Quiz 11

by Björn Bokelmann

Given that $X_n = AN(n, 2n)$, why does $\frac{n-1}{n} X_n = AN(n, 2n)$ hold and why is $\frac{n^{1/2}-1}{n^{1/2}} X_n \not\sim AN(n, 2n)$?

First we define $a_n = \frac{n-1}{n}$, $b_n = 0$, $\mu_n = n$, $\sigma_n = \sqrt{2n}$.
Then $X_n = AN(\mu_n, \sigma_n)$. Further it holds

$$a_n = \frac{n-1}{n} = 1 + \frac{1}{n} \overset{n\to\infty}{\to} 1$$

and

$$\frac{\mu_n(a_n - 1) + b_n}{\sigma_n} = \frac{n(\frac{n-1}{n} - 1)}{\sqrt{2n}} = \frac{-1}{\sqrt{2n}} \overset{n\to\infty}{\to} 0 \ .$$

After Lemma 23 $\frac{n-1}{n} X_n$ is $AN(\mu_n, \sigma_n)$.

Given that $X_n = AN(n, 2n)$, why does $\frac{n-1}{n} X_n = AN(n, 2n)$ hold and why is $\frac{n^{1/2}-1}{n^{1/2}} X_n \not\sim AN(n, 2n)$?

Again we define $a_n = \frac{n^{1/2}-1}{n^{1/2}}$, $b_n = 0$, $\mu_n = n$, $\sigma_n = \sqrt{2n}$.
Then it holds

$$\frac{\mu_n(a_n - 1) + b_n}{\sigma_n} = \frac{n(\frac{n^{1/2}-1}{n^{1/2}} - 1)}{\sqrt{2n}} = \frac{n - n^{1/2} - n}{\sqrt{2n}} = -\frac{1}{\sqrt{2}} \overset{n\to\infty}{\not\to} 0$$

Again after Lemma 23 we can conclude that $a_n X_n \not\sim AN(n, 2n)$.

An example for a sequence $X_n \sim AN(n, 2n)$ is
$X_n = 3\sum_{i=1}^{n} Y_n$ with $Y_n \sim Ber(\frac{1}{3})$.

Proof.
Because $E[3Y_i] = 3 \cdot \frac{1}{3} = 1$ and $Var[3Y_i] = 9 \cdot \frac{1}{3}(1 - \frac{1}{3}) = 2$ it holds after the Central Limit Theorem

$$\frac{\sum_{i=1}^{n} 3Y_n - n}{\sqrt{2n}} \xrightarrow{\mathcal{L}} N(0,1)$$

Thus $X_n = \sum_{i=1}^{n} 3Y_n \sim AN(n, 2n)$. $\square$