



Graduate School
of **BUSINESS**
UNIVERSITY OF CAPE TOWN

Active portfolio management: Improving factor-based portfolio construction by applying machine learning to classify stock performance

Research Report

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University of Cape Town

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of the requirements for the degree
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Submitted by:

Anton Johan Runhaar

Supervisor:

Dr. Kutlwano Ramaboa

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Abstract

This study investigated the application of machine learning to active portfolio management by comparing the performance of a factor based investment strategy to one that employs support vector machine classification. Machine learning prediction models were trained on historical factor data and used to determine the probability of future upward stock price movement. Stocks were ranked based on this probability and divided into equally weighted quintiles. The performance of first quintiles, for both factor based and machine learning based investment strategies, was used as the basis for comparison.

Portfolio comparison was conducted in the context of the Johannesburg Stock Exchange, using 100 companies ranked by size and filtered for liquidity and availability of historical data. Data was collected for a 15-year period, March 2001 to February 2016, of which 10 years were used for model training and validation, and 5 years for prediction and subsequent portfolio performance comparison. Seven factors, spanning the broad categories of momentum, value and quality, were used individually as the bases for portfolio construction and comparison. Combinations of factors were subsequently used to investigate the effect of additional inputs on the performance of machine learning based portfolios.

It was found that factor-based investing is effective in constructing benchmark-beating portfolios for only three of the seven factors: book-to-market value, return on equity and return on invested capital. Factor-based investing underperformed the benchmark when using 6-month price rate of change and dividend yield. Comparatively, the machine learning-based investment strategy outperformed the benchmark for six of the seven factors and performed on par with the benchmark using the seventh, return on invested capital. When compared directly, the single-factor machine learning-based portfolios yielded average monthly returns similar to, or better than, their factor-based investing counterparts, with the exception of return on invested capital. Furthermore, machine learning-based portfolios using multiple factors as inputs showed greater average portfolio yield than all single factor implementation, with the best performing portfolio using all seven factors as inputs.

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1 Introduction

1.1 Research Area and Problem

The role of a portfolio manager is to select stocks of such a nature that portfolio returns are maximised given a particular risk tolerance. Passive managers believe that this can only be achieved by constructing a fully diversified portfolio that tracks the performance of the market. Active portfolio managers, however, believe that returns in excess of the market can be achieved by capitalising on inefficiencies inherent in the market.

The Efficient Market Hypothesis states that all past and present information regarding a stock is captured within the current stock price and therefore no stock is ever mispriced. The hypothesis argues that stock movement is Brownian and follows a random walk evolution, and therefore cannot be predicted (Berk, 2005; Teixeira & De Oliveira, 2010). Passive portfolio managers agree with this sentiment and create funds that track the performance of a selected market by matching the portfolio beta to that of the market.

There are however those who debate the credibility of the Efficient Market Hypothesis. Haugen (1999) argues against the hypothesis and provides evidence supporting the notion that markets are inefficient and over reactive and that stock prices do not necessarily reflect their true value. An active strategy is therefore driven by an analysis of stock indicators, variables that allude to the true value of a stock, in an attempt to capitalise on market inefficiencies, such as mispriced stocks, thereby realising improved returns (Grinold & Kahn, 1996). By definition, active portfolio managers purposefully shift risk exposure away from market beta by altering the construction of the portfolio, either through alternate stock weighting or alternate fund composition (Silva, Philips, Walker, & Thomas, 2011). Therefore, active portfolio management is the discipline of applying rigorous analysis and a rigorous process, simply and persistently, to outperform the market, therefore outperforming passively managed funds (Grinold & Kahn, 1996; Muller & Ward, 2013).

According to Ang (2013, as cited by Tajbai, 2015), the key to success for active managers is to understand, identify and capitalise on market inefficiencies before information diffuses to other investors and before a price equilibrium is reached. One approach that aims to exploit such market

inefficiencies is factor-based investing, an investment strategy that constructs investment portfolios based on how companies perform with respect to particular factor criteria. Such factors can be thought of as any characteristic relating to a security that is important in explaining its return and risk. Well-known categories of factors include value, size, momentum, volatility, dividend yield and quality (Bender, Briand, Melas, & Subramanian, 2013). Each category is commonly captured by multiple factors, which are obtained or constructed from stock price history or underlying company financial statements. Active managers use these factors as proxies for future stock performance and construct portfolios accordingly. Despite its stark contradiction to the Efficient Market Hypothesis, many empirical studies have demonstrated the ability of active portfolio management to outperform the market (Hart, Slagter, & Dijk, 2003; Piotroski, 2001). As evidence of the practical applicability of actively-managed funds, both ABSA and Satrix have created momentum factor-based active index funds traded on the Johannesburg Stock Exchange (JSE), the NewFunds Equity Momentum Exchange Traded Fund and the Satrix Momentum Index Fund, respectively (ABSA, 2013; Satrix, 2016).

However, despite the empirical success of factor-based portfolio construction, there are several criticisms to the approach. Firstly, due to the multitude of factors available for evaluation, value, size, momentum, volatility, dividend yield and quality, no single approach is noted as one that performs best. Various academic studies have explored the use of different factors and combinations. Jegadeesh and Titman (1993) implement a momentum factor-based approach, comparing multiple analysis and buy-hold time frames to uncover the most profitable approach. Grinblatt, Titman and Wermers (2016) use a similar momentum factor-based approach but base it on a 6-month weighted average stock direction change, rebalanced quarterly. Conversely, Piotroski (2001) implements a value factor-based approach reliant on the quarterly book-to-market indicator and also rebalances portfolios quarterly. Asness, Moskowitz and Pedersen (2013) use a combined value and momentum factor-based approach, rebalanced monthly. Confirmation of the existence and extent of market premiums for various factors appear to vary from study to study.

Although there are many that agree that factors other than volatility (β), proposed by CAPM, can be used in asset pricing, there is little consensus in empirical studies about which factors should be included and how these factors should be assimilated. Within the recent context of the JSE alone Van Rensburg (2001) considers twenty seven factors measures spanning the categories of

value, earnings growth, bankruptcy improbability, price momentum and “neglect”, finding evidence of excess return for ten factor measures; Van Rensburg and Robertson (2003) consider only size (market capitalisation), price to earnings and volatility (β) and suggest predictive abilities in both size and value factors; Strugnell, Gilbert and Kruger (2011) also consider size, price to earnings and volatility, but differ in their conclusion regarding the predictive power of β ; and Muller and Ward (2013) consider eleven factor measures that exclude any measure of momentum. These studies use a static view of factor for portfolio construction i.e. the current factor value is used for stock ranking. The historical significance of a factor, as it relates to stock return, is not accounted for (typical of regression analysis) and the assumption is that a “high” factor value is a proxy for improved return. To simplify and improve this factor-based approach, it is necessary to use a method that can assimilate a wide range of input variables, as well as incorporate historical trends in predicting future performance to subsequently exploit inefficiencies in the market to yield high returns.

Machine learning, which has historically been strongly associated with cognitive science and engineering, has become a popular field of study as a method for predicting future stock performance. This is due to its inherent ability to predict select outcomes given large sets of complex correlated information. Popular fields of research for machine learning include time series forecasting, classification and pattern recognition (Ahmed, Atiya, El Gayar, & El-Shishiny, 2010; Chiam, Tan, & Al Mamun, 2009; Coakley & Brown, 2000; Lam, 2004; Patel, Shah, Thakkar, & Kotecha, 2015; Teixeira & De Oliveira, 2010). Within the scope of factor-based investing, most of the research efforts surrounding machine learning have focused on improving momentum strategies, where difficulties and opportunities arise from the wide range of measures that can be created from historical price data (Kara, Acar Boyacioglu, & Baykan, 2011; Kim & Han, 2000; Patel et al., 2015). Most such studies, however, focus on the prediction of index direction, and not on portfolio construction. Fan and Palaniswami (2001), followed by Huerta, Corbacho and Elkan (2013), perform an influential study on the application of machine learning in the identification of stocks expected to yield exceptional return and outperform the market. These studies consider a wide range of factor inputs, spanning numerous technical and fundamental factors, and use historical data to classify future stock performance. Portfolios constructed using this approach yielded significant portfolio returns above market benchmarks. Although these studies highlight the ability of machine learning to identify high performing stock, they do not relate their findings

directly to those of a “traditional” factor-based approach. Furthermore, the application of their machine learning portfolios is confined to developed markets, and have not yet been evaluated in the context of the South African market.

1.2 Purpose of Research

In this study, a machine learning algorithm is used to form predictive models after being trained on relevant factor data, thereby assimilating all available data sources and time information. These predictive models allow stocks to be ranked based on expected future performance which ultimately forms the basis for portfolio construction.

Numerous researchers, such as Asness, Moskowitz and Pedersen (2013), Drew, Veeraraghavan and Ye (2007), Grinblatt, Titman and Wermers (2016), and Takeuchi and Lee (2013) have found that factor-based inefficiencies in established equities market do exist and can be exploited to generate returns in excess of a market benchmark. Locally, Auret and Sinclair (2006), Muller and Ward (2013), Strugnell et al. (2011), Van Rensburg (2001), and Van Rensburg and Robertson (2003) have found similar inefficiencies present on the JSE. These studies are limited to constructing portfolios based on single factors, or select combination of multiple factors, and consider only the present value of factors in their portfolio rules, as opposed to historical data for the purpose of classifying future stock performance. By applying machine learning classification to the stock selection problem, this study aims to categorically improve the accuracy with which stock performance is predicted when compared to the ‘standard’ factor-based approach. The purpose on this study is therefore to construct portfolios will outperform their factor-based counterpart and highlight the application of machine learning to active portfolio manage in achieving superior returns.

A further purpose of this study is to illustrate the ability of an active portfolio management strategy, based on machine learning, to incorporate multiple factors in a portfolio rule without consideration for specific factor combinations. This study therefore aim to illustrate and explain how superior prediction accuracy and portfolio returns can be achieved by using all available factor data with the machine learning algorithm.

This study is explanatory by nature as it aims to validate a hypothesis and justify the findings. A comparison will be drawn between the percentage monthly returns of portfolios constructed using

a standard factor-based approach and those constructed following the prediction of a machine learning algorithm. A comparison of mean percentage monthly return using a t-test will provide statistical significance to a claim of similar performance.

1.3 Research Questions and Scope

This study aims to answer the following main question:

-
1. *Does the application of a machine learning algorithm (SVM) to a factor-based investment portfolio approach lead to comparable or higher returns?*
-

It has been shown that constructing a portfolio based on stock performance in single or multiple factors can consistently produce a return in excess of a market benchmark. Although this has been demonstrated empirically, both in developed markets and on the JSE, it will be necessary to test this hypothesis and establish a baseline for comparison. This question subsequently aims to compare the active portfolio management strategy of factor-based investing to that of an improved active portfolio management strategy which incorporates machine learning.

A portfolio's performance is measured by its monthly percentage return. In response to the research question, the monthly percentage returns of a portfolio constructed using the predictive outputs of a machine learning based investment strategy, μ^{MLI} , are compared to the monthly percentage returns of a portfolio constructed using a factor-based investment strategy, μ^{FBI} . This study therefore puts forth the null hypothesis that the difference between μ^{MLI} and μ^{FBI} is not statistically significant.

$$H_0: \mu^{MLI} - \mu^{FBI} = 0$$

$$H_i: \mu^{MLI} - \mu^{FBI} \neq 0$$

If it is found, using a t-test, that there is sufficient evidence to reject the null hypothesis, the alternate hypothesis of significant mean difference must be accepted. A further one tailed t-test will conclude whether the machine leaning based investment strategy yields returns greater than its factor based counterpart.

The questions that add towards the main question are as follows:

1. Does factor based investment portfolio construction generate sustained returns in excess of market returns?

The relevance of researching the use of machine learning as a tool for portfolio construction is anchored in the notion that factor-based portfolio construction is an effective approach to active portfolio management. It is therefore first necessary to review potential approaches to factor-based portfolio management and compare the resultant monthly percentage returns to that of a market benchmark. Once a factor-based portfolio approach has been established as a benchmark, further questions towards improvement can be addressed.

2. Does machine learning based investment portfolio construction generate sustained returns in excess of market returns?

In addition to comparing a factor based approach to a market benchmark, it is also necessary to validate the performance of the machine learning based investment strategy to that of the market benchmark. This will provide an independent point of reference for comparing two strategies.

3. Does the application of machine learning algorithms to factor-based investment data increase the accuracy of predicting over- and under-performing stock?

Factor-based portfolio construction approaches work by accurately picking future over- and under-performing stock. The basis of improving this approach therefore lies in improving the accuracy of prediction and subsequently increasing the returns achieved by a portfolio. This question aims to identify whether the machine learning based strategy can be used to improve stock over/under performer prediction and quantify the extent to which the accuracy of prediction can be improved.

1.4 Research Assumptions

- It is assumed that the Efficient Market Hypothesis is somewhat flawed and that stocks do not follow a random time evolution. An inefficient market allows for the notion that stocks are periodically over or under-priced and that knowledge of such price inefficiencies can be exploited to construct a portfolio that will consistently outperform the market.
- Machine learning algorithms can predict future behaviour after being trained on historical data. In this study, the time period considered is from 2001 to 2016. Monthly data from 2001 to 2011 will be used for model training and cross-validation. Out of sample prediction and subsequent portfolio construction occurs 2011 to 2016. It is assumed that the smallest dataset of 120 months (2001 – 2011) is sufficient to achieve “fit” in the machine learning algorithm, thereby allowing it to perform generalised predictions. It is also assumed that the portfolio comparison period of 60 months (2011 – 2016) is sufficient to highlight the effects of a factor based investment strategy and produce results that can be compared to a machine learning based strategy with statistical significance.
- It is assumed that the Support Vector Machine (SVM), identified as an appropriate machine learning algorithm in the literature review, is a sufficiently appropriate machine learning technique for classifying the future performance of stock. The literature review does consider alternative techniques but it is difficult to choose the most appropriate technique without performing an empirical study. Future work is likely to include a comparison of classification algorithms within this test framework.
- It is assumed that the liquidity of traded stocks is sufficient to limit the costs incurred to a large bid-ask spread. This assumption is addressed by limiting the selection of stock to the largest 100 stocks listed on the JSE by market capitalisation, and then filtered to exclude all stocks with less than 15% free float.

1.5 Research Scope and Limitations

This research is confined to application within the South African context, specifically to securities listed on the JSE. Active portfolio management will be applied to a portfolio consisting of the top 100 companies filtered by size, liquidity and availability of historical data. Together, these companies comprise roughly 98% of the total value of nearly 400 companies listed on the JSE.

Monthly stock data from March 2001 to February 2011 will be used exclusively for machine learning algorithm training and cross validation, whilst portfolio comparison commences monthly from March 2011 to February 2016.

Additional limitations stem from the application of machine learning algorithms in stock prediction. There are a multitude of algorithms that could be tasked with stock classification. The literature, as discussed further in the review, does not offer a single best-case solution. This study, however, is limited to the use of one algorithm and will not involve any empirical comparison. Future recommended work could include further studies about a more optimally predictive algorithm given the tested set of factors.

Furthermore, the literature highlights a large variety of factors that can be used as inputs to both a factor based and machine learning based investment strategy. It is impractical, within the scope of this research, to attempt to compare the performance of investment strategies using all available factors. Those factors that have been identified as the most prevalent in literature are used for portfolio construction, the results of which act as a baseline for future research to test the applicability of this research to a wider range of input variables.

1.6 Research Ethics

The ethical concerns in this study surround the validity of data used as inputs for selecting stocks; the unbiased statistical comparison of results; and the unbiased interpretation of said results. An ethics form, requesting ethical clearance for this research, has been completed as required by the University of Cape Town, and has subsequently been approved.

Adams, Khan, Raeside and White (2007) state that it is the researcher's ethical responsibility to do the work honestly and with integrity. They highlight three prominent ethical concerns prominent in quantitative research that this study aims to uphold:

- “Falsifying results—to make them fit your conclusion” (p. 35).
- “Trimming—removing data that does not fit in with your analysis” (p. 35).
- “Biased or inappropriate analysis” (p. 35).

All sources used when gathering the secondary data have been credited. All data identified for use in this research relates to public listed companies and is therefore in the public domain. Data has been obtained from a reputable source, INET BFA, and as such is considered correct. Where data was not available, measure where taken to either exclude the data from the analysis or infer values that would enable the research to be conducted without significantly affecting the results. Furthermore, all literature has been sufficiently referenced and credited to the original authors.

2 Literature Review

The literature review provides context to the research questions and justifies the significance of this study by highlighting the areas of research that require further exploration. Section 2.1 provides context for portfolio management and build the argument for active portfolio management from a historical perspective. Section 2.2 explores the field of active portfolio management and its conflict with the Efficient Market Hypothesis. The evolution of factor-based investing, as a form of active portfolio management, is discussed with the goal of identifying the most effective approach to portfolio construction with the highest likelihood of outperforming the market. Section 2.3 considers the implementation of machine learning in finance with a focus on portfolio construction. Emphasis is placed on the identification of machine learning algorithms relevant to improving the factor-based portfolio construction approach.

2.1 Portfolio Theory, CAPM and Passive Investment

Arguably one of the most significant theories in modern finance is that of Modern Portfolio Theory (MPT) put forth by Harry Markowitz in his paper *Portfolio Selection* published in 1952. Markowitz (1952) states that selecting a portfolio can be divided into two stages: “The first stage starts with observations and experience and ends with beliefs about the future performance of available securities. The second stage starts with the relevant beliefs about future performances and ends with the choice of portfolio” (Markowitz, 1952, p. 77). Markowitz (1952) therefore clearly distinguishes between a prediction aspect and a subsequent construction aspect of portfolio management. He further claims that investors are risk averse in constructing portfolios and aim to maximise expected return whilst minimizing the variance of that return. A crucial contribution by Markowitz (1952) occurs in his calculation of portfolio risk, stating that it is not only a function of individual stock variances but also of their covariances. An efficient portfolio is therefore one that, for a given expected return, minimises portfolio risk through the appropriate selection of risky assets. A sufficiently diversified portfolio eliminates unsystematic risk and exposes the portfolio only to systematic risk, risk inherent in the market. Markowitz (1952) states, “A rule of behaviour that does not imply diversification must be rejected as a hypothesis” (p. 77). This laid the foundation for the development of the Capital Asset Pricing Model (CAPM) by William Sharpe (1964) and John Lintner (1965) and the birth of asset pricing theory (Fama & French, 2004).

CAPM offers a model for measuring risk and the relation between expected return and risk. In Sharpe's model, expected return of a portfolio is a function of the combined risk of all assets held within that portfolio and is maximised for a given risk profile when assets are sufficiently diverse. The *minimum variance frontier* tells the story of CAPM – if we ignore risk free investments – and indicates the optimal weighted combination of equity assets to maximise returns for a given risk profile. When including the option of risk free assets, Sharp (1964) identifies a linear relationship between risk and return termed the “market line”. The essence of CAPM, as noted by Fama and French (2004), is that all investors see that same opportunity and will combine the same set of risky assets to construct the most efficient portfolio. Sharp (1964) makes the point that if all investors hold the same portfolio of risky assets, it must be the value-weight market portfolio of risky assets. This forms the basis for passive investment strategies.

A passive investment strategy is one where assets in a portfolio are chosen to match the portfolio beta (β) of a given market, that is to say, the portfolio return and market return have unity correlation. Portfolio return is therefore directly linked to market return. In a theoretical environment, an optimal passive portfolio would be constructed using all stocks present within a market, weighted in real-time to their market capitalisation. Transaction costs, however, make this form of portfolio impractical. Managers of passive portfolios choose subsets of stocks that represent a particular market and choose a frequency with which to rebalance the portfolio based on individual stock market-capitalisation. The use of a passive investment strategy is aligned with the view of the Efficient Market Hypothesis (EMH), introduced by E.F Fama in a 1965 paper, where it is stated that competition in the stock market will cause the full effects of information to be instantaneously reflected in its price. Therefore, no form of analysis, be it technical (which is the study of past stock prices to predict future stock prices), or fundamental (which is the analysis of financial information to determine whether a stock is under- or overvalued), would enable an investor to achieve return greater than that obtained by holding a randomly selected portfolio of risk comparable stock (Malkiel, 2003).

Despite the popularity and ubiquity of the Efficient Market Hypothesis and the Capital Asset Pricing Model, numerous studies provide evidence against market efficiency and advocate alternate factor-based approaches to pricing assets. This research falls within the realm of active portfolio management.

2.2 Active Portfolio Management and Factor Based Investing

Active fund managers, by definition, shift risk exposure away from market beta by altering the construction of the fund either through alternate stock weighting, or alternate fund composition (Silva et al., 2011). Where Modern Portfolio Theory, the Capital Asset Pricing Model and the Efficient Market Hypothesis advocate the use of stock mean return and risk (volatility) as the basis for portfolio construction, active portfolio management includes the consideration of other fundamental and technical factors in pricing assets. Such factors can be used in a portfolio strategy on a single-factor basis, using one factor as the foundation for portfolio construction, or on a multi-factor basis, combining numerous factors into a portfolio construction rule, to maximise risk adjusted return.

Basu (1977) performed an empirical study to determine whether the investment performance of a common stock is related to its price-to-earning (P/E) ratio, which is an indicator of firm value. The study states that proponents of the price-ratio hypothesis believe it to be an indicator of future investment performance, and that low P/E securities will tend to outperform high P/E securities. The study found that the P/E ratio of a stock was not fully reflected in the price thereby alluding to the possible mispricing of stocks and providing supporting evidence in contradiction to the EMH and CAPM.

Reinganum (1981) presented similar findings, pointing out that portfolios based on the P/E ratio, as well as those based on firm size, experience systematically different returns than those predicted by CAPM. Banz (1981) found that the 'size effect' has been in existence for at least forty years and presents this as evidence to the notion that CAPM is misspecified. Keim (1983) found a similar size-based mispricing, also noting that stock price movement experiences seasonality, with size effects being more prominent at specific times within the financial year.

This early research was followed by Fama and French (1993, 1996), who published several papers challenging the underlying assumption on which CAPM is based. They state that a cross-section of average returns on U.S. common stocks shows little relation to beta (β), concluding that asset risk, in relation to the market, is not enough to account for the return on assets. They propose a three-factor model for determining the expected return on assets which includes the market factor, proposed by CAPM, as well as both a firm size factor and book-to-market ratio, where book-to-

market is an indicator of firm value, similar to the P/E ratio. A further influential study by Jegadeesh and Titman (1993) also contradict CAPM but base their research on the notion that stock prices either overreact or underreact to information. Using past stock performance, a strategy akin to momentum investing, Jegadeesh and Titman found that ranking stocks based on a 12-month average return, paired with a 3 month holding period, yielded the most significant return. Importantly, Fama and French (1996) noted that their model does not account for the short term returns shown to be present in the findings of Jegadeesh and Titman (1993). Following this, Carhart (1997) presented a model that included the market factor, proposed by CAPM, as well as firm size, value and momentum factors. Having laid the foundations for factor-based investing, numerous studies have since been undertaken to both support and refute its efficacy.

In a report on the fundamentals of factor based investing, Bender, Braid, Melas and Subramanian (2013) note that a factor can be defined as “any characteristic relating to a group of securities that is important to explaining their return and risk” (p. 2). They note that the most popular factors today are value, growth, size and momentum, but that more recently, volatility, yield and quality factors have become increasingly accepted in literature. Empirical studies illustrating the success of factor-based portfolio construction extends to South Africa and the JSE. Van Rensburg (2001) conducted an extensive single-factor study evaluating measures of value, earnings growth, bankruptcy improbability, momentum and neglect. The study found that earnings yield as a factor measure of value; market capitalisation as a factor measure of quality; and, 12 month past positive return as a factor measure of momentum, formed a somewhat accounted for factor based risk on the JSE. This conclusion is followed by a further study by Van Rensburg and Robertson (2003), supported by Strugnell, Gilbert and Kruger (2011), providing empirical support that the size effect and value effect are relevant factors for explaining cross-sectional returns on the JSE, and that market beta (β) has no predictive power on the JSE, a notion that invalidates CAPM.

Finally, a combination of factors in a multi factor-based portfolio construction approach has been shown to improve the efficacy of active portfolio management. Van der Hart, Slagter and van Dijk (2003) compared the performance of value and momentum strategies, and included strategies using short term and long term mean reversion, earning revision by analysis, size and liquidity. Importantly, they considered multivariate strategies that rank stocks according to indicators of value, momentum and earnings revisions jointly, and found that this improves overall portfolio

performance. Miller and Ward (2013) similarly found that portfolios formed using a combination of factors consisting of 12-month momentum, return on capital, cash flow to price ratio and earnings yield outperform all other factors tested.

Critical to this study is the apparent improvement in portfolio returns that can be realised by incorporating multiple factors into the portfolio construction rule. This can, however, be limited by the vast choice of factors available for use, and the ability to effectively assimilate multiple factors simultaneously in to a single portfolio rule. There is no general rule for the combined use of multiple factors, other than the CAPM extensions to the 3 factor and 4 factor rules introduced by Fama and French (1996) and Carhart (1997) respectively. This is evident in studies such as Asness (1997), Asness et al. (2013) and Tajbani (2015) where factors are combined using somewhat arbitrary weightings to combine factors and establish a single portfolio rule.

This problem of incorporating multiple inputs to evaluate a single outcome is one uniquely suited to machine learning, which presents itself as a solution to the problem of using multiple factors for portfolio construction, without losing performance as a result of ‘bad’ indicators.

2.3 Machine learning and Portfolio Construction

An early definition of machine learning is given by Arthur Samuel in 1959 as a “field of study that gives computers the ability to learn without being explicitly programmed” (Munoz, 2016, p. 1). It is a discipline that has historically been strongly associated with cognitive science and engineering. Recently, machine learning has become popular in finance due to its inherent ability to predict select outcomes given large sets of complex correlated information. Popular fields of research for machine learning include time series forecasting, classification and pattern recognition (Kaastra & Boyd, 1996). The practical implementation of machine learning in finance is widespread and includes, but is not limited to, economic prediction, risk rating of exchange-traded fixed-income investment, portfolio selection and diversification and index construction (Ahmed et al., 2010; Chiam et al., 2009; Coakley & Brown, 2000; Lam, 2004; Patel et al., 2015; Teixeira & De Oliveira, 2010; Trippi & DeSieno, 1992).

The research question in this study is one of portfolio selection and construction. Building on the fundamentals of factor-based portfolio management, stocks would be classified as either over- or under-performing based on single or multi-factor inputs. Egbo, Onyeagu, Ekezie and Uzoma

(2014) state that “binary classification is the task of classifying elements of a given set into two groups on the basis of a classification rule” (p. 124). Common methods used for binary classification are decision trees, random forest, Bayesian networks, support vector machines, neural networks and logistic regression. The use cases for each of these methods, of which there are many variants, are highly dependent on the type of probabilistic observations and the criteria with which the performance is measured (Peng, Wang, Kou, & Shi, 2011).

When considering the literature on machine learning as it applies to factor-based portfolio management, we must consider both historical evidence of machine learning in portfolio management as well as empirical comparisons of machine learning algorithms in classification problems. Arik, Eryilmaz and Goldberg (2013) use SVMs to classify stocks as either bullish or bearish: Bull stocks outperform the market and bear stocks underperform. They use 52 fundamental indicators obtained from yearly financial statements and stock performance 3 months after the financial statements have been made publicly available to train the algorithm. They limit the number of stocks considered by eliminating those with more than 5% missing data in any one factor, and also companies that are no longer traded following bankruptcy. The research initially considers supervised classification using Decision Trees, k-Nearest Neighbours, Naive Bayes and SVM techniques. Results suggest that SVM outperforms the other three classification techniques with more than 3% prediction accuracy, providing more robustness to the changes in the training data set, and is more computationally efficient for large data sets. The investment portfolio is constructed by equally weighting all stocks predicted to be bullish. It was found that the average return of the resultant portfolio outperformed the market average by 3%.

Caruana and Niculescu-Mizil (2006) performed an empirical comparison of classification algorithms using data from eleven standard binary classification problems. In addition to Support Vector Machines they found that Boosted Trees and Bagged Trees (complex variations of Decision Trees), Random Forests and Artificial Neural Networks performed best, whereas on average, Single Decision Trees (a simple variation of Decision Trees), Logistic Regression, and Naïve Bayes Classifiers were not competitive. They do however note that this generalisation does not always hold, and that uncompetitive algorithms achieve top performance under specific circumstances.

Imandoust and Bolandraftar (2014) provide a practical implementation of machine learning when comparing the performance of Decision Trees, Random Forests and Naïve Bayesian Classifiers to predict the movement of the Tehran Stock Exchange. Stock direction prediction is a typical classification problem where stocks are predicted to move either up or down in the predicted time frame. The study differs from those predicting individual stock movements as it attempts to estimate the movement of an entire market. As such, the input data used for training and prediction is market specific and not stock specific. Ten microeconomic variables are used for prediction in the proposed models. Experimental results show that the Decision Tree outperforms the Random Forrest, followed by the Naive Bayesian Classifier, with up to 80% direction prediction accuracy.

Patel, Shah, Thakkar and Kotecha (2015) attempt to predict the direction of stock price movement using Artificial Neural Networks, SVM, Random Forrest and Naïve Bayes Classifiers. Their results suggest that Random Forrest outperform the other prediction models, with Naïve Bayes Classification yielding lowest accuracy. They further note that the performance of all algorithms improves significantly when trained using trend-deterministic data, in which case, the Naïve Bayes Classifier performed best.

Binary classification is a popular field of study when predicting credit and financial risk. Peng et al. (2011) conducted a study to determine the most effective classification algorithms for the early detection of financial risk and found that Logistic Regression and Naïve Bayes Classifiers outperformed SVMs and Repeated Incremental Pruning to Produce Error Reduction (a form of Decision Tree).

From the literature, it is clear that Artificial Neural Networks, Random Forests, SVMs, as well as complex variants of Decision Trees, generally perform well as binary classification methods, but that underperforming methods such as Naïve Bayes Classifiers and Logistic Regression do perform well under select conditions. It is therefore pertinent to select an algorithm with proven performance in a scenario similar to that being tested in this research.

The SVM algorithm has been used in numerous studies (Fan & Palaniswami, 2001; Huerta et al., 2013; Kim, 2003; Udomsak, 2015) and is noted for its simplicity and its effectiveness. Importantly, the SVM algorithm is not based on the assumption that input variables are uncorrelated. This assumption is a prerequisite for competing algorithms such as Naïve Bayes Classification and limiting to a study that is attempting to use multiple factors, likely somewhat correlated, as

inputs. Furthermore, the solutions of SVMs are unique, optimal and absent from local minima (Tay & Cao, 2001, as cited by Fethi & Pasiouras, 2010). SVMs have most notably been used in the prediction of index returns and index direction (Kim, 2003; Kara, Acar Boyacioglu & Baykan, 2011; Sewell, 2012; Udomsak, 2015; Imandoust & Bolandraftar, 2014). However, recent research by Huerta, Corbacho and Elkan (2013), who closely followed the work of Fan and Palaniswami (2001) and Ruie-Shan (2008, as cited by Huerta et al., 2013), shows significant return can be realised when using SVM classification to construct portfolios from stocks that are predicted to yield abnormal returns in excess of the market. This validated the use of the SVM algorithm to construct portfolios in the US, Australian and Taiwanese markets respectively. No such research, however, has been found in the context of the South African market nor has there been a direct comparison between machine learning (SVM) based portfolios and those based on standard factor based investing, as presented by Muller and Ward (2013); Strugnell et al. 2011; Van Rensburg (2001); and Van Rensburg and Robertson (2003).

2.4 Conclusion

The literature reveals the ubiquity of concepts such as the Efficient Market Hypothesis and the Capital Asset Pricing Model that form the base rationalisation for a passive investment strategy. Numerous studies, however, have presented empirical evidence that active portfolio management, through the exploitation of mispriced stocks, can result in sustained returns in excess of the market. Furthermore, these studies have shown that multiple factors contribute to the identification of mispriced stocks and should be combined in analysis to yield the best results. The combination of factors, however, is not trivial and it is difficult to construct portfolios that combine all available factors in the portfolio rule.

Machine learning is a field of study that shows promise in its ability to combine numerous sources of complex related data to predict select outcomes. The literature shows that a multitude of algorithms exist and have been used for the purpose of financial prediction and classification. Artificial Neural Networks, Random Forests and Support Vector Machines have empirically been found to perform best in financial classification problems and have shown the ability to construct portfolios that outperform the market using select factors. Classification using Support Vector Machines have been noted as the easiest to implement with the greatest chance of success, given a wide range of inputs, regardless of correlation.

Similar to factor-based portfolio management research, most portfolio construction research based on machine learning, attempts to outperform the returns of a select market using select factors such as momentum or values. Few studies however, consider the relative performance of both active portfolio management approaches using all available factor data. This study therefore considers the relative performance of active portfolio management approaches, comparing standard factor-based portfolio management against one that applies SVM in performance prediction within the context of the JSE, and considers the most noteworthy factors in relevant literature, namely value, momentum and quality.

3 Research Methodology - Design

Saunders, Lewis and Thornhill (2008) provide a framework for understanding research methodologies, dubbed the research ‘onion’, shown in Figure 1. The onion consists of many layers, but can fundamentally be divided into two stages noted as research design and research tactics. Research design is concerned with the overall plan for your research and considers the choice of research philosophy, approach, strategy, methodology choice and time horizon. Research tactics is concerned with quantitative and qualitative data collection techniques and subsequent analysis procedures specific to this study. The research onion shares structural similarities with the three-question framework presented by Creswell (2003), which addresses the *elements of inquiry*, *approach to research* and the *design process of research*.

Due to the distinct separation between design and implementation, the research methodology will be spread over two section. This section, Section 3, will consider each of aspects of research design, whilst the following section, Section 4, will address research techniques and procedures, an in-depth review of how data is collected and results obtained and analysed.

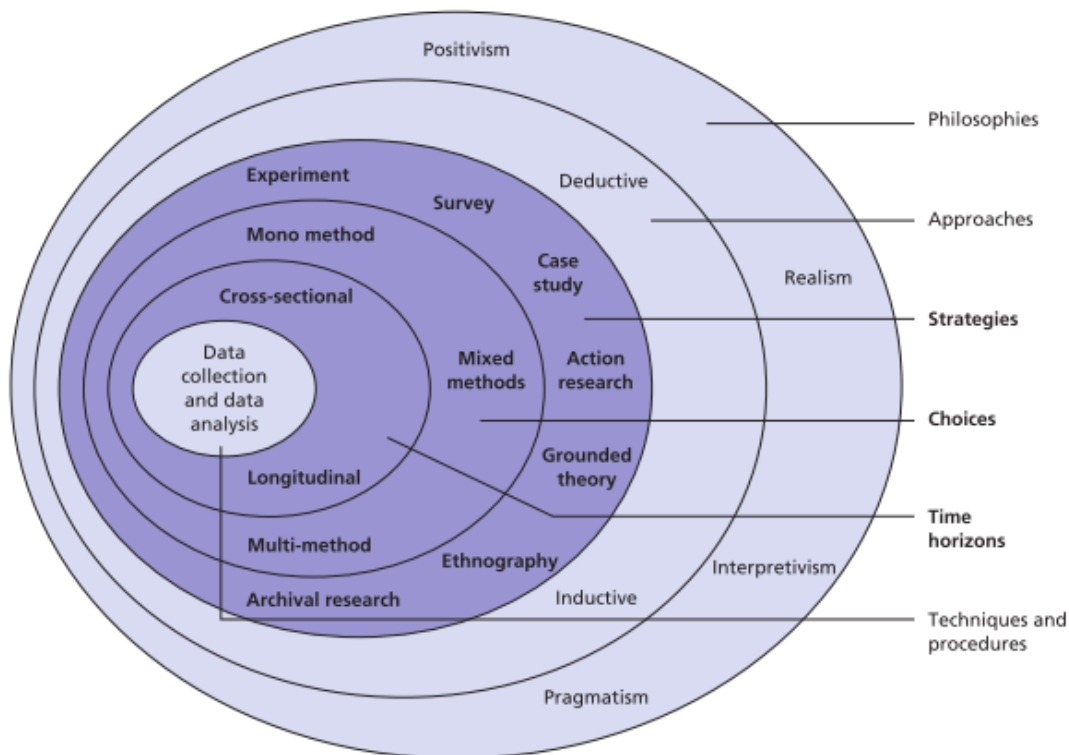


Figure 1: The research ‘onion’ (Saunders et al., 2009)

3.1 Research Philosophy

Research philosophy is an over-arching term that relates to the development of knowledge and the nature of that knowledge and can be broadly categorised as either positivist, realist, interpretivist or pragmatist (Saunders et al., 2009). Remenyi (1998, cited by Saunders et al., 2009) equates positivism to working with an observable reality such that the research culminates with law-like generalisations similar to those produced by physical and natural scientists. They state that only observable phenomena lead to the production of credible data, and that there is little to be done to alter that data. Typically, existing theories are used to develop hypotheses, followed by the gathering of facts, which leads to hypothesis testing. Creswell (2003) share a similar view noting that empirical observation and measurement as well as theory verification are critical elements of positivism. In contrast, interpretivism argues that insight is lost if complexity is reduced to law like generalisations. This philosophy requires researchers to adopt an empathic stance, as the research often involves people as opposed to objects (Saunders et al., 2009). Creswell (2003) does not explicitly note interpretivism, but presents an overlapping philosophy, constructivism, which highlights the aspects of understanding, participant meaning and theory construction as dominant elements of the philosophical position. Realism, direct or critical, is a philosophical position that relates to scientific inquiry, and argues that objects have existence regardless of human observation. They note that it is through observations that we gather knowledge, which is influenced by social conditioning and cannot be understood independently of knowledge actors in the knowledge derivation process (Dobson, 2002, as cited by Saunders et. al., 2009). Finally, pragmatism argues that variations of epistemology, ontology and axiology are possible, given an unambiguous research question.

This study aligns itself strongest with that of the positivist philosophy because it aims to validate a generalised claim, informed by existing theories, based on objectively measured data. To reiterate the research question, this study claims that a factor-base portfolio investment strategy can be universally improved by applying the SVM machine learning algorithm to a similar set of input factors. At the root of this claim is the notion that the SVM algorithms can, under all circumstances given the same set of data, more accurately predict future stock performance than a standard factor-based approach.

3.2 Research Approach

Saunders et al. (2009) discuss two approaches to research, deduction and induction, noting that deduction aligns itself more with positivism, and induction with interpretivism. Deduction, in line with positivism, has its roots in scientific research and starts with the formulation of a hypothesis based on theory. It proceeds to testing the hypothesis and examining the outcomes of the inquiry and ends with a confirmation of the hypothesis, or a need for modification. Induction, with roots in interpretivism, starts with the intention of understanding a problem and the collection of data, followed by a formulation of theory. When applying induction, theory is followed by data, rather than the inverse for deduction.

The study follows a deductive approach, typical of quantitative research. Deductive research follows five sequential stages (Robson, 2002, cited by Saunders et al., 2008) as show in **Table 1**.

Table 1: The five sequential stages of deductive research

	Stage	Study specific approach
1	Deducting a hypothesis from theory.	The application of the SVM machine learning algorithm to factor-based portfolio construction can improve returns.
2	Expressing the hypothesis in operational terms.	Is there a difference in the percentage monthly return of a multi factor-based portfolio and a portfolio constructed using the predictive outcome of the SVM algorithm using the same inputs?
3	Testing the operational hypothesis.	The monthly percentage returns of each portfolio will be compared using a t-test to evaluate the statistical significance of similarity.
4	Examining the outcome of the inquiry.	If there is a significant difference in the mean monthly percentage return, i.e. 95% confidence, the null hypothesis of similarity is rejected.

5	If necessary, modify the theory based on the results.	Are the results aligned with what is found in literature?
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3.3 Research Purpose and Strategy

Strategically, the study is considered explanatory in that it aims to establish a causal relationship. As noted by Saunders et al. (2009) the emphasis of explanatory research is to explain relationships between variables, using the experimentation strategy for proving causal links; whether changes in independent variables result in changes in dependent variables. In this study, the hypothesis invites the application of a new technique (machine learning) to an existing approach (factor-based portfolio construction) with the goal of comparing the portfolio return. There is therefore a causal relationship between the application of the algorithm and a change in the dependent variable, measured by the monthly percentage return of a portfolio. A change in the results will ultimately elicit an explanation as to the cause.

3.4 Research Method

Quantitative analysis is the overarching term used for any data collection technique or data analysis procedure that uses numerical data, whereas qualitative analysis generates non-numerical data (Saunders et al., 2009). When using one method in isolation, data analysis is deemed to be mono-method, whilst using a combination of more than one method of collection of analysis is referred to as multi-method analysis. In a multi-method analysis, when both quantitative and qualitative data collection and analysis methods are used it is referred to as a mixed method analysis.

The research follows a single, purely quantitative method of collecting and analysing data due to the fact that all data is secondary and numerical by nature. Subsequently, there is no risk of bias in the interpretation of raw data, and the application of data in the construction of a factor-based portfolio has been prescribed by prior research. Furthermore, considering the application of machine learning, the SVM algorithm is deterministic as well as data-source agnostic. In other words, it will predictably and repeatedly produce a set of results given a set of inputs, and it is not affected by the source or the nature of the data. For these reasons the study can be considered inherently objective.

3.5 Research Time Horizon

Finally, considering the time horizon of the study, it is imperative in the research design to consider whether the research relates to a single instance ('snapshot') in time or a series of instances ('diary') representing events over a given period. Research considering a single instance in time is cross-sectional, whereas research considering events over a given period is longitudinal, (Saunders et al., 2009). Adams, Khan, Raeside and White (2007) define a longitudinal study as one that "covers a long period of time, at times several decades, and follows the sample a repeated number of times" (p. 27)

For this study, a cross-sectional approach would not be appropriate due to the nature of the data collected and the time span over which portfolios are constructed and compared. Ten years of historical monthly factor indicators are used to train the SVM predictive models and allow for the future performance of a stock to be evaluated. Training is followed by 5 years of stock performance prediction, portfolio construction and performance comparisons. This type of time series forecasting and comparison aligns itself strongly with the definition of a longitudinal study by Adams et. al. (2007).

4 Research Methodology – Techniques and Procedures

4.1 Research Criteria

Reliable research requires that the same information be produced regardless of the individual or organisation conducting it, or the time at which research is conducted (Roberts & Priest, 2006). Similarly, Adams, Khan, Raeside and White (2007) define reliable research as an ability to produce consistent results. Following these definitions, this study can be considered reliable due to the objective way in which data is collected and the consistency of results obtained by executing prediction algorithms. Any individual following the same research methodology would obtain the same results. A further aspect of consideration is the reliability of the source of data. This study uses secondary data as input to the portfolio construction rules and as such will yield different results if this data changes between studies. Stock data is obtained via the INET BFA portal, made available by the UCT Graduate School of Business, which allows for historical stock data, financial ratios and financial statements to be obtained. Strugnell et al. (2011) collect data from the same source, for a similar purpose, and raised concerns regarding the quality of the data. They note that data for currently listed stocks appear to be accurate, though an analysis of the data of delisted stock yielded numerous anomalies and inaccuracies. This study found some missing when calculating financial ratios from financial statements. The ‘gaps’ in time series data appeared to stem specifically from changes in the date of statement publication. Where data was found missing, linear interpolation and extrapolation was employed. There were also discrepancies in the reported ratios compared to ratios calculated directly from financial statements. Where possible, financial statements were used as the basis for ratio calculation. The accuracy of the stock data and financial statements obtained from INET BFA was not verified in relation to any other reporting platform and therefore could contain unidentified inaccuracies, potentially jeopardizing the reliability of this study.

Validity describes the extent to which a measure accurately represents the concept it claims to measure (Punch, 1998, as cited by Roberts & Priest, 2006). Adams et al. (2007) echo this by defining validity as involving “the degree to which you are measuring what you are supposed to, more simply, the accuracy of your measurement” (p. 237). Roberts and Priest (2006) draw a further distinction between external and internal validity, where external validity is stated to “addresses

the ability to apply with confidence the findings of the study to other situations” (p. 43); and internal validity “addresses the reasons for the outcomes of the study, and helps to reduce other, often unanticipated, reasons for these outcomes” (p. 43).

Considering the external validity of this study, it is argued that the application of machine learning algorithms to improve the performance of actively managed portfolios is valid within the context of investment management, and that the measured and compared variable, namely portfolio return, is an accurate and correct reflection of fund performance. There are, however, those that would argue that alternate measure of portfolio performance, such as Jensens’s α or the information ratio, would be a more appropriate. The choice of portfolio performance measure is somewhat arbitrary, but can easily be substituted into this study to yield a new set of conclusions. Furthermore, This study is conducted within the scope of the JSE, which is considered an emerging market. The findings of the study are not necessarily extendable to other emerging markets or more established markets; however, a similar method of study has been applied in established markets such as the U.S by Huerta et al. (2013), and Australia by Fan and Palaniswami (2001). The study is therefore considered externally valid.

When considering internal validity, there are three approaches: content validity, criterion-related validity, and construct validity (Eby 1993, Punch 1998, cited by Roberts and Priest, 2006). (1) “Content validity is concerned with the relevance and representativeness of data” (p. 43). This study aims to improve on an existing portfolio management strategy by using the same secondary data; the data used is therefore considered relevant and representative in obtaining the objective. (2) “Criterion related validity requires that the tool for validation be compared to an alternate validation method” (p. 43). A t-test is used to compare the average monthly stock performance. The Mann-Whitney U test is an alternate test that will be used to confirm the result of the t-test. (3) “Construct validity involves demonstrating relationships between the concepts under study and the construct or theory that is relevant to them” (p. 43). Conceptually, this study theorises that improved stock performance can be realised by including machine learning tools specifically designed for improving the accuracy of classification. The study therefore achieves construct validity by demonstrating the relationship between improved portfolio returns and the use of machine learning algorithms.

4.2 Input Data Consideration

4.2.1 Stock Selection

This study applies machine learning techniques to classify stock performance and construct portfolios within the South African market context, and as such uses stocks listed on the JSE. Other notable studies, using factor based approaches, apply their techniques to a broad range of stocks in order to gain an appropriate cross-sectional view of the market. Note that the sample of securities is limited to equities, and excludes the trade of bonds, preference share and commodities.

Muller and Ward (2013) use 27 years of historical stock price data and financial statements for the top 160 companies listed on the JSE, ranked by a custom metric based on market capitalisation, earning yield and cash-flow to price. Strugnell et al. (2011) conducted their study using 19 years of historical data for all stock listed on the JSE main board. Van Rensburg (2001) limited their data set to the all industrial securities listed on the JSE Main board, using 16 years of historical data.

This study, somewhat subjectively, will use the 100 stocks listed on the JSE main board using 15 years (March 2001 to February 2016) of historical stock data and financial statements. The JSE has 306 tradeable securities listed on the main board, 260 of which have sufficient free float to be used in portfolio construction. Within the 260, the top 100 securities, as ranked by market capitalization, constitute 98% of the total market value and is considered a sufficient representation of the market.

Critical to the implementation of machine learning is the availability of sufficient historical data for training and model validation. Therefore, an additional criterion for the 100 securities selected is availability of data for the 15-year period. This criterion could bias the findings of this study to only be applicable to ‘mature’ companies, and discount the effects introduced by younger companies experiencing more significant growth. Not accounting for ‘young’ companies is a limitation of this study that originates from the nature of machine learning training. It is suggested that future studies attempt to incorporate younger stocks basing their predictions on training from proxy companies, or using alternate input measures for stock prediction.

4.2.2 Survivorship and look-ahead bias

Survivorship bias is a logical error that can occur when a study only considers companies that have survived, and overlooks companies that did not due to lack of visibility. It is pertinent that all relevant and listed companies be considered as part of the investment portfolio at the time it is constructed, regardless of potential future delisting. Note that this ties in closely with the selection of data with sufficient historical data for training, cross validation and out of sample prediction. Prediction and portfolio construction only occurs from 2011 to 2016, therefore only stocks listed during that time are considered. Furthermore, stocks that do not have historical data dating back to 2001 are also omitted due to the training requirement for the machine learning algorithms. This could introduce some form of survivorship, or ‘mature’ stock bias. This limitation is not fully addresses in this study, but is deemed acceptable because of relative comparison between investment strategies. The portfolio performance of factor based investing is compared to machine learning based investing, not to a real-world market benchmark such as the FTSE/JSE Top 100. Both investment strategies will therefore have access to the same set of stock, although slightly biased towards mature companies.

Look ahead bias, also known as multi-period sampling bias, occurs when information used in a study would not have been known at the time of analysis. Studies that make use of back testing to validate the performance of a portfolio must take care not to use data that would not have been available at the time of the trade. The release timing of financial statements is a typical example. Statements for the financial year 2015/2016 ending February 2016 would only be made public several months later following a formal audit. Releasing this information earlier would produce erroneous results. This study avoids look-ahead bias by lagging financial return information by 3 months following the end of a financial semester.

4.2.3 Raw Data

All data is secondary and is collected from third party organisations. Stock data is owned and published by the JSE and made available through portals such as INET BFA under license of the University of Cape Town. The raw data collected, used for both factor and machine learning based portfolio construction is summarised in Table 2.

Table 2: Raw data collected for factor and machine leaning based portfolio construction

Parameter	Symbol	Description
Open	<i>O</i>	Daily stock opening price
Close	<i>C</i>	Daily stock closing price
Volume	<i>V</i>	Daily volume of stock traded
High	<i>H</i>	Highest price of stock traded on a given day
Low	<i>L</i>	Lowest price of stock traded on a given day
Dividend Yield	<i>DY</i>	Dividend expresses as percentage of closing share price
Earning Yield	<i>EY</i>	Earning expresses as percentage of closing share price
Market Capital	<i>MC</i>	Shares in issue multiplied by the closing share price
Book Value	<i>BV</i>	Equity value of organisation
ROE	<i>ROE</i>	Net profit after tax as a percentage of equity
ROIC	<i>ROIC</i>	Return on Invested Capital

These indicators are sufficient to apply the factor-based portfolio construction approach. The frequency with which the data is collected is discussed in the following section.

4.2.4 Sample Frequency

Investment factors are available in a range of time scales. Stock prices and volume data for instance, are available daily, while fundament factors such as those reported in financial statements are published once every year months. The decision was made to only used audited financial statements with result in significant time between changes in information. Future studies could improve the fidelity of this study by increasing the frequency with which the financial statement based factor are measured. Portfolios are rebalanced once a month, and therefore the data used as inputs to the prediction algorithms must be representative of the company’s current financial position. Monthly indicators are obtained directly from the INET BFA portal. Low frequency indicators, such as those recorded in financial statements, are obtained annually and related to a monthly time scale by applying a zero-order-hold, i.e. constant value.

4.3 Generalised Portfolio Construction

This section will introduce the portfolio construction methodology, agnostic to the method used for ranking stocks. This allows for a standardised environment in which both the factor-based investment and machine learning-based investment strategies can be tested and compared.

4.3.1 Stock Ranking and Quintiles

Portfolios are constructed from a pool of stocks, in this case the top 100 stock listed on the JSE filtered by size and availability of historical data. When constructing a portfolio, each stock must be ranked based on a set of rules, called portfolio rules, which dictate the nature of a specific portfolio. As an example, a market capitalisation based portfolio would rank each stock based on its normalised market capitalisation – an indicator of size – relative to other stocks, and weight stocks accordingly. Similarly, an SVM algorithm would rank stocks based on the statistical probability of positive future return, given historical market capitalisation, and weight stocks accordingly.

Regardless of the ranking strategy, each strategy outputs a ranked list of stocks from which a portfolio must be constructed. Once ranked, stocks are subsequently divided into quintiles, Q1 to Q5, based on their rank. This is a common strategy implemented in numerous studies, most notably, Asness (1997), Bollen and Busse (2005), Muller and Ward (2013), Strugnell et al. (2011) and Van Rensburg and Robertson (2003). Given a pool of 100 stocks, each quintile consists of 20 stocks that are assigned equal weight. The monthly returns produced by each quintile can be compared to one another to assess the efficacy of the portfolio ranking strategy. Furthermore, the quintiles between different investment strategies - factor-based investing vs. machine learning-based investing - can be compared to evaluate the ability of an investment strategy ability to outperform its counterpart.

4.3.2 Rebalancing Timeframe

The factors that form the basis for classification change continuously. Therefore, for a portfolio to truly adhere to its classification rule, it must continuously be rebalanced i.e. re-rank stocks and allocate stock quintiles. This, however, is not practical for numerous reasons, the most prevalent of which is the cost involved in buying and selling stocks. These costs include brokerage fees, Securities Transfer Tax (STT), Security Transactions Transfer Totally Electronic (STRATE), Insider Protection Levy and VAT. It is also necessary to consider the additional indirect cost incurred as a result of the bid-ask spread. All costs become more relevant as the frequency of rebalancing increases.

Numerous studies span a wide range of time frames. Momentum strategies tend to have short time horizons ranging from intra-day rebalancing (Kaucic, 2010; Patel et al., 2015), to weekly (Schumaker & Chen, 2008), monthly (M. E. Drew et al., 2007; Takeuchi & Lee, 2013) and quarterly rebalancing (Zhang & Kline, 2007). Studies that include other factors such as value, quality and size tend to have longer prediction and rebalancing periods, such as the classic study by Jegadeesh and Titman (1993), that employs a range of holding strategies from 1 to 12 months. A local JSE example study by Strugnell et al. (2011) also considers returns for buy hold strategies ranging from 1 to 12 months while evaluating the effects of volatility, size and value factors.

The frequency and the nature of the data used for the study is tightly coupled to the timeframe of rebalancing. Studies with short timeframes attempt to predict the future state of single stock or indices and use high frequency data to do so. Data can be gathered from as little as every 15 minutes, and may be used to make high frequency buy-sell decisions. Stock price and volume are the two factor indicators reported at this frequency, along with daily high, low, open and close value, typically used in momentum trading. Longer timeframes require more historical data to be incorporated in order to observe trends. It also allows for sources of data such as earning, cash flows, and other balance sheets items, to be incorporated. However, a longer timeframe potentially brings with it a higher degree of uncertainty.

This study follows an approach similar to Drew and Murtagh (2005), who implemented a one month portfolio rebalancing with a three month holding period. The three-month holding period has been shown empirically to be particularly successful in yielding market premium. However, buying and holding for three months would result in two untraded months. The solution is to split the total amount traded into three equal parts, and stagger the three-month portfolios one month apart, i.e. portfolio 1 at time t , P_t^1 , is rebalanced at $t+3$, P_{t+3}^1 , portfolio 2, P_{t+1}^2 , rebalanced at P_{t+4}^2 and portfolio 3, P_{t+2}^3 , rebalanced at P_{t+5}^3 . This allows for a higher frequency of portfolio construction and more data for training, without sacrificing the buy-hold timeframe. It would be possible to consider weekly rebalancing and stagger 12 portfolios, but this will not be tested in this study.

4.4 Factor Based Ranking and Classification

As noted in Section 2, the literature does not provide consensus on the single best factor combination that would yield maximum portfolio returns. It is therefore necessary to test multiple factors-based portfolios, select those that perform best and compare them to the machine learning-based portfolio. The factor-based portfolios will be constructed using value, momentum and quality factor measures, similar to the study performed by Tajbani (2015) and Muller and Ward (2013).

Value

1. $DY = \text{Dividend Yield} = \frac{\text{Annual Dividends per Share}}{\text{Market Price Per Share}}$
2. $EY = \text{Earnings Yield} = \frac{\text{Earnings per Share (most recent 12 month period)}}{\text{Market Price Per Share}}$
3. $BTM = \text{Book to Market Value} = \frac{\text{Net Asset Value}}{\text{Market Value}}$

Momentum

1. $M6M = \text{6 Month Price Rate of Change} = \frac{\text{Share Price}_t - \text{Share Price}_{t-6}}{\text{Share Price}_t}$
2. $M12M = \text{12 Month Price Rate of Change} = \frac{\text{Share Price}_t - \text{Share Price}_{t-12}}{\text{Share Price}_t}$

Quality

1. $ROE = \text{Return on Equity} = \frac{\text{Net Income}}{\text{Shareholders Equity}}$
2. $ROIC = \text{Return on Invested Capital} = \frac{\text{Net Income} - \text{Dividends}}{\text{Working Capital}}$

Each of the factor measures have been individually used in both factor and machine learning based investment strategies. Comparisons are drawn between the performance of each approach, given a particular factor, followed by conclusions on similarities and differences.

4.4.1 The Support Vector Machine Algorithm

The Support Vector Machine, first introduced by Cortes and Vapnik (1995), is the machine algorithm of choice for classifying future stock performance given single and multiple factors as inputs. It is considered simple to implement, easy-to-use and suitable for a large variety of input parameters. This study follows a similar approach to Huerta, Corbacho and Elkan (2013) in terms of how the model is applied to a classification problem. This section will describe the technical detail of the algorithm and how it is used to classify and rank stocks based on expected future performance.

The SVM algorithm uses a linear model to implement a non-linear class boundary; a boundary that separates positive and negative classifier outcomes, through non-linear mapping of the input vector to a high-dimensional feature space (Huang, Chiang, Chang, Tzeng, & Tseng, 2011). This mapping allows the SVM algorithm to implement a linear model in the new high-dimensional space that represents a non-dimensional boundary in the original space. Within this newly formed high-dimensional feature space, the SVM algorithm calculates the maximum margin hyperplane i.e. an n -dimensional plane that maximises the distance between decision classes. Training datasets closest to the maximum margin hyperplane are called support vectors.

Simplistically, what this means is that each input vector, comprised of factor indicators at a given time, is accompanied by a specific binary outcome: either the stock price went up or it went down. During SVM training, the binary outcome is mapped in n -dimensional space and a boundary is calculated that separates the positive outcomes from the negative outcomes as best as possible. Following training, during SVM prediction when an input vector results in a point on a side of the decision boundary, the prediction algorithm assigns it to that binary classifier. The distance of that data point from the boundary is proportional to the probability of accurate classification. This probability can be used to rank stocks, i.e. the probability of upward stock movement for stock A is higher than stock B, therefore stock A should receive a higher weight in the portfolio. Figure 2 illustrates an simple example with 2-dimensional input vector $[a_i, b_i]^T$, and binary outcome; 1, stock price increase; and 0, stock price decrease. If a new vector produces an outcome above the line, the prediction will be that $y = 1$, i.e. the stock price will increase.

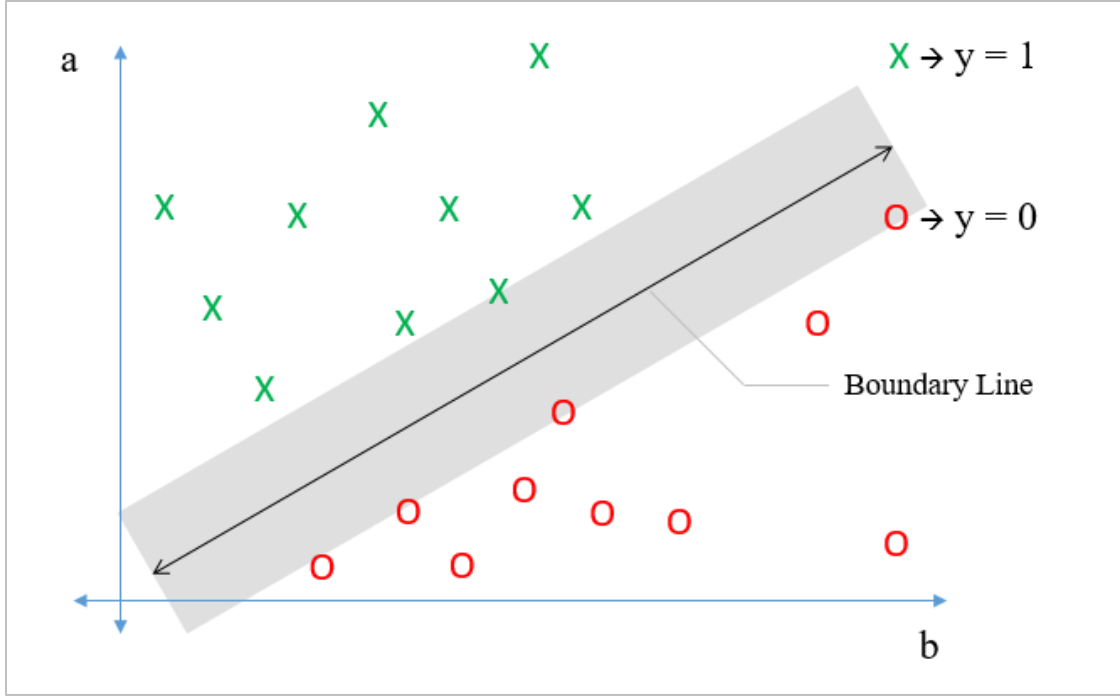


Figure 2: 2 Dimensional SVM feature space

Huerta, Corbacho and Elkan (2013) note that there are two types of Support Vector Machines: linear or non-linear. The example above denotes a linear example, whereas a non-linear example uses a non-linear function to separate the binary classifications. This is useful when the classifications are not clearly separated by a linear function. Huerta et. al. (2013) note that for this reason the linear type requires less data to train and execute but the non-linear type achieves superior performance. The non-linear implementation also requires more careful selection of tuning parameters to avoid overfitting. Erring on the side of simplicity and the requirement of less data, this study implements a linear SVM algorithm

The SVM classification function is as follows:

$$f(x) = \sum_{i=1}^N \alpha_i y_i K(x, x_i) - b \quad (1)$$

where

- x is the feature vector consisting of M components or input variables. In this study, this vector will comprise investment factors. As an example, if SVM is used in conjunction with a momentum investment strategy, 6 and 12-month price rate of change factors will be used in the feature vector.
- x_i is all historical feature vectors of the training set.
- N is the number of training samples used to fit the SVM parameters. In this study, 15 years of data was used for both training and prediction. 10 years were used for training and 5 were held out for prediction and portfolio construction. Therefore, there were 120 data points, assuming monthly predictions, used for prediction and validation.
- α_i is a scalar tuning parameter with a value between 0 and C . The value of C is important because it indicates the level of emphasis given to the accuracy with which the model is fitted to the training data. Increasing the value improves the accuracy of the model but increases the risk of overfitting, diminishing the model's ability to generalise out-of-sample data.
- y_i is very important as it accompanies x_i by indicating the stock classification of winner or loser. It holds the value of +1 or -1. In other words, if measurement x_j results in a positive stock return at time $j+3$ - one quarter ahead - y_i is equal to +1. Similarly, if the return is negative, y_i is -1.
- b is a scalar value obtained by the training process used to shift the output of the output of the SVM function by a constant.
- $K(x, x_i)$ is the kernel function which dictates how data points are to be separated on the hyperplane. In the example of Figure 2, the kernel function is linear, and as can be seen, positive and negative data points are separated by a straight line. The choice of kernel function is central to the functioning of the SVM. Choosing a polynomial kernel function when data is linearly separable, will determine the accuracy of classification. Gaussian kernels, also known as Radial Bias Functions, are effective when similar classifications are clustered and not divisible from other classifications by a single line. Muller and Ward

(2013, as cited by Huerta et al., 2013) note that the properties of the kernel function are central to the functioning of the SVM algorithm, and that the selection of kernel function can greatly affect classification accuracy. The kernel function employed for this study will be a simple linear kernel.

- a. Linear: $K(x, x_i) = x^T x_i$
- b. Polynomial: $K(x, x_i) = (\tau - x^T x_i)^d$
- c. Gaussian (Radial Bias): $K(x, x_i) = e^{\frac{-\|x-x_i\|^2}{\sigma^2}}$
- d. Sigmoid: $K(x, x_i) = \tanh(\kappa_1 x^T x_i - \kappa_2)$

Note the additional tuning parameters, d , σ , κ_1 , κ_2 , for the non-linear kernel functions. These tuning parameters can improve the accuracy of prediction at the cost of introducing significant complexity in model design.

The SVM function is trained on historical data such that $f(x)$ is larger or equal to 1 when x belongs to the class +1, i.e. when the stock indicators at a time correspond a future stock performance above the set benchmark. Similarly, $f(x)$ becomes less or equal to -1 when x belongs to the class 0, i.e. stock indicators correspond to a future stock performance less than a set benchmark. C and γ are meta-parameters that must be carefully chosen to optimise the performance of the linear SVM algorithm.

Monthly stock data as identified in Section 3.2 for the period March 2001 to February 2011 was used as the initial data set for training and cross validation. The measurement vector x consisted all data noted in Section 3.2. The study assumed the start of an actively managed portfolio in March 2011. The algorithm was therefore initially trained and validated on data up to February 2011 and used to classify stock for March 2011 onwards.

4.4.2 Feature and Target Vector

The feature vector is the set of inputs on which the classification of future stock performance is based. When comparing the performance of factor base investment strategy with one that uses machine learning it is necessary to populate the feature vector with the indicators used in the factor

based classification rule. In other words, the feature vectors for the machine learning approach to value (x_v), momentum (x_m) and quality (x_q) investing are:

$$x_i^v = \begin{bmatrix} DY \\ EY \\ BTM \end{bmatrix} \quad x_i^m = \begin{bmatrix} M6M \\ M12M \end{bmatrix} \quad x_i^q = \begin{bmatrix} ROE \\ ROIC \end{bmatrix}$$

During SVM training, the input vector is used in combination with a known target future state y . In this study the percentage change in closing share price, one month in the future is to be used at the target state.

$$y_i = \text{sign} \left(\frac{C_{t+3} - C_t}{C_t} \right) \quad (2)$$

C_t in Equation 2 is the closing stock price on the first trading day of a given month. As with the feature based investment approach, it is necessary to scale the vectors to allow for a more effective incorporation of variables of differing magnitude.

4.4.3 Scaling

Input data scaling before applying SVM is very important. Kim (2003) notes that scaling independently normalises each feature component which ensures that the larger absolute values do not ‘overwhelm’ smaller value feature components. More clearly stated, the main advantage of scaling is to avoid attributes in greater numeric ranges dominating those in smaller numeric ranges.

Three main types of scaling are typical in machine learning preprocessing, namely, rescaling, standardisation and scaling to unit length. Rescaling scales features to within a specified range [0,1] or [-1,1] using minimum and maximum values, standardisation scales the feature vector to have zero mean and unity standard deviation, and scaling to unit length divides each item of the feature vector by the vector’s Euclidean length. It is informally noted that there is no correct choice of scaling method in SVM input data preprocessing (Raschka, 2014). The choice of scaling method is dependent on the preservation or extraction of informational content. This study will employ standardisation scaling, noted in Equation 3, where the input vector is scaled to have zero mean and unity variance.

$$x' = \frac{x - \bar{x}}{\sigma} \quad (3)$$

Note that it is critical that the training and prediction data be similarly scaled. Without similar scaling, the SVM algorithm will have trained on data not representative of future inputs, which will yield erroneous predictions.

4.4.4 Parameter Selection, Cross Validation and Overfitting

The goal of parameter selection is to identify appropriate values for C and γ which will allow the classifier, Support Vector Machine, to accurately predict unknown data after being trained on known data.

Arguably the most important parameter in “tuning” the Support Vector Machine is C . As noted in Equation 1, α_i is a scalar tuning parameter with a value between 0 and C . The value of C is important because it indicates the level of emphasis given to the accuracy with which the model is fitted to the training data. Staelin (2003) notes it as the amount of “slack” that the model allows a classified sample to be on the “wrong” side of the decision boundary. Increasing the C value improves the accuracy of the model but increases the risk of overfitting, diminishing the model’s ability to generalise out-of-sample data. The kernel parameter γ can intuitively be understood as the reach of influence of a single training sample. Low values, indicating far influence, will result in a larger training penalty being incurred if the decision boundary is close to the sample point. A grid search method is employed to select the values of C and γ . The combination of parameters that yield the highest validation accuracy is selected for future training and prediction. The test yielded values of $C = 1$ and $\gamma = 0.1$.

During parameter selection, and the evaluation of prediction performance, it is necessary to consider how the accuracy of SVM predictions are to be measured and whether they are considered to be generalised over an independent data set. This is a process called validation. As noted in Hsu, Chang and Lin (2008) a common strategy for validation is to separate the historical training data set into two parts, where the one set is considered as the known training set, and the second, considered as the unknown validation set. After training, predictions are performed on the

validations set and compared against the true values, yielding a single measure of model prediction accuracy. The limitation to this approach is that there is an inherent conflict in the tradeoff between the amount of data used for training and the amount used for validation. Maximising training data will produce a better predictive model, and maximising testing data will yield improved confidence in the resultant accuracy measure of the model. Furthermore, this method only considers a single validation set, which could prove problematic if the arbitrarily chosen validation set is not representative of a general set. An improved version of this procedure, that solves these limitations, is known as k-fold cross-validation. In k-fold cross-validation, the dataset is divided into k training sets. The validation procedure is then applied k times, each time using $k-1$ subsets for training, and the remaining set applied for validations. This yields k measurement of model prediction accuracy as an indicator of model performance (Ahmed et al., 2010; Cheng & Greiner, 2001).

4.4.5 SVM support software

Much work has been done in the field of machine learning and support vector machines. Libraries such as Sci-Lear-Kit, built in the Python framework, are available for public use. This study uses this library extensively in the implementation of the SVM algorithm. More specifically Sci-Lear-Kit is used in parameter selection, γ and C , input data scaling, model training and k-fold cross validation, prediction, and evaluating predicted probabilities.

4.5 Comparing Portfolio Performance

This study compares the performance of factor based investing to that of machine learning based investing (using the support vector machine). As noted in Section 4.3, each investment strategy ranks stocks based on a select factor, dividend yield, earning yield etc. and divides the ranked stocks into equally weighted quintiles, Q1 to Q5. A successful investment strategy ranks stocks in such a manner that the first quintile, Q1, performs best, and the fifth quintile, Q5, performs worst. The performance of first quintiles between investment strategies are subsequently compared and discussed.

The simplest method for evaluating stock performance is calculating the average return of the portfolio, or quintile, over the period of evaluation. This study rebalances stocks monthly, and therefore uses the average monthly return as a measure of performance, evaluated over 60 months, March 2011 to February 2016.

$$\bar{R}_i = \frac{\sum_{t=1}^n \left(\frac{P_i(t+1) - P_i(t)}{P_i(t)} \right)}{n} \quad 4$$

Where, \bar{R}_i is the average percentage monthly return for a given portfolio i , and $P_i(t)$ is the accumulated value of the portfolio at the given month t . The average monthly return and the standard deviation of individual monthly returns are used in the calculation of the t-statistic, which forms the basis of comparison for evaluating differences of significance in portfolio performance.

An additional method for evaluating the performance of a stock is called the Sharpe ratio, Equation (5), which is a measure of risk-adjusted-return. Simplistically, the Sharpe ratio is the average return in excess of the risk-free rate, per unit of volatility (Vanstone & Finnie, 2007). A portfolio with a greater Sharpe ratio has a greater risk-adjusted return and therefore is considered a better performing portfolio.

$$R_i^s = \frac{R_i - R_f}{\sigma_i} \quad (5)$$

More complex comparisons, as discussed by Ingersoll, Spiegel, Goetzmann, Box and Haven (2016), claim that such measures can be biased and manipulated and advocate the used of more sophisticated algorithms. They claim that the Sharpe ratio suffers from the assumption of variable independence and normality, and that “exact” performance measures can only be calculated in theory, and further propose the use of a manipulation proof performance measure (MPPM).

Alternatively, measures such as the Treynor measure or Jensen’s α , noted by Huerta et al. (2013; Sewell (2012), can also be used to compare portfolio performance. While these can be used, they similarly adjust the return to account for risk. Having considered these alternatives, the Sharpe ratio still is the most widely used measure for portfolio performance and is therefore deemed appropriate for the purposes of this study.

5 Research Findings, Analysis and Discussion

The performance of both factor-based and machine learning-based investment strategies are compared using statistical and financial metrics as discussed in Section 4.5. The financial performance comparison of portfolios that were constructed using single factors are compared in Section 5.1, followed by a comparison of multi-factor portfolios in Section 5.2. Section 5.3 provides an analysis and discussion regarding the prediction accuracy of the factor based and machine learning based investment strategies and relates the findings back to the financial performance of relevant portfolios.

5.1 Single Factor Comparison

In the single factor strategy, a single variable was used to rank stocks and construct a portfolio. Each portfolio strategy, represented by 5 quintiles, intended to highlight the difference in return of highly ranked stocks to those ranked lower. A successful strategy would show quintiles Q_1 and Q_2 yielding higher returns than quintiles Q_4 and Q_5 , as well as higher returns than benchmark portfolio constructed using all stocks equally weighted, EW. This is a similar approach in evaluating factor based investment strategies to that of Muller and Ward (2013). The superscript SVM and FBI indicate the investment strategy employed, i.e. $Q_1^{SVM}(6\text{ Month } ROC)$ is the first quintile portfolio performance of the Support Vector Machine learning strategy as a function of the 6-month price rate of change momentum factor.

5.1.1 Momentum

Price momentum strategies are founded on the principle that a change in price is an indication of future earnings, and subsequently rank stocks based on the magnitude of price change. 6 and 12-month price rate of change, as single factors, are considered in this section as considered by Huerta et al. (2013), Muller and Ward (2013) and Van Rensburg (2001).

5.1.1.1 6 Month Price Rate of Change

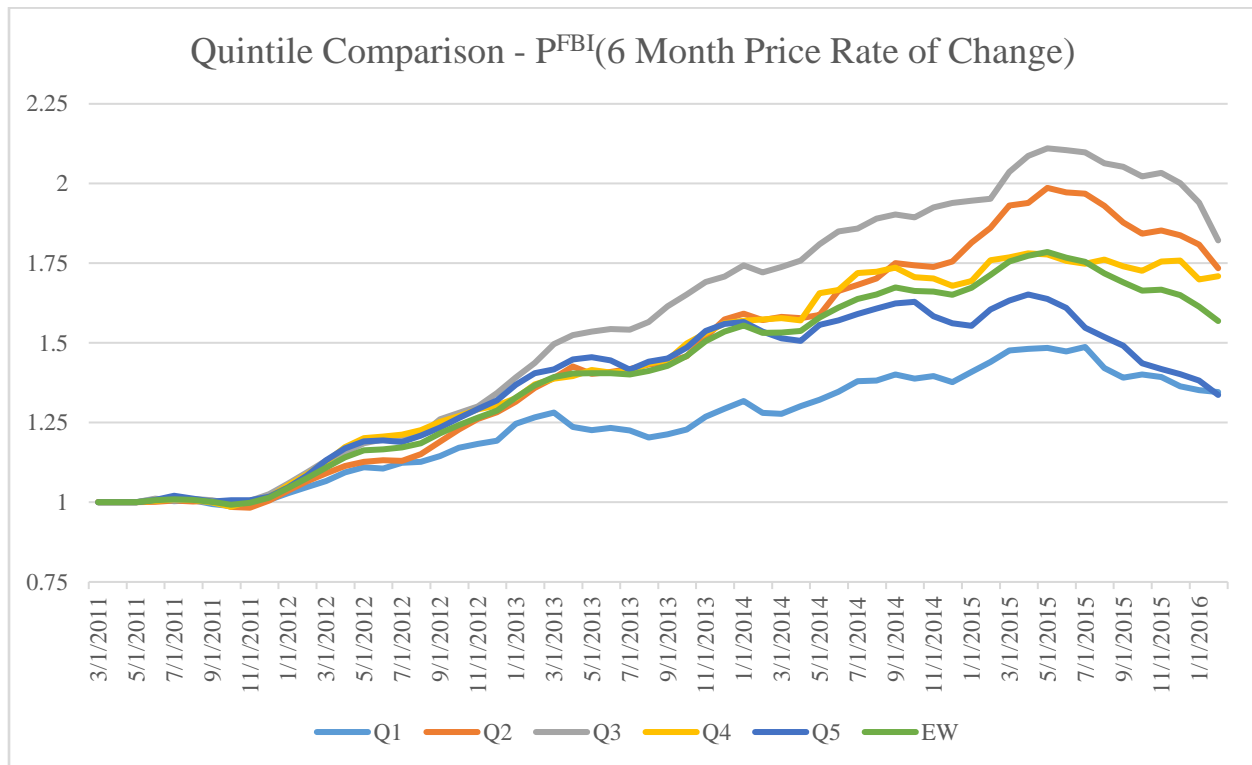


Figure 3: 6 Month ROC Quintile Comparison - Factor Base Investing

Figure 3 shows the growth in total value of each quintile (Q1 to Q5) for a factor-based investment strategy that ranks stocks based on a 6 Month Price Rate of Change. Table 3 notes the average return, standard deviation and Sharpe ratio for each quintile, as well as the benchmark equally weighted portfolio. The final column is the one-tailed test-statistic probability of each quintile, comparing, in pairs, the monthly return of each quintile to the equally weighted portfolio.

Table 3: 6 Month ROC Performance Statistics – Factor Based Investing

	Average Return	Standard Deviation	Sharpe Ratio	P(T<=t) One Sided
EW	0.8%	1.5%	0.19	
Q1	0.5%	1.7%	0.02	9.2%

Q2	1.0%	1.8%	0.26	16.6%
Q3	1.0%	1.9%	0.30	4.8%
Q4	0.9%	1.6%	0.27	30.4%
Q5	0.5%	1.9%	0.01	4.1%

When the average return of each quintile was compared to the return of the equally weighted portfolio, using a one tailed paired t-Test, only Q3 was noted to outperform the benchmark equally weighted portfolio at the 95% confidence level. However, Q2, Q3 and Q4 were noted to have a higher average return and higher Sharp ratio (risk adjusted return). Quintile Q1 was not the top performing quintile, and correct quintile ordering was not maintained, i.e. Q1 performing best and Q5 worst. This lead to the conclusion that a 6-month rate of change is not a sufficient measure of future (next quarter) stock performance when used in a factor-based investment strategy. This contradicts the conclusion drawn by Van Rensburg (2001), which claims that a 6-month momentum yields a 1% monthly premium above a benchmark JSE All share market index. Van Rensburg employed a monthly rebalancing and holding period, which differs from the 3-month holding period employed in this study; this could account for the difference in findings. However, Muller and Ward (2013) and Brown, Yan Du, Rhee, and Zhang (2008) implemented a similar rebalancing and buy-hold strategy to this study and found evidence of sufficient Q1 return and Q1-Q5 return difference to substantiate claims of a 6 month momentum effect. Both studies employ this single factor based strategy over a significantly longer period than employed in this study, which could account for difference in results and conclusions.

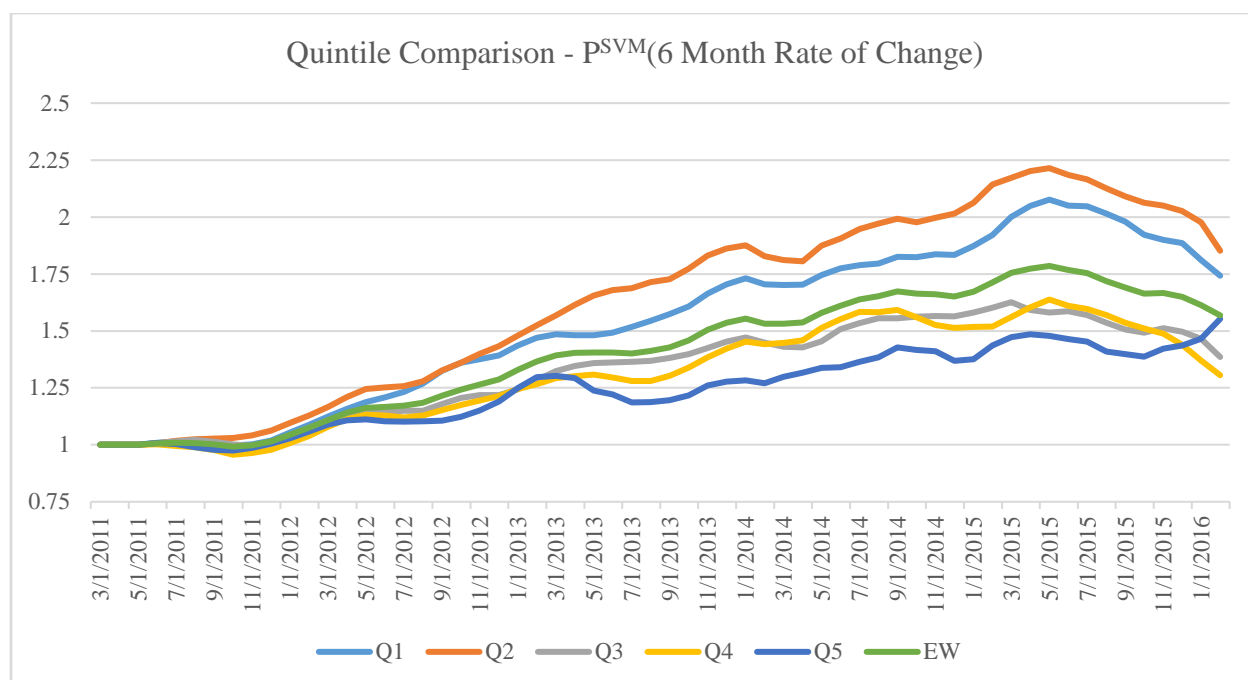


Figure 4: 6 Month ROC Quintile Comparison – Machine Learning (SVM) Based Investing

Figure 4 shows the growth in total value of each quintile (Q1 to Q5) using the machine learning investment strategy, which ranks stocks based on historical 6-month rate of change and the statistical likelihood that the most recent 6-month price rate of change will result in positive future returns. Table 4 notes the average return and Sharpe ratio for each quintile, as well as for a benchmark equally weighted portfolio. The final column is the one-sided test-statistic probability of each quintile, comparing, in pairs, the monthly return of each quintile to the equally weighted portfolio.

Table 4: 6 Month ROC Performance Statistics – Machine Learning (SVM) Based Investing

	Average Return	Standard Deviation	Sharpe Ratio	P(T<=t) One Sided
EW	0.8%	1.5%	0.19	
Q1	1.0%	1.8%	0.26	4.8%
Q2	1.1%	1.9%	0.30	0.9%
Q3	0.6%	1.6%	0.05	4.7%
Q4	0.5%	2.0%	-0.01	0.9%
Q5	0.8%	2.0%	0.14	48.5%

When the average return of each quintile was compared to the return of the equally weighted portfolio using a one-sided paired t-Test, both Q1 and Q2 outperformed the benchmark in average

return and risk adjusted return. Quintile performance was exactly correctly ranked, with Q2 outperforming Q1, which indicated that the SVM algorithm was not entirely successful in ranking stock based on the 6-Month price rate of change. However, given that the top 2 performing quintiles were in fact Q1 and Q2, ranking accuracy was deemed somewhat successful. This lead to the conclusion that an historical 6-month momentum contains sufficient information to allow a machine learning algorithm to predict future price movement. Huerta et al. (2013) does not evaluate the individual performance of using 6 month price rate of change, but find that a combination of momentum factors do yield significant portfolio return when compared to a benchmark, which align itself with the finding of this study.

Table 5 and **Table 6** show the difference in return, and one tailed $P(T \leq t)$ comparing the first and second SVM quintiles, Q_1^{SVM} and Q_2^{SVM} , to the factor-based investing quintiles. Both Q_1^{SVM} and Q_2^{SVM} outperformed Q_1^{FBI} by 0.4% and 0.5% respectively, and performed at least as well as Q_2^{FBI} . This confirms the hypothesis that SVM investment strategy outperform its FBI based counterpart when considering only a 6-month price momentum.

The poor performance of the factor base approach compared to literature is noted. A higher fidelity 6-month price rate of change model for factor base investing could positively influence average return and affect the conclusion of superior SVM portfolio performance.

Table 5: Average paired difference, SVM - FBI (6 Month Price Rate of change)

Mean Difference $Q_{\#}^{SVM} - Q_{\#}^{FBI}$		FBI	
		Q1	Q2
SVM	Q1	0.40%	0.01%
	Q2	0.50%	0.10%

Table 6: Paired difference on-tailed probability, SVM - FBI (6 Month Price Rate of change)

$P(T \leq t)$ One sided $Q_{\#}^{SVM} - Q_{\#}^{FBI}$		FBI	
		Q1	Q2
SVM	Q1	1.1%	47.8%
	Q2	0.8%	24.9%

5.1.1.2 12 Month Price Rate of Change

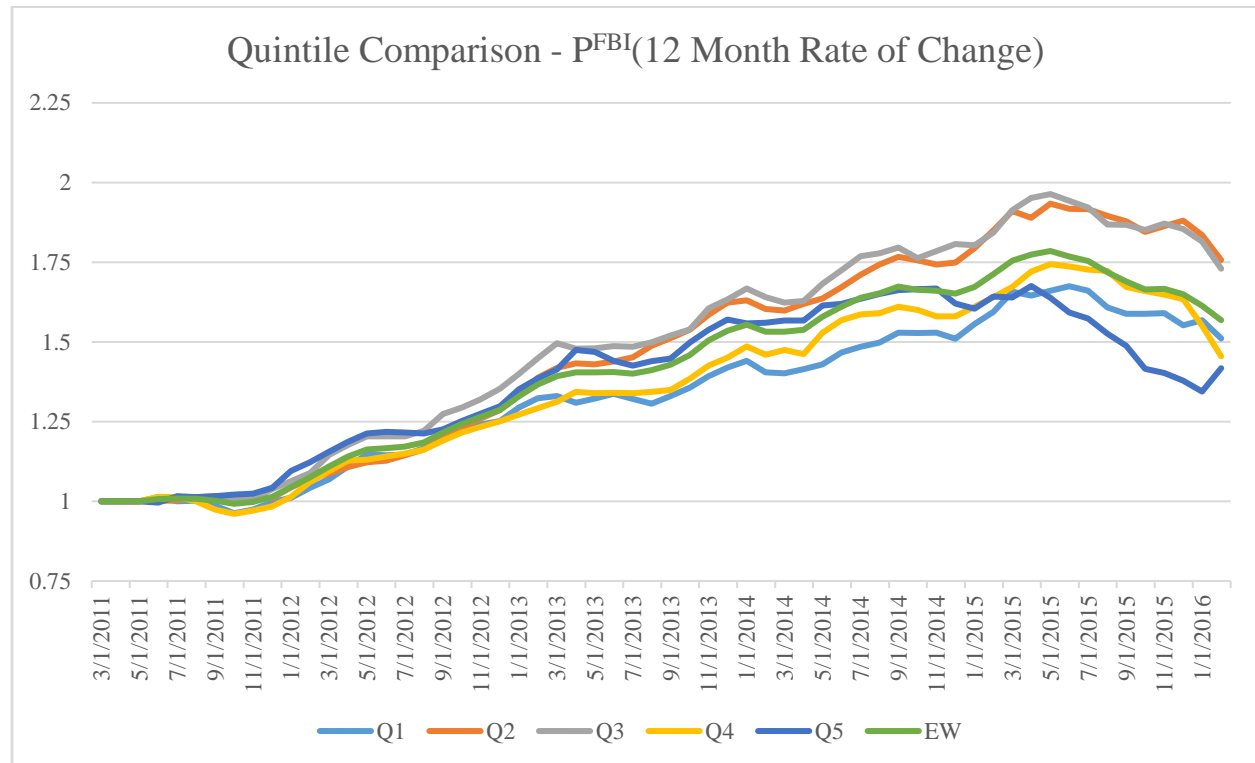


Figure 5: 12 Month ROC Quintile Comparison - Factor Base Investing

Figure 5 shows the growth in total value of each quintile (Q1 to Q5) for a factor-based investment strategy that ranks stocks based on a 12-month price rate of change. Table 7 notes the average return and Sharpe ratio for each quintile as well as for the benchmark equally weighted portfolio.

Table 7: 12 Month ROC Performance Statistics – Factor Based Investing

	Average Return	Standard Deviation	Sharpe Ratio	P(T<=t) One Sided
EW	0.8%	1.5%	0.19	
Q1	0.7%	1.8%	0.13	67.4%
Q2	1.0%	1.6%	0.30	10.6%
Q3	1.0%	1.9%	0.25	14.7%
Q4	0.7%	2.0%	0.09	39.4%
Q5	0.6%	2.1%	0.06	47.1%

When the average return of each quintile was compared to the return of the equally weighted portfolio, using a one-sided paired t-Test, no quintile was noted to outperform the benchmark equally weighted portfolio at the 95% confidence level. Furthermore, clear and correct ordering of quintiles, i.e. Q1 performing best and Q5 worst, did not appear to be prevalent. This led to the

conclusion that a 12-month rate of change, similar to its 6-month counterpart, was not an appropriate strategy for ranking stock as it fails to produce returns significantly higher than an equally weighted portfolio. Again, this conflicted with the findings of Van Rensburg (2001), Hart et al. (2003) and Muller and Ward (2013). The difference in findings could be attributed to differences in rebalancing and buy-hold time-frame, the difference in time over which the portfolios are constructed, or differences in sample companies. Muller and Ward (2013) specifically note 12 month price momentum as a significant indicator of future performance. A failure in both 6 and 12 month momentum to command a price premium highlights a lack of fidelity in the application of factor based momentum strategies. It is suggested that future research attempt to reevaluate factor based momentum to a point where significant return are found, and subsequently compares it machine learning momentum.

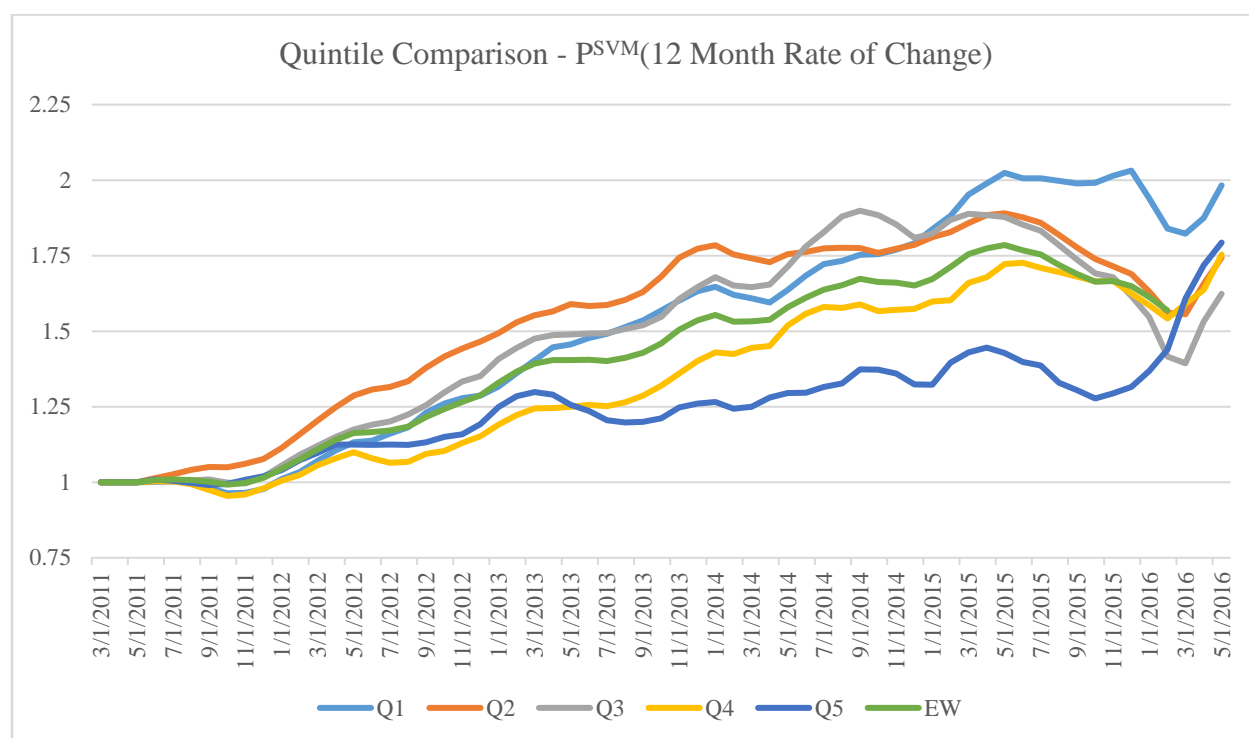


Figure 6: 12 Month ROC Quintile Comparison – Machine Learning (SVM) Based Investing

Figure 5 shows the growth in total value of each quintile (Q1 to Q5) using the machine learning investment strategy, which ranks stocks based on the historical 12-month rate of change and the statistical likelihood that the most recent 12-month rate of change will result in positive future returns. Table 8 notes the average return and Sharpe ratio for each quintile as well as for a benchmark equally weighted portfolio.

Table 8: 12 Month ROC Quintile Performance Statistics – Machine Learning (SVM) Based Investing

	Average Return	Standard Deviation	Sharpe Ratio	P(T<=t) One Sided
EW	0.8%	1.5%	0.19	
Q1	1.1%	1.8%	0.32	2.1%
Q2	0.8%	1.8%	0.16	48.8%
Q3	0.6%	2.3%	0.06	15.6%
Q4	0.8%	1.7%	0.16	41.4%
Q5	0.6%	2.0%	0.08	28.8%

When the average return of each quintile was compared to the return of the equally weighted portfolio, using a one-sided paired t-Test, only Q1 outperformed the benchmark. Though Q1 was correctly ranked as the top performing quintile, the other quintiles did not appear to yield returns in descending order. This indicates that the historical 12-month rate of change does contain some information that allows the SVM algorithm to predict future positive return, but that the SVM algorithm was only somewhat successful in correctly ranking stocks based on historical 12-month rate of change. As noted previously, Muller and Ward (2013) find that a multi-factor momentum approach does yield significant return, which supports these findings.

Table 9 and Table 10 show the difference in return, and one tailed $P(T \leq t)$ comparing the first and second SVM quintiles, Q_1^{SVM} and Q_2^{SVM} , to the factor-based investing quintiles. Q_1^{SVM} outperformed Q_1^{FBI} , and yielded comparable returns to Q_2^{FBI} . Q_2^{SVM} performed comparably to both Q_1^{FBI} and Q_2^{FBI} . This confirms the hypothesis that SVM performs at least as well if not better than its FBI based counterpart, when considering only a 12-month price momentum.

Table 9: Average paired difference, SVM - FBI (12 Month Price Rate of change)

Mean Difference $Q_{\#}^{SVM} - Q_{\#}^{FBI}$		FBI	
		Q1	Q2
SVM	Q1	0.31%	0.07%
	Q2	0.05%	-0.18%

Table 10: Paired difference on-tailed probability, SVM - FBI (12 Month Price Rate of change)

P(T<=t) One sided $Q_{\#}^{SVM} - Q_{\#}^{FBI}$		FBI	
		Q1	Q2
SVM	Q1	5.1%	28.3%
	Q2	38.4%	11.9%

5.1.2 Value

Value based investing assumes that indicators of firm value, such as dividend yield, earning yield, and book value are better indicators of stock value than price history. These factor provide information both on the fundamental of an organisations as well as the current market prices and have been noted to yield price premiums, to differing extents, by Aurret and Sinclair (2006), Muller and Ward (2013), Strugnell et al. (2011), and Van Rensburg (2001).

5.1.2.1 Dividend Yield

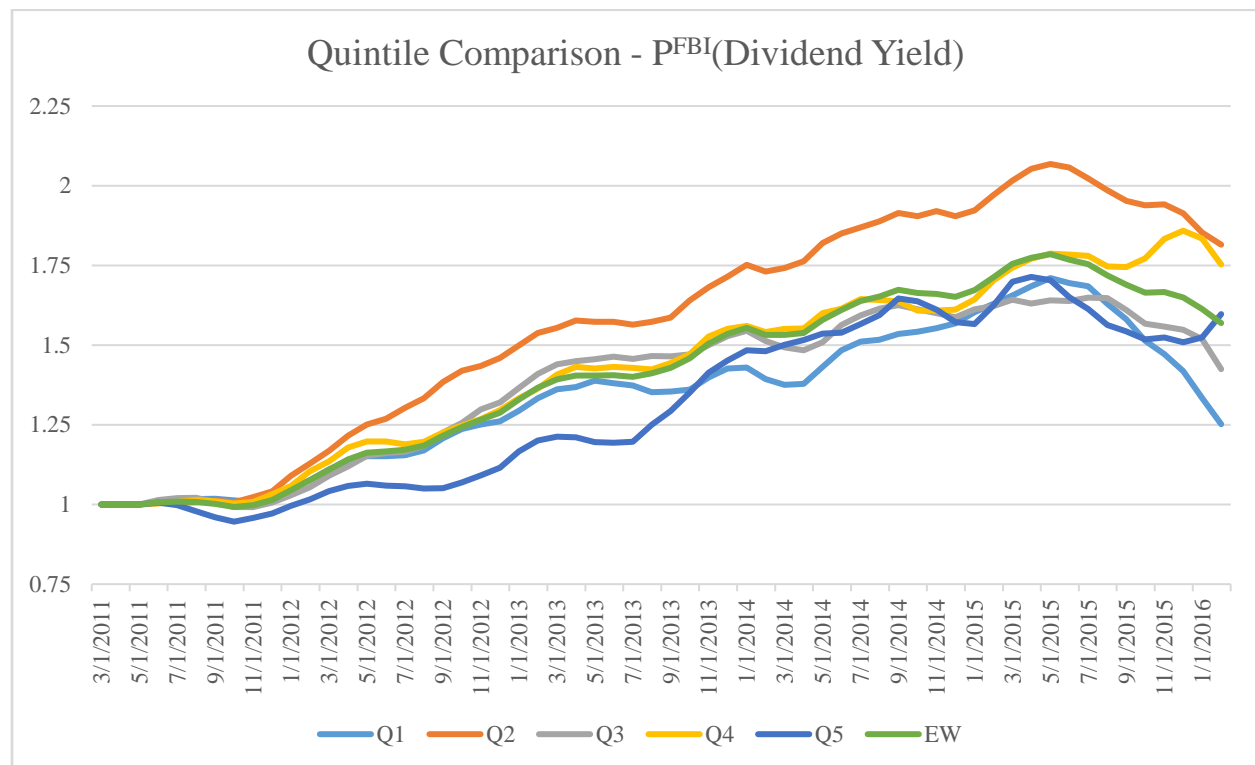


Figure 7: Dividend Yield Quintile Comparison - Factor Base Investing

Figure 7 shows the growth in total value of each quintile (Q1 to Q5) for a factor-based investment strategy that ranks stocks based on dividend yield. Table 11 notes the average return and Sharpe ratio for each quintile as well as for the benchmark equally weighted portfolio.

Table 11: Dividend Yield Quintile Performance Statistics – Factor Based Investing

	Average Return	Standard Deviation	Sharpe Ratio	P(T≤t) One Sided
EW	0.8%	1.5%	0.19	
Q1	0.4%	2.2%	-0.04	2.2%
Q2	1.0%	1.7%	0.32	0.8%

Q3	0.6%	1.8%	0.07	20.8%
Q4	1.0%	1.6%	0.30	14.4%
Q5	0.8%	2.0%	0.16	85.9%

When the average return of each quintile was compared to the return of the equally weighted portfolio, using a one-sided paired t-Test, only quintile Q2 was noted to outperform the benchmark equally weighted portfolio at the 95% confidence level, and yielded a significantly higher Sharpe ratio compared to the equally weighted benchmark. Q1 significantly underperformed the market benchmark, which contradicts the conclusion that high dividend yield is a signal of future stock performance. The fact that Q1 was not the top performing quintile, and that correct quintile ordering was not maintained, i.e. Q1 performing best and Q5 worst, lead to the conclusion that dividend yield is not a sufficient measure of future (next quarter) stock performance, when used in a factor-based investment strategy. Again, this was dissimilar to the findings of Van Rensburg (2001), Hart et al. (2003) and Muller and Ward (2013), who concluded that factor-based investing focused on dividend yield offers a yield premium.

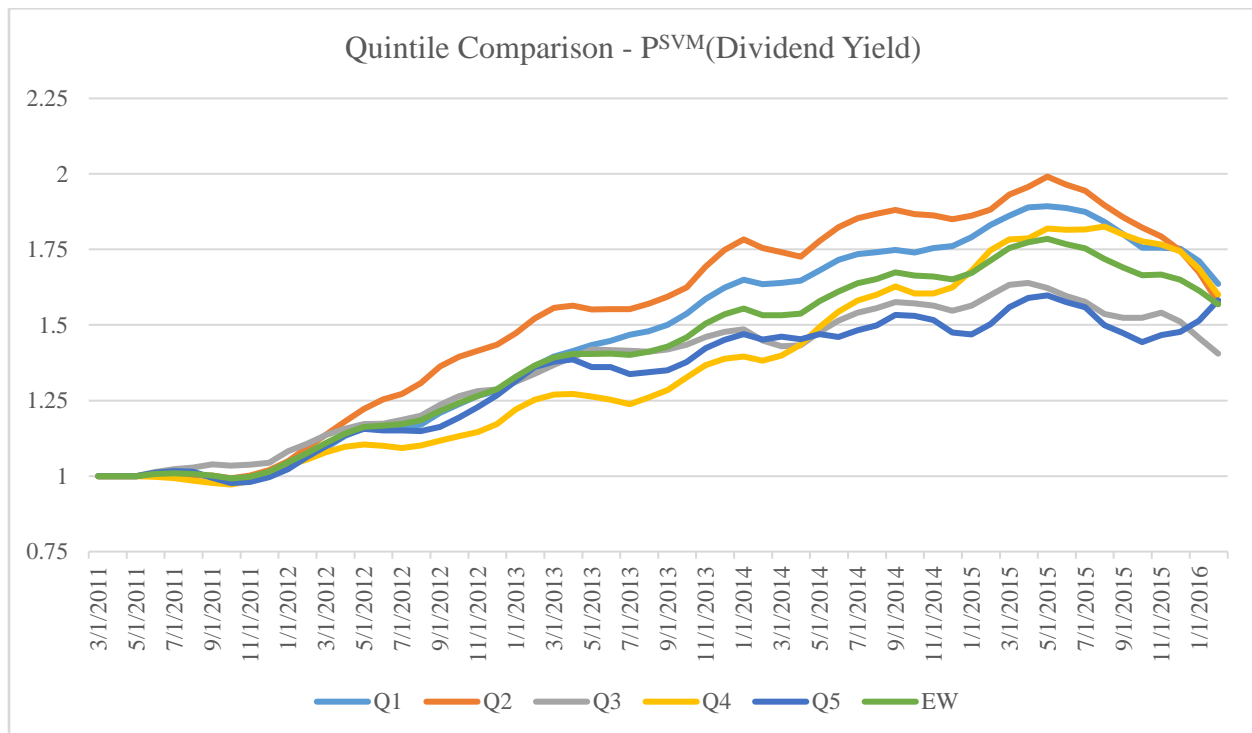


Figure 8: Dividend Yield Quintile Comparison – Machine Learning (SVM) Based Investing

Figure 8 shows the growth in total value of each quintile (Q1 to Q5) using the machine learning investment strategy that ranks stocks based on the historical dividend yield and the statistical

likelihood that the most recent dividend yield will result in positive future returns. Table 12 notes the average return and Sharpe ratio for each quintile as well as for a benchmark equally weighted portfolio.

Table 12: Dividend Yield Quintile Performance Statistics – Machine Learning (SVM) Based Investing

	Average Return	Standard Deviation	Sharpe Ratio	P(T≤t) One Sided
EW	0.8%	1.5%	0.19	
Q1	0.9%	1.6%	0.22	17.5%
Q2	0.8%	2.1%	0.15	44.4%
Q3	0.6%	1.6%	0.07	4.5%
Q4	0.8%	1.8%	0.18	38.9%
Q5	0.8%	1.9%	0.17	45.9%

When the average return of each quintile was compared to the return of the equally weighted portfolio, using a one-sided paired t-Test, no quintile outperformed the benchmark, nor was any clear quintile separation revealed. This brought into question the accuracy with which the SVM algorithm could rank stocks and utilise the informational content of historical dividend yield to predict future stock direction. Fan and Palaniswami (2001) claimed to have achieved significant excess returns using a multitude of input factors to the SVM algorithm, of which dividend yield was one. However, they did not attribute the success of the strategy specifically to dividend yield, nor did they quantify its contribution.

Comparing to the relative performance of FBI and SVM strategies, Table 13 shows that both the first and second quintiles of the SVM approach outperformed the first quintile of the FBI approach by 0.41% and 0.35% per month. Table 14 provides confirmation, highlighting that the differences in return are statistically significant. It should be noted that the SVM approach did not outperform the second quintile performance of the FBI approach. This is of lesser relevance, as the purpose of this study was predominantly to compare the return of first quintiles. These results indicated that, in an environment where dividend yield is not particularly effective in outperforming a benchmark portfolio, the SVM approach was better able to rank stock, select the likely top performers and construct a portfolio to achieve greater return when compared to the FBI approach.

Table 13: Average paired difference, SVM - FBI (Dividend Yield)

Mean Difference $Q_{\#}^{SVM} - Q_{\#}^{FBI}$	FBI	
	Q1	Q2

SVM	Q1	0.41%	-0.16%
	Q2	0.35%	-0.21%

Table 14: Paired difference on-tailed probability, SVM - FBI (Dividend Yield)

P(T<=t) One sided $Q_{\#}^{SVM} - Q_{\#}^{FBI}$		FBI	
		Q1	Q2
SVM	Q1	0.2%	8.2%
	Q2	0.5%	3.6%

5.1.2.2 Earnings Yield

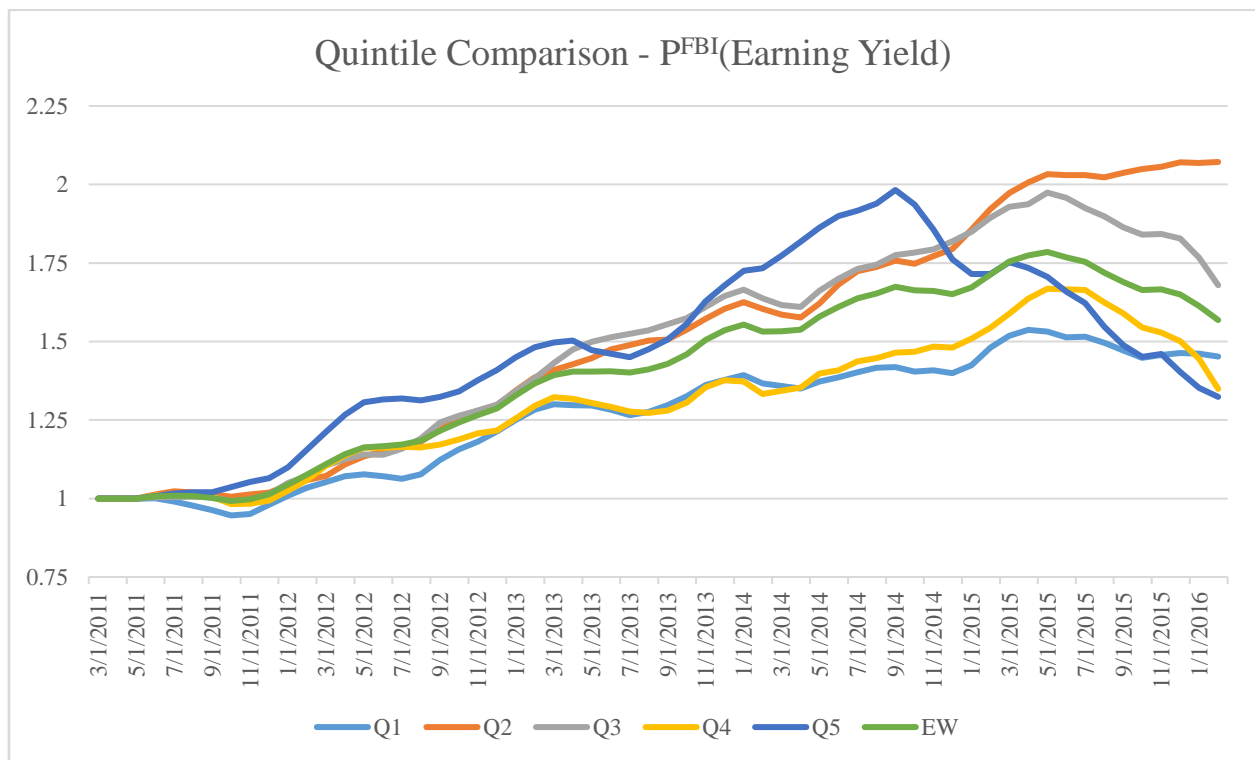


Figure 9: Earnings Yield Quintile Comparison - Factor Base Investing

Figure 9 shows the growth in total value of each quintile (Q1 to Q5) for a Factor Based Investment strategy that ranks stocks based on Earnings Yield. Table 15 notes the average return and Sharpe ratio for each quintile as well as for the benchmark equally weighted portfolio.

Table 15: Earnings Yield Quintile Performance Statistics – Factor Based Investing

	Average Return	Standard Deviation	Sharpe Ratio	P(T<=t) One Sided
EW	0.8%	1.5%	0.19	
Q1	0.6%	1.6%	0.10	27.5%

Q2	1.3%	1.3%	0.60	0.1%
Q3	0.9%	1.7%	0.24	30.6%
Q4	0.5%	2.0%	0.02	6.1%
Q5	0.5%	2.5%	0.01	22.6%

When comparing the average return of each quintile to the return of the equally weighted portfolio, using a one-sided paired t-Test, only Q2 outperforms the benchmark equally weighted portfolio. Q1 does not outperform the benchmark. Clearly the correct quintile ordering is not achieved, i.e. Q1 performing best and Q5 worst, which leads to the conclusion that earning yield is not a sufficient measure of future (next quarter) stock performance, when used in a factor base investment strategy. This contradicts what is found by Muller and Ward (2013) who note a significant spread between the best and worst performing quintiles. Their results suggest that there is a significant premium attached to value stocks, with low PE, as compared to growth stocks, with high PE. Strugnell et al. (2011) similarly note that PE is a statistically significant predictor of excess return for a one month holding period. They note that such evidence weakens over longer holding periods. This study implements a 3-month holding period, which could in part account for the difference in findings.

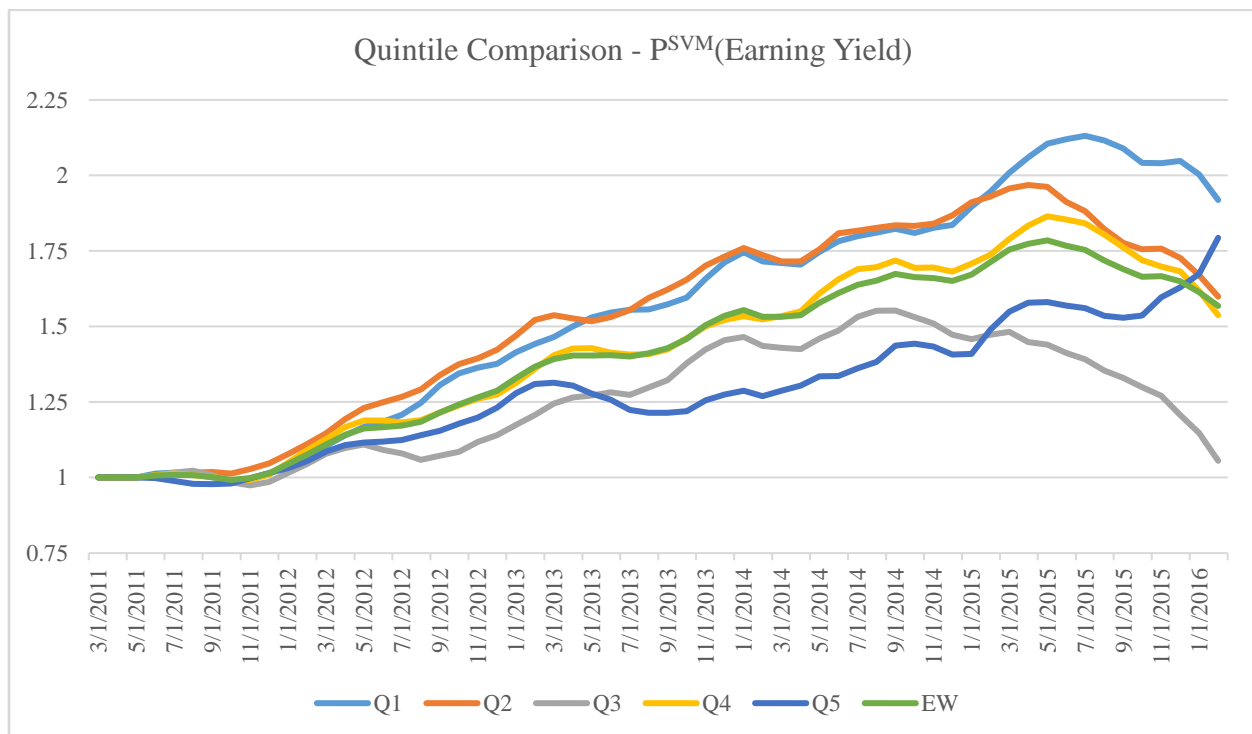


Figure 10: Earnings Yield Quintile Comparison – Machine Learning (SVM) Based Investing

Figure 10 shows the growth in total value of each quintile (Q1 to Q5) using the Machine Learning Investment Strategy that ranks stocks based on historical earning yield and the statistical likelihood that present earnings yield will result in positive future returns. Table 16 notes the average return and Sharpe ratio for each quintile as well as a benchmark equally weighted portfolio.

Table 16: Earnings Yield Quintile Performance Statistics – Machine Learning (SVM) Based Investing

	Average Return	Standard Deviation	Sharpe Ratio	P(T<=t) One Sided
EW	0.8%	1.5%	0.19	
Q1	1.1%	1.7%	0.37	0.3%
Q2	0.8%	1.8%	0.18	37.6%
Q3	0.1%	2.3%	-0.16	0.0%
Q4	0.7%	1.9%	0.14	38.9%
Q5	1.0%	1.9%	0.28	18.8%

When comparing the average return of each quintile to the return of the equally weighted portfolio, using a one-sided paired t-Test, only Q1 outperforms the benchmark. Importantly it is Q1 that performs best, followed by Q2, which suggests somewhat successful quintile ordering. However, it is noted that Q5 does not perform worst, it is in fact Q3 that underperforms the equally weighted benchmark. The SVM strategy has therefore been somewhat successful in using earning yield to classify stocks that generate positive return.

Comparing the relative performance of the SVM and FBI strategies, Table 17 shows that the average monthly return of the first and second SVM based quintiles, Q_1^{SVM} and Q_2^{SVM} , are both greater than the first FBI quintile, 0.43% and 0.15% respectively. Table 18 notes that only Q_1^{SVM} outperforms Q_1^{FBI} at the 95% confidence level. These results suggest that the FBI based strategy is unable to utilise earning yield when formulating a high performing first quintile, but that the SVM approach is more effective in predicting future stock performance, and therefore yields a significantly higher Q1 average return.

Again, it is noted that the factor base investment strategy underperforms the benchmark and contradicts the finding of particular studies. However, the SVM strategy is able to yield return in excess of the benchmark and outperform its factor based counterpart in similar conditions.

Table 17: Average paired difference, SVM - FBI (PE Ratio)

Mean Difference $Q_{\#}^{SVM} - Q_{\#}^{FBI}$		FBI	
		Q1	Q2
SVM	Q1	0.43%	-0.11%
	Q2	0.15%	-0.39%

Table 18: Paired difference on-tailed probability, SVM - FBI (PE Ratio)

P(T<=t) One sided $Q_{\#}^{SVM} - Q_{\#}^{FBI}$		FBI	
		Q1	Q2
SVM	Q1	0.2%	19.7%
	Q2	18.2%	0.9%

5.1.2.3 Book-to-Market Value

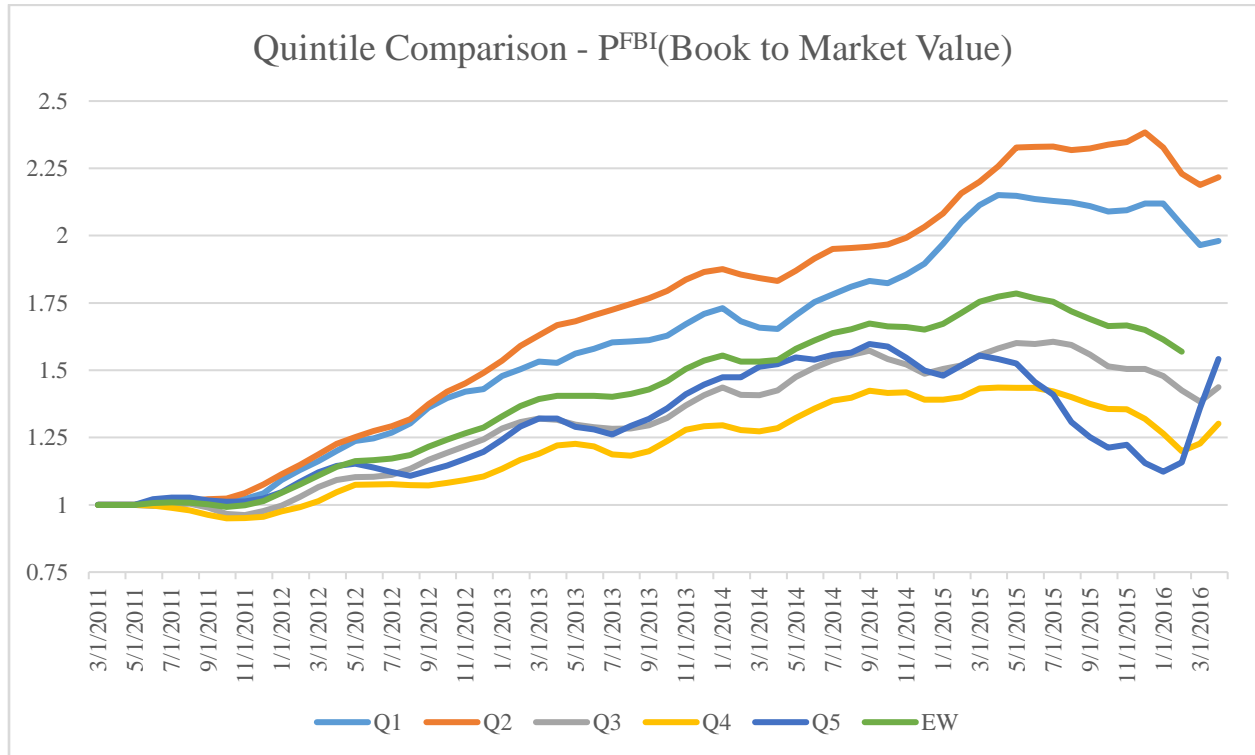


Figure 11: Book-to-Market Quintile Comparison - Factor Base Investing

Figure 11 shows the growth in total value of each quintile (Q1 to Q5) for a Factor Based Investment strategy that ranks stocks based on book to market value. Table 19 notes the average return and Sharpe ratio for each quintile as well as a benchmark equally weighted portfolio.

Table 19: Book to Market Quintile Performance Statistics – Factor Based Investing

	Average Return	Standard Deviation	Sharpe Ratio	P(T<=t) One Sided
EW	0.8%	1.5%	0.19	
Q1	1.2%	1.7%	0.44	0.2%
Q2	1.4%	1.5%	0.58	0.0%
Q3	0.6%	1.7%	0.07	13.5%
Q4	0.3%	1.8%	-0.09	0.0%
Q5	0.3%	2.5%	-0.08	3.9%

When comparing the average return of each quintile to the return of the equally weighted portfolio, using a one-sided paired t-Test, both Q1 and Q2 outperform the benchmark at the 95% confidence level. Importantly, correct quintile ordering is maintained, i.e. Q1/Q2 performing best and Q4/Q5 worst, which leads to the conclusion that book to market value is a sufficient measure of future (next quarter) stock performance, when used in a Factor Base Investment strategy. This conclusion is similar to the finding of Muller and Ward (2013) and Auret and Sinclair (2006) who found that low Price-to-Book value stocks significantly outperform those with high Price-to-Book values, where Price-to-Book can be seen as an inverse proxy for book to market value, not accounting for shares in circulation. These conclusions do however differ from those presented by Van Rensburg (2001), who does not find statistically significant excess return when considering single factor price to net asset value.

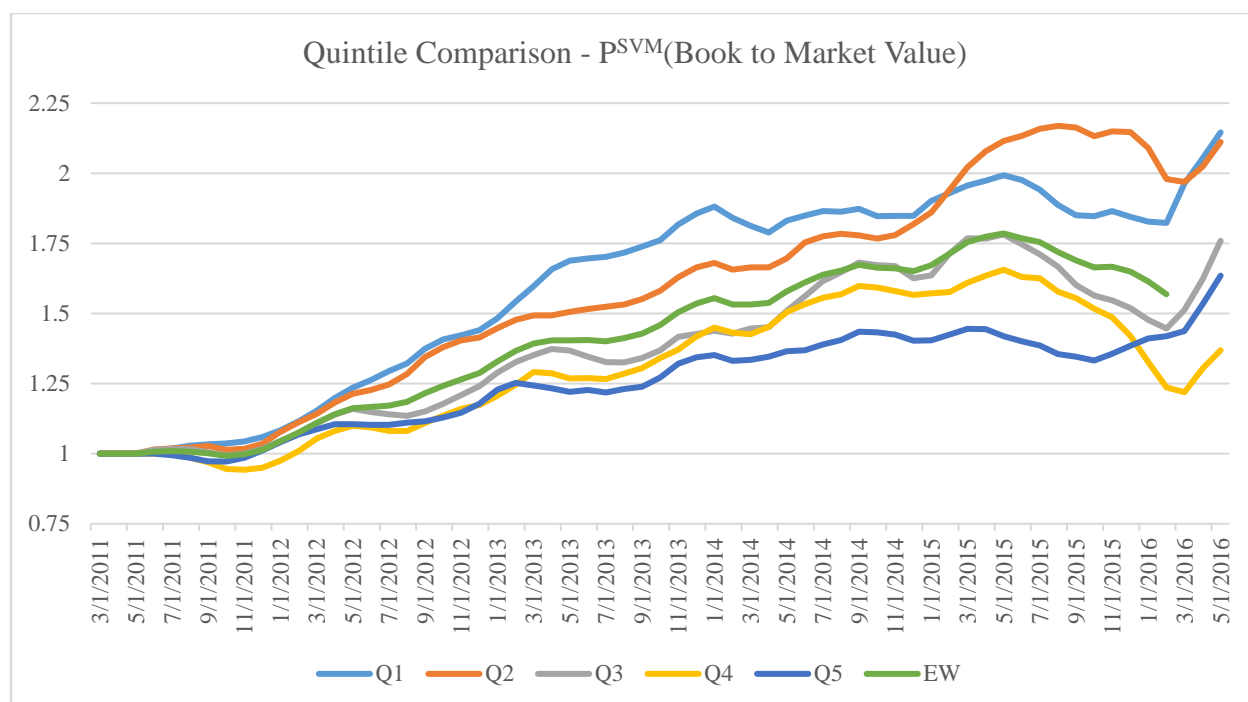


Figure 12: Book to Market Value Quintile Comparison – Machine Learning (SVM) Based Investing

Figure 12 shows the growth in total value of each quintile (Q1 to Q5) using the Machine Learning Investment Strategy that ranks stocks based on historical book to market value and the statistical likelihood that a present book to market value will result in positive future returns. Table 20 notes the average return and Sharpe ratio for each quintile as well as a benchmark equally weighted portfolio.

Table 20: Book to Market Value Quintile Performance Statistics – Machine Learning (SVM) Based Investing

	Average Return	Standard Deviation	Sharpe Ratio	P(T<=t) One Sided
EW	0.8%	1.5%	0.19	
Q1	1.0%	1.6%	0.33	3.3%
Q2	1.2%	1.7%	0.40	0.3%
Q3	0.6%	2.0%	0.08	15.6%
Q4	0.4%	2.3%	-0.04	1.0%
Q5	0.6%	1.4%	0.08	15.8%

When comparing the average return of each quintile to the return of the equally weighted portfolio, using a one-sided paired t-Test, both Q1 and Q2 outperform the benchmark. Importantly, similar to the factor based approach, correct quintile ordering is achieved, with Q1/Q2 performing best, and Q4/Q5 performing worst. The SVM strategy has therefore been successful in using book to market value to separate stock that generate positive return from those that do not.

Comparing the performance of both strategies, Table 21 shows that the average monthly return of both first and second SVM based quintiles, underperform their FBI counterparts in terms of average return. Table 22 notes that the difference in return between the SVM and FBI approach do not appear to be statistically significant, except for Q_1^{SVM} underperforming Q_2^{FBI} . Comparing only Q1 performance there appears to be little difference in average return between SVM and FBI strategies. Though the SVM strategy does not perform significantly worse than the FBI strategy, it was not able to better predict future price movement given historical book to market value.

Table 21: Average paired difference, SVM - FBI (Book to Market Value)

Mean Difference $Q_{\#}^{SVM} - Q_{\#}^{FBI}$		FBI	
		Q1	Q2
SVM	Q1	-0.18%	-0.31%
	Q2	-0.05%	-0.18%

Table 22: Paired difference on-tailed probability, SVM - FBI (Book to Market Value)

P($T \leq t$) One sided $Q_{\#}^{SVM} - Q_{\#}^{FBI}$		FBI	
		Q1	Q2
SVM	Q1	13.6%	1.7%
	Q2	33.4%	5.1%

5.1.3 Quality

Quality based investing derives stock value from fundamental factors obtained from financial statement and no consider current market prices in the ranking of stocks. Quality factor are reported on are obtained from financial statements and held constant during monthly rebalancing.

5.1.3.1 Return on Equity

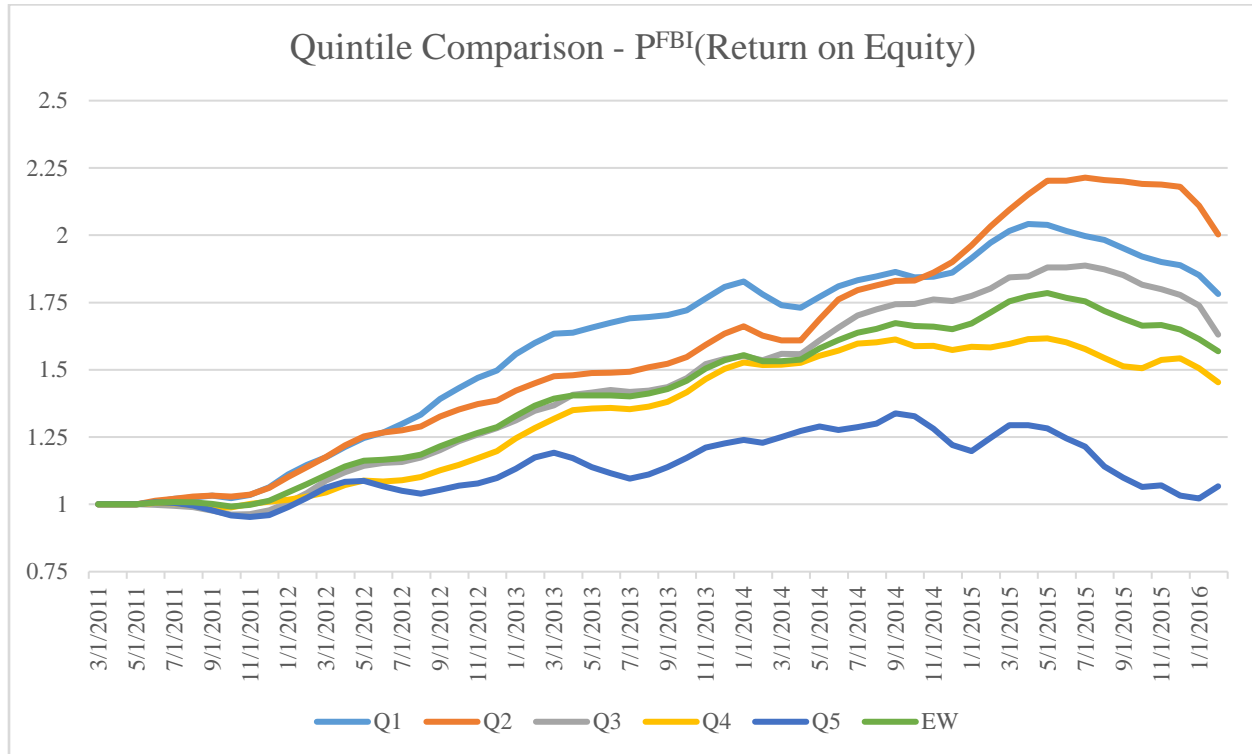


Figure 13: Return on Equity Quintile Comparison - Factor Base Investing

Figure 13 shows the growth in total value of each quintile (Q1 to Q5) for a factor based investment (FBI) strategy that ranks stocks based on Return on Equity. Table 23 notes the average return and Sharpe ratio for each quintile as well as a benchmark equally weighted portfolio.

Table 23: Return on Equity Quintile Performance Statistics – Factor Based Investing

	Average Return	Standard Deviation	Sharpe Ratio	P(T<=t) One Sided
EW	0.8%	1.5%	0.19	
Q1	1.0%	1.8%	0.29	6.5%
Q2	1.2%	1.8%	0.41	0.1%
Q3	0.8%	1.8%	0.20	52.9%
Q4	0.6%	1.5%	0.11	25.7%
Q5	0.1%	2.4%	-0.15	1.1%

When comparing the average return of each quintile to the return of the equally weighted portfolio, using a one-sided paired t-Test, only Q2 outperforms the benchmark to a 95% confidence level. The yield of Q1 is also more than that of the equally weighted benchmark, but can only be considered somewhat statistically significant at the 93% confidence level. Correct quintile order is achieved with Q1/Q2 yielding the highest returns, and Q4/Q5 yielding the lowest. This leads to the conclusion that Return on Equity is a somewhat sufficient measure of future (next quarter) stock performance, when used in a factor base investment strategy. This is in contradiction to Van Rensburg (2001), who did not find portfolios based on return on equity to yield statistically significant excess return above an equally weighted benchmark. Similarly, Muller and Ward (2013) did not notice significant ordering of quintiles and separation between first and fifth quintiles, suggesting that the market does not underprice high ROE stocks. However, Tajbani (2015) notes a significant premium attached to ROE, with clear quintile separations, citing that their findings are consistent with that of Gordan (2013), which lend credence to the findings in this study.

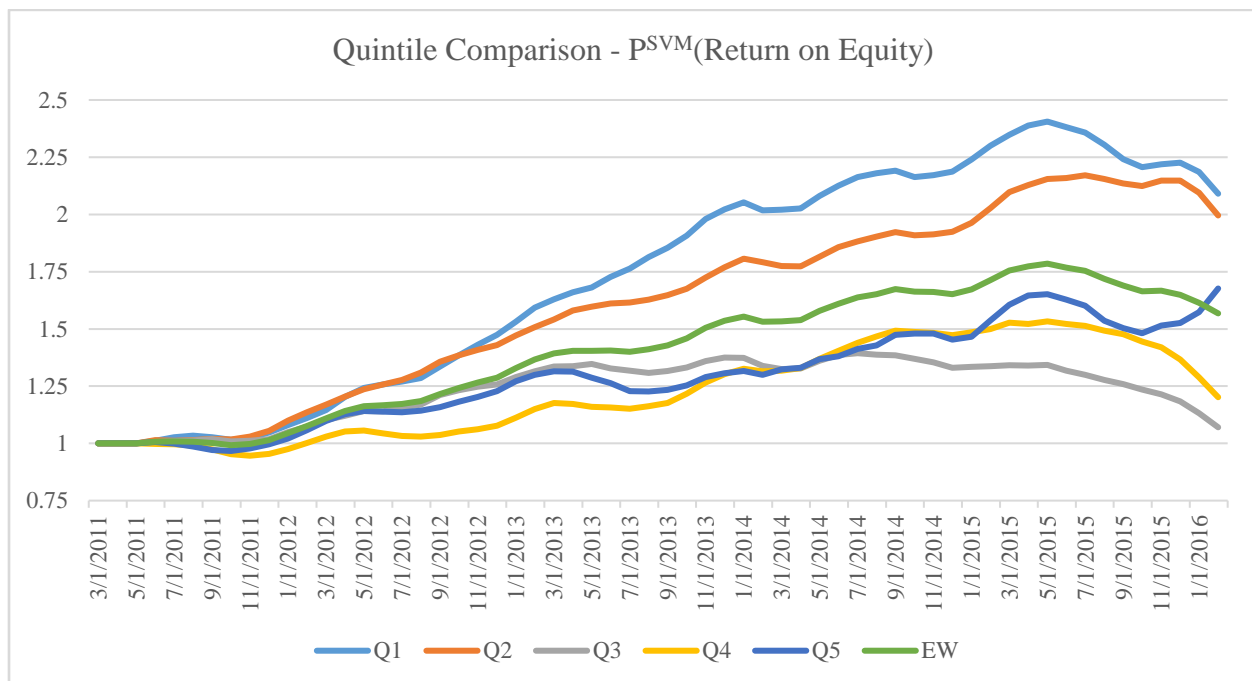


Figure 14: Return on Equity Quintile Comparison – Machine Learning (SVM) Based Investing

Figure 14 shows the growth in total value of each quintile (Q1 to Q5) using the machine learning investment (SVM) strategy that ranks stocks based on historical return on equity and the statistical likelihood that present return on equity will result in positive future returns. Table 24 notes the

average return and Sharpe ratio for each quintile as well as a benchmark equally weighted portfolio.

Table 24: Return on Equity Quintile Performance Statistics – Machine Learning (SVM) Based Investing

	Average Return	Standard Deviation	Sharpe Ratio	P(T<=t) One Sided
EW	0.8%	1.5%	0.19	
Q1	1.3%	1.9%	0.42	0.0%
Q2	1.2%	1.6%	0.44	0.0%
Q3	0.1%	1.7%	-0.21	0.0%
Q4	0.3%	2.0%	-0.07	0.2%
Q5	0.9%	2.0%	0.21	30.3%

When comparing the average return of each quintile to the return of the equally weighted portfolio, using a one-sided paired t-Test, both Q1 and Q2 outperform the benchmark more than the 95% confidence level. Importantly, Q1 and Q2 perform best, yielding the highest returns. A fully successful SVM implementation would have had a correct quintile ranking, however it is noted that Q5 yielded similar returns to those of the equally weighted benchmark, and in fact Q3 and Q4 underperformed the benchmark. This ranking is not ideal, but does indicate the SVM approach was somewhat able to separate stocks yielding future positive returns, from those that do not.

Comparing the relative performance between the SVM and FBI quintiles, Table 25 shows that the average monthly return of the first SVM based quintile, Q_1^{SVM} , is greater than both the first and second FBI quintiles. This is confirmed in Table 26 which notes the statistical significance in the paired difference above the 95% confidence level.

Table 25: Average paired difference, SVM - FBI (Return on Equity)

Mean Difference $Q_{\#}^{SVM} - Q_{\#}^{FBI}$		FBI	
		Q1	Q2
SVM	Q1	0.25%	0.07%
	Q2	0.17%	-0.01%

Table 26: Paired difference on-tailed probability, SVM - FBI (Return on equity)

P(T<=t) One sided $Q_{\#}^{SVM} - Q_{\#}^{FBI}$		FBI	
		Q1	Q2
SVM	Q1	1.5%	31.9%
	Q2	3.5%	46.4%

5.1.3.2 Return on Invested Capital

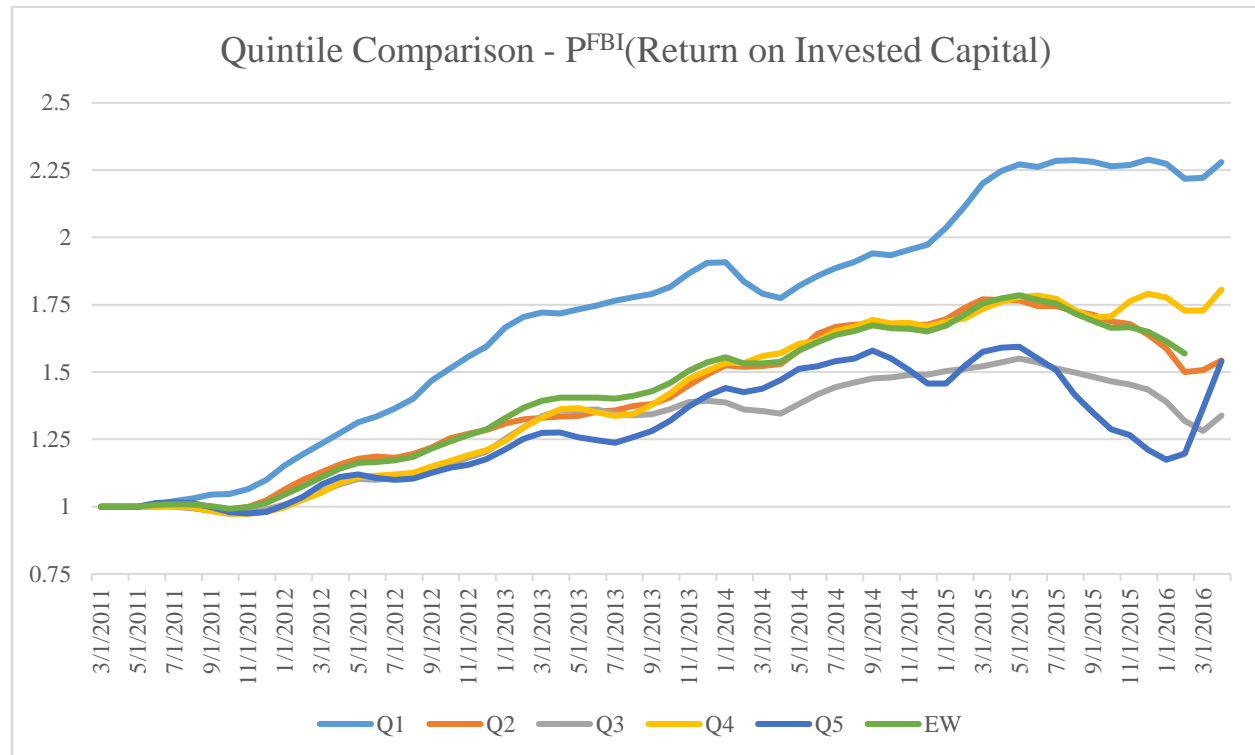


Figure 15: Return on Invested Capital Quintile Comparison - Factor Base Investing

Figure 15 shows the growth in total value of each quintile (Q1 to Q5) for a factor based investment (FBI) strategy that ranks stocks based on return on invested capital. Table 27 notes the average return and Sharpe ratio for each quintile as well as a benchmark equally weighted portfolio.

Table 27: Return on Invested Capital Quintile Performance Statistics – Factor Based Investing

	Average Return	Standard Deviation	Sharpe Ratio	P(T<=t) One Sided
EW	0.8%	1.5%	0.19	
Q1	1.4%	1.7%	0.51	0.0%
Q2	0.7%	1.7%	0.13	51.1%
Q3	0.5%	1.6%	0.00	0.7%
Q4	0.9%	1.5%	0.30	18.8%
Q5	0.3%	2.4%	-0.06	2.6%

When comparing the average return of each quintile to the return of the equally weighted portfolio, using a one-sided paired t-Test, Q1 far outperforms the equally weighted benchmark, and is the only quintile to do so above the 95% confidence level. Correct quintile ordering is achieved, i.e. Q1 performing best and Q5 worst, which suggests that return on invested capital is an adequate

measure of future (next quarter) stock performance, when used in a Factor Base Investment strategy. This is again similar to the findings by Tajbai (2015), who claim similar results found by Gordan (2013), both noticing market undervaluation for stocks with high return on invested capital that subsequently achieve high first quintile portfolio returns.

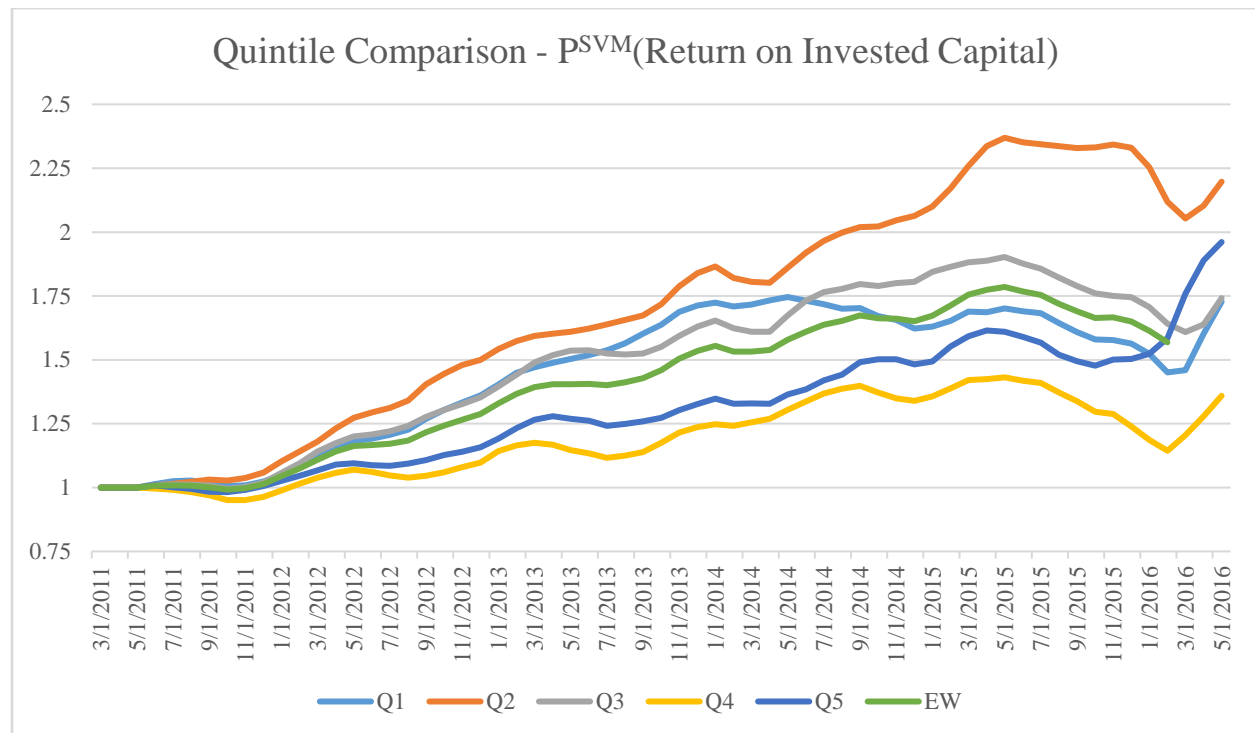


Figure 16: Return on Equity Quintile Comparison – Machine Learning (SVM) Based Investing

Figure 16 shows the growth in total value of each quintile (Q1 to Q5) using the machine learning investment (SVM) strategy that ranks stocks based on historical return on invested capital and the statistical likelihood that present earnings yield will result in positive future returns. Table 28 notes the average return and Sharpe ratio for each quintile as well as a benchmark equally weighted portfolio.

Table 28: Return on Equity Quintile Performance Statistics – Machine Learning (SVM) Based Investing

	Average Return	Standard Deviation	Sharpe Ratio	$P(T \leq t)$ One Sided
EW	0.8%	1.5%	0.19	
Q1	0.6%	1.7%	0.09	15.1%
Q2	1.3%	1.9%	0.43	0.0%
Q3	0.9%	1.7%	0.21	17.8%
Q4	0.2%	1.9%	-0.13	0.0%
Q5	0.8%	1.5%	0.20	45.9%

When comparing the average return of each quintile to the return of the equally weighted portfolio, using a one-sided paired t-Test, only Q2 outperforms the benchmark with statistical significance. Q1 underperforms the equally weighted benchmark, but it is noted that it does not differ from the benchmark with statistical significance. Correct quintile ordering, i.e. Q1 performing best and Q5 worst, has not been achieved, and therefore the SVM strategy is deemed to have been unsuccessful in using historical return on invested capital to separate stocks that generate positive return from those that do not.

Comparing the performance of the SVM and FBI strategies, Table 18 shows that the average monthly return of the first SVM based quintile, Q_1^{SVM} , is far less than its FBI counterpart. Table 4 shows that there is a significant statistical difference in Q_1^{SVM} and Q_1^{FBI} return. The results suggest that return on invested capital can be used to construct high return first quintiles when using a factor based investing strategy, but when implemented using a support vector machine to predict future stock performance, no excess returns are achieved.

Table 29: Average paired difference, SVM - FBI (Return on Invested Capital)

Mean Difference $Q_{\#}^{SVM} - Q_{\#}^{FBI}$		FBI	
		Q1	Q2
SVM	Q1	-0.66%	-0.05%
	Q2	-0.07%	0.54%

Table 30: Paired difference on-tailed probability, SVM - FBI (Return on Invested Capital)

P(T<=t) One sided $Q_{\#}^{SVM} - Q_{\#}^{FBI}$		FBI	
		Q1	Q2
SVM	Q1	0.0%	36.5%
	Q2	27.4%	0.0%

5.1.4 Single Factor Summary and Hypothesis Testing

The research question, outlined in Section 1.3, requires an investigation and comparison of the return generated by a Factor Based Investment as well as Machine Learning (SVM) Based Investment strategy to an equally weighted portfolio benchmark as well as to each other.

To reiterate, the main questions read:

-
1. *Does factor based investment portfolio construction generate returns greater than an equally weighted benchmark?*
 2. *Does machine learning based investment portfolio construction generate returns greater than an equally weighted benchmark?*
 3. *Does the application of a machine learning algorithm (SVM) to a factor-based investment portfolio approach lead to comparable or higher returns?*
-

The null and alternate hypotheses for each question are as follows:

$$\begin{array}{ccc|ccc} H_0^1: \mu_{Q1}^{FBI} - \mu_{EW} = 0 & & H_0^2: \mu_{Q1}^{SVM} - \mu_{EW} = 0 & & H_0^3: \mu_{Q1}^{SVM} - \mu_{Q1}^{FBI} = 0 \\ H_i^1: \mu_{Q1}^{FBI} - \mu_{EW} \neq 0 & & H_i^2: \mu_{Q1}^{SVM} - \mu_{EW} \neq 0 & & H_i^3: \mu_{Q1}^{SVM} - \mu_{Q1}^{FBI} \neq 0 \end{array}$$

Where, μ_{Q1}^{FBI} and μ_{Q1}^{SVM} are the average monthly returns of the first quintiles of the FBI and SVM investment strategies respectively, and μ_{EW} is the average return of the equally weighed benchmark portfolio. The method for hypothesis testing is a one-tailed t-Test. Table 31, Table 32 and Table 33 show the one-tailed probability, $P(T \leq t)$, for each of the three hypotheses, given the single factor strategies considered in this section. Note that a positive average difference paired with a one-tailed t-statistical probability of less than 5% means that we can reject the null hypothesis for equality, and accept that the alternate hypothesis claiming significantly larger returns.

Table 31: Hypothesis test, FBI based Q1 portfolios vs equally weighted portfolio

$H_0^1: \mu_{Q1}^{FBI} - \mu_{EW} = 0$				
Factor		Average Difference	Std. Dev.	P(T<=t)
Momentum	6 Month Price Rate of Change	-0.23%	1.1%	4.62%
	12 Month Price Rate of Change	-0.05%	0.99%	33.7%
Value	Dividend Yield	-0.34%	1.10%	1.1%
	Earning Yield	-0.12%	0.83%	13.8%
	Book-to-Market Value	0.41%	0.99%	0.1%
Quality	Return on Equity	0.20%	0.82%	3.2%
	Return on Invested Capital	0.54%	1.00%	0.0%

Considering the first-quintile performance of each of the single-factor approaches using the Factor Base Investment strategy, **Table 31**, shows that portfolios based on book to market value, return on equity, and return on invested capital all, achieve returns above that of an equally weighted benchmark with statistical significance. Portfolios based on a 12-month price rate of change, and earning yield perform no differ from the benchmark, whilst a 6-month price rate of change and dividend yield underperform the benchmark with statistical significance.

A similar comparison using the first-quintile performance of each of the single-factor approaches using the Machine Learning (SVM) Investment strategy is shown in Table 32. μ_{Q1}^{SVM} outperforms the equally weighted benchmark, with statistical significance, using each of the momentum factors, the 6 and 12-month price rate of change, as well as with earnings yield, book to market value and return on equity. The other factors, dividend yield and return on invested capital, yield statistically similar returns to the benchmark equally weighted portfolio. Critically, none of the factors yield a first quintile average return that underperforms the benchmark. This indicates that the SVM strategy can rank the future performance of stocks based on single factors to consistently yield high performing first quintile portfolio, and will at worst, perform on par with the an equally weighted portfolio.

Table 32: Hypothesis test, SVM based Q1 portfolios vs equally weighted portfolio

$H_0^1: \mu_{Q1}^{SVM} - \mu_{EW} = 0$				
Factor		Average Difference	Std. Dev.	P(T<=t)
Momentum	6 Month Price Rate of Change	0.17%	0.76%	4.8%
	12 Month Price Rate of Change	0.25%	0.93%	2.1%
Value	Dividend Yield	0.07%	0.55%	17.5%

Quality	Earning Yield	0.32%	0.85%	0.3%
	Book-to-Market Value	0.23%	0.96%	3.3%
	Return on Equity	0.45%	0.78%	0.0%
	Return on Invested Capital	-0.12%	0.86%	15.1%

The comparison of average returns between the two strategies, SVM and FBI, is shown in Table 33. The null hypothesis, stating that the mean difference between SVM and FBI strategies is zero, H_0^3 , cannot be rejected for book to market value. It can however be rejected for the 6 and 12-month price rate of change, dividend yield, earnings yield, return on equity and return on invested capital, which leads to the conclusion that the average first quintile returns are statistically dissimilar. Return on Invested capital is the unique case where the FBI strategy outperforms the SVM strategy. The purpose of this research was to compare the performance of factor based investing to machine learning based investing. The research question: “Does the application of a machine learning algorithm (SVM) to a factor-based investment portfolio approach lead to comparable or higher returns?” requires that in all cases the machine learning based approach outperform, or perform similar, to the factor base approach. Evidence suggests that this is true for all factors except return on invested capital, in which machine learning based investing was outperformed by factor base investing.

It should however, be noted that many of the factor based approaches did not yield returns greater than that of the equally weighted benchmark, and contradict empirical evidence of excess returns found in literature. It is suggested that future work improve the fidelity of factor based investing to align findings with that of literature, and subsequently reevaluate the apparent superior performance of machine learning based investing found in this study.

Table 33: Hypothesis test, SVM based Q1 portfolios vs FBI based Q1 portfolios

$H_0^1: \mu_{Q1}^{SVM} - \mu_{Q1}^{FBI} = 0$				
Factor		Average Difference	Std. Dev.	P(T<=t)
Momentum	6 Month Price Rate of Change	0.40%	1.36%	1.1%
	12 Month Price Rate of Change	0.31%	1.45%	5.1%
Value	Dividend Yield	0.41%	1.08%	0.2%
	Earning Yield	0.43%	1.17%	0.2%
	Book-to-Market Value	-0.18%	1.25%	13.6%
Quality	Return on Equity	0.25%	0.89%	1.5%
	Return on Invested Capital	-0.66%	1.32%	0.0%

5.2 Multi Factor Comparison

In the multi-factor strategy, multiple variables are used to rank stocks and construct a portfolio. Each portfolio strategy is represented by 5 quintiles intended to highlight the difference between the return of highly ranked stocks and those ranked lower. A successful strategy will show quintiles Q_1 and Q_2 yielding higher returns than quintiles Q_4 and Q_5 , as well as a benchmark portfolio constructed using all stocks equally weighted, EW. This section only considers the application of the Machine Learning (SVM) Investment strategy, and attempts to evaluate which combination of factors result in the highest first quintile return.

5.2.1 Momentum, Value and Quality

The support vector machine learning algorithm can use multiple inputs for training and prediction purposes. The use of more information, i.e. input variables, in both training and prediction should have a positive effect on the prediction accuracy. At the very least, a portfolio constructed using a multi-factor SVM approach should perform at least as well as the best performing single factor. Figure 17 shows the first quintile (Q_1) performance of each of the three broad factor categories, momentum, value and quality, as well for a machine learning based portfolio constructed using all factors

Input vectors as follows:

- Momentum: $x = [6ROC \ 12ROC]^T$
- Value: $x = [DY \ EY \ BTM]^T$
- Quality: $x = [ROE \ ROIC]^T$
- Momentum, Value and Quality: $x = [6ROC \ 12 \ ROC \ DY \ EY \ BTM \ ROE \ ROIC]^T$

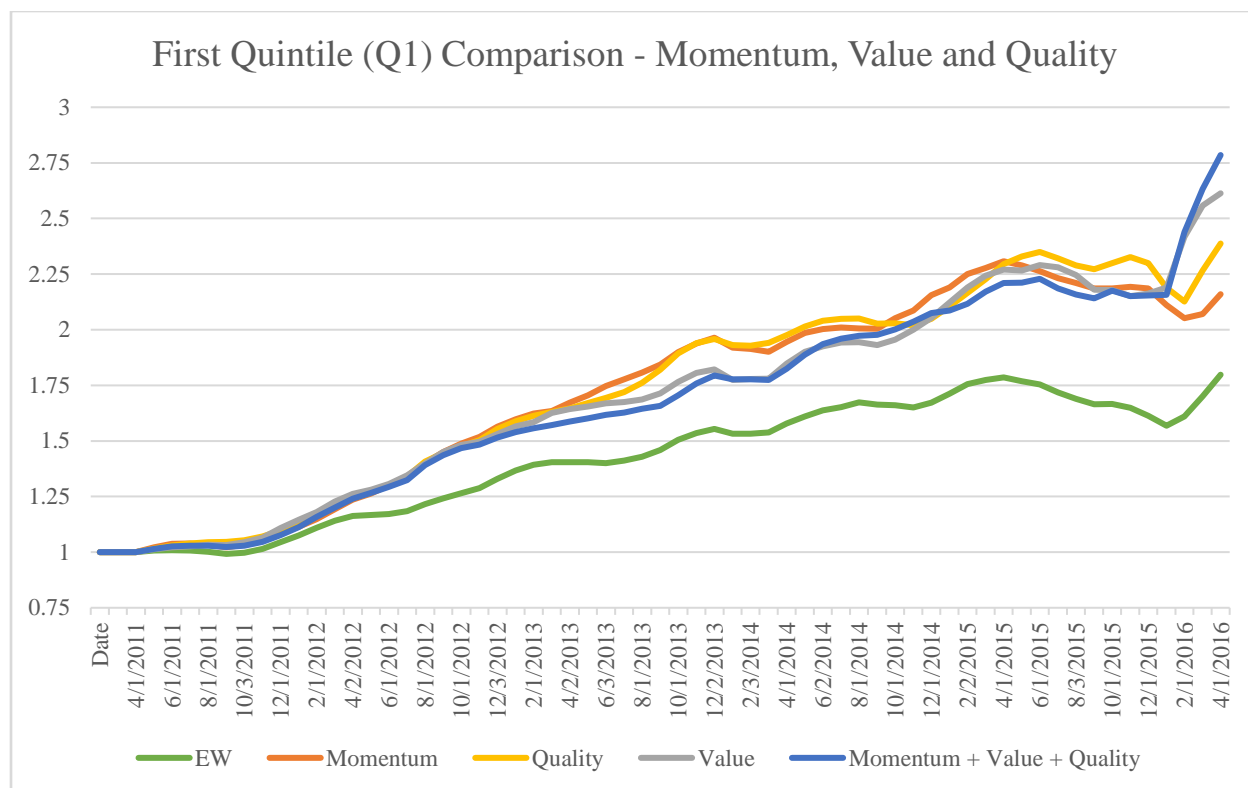


Figure 17: First Quintile Comparison - Momentum vs. Quality vs. Value

Clearly all three approaches outperform the equally weighted benchmark. and Table 35 note the positive difference in average return. The combination of value factors, with the SVM algorithm, appears to yield the highest average return, followed by quality and then momentum. The momentum approach fails to outperform the equally weighted benchmark at the 95% confidence level, but can still be considered somewhat significant at the 94% confidence level. Importantly, not only do these multi-factor portfolios each outperform the equally weighted benchmarks, they also yield higher returns than each of their constituent single factor portfolios, i.e. momentum Q1 yields a higher average return than the 6 and 12-month rate of change Q1's respectively. From this we can conclude that the use of the machine learning algorithm (SVM) is effective in ranking stocks, based on the probability of future positive returns, given multiple factor inputs. The use of multiple factors in the classification problem improve the efficacy of using the support vector machine in portfolio construction. These finding align with those of Fan and Palaniswami (2001), Huerta et al. (2013) and Sewell (2012). We note that the use of multiple input factors to the support vector machine classification model will outperform, or perform at as well as, their single factor constituents.

Table 34: Multi-factor SVM Q1 portfolio comparison

	Average Return	Standard Deviation	Sharpe Ratio
Momentum	1.26%	1.74%	0.45
Value	1.58%	1.97%	0.55
Quality	1.43%	1.87%	0.50
EW	0.96%	1.71%	0.28
Mom,Val and Qual	1.69%	2.24%	0.54

Table 35: Multi-factor SVM Q1 Portfolio Comparison vs. equally weighted portfolio

	Average Difference	Standard Deviation	P(T<=t)
Momentum - EW	0.29%	1.39%	5.5%
Value - EW	0.59%	1.42%	0.1%
Quality - EW	0.44%	1.16%	0.2%
(Mom,Val and Qual) - EW	0.69%	1.54%	0.0%

It is further noted that the combination of all factors from momentum, value and quality yield the highest average return of 1.7% per month, which equates to 22.3% per annum if compounded, and outperforms the benchmark equally weighted portfolio by the largest margin. This outcome was to be expected given the earlier discussion, which states that the support vector machine will perform better, or at least similarly, with the addition of input variables. The accuracy with which the SVM algorithm can classify stock performance is dependent on the correlation between input values and the classification of the output variable, i.e. positive or negative future return. If the addition of an input variable aids in more distinctly separating between classifications, the accuracy of prediction will increase. If, however, the addition of input factors does not aid in separating classifiers, prediction accuracy will remain unchanged and average portfolio yield will not improve. The first quintile performance of the full contingent of factors, does outperform the portfolios of momentum, value, quality which suggest that the combination of factors does improve the accuracy of prediction and portfolio return. However, the gain in portfolio yield relative to momentum, value, quality strategies is not significant. Huerta et al. (2013) similarly find that though considering fundamental factor and technical factors, akin to value/quality and momentum respective, do outperform a benchmark, the combination does not significantly outperform a portfolio based on the best performing subset of factors.

5.3 Prediction and Ranking Accuracy

The results from both the factor based and machine learning (SVM) strategies have been considered from a financial return point of view. Comparing the average return of first quintiles has shown the ability of the SVM classification based portfolio to consistently outperform their FBI counterparts, except for return on invested capital. The differences in portfolio performance is, however, routed in the accuracy with which the SVM strategy can rank stock and the extent to which that accuracy is better than the FBI strategy. A portfolio construction strategy that can exactly predict the top 20, first quintile, highest yielding stock will necessarily return the highest yield. Therefore, prediction accuracy is a proxy portfolio performance.

The first measure of prediction accuracy for the SVM algorithm appears during training and cross validation, noted in Section 4.4.4, where the model is trained and validated on historical data and known outcomes, i.e. next quarter return. Table 36 shows the accuracy of model validation, indicating that the SVM is between 63.0% and 64.3% accurate in determining whether a stock will yield positive return in the next quarter.

Table 36: Single-factor SVM cross validation

Factor	Average Accuracy	Standard Deviation	Mean/StdDev
6ROC	64.3%	6.9%	9.35
12ROC	64.2%	7.0%	9.12
BTM	63.6%	8.9%	7.17
EY	63.3%	9.4%	6.75
ROIC	63.0%	9.5%	6.60
ROE	63.0%	9.6%	6.55
DY	63.3%	9.7%	6.54

Training and validation is followed by prediction, where current factor data is applied to the trained model to predict future stock direction. Simplistically, prediction accuracy is determined through a post-hoc comparison of the predicted classification, positive or negative return, to the actual direction of return. The prediction accuracy for the SVM approach, shown in Table 37, ranges from 60% to 65%. When comparing the prediction accuracies of the SVM approach to that of the FBI approach the results indicate that SVM accuracy is superior for all factors except for but return on invested capital. This aligns well will the financial performance comparison in Section 5.1,

where those single-factor portfolios that achieved a higher accuracy also yield a higher average Q1 portfolio return. This suggesting that prediction accuracy and first quintile portfolio performance are closely related. The difference in prediction accuracy of the book to market based portfolio does not align with the difference in portfolio return, suggesting that accuracy is not an absolute measure of portfolio success. This discrepancy can potentially be attributed to the magnitude of magnitude of stock movement. The SVM algorithm ranks stock base on the probability of directional movement, and not the magnitude of movement. High accuracy portfolios with non-performing returns could be attributed to misclassification of a positive yielding stock to a lower quintile, that subsequently performs well.

Table 37: SVM vs FBI prediction accuracy comparison

	SVM Q1 Accuracy	FBI Q1 Accuracy
6 Month Price Rate of Change (6ROC)	60%	55%
12 Month Price Rate of Change (12ROC)	61%	57%
Book to Market Value	65%	56%
Dividend Yield	62%	59%
Earnings Yield	64%	61%
Return on Equity	64%	60%
Return on Invested Capital	62%	64%
Momentum (6ROC, 12ROC)	66%	58%
Value (DY, EY, BTM)	64%	56%
Quality (ROE, ROIC)	64%	60%
All factors (6ROC, 12ROC, DY, EY, BTM, ROE, ROIC)	62%	58%

When comparing these finding to related studies, Udomsak (2015) find an average SVM prediction accuracy of 56% on the Stock Exchange of Thailand, though it should be noted that they predict the direction of an entire exchange and not individual stocks. Arik et al. (2013) achieved a training and validation accuracy of 71.2% when classifying bullish stocks, yet found that prediction was less accurate at 58.8%. Similarly, Kim (2003) achieve their best training and validation performance, following parameter tuning, of 64.7% yet find that prediction accuracy diminished to an average of 57.8%.

A large disparity between cross validation accuracy and prediction accuracy is indicative of an overfitted model that is not able to generalise is training to out of sample data. The cross-validation accuracy noted in this study appears to be within range, all be it slightly lower, than those found

in similar studies. However, prediction the accuracy found in this study, of between 60% and 65% is somewhat higher than found similar studies. This indicates that, in this study, the choice of input data, SVM kernel and model parameter, C , have been selected with sufficient care to yield a predictive model capable of generalised prediction.

5.4 Research Limitations and Future Research

This study has presented challenges that could not be fully evaluated due to scope and time constraints. Each limitation, along with attempts to mitigate negative effects are discussed. These are followed by suggestions for future research to improve the fidelity of this study.

5.4.1 Misalignment of factor based investment with empirical studies

The evidence presented in this study suggests that there is no price premium present in the 6 and 12-month price rate of change, as well as dividend and earning yield factors. This conflict with findings by Muller and Ward (2013), Strugnelli et al. (2011), Van Rensburg (2001) and Van Rensburg and Robertson (2003) among others. This discrepancy could be attributed to differences in data used, the timeframe of portfolio construction or the method of portfolio construction. It is suggested that the results of a benchmark study be recreated and the performance of machine learning based portfolios subsequently be compared to factor based portfolios that do yield significant return.

5.4.2 Using a single machine learning algorithm

A primary limiting factor to the research lies in the complexity associated with the machine learning algorithm and its application to a classification problem. The literature review has highlighted numerous algorithms which have been used in financial forecasting and portfolio construction. Each algorithm has several possible variations addressing circumstantial shortcomings to improve prediction accuracy. In this study, only a single algorithm, the Support Vector Machine, has been used to illustrate the effect of improved prediction accuracy on portfolio construction. There is a possibility that a machine learning algorithm exists that is capable of producing higher accuracy classification. Finding such an algorithm would only be possible following an extensive empirical analysis, which is not in the scope of this study. This limitation has been minimised by conducting a thorough literature review and selecting an algorithm has

been used in a similar application. It is suggested that future work include further empirical comparison of machine learning algorithms in classification and portfolio construction using factor-based inputs.

5.4.3 Transaction Costs

This study is further limited by its inability to fully simulate the costs involved in rebalancing portfolios, i.e. in buying and selling stocks. Although broker and transaction fees can be established, the costs inherent in the bid-ask spread can only be estimated, due to the retrospective nature of the research. It is impossible to know at what exact price certain stocks could be purchased or sold for. This simulated marketplace does not accurately account for the costs associated with the bid-ask spread and therefore makes it difficult to directly compare the simulator returns to real-world returns. This limitation is minimised by constructing a portfolio from the largest 40 companies listed on the JSE, as identified by the FTSE/JSE Africa Top 40 Index. The stock liquidity inherent in these companies minimises the risk where sought-after quantities of stock would not be available at market-ask prices, and where sufficient buyers would not be available at market-bid prices.

5.4.4 Historical Availability Bias

Under ideal portfolio management circumstances, an investment strategy can select stocks from the full range traded on the JSE. For reasons of practicality a subsample of 100 listed stocks, ranked by Market Capitalisation, and filtered for liquidity and the availability of the 15-year historical data, was chosen for this study. Survivorship bias was avoided by including stocks up to the date that they were listed. The top 100 stocks account for 96% of the total market value, and therefore represent a significantly sizeable cross-section, however, the omission of the remaining 4% small cap stocks potentially introduces a bias in the findings. Furthermore, by omitting stocks with price histories of less than 15 years, this study potentially introduces a bias only applicable to companies that have existed for a longer time. This study did not evaluate the shortest historical dataset necessary for training, machine learning model validation and prediction. Further research could use the full contingent of stocks listed on the JSE, still filtering for liquidity, and apply machine learning to stocks with shorter histories.

5.4.5 Buy-Hold Time Horizon

This study employs a monthly rebalancing strategy with a 3 month buy-hold period. In other words, on the first day of each month a portfolio is constructed, using either the Factor Bases or Machine Learning investment strategies, and sold three months later. This method is generally considered to allow for the highest frequency of portfolio rebalancing, whilst limiting the costs associated with stock trading, which in turn allows for portfolio comparison to index funds. If trading cost is to be considered, and the performance of Machine learning based portfolios compared to market benchmarks, this buy-hold strategy would need to be evaluated for its cost efficiency. Furthermore, studies such as Brown et al. (2008), evaluate the efficacy of factor based investing, compare the results using different buy-hold time horizons. Future research could include comparisons of longer- and shorter-term predictions to evaluate the efficacy of machine learning based portfolio construction given such conditions.

5.4.6 Factor Selection

This study considered 10 single-factor and 4 multi-factor scenarios for evaluating the efficacy of machine learning based portfolio construction. Studies focused primarily on factor based investing such as Muller and Ward (2013) consider up to 27, and studies focused solely on machine learning based portfolio performance such as Huerta et al. (2013) consider up to 25, split between technical and fundamental factors. The reduced set of factors considered in this study is due to its comparative nature, factor based investing vs. machine learning based investing. A more thorough study would include the full range of factors noted in literature to highlight the superior performance of machine learning as it compare to factor based investing

6 Conclusions

This study applies machine learning to a factor based investment strategy with the goal of investigating the potential for improved portfolio return. The support vector machine learning algorithm is chosen as an appropriate model for classification.

The SVM algorithm is trained and validated on historical factor data and stock yield, and used to rank stocks based on the likelihood of share price increase, given a 3-month holding period. The ranking was applied to the 100 largest stocks on the JSE, filtered by size, liquidity and availability of historical data. Ranked stocks are divided into quintiles which form the basis for comparison. First quintile comparisons between Factor Based Investment and Support Vector Machine portfolios are used as a measure for the success of an investment strategy.

The following questions were investigated, and conclusions drawn:

Does a factor based investment portfolio yield higher returns than an equally weighted market benchmark for single factor implementations?

This question has been widely explored in literature without achieving consensus about what factors are most indicative of mispriced stock and how subsequently exploit them to improve portfolio yield. This study applies techniques, similar to those implemented by Muller and Ward (2013); Strugnell et al. (2011); Van Rensburg (2001); and Van Rensburg and Robertson (2003), to add to empirical findings relevant to the JSE, while also establishing a baseline for comparison with machine learning based investment portfolios.

We found that of the seven factors tested, only Book-to-Market Value (BTM), return on equity (ROE) and return on invested capital (ROIC), yielded first quintile average returns greater than the average return of an equally weighted benchmark. First quintile returns based on a 12-month price rate of change (12ROC) and earning yield (EY), respectively, did not differ significantly from the benchmark, whereas those based on a 6-month price rate of change (6ROC) and dividend yield (DY) underperformed the benchmark. These findings appear to support some studies, but in contrast with others, highlight the variability of factor based investing and sensitivity of success to the parameters used in a specific empirical study. This was followed by the application of machine learning in portfolio construction using single factor inputs.

Does a machine learning (support vector machine) based investment portfolio yield higher returns than an equally weighted market benchmark for single factor implementations?

A support vector machine, using historical single factor input data and used to rank stock, based on the probability of stock price improvement, resulted in first quintile average returns greater than the equally weighted benchmark for five of the seven factor tested; 6 and 12-month price rate of change; earning yield; Book-to-Market Value; and return on equity. The remaining two factors, dividend yield and return on invested capital yielded average returns that were statistically similar to the equally weighted benchmark. Critically, no single factor strategy implemented using the SVM approach underperformed the benchmark. This leads to the question of how the two strategies compare directly.

Does a machine learning (support vector machine) based investment portfolio yield higher returns than its factor based counterpart for single factor implementations?

A pared difference test showed that the SVM approach outperforms the FBI approach, in terms of the first quintile average monthly return, for a 6 and 12-month price rate of change, dividend yield, earnings yield and return on investment. The Book-to-Market portfolios performed similarly, whereas FBI yielded a higher average return when using return on invested capital. We therefore cannot conclude, categorically, that SVM based portfolios always perform as well as or better than FBI based portfolios, due to the ROIC exception. This is however the case for the majority of factors tested. This prompted a further investigation into the effect of portfolio return using multiple factors as inputs to the SVM.

Does the use of multiple factors as inputs to the machine learning base investment portfolio improve the efficacy of portfolio construction?

When combining factors for momentum, value and quality separately we find that return for both value and quality first quintiles significantly outperform the benchmark. The momentum based portfolio also achieved a greater return than the equally weighted benchmark, but only to the 94% confidence level. Critically, none of the three combined portfolios outperform their best performing constituent factors, indicating that the SVM algorithms is not effective in combining multiple factor to improve prediction accuracy. The combination of all factors yielded the highest average monthly return of all single or multi factors tested. The application of a multi-factor input

to a machine learning based investment portfolio did have an impact of portfolio performance in that it yielded the highest return whilst using all available information.

This study is significant to literature because it illustrates the application of machine learning the problem of factor based investing. It shows how all available information can be combined, without arbitrary weighting, to a single portfolio construction rule that utilises all available data. Limitations to these findings stem from a limited set of stocks used to construct portfolios, especially for the requirement of historical data used in model training. Furthermore, by discounting the cost of stock purchase, it is difficult to relate portfolio returns to real-world market benchmarks.

Further research should improve the fidelity of the SVM application by incorporating all stock traded on the JSE; using the history of each stock up to the date of prediction for model training allows for companies with shorter stock histories to be used; testing different buy-hold timeframes; and finally, including more factors in the analysis with the explicit goal of improving prediction accuracy.

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