ is $\mathrm{AN}(n, 2n) \iff a_n \to 1, \frac{\mu_n(a_n-1)+b_n}{\sigma_n} \to 0.$

- For $\frac{n-1}{n}X_n$:
  $a_n = \frac{n-1}{n}, b_n = 0$, we have obviously
  $a_n = \frac{n-1}{n} \to 1, \frac{\mu_n(a_n-1)+b_n}{\sigma_n} = \frac{n(\frac{n-1}{n}-1)+0}{\sqrt{2n}} = \frac{-1}{\sqrt{2n}} \to 0$
  By Lemma 23, $\frac{n-1}{n}X_n$ is also $\mathrm{AN}(n, 2n)$.

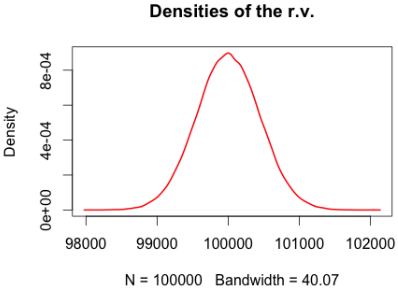
Quiz 11: If X_n is $\mathrm{AN}(\mu_n, \sigma_n^2)$, then so is $\frac{n-1}{n}X_n$, but NOT $\frac{n^{1/2}-1}{n^{1/2}}X_n$, why? and find a $r.v. X_n s.t. X_n \sim \mathrm{AN}(n, 2n)$

Proof:

- For $\frac{n^{1/2}-1}{n^{1/2}}X_n$:
  $a_n = \frac{n^{1/2}-1}{n^{1/2}}$, $b_n = 0$, we have obviously
  $a_n = \frac{n^{1/2}-1}{n^{1/2}} \to 1$, $\frac{\mu_n(a_n-1)+b_n}{\sigma_n} = \frac{n(\frac{n^{1/2}-1}{n^{1/2}}-1)+0}{\sqrt{2n}} = -\sqrt{2}/2 \nrightarrow 0$
  By Lemma 23, $\frac{n^{1/2}-1}{n^{1/2}}$ is not $\mathrm{AN}(n, 2n)$.

Quiz 11: If X_n is $\text{AN}(\mu_n, \sigma_n^2)$, then so is $\frac{n-1}{n}X_n$, but NOT $\frac{n^{1/2}-1}{n^{1/2}}X_n$, why? and find a $r.v. X_n s.t. X_n \sim \text{AN}(n, 2n)$

An example $r.v. X_n$ satisfies $X_n \sim \text{AN}(n, 2n)$



Quiz11-MSM.R