Towards Prediction of Financial Crashes with a D-Wave Quantum Computer

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Prediction of financial crashes in a complex financial network is known to be an NP-hard problem, i.e., a problem which cannot be solved efficiently with a classical computer. We experimentally explore a novel approach to this problem by using a D-Wave quantum computer to obtain financial equilibrium more efficiently. To be specific, the equilibrium condition of a nonlinear financial model is embedded into a higher-order unconstrained binary optimization (HUBO) problem, which is then transformed to a spin-1/2 Hamiltonian with at most two-qubit interactions. The problem is thus equivalent to finding the ground state of an interacting spin Hamiltonian, which can be approximated with a quantum annealer. Our experiment paves the way to study quantitative macroeconomics, enlarging the number of problems that can be handled by current quantum computers.

I. INTRODUCTION

Even though economics is not an exact science, there exist theoretical analyses which can, in principle, quantify and model certain processes in the financial markets like physical laws do for the universe. Following this idea, quantitative finance and economics emerged. They were applied to understand the evolution of financial markets and economies, as well as to forecast their possible future. A realistic question in risk management is: would there be a drastic drop in the market values if the prices of assets suffer some small perturbation? The cross-holdings and nonlinear character of financial network dynamics will cause chain reactions, implying that sudden drops of a market value might affect other nodes in the network resulting in a financial crisis. Presently, the prediction of crashes is mainly performed by studying previous cases in history and comparing with the current configuration [1–6]. While this empirical approach has been succesful [7], the economic environment is constantly evolving. Hence, we cannot limit ourselves to predicting economic disasters which are qualitatively similar to past events. Therefore, ab initio simulations of financial networks will become essential for avoiding financial crises. This problem was recently shown to be NP-Hard [8], i.e., virtually impossible to be solved with classical computers. Indeed, given the global knowledge of a financial network, the time to compute the consequences of a perturbation would by far exceed the age of the universe.

An alternative approach to this problem was presented in Ref. [9] due to its efficiency to find the ground state of a complex system. In particular, a mathematically identical problem is simulated and the corresponding result measured [10–13]. Specifically, it was shown that obtaining the equilibrium configuration of a financial network is equivalent to solving a higher-order unconstrained binary optimization (HUBO) problem, which should be feasible for a quantum annealer allowing for multi-qubit interactions. Unfortunately, this hardware has not been realized yet, as state-of-the-art quantum annealers are restricted to two-qubit interactions [14]. A possible workaround, which comes at the price of introducing ancillary qubits, is to find an effective Hamiltonian with the same low-energy subspace and two-qubit interactions at most. This leaves us with a quadric unconstrained binary optimization (QUBO) problem to solve, which encodes the equilibrium configuration of a financial network. This problem can be solved using a quantum annealer, such as the D-Wave 2000. An analysis of the changes experienced by the financial network to reach its equilibrium configuration will tell whether a crash has occurred.

This paper experimentally validates the study presented in Ref. [9]. Specifically, we compute the equilibrium configuration of a financial network after perturbation with a D-Wave 2000 quantum annealer, and compare the result to alternative methods. Although the D-Wave machine has been successfully used to solve problems in condensed matter physics [15–19], engineering [20], cryptography [21], biology [22], and quantitative finance [23, 24] among others, it is the first time that quantum annealing is applied to solve a problem of macroeconomics. This should attract more attention from the finance and economic disciplines towards quantum computing [25, 26], as well as enlarge the amount of feasible problems for quantum annealers.

The contents are organized as follows: in Sec. II, we introduce the model of financial network that will be considered. Sec. III reviews the quantum annealing algorithm to find financial equilibrium. Sec. IV experimentally proves the validity of the scheme by finding the financial equilibrium of a random network of the largest implementable size with a D-Wave 2000 quantum an-

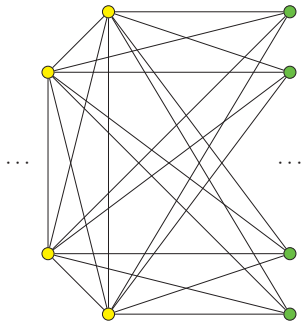

FIG. 1. Example of a financial network: the yellow nodes and green nodes denote institutions and assets, respectively. Links denote ownerships and cross-holdings.

nealer; for this network, we also show experimentally how the scheme allows to compute the financial equilibrium. Sec. V analyzes the achieved results and discusses further possible improvements. The conclusions drawn from the work are shown in Sec. VI.

II. FINANCIAL NETWORK MODEL

A simple network model for financial markets is proposed in Ref. [27]. It is made up of n institutions and m assets, and aims at representing the market values of institutions. The concept of institution and asset is quite general, i.e., institutions could be countries, banks, companies, and the like, while assets are any type of object (or derivative object) with intrinsic value. The ownership of assets is shared by the institutions. Meanwhile, there are cross-holdings between institutions, modeling the fact that part of an institution could be owned by other institutions, e.g., sovereign bonds between countries, or debt contracts between companies. Under these assumptions, the model can be mapped onto a network, as shown in Fig. 1. We codify the prices of the m assets by an $m-$dimensional vector $\vec{p}$, where the element p_k represents the price of asset k. Moreover, an $n \times m$ ownership matrix $\mathbf{D}$ can be defined such that the element $D_{ik} \geq 0$ corresponds to the percentage of asset k owned by institution i. There is also an $n \times n$ ownership matrix $\mathcal{C}$ that describes the cross-holdings and self-ownerships between institutions. The coefficients $\mathcal{C}_{ij}$ denote the percentage of institution j owned by institution i. By considering all self-ownerships (i.e., the diagonal elements) from $\mathcal{C}$ one forms a new diagonal matrix $\tilde{\mathbf{C}}$ which represents the self-ownership only, such that matrix $\mathbf{C} = \mathcal{C} - \tilde{\mathbf{C}}$ codifies all cross-holdings. The equity value V_i of institution i is defined by summing up its ownership of all assets and cross-holdings, $V_i = \Sigma_k D_{ik} p_k + \Sigma_j C_{ij} V_j$. One thus obtains a matrix equation $\vec{V} = \mathbf{D}\vec{p} + \mathbf{C}\vec{V}$, where equity value vector $\vec{V}$ is an $n-$dimensional vector. Accordingly, the market value is the equity value rescaled with its self-ownership, resulting in the market value vector $\vec{v} = \tilde{\mathbf{C}}\vec{V}$.

The solution to the linear matrix equation thus reads

$$\vec{v} = \tilde{\mathbf{C}}(\mathbf{I} - \mathbf{C})^{-1}\mathbf{D}\vec{p}. \tag{1}$$

The next important aspect to be considered in the model is *panic*. This is a nonlinear effect such that, if the market value of an institution drops below some threshold, then it discontinually drops further by some quantity. This models two well-known properties of the financial market: (i) the income of an institution will drop suddenly once it cannot afford its operating costs; and (ii) the market value of an institution will be reduced drastically due to the loss of trust from investors, if the credit rating is downgraded. This may cause an institution's value to *crash*, a behavior which can infect other nodes in the network. This can be simulated with a Heaviside-theta function Θ; if the market value v_i drops below the critical value v_c^i, *failure* of institution i occurs and its equity value drops by $\beta_i(\vec{p})$ which is governed by the price vector of assets. Once we define the failure vector $\vec{b}(\vec{v}, \vec{p}) = \beta_i(\vec{p})(1 - \Theta(\vec{v} - \vec{v_c}))$, the market value vector with nonlinearity can be written as

$$\vec{v} = \tilde{\mathbf{C}}(\mathbf{I} - \mathbf{C})^{-1}(\mathbf{D}\vec{p} - \vec{b}(\vec{v}, \vec{p})). \tag{2}$$

Mathematically, it is the nonlinearity of $\vec{b}(\vec{v}, \vec{p})$ which makes financial networks so hard to be predicted.

III. QUANTUM ANNEALING ALGORITHM

The algorithm proposed in Ref. [9] was employed to find the equilibrium configurations of financial networks. In particular, finding financial equilibrium can be represented as the minimization of an objective function for which the optimal configuration of variables represents the equilibrium state, and this is, in turn, equivalent to finding the ground state of a classical spin Hamiltonian.

By squaring Eq. (2), we obtain an objective function that meets its minimum value when the market value state is set to be the equilibrium state

$$Obj(\vec{v}) = (\vec{v} - \tilde{\mathbf{C}}(\mathbf{I} - \mathbf{C})^{-1}(\mathbf{D}\vec{p} - \vec{b}(\vec{v}, \vec{p})))^2. \tag{3}$$

Thus, our task is now to find the $\vec{v}$ that minimizes $Obj(\vec{v})$ for a given financial network.

Next, we need to deal with the nonlinear terms (modeling failure) of the objective function. The reason is that once the objective function is transformed to a spin-1/2 Hamiltonian, it should ideally be made of polynomial terms only, due to the limitations of quantum annealers. Thus, one expands the failure terms with Heaviside-theta functions in terms of polynomials. This expansion is not unique, and here we choose the Legendre expansion [9],

$$\Theta(x) = \frac{1}{2} + \sum_{l=1}^{\infty}(P_{l-1}(0) + P_{l+1}(0))P_l(x), \tag{4}$$

in the domain $[-1,1]$, with $P_l(x)$ to be the l-th Legendre polynomial. By setting $x=(v_i-v_i^c)/v_{max}^i$, Eq. (4) enables us to expand $\Theta(v_i-v_i^c)$ in the range of $v_i\in[0,v_{max}^i]$. Using this expansion as an example, we take the approximation $b_i(v_i,\vec{p})\approx\beta_i(\vec{p})\mathrm{Poly}_r(v_i-v_i^c)$, where Poly_r denotes some polynomial of degree r. The polynomial expansion removes the discontinuity while maintaining the strong nonlinearity of the network.

We now encode the continuous variables v_i with classical bits. This will allow rewriting the resulting objective function in digital form. The expansion is straightforward, and reads $v_i=\sum_{\alpha=-\infty}^{\infty}x_{i,\alpha}2^\alpha$. However, due to the limited resources in real-world devices, one must truncate this expansion, i.e., $v_i\approx\sum_{\alpha=-q}^{q}x_{i,\alpha}2^\alpha$, where $x_{i,\alpha}$ are classical bits with binary values 0 or 1. In this way, the market value of institution i is encoded with $2q+1$ classical bits. The maximal market value v_i^{max} is given by $\sum_{\alpha=-q}^{q}2^\alpha$.

The resulting objective function is a polynomial of binary variables $x_{i,\alpha}$ of degree $2r$. To express it as a spin-1/2 Hamiltonian, we replace the binary variables $x_{i,\alpha}$ by qubit operators $\hat{x}_{i,\alpha}$ with eigenvalues 0 and 1, i.e., $\hat{x}_{i,\alpha}|0\rangle=0$, $\hat{x}_{i,\alpha}|1\rangle=1$. The Pauli-$z$ operator satisfies $\hat{x}_{i,\alpha}=(1+\hat{\sigma}_{i,\alpha}^z)/2$, and therefore the Hamiltonian reads

$$\hat{H}=\mathrm{Poly}_{2r}(\hat{\sigma}_{i,\alpha}^z), \tag{5}$$

which is equivalent to our objective function but written with Pauli matrices. This Hamiltonian includes all types of multi-spin interactions, up to $2r$-body terms.

The Hamiltonian obtained in Eq. (5) is appropriate for a quantum annealer that allows many-qubit interactions. However, state-of-the-art quantum annealers only accept inputs with at most two-qubit interactions. Finding the ground state of a spin-1/2 Hamiltonian, as the one obtained in Eq. (5), is equivalent to solving a Quadratic Unconstrained Binary Optimization (QUBO) problem, which is the input of the quantum annealer. Thus, we should recast our quantum Hamiltonian into a modified, effective Hamiltonian with two-qubit interactions at most. A protocol achieving exactly this is proposed in Ref. [28], where k ancilla qubits are introduced to implement an effective k-qubit interaction. Suppose that there is a k-qubit interaction term $\hat{H}_k=J_k\Pi_{i=1}^k\sigma_i^z$ with the same low-energy spectrum of another Hamiltonian term $\tilde{H}_k$ with at most two-qubit interactions. We can express $\tilde{H}_k$ with k logical qubits and k extra ancilla qubits as

$$\begin{aligned}\tilde{H}_k=&J\sum_{i=2}^{k}\sum_{j=1}^{i-1}\hat{\sigma}_i^z\hat{\sigma}_j^z+h\sum_{i=1}^{k}\hat{\sigma}_i^z\\&+J^a\sum_{i=1}^{k}\sum_{j=1}^{k}\hat{\sigma}_i^z\hat{\sigma}_{j,a}^z+\sum_{i=1}^{k}h_i^a\hat{\sigma}_{i,a}^z,\end{aligned} \tag{6}$$

as represented in Fig. 2. This two-qubit Hamiltonian has the same low-energy spectrum than $\hat{H}_k$ when J, J^a, h and h_i^a are set to some appropriate values. As Ref. [28] suggested, this can be achieved once $q_i=(-1)^{k-i+1}J_k+q_0$, $h=-J^a+q_0$, $h_i^a=-J^a(2i-k)+q_i$ and $J=J^a$, with any q_0 that satisfies $|J_k|\ll q_0<J^a$ and $|J_k|\ll J^a-q_0<J^a$. These conditions can be relaxed to $|J_k|<q_0<J^a$ and $|J_k|<J^a-q_0<J^a$ if one aims at having the same ground state only, instead of the whole low-energy sector.

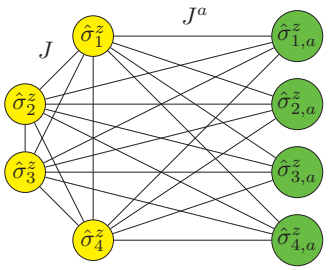

FIG. 2. Hamiltonian with two-qubit interactions only and with the same low-energy spectrum than that of some Hamiltonian with k-qubit interactions, where k ancilla qubits are introduced (k=4 in the illustration).

IV. IMPLEMENTATION IN A D-WAVE 2000 QUANTUM ANNEALER

Once shown that it is possible to recast the problem of finding financial equilibrium into a language that is amenable to quantum annealers, this section deals with its implementation using a state-of-the-art quantum annealer, namely, the D-Wave 2000. This quantum annealer consists of 2048 qubits connected according to the Chimera graph topology (see Fig. 3). It is designed to solve embedded Ising problems or QUBO problems.

Two simulations were produced:

1. A financial network without failure term, which is simple to solve on a classical computer in order to benchmark the performance of the quantum processor.

2. A financial network with the inherently nonlinear risk of failure. We will perturb the asset price vector in this network to compute the new equilibrium configuration using the quantum annealing algorithm.

We initially generate a financial network with 10 institutions and 15 assets. To demonstrate the algorithm, we randomize the ownership matrix $\mathbf{D}$ with a Dirichlet distribution that satisfies $\sum_{i=1}^{n}D_{ij}=1$, where D_{ij} are random variables. The cross-holding matrix $\mathcal{C}$ is generated in a similar way but with a constraint that all diagonal elements should be larger than 0.5, ensuring that all institutions can make decisions according to their own wills. Thus, we randomize $\tilde{C}_{ii}$ between 0.5 and 1 and randomize $\sum_{i=1}^{n}C_{ij}=1-\tilde{C}_{jj}$ with a rescaled Dirichlet distribution. The price vector $\vec{p}$ is also random, with $p_i\in[10,40]$. The network configuration is shown in Figs. 4 and 5.

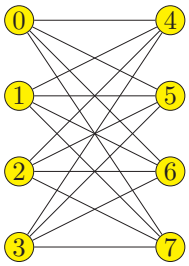

FIG. 3. Chimera graph topology implemented by the D-Wave 2000 quantum annealer. The 2048 qubits are partitioned into subgraphs of 8 qubits. The connection between subgraphs is sparse, as each qubit is connected only to an adjacent qubit of a different subgraph.

We can calculate the equilibrium state $\vec{v_q}$ and the equity value vector $\vec{V}$ on a classical computer using

$$\vec{v_q} = \tilde{\mathbf{C}}(\mathbf{I} - \mathbf{C})^{-1}\mathbf{D}\vec{p}, \tag{7}$$

$$\vec{V} = (\mathbf{I} - \mathbf{C})^{-1}\mathbf{D}\vec{p}. \tag{8}$$

The objective function shown in Eq. (3) was implemented, for benchmarking reasons, both in a quantum annealer and a classical simulator. Variables v_i were encoded, $v_i = \sum_{\alpha=0}^{7} 2^\alpha x_{i,\alpha}$, on seven qubits. As such, this constrains the v_i to be integers smaller than 127. A quantum implementation of this algorithm does not require ancilla qubits, as there are no many-qubit interactions.

The QUBO for this linear problem is a 70×70 matrix, with 210 couplers which cannot be solved directly due to the topology structure of the quantum annealer. D-Wave provides a software named *qbsolv* that allows to combine the classical computer with its quantum annealer by splitting the QUBO matrix into partition matrices that can be embedded in the quantum annealer. As a decomposing solver, it finds a minimum value of a large QUBO problem by splitting it into pieces solved either via a D-Wave system or a classical tabu solver (both approaches were considered here for comparison purposes). Since the D-Wave 2000 processor is a quantum annealer, 20 results would be obtained from a *qbsolv* process with a default setting; these results should be handled by a correction process, e.g., majority voting, to help us identify the most plausible answer. The result of this QUBO problem is shown in Fig. 6, where the exact solution via solving a linear matrix equation, *qbsolv* solution with classical tabu solver, and *qbsolv* solution with D-Wave quantum annealer, are compared. It is straightforward to observe that a quantum annealer provides a similar solution to the exact solution than the classical tabu solver.

While the failure-free model only has linear terms in v_i, the nonlinear model has powers of v_i up to order $2r$, as shown in Eq. (5). For large r, this can be extremely expensive in ancilla qubits. An estimation of the number of qubits can be made by counting the number of interaction terms; Eq. (5) indicates that $\hat{H}$ can have up to $\sum_{\alpha=0}^{2r} C_\alpha^{n(2q+1)}$ terms, where $n(2q+1)$ denotes the

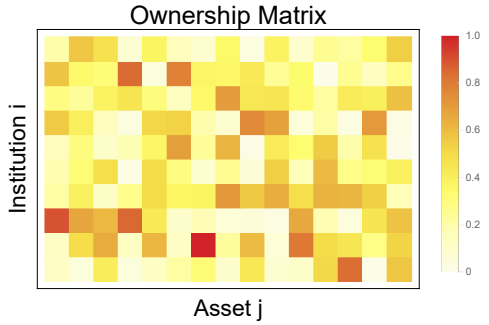

FIG. 4. Ownership matrix.

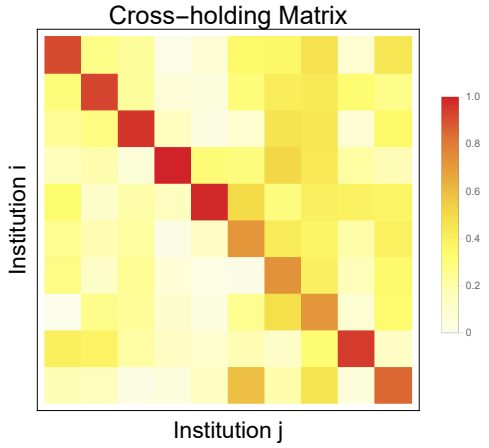

FIG. 5. Cross-holding matrix.

logical qubits that are required. In each term, 3-to-$2r$ new ancilla qubits are needed, depending on the number of logical qubits in this term. Therefore, the number of necessary qubits grows rapidly with the degree of the polynomial expansion r. Note that this problem is NP-hard for any $n \geq 2$. In practice, this is an upper bound to the required resources, calculated assuming that $\hat{H}$ has all possible terms up to order $O(2r)$.

Here, we implement an enhanced model with failure terms on the basis of the linear model previously simulated. We perturb the vector of asset prices, leaving the ownership matrix $\mathbf{D}$ and cross-holding matrix $\mathcal{C}$ invariant, and recompute the equilibrium state. Specifically, we set the price of some random assets to zero (to simulate, e.g: the assets' destruction). In this study, we will use an expansion of $\hat{H}$ to third order, which still characterizes the phenomenon of sudden drop near the critical value. Moreover, this approach provides strong nonlinearity while saving plenty of qubit resources. As a result, 70 logical qubits and 872,690 ancilla qubits are required with 4,446,575 couplers in the problem. This results in the requirement of about 6TB memory, since each element has an accuracy of double float in *qbsolv*. Due to the limitations of state-of-the-art techniques, the network is reduced to three institutions and each market value v_i is encoded by five qubits,

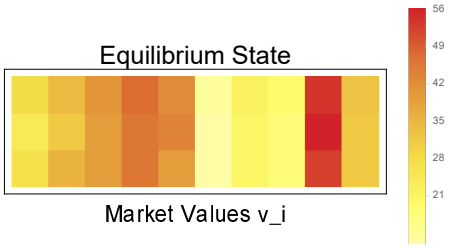

FIG. 6. Linear-model result. The first row shows the result if the matrix equation is solved exactly, the second row if *qbsolv* with tabu classical solver is used, and the third row if *qbsolv* with D-Wave 2000 solver is employed.

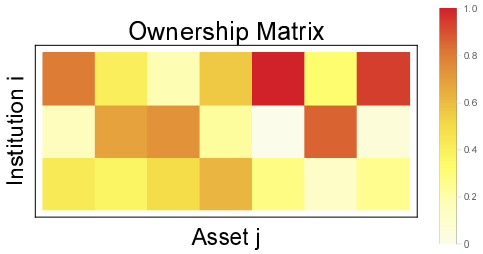

FIG. 7. Ownership matrix for the implemented network.

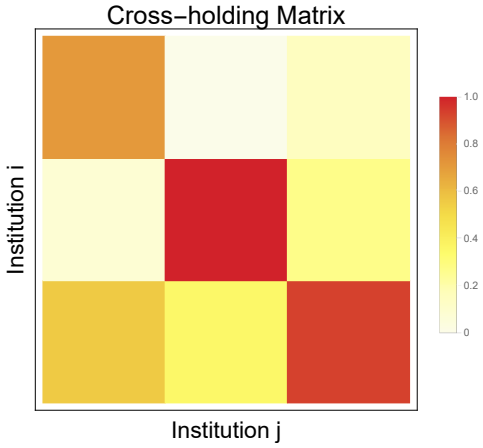

FIG. 8. Cross-holding matrix for the implemented network.

bounding the maximum to be 31. New 3×7 ownership matrix $\mathbf{D}$ and 3×3 cross-holding matrix $\mathcal{C}$ are generated while the price vector $\vec{p}$ before perturbation is $\vec{p} = \{8.43,\ 14.47,\ 6.75,\ 8.09,\ 19.11\ ,11.32,\ 7.19\}^T$. The network configuration is shown in Figs. 7 and 8. The equilibrium state before perturbation without nonlinearity is given as $\vec{v_q} = \{21.18\ \ 23.33,\ \ 30.83\}^T$, and the critical value vector is still set to be 80% of the original equilibrium state, while the failure strength $\vec{\beta}$ is considered to be 30% of the original equity value. The corresponding perturbed price vector is given as $\vec{p} = \{8.43,\ 14.47,\ 0,\ 8.09,\ 0\ ,11.32,\ 7.19\}^T$. Before calculating the new equilibrum state with nonlinearity and perturbation, some parameters, like J^a and q_0, must be set. For the minor embedding of a submatrix in the D-Wave quantum annealer, this is done by introducing a penalty function between qubits in the Chimera graph requiring $J^m \geq J^a$, which means that the J^a for mapping multi-qubit interactions to two-qubit interactions should be in the proper scale. Meanwhile, as we mentioned in the theory part, we need to sample out the thermal fluctuation by assuming that $|\hat{H}_k|$ is much smaller than J^a, or the protocol will break down because those ancilla qubits will not be in the corresponding ground state anymore. Thus, in the implementation we took $J^a = 20J_k$ and $q_0 = 10J_k$, such that this could ensure that either q_0 or $J^a - q_0$ would be at least 10 times larger than J_k.

For this problem, the QUBO matrix had the size of 8280×8280, with 15 logical qubits, 8265 ancilla qubits and 38,790 couplers. Remark that the available quantum annealer structure is not optimized for this problem and, also, that the translation is not efficient because of sparse connectivity of the quantum processor. Therefore, the only benefit of a quantum annealer here is to implement exponential acceleration [29] in optimizing submatrices generated by the partition algorithm in *qbsolv*. Finally, we compare our results from the quantum annealer with the integer equilibrium solution calculated in a straightforward method by trying 32^3 times in Fig. 9, which shows a good agreement.

V. RESULTS AND DISCUSSION

D-Wave is a quantum annealer designed to deal with QUBO problems, e.g., Ising model. However, the problem faced in this paper, namely, financial crisis prediction with nonlinearity associated to panic, is not QUBO but HUBO instead, thus requiring multi-qubit interactions. In order to approximate this HUBO problem with two-

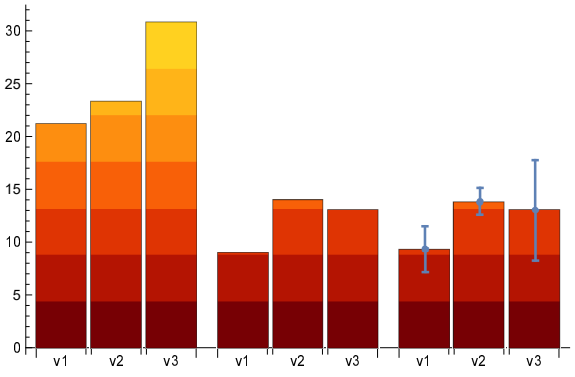

FIG. 9. The first group (left) is the equilibrium without failure term before perturbation. The second group is the integer result of the implemented network with polynomial expansion (center). The third group is the result from *qbsolv* software with D-Wave 2000 (right). The error bar characterizes a 95% confidence interval.

qubit interactions, at the current stage of hardware and software we were limited to simulate a small financial network, made up of three institutions and cross-holdings.

An effective two-qubit quantum Hamiltonian could still not be read directly in D-Wave system which requires QUBO type input or Ising type input. Although some open-source software like *pyqubo* can generate it, the input size must be very small in order to avoid a stack overflow associated with recursion errors. A possible solution is to produce a Mathematica script that reads each term, write it as a string of coefficients and qubits in an input file for the D-Wave system. Once we generate the input for this problem, this is still too large to be embedded in the D-Wave 2000 quantum annealer because of the graph structure. Thus, *qbsolv* is an inevitable option for us, which works by separating the large matrix to submatrices and solve them by a classical tabu solver or D-Wave solver. This kind of hybrid computation provides the possibility to solve the complicated problem but brings some new constraints, namely: (i) *Local hardware.* Once the QUBO matrix is provided, *qbsolv* allocates dynamic memory before separating it to submatrices with elements of double precision floats, by requiring a size of $8n \times n$ bytes of memory. However, the bottleneck is not the memory size but the performance of CPU since a large QUBO matrix will consume exhaustive CPU time if one needs high accuracy of the optimized result; (ii) *Algorithm.* Instead of a real quantum annealing process for the whole matrix, *qbsolv* provides a tabu algorithm or D-Wave 2000 quantum annealer for submatrices. The partition strategy for generating submatrices may get stuck in a local minimum instead of the global minimum that quantum annealing guarantees. Considering that the logical qubits only encode less than 1% in the QUBO matrix, the risk of getting stuck is still high, even if we sample over the thermal distribution or give a huge repeat limitation in the main loop to improve its accuracy. We would have to customize a random seed for the separation, and check the final result manually, to see whether the result is near from the equilibrium. Another option is that one may send the QUBO matrix to the solver many times and average the result to obtain the best solution. (iii) *Quantum annealer.* The submatrices will be sent to D-Wave 2000 quantum annealing device for optimization after they are generated by Glover's algorithm [30]. In the quantum annealing process, magnetic fields are applied to the processors and the strength should be accurate because J^k, J^a in the QUBO matrix and J^m for the embedding belong to different magnitudes. Any imprecision in the system preparation will cause significant deviation from the correct result.

In this implementation, the accuracy is not especially high, since we are not optimizing the objective function rigourously because the market values are integers $v_i \in [0, v_{max}]$ constrained by the qubits we take to encode them. The computation time is also long, considering that there is a straightforward but equivalent classical algorithm by testing the value of the objective function

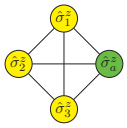

FIG. 10. An efficient encoding of three qubits, making use of only one ancilla qubit.

32^3 times. Although mapping it to a QUBO problem and optimizing it with a general quantum annealer is not efficient enough for current technology, we believe it is a valuable example of how one can solve an NP-hard problem via quantum computation. With quantum annealers designed for solving HUBO problems that allow multi-qubit interactions, an exponential acceleration can be realized in forecasting the behavior of complex financial networks. We expect a HUBO quantum solver may be available in the near future. Meanwhile, D-Wave has recently considered its next generation of quantum annealers [31]. It would consist of more than 5000 qubits connected with each other according to the Pegasus topology. In this manner, one could improve the number of qubits and the connectivity by a factor of 2.5.

Considering that a specialized quantum annealer for HUBO problems would not be available to the public anytime soon, we now analyze the possible ways to enhance the performance of D-Wave 2000 quantum annealer on this problem. After compromising on the maximum two-qubit interactions in hardware, the subsequent strategy will be reducing the number of ancilla qubits. With fewer ancilla qubits, the size and accuracy of a solvable network can be improved. As proposed in Ref. [28], the multi-to-two mapping is a general method, but for three-to-two, for example, a more efficient mapping can be constructed with only one ancilla qubit. Suppose there is a sub-Hamiltonian of three-qubit interactions

$$\hat{H}_3 = J_3 \hat{\sigma}_1^z \hat{\sigma}_2^z \hat{\sigma}_3^z. \tag{9}$$

A subgraph with full connectivity of three logical qubits and one ancilla qubit is shown in Fig. 10, where the equivalent Hamiltonian is given as

$$\tilde{H}_3 = J \sum_{i=2}^{3} \sum_{j=1}^{i-1} \hat{\sigma}_i^z \hat{\sigma}_j^z + h \sum_{i=1}^{3} \hat{\sigma}_i^z + J^a \sum_{i=1}^{3} \hat{\sigma}_i^z \hat{\sigma}_a^z + h^a \hat{\sigma}_a^z. \tag{10}$$

At variance with the previous protocol, $J^a = 2J > h$ and $h^a = 2h = 2J_3$. Also, for sampling out the thermal fluctuation, we take $J^a \geq J_3$, to prevent the protocol to fail for the same reason. The ancilla qubits can be reduced to about 7000 with this method. Meanwhile, the partition method in *qbsolv* may cause the system to get stuck in local minima which requires a better algorithm in the main loop.

VI. CONCLUSION

We have implemented in a D-Wave quantum computer the algorithm proposed in Ref. [9], to solve the equilibrium state of a complex financial network that predicts financial crashes. Although the size of the studied financial network is limited, this proof of principle is in agreement with the result of an exhaustive search. Moreover, this work is a convincing evidence that quantum computation can be used to study quantitative finance and help institutions anticipate risks. This result may be improved with the design of a customized "financial quantum annealer": a quantum processor with suitable connectivity for efficient embedding of this kind of problems. Such coherent quantum annealers can be built with current technology [32–34], providing convenient multi-qubit couplings.

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