

Joint UV Optimization and Texture Baking

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Level of detail (LOD) has been widely used in interactive computer graphics. In current industrial 3D modeling pipelines, artists rely on commercial software to generate highly detailed models with UV maps, and then bake textures for low-poly counterparts. In these pipelines, each step is performed separately, leading to unsatisfactory visual appearances for low polygon count models. Moreover, existing texture baking techniques assume the low-poly mesh has a small geometric difference from the high-poly, which is often not true in practice, especially with extremely low poly count models.

To alleviate the visual discrepancy of the low-poly mesh, we propose to jointly optimize UV mappings during texture baking, allowing for low-poly models to faithfully replicate the appearance of the high-poly even with large geometric differences. We formulate the optimization within a differentiable rendering framework, allowing the automatic adjustment of texture regions to encode appearance information. To compensate for view parallax when two meshes have large geometric differences, we introduce a Spherical Harmonic (SH) parallax mapping, which uses SH functions to modulate per-texel UV coordinates based on the view direction. We evaluate the effectiveness and robustness of our approach on a dataset composed of online downloaded models, with varying complexities and geometric discrepancies. Our method achieves superior quality over state-of-the-art techniques and commercial solutions.

CCS Concepts: • Computing methodologies → Modeling and simulation.

Additional Key Words and Phrases: Texture Baking, Differentiable Rendering, UV Optimization

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1 INTRODUCTION

The tradeoff between visual fidelity and high performance has been a central topic in interactive computer graphics for decades and will continue to be. Despite tremendous strides in graphics hardware, user requirements for highly realistic content always necessitates rendering more polygons than what is affordable for both high-end and low-end platforms. In light of that, level of detail (LOD) techniques have been widely used to render a low-poly model with fewer details when the model is small, distant, or unimportant, while rendering a high-resolution, detailed model when the viewer is close.

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In industrial 3D modeling pipelines, artists create a high-poly model and rely on commercial software to generate low-poly counterparts to replicate the appearance. Typically, 3D artists first generate a low-poly mesh from its high-poly counterpart, then create a UV mapping for the low-poly mesh, and finally bake textures from the high-poly using commercial tools, such as Marmoset [Marmoset, 2022], Blender [Blender Online Community, 2018], Unreal Engine (UE) [Epic Games, 2022], Geogram [Lévy, 2019], InstaLOD [Nerurkar, 2021], Rapid Compact [DGG, 2018] and Substance Painter [Adobe, 2014]. This pipeline can often generate unsatisfactory results, requiring manual intervention due to several issues. First, the results of texture baking heavily rely on the outcome of UV mapping, but the quality of a UV mapping is not taken into account during texture baking, as they are performed independently. Without appearance-aware UV mappings, texels can be wasted on small or unimportant mesh regions, leaving salient regions ill-represented. Moreover, existing texture baking techniques assume there are small geometric differences between the low-poly and high-poly models, which is often not the case in practice. With large geometric discrepancies, both raycasting-based [Engel, 2019] and differentiable-rendering-based [Hasselgren et al., 2021, Li et al., 2018a, Liu et al., 2019, Loper and Black, 2014, Nimier-David et al., 2019] texture bakers suffer from view-parallax artifacts [Kaneko et al., 2001]. These artifacts manifest as strange copies from other parts of the model or blurry regions with unrecognizable features (Fig. 1). Given these issues, users must manually tune the interwoven factors of the model including the geometry, the UV parameterization, and the texture map, to achieve a satisfactory appearance.

Inspired by recent advances in differentiable rendering [Hasselgren et al., 2021], we propose a novel appearance baker that is robust to geometric differences. The novelty of our method lies in two aspects. First, our method jointly optimizes the texture map and UV coordinates guided by a visual difference metric. We alternatively update the texture content and UV parameterization, which effectively enlarges the solution space for high visual quality. Second, our method compensates for large geometric discrepancies using an optimized view parallax mapping. Specifically, we introduce *Spherical Harmonic (SH) Parallax mapping*, which uses a SH texture map to store coefficients for nonlinear Spherical Harmonic functions that shifts per-texel UV coordinates based on the view direction. The SH map is optimized jointly with the UV and texture maps to minimize the visual differences. The introduced SH map adds only a marginal overhead to the rendering pipeline but offers additional degrees of freedom to significantly improve visual similarity between the low-poly and the high-poly.

We have evaluated our method with 26 high-poly meshes and 78 meshes with different level-of-details, geometric discrepancies,

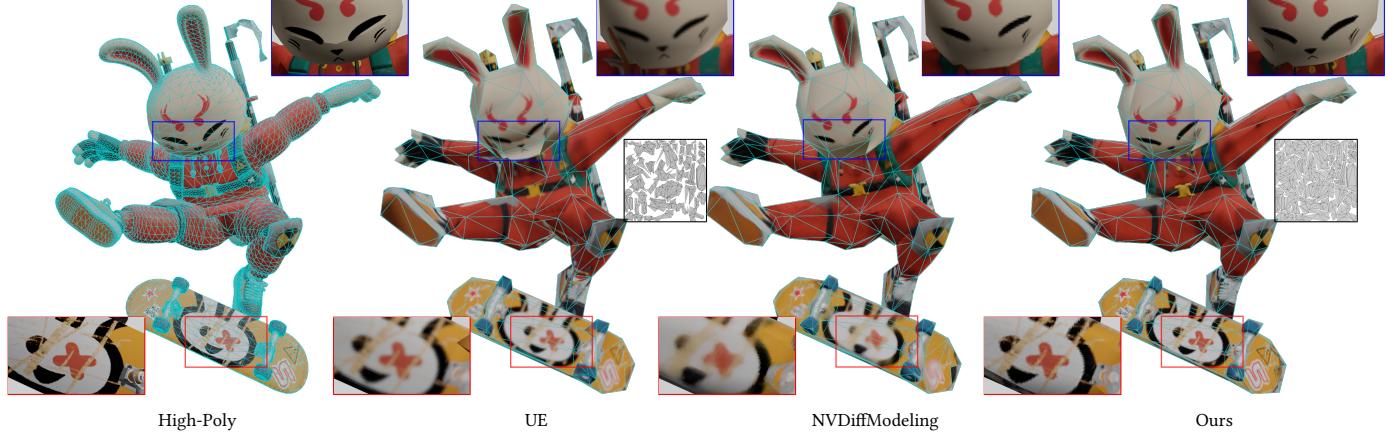


Fig. 1. Compared with the high-poly (#Faces = 34K), low-polys (#Faces = 704) with textures baked using either the raycasting approach from UE [Epic Games, 2022] or the differentiable rendering approach from NVDiffModeling [Hasselgren et al., 2021] that exhibit various blurry artifacts (red) or inaccurate geometric details (blue), our method demonstrates the best baking quality. The PSNR/MS-SSIM values (higher is better) of the results generated by UE, NVDiffModeling, and ours are 21.17/0.921, 23.46/0.953, and 27.05/0.979, respectively. We also visualize the UV mesh for the original low-poly mesh (middle) and ours after optimization (far-right).

and appearance complexities. Compared with state-of-the-art approaches and commercial solutions, our method consistently delivers higher visual similarity at the same textures resolutions, in both PSNR and MS-SSIM. We attach the tested dataset and the results corresponding to all the figures in an anonymous github repository at https://anonymous.4open.science/r/tex_baking_supplementary/, including meshes, textures, and video clips of the final rendered results of each model in the dataset.

2 RELATED WORKS

In the following section, we review prior works on UV parameterization, texture baking, and bump maps that are related to our work.

2.1 UV Parameterization

UV parameterization maps 2D textures to 3D meshes and plays a central role in visual appearances. Given a mesh, generating a UV map typically involves three steps: 1) cutting open the mesh into one or more “charts”, each of which can be flattened onto the 2D plane without introducing flips; 2) UV distortion minimization of all charts which involves reshaping and moving each chart; 3) packing optimized charts into an “atlas” [Sander et al., 2003, Young, 2017] according to texture resolutions desired by users. UV parameterization has been studied for decades, with early work [Lévy et al., 2002, Tutte, 1963] focusing on mapping from meshes isomorphic to disks to 2D planes. Recent work aims at improving the mapping quality by providing local injectivity [Rabinovich et al., 2017] or bijectivity [Jiang et al., 2017, Smith and Schaefer, 2015] guarantees. These works typically start from a valid initialization by Tutte’s approach [Tutte, 1963] and then improve the quality by minimizing geometric distortion energies such as conformal mapping, Symmetric Dirichlet, AMIPS, or ARAP [Rabinovich et al., 2017, Smith and Schaefer, 2015]. Local injectivity and bijectivity guarantees are generally achieved via free-form boundary-checked

optimizations [Smith and Schaefer, 2015] or scaffolding-based approaches [Jiang et al., 2017]. Among the abundant UV optimization techniques, few directly incorporate appearance attributes into the optimization process. The primary example of appearance aware optimization is [Sander et al., 2002], which optimizes the UV parameterization according to a measure of the complexity of given textures. We also note another recent work [Sun et al., 2022] with a similar goal of optimizing UV parameterization to improve texture efficiency. In this method, the shape of each parameterized 2D triangle is optimized by placing a camera facing each triangle to measure visual difference with the original parameterization, while our method focuses on a visual metric for the whole model from a specific distance. Unlike these works [Sander et al., 2002, Sun et al., 2022] that assume textures are constant throughout optimization, we jointly optimize both the texture content and the UV mapping to maximize measured visual similarity.

2.2 Texture Baking

Two techniques are widely used for texture baking: *Ray Casting* and *Differentiable Rendering*. The most widely used texture-baking technique is based on raycasting which has been the default approach in most popular 3D modeling software [Blender Online Community, 2018, Epic Games, 2022, Marmoset, 2022, Pixologic, 2022]. Given the high-poly and the low-poly model with its UV layout, this approach first casts a ray for each surface texel of the low-poly along its normal direction, then computes the interpolated attributes at the intersection point between the ray and the high-poly, and directly assigns the attribute to the texture of the low-poly. There are technical variants of how the origins and directions of the rays are determined [Engel, 2019], but regardless of the various choices of raycasting, ambiguity can happen for regions with significant geometric differences since multiple rays may intersect the high-poly at the same position, producing a baked color map with numerous artifacts. There have also been recent works such as [Jiang

et al., 2020] which produce a bijective mapping between a high-poly mesh and a generated low-poly mesh, but they cannot handle the general case for an arbitrary high-poly and low-poly mesh. Recent advances in differentiable rendering [Hasselgren et al., 2021, Johnson et al., 2020, Laine et al., 2020, Li et al., 2018a, Liu et al., 2019, Loper and Black, 2014, Nimier-David et al., 2019, Patow and Pueyo, 2003] propose to resolve mapping ambiguity by changing the problem to a least-squares minimization of a visual similarity metric. Through auto-differentiation, attributes for the 3D scene can be automatically optimized by stochastic gradient descent [Kingma and Ba, 2015] to minimize visual metrics such as the ℓ_2 pixel-wise loss between rendered images of the low-poly and the high-poly. However, all existing works only consider optimizing the texture while fixing the given UV parameterization. As a result, the optimized textures can oftentimes be sub-optimal, where a large number of texels are wasted on small or unimportant regions, which may not even be visible from a distant view. Although optimizing textures via differentiable rendering is not a new concept [Hasselgren et al., 2021, Laine et al., 2020, Li et al., 2018a, Patow and Pueyo, 2003], our method is the first to jointly optimize UV and texture maps using differentiable rendering, leading to a much larger search space.

2.3 Bump Mapping

In interactive computer graphics, bump mapping refers to a collection of techniques representing small-scale geometry details, which enrich the geometric details of 3D models via pixel or texel perturbations. The distinctions between different bump mapping techniques lay in how they add details. *Normal mapping* [Cohen et al., 1998] changes the geometry's normal to create fine shading details. Since normals do not create a strong illusion of depth, *parallax mapping* [Kaneko et al., 2001] is often added to planar surfaces to add depth-parallax by shifting texture coordinates in the UV space, corresponding to the depth of elements. This idea was further improved by offset limiting [Welsh et al., 2004], which reduces artifacts at steep angles. Later, *steep parallax mapping* was introduced to perform multiple iterations of the UV shift by small steps, where a new depth is queried at each step of a ray-march through the depth map. Such ray-marching stops when it is below the object surface [McGuire and McGuire, 2005] and is then backtracked to the exact ray-object intersection point [Policarpo et al., 2005] or linear interpolation [Tatarchuk, 2006] is used as a fast approximation. These approaches need to perform multiple texture queries, which can cause significant overhead. Also these methods assume that the UV translation is always parallel to the original view direction, even when the UV parameterization has cuts and warps across triangular boundaries. Instead of modifying per-pixel attributes in the screen space, *displacement mapping* [Thonat et al., 2021] uses a height texture to displace vertices' positions in the vertex shader. While this is effective as a general-purpose method for recovering high-quality appearances, it generally requires significantly more computations since it requires the subdivision of the mesh to fit within each pixel, requiring a geometry shader to be employed. In this work, we borrow SH functions from irradiance environment maps [Ramamoorthi and Hanrahan, 2001] to encode geometric details while reducing the number of texture fetches needed.

3 PROBLEM STATEMENT

Given a high-poly model with known textures and a target low-poly model with an initial UV parameterization but without any texture, our goal is to optimize the UV mesh and textures of the low-poly, such that its expected visual appearance under all view directions is as close to the high-poly as possible. One key difference from conventional texture baking lies in the fact that we jointly optimize several new parameters related to texture mapping. This section briefly reviews the forward rendering procedure and introduces related notations.

Texture Mapping. The low-poly mesh $M \triangleq (V, E)$ is composed of a set of vertices $V \subset \mathbb{R}^3$ connected into a graph via edges E representing a triangulation. M is cut open along selected seam boundaries into a set of local charts that can be embedded into \mathbb{R}^2 . The low-poly mesh is mapped through a UV parameterization function ψ to a planar mesh: $M_p \triangleq (U, E_{uv})$ with $U \subset \mathbb{R}^2$. Here ψ is typically chosen as a bijective function such that $\psi(V) = U$ and ψ is affine within each triangle.

Rasterization Pipeline. Following the definition introduced by NVDiffRast [Laine et al., 2020], the final color $I(p)$ of the pixel at screen coordinates $p = (x, y)$ in the forward rendering process can be written in the following form:

$$I(p) = \text{filter}(\text{shade}(d, C_p, N_p, \dots))(p), \quad (1)$$

where the filter is used to sample the color at the pixel center p from the continuous screen coordinates $p = (x, y)$ for antialiasing. The shading function is generic, and can be used for arbitrary rendering. It can takes arbitrary inputs, such as view direction d , color C_p , normal N_p at p . For our results, we use Lambertian shading:

$$\text{shade}(d, C_p, N_p) \triangleq (d \cdot N_p) C_p. \quad (2)$$

In the rasterization-based rendering pipeline, any visible 3D point v can be rasterized on the 2D screen space along the view direction d via a rasterization function R such that $p = R(v, d)$. Assuming R is locally invertible, given a view direction $d \in \text{SO}(3)$, each screen-space pixel p can be associated with a point $v = R^{-1}(p, d) \in \mathbb{R}^3$ on the input mesh. Given the screen coordinate p , the visible surface color C_p is defined as:

$$C_p \triangleq \text{fetch}(\psi(R^{-1}(p, d)), T_C) \quad (3)$$

where T_C is the diffuse texture, and the fetch function gets the texture data by interpolating neighboring texels. Similarly, we have $N_p \triangleq \text{fetch}(\psi(R^{-1}(p, d)), T_N)$ with T_N being the texture for the normal map. While we primarily demonstrate Lambertian shading in most experiments, our approach is able to handle other BSDF models. We demonstrate in Fig. 14 that we can optimize a mesh with specular texture under random lighting or with parallax mapping.

Differentiable Rendering. Recently, NVDiffmodeling [Hasselgren et al., 2021] proposed to resolve texture ambiguity during texture baking in a least-square sense. Differentiable rendering identifies T_C and T_N as the minimizer of the following visual similarity

metric among all view directions:

$$\operatorname{argmin}_{T_C, T_N} \mathbb{E}_{d \in \text{SO}(3)} \left[\int_S \mathcal{L}_{\text{diff}}(p, d) dp \right]$$

$$\mathcal{L}_{\text{diff}}(p, d) \triangleq \|I(p, d, T_C, T_N) - I(p, d, T'_C, T'_N)\|,$$

where S is the screen space and T'_C, T'_N are the textures of the ground truth high-poly. Here we slightly abuse notation and denote $I(p, d, T'_C, T'_N)$ as the rendering function of the high-poly. We also stipulate that, if a 3D point v is occluded, then $p = R(v, d)$ is undefined and $I(p, d, T_C, T_N) = I(p, d, T'_C, T'_N)$. However, all existing differentiable rendering frameworks only consider optimizing the texture while fixing the given UV parameterization. As a potential drawback of such an approach, a large number of texels could be wasted on small or unimportant portions.

4 JOINT OPTIMIZATION FRAMEWORK

Our method differs from existing approaches in two aspects. First, we unify two avenues of prior research, distortion-minimizing [Rabinovich et al., 2017, Smith and Schaefer, 2015] and content-aware [Sander et al., 2002, Sun et al., 2022, Tewari et al., 2004] UV parameterization, using differentiable rendering [Hasselgren et al., 2021]. Second, we propose to optimize a view-parallax mapping [Kaneko et al., 2001] to compensate for potentially large geometric differences between the two meshes.

4.1 Objective Function

It has been shown in prior works [Rabinovich et al., 2017, Sander et al., 2002] that UV parameterization ψ plays an important role in high-quality texture baking. There are two lines of research that use different criteria of a “good” ψ . First, distortion-minimization algorithms such as [Rabinovich et al., 2017] assume that an ideal ψ should be as isometric as possible, i.e., $\nabla\psi$ should be as close to the identity as possible. Follow-up work [Jiang et al., 2017, Su et al., 2020] further ensures ψ is globally bijective, preventing different 3D points from being ambiguously mapped to the same UV coordinate. The other line of research, as in [Sander et al., 2002, Sun et al., 2022], proposes to make ψ content-aware. In doing so, an ideal ψ should assign larger areas to T_C with “richer” color information. However, the area of content-aware texture baking has received less attention than the isometric approach due to the inherent difficulty in measuring the texture information entropy. With the growth of differentiable rendering, measures of visual similarity, $\mathcal{L}_{\text{diff}}$, are ideal candidates for content complexity measures, so we propose to jointly optimize UV coordinates U and texture color/normal maps T_C, T_N . Our augmented optimization problem takes the following form:

$$\operatorname{argmin}_{T_N, T_C, U} \mathbb{E}_{d \in \text{SO}(3)} [\mathcal{L}_{\text{diff}}] + \lambda_{\text{I} \circ \text{UV}} \mathbb{E}_{d \in \text{SO}(3)} [\mathcal{L}_{\text{I} \circ \text{UV}}]$$

s.t. ψ is bijective,

where $\mathcal{L}_{\text{diff}}$ plays a guiding role that provides content to T_N, T_C , and $\mathcal{L}_{\text{I} \circ \text{UV}}$ makes UV coordinates U content aware. $\lambda_{\text{I} \circ \text{UV}}$ denotes a weight coefficient.

A priori, one might assume the image loss, $\mathcal{L}_{\text{diff}}$, alone could recover content-aware UV coordinates, but we find that it does not provide sufficient gradient information to do so, as illustrated

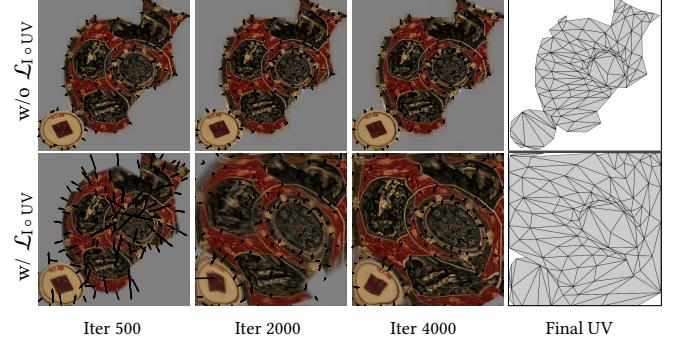


Fig. 2. As compared with our formulation (bottom row), the optimization without the content-aware term (top row) cannot provide sufficient gradient information to make the optimized texture content-aware. Arrows indicate gradient magnitude and direction.

in Fig. 2 and ablated in 5.3. Thus we propose a multiplicative, content-aware image-based symmetric Dirichlet energy $\mathcal{L}_{\text{I} \circ \text{UV}}$ formulated as:

$$\mathcal{L}_{\text{I} \circ \text{UV}}(d) \triangleq \int_M (\|\nabla\psi(v)\|^2 + \|\nabla\psi^{-1}(v)\|^2) \mathcal{L}_{\text{diff}} dv,$$

where we adopt the convention that $\mathcal{L}_{\text{diff}} = 0$ for invisible, occluded v . $\mathcal{L}_{\text{diff}}$ within $\mathcal{L}_{\text{I} \circ \text{UV}}$ is a fixed, per-triangle image loss and acts as a weight for the symmetric Dirichlet Energy, $\|\nabla\psi(v)\|^2 + \|\nabla\psi^{-1}(v)\|^2$. The symmetric Dirichlet energy [Smith and Schaefer, 2015] penalizes deviations of $\nabla\psi$ from the identity and helps prevent ψ from producing local inversions. In practice, the integral over the mesh surface M is a summation of triangles if linear shape functions are used in discretization, and v can be any point on M . Note that ψ^{-1} is well-defined as we require ψ to be bijective. The original symmetric Dirichlet energy is not content-aware by itself. By multiplying with $\mathcal{L}_{\text{diff}}$, $\mathcal{L}_{\text{I} \circ \text{UV}}$ will impose a higher distortion penalty if the surface point v induces a larger visual dissimilarity. Thus, $\mathcal{L}_{\text{I} \circ \text{UV}}$ becomes a content-aware UV mapping regularizer. We note that image loss is a per-pixel measure, whereas the symmetric Dirichlet energy is a per-triangle measure. In order to unify the two, we evaluate $\mathcal{L}_{\text{I} \circ \text{UV}}$ as a summation over triangles, where for each triangle, we compute a screen-space measure of the integral of $\mathcal{L}_{\text{diff}}$. The visual similarity loss contributed by the given triangle is computed using rasterization, as in NVDiffmodeling [Hasselgren et al., 2021].

Note that if some v is occluded from d , then $\mathcal{L}_{\text{diff}} = 0$, which will cause the triangle containing v to easily flip or become degenerate. To avoid this degenerate solution, we add the original symmetric Dirichlet energy \mathcal{L}_{uv} with the weight λ_{uv} to our objective to bias towards a distortion-minimizing ψ :

$$\mathcal{L}_{\text{uv}} \triangleq \int_M \|\nabla\psi(v)\|^2 + \|\nabla\psi^{-1}(v)\|^2 dv.$$

Empirically, we find $\mathcal{L}_{\text{I} \circ \text{UV}}$ and \mathcal{L}_{uv} useful in different scenarios, where $\mathcal{L}_{\text{I} \circ \text{UV}}$ increases quality in a content-aware sense, while \mathcal{L}_{uv} ensures a minimum baseline isometric quality for ψ . Because geometric distortion is less important, we typically choose $\lambda_{\text{I} \circ \text{UV}}$ much larger than λ_{uv} .

Finally, to ensure that ψ is bijective, we need the UV mesh M_p to be free of intersections, for which there have been two approaches.

The scaffolding approaches [Jiang et al., 2017] triangulate the ambient space of M_p in the UV space and transforms the constraint into locally injective symmetric Dirichlet regularizations for both internal and ambient triangles. Alternatively, primal interior point method such as the incremental potential function [Li et al., 2021] formulates a barrier function bounding the distance between any $v_0, v_1 \in \partial M_p$ away from zero. We adopt the latter approach and transform the constraint into the following barrier term:

$$\mathcal{L}_{IPC} \triangleq - \sum_{\substack{v_a \neq v_b, v_a \neq v_c \\ v_a, v_b, v_c \in \partial T \cup \partial M_p \\ <v_b, v_c> \in E}} clog(\text{dist}(v_a, v_b, v_c))$$

$$clog(x) \triangleq \max(-(x - d_0)^2 \log(x/d_0), 0),$$

where T represents the rectangle texture space, \mathcal{L}_{IPC} is a sum of the barrier distance function, $\text{dist}(\bullet)$, over all the vertex-edge pairs on $\partial T \cup \partial M_p$ and d_0 is an activation distance for efficiently pruning faraway geometric pairs from computation (we refer readers to [Li et al., 2021] for more details). We avoid the intersection between ∂M_p and ∂T since we need to constrain the UV mesh to remain inside the texture space T . The IPC energy allows us to impose bijectivity without the additional memory and computational cost of storing and re-triangulating a scaffolding mesh. Finally, we add a distance d_0 , which is used to separate different charts by a distance of several texel widths, ensuring enough texels for constructing independent interpolation stencils for neighboring charts. This reduces seam artifacts on the resulting model.

4.2 Spherical Harmonic Parallax Mapping

When geometric discrepancies between the low-poly and the high-poly mesh are large, optimizing UV parameterization may lead to large UV regions being allocated to mismatched surfaces. This exacerbates blurriness in regions with high geometric differences, leading to an even worse final quality. To address this issue, we built *SH parallax mapping* to reduce the influence of geometric discrepancy, where the texture mapping function is formulated as the following operation:

$$C(v) = \text{fetch}\left(\psi(v) + \frac{\text{fetch}(\psi(v), T_D)}{d_{\text{tangent}(z)}} \phi_{SH}(\psi(v), d), T_C\right), \quad (4)$$

Where $d_{\text{tangent}(z)}$ is the value in the local normal axis when projecting the view direction in the coordinate system defined by the tangent, bitangent, and normal of a triangle. T_D is a scalar depth map, and the SH-based UV coordinate shift function ϕ_{SH} of order N is defined as:

$$\phi_{SH}(t, d) = \sum_{\ell=0}^N \sum_{m=-\ell}^{\ell} \text{fetch}(t, T_{\ell,m}) Y_{\ell}^m(d), \quad (5)$$

where t is the UV coordinates, Y_{ℓ}^m is the real Legendre polynomial of order ℓ , and $T_{\ell,m}$ is the texture storing the corresponding coefficients. Note that all coefficients are stored in one multi-channel texture so that a single texture lookup fetches all the coefficients. Our UV coordinate bias function unlocks a larger solution space, leading to higher visual similarity, and the cost of deploying equation (5) in a forward renderer is marginal using low-order SH functions. Prior work utilizing SH in irradiance mapping [Ramamoorthi and

Hanrahan, 2001] directly computes the coefficients, but we optimize coefficients through auto-differentiation of the forward rendering pass with integrated SH parallax mapping.

4.3 Vertex Displacements Optimization

In addition to using parallax mapping, we found that constrained optimization of low-poly mesh vertices improves the silhouette mismatch between the low-poly and high-poly meshes. As we decrease the level of details, the silhouette mismatch between the high-poly and low-poly deteriorates further, which cannot be mitigated by any texture-based approach. To tackle this issue, we additionally optimize vertex displacements. Different from prior differentiable rendering works that perturb the vertex coordinates permanently at each iteration using a Laplacian smoothness term [Hasselgren et al., 2021, Luan et al., 2021, Nicolet et al., 2021], which may cause our low-poly meshes to deform in ways that do not minimize silhouette differences, we allow low-poly vertices to be perturbed only along their normal directions fetched from the normal map, and the amount of perturbations are additional optimization variables denoted as $\delta \in \mathbb{R}$. In other words, we only apply the displacements of our low-poly after the optimization procedure, moving $v_i \in V$ to $v_i + \delta_i n_i$, where all δ_i are assembled in a vector ΔV . In Fig. 13 and by comparing the columns Oursy and NV * of Table 11 in the Appendix, we demonstrate the quality degradation of prior works using Laplacian smoothing, and empirically show that our vertex optimization strategy leads to higher-quality results.

4.4 Optimization Technique

Put together, our optimization problem takes the following unconstrained form:

$$\underset{T_N, T_C, T_D, T_{l,m}, \Delta V, U}{\text{argmin}} \quad \mathcal{L} \triangleq \mathbb{E}_{d \in SO(3)} [\mathcal{L}_{\text{diff}}] + \lambda_{uv} \mathcal{L}_{uv} + \lambda_I \circ_{uv} \mathbb{E}_{d \in SO(3)} [\mathcal{L}_I \circ_{uv}] + \lambda_{IPC} \mathcal{L}_{IPC}, \quad (6)$$

where λ_{\bullet} denotes a weight coefficient for the corresponding energy term. Unfortunately, the original Adam optimizer [Kingma and Ba, 2015] in the differentiable rendering framework [Hasselgren et al., 2021, Laine et al., 2020] does not work for our problem for two reasons. First, UV optimization involves non-Lipschitz barrier energy with a non-trivial feasible domain which, once exited is hard to return to. Using a constant or shrinking learning rate as in [Hasselgren et al., 2021, Laine et al., 2020] could potentially leave the feasible domain of these functions. Second, some functions are much more costly to compute than others. For example, evaluating the $\mathcal{L}_{\text{diff}}$ term only involves efficient rasterization-based rendering, but the \mathcal{L}_{IPC} term involves costly geometric proximity querying, which scales with the number of vertices. To overcome these difficulties, we propose a feasibility-guaranteed alternating multi-stage optimizer, which is described below.

Line-Search Safety Guarantee. The unconstrained objective function is non-Lipschitz due to the barrier terms \mathcal{L}_{uv} and \mathcal{L}_{IPC} , ensuring the UV mapping function ψ is bijective and M_p has no-flipped triangles. Unlike standard differentiable rendering framework that uses stochastic gradient descend for Lipschitz functions, we need to resort to strict line-search to ensure \mathcal{L}_{uv} and \mathcal{L}_{IPC} stay in their

feasible domain. We follow prior work [Li et al., 2020, Rabinovich et al., 2017] that first estimates the largest step size α_0 and then find a suitable final step size α satisfying the first Wolfe condition. To keep \mathcal{L}_{uv} and $\mathcal{L}_{\text{IoUV}}$ well-defined, we compute α_0 to be the largest value such that the minimal singular value $\sigma_{\min}(\nabla\psi) \geq 0$, or the point at which triangle inversion would occur. This can be computed by solving a quadratic equation per triangle in parallel. To ensure that M_p and T remain intersection-free, we further compute α_1 , the time-of-impact where a vertex-edge pair on $\partial T \cup \partial M_p$ intersects. Then, we define the final $\alpha \triangleq \min(\alpha_0, \alpha_1)$.

UV and Texture Initialization. Since our method requires a bijective initialization for the UV coordinates U , we employ *xatlas* [Young, 2017] to generate an initial bijective mapping. We also perform a normalization step of the initial UV parameterization, such that it is centered and fills the 2D bounding box $[0.01, 0.99]^2$, to ensure that the UV has space to deform. We initialize texture colors as gray, tangent normal values to $[0, 0, 1]$ which corresponds to using the mesh's face normal, and Spherical Harmonic coefficients to a small uniformly random value.

Mesh Normalization. Our method requires both meshes to be well-aligned. We assume the two meshes are both contained within the bounding box of the input high-poly mesh, so we uniformly transform the high-poly to be contained in the unit box $[-1, 1]^3$. We then apply the same transformation to the low-poly mesh. We find this strategy enough to yield high-quality textures. We have also experimented with using independent transformation to the two meshes, but this can oftentimes lead to misaligned silhouettes.

Alternating Optimization. After initialization, we alternate updating the textures and UV coordinates to achieve joint optimality. We first fix the UV coordinates U and update all the textures, $T_N, T_C, T_D, T_{l,m}, \Delta V$, using k_0 iterations of Adam [Kingma and Ba, 2015] with batch-sampled approximations of the rendering loss over view directions $d \in \text{SO}(3)$. This gives a good initial estimate of the spatially varying weight of the symmetric Dirichlet texture regularization term $\mathcal{L}_{\text{IoUV}}$. We then perform another k_1 iterations, with each iteration having f steps of the Adam optimizer for the textures and one iteration of L-BFGS [Liu and Nocedal, 1989] with line-search for UV coordinates by fixing the per-triangle image loss. When updating $\mathcal{L}_{\text{IoUV}}$, we do not minimize the final rendering loss and do not update the texture, and focus primarily on updating the UV parameterization. The gradient of the spatially adaptive symmetric Dirichlet term takes the following form:

$$\begin{aligned} \nabla \mathcal{L}_{\text{IoUV}} = & \int_M \nabla [\|\nabla\psi(v)\|^2 + \|\nabla\psi^{-1}(v)\|^2] \mathcal{L}_{\text{diff}} dv + \\ & \int_M (\|\nabla\psi(v)\|^2 + \|\nabla\psi^{-1}(v)\|^2) \nabla \mathcal{L}_{\text{diff}} dv. \end{aligned}$$

Although this alternating scheme is not theoretically justified, it achieves an ideal balance between efficacy and accuracy in practice. We have experimented with exact line-search scheme, where $\mathcal{L}_{\text{diff}}$ in $\mathcal{L}_{\text{IoUV}}$ are updated at each step of line search. However, this exact scheme leads to twice more expensive iteration cost, with minimal noticeable improvements in results. Our alternating optimization

is terminated after a fixed iteration budget is reached or the gradient of the UV vertices and the change in diffuse texture is below a certain threshold. After this, the texture and UV coordinates are approximately jointly optimal. Finally, we use another k_2 iterations of Adam optimization to fine-tune textures, which can also be terminated early if textures have converged. Our final optimization is summarized in Algorithm 1.

Algorithm 1 Alternative Optimizer for Equation 6

Input: $k_0, f, k_1, k_2, k_3, \epsilon_0, \epsilon_1$, initial U and V
Output: $T_N, T_C, T_D, T_{l,m}, U, V$

- 1: Initialize T_N by setting all normals to $(0, 0, 1)$
- 2: Initialize T_C by setting all colors to gray $(0.5, 0.5, 0.5)$
- 3: Initialize T_D as 0
- 4: Initialize $T_{l,m} \sim \mathcal{U}(-\epsilon_0, \epsilon_0)$
- 5: Use k_0 iterations of Adam to update $T_N, T_C, T_D, T_{l,m}, \Delta V$
- 6: **while** $\Delta \|T_N\|, \Delta \|T_C\| > \epsilon_1$ and $\text{iters} < k_1$ **do**
- 7: Use f iterations of Adam to update $T_N, T_C, T_D, T_{l,m}, \Delta V$
- 8: Use 1 iteration of L-BFGS with line search to update U
- 9: Use k_2 iterations of Adam to fine-tune $T_N, T_C, T_D, T_{l,m}, \Delta V$
- 10: Set $V = V + \Delta V$
- 11: Return $T_N, T_C, T_D, T_{l,m}, U, V$

4.5 Implementation Details

We implement our algorithm in Python, using NVDiffraff [Laine et al., 2020] with CUDA for numerical computations and differentiable rendering. Details related to the implementation of our algorithms are described below.

Texture Padding and Scaling. We use d_0 in \mathcal{L}_{IPC} to impose a padding distance between neighboring charts, which ensures charts are far enough apart for texture interpolation. We set $d_0 = 5\Delta t$, where Δt is the texel width. Further, we note that the low-poly and high-poly can have drastically different UV mapping functions ψ . Intuitively, our method performs the best when the two mapping functions ψ contain the same number of texels. To approximately enforce this constraint, we scale the UV coordinates relative to texture resolution to ensure the low-poly and high-poly take up the same number of texels in the UV space and we do not recalculate the scaling during optimization.

Numerical Stability. Our method relies on automatic differentiation for computing the gradients, which may introduce numerical issues in several places. As compared to analytic gradients which are used in [Li et al., 2020, Rabinovich et al., 2017], we find automatic differentiation requires additional treatment for numerical stability of non-Lipschitz barrier energy terms. First, for the symmetric Dirichlet function, we use a log exponential trick and let $\psi(v) = \exp(\log(\psi(v)))$. The log-exponential trick is known to better utilize the limited floating point precision and prevent catastrophic gradient explosion when $\phi(v)$ takes on values very close to zero. Second, we find that boundary computation can quickly consume floating point precision, as edges and points get extremely close. To remedy this, we propose a mixed precision scheme, computing all gradients using 32-bit floats to save hardware bandwidths while

switching to 64-bit floats to compute boundary edge-vertex overlaps, which do not require gradients.

Parameter Settings. For the number of iterations, k_0, f, k_1, k_2 , we set a budget of $k_0 = 500$, $k_1 = 800$, $f = 5$, and $k_2 = 500$. We find that 500 iterations are often enough to approximate the output of NVDiffmodeling initially, and it is enough to clean up any artifacts caused by shifting UV coordinates. To initialize Algorithm 1, we set SH coefficients uniformly by setting $\epsilon_0 = 1 \times 10^{-5}$, as we do initially do not want parallax mapping. We also set $\epsilon_1 = 1 \times 10^{-6}$. For weights on loss terms, we set $\lambda_{UV} = 1 \times 10^{-2}$, $\lambda_{IPC} = 1$, and $\lambda_{\Delta V} = 5$. We use 3rd-order SHs as it is a good balance between efficiency and representability.



Fig. 3. Textured high-poly models used in our dataset (not to scale) from [Sketchfab, 2022]. Each model has a single texture atlas of size 2048×2048 , with varying geometric and texture complexities. Attributions for models are provided in the code repository, and are licensed under CC Attribution.

5 EXPERIMENTS

We conduct various experiments to validate our proposed approach and demonstrate its robustness. We run all our experiments on a workstation with an 8-core Intel processor clocked at 2.5 GHz, 64 GB of memory, and a single NVIDIA Titan XP GPU with 12 GB of RAM. All computations are performed on the GPU and in a single CPU thread. We evaluate the appearance quality of the generated textures by sampling a set of 48 camera views uniformly spaced on a sphere of radius 2 centered around the model, which is resized into the unit box $[-1, 1]^2$, and compare the visual similarity using both PSNR and MS-SSIM [Wang et al., 2003], for both metrics higher values are better. For conciseness, variations of our approach are denoted by a combination of UV , SH , and V , which stand for UV optimization, SH parallax mapping, and vertex displacement, respectively. We always optimize the diffuse color and normal textures for all the variations

of our method. For example, $Ours_{UV}$ means we optimize T_C , T_N , and U , $Ours_{SH}$ optimizes T_C , T_N , T_D , and $T_{l,m}$, $Ours_V$ optimizes T_C , T_N , and ΔV , while $Ours_{UV, SH}$ optimizes T_C , T_N , U , T_D , and $T_{l,m}$, etc. *Ours* indicates that we optimize all variables as described in Algorithm 1.

In the following sections, we will describe the dataset preparation, evaluation of our approach, and ablation studies of our algorithm. While only selected images are provided in the main text, for the curious reader, we release our dataset and the generated results for each rendered model in the figures on GitHub at https://anonymous.4open.science/r/tex_baking_supplementary/, which includes the high-poly and low-poly meshes, the associated diffuse color and normal textures, as well as depth map, vertex displacement vector and spherical harmonic coefficients in PyTorch “.pt” format. We also include a video for each result showing the baked appearance.

5.1 Dataset

To comprehensively evaluate our approach, we collect a dataset of 3D models found in the wild and generate a set of LOD models. We first obtain high-poly meshes by collecting a set of 3D models from Sketchfab [Sketchfab, 2022] and then convert each of them into a mesh with a single UV atlas $\in [0, 1]^2$ and a 2048^2 texture using xatlas [Young, 2017] for parameterization and Blender [Blender Online Community, 2018] for texture transfer. For each high-poly mesh, we then produce 3 low-poly meshes with varying geometric differences from the reference mesh using Simplygon [Donya Labs AB, 2022].

When generating meshes using Simplygon, we set the screen-size parameters to 50, 100, and 300, corresponding to large, moderate, and small geometry differences, denoted as LOD-3, LOD-2, and LOD-1, respectively. Screen-size roughly corresponds to the mesh resolution, where lower values will lead to coarser approximations with fewer vertices and faces. We use the UV parameterization that Simplygon generates alongside the produced meshes. While our set of high-poly meshes has a broad variety of shapes, geometric complexities, and texture qualities, the low-polys have dramatically different geometry and topology properties from their high-poly counterparts, see Fig. 4, Fig. 5, Table 5 for detailed statistics and Fig. 3 for visual renderings. Since the original models are from online, we release attributions to the original artists but not the original models themselves.

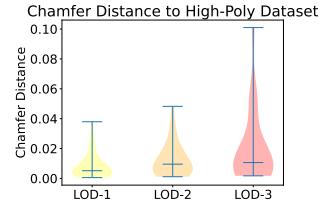


Fig. 4. Chamfer distance statistics between the LOD datasets and the high-poly meshes. For the computation of Chamfer distance, we center each mesh at the origin and resize it to fit in the unit box.

5.2 Evaluation

Comparisons with Competing Texture Bakers. We compare against the open-source ray-casting approach in Blender [Blender Online Community, 2018], the texture baker in a flagship 3D game engine,

Dataset	#Comp.	#Vertices	#Faces	Manifold	Genus	Holes
High-Poly	8/225/2k	502/8k/70k	723/9k/0.1M	5/26	0/0/0	0/29/174
LOD-1	1/41/312	244/1k/5k	306/1k/5k	24/26	0/3/39	0
LOD-2	1/19/82	43/354/1k	64/406/1k	26/26	0/2/28	0
LOD-3	1/6/40	28/114/349	32/137/368	26/26	0/0/12	0



Fig. 5. Statistics of our dataset: (top) topology properties of the high-poly and different LODs of the dataset, (bottom) visual rendering of LOD of the Mask model in our dataset. Within the table, we show the minimum/median/maximun of different properties of our dataset, including the number of components (#Comp.), the number of vertices and faces, the number of manifold meshes/the total number of models, the genus, and the number of holes contained in the models.

Unreal Engine (UE) [Epic Games, 2022], and the differentiable rendering approach NVDiffModeling [Hasselgren et al., 2021]. For all approaches, including ours, we use the same configuration and settings for all meshes. For Blender, we use the default setting for cage extrusion without additional resizing. For UE, we set the ray offset distance from the surface to be large (1×10^5), as their approach does not automatically rescale meshes before texture baking and operates in world space.

We batch process the competing approaches and our method on the three LOD datasets generated from Simplygon and produce new diffuse and normal textures with resolutions at 512^2 , 256^2 , and 128^2 . As shown in Fig. 6. Our approach is noticeably better by 2-4 PSNR than competing approaches across all datasets. Our approach significantly outperforms traditional raycasting, being up to 10 PSNR better on some meshes and consistently better on every mesh in our dataset independent of texture resolution.

On the LOD-3 dataset, where the low- and high-polys have high geometric discrepancies, we achieve around 3 PSNR improvements compared to NVDiffModeling. On LOD-2 with lower geometric discrepancies, our approach performs even better, achieving around 3.25 PSNR than NVDiffModeling, except with a texture resolution of 128^2 where our performance is about 2 PSNR better. When the meshes are extremely similar on LOD-1, our approach consistently performs about 3.5 PSNR better than NVDiffModeling, independent of texture resolution. All the reported PSNR improvement is the median improvement over NVDiffModeling, which is outlier resilient and demonstrates a consistent increase. Across texture resolutions, we find our relative PSNR gain slightly decreases when the texture is tiny, e.g. at a resolution of 128^2 , but increases as resolution increases. Intuitively, this is because all approaches need a minimal number of texels to encode the high-poly’s appearance. Full statistics for each mesh in our dataset at each resolution are attached in Table 8, Table 9, and Table 10 of Section A in the Appendix.

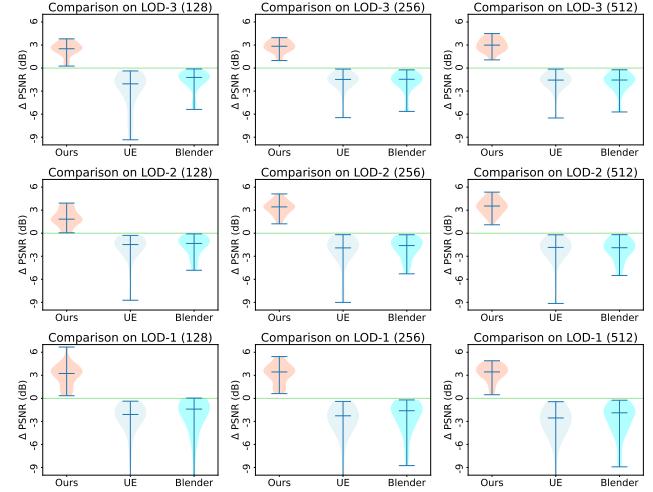


Fig. 6. We compare our approach with UV optimization, SH parallax, and vertex displacements (Red), relative to NVDiffModeling (Green Line), and include UE (Dark Blue) and Blender (Teal) for reference. We find that across all texture resolutions and mesh resolutions, our approach outperforms NVDiffModeling significantly. The horizontal green line denotes NVDiffModeling, which we use as the baseline, and the middle marker of the violin plot denotes the median improvement.

We show some visual comparisons of baked models in Fig. 7. On a single model, we compare multiple levels of detail, alongside the ground truth model and competing methods, with close-up insets of different portions of each model. At LOD-1, our approach is similar to NVDiffModeling but is able to capture finer features around edge borders, as can be seen in the insets around the eyes. Blender also performs well, but UE has artifacts due to casting the ground onto the penguin and the penguin onto the ground. At LOD-2, our approach retains the visual acuity of LOD-1, whereas the other approaches become more blurred. Notably, around the eyes, UE and NVDiffModeling begin to blur, significantly degrading the quality. At the lowest LOD, LOD-3, our approach still appears sharp but has more errors around regions where the low-poly and high-poly mesh have begun to diverge significantly, which can be seen on the cracking ground which has started to vanish in NVDiffModeling. UE performs surprisingly well, appearing sharp on the eyes, but has artifacts since the feet are now a cylinder connected to the ground, which is extremely different as compared to the original model. As can be seen, across different LOD our approach preserves sharp features and remains faithful to the original model. Our advantages at low LOD as compared with NVDiffModeling are mainly due to our SH parallax mapping and vertex displacement. At higher LOD, our advantages are due to UV optimization.

We also examine different texture resolutions in Fig. 8 for LOD-2 of a simple geometric model with a complex texture. We can see that across all texture resolutions, ours maintains key details, whereas the other approaches appear blurry or pixelated. The optimized UV is highly efficient, taking up almost the entire 0-1 space, whereas the original UV does not utilize much texture space. At the lowest texture resolution, 128^2 , our approach has some pixelation artifacts

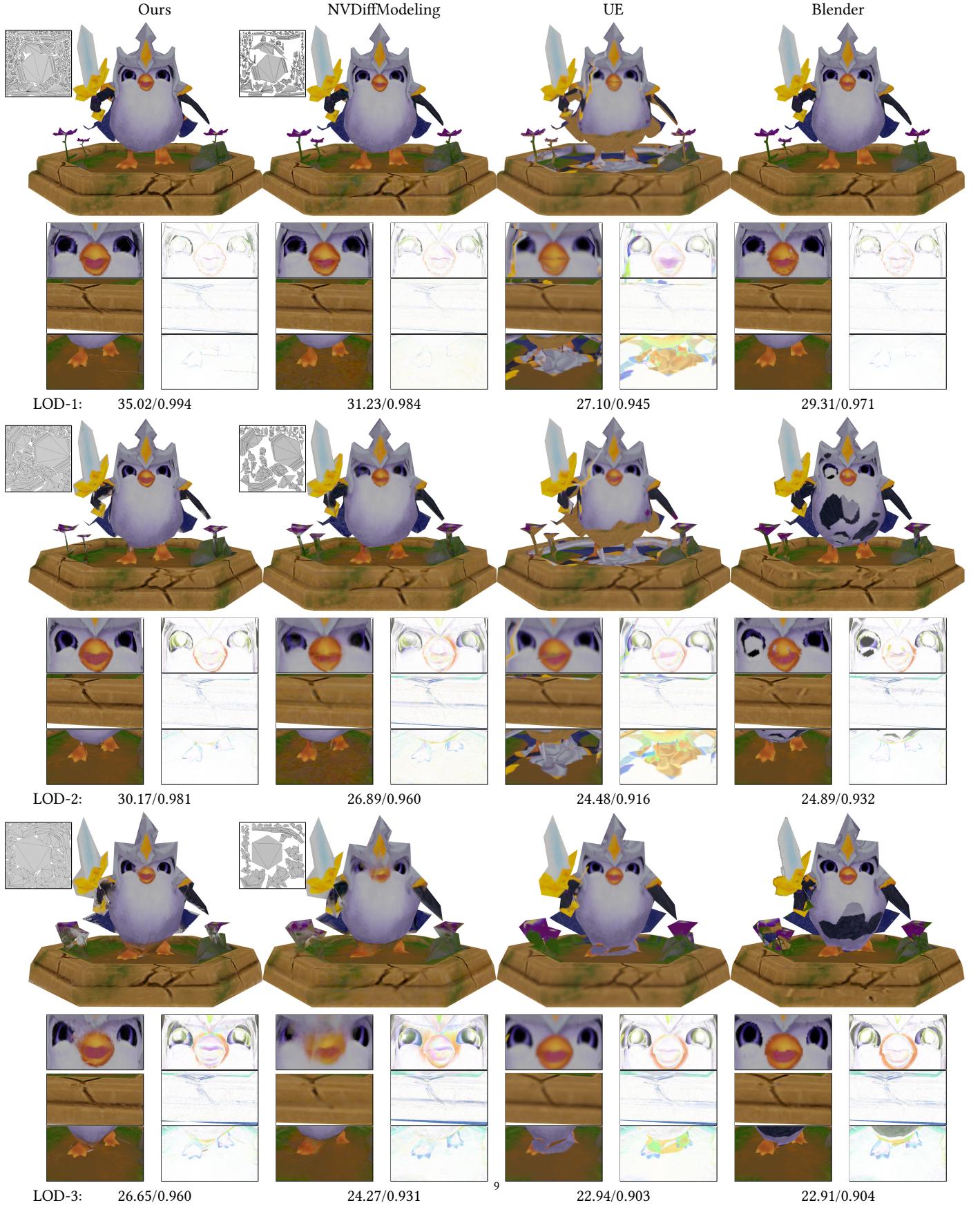


Fig. 7. Visualization of outputs of our texture baking approach on multiple levels of details of the Penguin model, where the number of faces for the high-poly, LOD-1, LOD-2, and LOD-3 are 18163, 2372, 664, and 292, respectively. Independent of LODs, our approach outperforms raycasting and differentiable rendering approaches, as shown by visual inspections and the corresponding PSNR/MS-SSIM values below each mesh. Note that UE and Blender have the same UV as NVDiffmodeling.

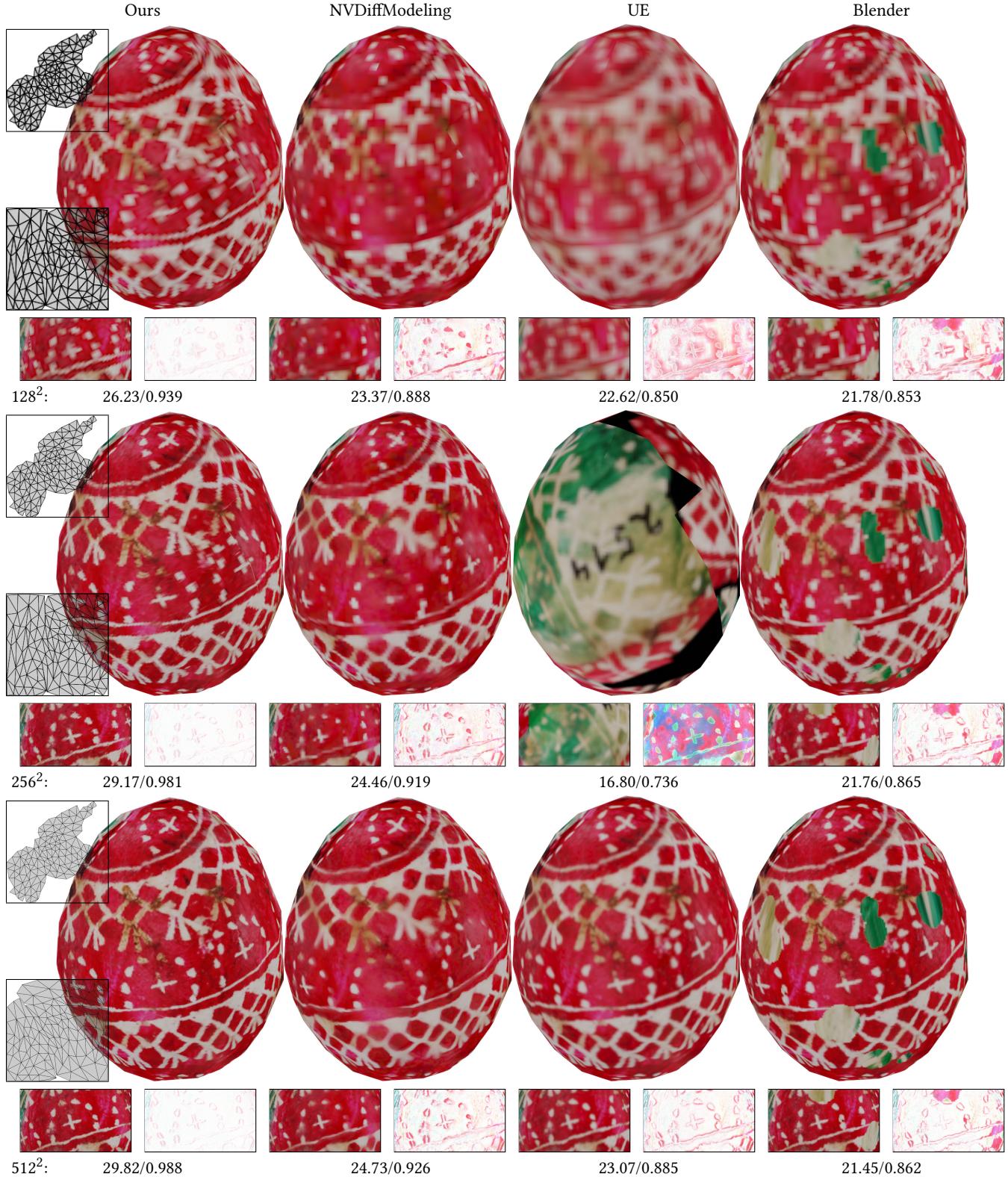


Fig. 8. Comparisons of the baked Egg LOD-2 models generated by raycasting, differentiable rendering, and our approaches at texture resolutions of 128, 256, and 512. In the left column we show the initial UV at the top, and our optimized UV at the bottom.

because there are insufficient texels to represent the original texture. Even so, our approach is able to recover some sharp features. NVDiffModeling loses many details, and some regions appear pixelated. UE blurs the features but maintains the general concept, whereas Blender instead leads to pixelization of features. At the middle texture resolution, 256^2 , our approach reduces some of the blurrings from 128^2 , capturing more precise details. NVDiffModeling begins to exhibit a higher quality but still misses many small details, and there is uneven blurring across the surface. UE and Blender both contain artifacts, but where Blender does not have artifacts it appears slightly pixelated. At the highest texture resolution, 512^2 , our approach appears very sharp. NVDiffModeling is still unable to recover some details and appears blurry in multiple places. UE appears very sharp but has a high discrepancy against the ground truth. Blender, on regions without artifacts, appears very sharp and has good quality as well. Across all resolutions, we demonstrate that our approach has superior visual quality and captures more details.

By having methods to handle varying levels of mesh and texture resolutions, our approach is robust and maintains high quality even when the low- and high-polys are significantly different. A video clip for each generated result with a 360° comparison of the target and reference models for all the methods is provided in the anonymous github repo as mentioned earlier in this section.

Comparisons with Content-Aware UV Parameterizations. While existing content-aware UV optimization methods [Sander et al., 2002, Sun et al., 2022] cannot be used to bake textures between meshes with geometric differences, they can be employed to reduce the texture resolution for the same model. Specifically, we examine the application of generating lower resolutions of the textures for high-poly models for both 2D and 3D applications.

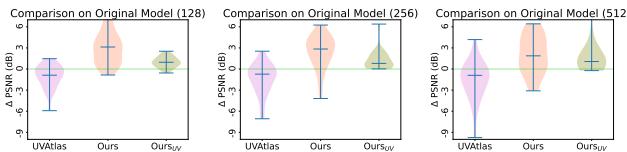


Fig. 9. We compare 4 methods: NVDiffModeling (green line), our method optimizing texture content, UV parameterization, SH and vertex displacement (Ours), our method without SH or vertex displacement (Ours_{UV}), and UVAtlas, where we first use UVAtlas [Sander et al., 2002] to generate a UV parameterization, and then using NVDiffModeling to generate texture contents. Our approaches (Ours and Ours_{UV}) achieve the best results over all comparisons on downsampled textures on the original model.

For 3D comparisons, we compare our method against a baseline of NVDiffModeling [Hasselgren et al., 2021] and Signal-Specialized Parameterization [Sander et al., 2002], where we first apply UVAtlas [Sander et al., 2002] to generate a UV parameterization, then bake textures using NVDiffModeling. To parameterize our approach while not using any signal-aware parameterization, we reparameterize each high-poly mesh using Blender’s smart projection and set it such that it produces many charts with large margins, utilizing few texels. This often produces poor parameterizations but usually generates a bijective mapping. For those which are not bijective, we manually clean them, triangulating, removing degenerate faces, and

reparameterizing them as necessary. As shown in Table 7 and Fig. 9, we find that UV optimization provides a performance boost and including SH and vertex displacement also helps reduce the visual differences, despite poor initialization as compared to NVDiffModeling, which uses Simplygon’s parameterization. Surprisingly, we find that UVAtlas performs even worse than only NVDiffModeling, and we suspect this is because while UVAtlas may allocate relatively more texels to important regions, it does not use the full texture space, so the absolute number of texels is less than the number of texels used by the original parameterization. This can be seen in the output parameterizations as provided in the GitHub repo.

We also demonstrate the effectiveness of our method of compressing textures on 2D meshes, a common application in 2D and 2.5D games. For 2D models, it is trivial to generate a UV parameterization which is perfectly isometric to the mesh, but we posit that deforming the UV parameterization can increase visual quality for compressed textures. We simplify our approach by fixing an orthographic camera that can view the entire plane and do not optimize the normal map, SH parallax, or vertex displacements, as we are focused only on recovering a compressed texture on the plane. We also set $\mathcal{L}_{UV} = 1 \times 10^{-5}$. As shown in Table 6 and Fig. 10,

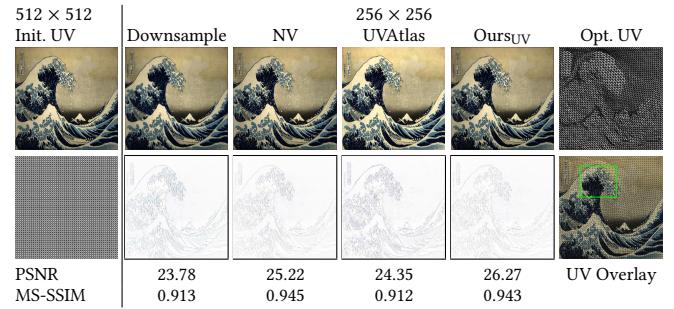


Fig. 10. Our method is able to compress textures on planar surfaces by giving regions with higher frequency detail a larger portion of UV space, such as the region highlighted by the green box on the rightmost image. In the bottom row, from left-to-right is the original UV, the pixel-wise difference between the reconstructed and the original, and the optimized UV overlaid on top of the original image.

our method is able to outperform direct bilinear downsampling, NVDiffModeling, and UVAtlas on a variety of images with different kinds, such as single icon images, paintings, pixel art, game textures, and maps, including those without high-frequency details, or those with extremely high information density. All the tested 2D data are provided in the aforementioned github repo. Similar to our approach on 3D meshes, across all 2D textures at varying target resolutions (three different resolutions are tested), our algorithm performs much better than UVAtlas, NVDiffModeling, and direct downsampling, achieving 1 dB, 3 dB, and 3.25 dB more on average over the 2D dataset at 64^2 , 128^2 , and 256^2 respectively.

Comparison with Other Parallax Mapping Variants. As an alternative to resolving view parallax, our framework can be used to optimize other parallax mapping variants, such as parallax mapping (Parallax) [Kaneko et al., 2001], parallax mapping with offset limiting (PMOL) [Welsh et al., 2004], steep parallax mapping

Table 1. A comparison of some existing approaches that handle parallax, in no particular order. Depth (h) and SH are evaluated at a specific texture UV coordinate, and v is the tangent space view direction transformed from the view direction d in the world space. We note that for approaches after the first two, it is possible to either include or not include the factor $\frac{1}{v_z}$. POM is based on SPM, but seeks to find a closer approximation to the correct position by using the linear interpolation between the last two steps.

Method	Formula	#Tex. Fetch
Parallax Mapping [Kaneko et al., 2001]	$UV + v_{xy} \frac{h}{v_z}$	1
PMOL [Welsh et al., 2004]	$UV + v_{xy}h$	1
SPM [McGuire and McGuire, 2005]	$UV_i + v_{xy} \frac{h}{k}, h > \frac{i}{k}, k \in \mathbb{Z}_+$	$O(N)$
POM [Tatarchuk, 2006]	SPM + Lerp	$O(N)$
Ours	$UV + SH(d) \frac{h}{v_z}$	1

(SPM) [McGuire and McGuire, 2005], and parallax occlusion mapping (POM) [Tatarchuk, 2006]. For SPM and POM, we multiply translation by a function of depth (depth/#layers), so that it can be optimized jointly alongside a depth map. Refer to Table 1 for different equations of each method. We use 3rd-order SH for our approach, 16 layers for SPM and POM, and a height scale of 1×10^{-2} for all parallax approaches. We use a minimum of 1000 iterations (k_1 in Algorithm 1) for optimizing Parallax, PMOL, and SH. We use 2500 iterations for SPM and POM. On the LOD-3 dataset, we demonstrate that our approach has the best visual quality for reconstructing textures amongst all the parallax variations, see Table 11. In Fig. 11, we also show the comparisons of our approach with other variants on a typical example, brick wall, where the high-poly has a complex geometry and the low-poly is a cube. Quantitatively, our approach achieves the best PSNR/MS-SSIM evaluations. Qualitatively, optimizing SH is able to capture occlusion, which is not recovered as clearly by optimizing other variants. In addition, our approach only requires a single texture fetch, whereas POM and SPM require multiple fetches for ray marching.

We also demonstrate how our approach compares visually on an example when the ground truth depth to a high-poly mesh is known in Fig. 12. Here, we apply different parallax variants to the same plane, and visualize the difference, and optimize our approach on the same plane. We produce a high-poly mesh by directly shifting vertices in the opposite direction of the normal by the given depth, and use the original plane to recover the input. In this case, our approach is still competitive with prior approaches, and provides a convincing perception of parallax.

Comparison with Displacement Mapping and Mesh Deformation. We compare our approach with two other approaches to reduce the geometric difference between the source and target meshes on the LOD-3 dataset. Specifically, we compare with 1) displacement mapping with 1 level of mesh subdivision that is optimized by NVDiffModeling, and 2) mesh deformation originally provided by NVDiffModeling. As illustrated in Table 11, among the three methods, our approach is competitive with 1 level of subdivision of the displacement mapping and performs much better than mesh deformation since NVDiffModeling employs Laplacian term that often leads to undesirable mesh deformations on most of the low-poly models. Note that the subdivision of the displacement mapping

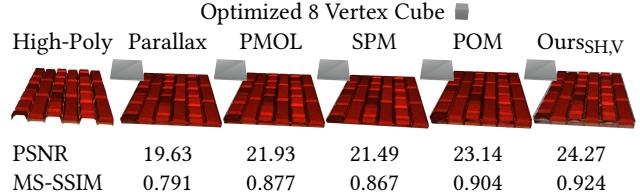


Fig. 11. We compare the baking results by optimizing different variants of parallax mapping on a brick wall, using a cube as the low-poly model. From left-to-right, Parallax Mapping [Kaneko et al., 2001], parallax mapping with offset limiting (PMOL) [Welsh et al., 2004], steep parallax mapping (SPM) [McGuire and McGuire, 2005], Parallax Occlusion Mapping (POM) [Tatarchuk, 2006], and our SH approach. Our proposed SH gives a convincing perception of depth, where the other variants do not appear as deep between bricks. We optimize each of the vertex positions of the cubes for all the variants, and note that the result of each approach is nearly identical.

requires a significantly different pipeline in practice than our approach, including a geometry shader which is known to be slower than fragment or vertex shaders. Fig. 13 demonstrates a visual comparison of these approaches on the Knight model.

Extensions to Other Texture Maps. While we demonstrate that our approach works for generating diffuse and normal maps, we also demonstrate in Fig. 14 that it works for other kinds of texture maps, such as a specular map, which can be achieved by modifying the forward rendering equation to include a specular term. As demonstrated, any map can be generated as long as it is used in the forward rendering equation in a differentiable form. As another example, we have shown in Fig. 11 that we can optimize the depth map for traditional parallax mapping. We test optimizing a depth map on all models with various parallax approaches in Table 11.

Timings. We compare the timings of our approach with NVDiffModeling, and demonstrate the per-iteration cost of our approach, as shown in Table 2. First, we note that it is essentially cost-free to include vertex optimization on top of UV and parallax mapping optimizations. While NVDiffModeling is mostly independent of texture resolution, our approach’s cost increases with texture size, due to larger memory usage for UV optimization. Our approach also requires more iterations for the UV parameterization to converge. We note that timings are always machine and implementation dependent, and since our approach is entirely implemented in Python, it does not have the implementation benefits and compiler optimizations that NVDiffModeling has, as it is mostly a wrapper around a CUDA library. We also omit comparisons with raycasting, because there is no “per iteration” cost for raycasting. Raycasting approaches are two orders of magnitude faster than any of the approaches listed though and are the most efficient.

Stage-wise Convergence. To justify the necessity of our three-stage pipeline, we plot the convergence history of $\mathcal{L}_{\text{diff}}$ and $\mathcal{L}_{\text{IoUV}}$ in Fig. 15, which is smoothed to filter out the noises. In these plots, the green and yellow vertical lines indicate the point where UV optimization starts and ends, respectively. Our first stage before the green line can significantly reduce the visual discrepancy and

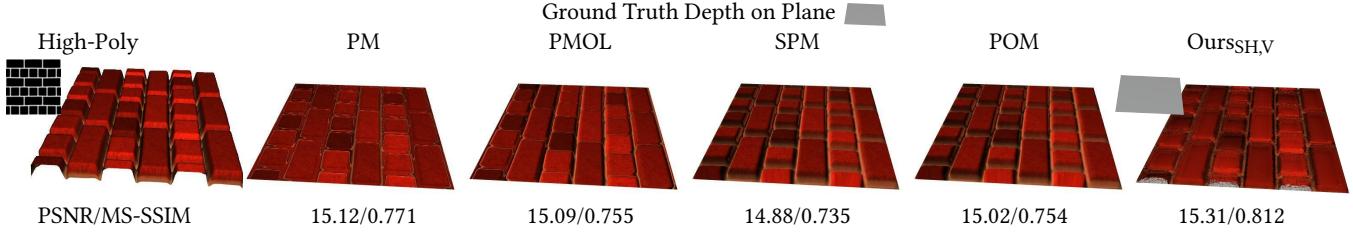


Fig. 12. We compare our approach against different parallax variants on the same textured mesh with a ground-truth depth map. Ours is competitive with the other approaches and has an illusion of depth. We do not show the geometry for each parallax variant, as we do not perform any optimization on those, only using the ground truth depth. We *do* optimize our own approach, as it does not work without optimization.

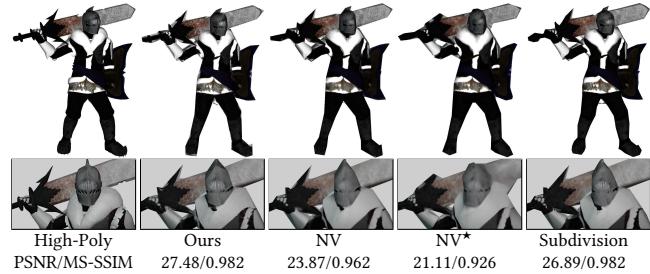


Fig. 13. We visualize the difference between our approach, NVDiffModeling (NV), NVDiffModeling with the vertex optimization enabled (NV*), and 1 level subdivision of the displacement mapping (subdivision) on the same low-poly mesh. We can see that our approach has much better visual quality than NV and NV* where both blur features on the reconstructed model, and slightly better than subdivision, despite subdivision quadrupling the number of triangles.



Fig. 14. Our approach can also optimize any map that is used during forward rendering, such as the specular map. We visualize a wall-console with optimized specular (T_S), normal (T_N), diffuse (T_C), and depth maps (T_D). Any map can be optimized if it can be used in a differentiable forward rendering equation.

Table 2. We compare the time required to perform 1 iteration of optimization for each approach at different texture resolutions in seconds, as well as the number of iterations until convergence. [†]The number of iterations may change per scene and for each approach, so we enforce a minimum number of iterations.

Texture Baking Timing (wall-clock)			
	Method (Tex. Res)	ms/iter	Iters [†]
NV	NV (128)	245	1183
	NV (256)	242	1105
	NV (512)	244	1050
Ours	OursUV,SH (128)	448	4504
	OursUV,SH (256)	450	4504
	OursUV,SH (512)	607	4504
	Ours (512)	608	4504
	OursSH,V (512)	486	1000
	OursSH,V (512)	507	1000

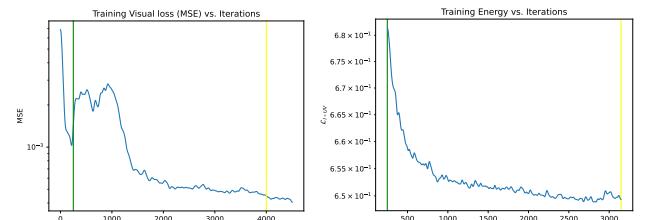


Fig. 15. Convergence, visualized as iteration versus MSE and $\mathcal{L}_{\text{IoUV}}$ for one model. UV optimization starts at the green line, and ends at the yellow line. We find this trend to be consistent across our dataset.

bootstrap the follow up stages. When UV optimization starts, it contributes to another considerable decrease in $\mathcal{L}_{\text{diff}}$. Our third stage after the yellow line does not reduce the visual discrepancy as much, but it is needed to fine-tune the texture image and remove noises.

5.3 Ablation Studies

In this section, we evaluate the necessity and importance of various components in our method.

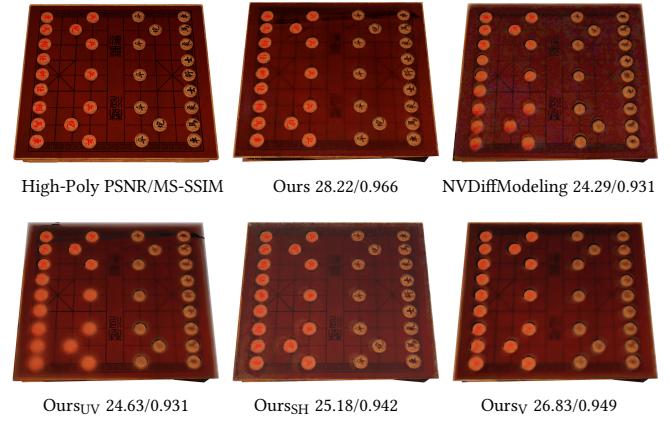


Fig. 16. We demonstrate the importance of each component (UV optimization, parallax mapping, and vertex displacement on a single model). Individually, each component of our approach helps minimize the visual difference between the low- and high-poly.

Vertex Displacement Optimization. In order to demonstrate the importance of our approach’s vertex displacement, we compare our approach with and without it in Table 8, Table 9, and Table 10. By including vertex displacement, our approach is able to perform better on most examples as compared to our approach with only SH and UV optimization. This is especially noticeable at LOD-3, as there is a more significant geometric difference, leading to a large increase for our approach.

Parallax Mapping. We demonstrate the importance of SH Parallax mapping in Table 11. Across all versions of parallax mapping, we demonstrate that texture baking quality is improved through the use of parallax mapping universally across all models. By examining SH Parallax compared with other parallax mapping techniques, we find their improvements comparable, but SH parallax has higher final quality. We find adding parallax mapping to differentiable rendering as important for final quality, as even alternative approaches such as standard parallax mapping [Kaneko et al., 2001] provide a consistent improvement.

UV Optimization. The importance of UV optimization is highlighted during texture compression, as SH parallax mapping and vertex displacements are not used during the optimization. In Fig. 10, we show the produced UV parameterization easily identifies the original image features, and demonstrate that UV optimization has a non-negligible impact on the visual quality.

The importance of each of the individual components can be visually observed on the chess model as shown in Fig. 16.

SH Function Order. For our approach it is not *a priori* clear what order of SH provides good quality, as there is a tradeoff between universality versus memory requirements and computational cost as shown in the inset. We would like to keep the number of coefficients as low as possible while providing the highest quality, so we compare the rendering quality of varying the order of SH coefficients versus quality on a single mesh, and show a comparison in Fig. 17. We find that there is a clear correlation between order and quality, and we select 3rd order SH as it still has significant visual quality improvements but is not expensive to include in the forward rendering equation.

Different UV Initializations. We also demonstrate that our approach works on arbitrary bijective UV mappings even those with poor initial distortions, as long as they are bijective. For example, given the Ogre model in Fig. 18, we initialize the UV parameterization of its low-poly mesh using the Tutte embedding [Tutte, 1963], and recenter and scale it within the bounds [0.25, 0.75]. We then bake the texture from the original model onto the remeshed version with NVDiffModeling and our approach, and observe that our approach can recover a higher quality UV embedding that can better represent the texture content. Then, we compare this to a different

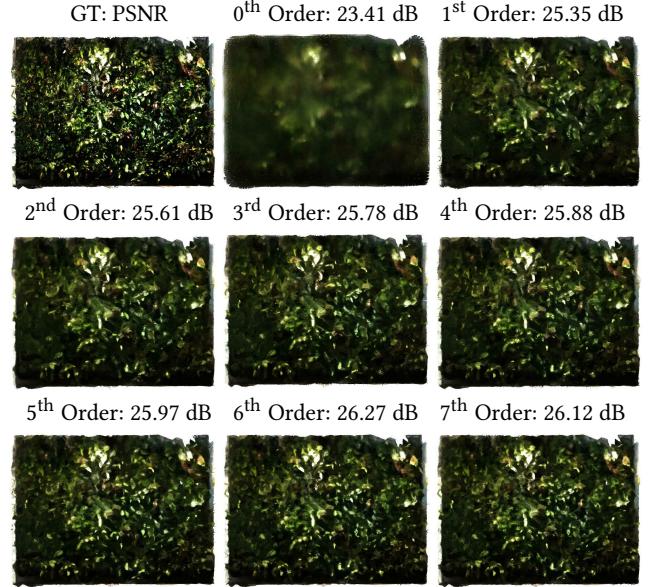
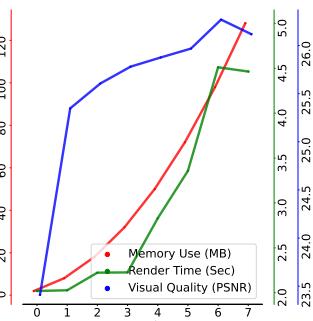


Fig. 17. We compare different orders of SH for our approach. The effect of increasing the order is negligible around order 5, but visible improvement can be seen in orders 0 to 4.



UV parameterization, using Blender’s smart unwrapping, and compare it with and without our UV optimization. Our optimized Tutte embedding achieves better PSNR as compared to an unoptimized Blender UV unwrapping, but by also optimizing Blender’s UV unwrapping we are able to achieve even better performance. In Fig. 18, we visualize the optimized Tutte embedding using our approach, as well as the parameterization output by Blender, optimized by our approach. Because there are many small charts in Blender’s output, there is some difficulty in optimization, but these small triangles are able to expand and take up more texels, and within the large chart, areas with internal cuts are expanded.

Content-aware Symmetric Dirichlet $\mathcal{L}_{I \circ UV}$. We validate the necessity of including $\mathcal{L}_{I \circ UV}$ in our objective function by comparing our approach with and without it, as summarized Table 3, showing our approach performs much better with this term. We observe that with only the image loss \mathcal{L}_{diff} to guide UV optimization, there is not sufficient gradient information to update the UV coordinates. We then demonstrate that our approach relies more on $\mathcal{L}_{I \circ UV}$ rather than \mathcal{L}_{UV} to achieve improved performance. To this end, we test our approach on two models (Jpn. Lamp and Hand Fan) with Spherical Harmonic Parallax and one of the two terms: \mathcal{L}_{UV} and $\mathcal{L}_{I \circ UV}$. We find that using $\mathcal{L}_{I \circ UV}$ as opposed to \mathcal{L}_{UV} improves the output by 0.25 and 0.4 PSNR, respectively. Finally, we experimented with only optimizing UV using \mathcal{L}_{diff} , where we found that \mathcal{L}_{diff} alone does not provide sufficient gradient information for the charts to expand, due to the discontinuous nature of the pixel gradients.

6 CONCLUSION & DISCUSSION

We demonstrate the efficacy of combining UV parameterization, view parallax mapping, vertex displacement, and texture-baking

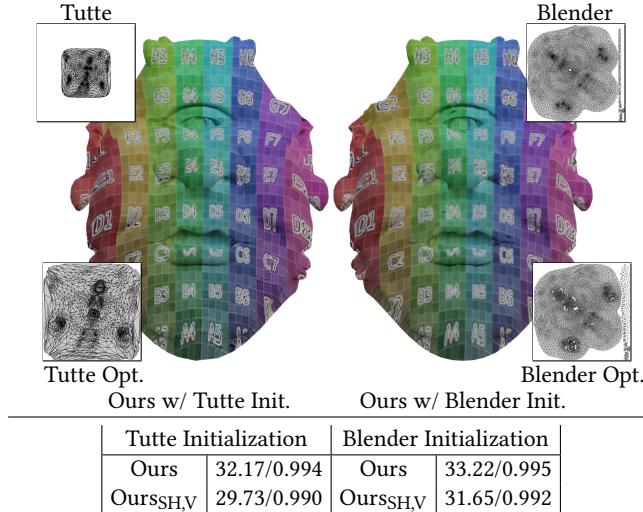


Fig. 18. We demonstrate how our approach handles different UV initializations. The left of the figure is our optimized textured result with the Tutte embedding initialization which is optimized to be space filling. The right is with the Blender’s UV unwrapping initialization which introduces additional seams. From the PSNR/MS-SSIM statistics table at the bottom, for the Tutte initialization, we see an increase of about 2.4 PSNR after optimizing UV, and for the Blender initialization, we see an increase of 1.6 PSNR. By optimizing the initial Tutte parameterization, our approach outperforms the initial Blender UV, and still improves when used on a better parameterization. Credits to Keenan Crane for the Ogre mesh.

Table 3. We ablate our approach with and without image-guided symmetric Dirichlet energy $L_{I \circ UV}$. Without this term, i.e. relying only on the image loss $\mathcal{L}_{\text{diff}}$, there is insufficient gradient information to guide the charts to expand, leading to inferior results.

Ablation of $L_{I \circ UV}$		
64x64	w/o $L_{I \circ UV}$	w/ $L_{I \circ UV}$
Cloud	32.19/0.956	33.32/0.960
Item Icons	21.69/0.907	23.77/0.939
Pixel Map	17.61/0.763	18.66/0.808
Painting	14.19/0.501	14.77/0.524
Sea Map	15.41/0.463	16.43/0.593
Rainforest	25.72/0.962	27.82/0.972
Texture 1	25.11/0.957	28.36/0.967
Texture 2	24.66/0.964	29.13/0.967
Texture 3	26.29/0.982	30.93/0.982
Hokusai	18.70/0.649	19.52/0.674

in a joint optimization formulation. Our approach builds on recent differentiable rendering advances to optimize a new parallax map and uses classic optimization techniques to ensure bijectivity of the UV parameterization. Over a dataset of complex 3D models, we demonstrate that combining these components leads to an increase in output visual quality, with little overhead.

Limitations. Our approach has a couple of limitations. First, the cost of optimizing UV coordinates is quite expensive, especially as compared to only baking textures without UV optimization. The bottleneck lies in the global intersection check and line search,

Table 4. Information about each 2D texture in our dataset.

2D Texture Information		
Name	Height	Width
Aliasing	1024	1024
Cloud	512	512
Item Icons	1024	1024
Pixel Map	2048	2048
Painting	1024	512
Sea Map	1024	2048
Mario	1024	1024
Rainforest	1024	1024
Texture 1	1024	1024
Texture 2	1024	1024
Texture 3	1024	1024
Hokusai	1024	2048

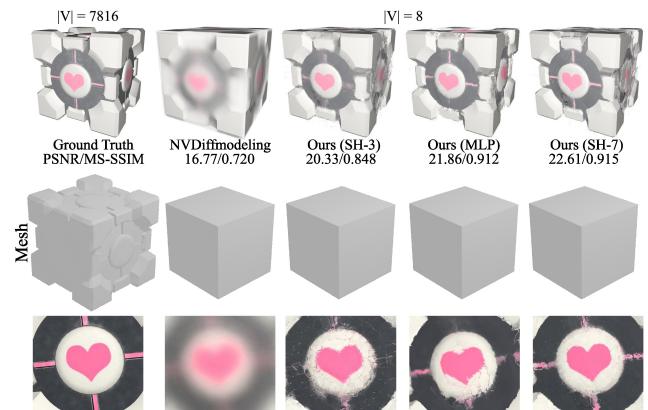


Fig. 19. We compare SH View Parallax and NVDiffmodeling on a challenging example. NVDiffmodeling cannot accurately capture the sharp details of the original model due to geometric differences. With only 3rd order SH, the low-poly model looks plausible from the front, but there are artifacts on the side. Using an MLP based approach reduces the number of artifacts on the side, but it takes significantly longer to train, and requires more resources to use in practice. We can increase the order of SH to 7 and find that it is able to outperform the MLP.

which we plan to speedup through re-engineering our implementation in CUDA. Second, our SH parallax mapping may introduce “cracks” if the low-poly and high-poly model are significantly different, see Fig. 19 for such an example. Due to bounds on frequency for our translation function, these cracks are inherent, as there will always be some region that it cannot fully capture, but they can be removed by introducing higher orders of SH or alternative representations such as neural nets. In practice, they will not be visible from a distance, and do not appear if the low-poly and high-poly models are similar. Thus, during practical use this drawback will be mitigated by switching LODs. Third, our method assumes that the cuts imposed by the original parameterization are fixed, and not optimized jointly with the learned texture map. It may be possible to incorporate OptCuts [Li et al., 2018b] or other cut reparameterizations with our work in order to optimize cuts, UV parameterization, and texture jointly.

Robustness & Failure Models. Our approach will never fail to output a model, even if it contains cracks in the output texture. If the input UV parameterization is bijective, the output will also be bijective as guaranteed by the safety of the line-search scheme. If the source and target mesh differ significantly, there may be issues in the optimized texture such as visible seams. For our test set where meshes are reasonably similar, cracks are never a prominent problem. We do not expect such problems when using mesh simplification tools to generate the output from the input mesh.

Downstream Applications. Our approach by default optimizes spherical harmonic view parallax maps, which might not have support in the downstream applications' rendering pipeline. To adapt our method to these applications, we propose a technique to "bake out" parallax mapping. Note that parallax mapping generates a view-dependent texture image, so the goal of "baking out" is to pick a single view direction d as a representative texel value. The differentiable rendering pipeline would choose to average the texel values over all view directions d , oftentimes leading to blurring data. Instead, we propose to follow the conventional approach and choose a single d along the normal direction, i.e. fetching the texel data along normal direction to yield a new "representative" texture to be used by a conventional rendering pipeline without parallax mapping functionality.

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Table 5. Different measures of complexity for each high-poly mesh and simplified mesh in our dataset.

LOD-1	# Comp.	# Vertices	# Faces	LOD-2	# Comp.	# Vertices	# Faces	LOD-3	# Comp.	# Vertices	# Faces	High-Poly	# Comp.	# Vertices	# Faces
Dragon Jar	20	866	1182	Dragon Jar	10	328	380	Garden Seat	2	61	84	Dragon Jar	40	11388	20412
Garden Seat	3	373	624	Garden Seat	1	122	194	Helmet	3	81	98	Garden Seat	746	12190	15000
Helmet	224	3673	4038	Helmet	37	624	684	Cat Statue	11	179	200	Helmet	213	5293	6509
Cat Statue	17	572	728	Cat Statue	6	169	214	Chn. Chess	4	62	80	Cat Statue	13	9454	17536
Chn. Chess	40	1076	1354	Chn. Chess	19	269	270	Hand Fan	3	42	44	Chn. Chess	244	18620	29660
Hand Fan	28	521	518	Hand Fan	13	192	192	Iron Cup	2	31	34	Hand Fan	134	3675	4048
Iron Cup	15	600	946	Iron Cup	8	243	304	Cut Fish	1	120	160	Iron Cup	440	15195	22258
Cut Fish	24	703	864	Cut Fish	12	241	268	Cut Fish	4	80	100	Cut Fish	57	5081	8168
Easter Egg	3	410	654	Easter Egg	1	135	214	Easter Egg	3	89	114	Easter Egg	8	4917	8964
Baguette	8	244	306	Baguette	2	53	64	Baguette	2	28	32	Baguette	89	16951	28640
Greek Vases	41	1181	1630	Greek Vases	28	487	560	Greek Vases	16	219	226	Greek Vases	273	23399	37524
Gundam	275	4308	4614	Gundam	47	816	886	Gundam	26	340	328	Gundam	1776	17948	16106
Italian Car	57	1198	1468	Italian Car	22	380	432	Italian Car	3	79	86	Italian Car	445	7923	8876
Teacup	8	462	692	Teacup	5	143	192	Teacup	3	92	104	Teacup	18	3311	5504
Jpn. Lamp	40	955	1042	Jpn. Lamp	32	611	660	Jpn. Lamp	12	349	368	Jpn. Lamp	200	3758	4586
Lego Fig.	36	760	872	Lego Fig.	14	262	304	Lego Fig.	4	89	108	Lego Fig.	117	3112	3680
Lemon	1	299	460	Lemon	1	113	178	Lemon	1	53	84	Lemon	17	502	723
Mask	42	1326	1726	Mask	16	440	522	Mask	13	203	218	Mask	102	3165	4082
Golem	125	2799	3202	Golem	20	574	716	Golem	15	251	276	Golem	225	5284	6677
White Tree	312	4937	5094	White Tree	82	1036	1018	White Tree	40	343	304	White Tree	598	8312	9398
Pingu	121	2188	2372	Pingu	34	617	664	Pingu	18	291	292	Pingu	655	14373	18163
Pony (Car)	42	1042	1232	Pony (Car)	18	307	334	Pony (Car)	7	107	108	Pony (Car)	331	6817	8338
Sand Arena	222	3799	3990	Sand Arena	50	808	836	Sand Arena	20	225	218	Sand Arena	1960	24209	23931
Skate Bunny	154	2977	3334	Skate Bunny	34	641	704	Skate Bunny	16	251	264	Skate Bunny	1216	27241	34444
Knight	184	2763	2872	Knight	32	560	578	Knight	7	207	222	Knight	1298	70291	103366
Umbrella	188	3107	3248	Umbrella	69	1187	1210	Umbrella	18	342	364	Umbrella	222	2511	2495

Table 6. We compare various algorithms in optimizing a 2D planar mesh with various texture images, where we initialize the UV mapping to the identity mapping. We perform texture baking to a lower-resolution texture image, using our method, NVDiffModeling, bi-linear downsampling. For NVDiffModeling, we initialize the UV parameterization using UVAtlas with signal-specialized parameterization [Sander et al., 2002]. For our mesh, we only optimize the diffuse map and UV coordinates (i.e. no parallax or displacement maps are optimized), and all other methods only optimize the diffuse map. All input images are at least 512^2 , and are not necessarily square. When performing extreme compression, such as to 64^2 images, our approach is able to consistently outperform other approaches. Our approach is even better when compressing to 256^2 images.

Texture Compression: $\mathbb{E}[\text{PSNR}]^\dagger/\text{MSSSIM}^\dagger$				Texture Compression: $\mathbb{E}[\text{PSNR}]^\dagger/\text{MSSSIM}^\dagger$				Texture Compression: $\mathbb{E}[\text{PSNR}]^\dagger/\text{MSSSIM}^\dagger$				Texture Compression: $\mathbb{E}[\text{PSNR}]^\dagger/\text{MSSSIM}^\dagger$			
64 × 64	NVDiffModeling	Ours(UV)	Downsample	128 × 128	NVDiffModeling	Ours _{UV}	Downsample	UVAtlas	256 × 256	NVDiffModeling	Ours _{UV}	Downsample	UVAtlas		
Aliasing	10.10/0.311	10.10/0.315	9.73/0.312	10.14/0.320	Aliasing	10.45/0.372	10.57/0.396	9.98/0.342	10.52/0.386	Aliasing	12.04/0.674	13.15/0.678	11.52/0.642	12.24/0.588	
Cloud	32.60/0.956	33.32/0.960	31.19/0.946	31.99/0.950	Cloud	37.33/0.985	39.31/0.990	36.24/0.980	36.18/0.978	Cloud	44.28/0.997	46.88/0.998	42.38/0.996	42.25/0.995	
Item Icons	22.49/0.921	23.77/0.939	21.72/0.902	22.03/0.907	Item Icons	25.31/0.969	28.24/0.983	23.98/0.947	24.88/0.959	Item Icons	30.09/0.994	35.67/0.998	28.71/0.988	28.95/0.988	
Pixel Map	18.22/0.786	18.66/0.808	17.74/0.756	18.04/0.770	Pixel Map	20.00/0.889	20.85/0.907	19.08/0.838	19.68/0.872	Pixel Map	22.89/0.966	24.16/0.972	22.11/0.944	22.26/0.950	
Painting	14.64/0.514	14.77/0.524	14.16/0.457	14.37/0.468	Painting	16.64/0.747	17.00/0.756	15.59/0.630	16.02/0.679	Painting	20.87/0.938	22.16/0.944	19.68/0.897	19.51/0.887	
Sea Map	16.17/0.560	16.43/0.593	16.03/0.553	15.80/0.491	Sea Map	17.44/0.745	18.06/0.784	17.02/0.700	16.83/0.647	Sea Map	20.05/0.925	21.28/0.942	19.46/0.904	18.62/0.827	
Mario	17.59/0.798	19.28/0.839	16.31/0.749	15.85/0.709	Mario	21.60/0.921	23.47/0.938	21.21/0.913	19.26/0.863	Mario	26.19/0.979	30.08/0.988	25.25/0.972	23.20/0.952	
Rainforest	27.05/0.966	27.82/0.972	27.10/0.964	26.45/0.961	Rainforest	30.17/0.985	30.74/0.987	29.61/0.980	28.73/0.980	Rainforest	32.23/0.994	36.05/0.997	32.44/0.993	31.75/0.992	
Texture 1	25.82/0.952	28.36/0.967	25.43/0.956	26.00/0.928	Texture 1	26.84/0.980	34.44/0.989	25.86/0.976	28.00/0.954	Texture 1	29.33/0.995	41.09/0.998	27.95/0.993	33.89/0.976	
Texture 2	25.50/0.956	29.13/0.967	25.08/0.960	26.02/0.928	Texture 2	26.64/0.980	36.24/0.987	25.52/0.976	28.58/0.953	Texture 2	30.06/0.993	40.84/0.997	28.65/0.993	37.10/0.973	
Texture 3	26.93/0.973	30.93/0.982	26.66/0.980	28.01/0.969	Texture 3	27.77/0.984	36.16/0.992	26.97/0.988	31.41/0.982	Texture 3	30.23/0.993	40.52/0.997	28.98/0.996	38.34/0.989	
Hokusai	19.15/0.656	19.52/0.674	17.42/0.539	18.68/0.615	Hokusai	21.46/0.816	21.99/0.823	20.58/0.771	20.82/0.769	Hokusai	25.41/0.946	26.15/0.940	23.78/0.913	24.35/0.912	

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A ADDITIONAL RESULTS

Table 7. Experiments with the same setting as Table 6 but instead we compress textures on 3D models, using their provided UV coordinates as initialization.

Resolution : 2048 ² → 128 ²	Original Model: E[PSNR] [†] /MSSSIM [†]				Resolution : 2048 ² → 256 ²	Original Model: E[PSNR] [†] /MSSSIM [†]				Resolution : 2048 ² → 512 ²	High-Poly: E[PSNR] [†] /MSSSIM [†]			
	UVAtlas	Ours	Ours _{UV}	NVDiffModeling		UVAtlas	Ours	Ours _{UV}	NVDiffModeling		UVAtlas	Ours	Ours _{UV}	NVDiffModeling
Dragon Jar	26.15/0.916	28.96/0.958	26.60/0.919	26.14/0.916	Dragon Jar	28.93/0.961	33.39/0.988	29.99/0.969	29.43/0.966	Dragon Jar	33.09/0.988	40.27/0.998	36.21/0.995	33.85/0.990
Garden Seat	22.17/0.807	25.61/0.913	23.33/0.838	22.28/0.810	Garden Seat	24.57/0.898	29.27/0.971	26.51/0.936	25.03/0.909	Garden Seat	27.78/0.959	33.12/0.989	30.38/0.980	28.43/0.966
Helmet	27.60/0.970	30.36/0.986	28.72/0.976	26.94/0.973	Helmet	29.45/0.982	33.65/0.995	31.20/0.990	29.27/0.982	Helmet	32.18/0.992	38.69/0.998	34.99/0.997	32.75/0.984
Cat Statue	29.95/0.933	32.43/0.969	31.24/0.953	31.11/0.953	Cat Statue	31.89/0.964	36.27/0.991	34.13/0.983	33.90/0.981	Cat Statue	34.69/0.986	41.25/0.998	39.37/0.997	37.76/0.994
Chn. Chess	34.88/0.98	37.28/0.989	35.58/0.982	36.14/0.984	Chn. Chess	35.88/0.987	38.77/0.993	36.99/0.990	36.85/0.988	Chn. Chess	36.86/0.992	41.42/0.997	39.34/0.995	38.88/0.994
Hand Fan	31.07/0.982	38.00/0.997	36.82/0.995	36.99/0.995	Hand Fan	32.23/0.987	39.67/0.998	39.08/0.997	38.87/0.997	Hand Fan	33.65/0.991	40.62/0.999	43.17/0.999	43.39/0.999
Iron Cup	23.11/0.922	26.64/0.965	24.15/0.932	23.39/0.924	Iron Cup	24.44/0.938	29.33/0.984	26.52/0.953	25.08/0.942	Iron Cup	26.07/0.957	32.54/0.994	27.91/0.975	27.32/0.964
Cut Fish	35.51/0.98	43.55/0.997	38.67/0.993	37.24/0.992	Cut Fish	39.11/0.994	45.45/0.998	42.69/0.997	42.14/0.996	Cut Fish	43.55/0.997	46.61/0.999	48.25/0.999	47.10/0.999
Easter Egg	25.02/0.900	34.85/0.985	30.11/0.963	27.59/0.942	Easter Egg	29.33/0.961	40.31/0.997	36.17/0.992	34.06/0.987	Easter Egg	34.87/0.989	44.21/0.999	43.23/0.999	41.05/0.998
Baguette	40.10/0.995	39.27/0.996	40.29/0.996	40.12/0.996	Baguette	40.27/0.996	39.35/0.996	40.38/0.997	40.32/0.997	Baguette	40.87/0.997	40.31/0.997	41.17/0.997	40.95/0.997
Greek Vases	30.87/0.979	34.94/0.993	31.72/0.982	29.85/0.982	Greek Vases	33.24/0.989	37.85/0.997	34.23/0.992	33.67/0.991	Greek Vases	36.46/0.995	40.53/0.999	37.88/0.997	37.50/0.997
Gundam	20.21/0.949	20.23/0.951	20.10/0.948	20.08/0.947	Gundam	20.34/0.952	21.76/0.967	20.23/0.952	20.21/0.951	Gundam	20.63/0.956	26.12/0.990	21.49/0.967	20.98/0.961
Italian Car	37.95/0.99	44.74/0.999	43.24/0.994	41.30/0.998	Italian Car	40.89/0.977	45.62/1.000	49.90/1.000	47.49/0.999	Italian Car	43.45/0.999	45.27/1.000	54.37/1.000	39.26/0.998
Teacup	24.95/0.925	37.27/0.979	34.33/0.960	32.55/0.950	Teacup	32.67/0.956	40.33/0.994	37.53/0.983	36.12/0.976	Teacup	36.39/0.979	43.59/0.998	40.90/0.995	39.48/0.991
Jpn. Lamp	29.28/0.937	33.88/0.979	31.26/0.959	30.45/0.953	Jpn. Lamp	30.66/0.954	36.14/0.988	33.12/0.976	32.05/0.969	Jpn. Lamp	32.42/0.970	38.38/0.995	35.11/0.987	34.13/0.983
Lego Fig.	35.69/0.992	39.40/0.999	39.44/0.998	39.02/0.998	Lego Fig.	39.09/0.998	38.27/0.998	42.73/0.999	42.46/0.999	Lego Fig.	42.12/0.999	41.89/0.999	44.80/0.999	44.99/0.999
Lemon	42.32/0.98	45.24/0.993	44.88/0.992	44.00/0.989	Lemon	42.44/0.990	48.82/0.998	49.35/0.996	46.95/0.995	Lemon	46.88/0.995	52.51/1.000	53.78/1.000	51.36/0.999
Mask	44.19/0.998	47.31/0.998	46.04/0.997	43.80/0.995	Mask	47.73/0.998	49.33/0.999	49.38/0.999	46.84/0.998	Mask	51.00/0.999	50.23/1.000	49.81/0.999	49.86/0.999
Golem	36.51/0.979	44.19/0.987	38.14/0.981	37.43/0.976	Golem	38.17/0.984	41.26/0.994	39.99/0.984	38.77/0.986	Golem	40.06/0.992	43.28/0.999	42.17/0.996	40.58/0.993
White Tree	17.32/0.834	17.77/0.809	17.61/0.826	17.26/0.823	White Tree	17.57/0.843	17.88/0.818	17.68/0.844	17.22/0.838	White Tree	17.41/0.843	17.86/0.815	17.71/0.847	17.32/0.842
Pengu	34.38/0.978	39.13/0.990	35.38/0.976	34.54/0.978	Pengu	37.14/0.987	41.88/0.996	38.85/0.999	38.69/0.991	Pengu	40.15/0.994	43.57/0.999	42.81/0.997	42.85/0.997
Pony (Car)	38.03/0.992	41.31/0.998	40.47/0.995	39.43/0.994	Pony (Car)	41.56/0.996	46.32/0.998	44.65/0.998	43.28/0.997	Pony (Car)	44.68/0.998	46.26/0.999	48.01/1.000	45.05/0.998
Sand Arena	39.75/0.996	43.30/0.999	45.33/0.998	42.46/0.998	Sand Arena	42.92/0.998	43.67/0.999	46.78/0.999	40.39/0.997	Sand Arena	44.71/0.999	43.93/0.999	48.32/0.999	42.79/0.998
Skate Bunny	28.79/0.986	32.63/0.989	30.91/0.990	29.62/0.988	Skate Bunny	31.77/0.992	34.87/0.997	34.19/0.997	33.72/0.996	Skate Bunny	34.57/0.997	37.70/0.999	39.93/0.999	39.61/0.999
Knight	26.77/0.970	28.54/0.970	29.41/0.982	28.33/0.977	Knight	28.07/0.983	28.98/0.984	29.73/0.986	28.69/0.989	Knight	29.34/0.987	28.88/0.986	29.94/0.989	29.06/0.983
Umbrella	29.81/0.969	35.31/0.990	29.84/0.962	28.34/0.949	Umbrella	33.46/0.988	37.82/0.996	34.93/0.992	32.42/0.983	Umbrella	37.55/0.995	36.47/0.995	38.02/0.996	36.60/0.995

Table 8. Comparison of texture baking on LOD-1 using NVDiffModeling, our approach, our approach with SH and UV optimization, Unreal, and Blender.

Resolution : 2048 ² → 128 ²	LOD-1: E[PSNR] [†] /MSSSIM [†]				Resolution : 2048 ² → 256 ²	LOD-1: E[PSNR] [†] /MSSSIM [†]				Resolution : 2048 ² → 512 ²	LOD-1: E[PSNR] [†] /MSSSIM [†]					
	LOD-1: E[PSNR] [†] /MSSSIM [†]	UVAtlas	Ours	UE	LOD-1: E[PSNR] [†] /MSSSIM [†]	UVAtlas	Ours	UE	LOD-1: E[PSNR] [†] /MSSSIM [†]	UVAtlas	Ours	UE	Blender			
Dragon Jar	24.57/0.887	27.71/0.956	29.11/0.968	22.48/0.838	13.88/0.766	Dragon Jar	26.51/0.937	30.61/0.984	31.95/0.988	23.37/0.882	24.23/0.911	Dragon Jar	28.79/0.970	32.30/0.992	33.30/0.993	24.38/0.925
Garden Seat	21.91/0.804	23.92/0.889	26.27/0.939	20.87/0.730	21.29/0.774	Garden Seat	24.08/0.893	27.34/0.940	29.08/0.978	22.36/0.882	23.09/0.862	Garden Seat	25.88/0.938	29.06/0.981	30.47/0.984	23.74/0.909
Helmet	25.55/0.960	26.84/0.972	26.96/0.969	22.84/0.920	22.77/0.928	Helmet	26.90/0.974	29.29/0.987	29.78/0.987	23.33/0.996	23.77/0.943	Helmet	28.03/0.982	30.79/0.992	32.55/0.994	23.71/0.947
Cat Statue	28.73/0.945	30.18/0.974	31.95/0.981	27.82/0.909	27.86/0.926	Cat Statue	29.78/0.968	31.47/0.988	33.44/0.992	28.06/0.938	28.68/0.951	Cat Statue	30.53/0.978	32.18/0.993	34.36/0.995	28.68/0.957
Chn. Chess	31.41/0.977	32.29/0.986	34.85/0.987	30.19/0.960	30.22/0.965	Chn. Chess	32.02/0.984	33.22/0.992	36.08/0.993	30.55/0.968	30.79/0.973	Chn. Chess	32.54/0.987	33.83/0.995	36.91/0.995	32.51/0.987
Hand Fan	28.92/0.986	31.65/0.992	33.51/0.993	25.97/0.963	26.29/0.994	Hand Fan	30.38/0.989	32.46/0.994	34.41/0.996	32.56/0.963	36.28/0.968	Hand Fan	30.74/0.990	32.82/0.995	34.58/0.996	25.94/0.965
Iron Cup	23.15/0.928	24.56/0.939	26.96/0.954	21.59/0.891	22.28/0.907	Iron Cup	24.35/0.939	26.64/0.964	28.38/0.988	22.34/0.908	23.01/0.926	Iron Cup	25.61/0.956	29.09/0.998	30.51/0.991	23.00/0.923
Cut Fish	31.67/0.985	33.21/0.992	35.53/0.994	28.78/0.964	29.60/0.971	Cut Fish	32.54/0.990	33.61/0.993	36.05/0.996	29.05/0.967	29.98/0.976	Cut Fish	32.89/0.992	33.80/0.995	35.98/0.999	29.16/0.970
Easter Egg	25.55/0.920	28.95/0.933	32.23/0.984	24.36/0.879	25.79/0.954	Easter Egg	28.79/0.967	30.22/0.994	34.22/0.994	28.23/0.953	28.73/0.976	Easter Egg	30.02/0.955	32.40/0.991	33.97/0.999	27.59/0.972
Baguette	34.28/0.993	36.62/0.994	37.57/0.995	32.05/0.989	34.05/0.991	Baguette	34.34/0.993	35.15/0.998	35.81/0.995	33.75/0.979	37.02/0.987	Baguette	34.56/0.994	35.60/0.997	36.40/0.996	32.40/0.991
Greek Vases	28.00/0.977	29.21/0.986	31.20/0.994	28.88/0.988	29.30/0.980	Greek Vases	28.68/0.983	29.99/0.990	32.87/0.997	27.08/0.978	28.77/0.967	Greek Vases	29.03/0.985	32.73/0.997	33.40/0.999	28.05/0.980
Gundam	19.47/0.930	19.74/0.939	19.81/0.941	18.29/0.886	18.88/0.901	Gundam	19.57/0.933	20.33/0.949	20.75/0.955	18.53/0.894	18.46/0.891	Gundam	20.03/0.945	22.73/0.973	23.23/0.977	18.54/0.896
Italian Car	28.39/0.983	30.36/0.991	31.20/0.994	29.43/0.950	29.71/0.959	Italian Car	29.32/0.988	31.20/0.993	33.60/0.996	24.56/0.951	26.11/0.963	Italian Car	27.09/0.989	31.68/0.994	33.69/0.996	24.54/0.952
Teacup	28.26/0.928	30.22/0.959	32.61/0.995	25.98/0.886	26.49/0.903	Teacup	3									

Table 10. Comparison of texture baking on LOD-3 using NVDiffModeling, our approach, our approach with SH and UV optimization, Unreal, and Blender.

Resolution : 2048 ² → 128 ²	LOD-3: E[PSNR] [†] /MSSSIM [†]					Resolution : 2048 ² → 256 ²	LOD-3: E[PSNR] [†] /MSSSIM [†]					Resolution : 2048 ² → 512 ²	LOD-3: E[PSNR] [†] /MSSSIM [†]				
	NVDiffModeling	Ours _{UVSH}	Ours	UE	Blender		NVDiffModeling	Ours _{UVSH}	Ours	UE	Blender		NVDiffModeling	Ours _{UVSH}	Ours	UE	Blender
Dragon Jar	21.84/0.847	22.04/0.869	24.42/0.927	20.48/0.807	20.54/0.814	Dragon Jar	22.08/0.859	23.76/0.919	25.09/0.943	20.06/0.813	20.34/0.816	Dragon Jar	22.13/0.862	24.00/0.927	25.17/0.946	19.85/0.809	20.03/0.811
Garden Seat	19.39/0.704	20.27/0.784	21.80/0.839	18.65/0.647	18.50/0.655	Garden Seat	19.91/0.745	21.11/0.832	23.66/0.914	18.67/0.665	18.28/0.656	Garden Seat	20.11/0.761	22.24/0.892	24.51/0.936	18.21/0.655	17.80/0.644
Helmet	20.62/0.888	20.96/0.898	22.88/0.929	19.25/0.859	19.27/0.865	Helmet	20.83/0.896	22.10/0.925	23.89/0.946	19.15/0.869	19.37/0.873	Helmet	20.98/0.900	22.15/0.934	23.92/0.948	19.11/0.871	19.30/0.873
Cat Statue	24.43/0.899	24.89/0.921	23.68/0.905	23.13/0.868	23.41/0.874	Cat Statue	24.58/0.905	25.32/0.939	26.73/0.959	23.22/0.876	23.35/0.878	Cat Statue	24.61/0.907	25.69/0.948	26.74/0.959	23.06/0.876	23.16/0.876
Chn. Chess	24.12/0.926	24.65/0.939	27.85/0.958	23.74/0.914	23.46/0.913	Chn. Chess	24.24/0.928	24.91/0.947	28.17/0.965	23.75/0.915	23.44/0.913	Chn. Chess	24.29/0.931	25.23/0.954	28.22/0.966	23.68/0.913	23.40/0.911
Hand Fan	25.49/0.956	26.54/0.968	27.83/0.976	23.52/0.933	23.83/0.936	Hand Fan	25.57/0.957	25.89/0.972	28.15/0.979	23.36/0.933	23.83/0.937	Hand Fan	25.59/0.957	26.92/0.972	27.89/0.976	23.24/0.932	23.93/0.939
Iron Cup	19.70/0.846	19.70/0.857	21.68/0.900	16.71/0.776	17.33/0.794	Iron Cup	19.34/0.852	19.79/0.862	21.85/0.926	16.53/0.774	17.07/0.788	Iron Cup	19.49/0.857	21.17/0.888	23.25/0.937	16.33/0.770	16.83/0.782
Cut Fish	25.43/0.951	25.96/0.956	27.28/0.955	23.30/0.917	24.15/0.931	Cut Fish	25.55/0.953	26.65/0.966	28.62/0.975	24.08/0.928	24.09/0.932	Cut Fish	25.58/0.954	26.85/0.969	28.62/0.976	24.01/0.928	24.01/0.931
Easter Egg	22.19/0.861	23.86/0.921	25.88/0.951	14.86/0.738	17.66/0.763	Easter Egg	22.71/0.881	24.71/0.946	26.67/0.968	21.57/0.839	17.42/0.766	Easter Egg	22.83/0.886	24.75/0.949	26.83/0.971	21.18/0.834	17.24/0.766
Baguette	28.24/0.980	28.44/0.987	30.69/0.987	24.96/0.954	28.13/0.972	Baguette	28.25/0.980	28.52/0.982	30.66/0.987	28.12/0.975	28.02/0.972	Baguette	28.25/0.981	28.66/0.983	30.32/0.981	28.11/0.975	28.03/0.977
Greek Vases	22.54/0.927	23.27/0.943	25.52/0.959	20.41/0.901	21.84/0.915	Greek Vases	22.62/0.929	23.35/0.948	25.77/0.964	21.83/0.915	21.73/0.914	Greek Vases	22.65/0.930	23.66/0.950	25.72/0.964	21.88/0.917	21.59/0.912
Gundam	18.04/0.871	18.35/0.881	18.69/0.897	14.31/0.805	17.23/0.833	Gundam	18.07/0.873	18.61/0.894	19.04/0.904	16.91/0.829	16.92/0.829	Gundam	18.14/0.875	19.03/0.905	19.64/0.920	16.89/0.830	16.85/0.836
Italian Car	23.03/0.942	23.79/0.950	26.28/0.969	13.69/0.860	17.64/0.893	Italian Car	23.20/0.945	24.72/0.962	26.73/0.974	20.90/0.966	17.56/0.894	Italian Car	23.21/0.945	24.96/0.965	26.68/0.974	20.74/0.905	17.50/0.894
Teacup	23.75/0.897	24.35/0.922	24.01/0.918	22.48/0.865	22.59/0.864	Teacup	24.02/0.912	24.66/0.941	26.10/0.954	22.34/0.872	22.73/0.876	Teacup	24.10/0.915	25.28/0.956	26.38/0.966	22.61/0.892	22.69/0.879
Jpn. Lamp	21.24/0.842	21.78/0.869	24.52/0.913	16.45/0.780	20.39/0.816	Jpn. Lamp	21.42/0.854	22.38/0.893	25.19/0.933	20.45/0.819	20.41/0.819	Jpn. Lamp	21.47/0.858	22.79/0.910	25.32/0.940	20.53/0.832	20.31/0.816
Lego Fig.	23.55/0.957	24.54/0.964	26.72/0.974	21.30/0.935	21.74/0.939	Lego Fig.	23.61/0.958	25.10/0.969	26.98/0.978	21.34/0.936	21.70/0.940	Lego Fig.	23.63/0.958	25.13/0.969	27.00/0.977	21.37/0.936	21.67/0.939
Lemon	25.70/0.970	25.82/0.975	25.88/0.971	18.11/0.894	25.35/0.947	Lemon	25.70/0.952	25.89/0.961	26.60/0.975	24.94/0.942	25.35/0.947	Lemon	25.71/0.953	25.90/0.961	28.34/0.974	24.92/0.941	25.34/0.946
Mask	23.48/0.942	24.18/0.954	26.28/0.964	22.50/0.929	22.82/0.934	Mask	23.56/0.944	24.57/0.960	26.33/0.967	22.81/0.933	22.96/0.935	Mask	23.62/0.944	24.65/0.961	26.22/0.966	22.78/0.931	22.96/0.934
Golem	22.93/0.910	23.27/0.924	25.04/0.934	21.80/0.864	22.54/0.896	Golem	22.98/0.914	23.62/0.934	25.09/0.942	22.39/0.889	22.52/0.897	Golem	22.99/0.915	23.86/0.942	25.17/0.945	22.48/0.892	22.50/0.896
White Tree	16.46/0.771	16.42/0.762	17.29/0.781	14.30/0.724	12.10/0.726	White Tree	16.42/0.773	16.38/0.757	17.42/0.793	14.77/0.725	12.18/0.720	White Tree	16.39/0.769	16.44/0.767	17.45/0.797	14.22/0.722	12.30/0.724
Pengu	24.08/0.922	24.37/0.933	26.32/0.951	22.77/0.887	22.96/0.899	Pengu	24.21/0.930	25.30/0.951	26.64/0.959	22.71/0.891	22.95/0.903	Pengu	24.27/0.931	25.30/0.957	26.65/0.966	22.94/0.903	22.91/0.904
Pony (Car)	23.77/0.931	24.56/0.942	27.57/0.962	17.46/0.859	21.46/0.903	Pony (Car)	23.85/0.935	24.35/0.939	27.49/0.963	17.40/0.861	21.32/0.903	Pony (Car)	23.88/0.937	24.70/0.943	27.48/0.972	17.39/0.864	21.10/0.898
Sand Arena	25.48/0.960	26.43/0.970	28.10/0.976	24.45/0.950	24.51/0.951	Sand Arena	25.54/0.961	26.61/0.972	28.11/0.976	24.38/0.950	24.52/0.951	Sand Arena	25.54/0.961	26.69/0.973	27.96/0.976	24.32/0.949	24.34/0.949
Skate Bunny	21.05/0.916	22.33/0.937	23.95/0.952	19.64/0.886	19.49/0.888	Skate Bunny	21.20/0.919	22.81/0.944	24.32/0.957	19.46/0.884	19.34/0.889	Skate Bunny	21.21/0.919	23.05/0.948	24.39/0.958	19.30/0.888	19.20/0.888
Knight	22.02/0.924	22.84/0.939	24.47/0.953	20.69/0.903	19.94/0.898	Knight	22.10/0.927	23.10/0.945	24.69/0.958	20.52/0.902	19.95/0.898	Knight	22.11/0.927	23.28/0.948	24.68/0.958	20.38/0.900	19.91/0.897
Umbrella	21.13/0.859	22.10/0.897	24.18/0.928	17.45/0.791	18.57/0.811	Umbrella	21.31/0.868	22.78/0.916	25.00/0.945	17.36/0.796	18.46/0.815	Umbrella	21.38/0.870	23.09/0.924	25.38/0.952	17.54/0.805	18.34/0.813

Table 11. From left to right, we compare our full method, our method with only SH and vertex displacement, our method with only vertex displacement, 1 level of subdivision using NVDiffmodeling, NVDiffmodeling with Laplacian smoothing, NVDiffmodeling, parallax mapping [Kaneko et al., 2001], parallax mapping with offset limiting [Welsh et al., 2004], our 3rd order Spherical Harmonics, steep parallax mapping [McGuire and McGuire, 2005], and parallax occlusion mapping [Tatarchuk, 2006]. We note subdivision introduces additional vertices, either as pre-processing or at runtime, thus does not pose a fair computational comparison, though it is still a common tool for rendering low-poly models. Our approach remains competitive with 1 level of subdivision which effectively increases the number of triangles 4 times. POM and SPM are also optimized with the term $\frac{1}{v_z}$, which slightly outperforms NVDiffModeling.

Resolution: 512 × 512	LOD-3: E[PSNR] [†] /MSSSIM [†]										
	Ours	Ours _{UVSH}	Ours _V	NV Subdiv.	NV*	NV	Parallax	PMOL	Ours _{SSH}	SPM	POM
Dragon Jar	25.07/0.944	24.60/0.933	23.14/0.890	25.58/0.941	18.74/0.757	22.13/0.862	23.13/0.897	22.37/0.913	23.06/0.897	23.04/0.897	
Garden Seat	24.53/0.935	24.10/0.922	21.99/0.841	24.92/0.928	18.49/0.685	20.12/0.762	21.78/0.868	21.41/0.846	22.21/0.874	21.70/0.864	21.65/0.862
Helmet	23.92/0.948	24.06/0.949	22.79/0.928	24.06/0.950	19.53/0.876	20.98/0.900	21.53/0.911	21.57/0.916	22.37/0.925	21.59/0.917	21.57/0.917
Cat Statue	26.74/0.959	26.55/0.955	25.59/0.928	27.40/0.953	23.27/0.864	24.61/0.907	25.29/0.940	25.17/0.934	25.35/0.929	25.23/0.937	25.25/0.937
Chn. Chess	28.22/0.966	27.94/0.963	26.83/0.949	27.67/0.961	23.69/0.918	24.29/0.931	24.54/0.937	24.64/0.939	25.18/0.942	24.69/0.941	24.68/0.941
Hand Fan	27.89/0.976	27.95/0.977	26.97/0.969	26.96/0.970	22.55/0.916	25.59/0.957	26.12/0.964	26.25/0.965	26.89/0.969	26.03/0.962	26.03/0.962
Iron Cup	23.25/0.937	23.90/0.943	22.01/0.909	23.93/0.934	18.46/0.833	19.49/0.857	19.98/0.870	20.22/0.876	21.25/0.901	20.14/0.876	20.16/0.877
Cut Fish	28.62/0.976	28.45/0.975	27.58/0.965	28.68/0.976	22.73/0.920	25.58/0.954	25.95/0.960	25.98/0.960	26.73/0.964	25.95/0.960	25.93/0.960
Easter Egg	26.83/0.971	26.70/0.969	24.74/0.929	31.12/0.987	23.92/0.906	22.83/0.886	24.49/0.942	24.25/0.934	24.73/0.941	24.55/0.944	24.52/0.943
Baguette	30.32/0.986	30.21/0.986	29.69/0.983	33.00/0.992	27.89/0.978	28.25/0.980	28.36/0.981	28.31/0.981	28.52/0.980	28.35/0.981	28.34/0.981
Greek Vases	25.72/0.964	25.42/0.960	24.54/0.946	26.41/0.971	21.75/0.914	22.91/0.936	22.87/0.936	23.39/0.939	22.88/0.936	22.88/0.936	22