17 - Spatial And Skeletal Deformations

Acknowledgement: Daniele Panozzo

Florida State University

Space Deformations

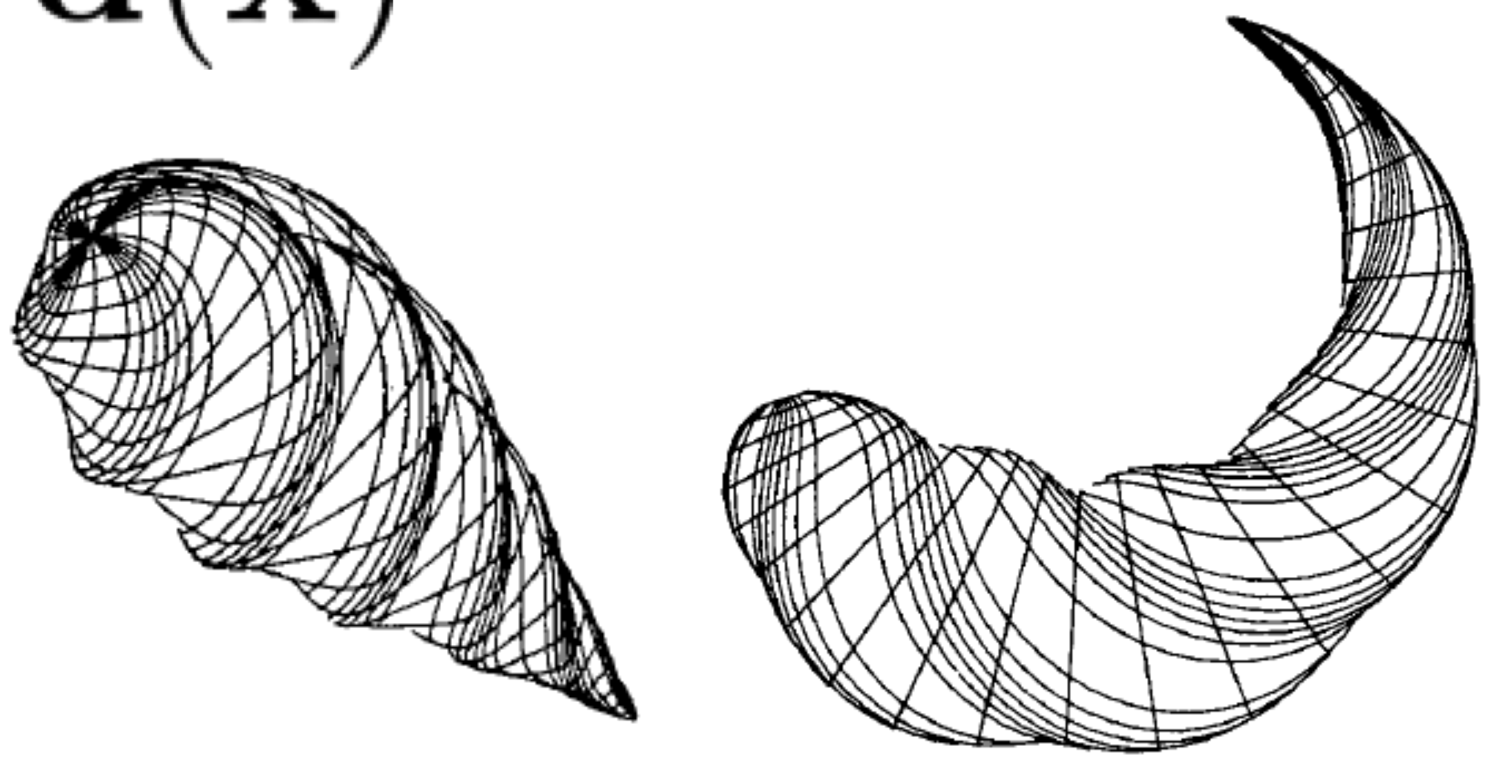
Space Deformation

- Displacement function defined on the ambient space

$$\mathbf{d} : \mathbb{R}^3 \rightarrow \mathbb{R}^3$$

- Evaluate the function on the points of the shape embedded in the space

$$\mathbf{x}' = \mathbf{x} + \mathbf{d}(\mathbf{x})$$



Twist warp
Global and local deformation of solids
[A. Barr, SIGGRAPH 84]

Freeform Deformations

- Control object
- User defines displacements $\mathbf{d}_i$ for each element of the control object
- Displacements are interpolated to the entire space using basis functions

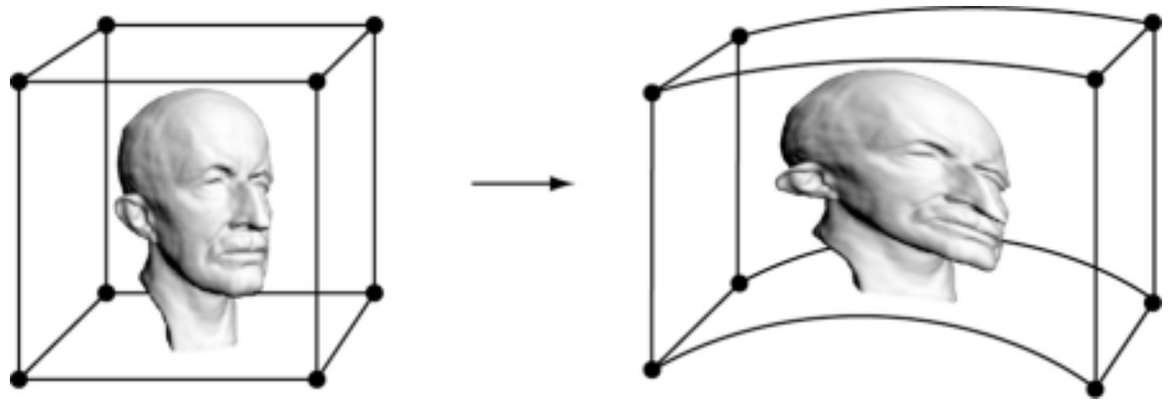
$$B_i(\mathbf{x}) : \mathbb{R}^3 \to \mathbb{R}$$

$$\mathbf{d}(\mathbf{x}) = \sum_{i=1}^{k} \mathbf{d}_i \, B_i(\mathbf{x})$$

- Basis functions should be smooth for aesthetic results

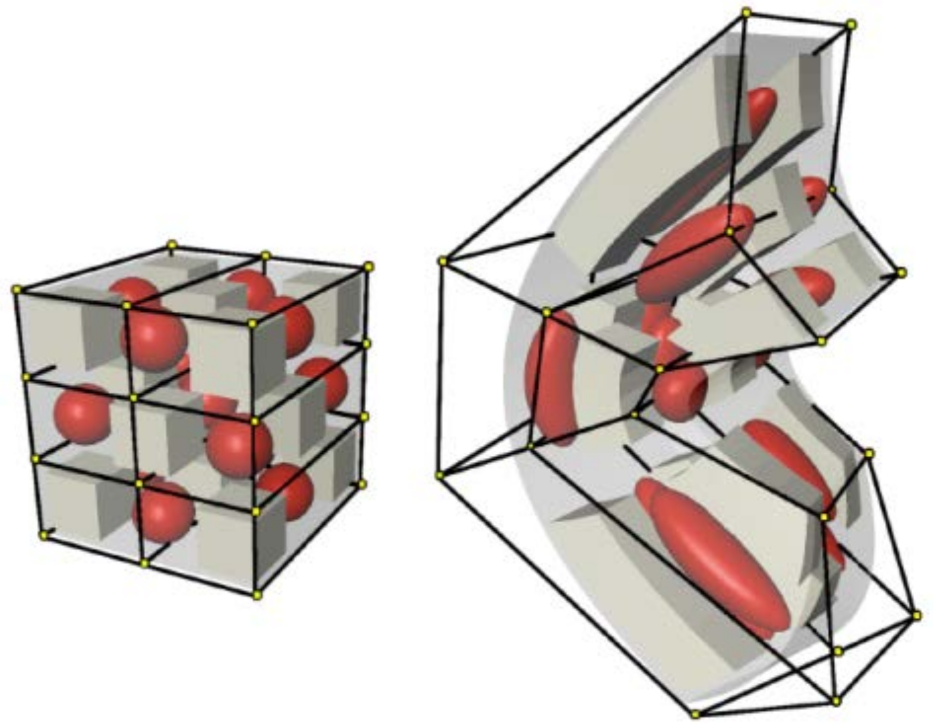
Freeform Deformation

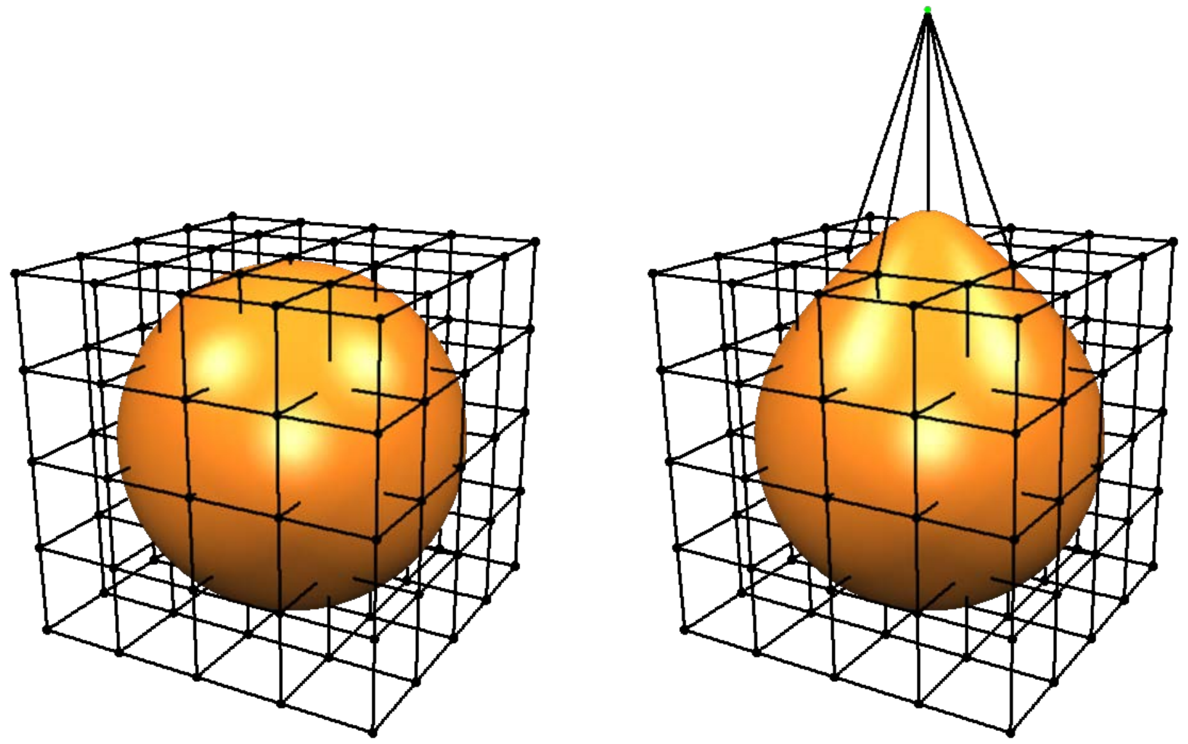
[Sederberg and Parry 86]



- Control object = lattice
- Basis functions $B_i(\mathbf{x})$ are trivariate tensor-product splines:

$$\mathbf{d}(x,y,z) = \sum_{i=0}^{l} \sum_{j=0}^{m} \sum_{k=0}^{n} \mathbf{d}_{ijk} N_i(x) N_j(y) N_k(z)$$



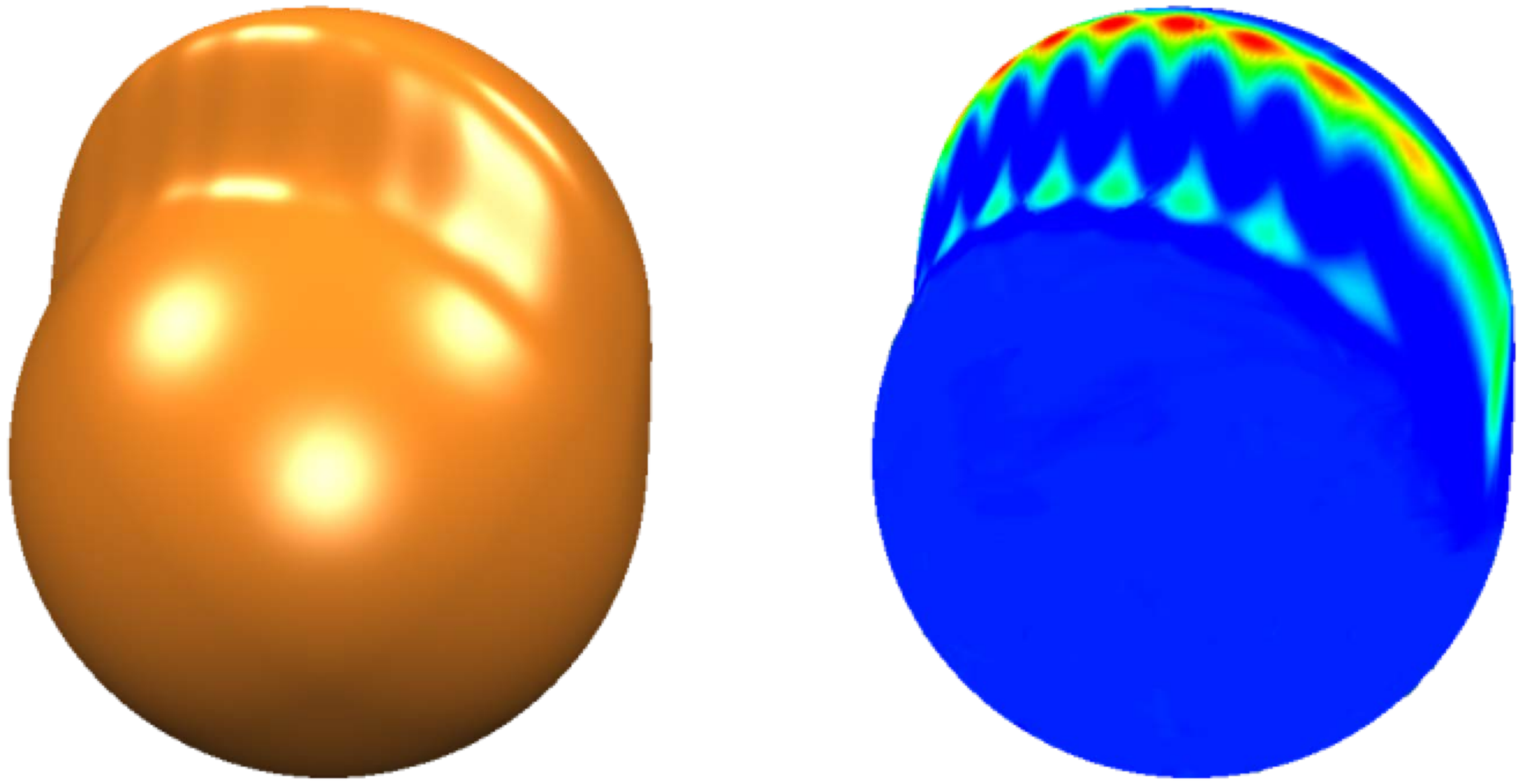


http://tom.cs.byu.edu/~tom/papers/ffd.pdf

Freeform Deformation

[Sederberg and Parry 86]

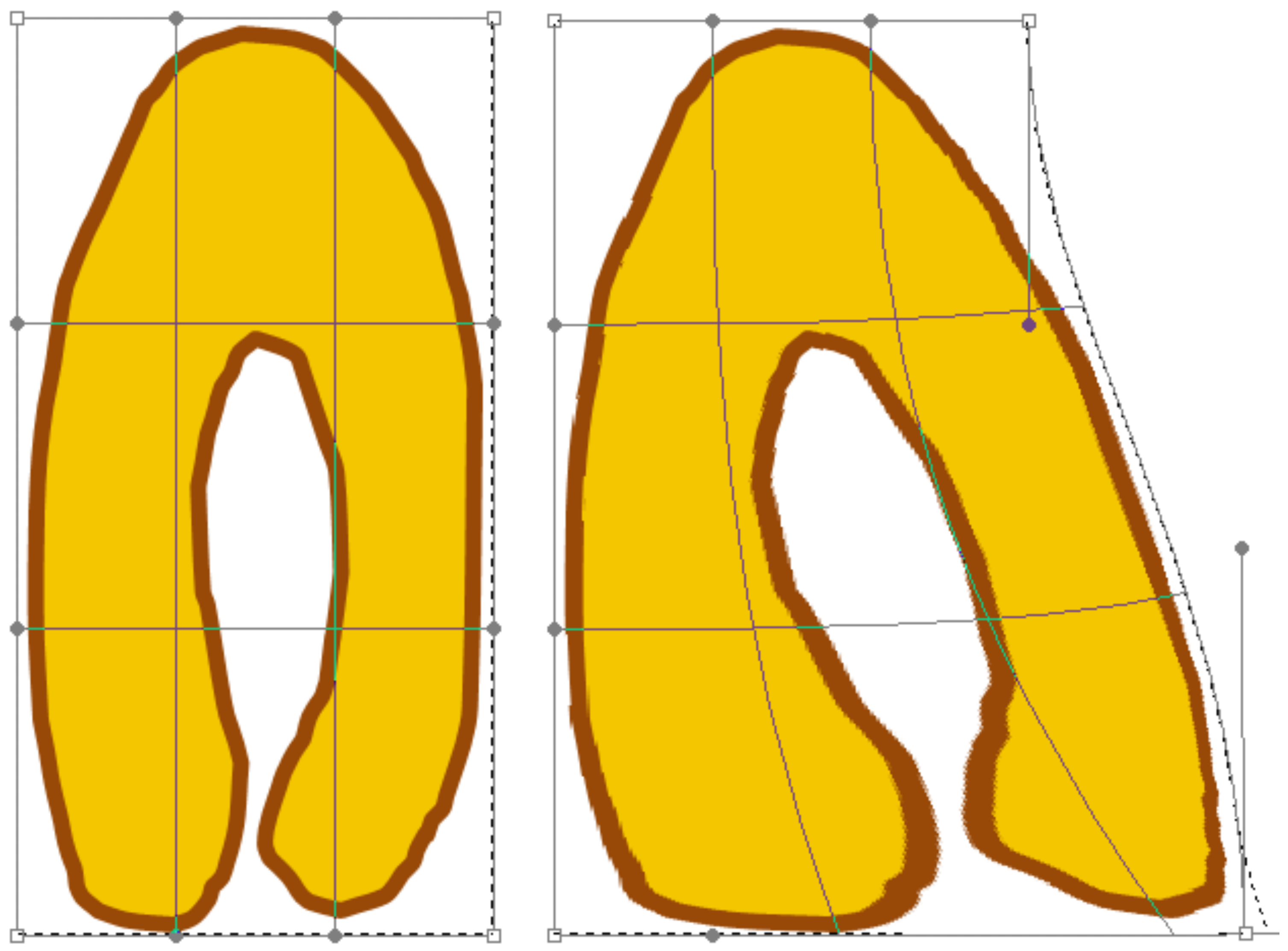
- Aliasing artifacts
- Interpolate deformation constraints?
  - Only in least squares sense



http://tom.cs.byu.edu/~tom/papers/ffd.pdf

Limitations of Lattices as Control Objects

- Difficult to manipulate
- The control object is not related to the shape of the edited object
- Parts of the shape in close Euclidean distance always deform similarly, even if geodesically far

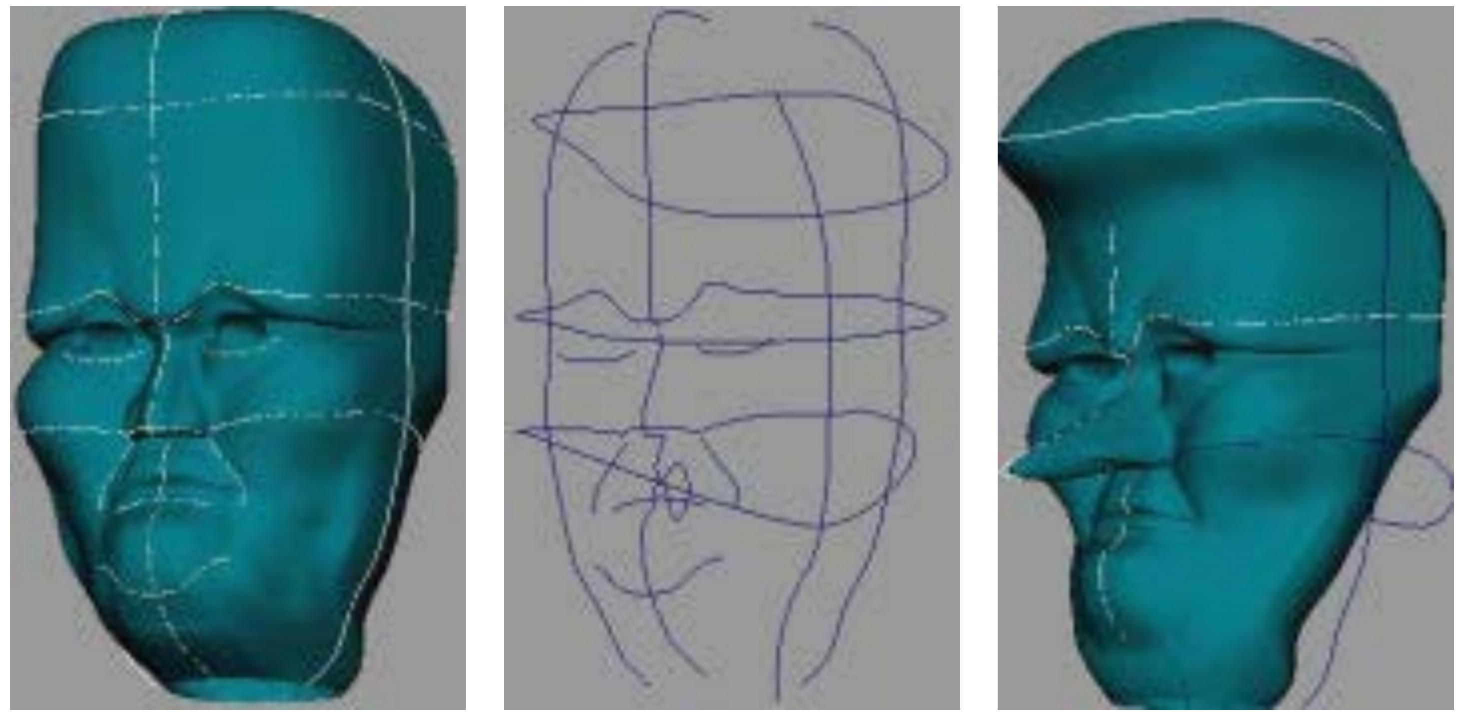


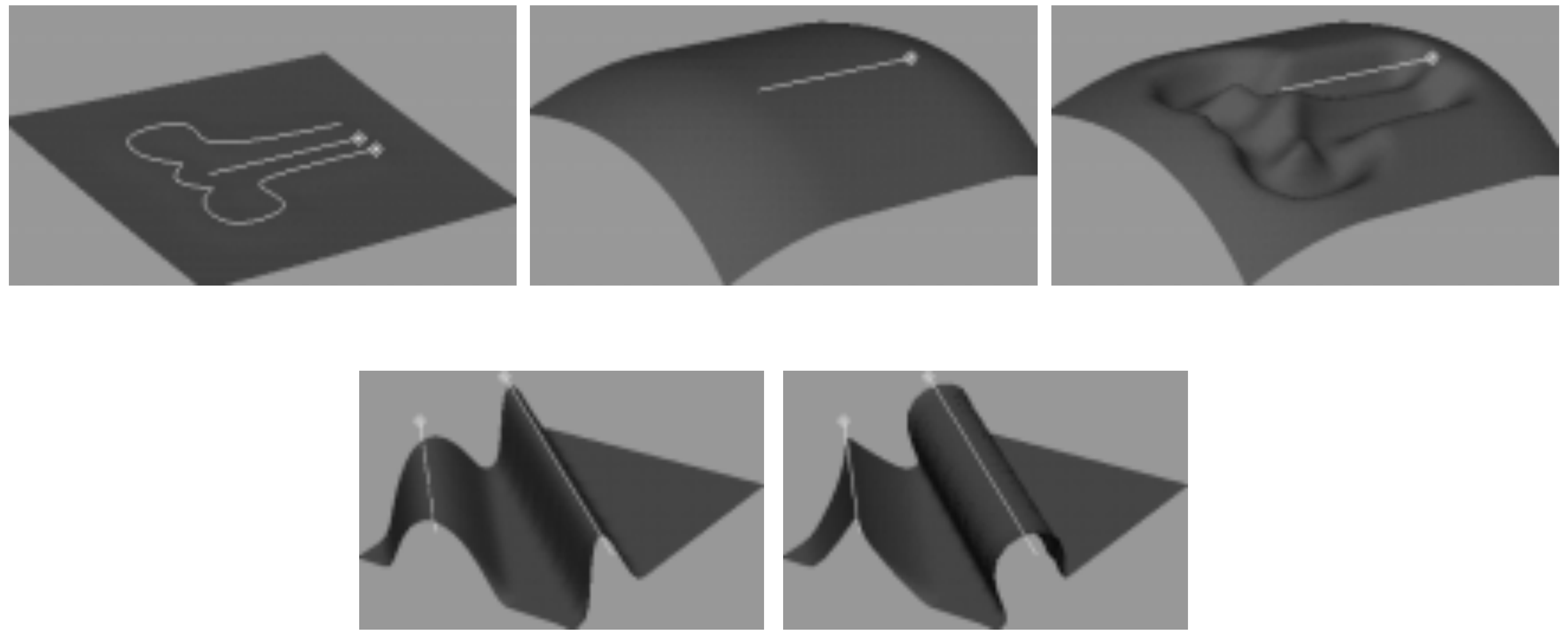
http://tom.cs.byu.edu/~tom/papers/ffd.pdf

Wires

[Singh and Fiume 98]

- Control objects are arbitrary space curves
- Can place curves along meaningful features of the edited object
- Smooth deformations around the curve with decreasing influence



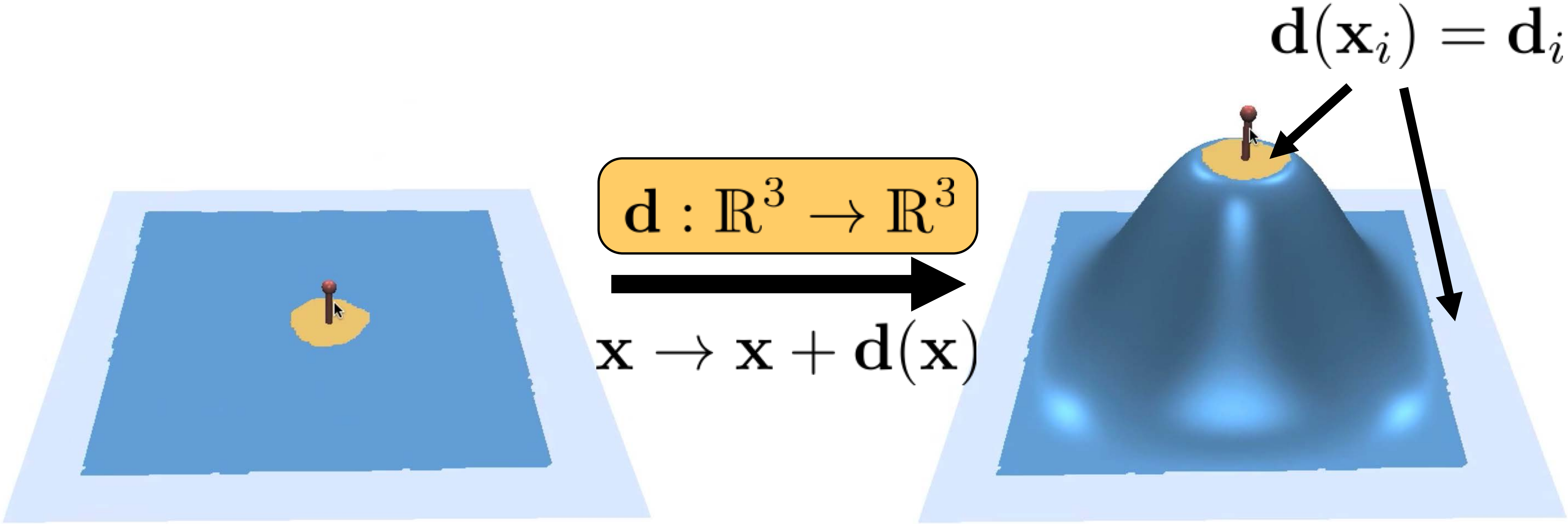


http://www.dgp.toronto.edu/~karan/pdf/ksinghpaperwire.pdf

Handle Metaphor

[Real-Time Shape Editing using Radial Basis Functions, Botsch and Kobbelt, EUROGRAPHICS 2005]

- Wish list for the displacement function $\mathbf{d}(\mathbf{x})$:
  - Interpolate prescribed constraints
  - Smooth, intuitive deformation



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Radial Basis Functions

[Real-Time Shape Editing using Radial Basis Functions, Botsch and Kobbelt, EUROGRAPHICS 2005]

- Represent deformation by RBFs

$$\mathbf{d}(\mathbf{x}) = \sum_j \mathbf{w}_j \, \varphi(\|\mathbf{c}_j - \mathbf{x}\|) + \mathbf{p}(\mathbf{x})$$

- Basis function $\varphi(r) = r^3$
  - C^2 boundary constraints
  - Highly smooth / fair interpolation

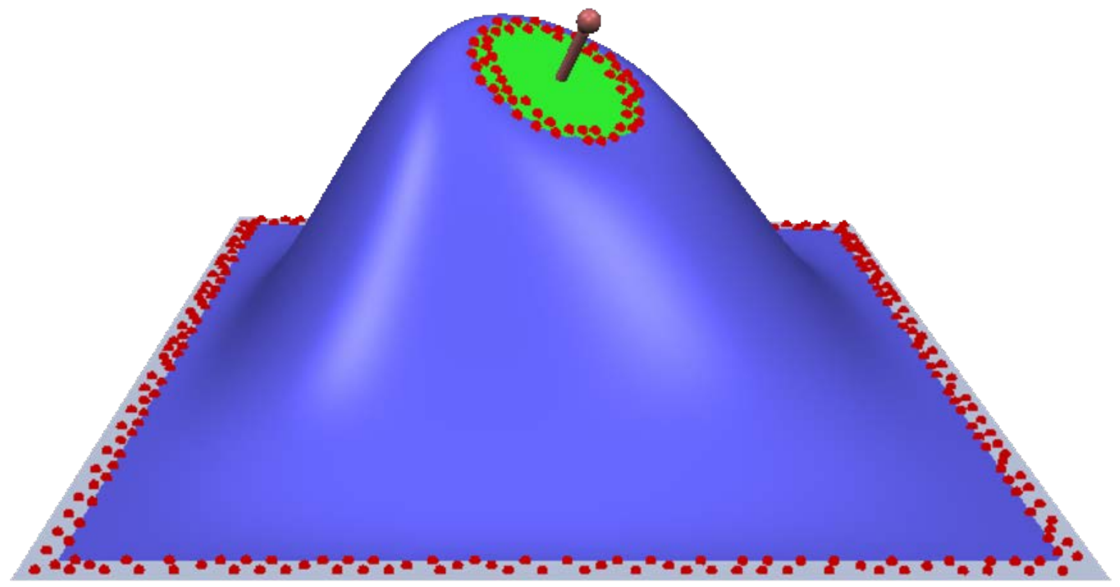
Radial Basis Functions

[Real-Time Shape Editing using Radial Basis Functions, Botsch and Kobbelt, EUROGRAPHICS 2005]

- Represent deformation by RBFs

$$\mathbf{d}(\mathbf{x}) = \sum_j \mathbf{w}_j \, \varphi(\|\mathbf{c}_j - \mathbf{x}\|) + \mathbf{p}(\mathbf{x})$$

- RBF fitting
  - Interpolate displacement constraints
  - Solve linear system for $\mathbf{w}_j$ and $\mathbf{p}$



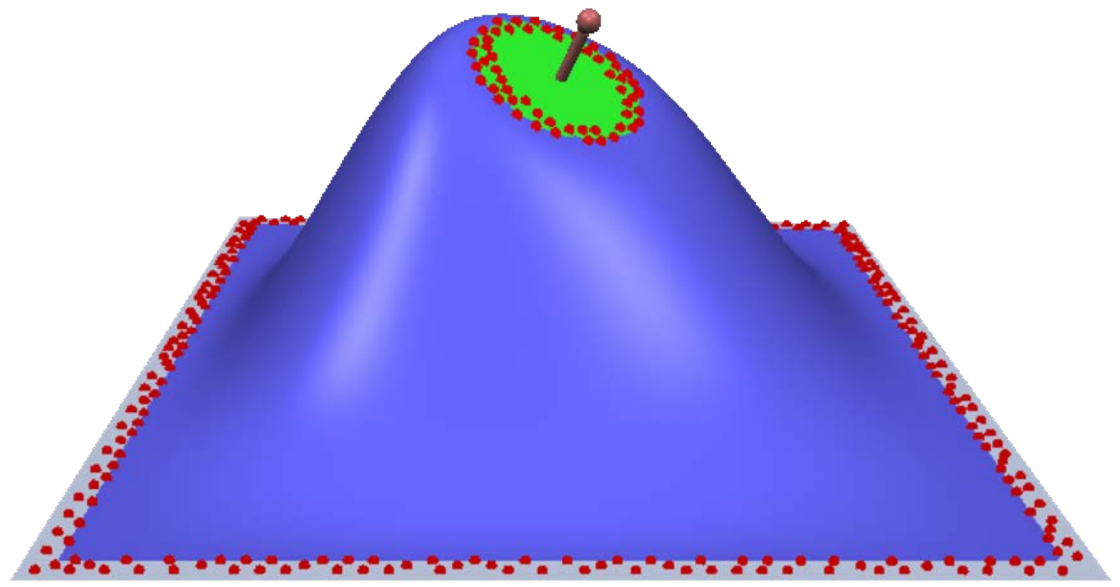
Radial Basis Functions

[Real-Time Shape Editing using Radial Basis Functions, Botsch and Kobbelt, EUROGRAPHICS 2005]

- Represent deformation by RBFs

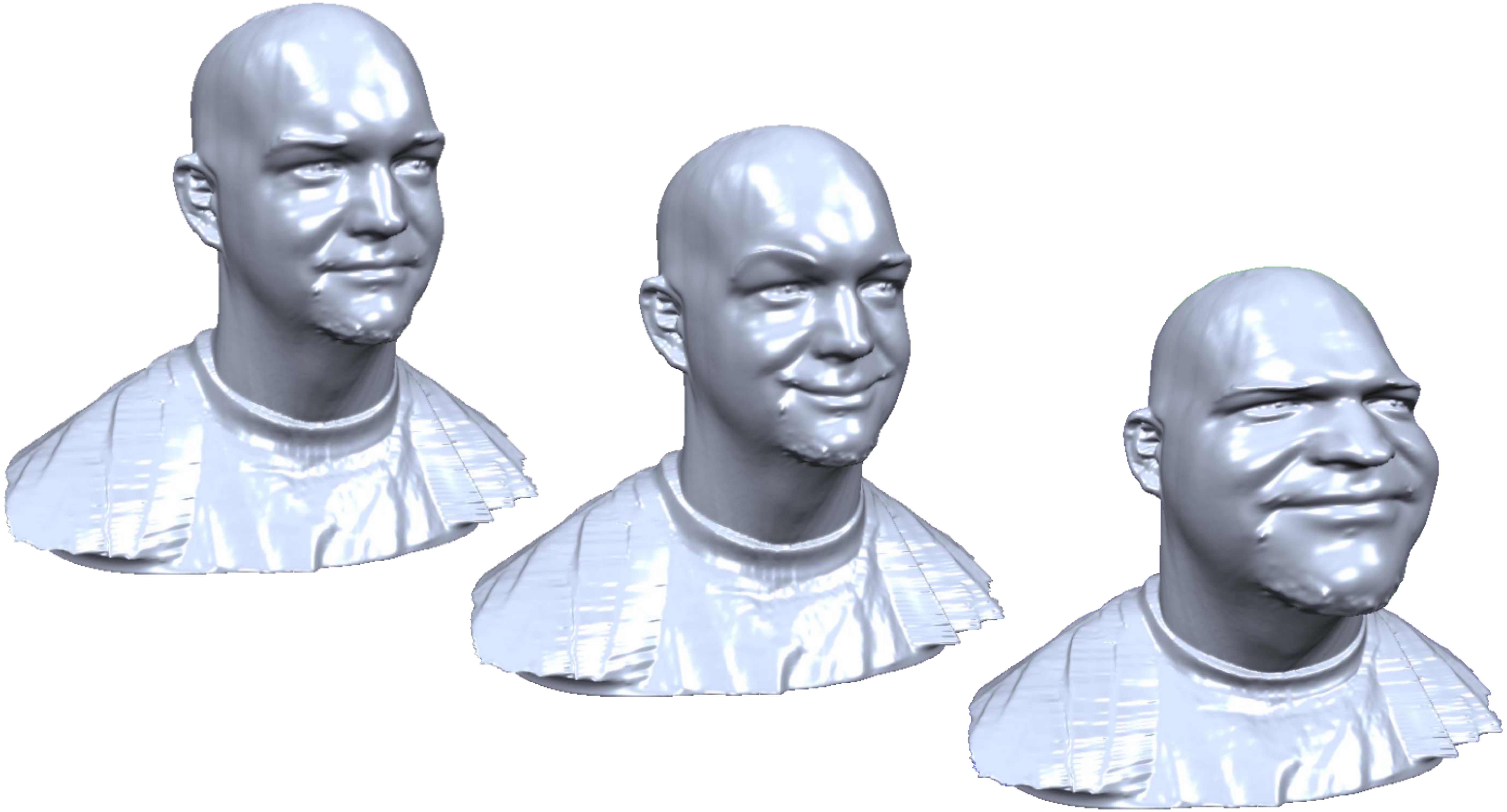
$$\mathbf{d}(\mathbf{x}) = \sum_j \mathbf{w}_j \, \varphi(\|\mathbf{c}_j - \mathbf{x}\|) + \mathbf{p}(\mathbf{x})$$

- RBF evaluation
  - Function $\mathbf{d}$ transforms points
  - Jacobian^{-T} $\nabla\mathbf{d}^{-T}$ transforms normals
  - Precompute basis functions
  - Evaluate on the GPU!



Local & Global Deformations

[Real-Time Shape Editing using Radial Basis Functions, Botsch and Kobbelt, EUROGRAPHICS 2005]



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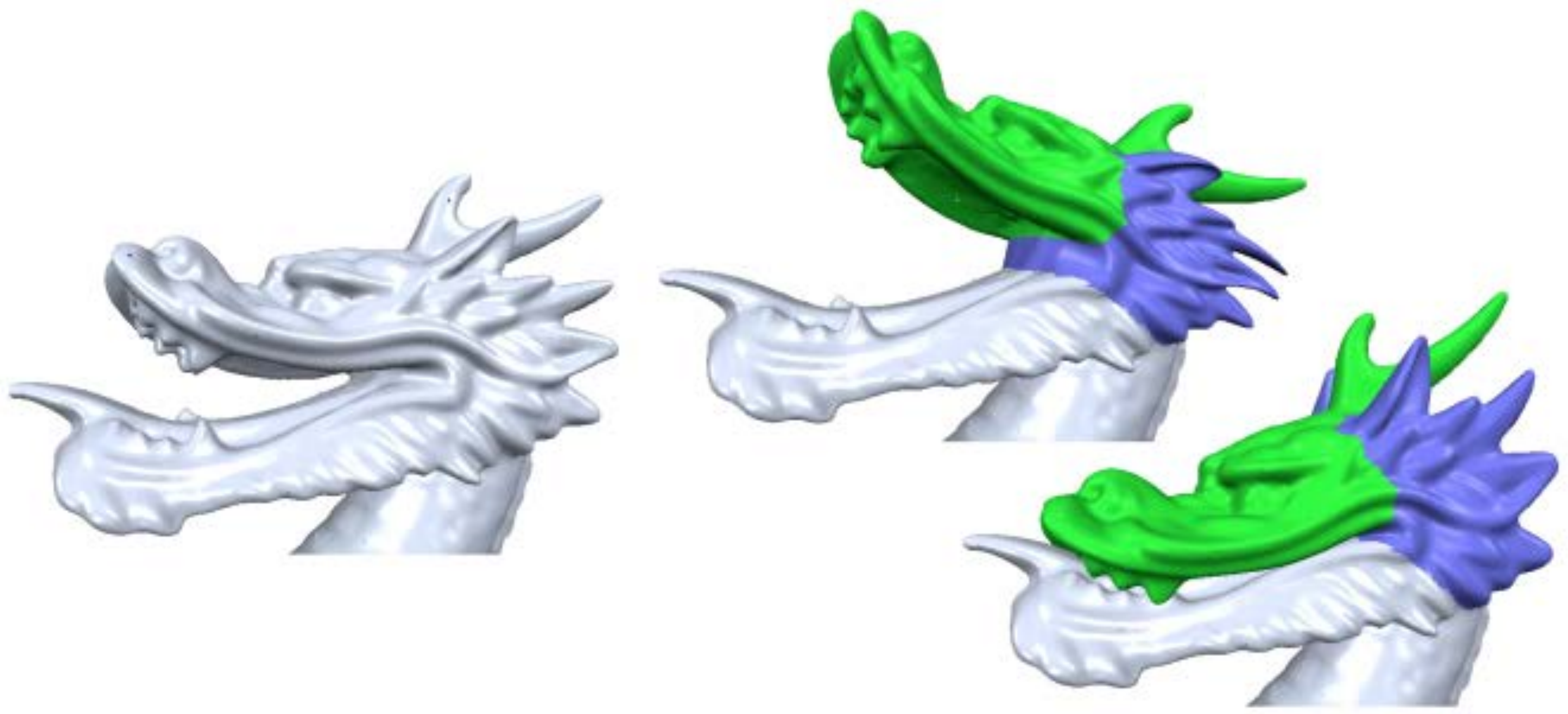
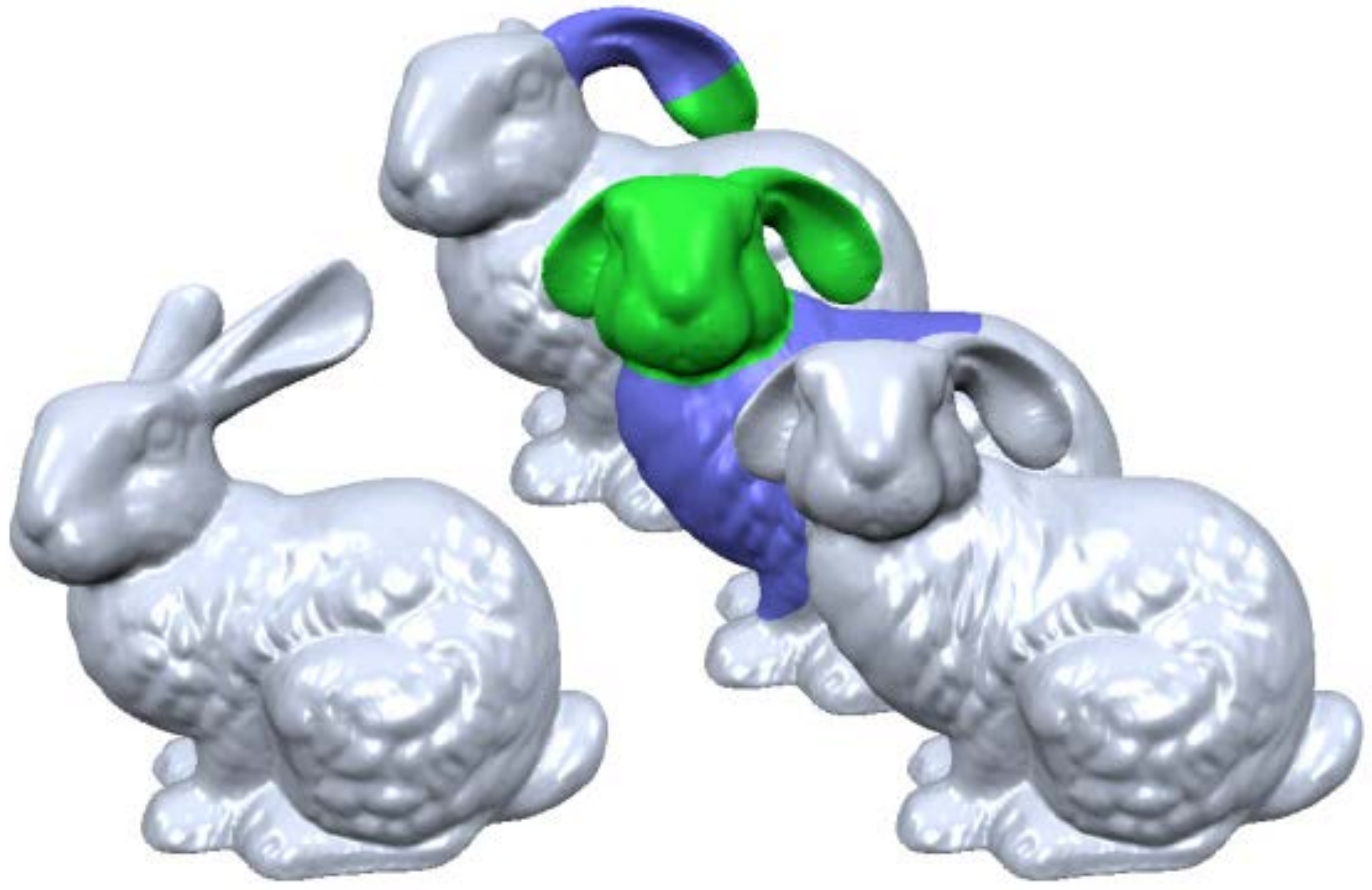
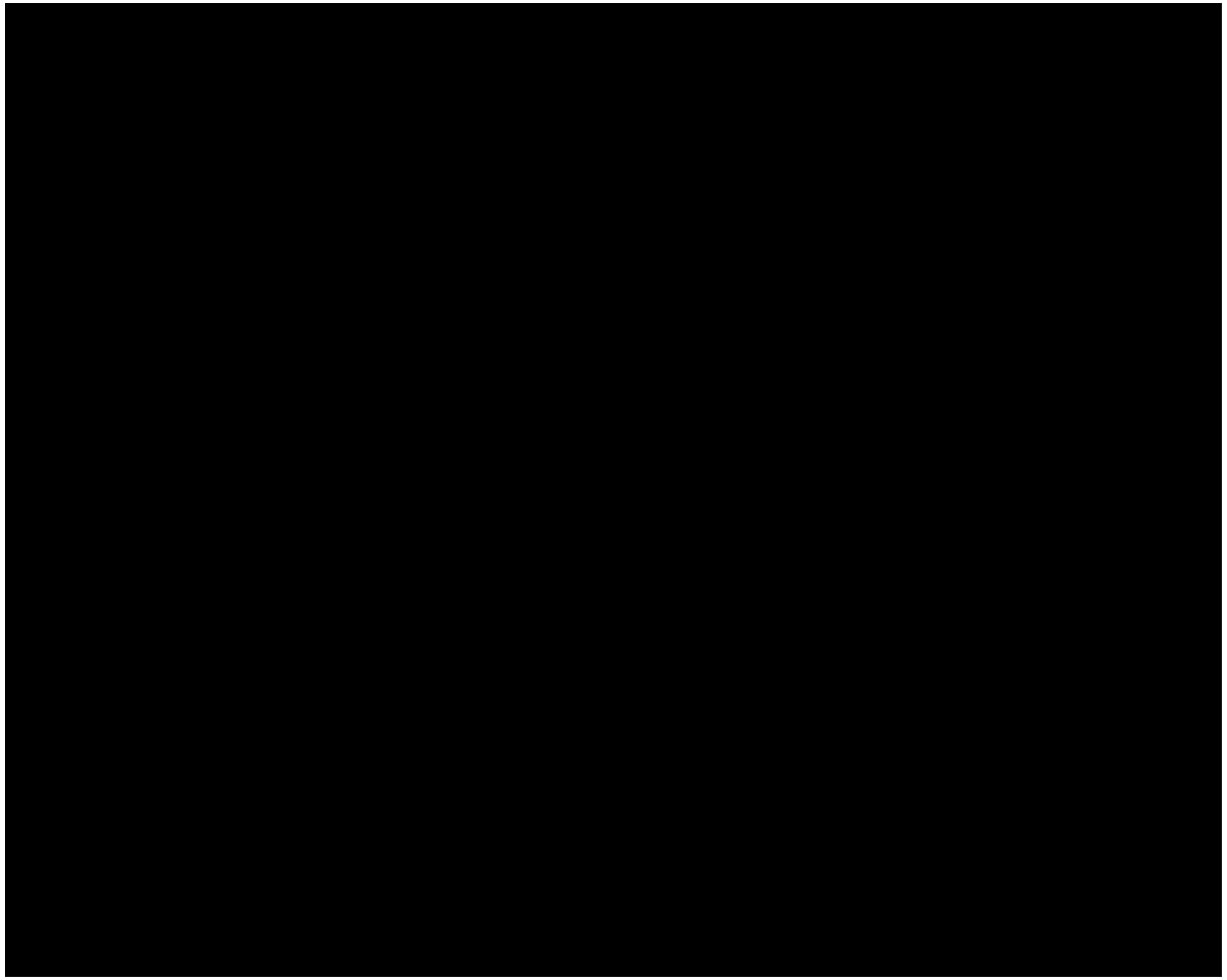




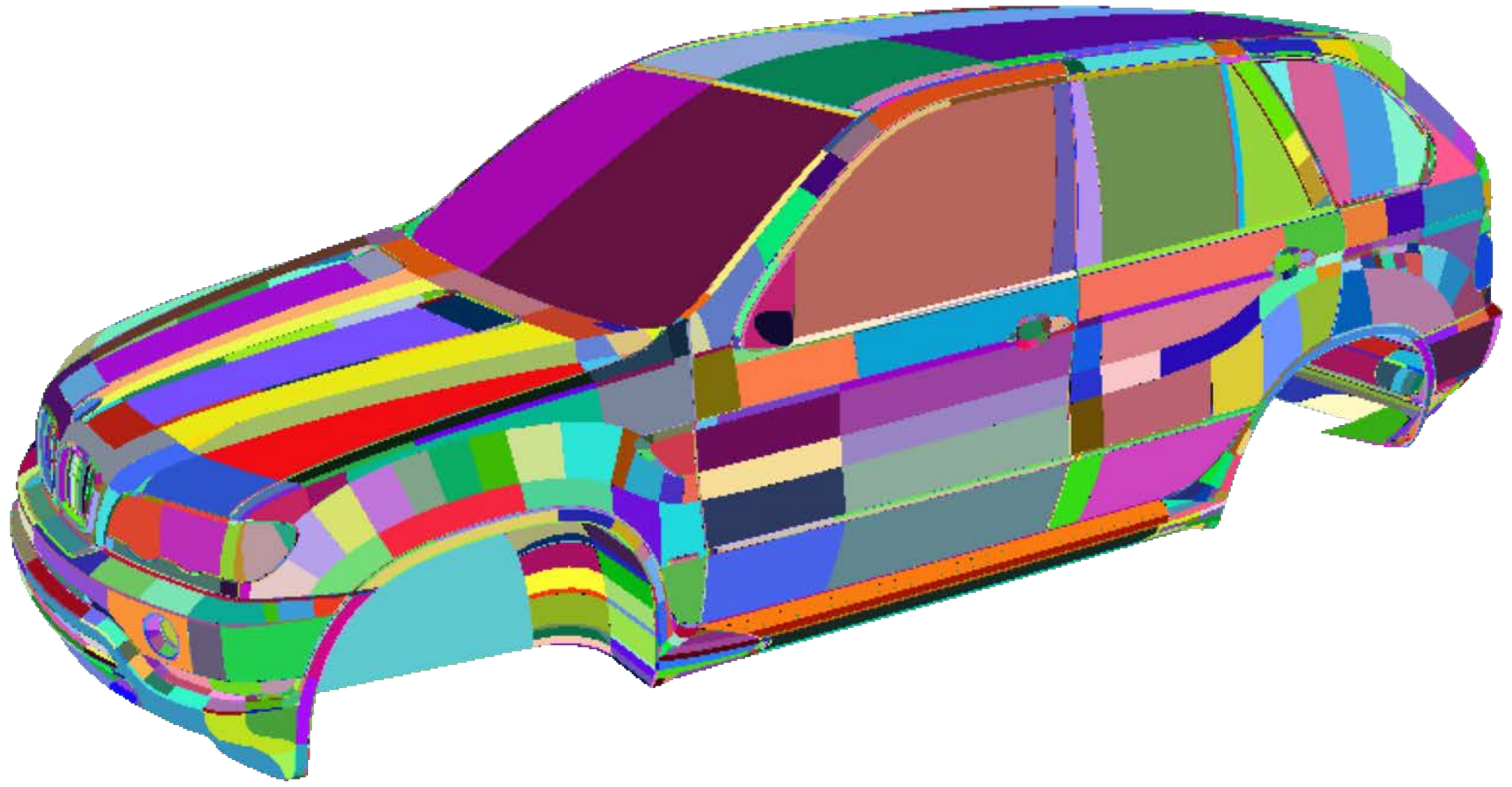


1M vertices

Space Deformations

Summary so far

- Handle arbitrary input
  - Meshes (also non-manifold)
  - Point sets
  - Polygonal soups
  - …
- Complexity mainly depends on the control object, not the surface

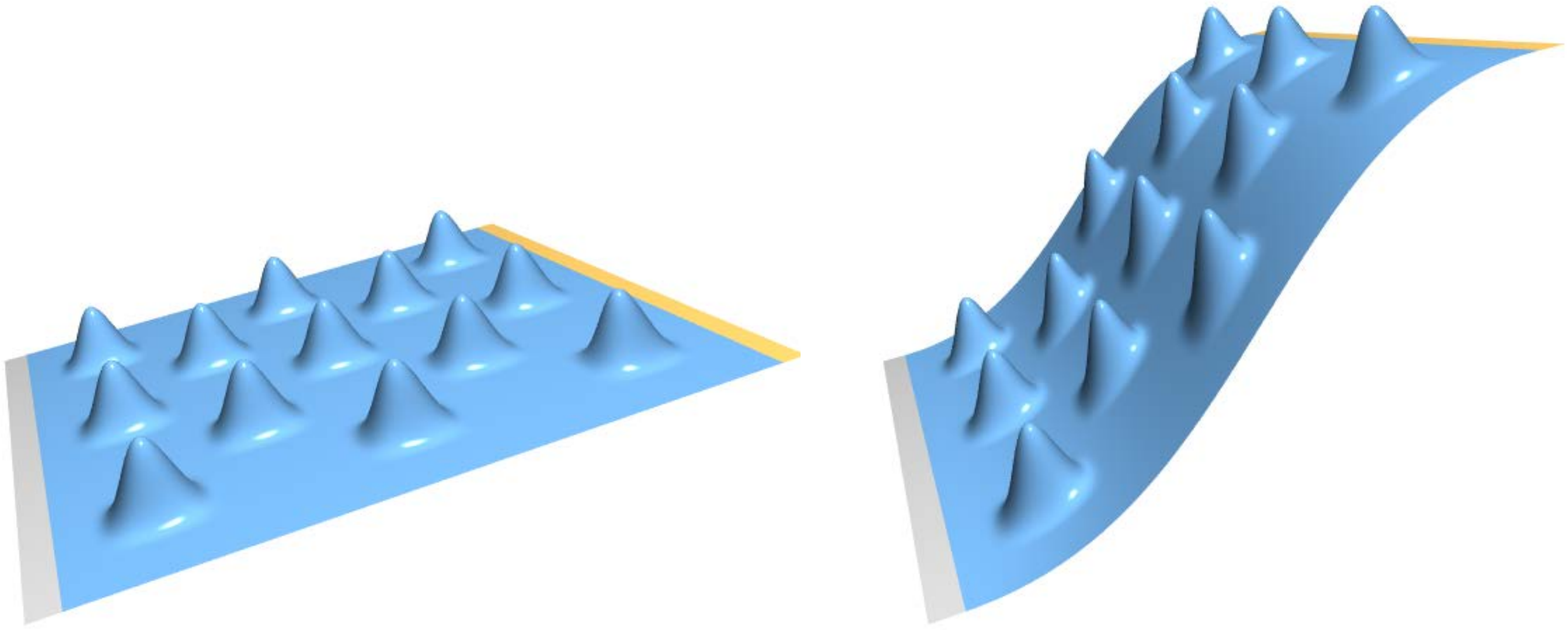


- 3M triangles
- 10k components
- Not oriented
- Not manifold

Space Deformations

Summary so far

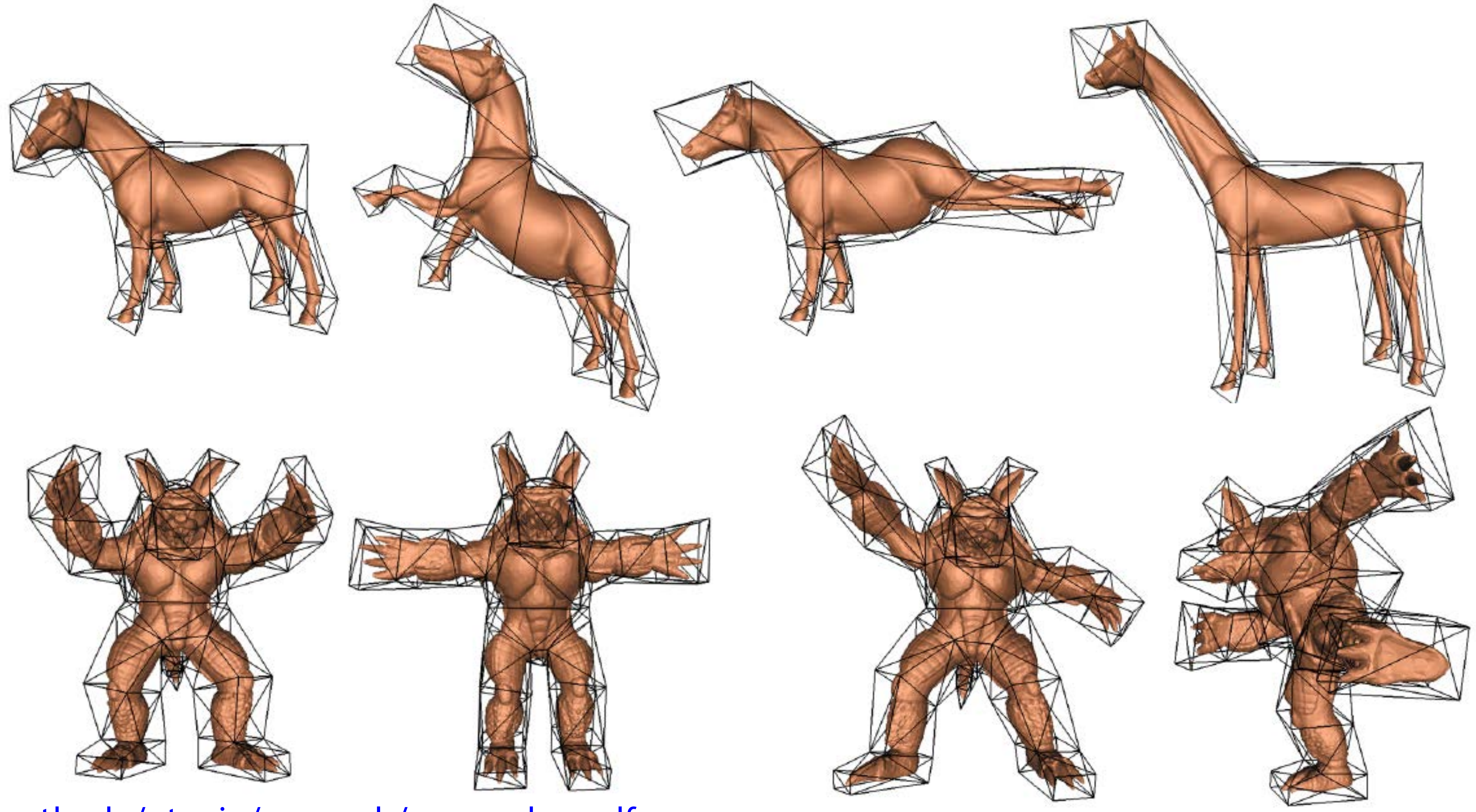
- The deformation is only loosely aware of the shape that is being edited
- Small Euclidean distance → similar deformation
- Local surface detail may be distorted



Cage-based Deformations

[Ju et al. 2005]

- Cage = crude version of the input shape
- Polytope (not a lattice)

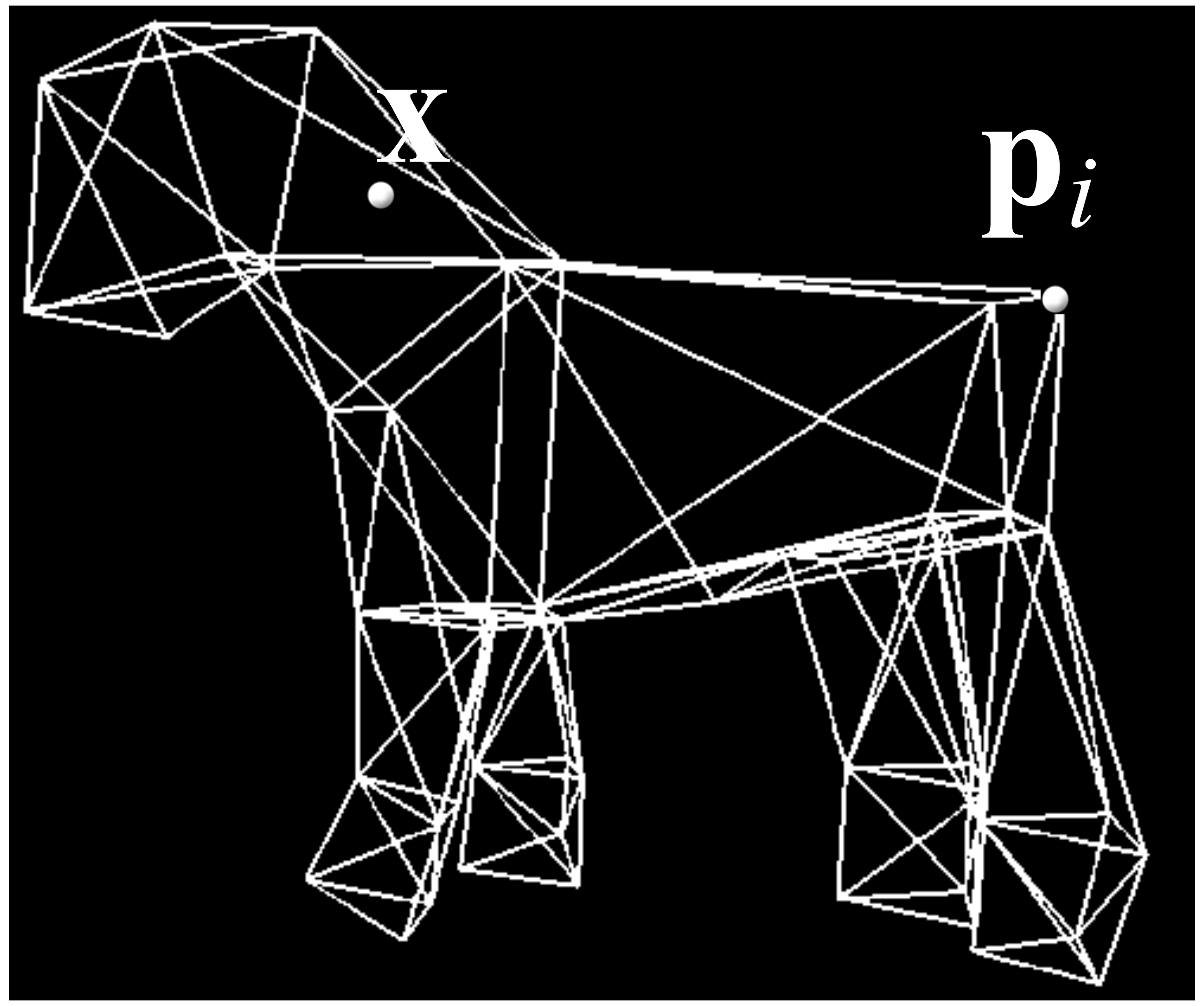


http://www.cs.wustl.edu/~taoju/research/meanvalue.pdf

Cage-based Deformations

[Ju et al. 2005]

- Each point $\mathbf{x}$ in space is represented w.r.t. the cage elements using coordinate functions



$$\mathbf{x} = \sum_{i=1}^{k} w_i(\mathbf{x})\,\mathbf{p}_i$$

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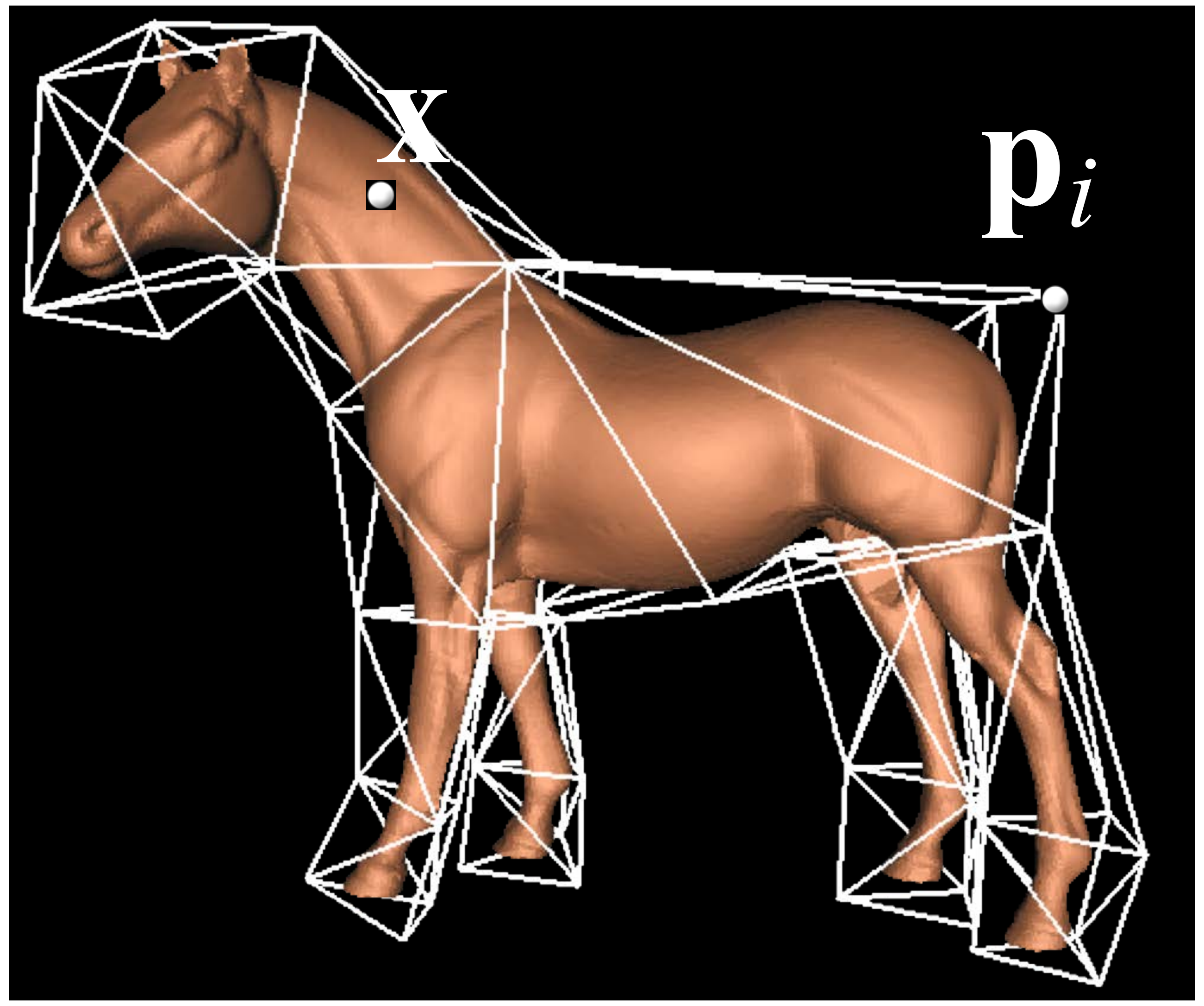

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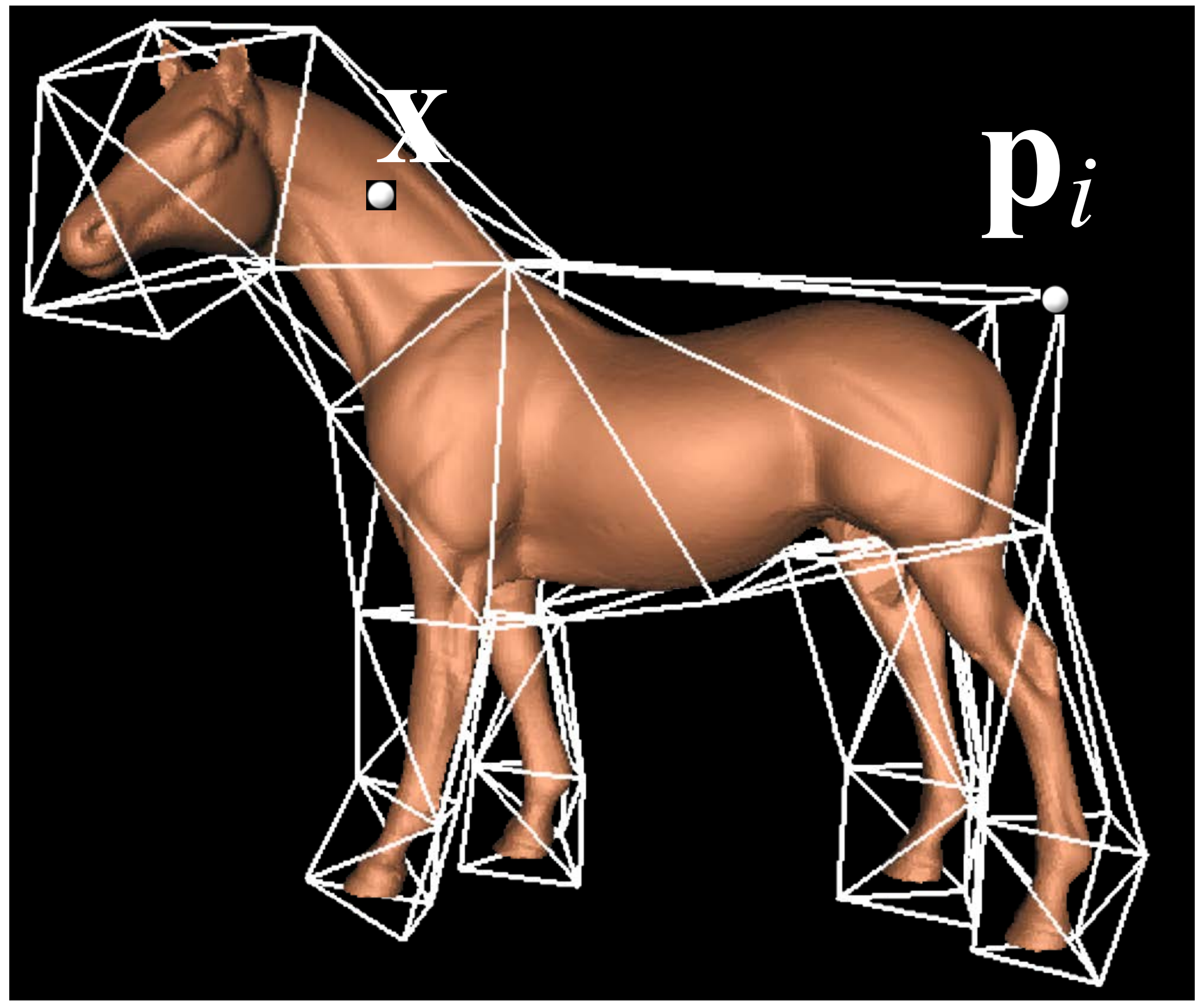

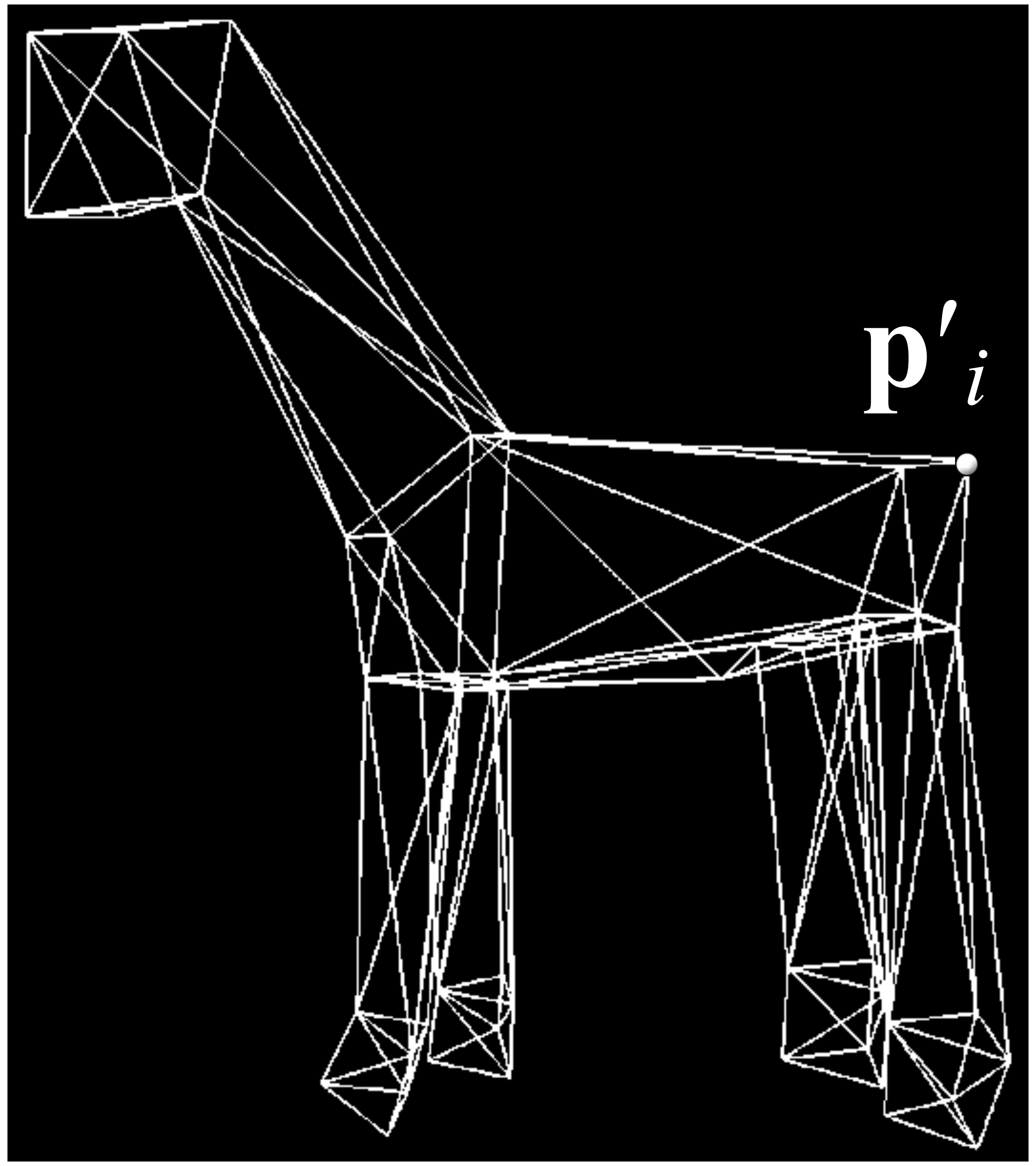

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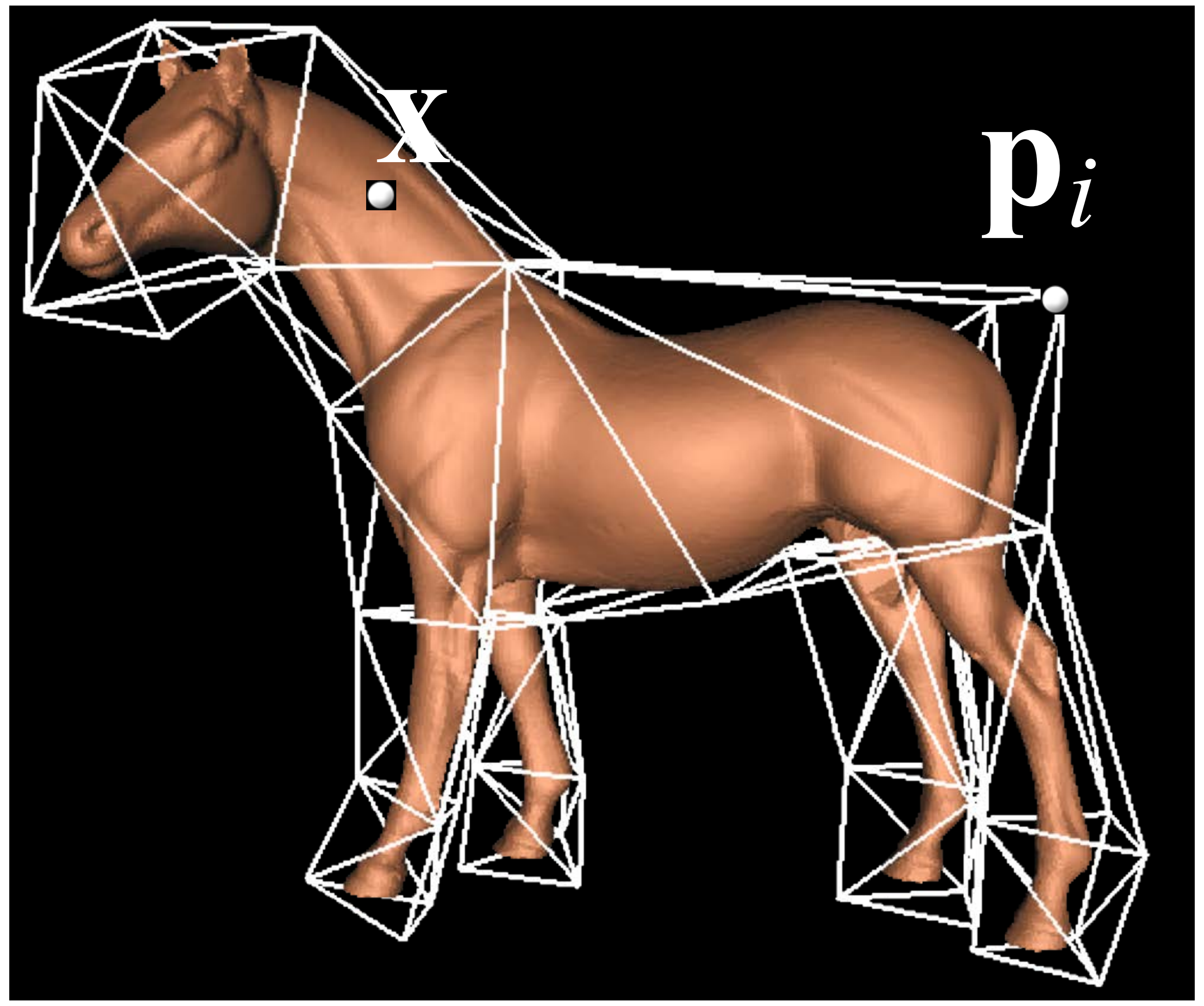
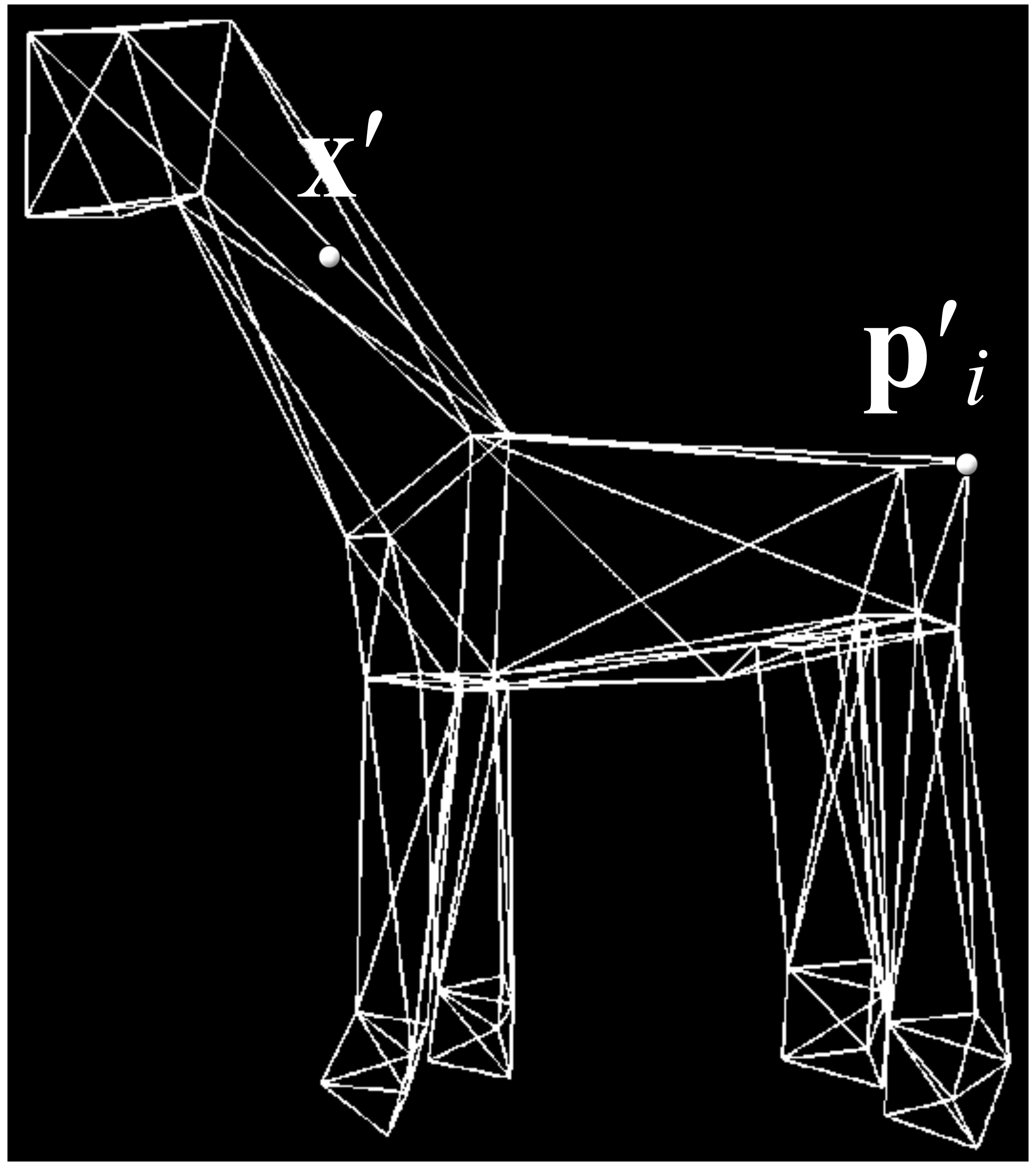
Cage-based Deformations

[Ju et al. 2005]

$$\mathbf{x}' = \sum_{i=1}^{k} w_i(\mathbf{x})\,\mathbf{p}_i'$$





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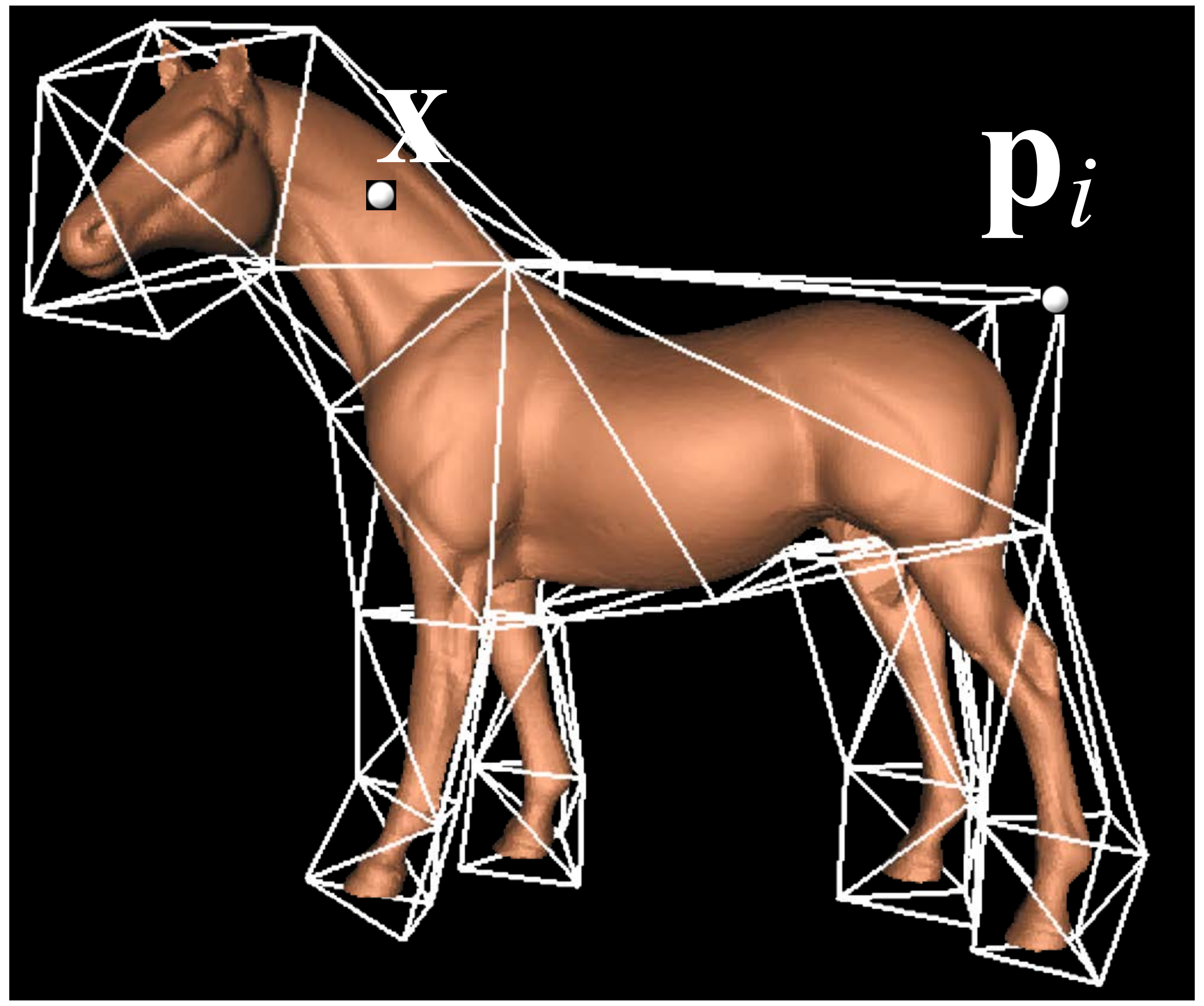
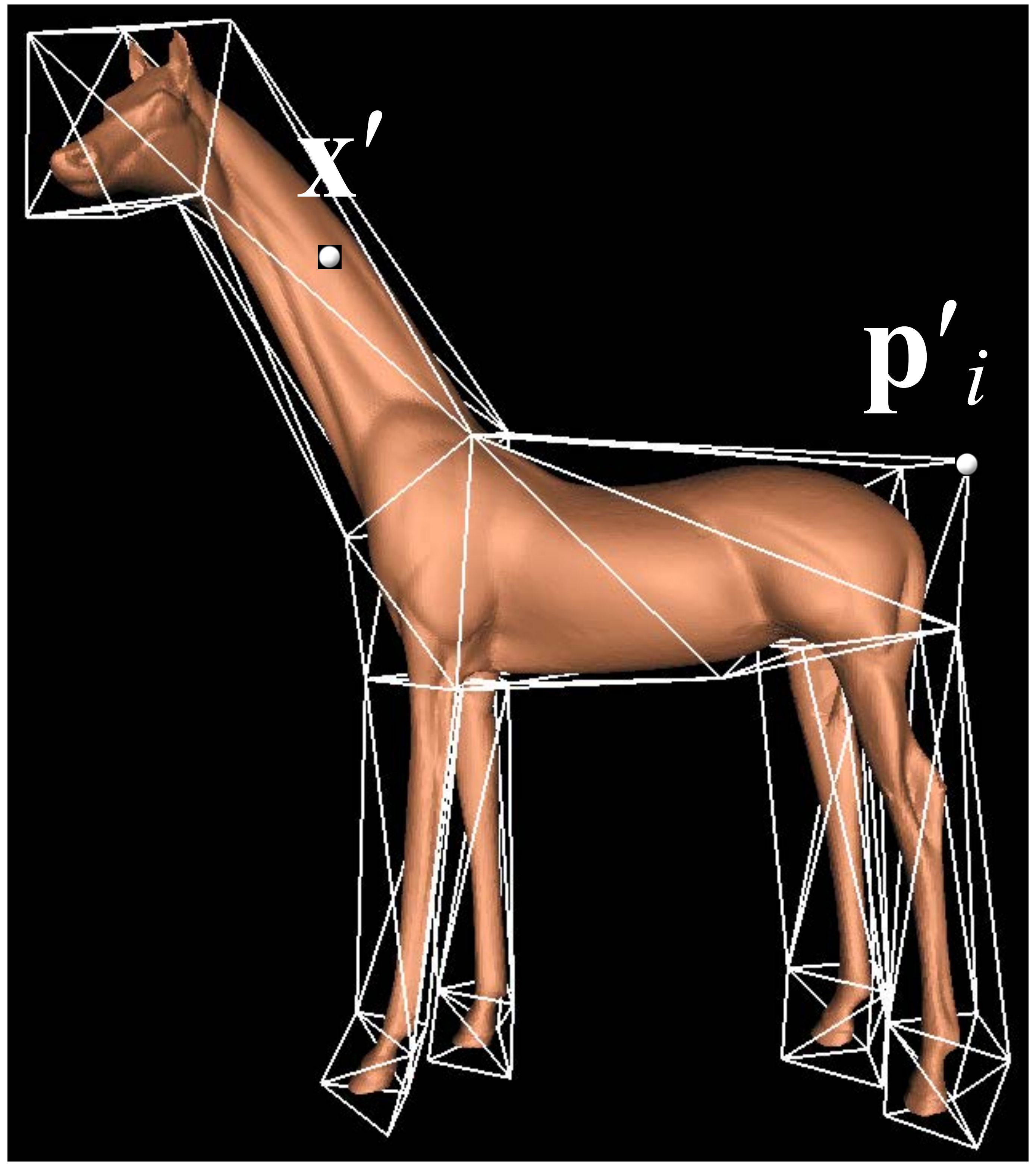
[Ju et al. 2005]

$$\mathbf{x}' = \sum_{i=1}^{k} w_i(\mathbf{x})\, \mathbf{p}'_i$$





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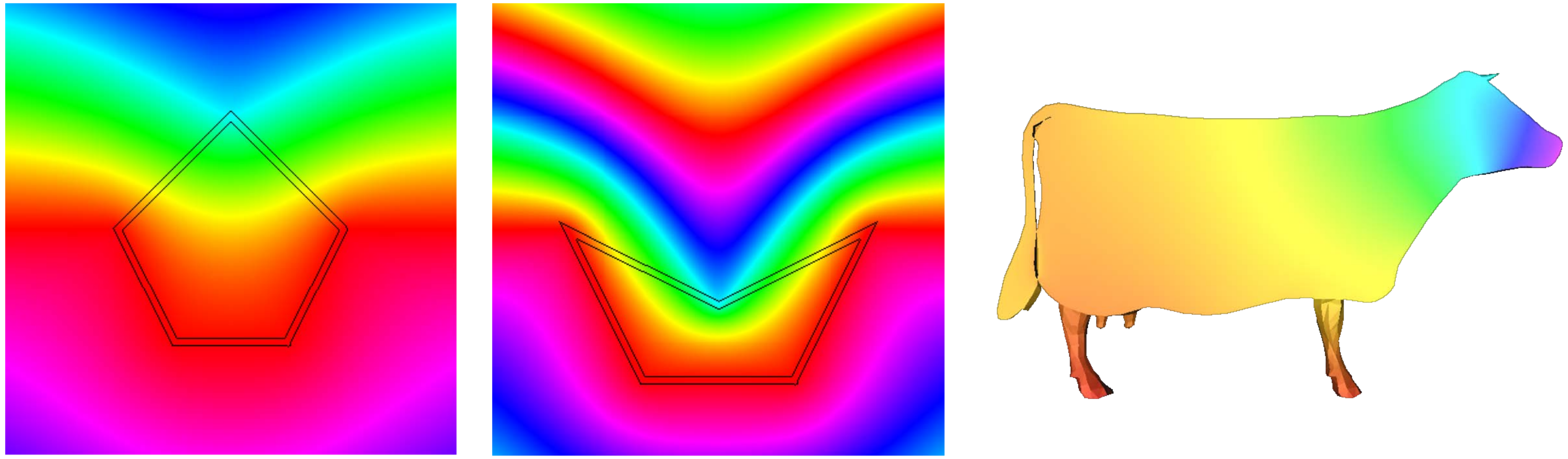
Generalized Barycentric Coordinates

- Lagrange property: $$w_i(\mathbf{p}_j) = \delta_{ij}$$
- Reproduction: $$\forall \mathbf{x},\ \sum_{i=1}^{k} w_i(\mathbf{x})\,\mathbf{p}_i = \mathbf{x}$$
- Partition of unity: $$\forall \mathbf{x},\ \sum_{i=1}^{k} w_i(\mathbf{x}) = 1$$

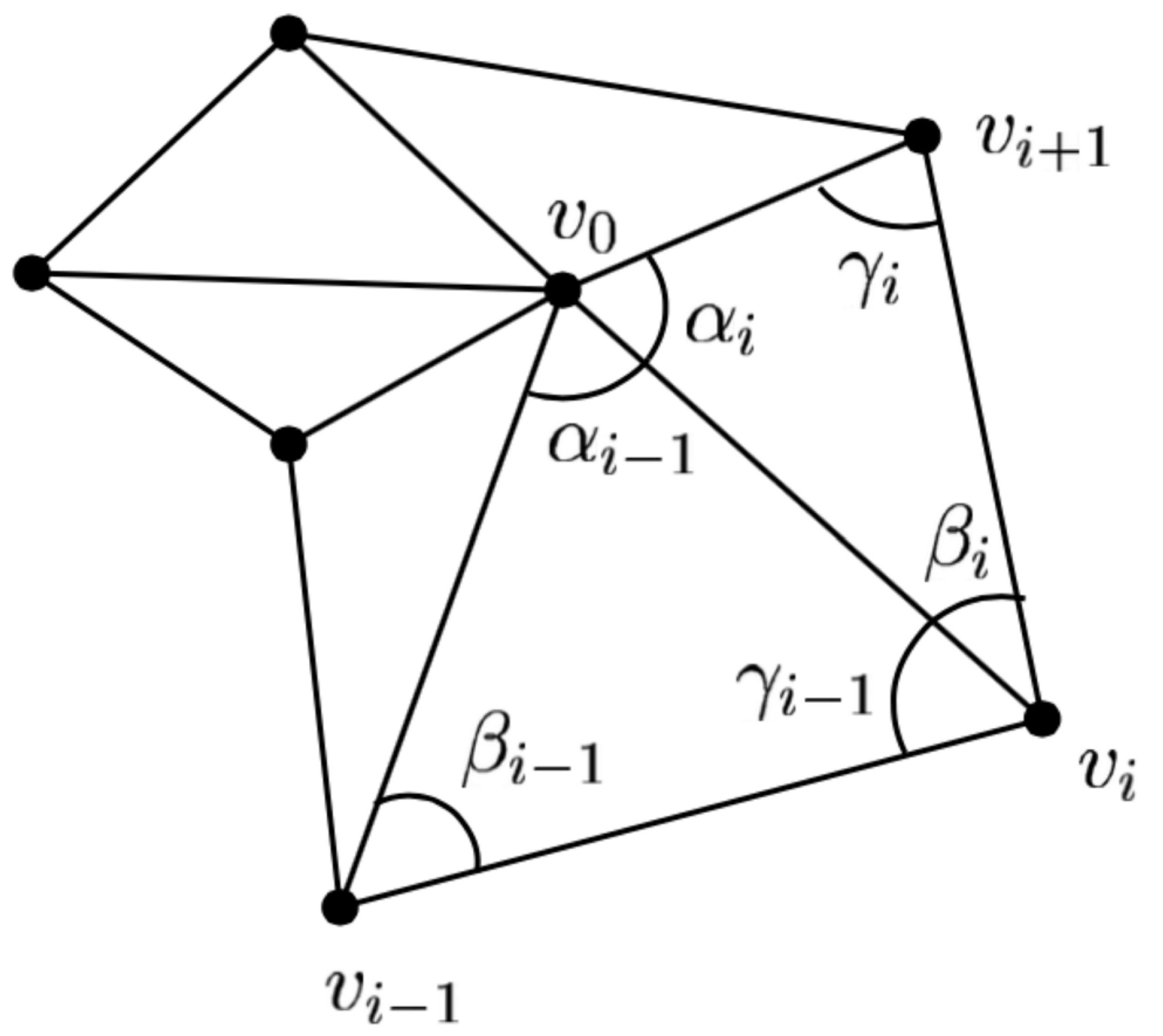
Coordinate Functions

- Mean-value coordinates [Floater 2003*, Ju et al. 2005]
  - Generalization of barycentric coordinates
  - Closed-form solution for $w_i(\mathbf{x})$



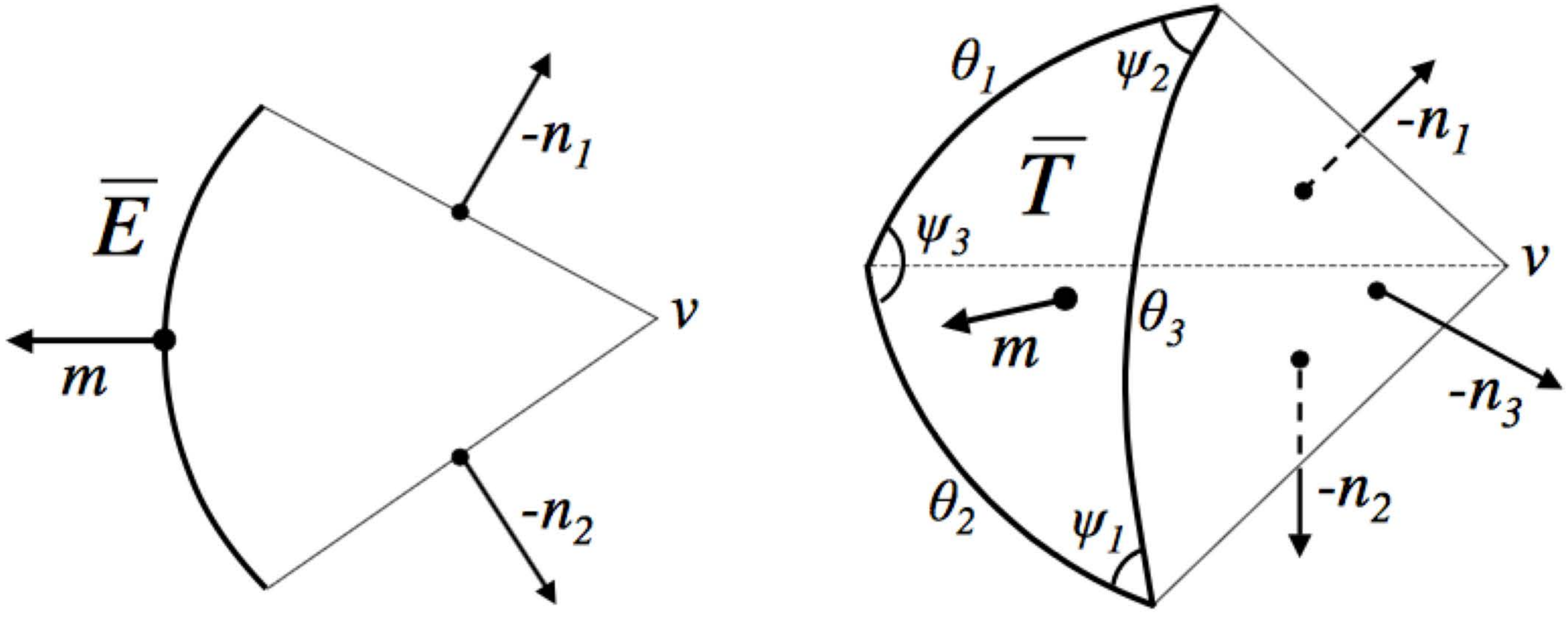
* Michael Floater, “Mean value coordinates”, CAGD 20(1), 2003

2D Mean Value Coordinates



$$\lambda_i = \frac{w_i}{\sum_{j=1}^{k} w_j}, \qquad w_i = \cot \beta_{i-1} + \cot \gamma_i.$$

3D Mean Value Coordinates



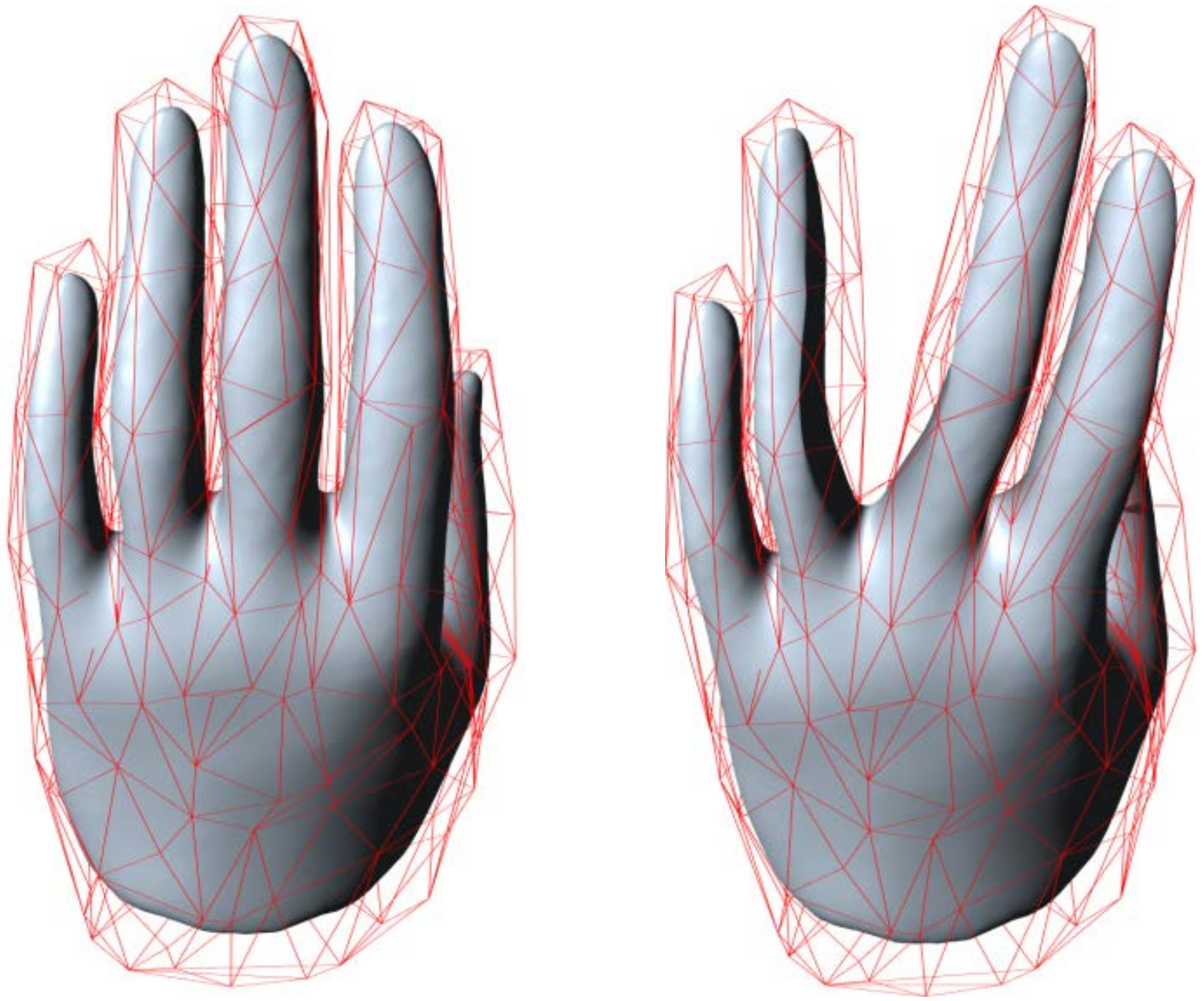
(a) (b)

Mean Value Coordinates for Closed Triangular Meshes
Tao Ju, Scott Schaefer, Joe Warren

// Robust evaluation on a triangular mesh
for each vertex p_j with values f_j
  $d_j \leftarrow \|p_j - x\|$
  if $d_j < \varepsilon$ return f_j
  $u_j \leftarrow (p_j - x)/d_j$
$totalF \leftarrow 0$
$totalW \leftarrow 0$
for each triangle with vertices p_1, p_2, p_3 and values f_1, f_2, f_3
  $l_i \leftarrow \|u_{i+1} - u_{i-1}\|$ *// for* $i = 1,2,3$
  $\theta_i \leftarrow 2\arcsin[l_i/2]$
  $h \leftarrow (\sum \theta_i)/2$
  if $\pi - h < \varepsilon$
    // x lies on t, use 2D barycentric coordinates
    $w_i \leftarrow \sin[\theta_i] d_{i-1} d_{i+1}$
    return $(\sum w_i f_i)/(\sum w_i)$
  $c_i \leftarrow (2\sin[h]\sin[h-\theta_i])/(\sin[\theta_{i+1}]\sin[\theta_{i-1}]) - 1$
  $s_i \leftarrow \mathrm{sign}[det[u_1, u_2, u_3]]\sqrt{1 - c_i^2}$
  if $\exists i, |s_i| \leq \varepsilon$
    // x lies outside t on the same plane, ignore t
    continue
  $w_i \leftarrow (\theta_i - c_{i+1}\theta_{i-1} - c_{i-1}\theta_{i+1})/(d_i \sin[\theta_{i+1}] s_{i-1})$
  $totalF += \sum w_i f_i$
  $totalW += \sum w_i$
$f_x \leftarrow totalF/totalW$

Coordinate Functions

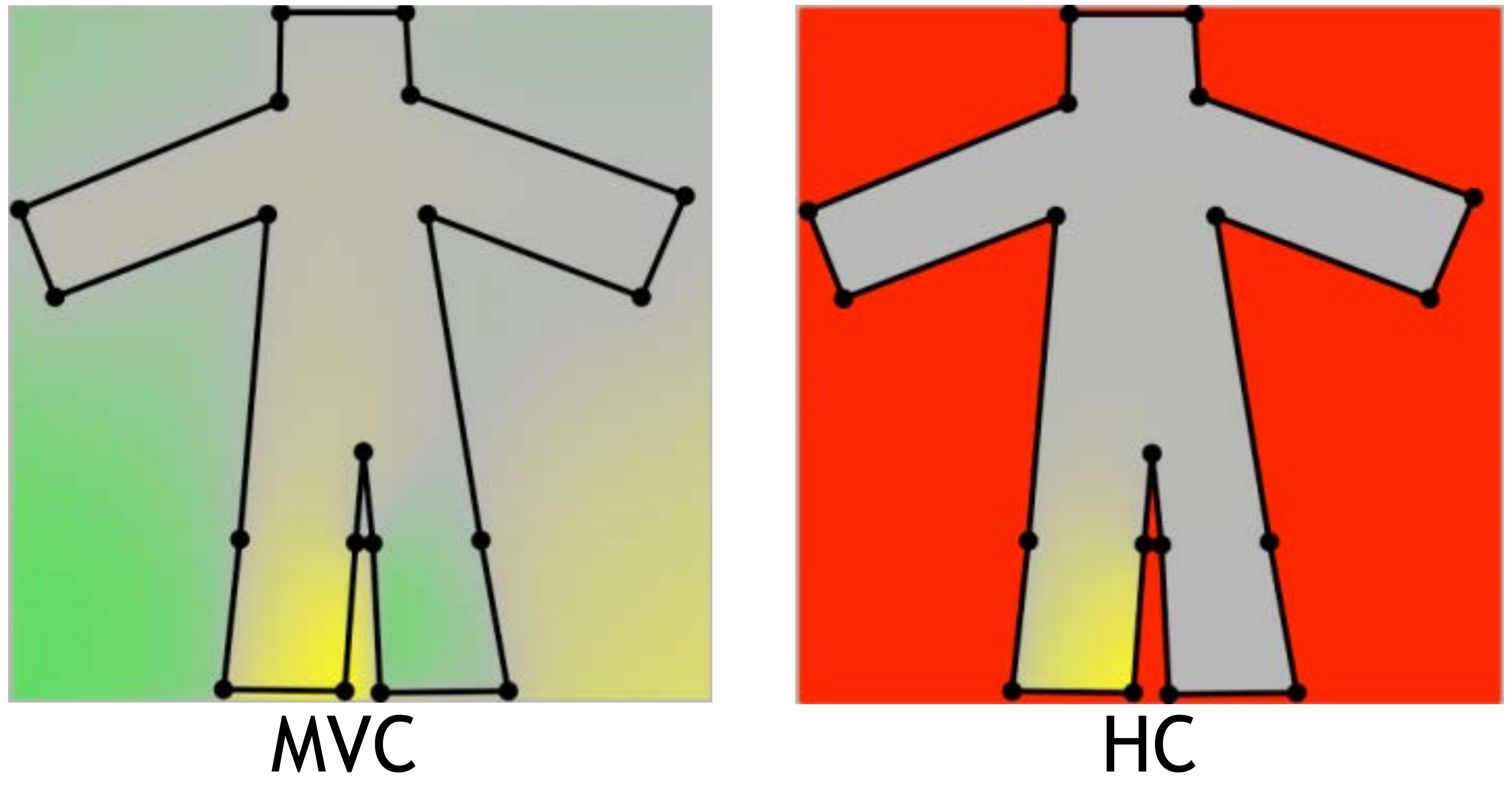
- Mean-value coordinates [Floater 2003, Ju et al. 2005]
  - Not necessarily positive on non-convex domains



Coordinate Functions

- Harmonic coordinates (Joshi et al. 2007)
  - Harmonic functions $h_i(\mathbf{x})$ for each cage vertex $\mathbf{p}_i$
  - Solve $\Delta h = 0$

  subject to: h_i linear on the boundary s.t. $h_i(\mathbf{p}_j) = \delta_{ij}$



http://www.cs.jhu.edu/~misha/Fall07/Papers/Joshi07.pdf

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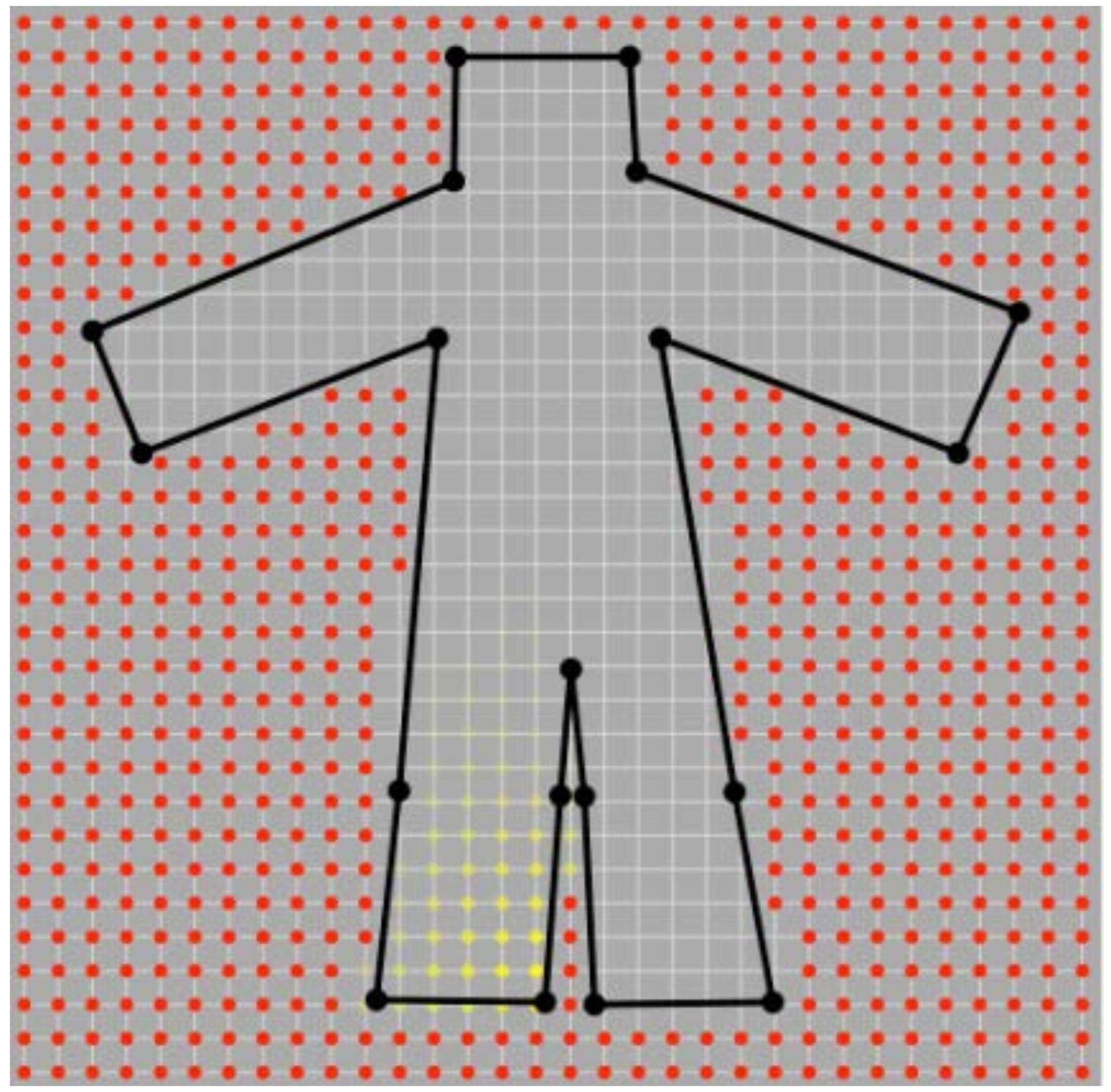
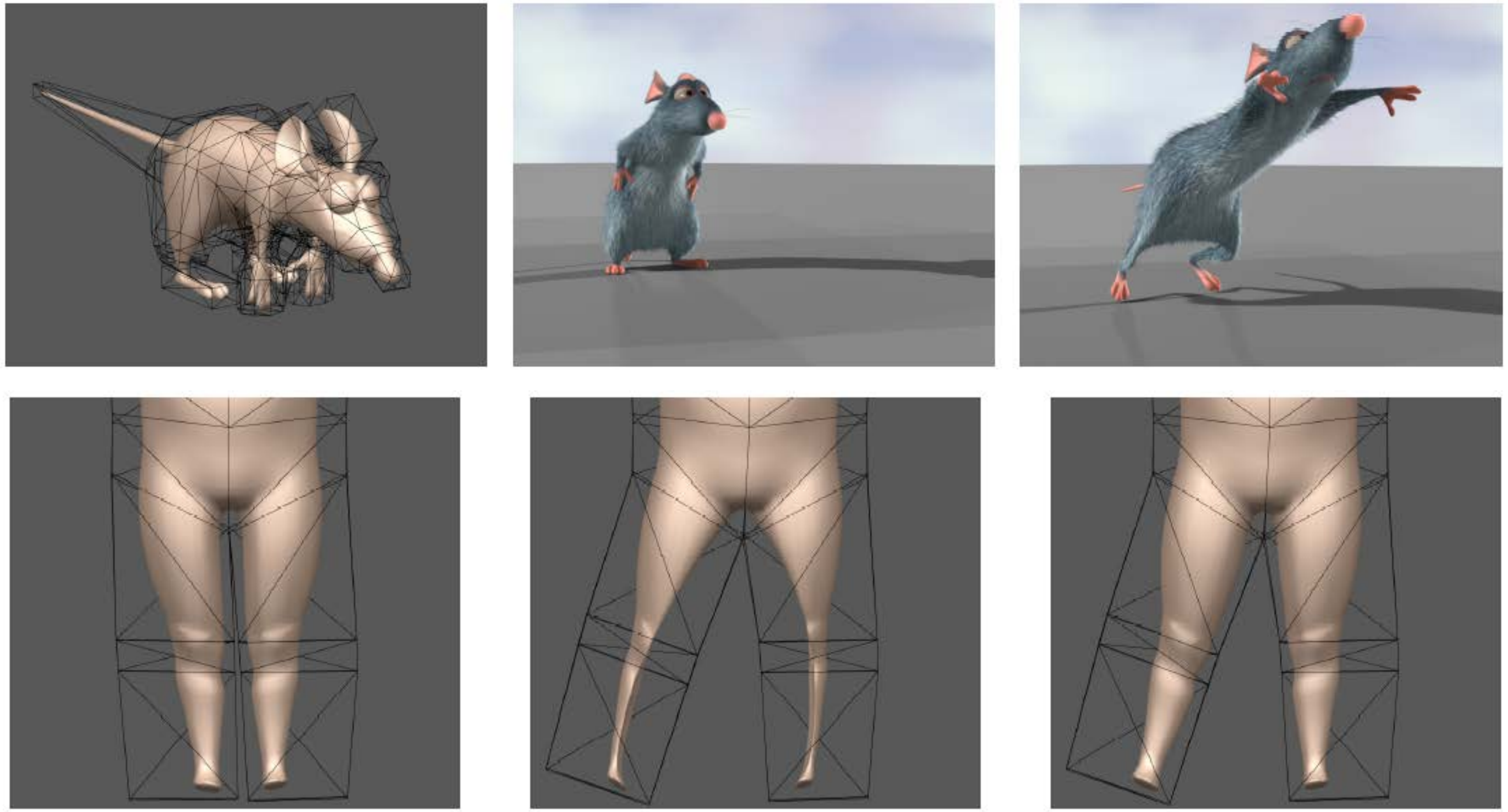
- Volumetric Laplace equation

- Discretization, no closed-form



Coordinate Functions

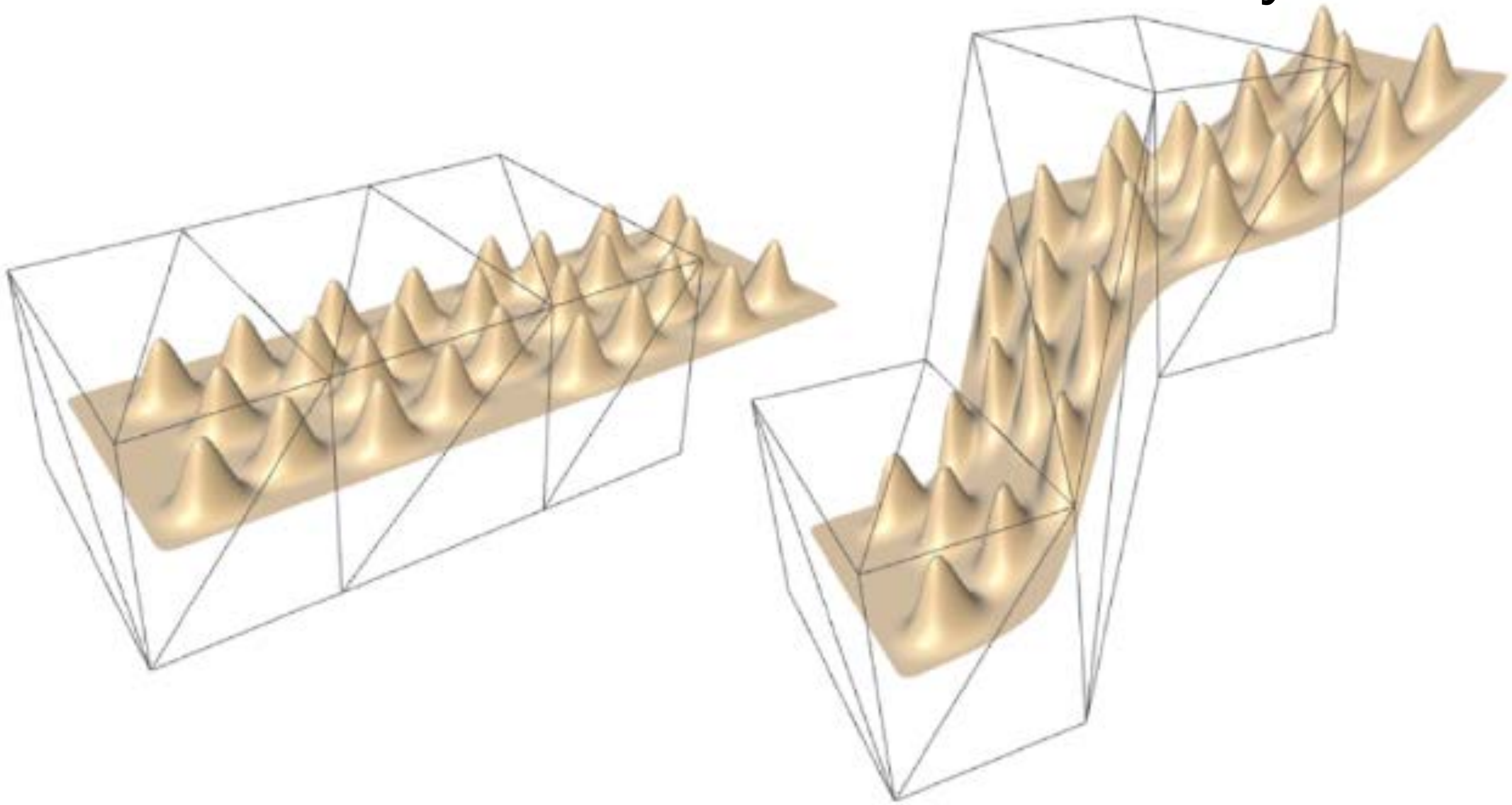
- Harmonic coordinates ([Joshi et al. 2007]())



MVC HC

Coordinate Functions

- **Green coordinates** (Lipman et al. 2008)
- Observation: previous vertex-based basis functions always lead to affine-invariance!



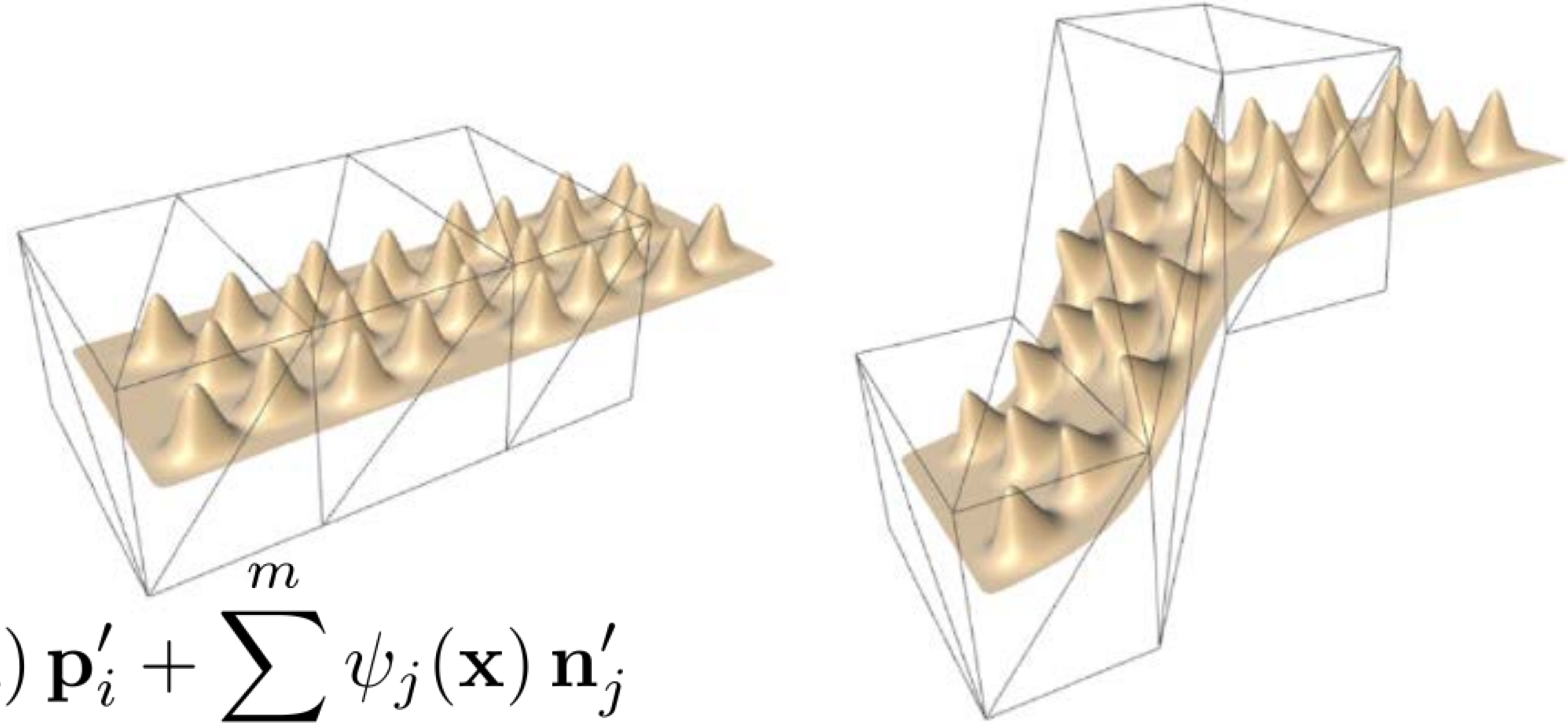
$$\mathbf{x}' = \sum_{i=1}^{k} w_i(\mathbf{x})\, \mathbf{p}'_i$$

http://www.wisdom.weizmann.ac.il/~ylipman/GC/gc.htm

Coordinate Functions

- **Green coordinates** (Lipman et al. 2008)

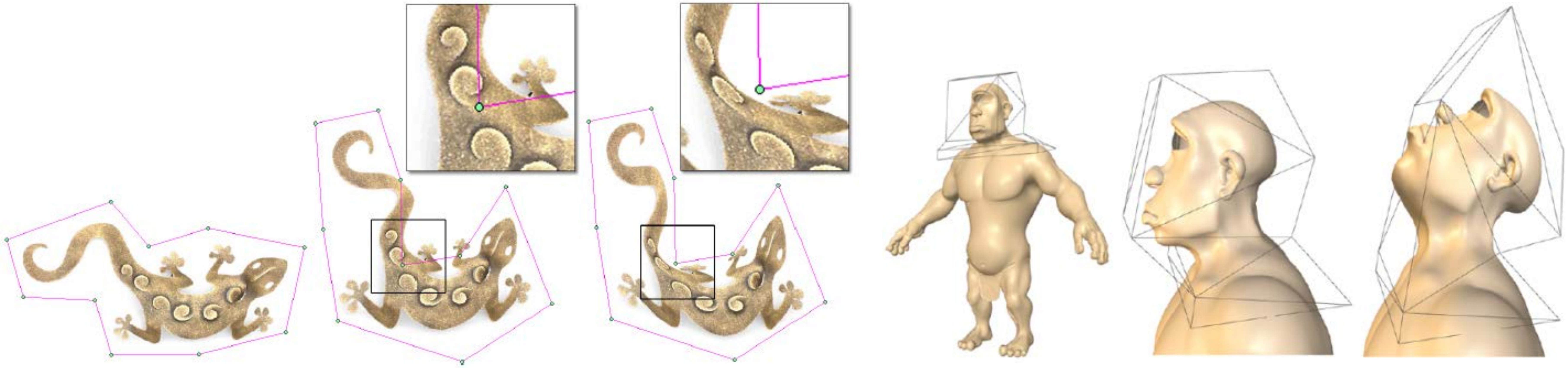
- Correction: Make the coordinates depend on the cage faces as well



$$\mathbf{x}' = \sum_{i=1}^{k} w_i(\mathbf{x})\,\mathbf{p}'_i + \sum_{j=1}^{m} \psi_j(\mathbf{x})\,\mathbf{n}'_j$$

Coordinate Functions

- **Green coordinates** (Lipman et al. 2008)
- Closed-form solution
- Conformal in 2D, quasi-conformal in 3D



GC MVC GC

Cage-based methods: Summary

Pros:

- Nice control over volume
  - Squish/stretch

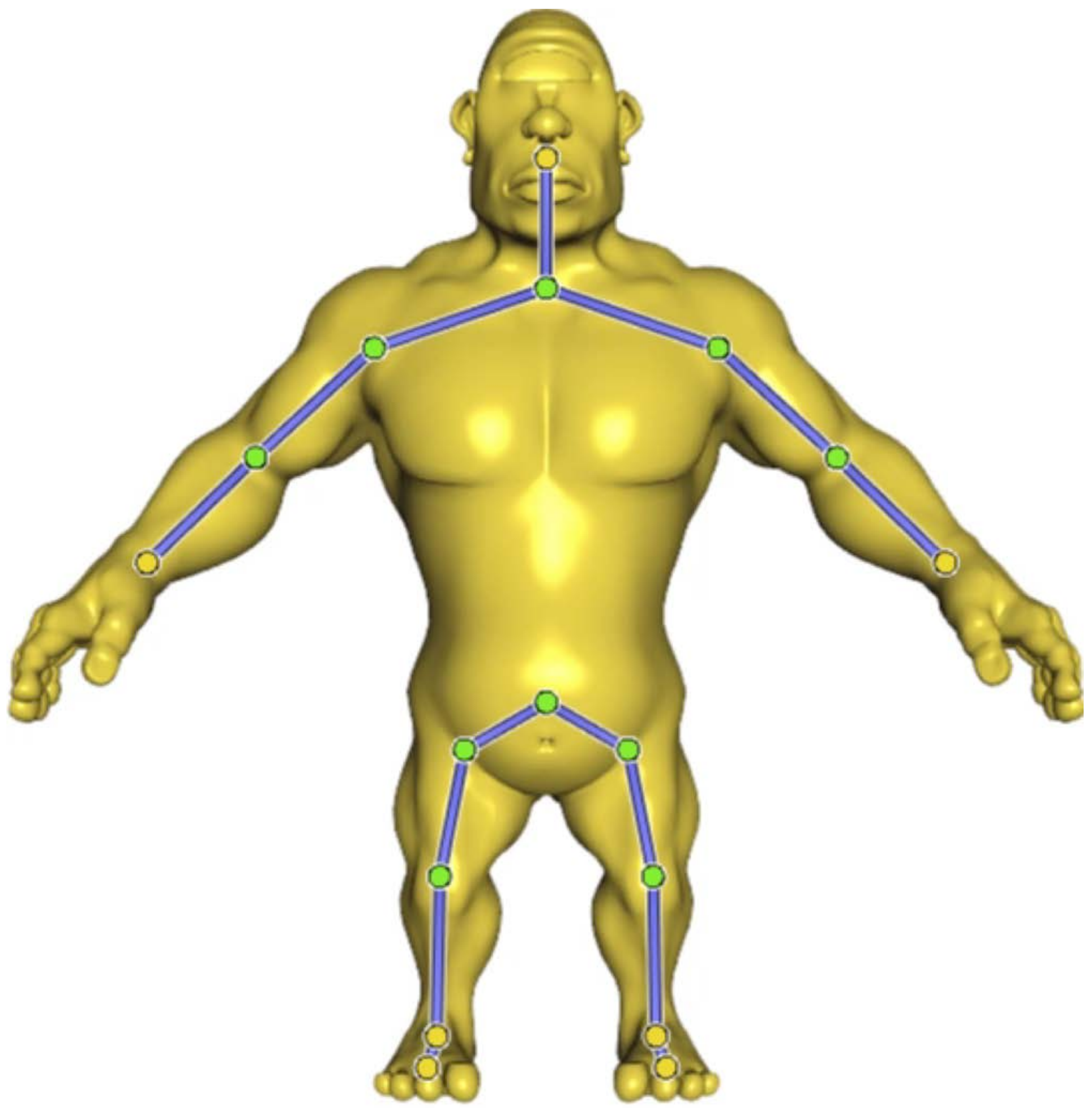
Cons:

- Hard to control details of embedded surface

Linear Blend Skinning (LBS)

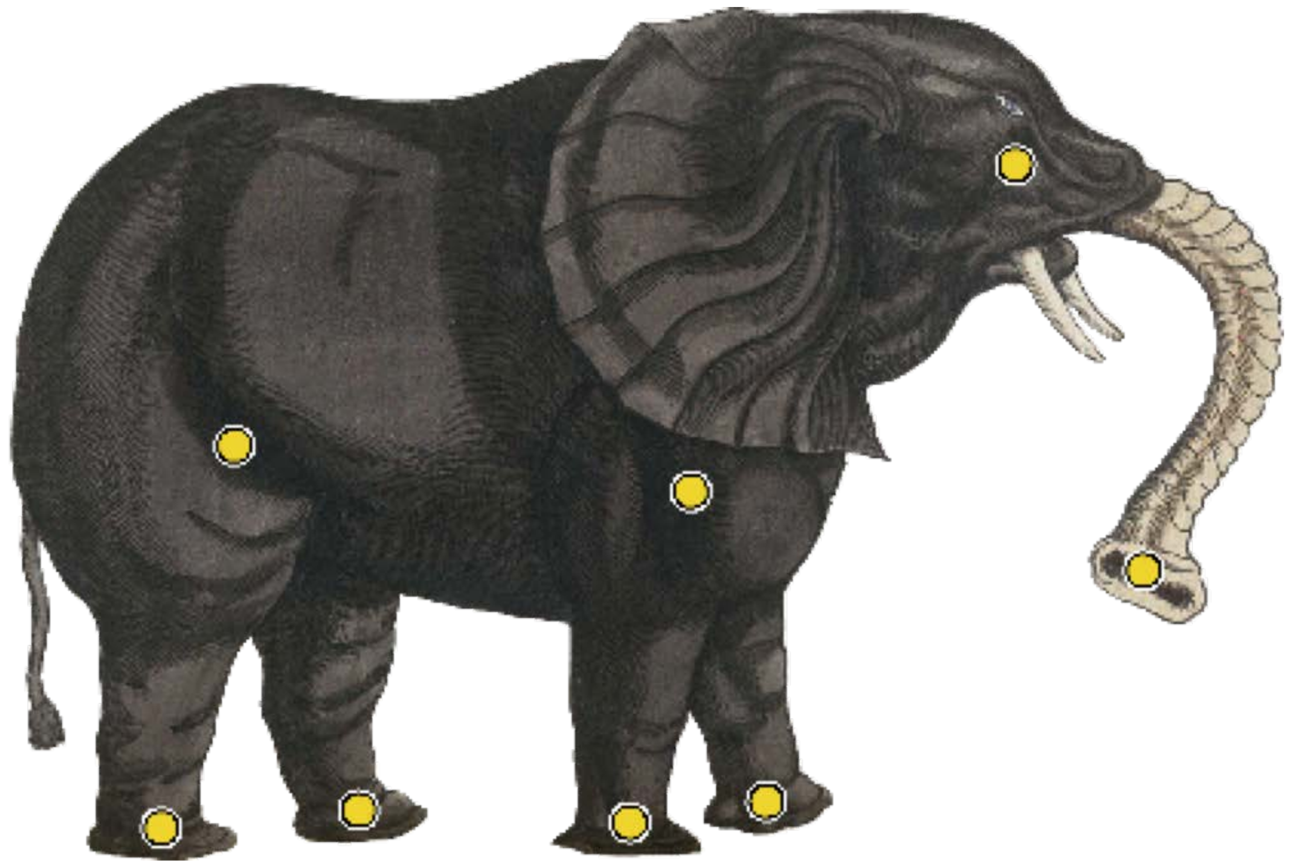
Acknowledgement: Alec Jacobson

LBS generalizes to different handle types

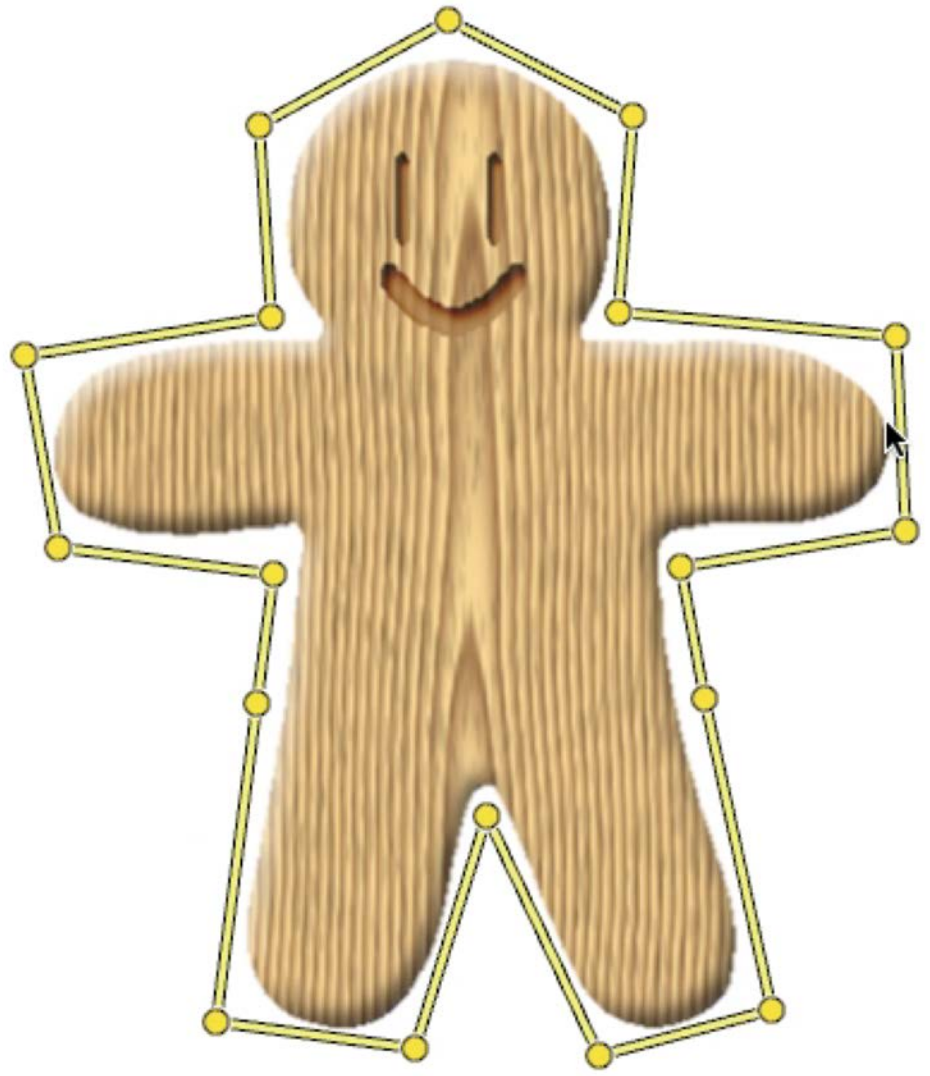


skeletons

regions

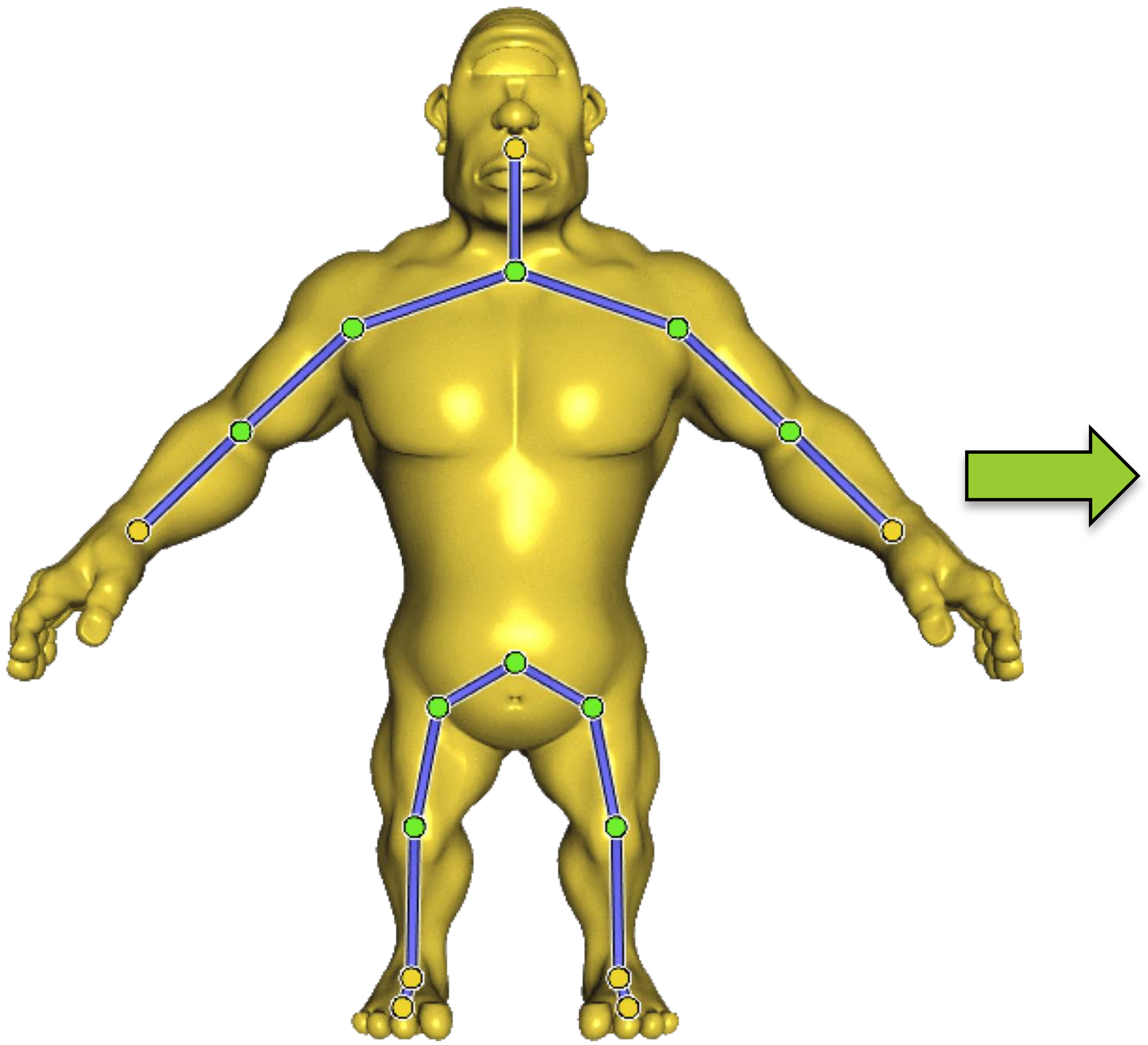


points



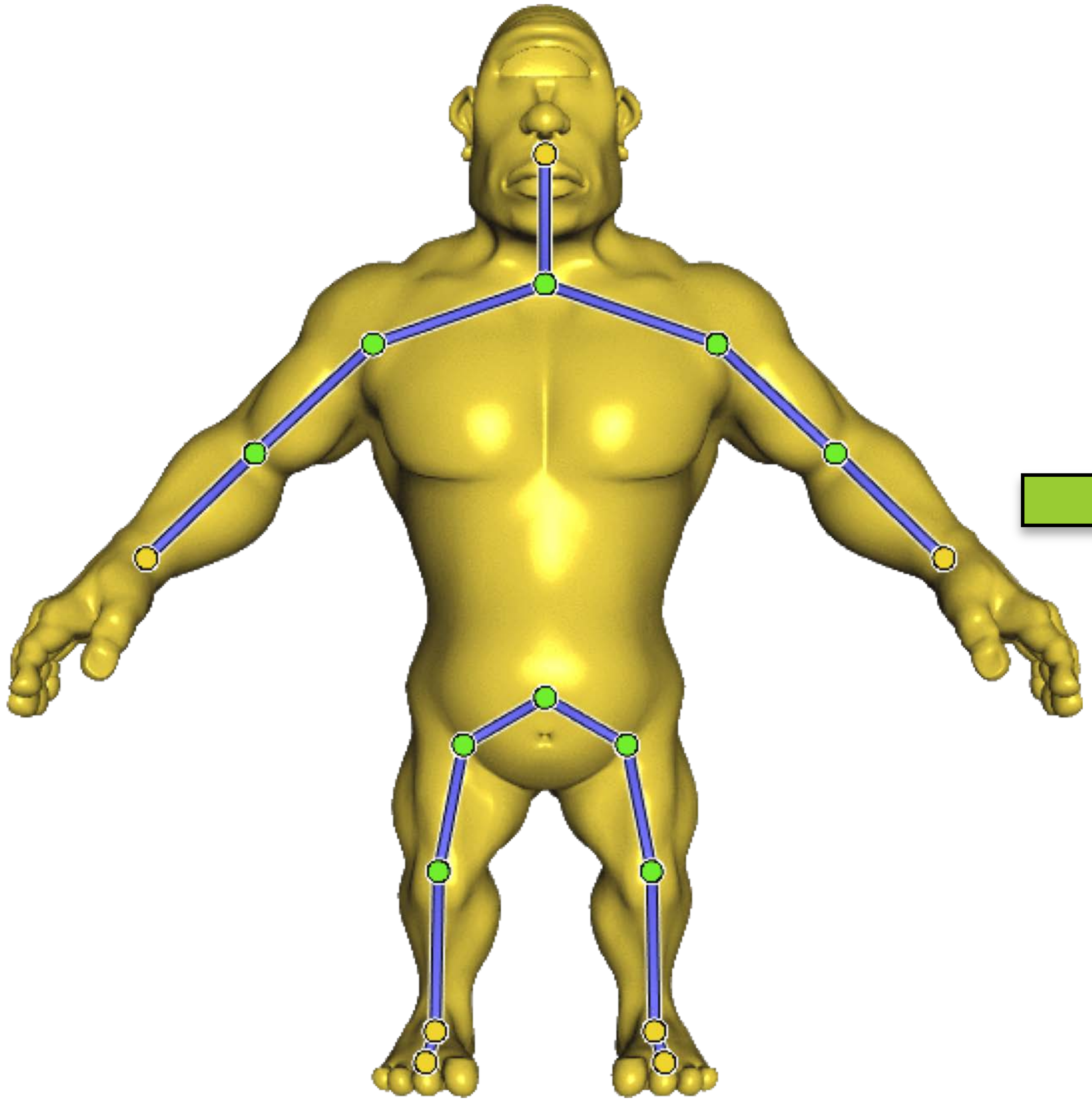
cages

Linear Blend Skinning rigging preferred for its real-time performance

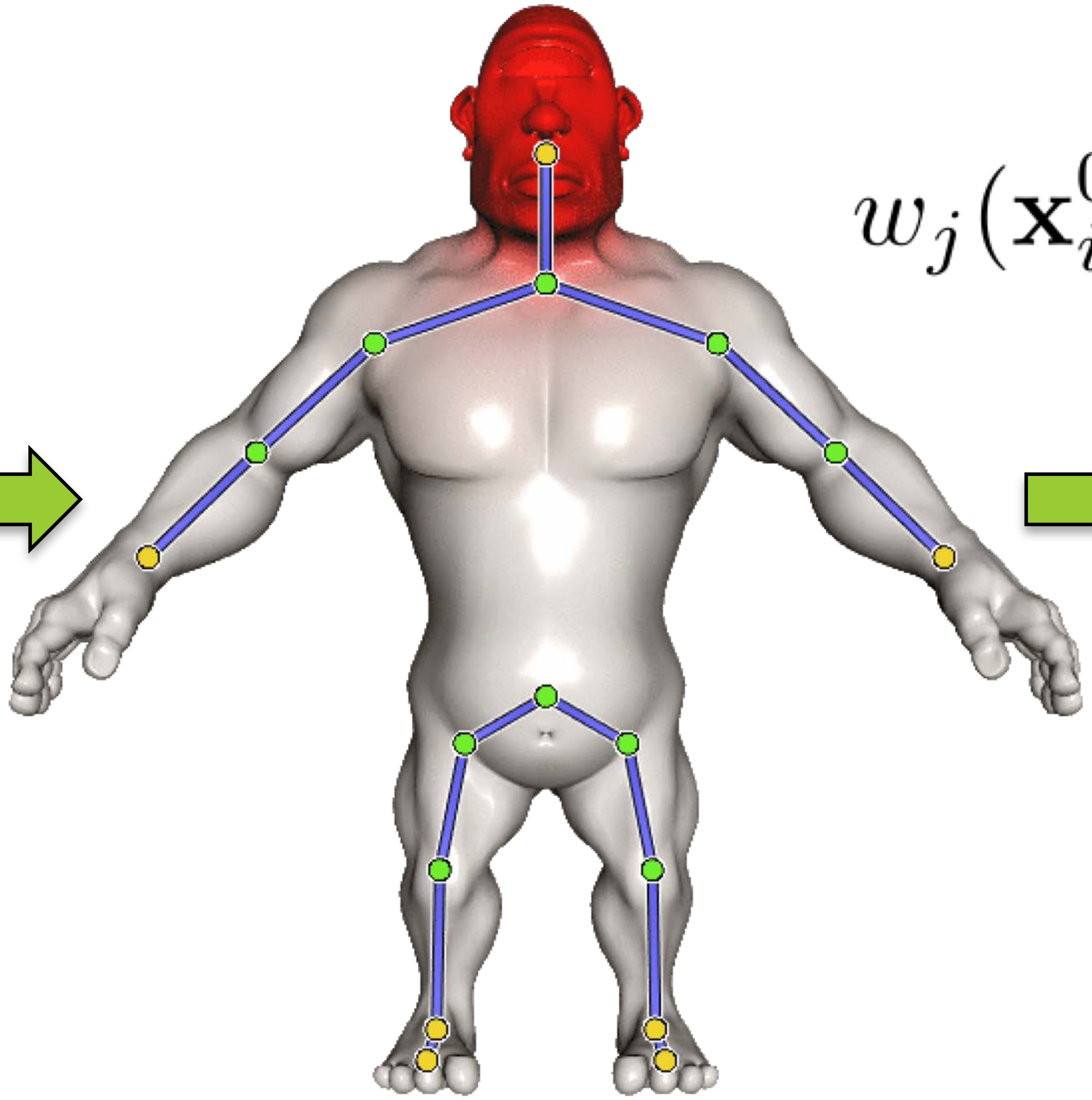


place handles in shape

Linear Blend Skinning rigging preferred for its real-time performance



place handles in shape



$w_j(\mathbf{x}_i^0)$

paint weights

Linear Blend Skinning rigging preferred for its real-time performance

place handles in shape | paint weights | deform handles

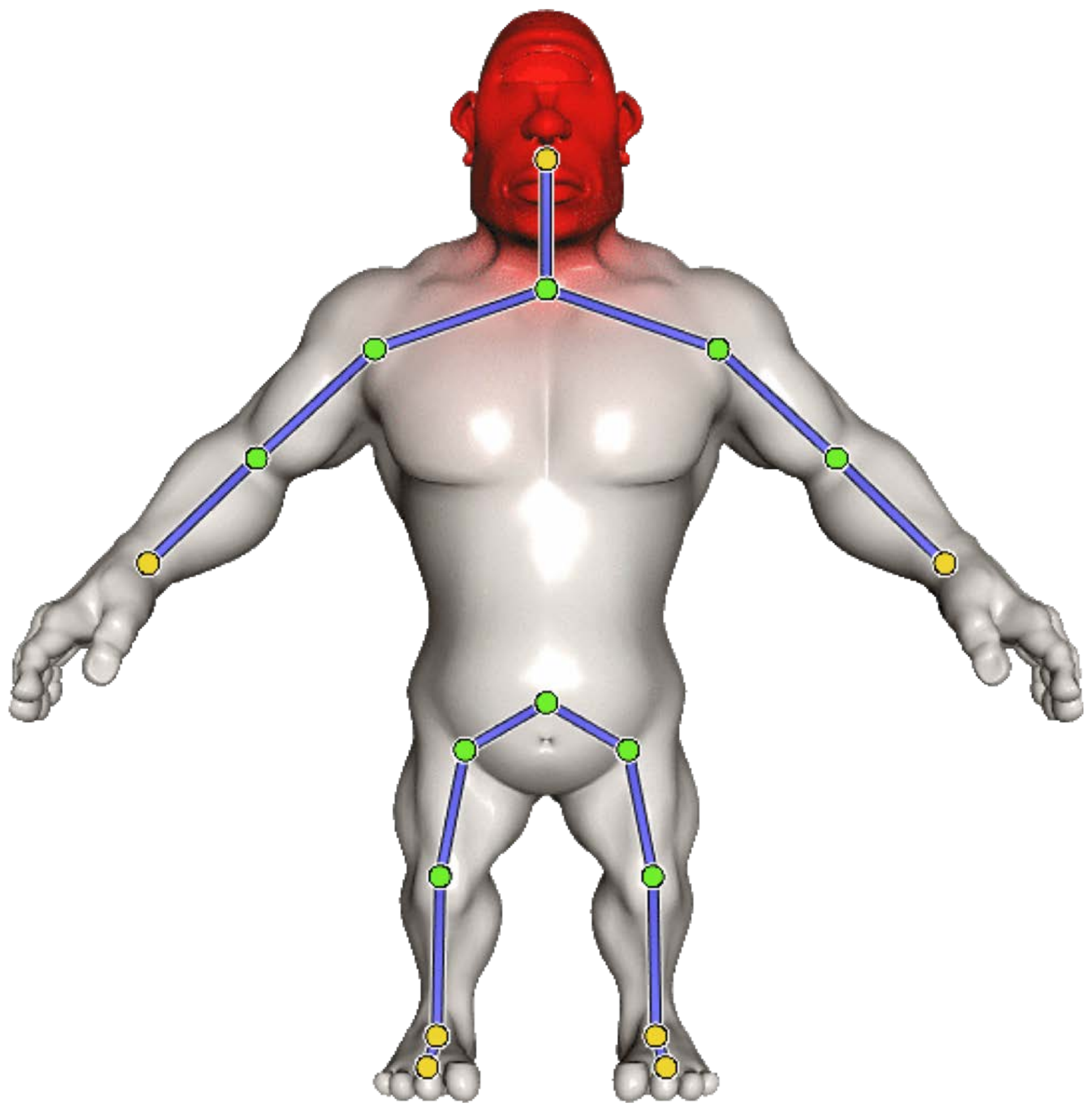
Linear Blend Skinning rigging preferred for its real-time performance

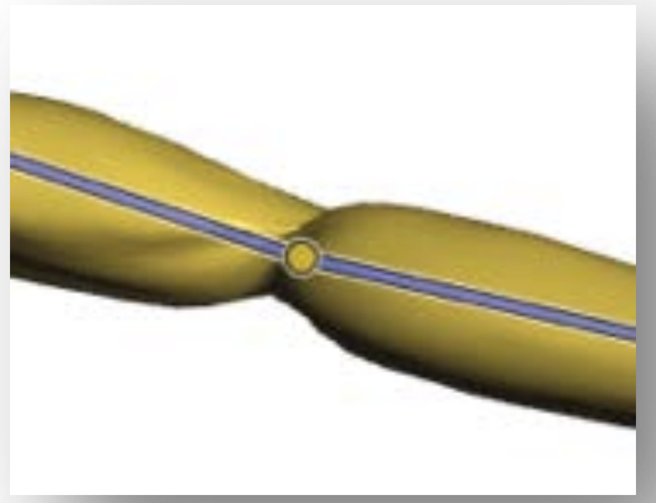
Linear Blend Skinning rigging preferred for its real-time performance

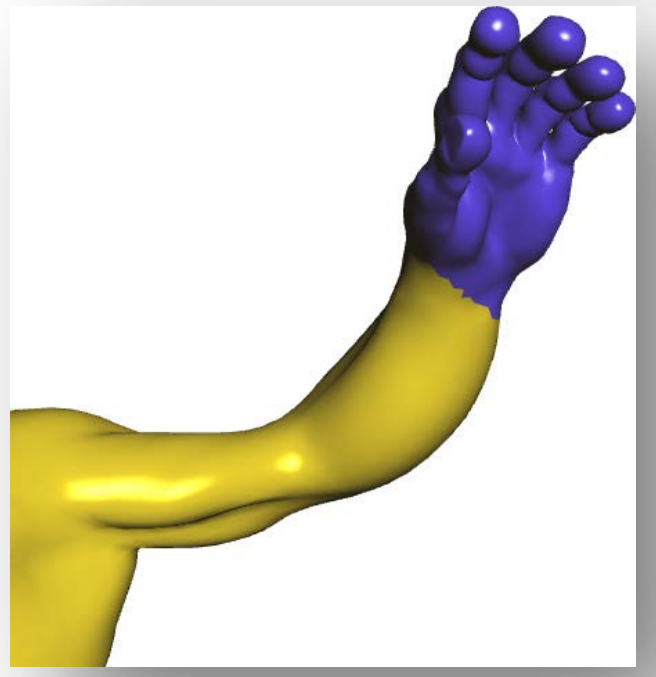
Challenges with LBS

$$\mathbf{x}_i = \sum_{j=1}^{m} \underline{w_j(\mathbf{x}_i^0)}\, \underline{\mathbf{T}_j} \begin{pmatrix} \mathbf{x}_i^0 \\ 1 \end{pmatrix}$$

- Weight functions w_j
  - Can be manually painted or automatically generated
- Degrees of freedom $\mathbf{T}_j$
  - Exposed to the user (possibly with a kinematic chain)
- Richness of achievable deformations
  - Want to avoid common pitfalls – candy wrapper, collapses







Properties of the Weights

$$\sum_{j \in H} w_j(\mathbf{x}^0) = 1$$

Partition of unity

Handle vertices

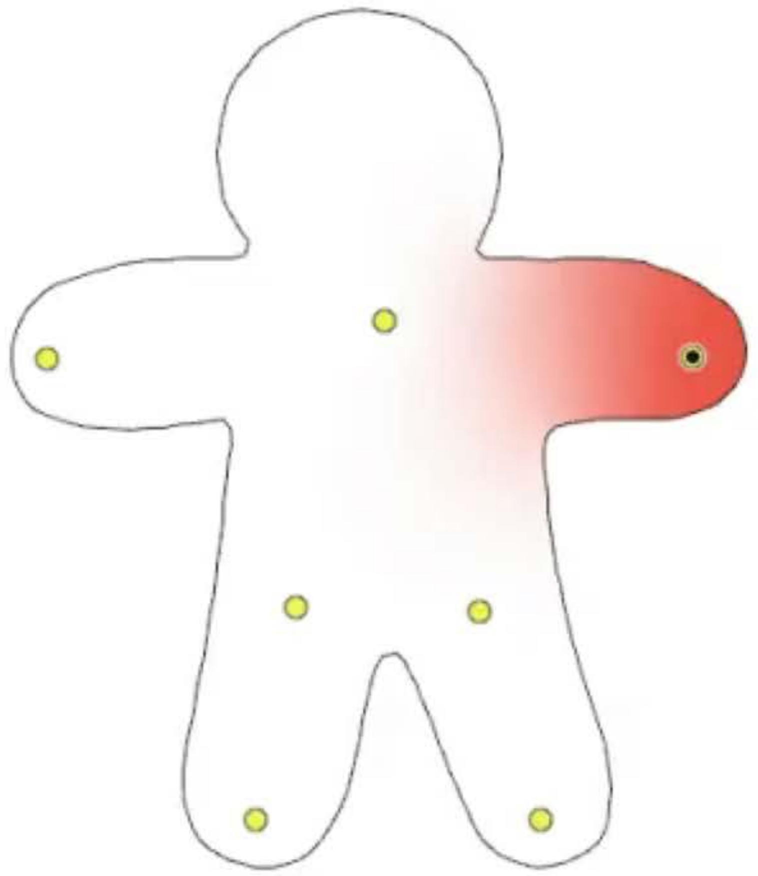
$$w_j\Big|_{H_k} = \delta_{jk}$$

w_j is linear along cage faces

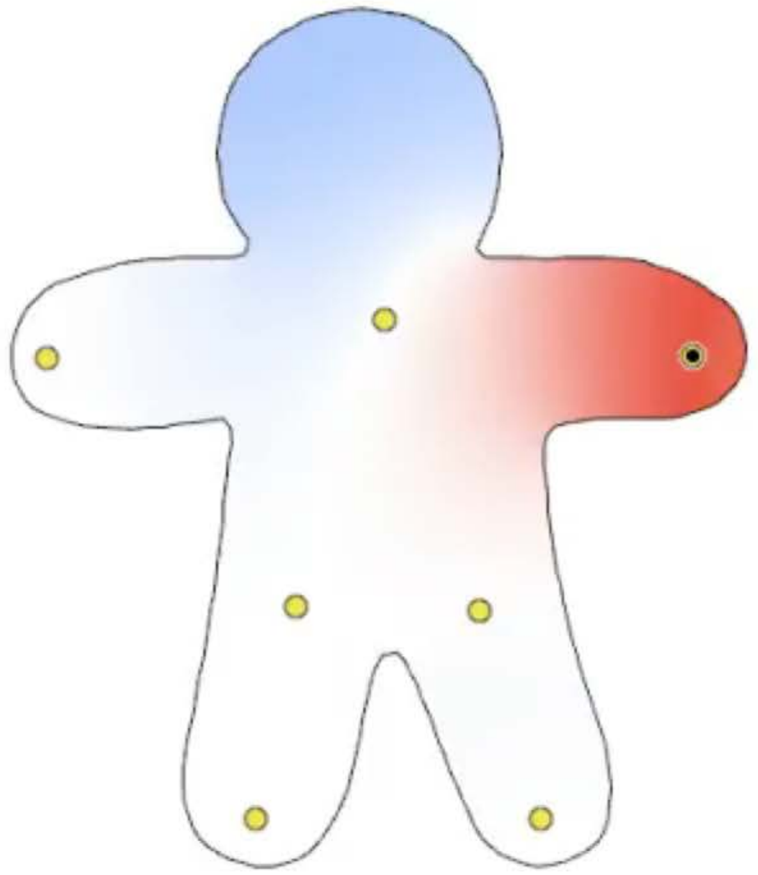
Interpolation of handles

Weights Should Be Positive

Bounded Biharmonic Weights
[Jacobson et al. 2011]



Unconstrained biharmonic
[Botsch and Kobbelt 2004]

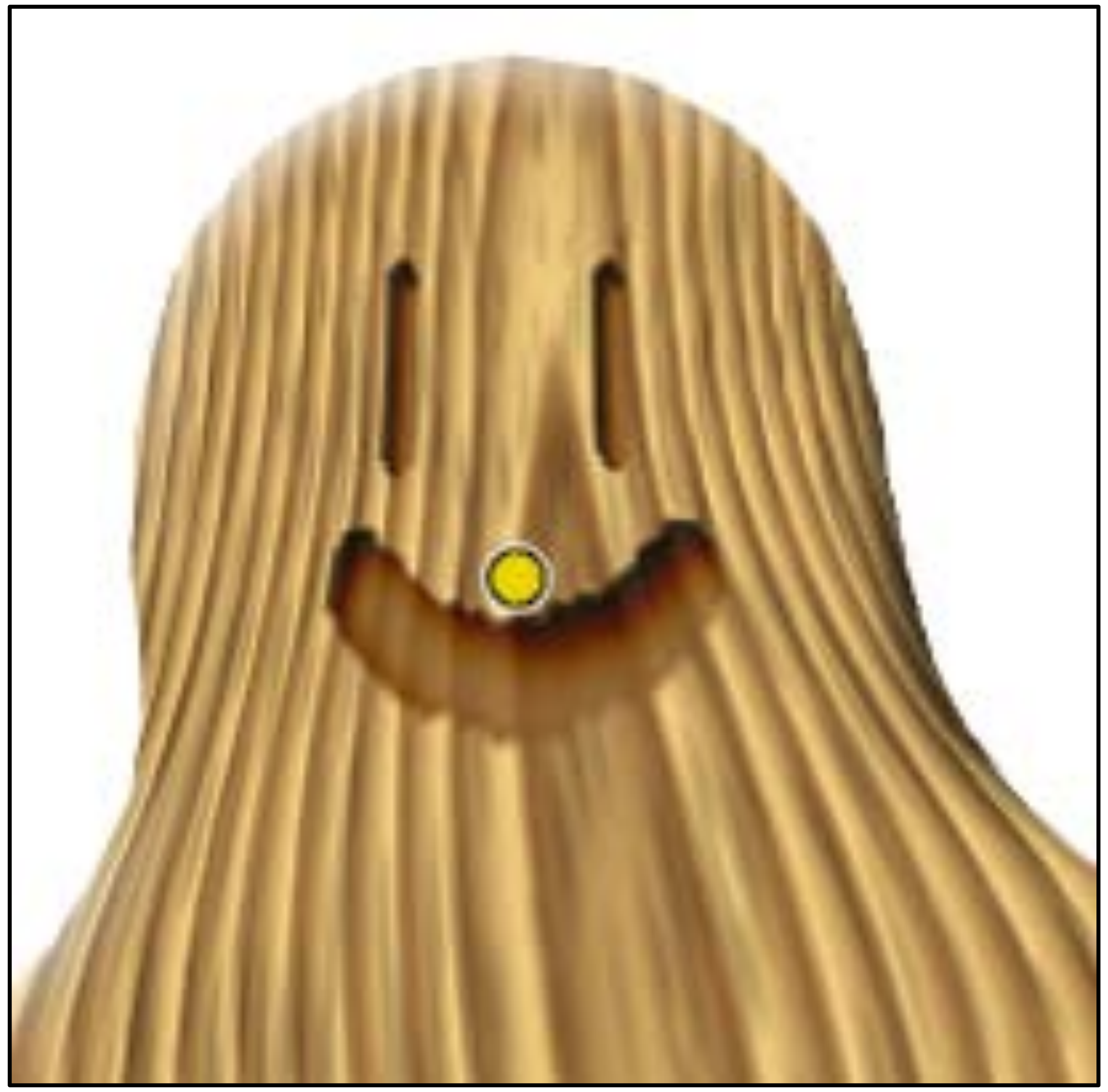


Weights Should Be Smooth

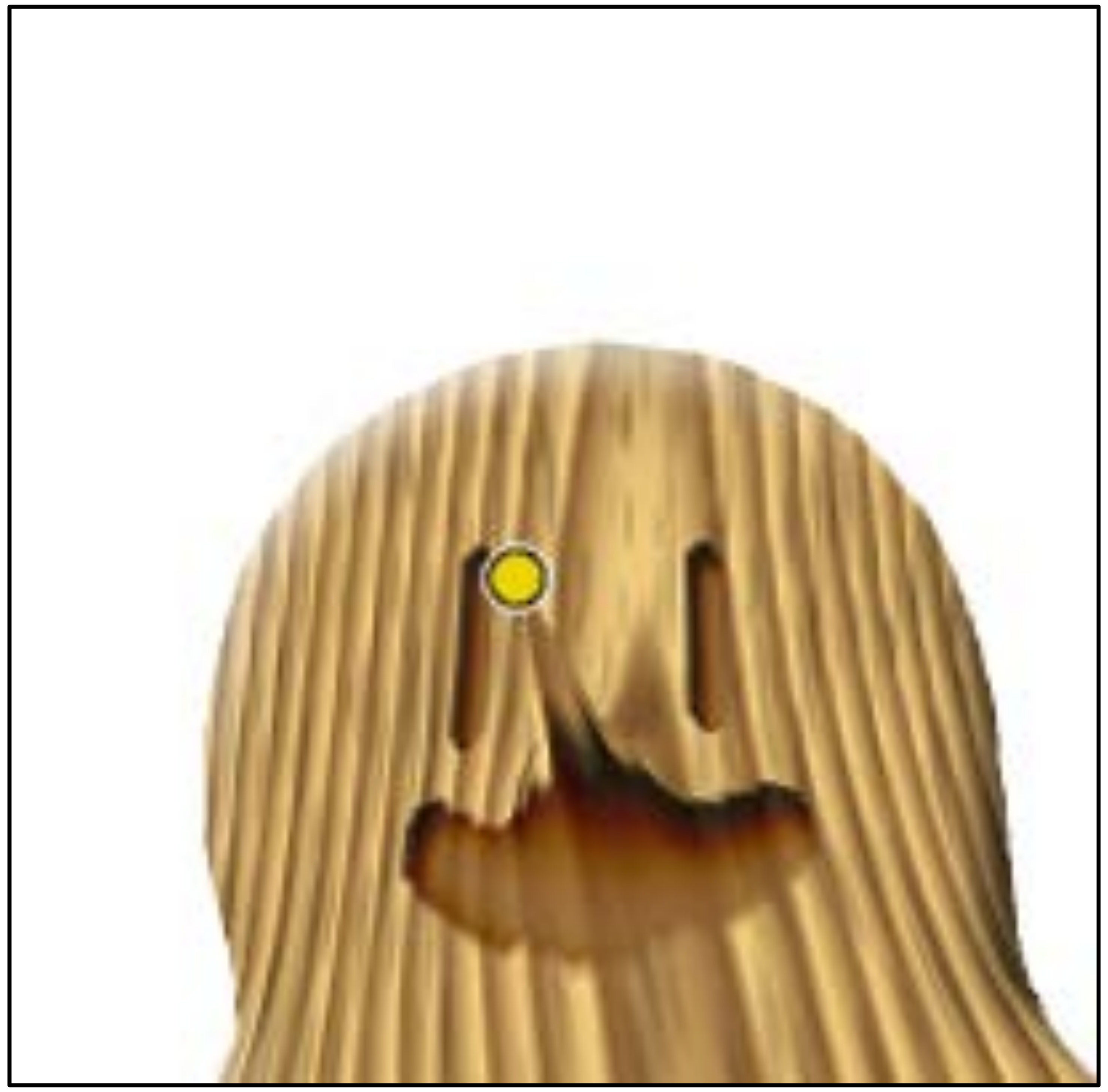
Bounded Biharmonic Weights
[Jacobson et al. 2011]

Extension of Harmonic Coordinates
[Joshi et al. 2005]

Weights Should Be Smooth



Bounded Biharmonic Weights



Extension of Harmonic Coordinates
[Joshi et al. 2005]

Bounded biharmonic weights enforce properties as constraints to minimization

$$\arg\min_{w_j} \frac{1}{2}\int_\Omega |\Delta w_j|^2 dV$$

$$w_j\Big|_{H_k} = \delta_{jk}$$

w_j is linear along cage faces

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$$0 \le w_j(\mathbf{x}^0) \le 1$$

Partition of unity

$$\sum_{j \in H} w_j(\mathbf{x}^0) = 1$$

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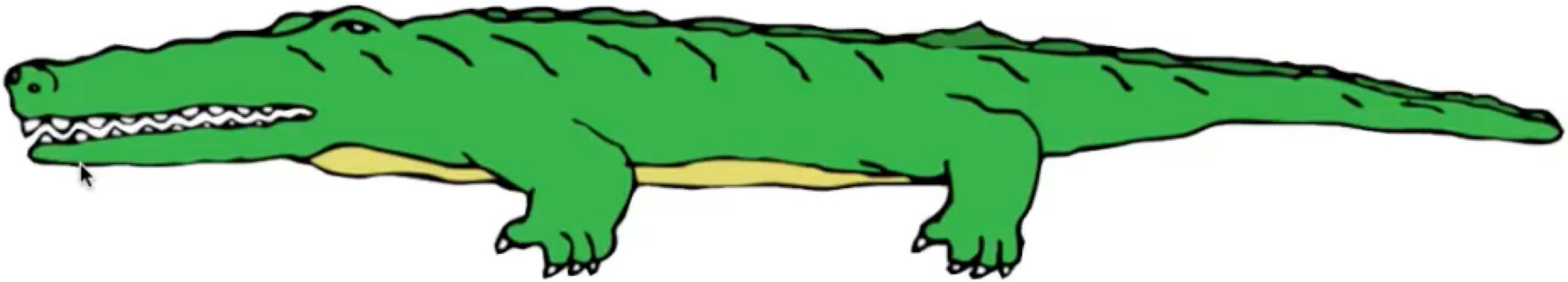
Constant inequality constraints

$$0 \le w_j(\mathbf{x}^0) \le 1$$

Solve independently and normalize

$$w_j(\mathbf{x}^0) = \frac{w_j(\mathbf{x}^0)}{\sum_{i\in H} w_i(\mathbf{x}^0)}$$

Some examples of LBS in action



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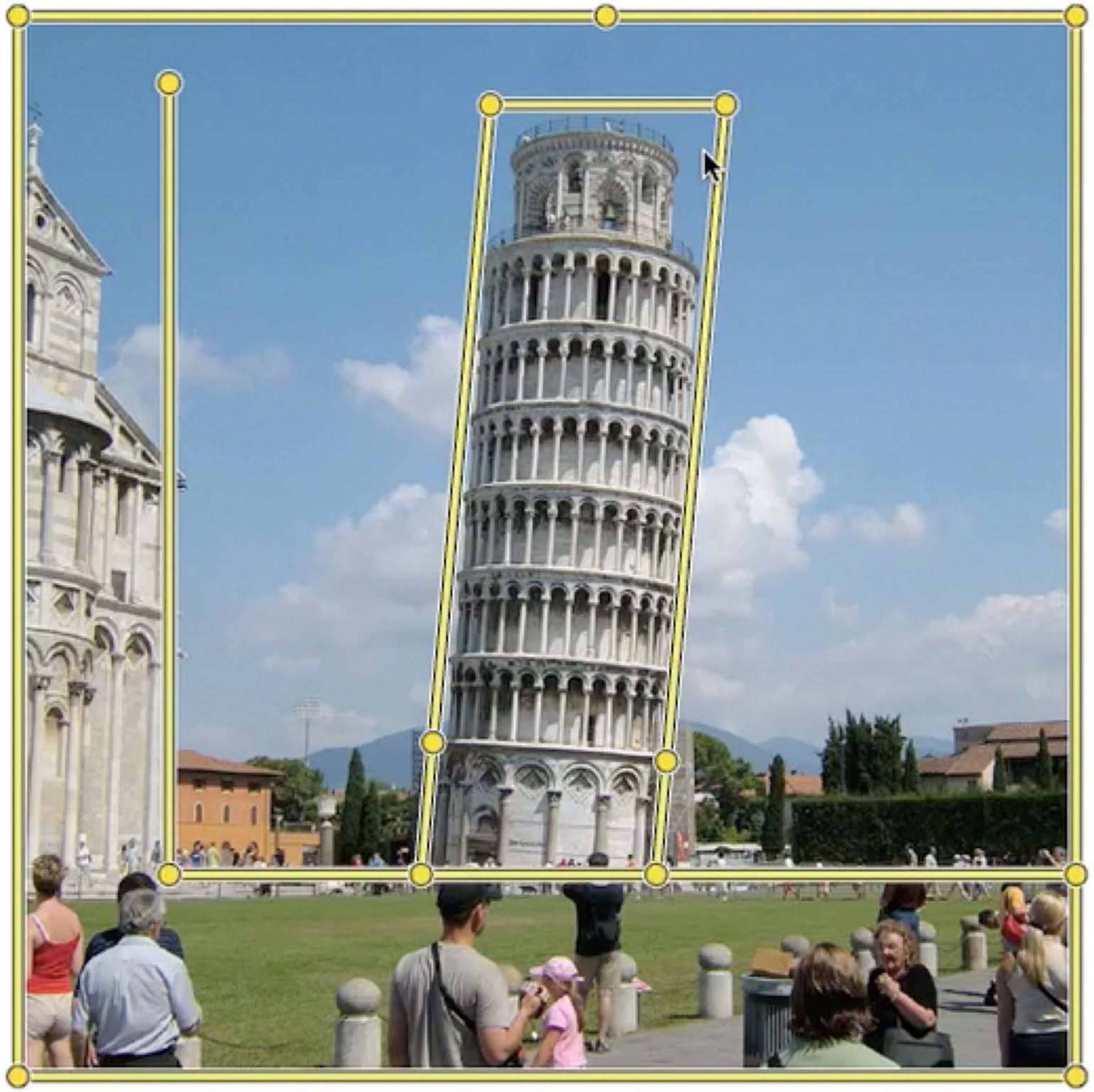
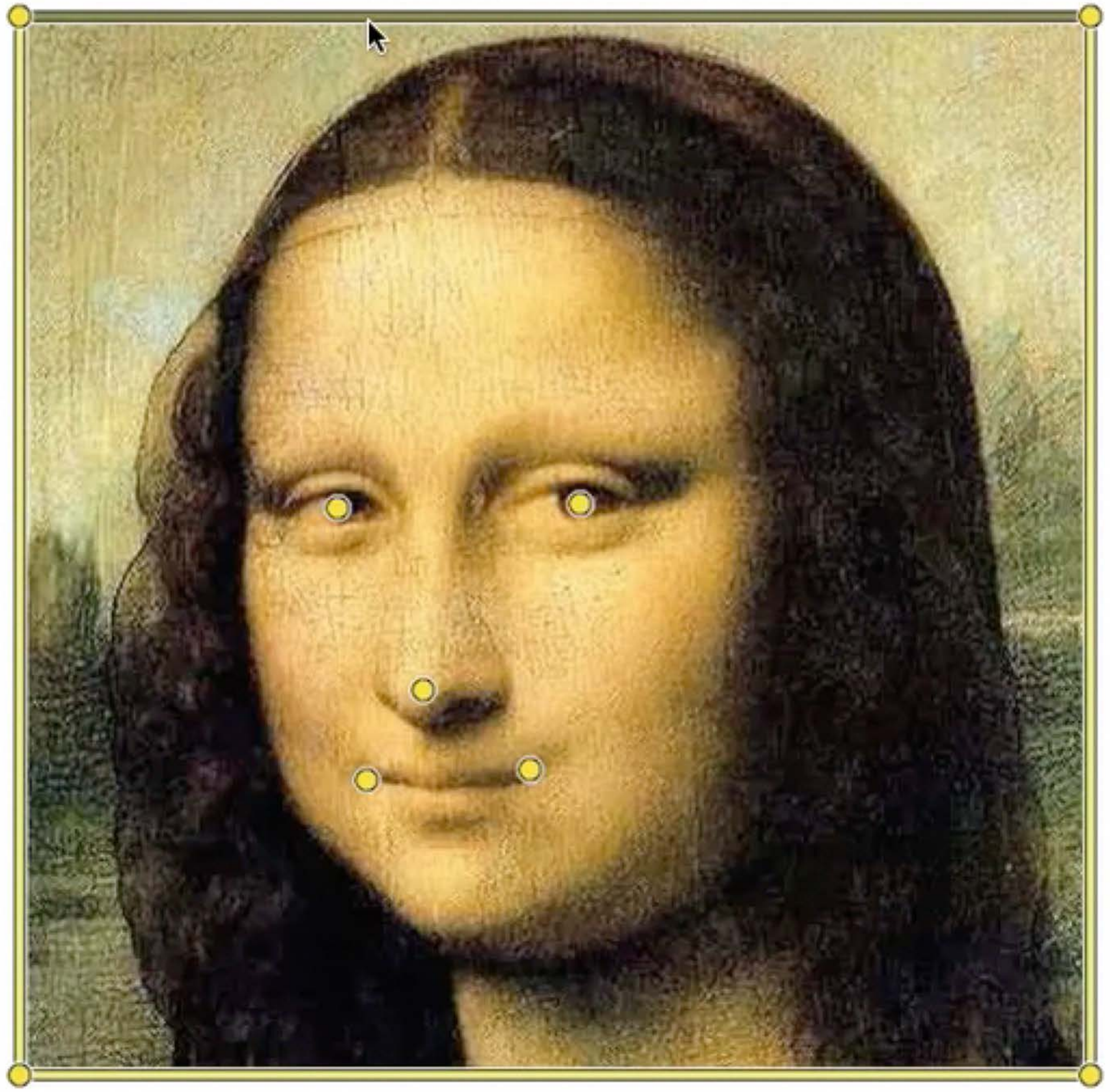
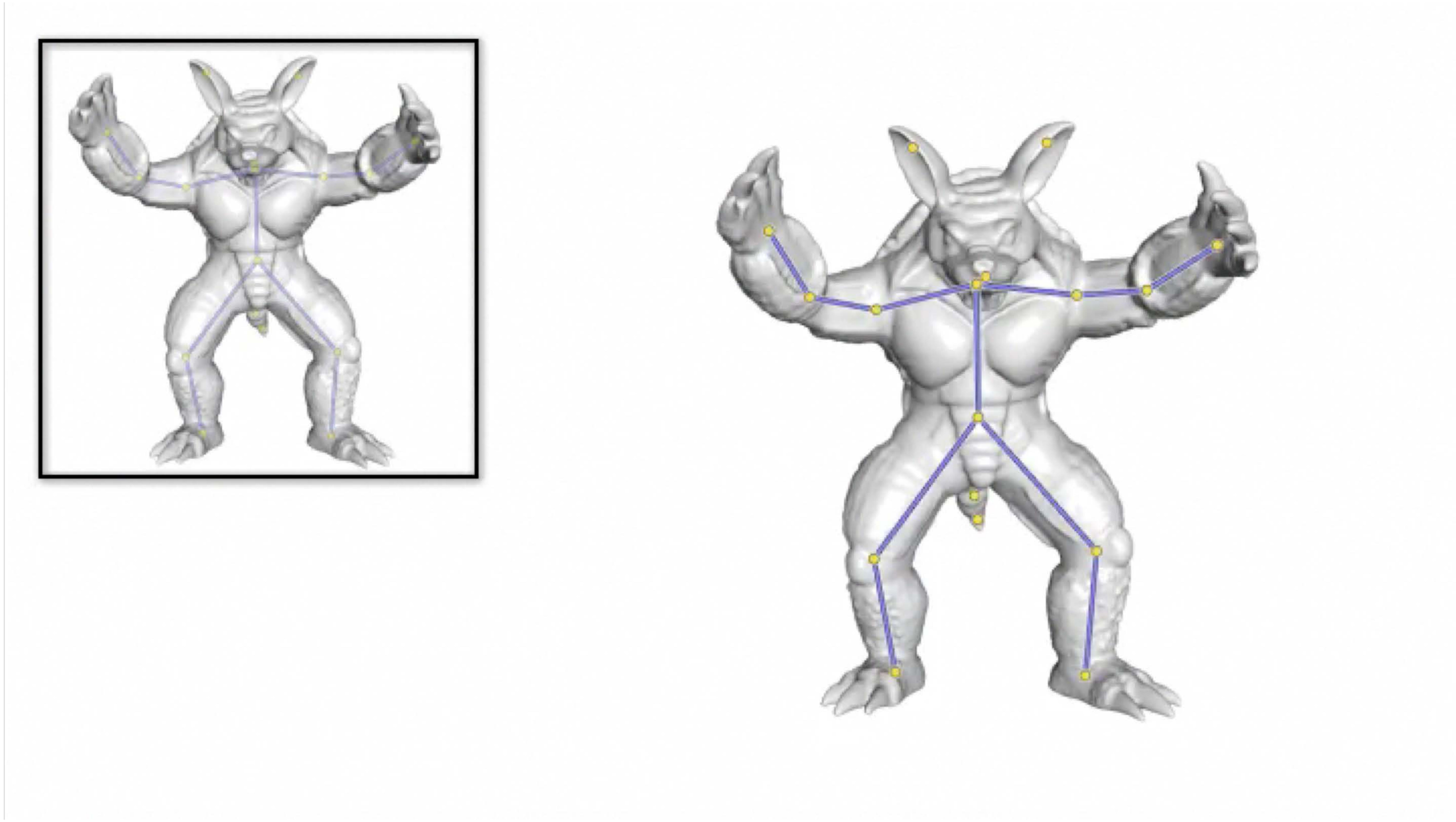


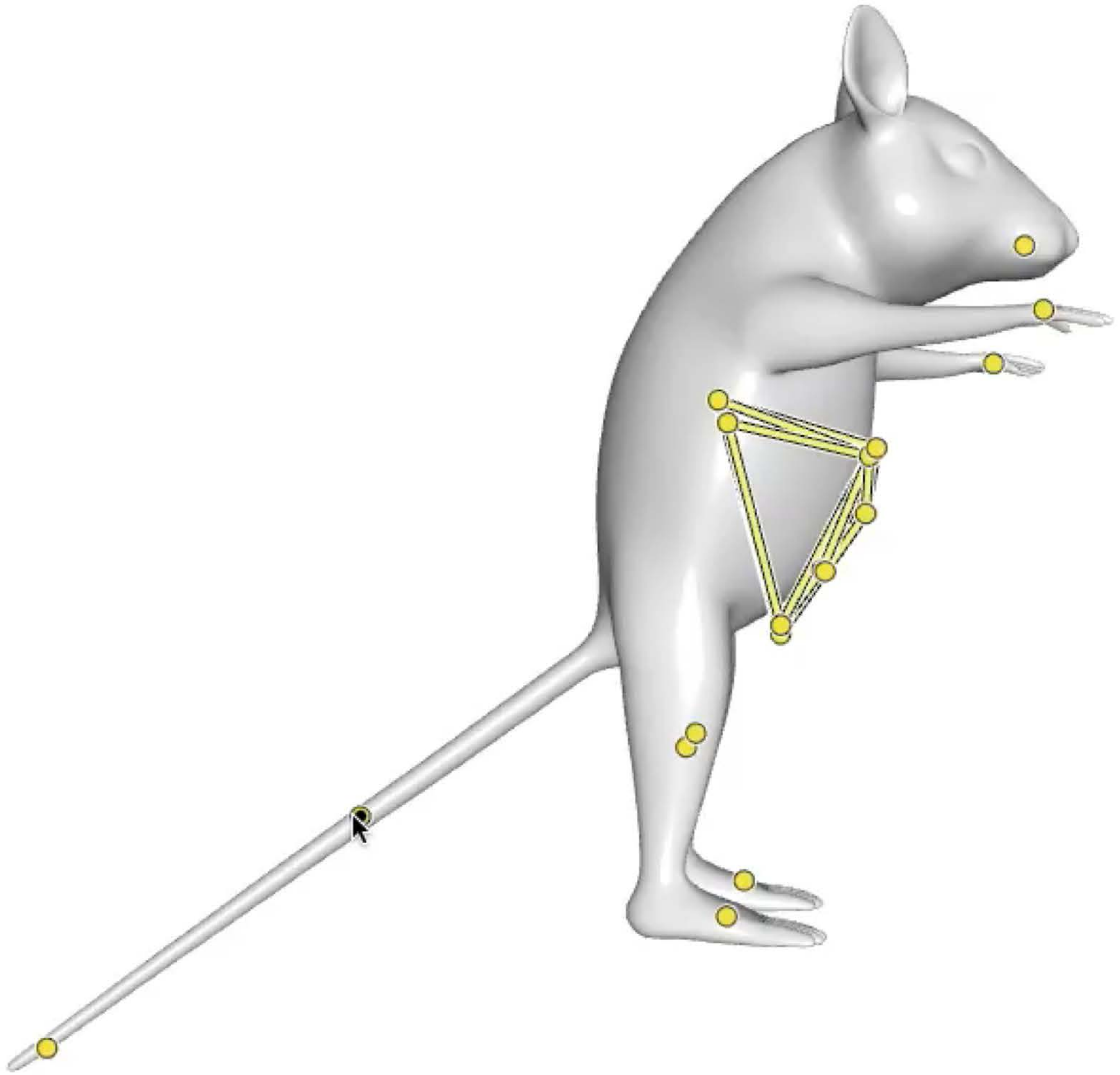
Some examples of LBS in action



3D Characters



Mixing different handle types



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