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Interactive multiple model ensemble Kalman filter for traffic estimation and incident detection

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***Abstract*— This paper studies the problem of real–time traffic estimation and incident detection by posing it as a hybrid state estimation problem. An interactive multiple model ensemble Kalman filter is proposed to solve the sequential estimation problem, and to accommodate the switching dynamics and nonlinearity of the traffic incident model. The effectiveness of the proposed algorithm is evaluated through numerical experiments using a perturbed traffic model as the *true* model. The supporting source code is available for download at `https://github.com/Lab-Work/IMM_EnKF_Traffic_Estimation_Incident_Detection`.**

I. Introduction

A. Traffic estimation and incident detection as a hybrid state estimation problem

Traffic incidents are a major safety concern and a cause of congestion in highway networks. Real–time traffic incident detection is an effective way to reduce the impacts of a traffic incident, since prompt actions can be taken to save lives and recover normal traffic operations. While traffic estimation techniques have become widely used in recent decades for traffic monitoring, an estimation technique that simultaneously provides traffic state estimation and traffic incident detection is wanting. In this paper, we estimate the traffic state and traffic incidents in real–time on a freeway segment using a multiple model nonlinear filter.

The traffic estimation and incident detection problem is posed as a hybrid state estimation problem. The traffic evolution equations are constructed from a scalar macroscopic traffic flow model denoted by f, which evolves the traffic state x_{n-1} (i.e. a vector of densities along the roadway) at discrete time $n-1$ to time n. The system evolution and observation equations of the hybrid system are given by [1]:

$$\begin{aligned} x_n &= f\left(x_{n-1}, \gamma_n\right) + \omega_{n-1} \\ z_n &= h_n(x_n, \gamma_n) + \nu_n. \end{aligned} \tag{1}$$

For our specific problem, the model variable $\gamma \in \Gamma$ is a time varying vector and it denotes the integer number of lanes open at each discretized location along the freeway segment during the time period $(t^{n-1}, t^n]$, where Γ is the set of all possible road operating conditions. The term z_n is a vector of speed or density measurements, h_n is a nonlinear observation operator that relates the system state with the measurements, ω_n is the noise associated with the traffic model, and ν_n is the measurement noise. We use an additive noise model for both the evolution and observation equation.

Given the evolution observation system (1), the traffic estimation and incident detection problem can be posed as the problem of estimating the traffic state x_n and the model γ_n given measurements $\{z_1, \cdots, z_n\}$. Because of the nonlinearity of the traffic model and the switching dynamics in the system model, we propose an *interactive multiple model* (IMM) *ensemble Kalman filter* (EnKF) to solve the hybrid state estimation problem.

B. Related work on incident detection

The main challenges for real–time incident detection are related to sensing and interpretation of the sensor data. Due to the traditionally high cost of monitoring infrastructure, the number of sensors on the freeways is limited. If fixed sensors are located far from each other and a traffic incident occurs on the roadway between the sensors, it may take a long time for the effect of the incident to propagate to a sensor, where it can be registered as an anomaly. This also contributes to the difficulties in estimating the traffic state between the sensors since the true location of the incident may not be uniquely determined.

The traffic incident detection problem has attracted attention in the research community for several decades. A comprehensive review of incident detection efforts can be found in the review papers [2], [3]. The most well known incident detection algorithms are variants of the California algorithm [4]–[7]. These techniques exploit the idea that an incident will cause a significant increase in the occupancy recorded by an upstream loop detector, and a decrease in the occupancy recorded by a downstream loop detector. Then, a decision tree structure is used to determine the existence of an incident by comparing the difference and relative difference between the upstream and downstream occupancies. While the algorithms can detect incidents, they do not provide traffic estimates near the incidents.

The fault detection approach [8] estimates the traffic state and the traffic incident by using the scalar macroscopic traffic flow model [9], [10] to predict the traffic state. The work shows that a traffic incident leads to a drop of the traffic flow. A fault detection algorithm is exploited to detect the incident by comparing the estimated residual with a defined threshold. While this approach is able to predict the traffic state and estimate the traffic incident, the traffic estimates are no longer accurate when a traffic incident occurs, since the algorithm does not include any dynamics to describe the traffic evolution under incident conditions.

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The technique most related to this work is the *dynamic model* [11], which uses the second order macroscopic traffic model proposed by Payne [12], [13]. Multiple models are generated by instantiating a new equilibrium fundamental diagram for each incident severity. Then, a *multiple model* (MM) extended Kalman filtering approach is used to select the most likely model (similarly incident severity) and to produce filtered traffic states. The main limitation of [11] is the assumption that sensors are available everywhere to directly measure the traffic state. While the framework can certainly support a different observation equation, sparse measurements can lead to poor performance of multiple model filtering for traffic incident detection. In this work, we use the IMM to accommodate the switching dynamics of the traffic incident model. We show the IMM approach can detect traffic incidents when sensors are limited, and it performs better than the MM approach when the traffic model is not perfect. Moreover, we consider a scalar traffic model based on the *Lighthill–Whitham–Richards* (LWR) *partial differential equation* (PDE) [14], [15], which is simpler than the Payne model used in [11]. Due to the non–differentiability of the discretized LWR model [16], we solve the sequential estimation problem with ensemble Kalman filter. Thus, no linearization of the traffic model is needed.

C. Outline and contributions of this article

The contributions of this article are summarized as follows:

- We formulate the traffic estimation and incident detection problem as a hybrid state estimation problem and propose an IMM EnKF algorithm to solve the sequential estimation problem.
- The IMM EnKF algorithm is evaluated through numerical experiments using a perturbed traffic model as the true model. The results show that the proposed algorithm is able to detect traffic incidents with good accuracy when the remaining lane(s) cannot accommodate all the traffic (e.g., when congestion is created).
- The performance of the IMM EnKF and the MM EnKF are compared. Simulation results show the IMM method performs better than the MM method, especially when the traffic model is not perfect.

The remainder of this article is organized as follows. In Section II, the traffic flow model is introduced. In Section III, the IMM EnKF is presented. In Section IV, the proposed method is tested with synthetic traffic incident data. Conclusions are presented in Section V.

II. Description of traffic state and incident evolution equations

In this section, the traffic evolution equation used for the hybrid state estimation is described. The scalar traffic model is parameterized with a model variable that identifies the location and severity of incidents. An evolution of the model variable is also provided.

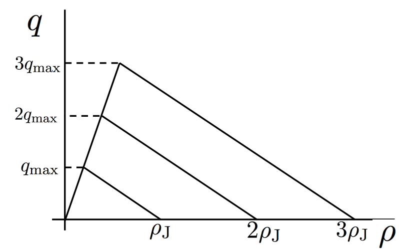


Fig. 1: Relationships between traffic density and flow under a traffic incident on a three-lane freeway (solid line). The parameters $q_{\max}$ and ρ_J are the maximum flow and jam density for one lane, respectively.

A. Traffic evolution equation

The LWR PDE [14], [15] is used to describe the evolution of the density $\rho(x,t) \in [0, \gamma(x,t)\rho_{\max}]$ at location x and at time t on a roadway. The variable $\gamma(x,t)$ denotes the number of lanes open at location x and time t, and $(x,t) \in (0,D)\times(0,T)$, where D is the road length and T is the time length. This model expresses the conservation of vehicles on the roadway, and is given by:

$$\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial \left(\rho(x,t) v\left(\rho(x,t), \gamma(x,t)\right)\right)}{\partial x} = 0 \tag{2}$$

with the following initial and boundary conditions:

$$\begin{aligned} &\rho(x,0) = \rho_0(x) \\ &\rho(0,t) = \rho_l(t),\ \rho(D,t) = \rho_r(t) \end{aligned} \tag{3}$$

and where ρ_0, ρ_l, and ρ_r are the initial, left, and right boundary conditions. To close the model, a constitutive relationship between density and velocity, denoted by v, must be specified. One common assumption is [9], [10]:

$$v(\rho,\gamma) = \begin{cases} v_{\max} & \text{if } \rho \le \gamma\rho_c \\ -w_f\left(1 - (\gamma\rho_{\max}/\rho)\right) & \text{otherwise,} \end{cases} \tag{4}$$

where $v_{\max}$ and w_f are the maximum velocity and the maximum backward propagating wave speed. The parameters ρ_c and $\rho_{\max}$ are the critical density and the maximum density for a single lane respectively. Figure 1 shows the resulting fundamental diagrams under different traffic incidents for a three–lane freeway. When a traffic incident occurs, the capacity and jam density will drop depending on how many lanes are blocked.

For numerical implementation, (2) is discretized using a Godunov scheme [17], yielding the *Cell Transmission Model* (CTM) [10], [18]. Specifically, the time and space domains are discretized by introducing a discrete time step ΔT, indexed by $n \in \{0, \cdots, n_{\max}\}$ and a discrete space step Δx, indexed by $i \in \{0, \cdots, i_{\max}\}$. The discretized system is given by:

$$\begin{aligned} \rho_{n+1}^{i} = \rho_n^i &+ \frac{\Delta T}{\Delta x} G\left(\rho_n^{i-1}, \rho_n^i, \gamma_{n+1}^{i-1}, \gamma_{n+1}^{i}\right) \\ &- \frac{\Delta T}{\Delta x} G\left(\rho_n^{i}, \rho_n^{i+1}, \gamma_{n+1}^{i}, \gamma_{n+1}^{i+1}\right). \end{aligned} \tag{5}$$

In (5), ρ_n^i denotes the value of the traffic density at time step n and in cell i. The numerical flux $G\left(\rho_n^i, \rho_n^{i+1}, \gamma_{n+1}^i, \gamma_{n+1}^{i+1}\right) = \min\left\{S\left(\rho_n^i, \gamma_{n+1}^i\right), R\left(\rho_n^{i+1}, \gamma_{n+1}^{i+1}\right)\right\}$. The functions S and R are known as the sending and receiving functions, which are given by:

$$S(\rho,\gamma) = \begin{cases} q(\rho,\gamma) & \text{if } \rho < \gamma\rho_c \\ q(\rho_c,\gamma) & \text{if } \rho \geq \gamma\rho_c \ , \end{cases} \tag{6}$$

$$R(\rho,\gamma) = \begin{cases} q(\rho_c,\gamma) & \text{if } \rho < \gamma\rho_c \\ q(\rho,\gamma) & \text{if } \rho \geq \gamma\rho_c \ . \end{cases} \tag{7}$$

Here, the function $q(\rho,\gamma) = \rho \times v(\rho,\gamma)$. In order to ensure numerical stability, the time and space steps are coupled through the CFL condition [19] : $v_{\max}\frac{\Delta T}{\Delta x} \leq 1$.

The boundary conditions of (5) are given by:

$$\begin{aligned} \rho_{n+1}^0 &= \rho_n^0 + \frac{\Delta T}{\Delta x} G\left(\rho_n^l, \rho_n^0, \gamma_{n+1}^l, \gamma_{n+1}^0\right) \\ &\quad - \frac{\Delta T}{\Delta x} G\left(\rho_n^0, \rho_n^1, \gamma_{n+1}^0, \gamma_{n+1}^1\right) \\ \rho_{n+1}^{i_{\max}} &= \rho_n^{i_{\max}} + \frac{\Delta T}{\Delta x} G\left(\rho_n^{i_{\max}-1}, \rho_n^{i_{\max}}, \gamma_{n+1}^{i_{\max}-1}, \gamma_{n+1}^{i_{\max}}\right) \\ &\quad - \frac{\Delta T}{\Delta x} G\left(\rho_n^{i_{\max}}, \rho_n^r, \gamma_{n+1}^{i_{\max}}, \gamma_{n+1}^r\right), \end{aligned} \tag{8}$$

where $\rho_n^l, \rho_n^r, \gamma_{n+1}^l, \gamma_{n+1}^r$ are the traffic density and model variable boundary conditions.

B. Model variable evolution equations to model incidents

In the problem described by (1), the model variable γ is used to model incidents through changes in the fundamental diagram. Specifically, the model variable γ is defined as an $i_{\max}+1$ dimensional vector, where the value in each element denotes the number of lanes open in the corresponding cell. The variable is modeled as a u-state first-order Markov chain [1] with transition probabilities defined by:

$$\pi_{kj} = p\left\{\gamma_n = j | \gamma_{n-1} = k\right\}, \quad k, j \in \Gamma, \tag{9}$$

where the set $\Gamma = \{1, 2, \cdots, u\}$ defines all possible incident conditions. The transition probability matrix is defined as $\Pi = [\pi_{kj}]$, which is a $u \times u$ matrix satisfying

$$\pi_{kj} \geq 0 \quad \text{and} \quad \sum_{j=1}^{u} \pi_{kj} = 1. \tag{10}$$

The model probabilities are defined as:

$$\mu^{(k)} = p\left\{\gamma = k\right\}, \tag{11}$$

for $k \in U$, such that

$$\mu^{(k)} \geq 0 \quad \text{and} \quad \sum_{k=1}^{u} \mu^{(k)} = 1. \tag{12}$$

Equation (9) indicates the probability of the transition from one model to another. In the traffic incident detection problem, it specifies how many lanes will likely be open at each time step.

The traffic model defined by (5) and (8) define the evolution operator f in (1), while (9) defines the evolution of the system models.

C. Observation equation

Since traffic density measurements from inductive loops and speed measurements from GPS equipped probe vehicles are assumed to be available, the nonlinear operator h in (1) needs to be defined to link the system state to the measurements. The system state at time n is defined by the vector $x_n = [\rho_n^0, \cdots, \rho_n^{i_{\max}}]$. The observation operator h is given by:

$$h_n(x_n, \gamma_n) = H_n \begin{bmatrix} x_n \\ v(x_n, \gamma_n) \end{bmatrix}. \tag{13}$$

The matrix H_n is constructed based on the locations where the measurements are acquired. Note, however, that the observation operator h_n is in general nonlinear, due to v. It is time varying because the locations of GPS vehicles are not fixed, and the number of equipped vehicles may change over time. The observation noise term in (1),

$$\nu_n = \begin{bmatrix} \nu_n^{\text{density}} \\ \nu_n^{\text{speed}} \end{bmatrix},$$

is composed of two parts, ν^{density} and ν^{speed}, to emphasize that different error models are assumed for density and speed measurements.

III. IMM EnKF

The hybrid state estimation problem in (1) is hard to solve because the traffic model is nonlinear, and the model contains multiple models that switch between each other. To accommodate the nonlinearity of the model, a nonlinear filter is used. In particular, the ensemble Kalman filter has been applied to solve the traffic estimation problem [16], where time invariant traffic parameters are assumed. To solve the estimation problem with multiple switching models, the IMM method [20] can be applied. In this section, we extend the EnKF into the IMM framework, and propose an IMM EnKF algorithm to solve the traffic estimation and incident detection problem in (1). The proposed algorithm can also be viewed as a mixture EnKF following the definition by [21], with the distinction that the models in this work correspond to incident scenarios while the models in [21] denote switched linear models that approximate the CTM.

The IMM EnKF algorithm is summarized in Algorithm 1, where l denotes the ensemble index, M is the total number of ensembles for each model, $\mu^{(\gamma)}$ is the probability of model γ, $x^{(\gamma,l)}$ is the state generated by model γ and ensemble l, $\Sigma^{(\gamma)}$ is the predicted covariance matrix, $K_n^{(\gamma)}$ is the Kalman gain of model γ at time n, and $L^{(\gamma)}$ is the likelihood of each model calculated from the EnKF. The subindex $n|n-1$ denotes the *prior* of a variable (before the measurements are obtained), and the subindex $n|n$ denotes the posterior of a variable (after the measurements are obtained).

At each time step, the algorithm first determines the probability of each model γ_n based on the previous system model γ_{n-1} and the transitional probability matrix defined in (9). Then, for each model, we run an EnKF to estimate the state. The EnKF algorithm first computes the one step

Algorithm 1 Interactive multiple model ensemble Kalman filter

1. Model–conditioned reinitialization (for all $\gamma \in \Gamma$):
Predicted model probability: $\mu_{n|n-1}^{(\gamma)} = \Pi\left(\gamma_{n-1|n-1}\right)$
2. Model–conditioned EnKF (for all $\gamma \in \Gamma$):
Predicted state: $x_{n|n-1}^{(\gamma,l)} = f(x_{n-1|n-1}^{(l)}, \gamma) + \omega_{n-1}^{(\gamma)} \quad l = 1, \cdots, M$
Predicted covariance: $\Sigma_{n|n-1}^{(\gamma)} = \frac{1}{M-1}\sum_{l=1}^{M}\left(x_{n|n-1}^{(\gamma,l)} - \frac{1}{M}\sum_{l=1}^{M} x_{n|n-1}^{(\gamma,l)}\right)\left(x_{n|n-1}^{(\gamma,l)} - \frac{1}{M}\sum_{l=1}^{M} x_{n|n-1}^{(\gamma,l)}\right)^T$
Kalman Gain: $K_n^{(\gamma)} = \Sigma_{n|n-1}^{(\gamma)} {h_n^{(\gamma)}}^T \left(h_n^{(\gamma)}\Sigma_{n|n-1}^{(\gamma)}{h_n^{(\gamma)}}^T + \nu_n^{(\gamma)}\right)^{-1}$
Updated state: $x_{n|n}^{(\gamma,l)} = x_{n|n-1}^{(\gamma,l)} + K_n^{(\gamma)}\left(z_n - h_n^{(\gamma)} x_{n|n-1}^{(\gamma,l)} - \nu_n^{(\gamma)}\right) \quad l = 1, \cdots, M$
Updated covariance: $\Sigma_{n|n}^{(\gamma)} = \Sigma_{n|n-1}^{(\gamma)} - K_n^{(\gamma)} h_n^{(\gamma)} \Sigma_{n|n-1}^{(\gamma)}$
3. Model probability update (for all $\gamma \in \Gamma$):
Model likelihood: $L_n^{(\gamma)} = p(z_n | x_{n|n}^{(\gamma,l)}) \quad l = 1, \cdots, M$
Model Probability: $\mu_{n|n}^{(\gamma)} = \frac{\mu_{n|n-1}^{(\gamma)} L_n^{(\gamma)}}{\sum_{\gamma' \in \Gamma} \mu_{n|n-1}^{(\gamma')} L_n^{(\gamma')}}$
4. Model inference (for all $\gamma \in \Gamma$):
Model selection: $\gamma_{n|n} = \text{argmax}_{\gamma}\left(\mu_{n|n}^{(\gamma)}\right)$
State selection: $x_{n|n}^{(l)} = x_{n|n}^{(\gamma_{n|n}, l)}$

Note: when the observation equation is nonlinear, the calculation of the covariance in the Kalman gain follows [22]:

$$\Sigma_{n|n-1}^{(\gamma)} {h_n^{(\gamma)}}^T = \frac{1}{M-1}\left(x_{n|n-1}^{(\gamma,l)} - \frac{1}{M}\sum_{l=1}^{M} x_{n|n-1}^{(\gamma,l)}\right)\left(h_n\left(x_{n|n-1}^{(\gamma,l)}, \gamma_n\right) - \frac{1}{M}\sum_{l=1}^{M} h_n\left(x_{n|n-1}^{(\gamma,l)}, \gamma_n\right)\right)^T$$

$$h_n^{(\gamma)} \Sigma_{n|n-1}^{(\gamma)} {h_n^{(\gamma)}}^T = \frac{1}{M-1}\left(h_n\left(x_{n|n-1}^{(\gamma,l)}\right) - \frac{1}{M}\sum_{l=1}^{M} h_n\left(x_{n|n-1}^{(\gamma,l)}\right)\right)\left(h_n\left(x_{n|n-1}^{(\gamma,l)}, \gamma_n\right) - \frac{1}{M}\sum_{l=1}^{M} h_n\left(x_{n|n-1}^{(\gamma,l)}, \gamma_n\right)\right)^T$$

predicted state and covariance. After the measurements are received, it updates the predicted state and covariance using the measurements at the current time step and the computed Kalman gain. The probability of each model γ_n is updated by considering the model probability $\mu_{n|n-1}$ and the model likelihood $L_n^{(\gamma)}$, which indicates how well the state generated by γ_n matches with the measurements. It is calculated by using the mean of the estimated state $\left(\Sigma_{l=1}^{M} x_{n|n}^{(\gamma,l)}\right)/M$, the measurements z_n, and the noise model ν_n. The model γ_n with the highest probability is used to estimate the true state.

The four steps in Algorithm 1 shows the filtering steps when only density measurements are obtained. When speed measurements are available, the observation equation (13) becomes nonlinear due to v, and the calculation of the covariance in the Kalman gain is modified according to the equations at the end of the algorithm.

The main difference between the IMM approach and the MM approach is that the IMM approach uses a Markov chain to model the evolution of the model variable, while the MM approach assumes the choice of the current model is independent from the previous model. Observe that if there is no incident at the current time step, it is unlikely to have an incident in the next time step, and if an incident occurs, it will likely remain in the next time step. Thus, the choice of the current model variable depends on the previous model variable. The IMM approach is able to model this feature, while the MM approach cannot.

IV. Numerical simulation

The proposed algorithm is tested by using numerical simulation. The simulation results from a perturbed traffic model are used as the source of the traffic measurements, and also the definition of the *true* state, to be estimated by the proposed algorithm.

A. Simulation description and assumptions

The parameters used for the discretized LWR model and noise models within the estimation algorithms and the *true* model are summarized in Table I. In the approximate model used in the filter, the speed limit (v_{max}), the maximum flow (q_{max}), backward propagation speed w_f, and the left boundary condition are assumed to be different from the *true* model, since a traffic model can never perfectly model the *true* traffic evolution in practice. In this simulation, the speed limit is perturbed by about 12 percent, the maximum flow and the backward propagation speed are perturbed by approximately 5 percent. The right boundary condition is assumed to be in free flow. The initial condition in all cells are assumed to follow a normal distribution, where the mean is the average of the density measurements from the inductive loop detectors located near both ends of the freeway, and the standard deviation is five percent of the mean. All of the noise models are specified by a Gaussian distribution.

We make several assumptions on the evolution of the model variable. First, we assume there is at most one traffic incident on the freeway at any given time. We make this assumption to reduce the number of models in the system.

Traffic model	Estimation model	True model
Link length (miles)	4	4
Number of cells	11	11
ΔT (seconds)	20	20
Δx (miles)	0.36	0.36
w_f (mph)	28	30
v_{max} (mph)	65	60
q_{max} (veh/hour/lane)	2000	1900
ρ_l (veh/mile/lane)	$\mathcal{N}(5000, 150^2)$	5000
ω	$\mathcal{N}(0, 1^2)$	
ν_{density}	$\mathcal{N}(0, 3.0^2)$	
ν_{speed}	$\mathcal{N}(0, 3.0^2)$	

TABLE I: Setup for the traffic models and noise model

In principle, the proposed approach can support the case when multiple incidents occur simultaneously, with increased computational cost. Second, we assume there is a one percent probability for the occurrence of a traffic incident at next time step, provided the freeway does not have any incidents at the current time. If an incident occurs, it has an equal probability to occur anywhere between the two inductive loop detectors with two possible severities: one or two lanes blocked. Third, if there is an incident on the freeway at the current time step, there is a 99% probability for the incident to remain in the next time step, and a one percent probability for the incident to be cleared. With these assumptions, the transition matrix Π can be constructed.

The simulation is performed on a four mile long, three–lane freeway segment for one hour (180 time steps). One incident is created in cell four, which is 1.36 miles from the starting point of the freeway segment. The incident occurs between time steps 60 and 120, and it blocks two lanes. The density and the system model in the time and space domain for the *true* condition are shown in Figures 2a and 2b.

For the proposed IMM EnKF algorithm, the number of ensembles for each model is set as M =100. Two inductive loop detectors are assumed available and they are located in cell one and cell nine. The proposed algorithms are tested by assuming different penetration rates of GPS vehicles, which is adjusted by changing the headway between GPS vehicles.

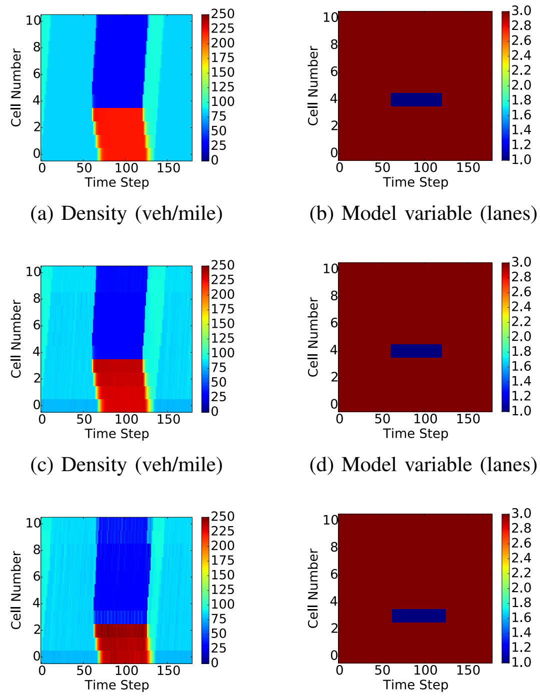
The IMM EnKF algorithm is implemented in Python and run on a 2.7GHz Intel Core i7 Macbook Pro. Each one hour numerical experiment can be run in about six minutes, and thus is suitable for real-time applications.

B. Simulation of GPS vehicle trajectory

In the *true* model, the trajectory of the jth GPS vehicle χ^{jth} is modeled according to [23]:

$$\dot{\chi_j}(t) = v(\rho(\chi_j(t), t), \gamma), \tag{14}$$

where $\chi_j(t)$ denotes the location of the jth vehicle at time t. We solve this ordinary differential equation by integrating both sides over ΔT. Following discretization by the Godunov scheme [17], the velocity function v and density ρ is constant in each cell. With each time step ΔT, a GPS vehicle may travel in at most two consecutive cells depending on its starting location within the cell. Then, the solution can be obtained by integrating two velocity functions over the corresponding time periods that the GPS vehicle stays in each cell.



(a) Density (veh/mile) (b) Model variable (lanes)

(c) Density (veh/mile) (d) Model variable (lanes)

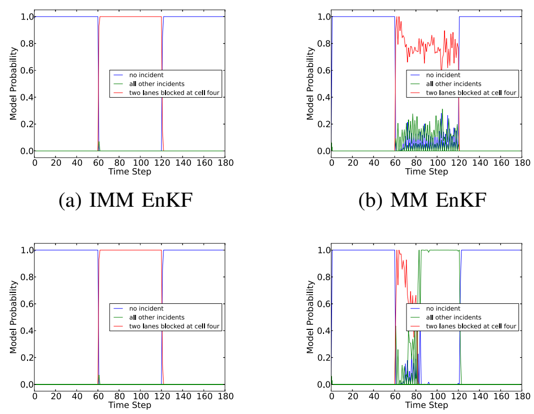
(e) Density (veh/mile) (f) Model variable (lanes)

Fig. 2: True evolution of the traffic density and the model variable (first row). Estimate of the IMM EnKF, probe vehicle headway 40 seconds (second row) and 80 seconds (third row). The values of the traffic state (left) and model variable (right) estimate at each time and space domain are described by the color bar.

C. Estimation results

The algorithm is tested by assuming probe headways of 40 and 80 seconds, and the estimation results are shown in Figures 2c, 2d, 2e, and 2f. As the result shows, when the headway is 40 seconds, the algorithm is able to correctly estimate the incident. In this case, a 40 second headway means 1.8 % of the vehicles have a GPS device. When the headway is increased to 80 seconds, the estimate of the location of the traffic incident is off by one cell, and the state estimation accuracy also decreases. In particular, when the probe headway is 40 seconds, the average absolute density error of the estimation algorithm is approximately 1 veh/mile. When the probe headway decreases to 80 seconds, the average absolute density error is about 11 veh/mile. The reason for this is because when the headway is high, there are fewer measurements from the freeway, and there is not enough information for the algorithm to correctly estimate the system model at each time iteration. Thus, the proposed algorithm requires some measurements from GPS vehicles to ensure the accuracy of the estimates.

Next, we evaluate the effectiveness of IMM approach by



(a) IMM EnKF (b) MM EnKF

(c) IMM EnKF (d) MM EnKF

Fig. 3: Comparison of model probabilities between MM EnKF and IMM EnKF. Maximum flow ($q_{\max}$) of the *true* model is 1900 veh/h (row one). Maximum flow ($q_{\max}$) of the *true* model is 1800 veh/h (row two).

comparing it with the MM approach. The MM approach assumes the selection of the model variable for the current time step is independent from the previous selected model. When the MM approach is applied, the predicted model probability $\mu^{(\gamma)}_{n|n-1}$ in Algorithm 1 is equal for each model γ. The simulations are conducted by assuming probe headways of 40 seconds and all other parameters are the same as our previous simulations.

Figure 3 shows a comparison of the IMM and MM methods. If we compare Figure 3a and 3b, although both approaches correctly estimate the incident, the IMM approach is more certain about the correct model when the incident occurs. Figures 3c and 3d shows the simulation results when the maximum flow of the *true* is set as 1800 veh/h, which means the estimation model is less accurate. In this case, the IMM EnKF is able to estimate the correct model, while the MM EnKF cannot.

V. Conclustion

This paper formulates the traffic estimation and incident detection problem as a hybrid state estimation problem. An IMM EnKF algorithm is proposed to estimate the traffic state and to detect the location and severity of traffic incidents. The algorithm is tested on synthetic traffic incident data generated by a perturbed traffic model, and the results show the proposed method is able to detect traffic incidents with good accuracy compared to a pure multiple model filtering approach.

One limitation for the IMM EnKF algorithm (as well as most existing incident detection algorithms) is that when the inflows are small, the the algorithm is unable to detect the traffic incident, since the remaining lanes have enough capacity to accommodate the traffic. Consequently, there is no congestion on the road, and the measurements from the sensors are unaffected.

Next steps of this work include additional testing in a traffic microsimulation software or on experimental field datasets.

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