Analytical Optimal Solution of Selfish Node Detection with 2-hop Constraints in OppNets: A Pontryagin's Maximum Principle Approach

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***Abstract*—Selfish node detection offers an effective means to mitigate the routing performance degradation caused by selfish behaviors in Opportunistic Networks (OppNets), but leads to the extra network overload and computation cost. Most existing effort in the literature focuses on exploring the detection methods based on the traffic analysis or the cooperations among nodes. In this paper, we investigate the state transition of nodes in the message dissemination without detection. Specifically, the Ordinary Differential Equation (ODE) is constructed to approximatively model the periodic detection with complete detection requirement. Then we propose the optimal detection solution with the Pontryagin's maximum principle, and mathematically deduce the right detection time during the message lifetime. The model soundness is verified statistically and the analysis accuracy is evaluated via extensive simulations. The experiments also show that our solution can achieve the tradeoff between the reward and the detection cost.**

***Index Terms*—Selfish Node Detection, Ordinary Differential Equation, Pontryagin's maximal principle**

I. Introduction

Opportunistic networks (OppNets) has been proposed for several years, and are used to describe the scenario, where the communication link may be established with contact opportunities among mobile nodes. At present, with a widespread use and availability of mobile communication devices, OppNets presents very broad development prospect, and numerous applications rely on it prospered. [1] builded a cloud-based recommendation system OmniSuggest based on opportunistic mobile social network. In this project, user activities, mobility patterns are tracked to generate optimal venue recommendations. The opportunistic computing technology utilize the shared resources of OppNets to provide a platform for the execution of distributed computing tasks [2]. More examples can be found in mobile data offloading, such as [3], [4].

Exploiting mobile nodes to transmit message in OppNets has been attracting increasing research attentions [5]–[8]. Traditional message dissemination approaches heavily rely on voluntary cooperation between mobile nodes, which excessively consumes the limited energy supply and lead to massive useless message copies due to the selfish behaviour. Adopting selfish node detection mechanisms to avoid selfish node involved in data forwarding, reduces and balances the communication loads of nodes (and thus their energy consumptions).

Much research effort on selfish node detection exists in the literature. The state-of-the-art detection schemes can be divided into two categories in light of their aims: watchdog systems [9]–[12] and social trust-based communications [13]–[15]. The former intends to detect selfish behavior by analyzing the traffic received from their encountered nodes while the latter establishes social trust relationships to select trusted and secured relay nodes. Most of the state-of-the-art work, either watchdog system method or social trust-based communicating approaches, are micro-perspective studies, leads to network management cost due to the detecting expense, and introduce extra detection traffic, degrading the overall performance of OppNets.

In this paper, we tackle the above issue from a different perspective, where

The main contributions are as follows:

- we formulate the ordinary differential equation model (ODE) to capture and analytically evaluate the state transition of nodes in OppNets without detection and with complete detection.
- we propose an optimal solution of selfish node detection based on the Pontryagin's maximum principle to achieve the tradeoff between the detection cost and the reward of selfish behaviors.
- we conduct experiments to evaluate the effectiveness of the proposed model and the optimal selfish detection solution in terms of the total cost, the wasted reward and the node state transition.

The rest of this paper is organized as follows. The literature is reviewed in Section II. We formulate the problem in Section III. The change of network state without detection and with fully detection is investigated in Section IV. The optimal solution of the selfish detection in OppNets is presented in Section V, and evaluated in Section VI. The paper concludes in Section VII.

II. Related Work

OppNets, which face two challenges, i.e., energy efficiency and network management cost minimization are expected to accommodate participants with low-delay and cost-effective services. Therefore, many research works targeted to address these issues.

A. Message Transmission in OppNets

In order to mitigate the performance degradation caused by the selfish behaviours in OppNets, much effort has been made to explore the methods of selfish node detection [7], [16]. An early investigation on the selfish behaviour detection is [9], where the watchdog nodes were proposed to analyze the traffic received from their encountered nodes. This work was extended for applications with the elimination of the limited knowledge on node detection by single watchdog, and the cooperative systems with multiple watchdogs were proposed in [10]–[12]. [10] proposed a collaborative approach (CoCoW-a, Collaborative Contact-based Watchdog), which considered the diffusion of local selfish nodes awareness, to conduct the selfish node detection in MANETs. Through accelerating the information propagation, the method improved the performance of selfish node detection in terms of the time and the precision. [12] proposed a social-based watchdog system (SoWatch), with a watchdog module to protect SoWatch against the wrong watchdogs manipulated by malicious nodes.

Another kind of approach tries to establish social trust relationships between mobile nodes in OppNets by leveraging their online social information (explicit trust) as well as their interactions or mobility properties (implicit trust). In [13], a probabilistic misbehavior detection scheme(iTrust), which introduced a periodically available Trusted Authority(TA), was presented to judge a node's behaviour. Another trust framework PROVEST (PROVEnance-baSed Trust model) that aimed to achieve accurate peer-to-peer trust assessment was presented in [14]. The partial selfishness was investigated and credit-based algorithm to measure the degree of selfishness was designed in [15].

[17] combined watchdog technique with trust-based communications and integrated with PRoPHET to build a global perception of forwarding behavior for detection of selfish nodes. [18] introduced ensemble learning for environment-adaptive malicious node detection. [19] integrated buffer-aided full-duplex/half-duplex relaying with non-orthogonal multiple access(BAHyNOMA) for relay selection.

Routing is a critical bottleneck when selfish behaviour is exhibited and a potential alleviation is to develop incentivizing mechanisms for message forwarding. Incentive-based protocols, such as SEIR [20], Multicent [21], were devised to increase node participation in message forwarding by opting for mechanisms that reward active participation of nodes in the forwarding of messages and penalize them otherwise. To balance the tradeoff between the delivery rate and forwarding cost, game theory was introduced to optimize the configuration in MANET for more efficient energy-aware routing in [22]. While geo-casting routing protocols like LoSeRo [23] exploited the location data to enhance the message routing performance, onion-based anonymous routing approach [24] and ePRIVO [25] were proposed to keep users' information private. For MOSNs, which exhibits a nested core-periphery hierarchy(NCPH), [26] presented an up-and down routing protocol to upload message from source node to the network core and then download to the destination. [27] proposed a context-aware self-adaptive routing protocol that is able to adapt to different scenarios.

B. Optimizations of OppNets

Optimization schemes of OppNets can be classified into several types, the most typical one tries to formulate the transmitting process in terms of a trade-off between the network management cost and the transmission performance. For example, on optimal neighbor discovery, PWEND [28] and Pharos [29] adopted time model for neighbor discovery and investigated the most energy efficient way and the least discovery latency, respectively. Then for a given energy budget, how to optimizing the number of discovered peers was researched in [8], what is the best achievable discovery latency was addressed by [30].

As for optimal data forwarding, [31] proposed an efficient time-aware data forwarding strategy(TCCB) for OMNs, based on temporal social contact patterns. The model performed a close delivery ratio to Epidemic but with significantly reduced delivery cost. [32] introduced a centralized heuristic algorithm which aimed to discover a tree for multicasting, with resource constrained (i.e. the delay-constrained least-const) in MONs. Both centralized and decentralized single-copy message forwarding algorithms were proposed in [33], which aimed to minimize the expected latencies from any node in the Opportunistic DTNs. However, aforementioned works just consider one part of the message transmission in OppNets, [34] mathematically characterized message transmission of the selfish and altruistic cases as an optimal control problem, whose controlling parameters were chosen according to the forwarding rate and beaconing rate, respectively. Then the Pontryagin's Maximum Principle was exploited to search the problem solution in multiple destinations scenario and the optimal control policies were proved to satisfy the threshold form.

Minimize the contact duration by optimizing mobile data offloading in OMNs is the objective of [35], [36]. A mathematical framework to study the problem of coding-based mobile data offloading was established in [35], the authors formulated the problem as a users' interest satisfaction maximization problem with multiple linear constraints of limited storage and efficient scheme was proposed to solve it. An optimal traffic offloading scheme through data partition, which generated forwarding paths with possible heterogeneous data chunks, was presented in [36].

Few these existing works focus on optimal control policy, while we introduce it for selfish node detection, where the scenario is different from [34] in this paper.

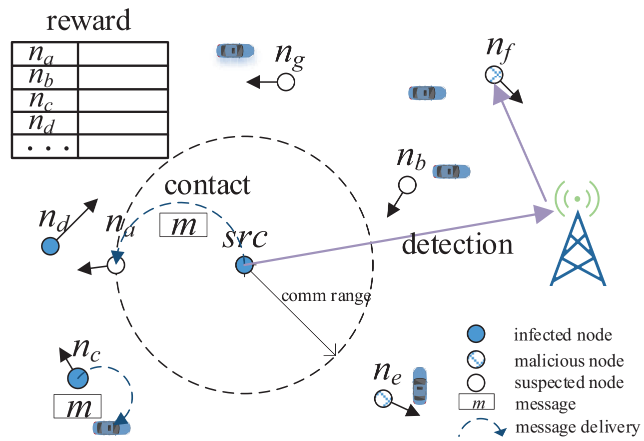

Fig. 1. Reward and Detection of the selfish nodes in OppNets.

III. Preliminaries

The source node src needs to disseminate its message m to vehicles or pedestrians. The N relay nodes can replicate m and send it to the vehicles, which is shown in Fig. 1. Thus the potential coverage area of the message is broadened by the opportunistic network. To encourage the collaboration of relay nodes, src should reward the relay node n_i $(1 \leq i \leq N)$ based on the time, when the message are carried by n_i. The time ranges from the replication time (τ_i) to the time-to-live of the message (T). τ_i can be recorded by src when n_i contacts src and replicates m. However, n_i may discard m immediately after the contact to earn the reward without carrying m, which is the selfish behavior. So src can check the checksum of m's specific part, which is store in the randomly selected relay node n_i. If the check failed, n_i will be identified as the selfish node and can not receive the reward. In this paper, we propose the optimal randomly detection strategy to achieve the tradeoff between the cost of the random detections and the wasted reward of the selfish behaviors.

$E(R(t))$ denotes the expected number of the relay nodes, which have not contacted src before time t. $E(I(t))$ denotes the expected number of infected relay nodes, which still carry the message at time t. $E(D(t))$ denotes the expected number of selfish relay nodes, which have discarded the message but are not known by src at time t. Similar to [37] and [38], the contacts between each pair of nodes including src are assumed to occur according to the Poisson process, in which the contact rate is λ. The total number of relay nodes is N, and $N = R(t) + I(t) + D(t)$, $\forall t$, $0 \leq t \leq T$. We also assume the change rate of becoming the selfish node is a constant value ρ. The detection rate is $U(t)$, $0 \leq U(t) \leq U_m$, $\forall t$, $0 \leq t \leq T$. which is the control function. For example, if the minimal circle of once detection T_m is that 2 seconds, the maximal detection rate is that $U_m = \frac{1}{T_m} = 0.5$ times per second. To simplify the denotations, we use $R(t)$, $I(t)$ and $D(t)$ to replace $E(R(t))$, $E(I(t))$ and $E(D(t))$, respectively. Then the main objective of our work is to to solve the following problem,

$$Min: J = \int_0^T (1-\alpha)D(t) + \alpha U(t)dt, \tag{1}$$

which minimizes the linear combination of the wasted reward and the detection cost through the weight α, $0 \leq \alpha \leq 1$. We can also get the total paid reward is

$$P = \int_0^T \beta(I(t) + D(t))dt, \tag{2}$$

where β is the reward paid for the one node's message carrying in a unit of time.

IV. Construction of ODE Model

We investigate the selfish detection in this and the following sections. Specifically, in this section, the ordinary differential equation model is constructed to capture the state change with time.

A. Case 1: without detection

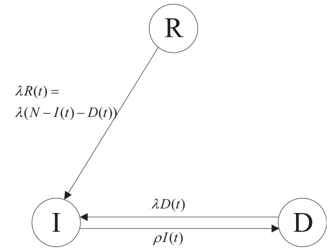

Fig. 2. State transition of the relay nodes without detection.

In the case without detection, the relay node with message can become the selfish node, but the selfish detection is not conducted. Then the state transition is shown in Fig. 2 with the following rules. The nodes change from state R to state I if they contact src. The corresponding incremental rate of state I is $\lambda R(t)$ at time t. The selfish node also may contact src in the opportunistic network. Then the total incremental rate of I is $\lambda(R(t) + D(t)) = \lambda(N - I(t))$. Additionally, the infected node may become the selfish node with rate ρ. Thus we can obtain the derivative of $I(t)$ with respect to t,

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} = \lambda(N - I(t)) - \rho I(t).$$

where λ and ρ are constants. Similar to $\frac{\mathrm{d}I(t)}{\mathrm{d}t}$, we can get the change rate of state D and state R, i.e. $\frac{\mathrm{d}D(t)}{\mathrm{d}t}$ and $\frac{\mathrm{d}R(t)}{\mathrm{d}t}$, and obtain the model,

$$\begin{aligned} \frac{\mathrm{d}I(t)}{\mathrm{d}t} &= \lambda(N - I(t)) - \rho I(t), \\ \frac{\mathrm{d}D(t)}{\mathrm{d}t} &= -\lambda D(t) + \rho I(t), \\ \frac{\mathrm{d}R(t)}{\mathrm{d}t} &= -\lambda(N - I(t) - D(t)). \end{aligned} \tag{3}$$

Since $I(t)$ in (3) is formed by the first-order first-power ordinary differential equations (ODE) [38], we can calculate the general solutions of $I(t)$, that is,

$$I(t) = C_I e^{-(\lambda+\rho)t} + \frac{\lambda N}{\lambda + \rho}.$$

Note that $I(0) = 0$, $D(0) = 0$ and $R(0) = N$, which means only src carries the message. Thus $C_I = \frac{-\lambda N}{\lambda + \rho}$, and

$$I(t) = \frac{\lambda N}{\lambda + \rho}(1 - e^{-(\lambda+\rho)t}),$$

where $0 \leq t \leq T$. Similarly, we can calculate the general solution of the first-order ODE $D(t)$ from $\frac{\mathrm{d}D(t)}{\mathrm{d}t} + \lambda D(t) = \rho I(t)$,

$$\begin{aligned} D(t) &= C_D e^{-\int \lambda dt} + e^{-\int \lambda dt} \int \rho I(t) e^{\int \lambda dt} dt \\ &= C_D e^{-\lambda t} + e^{-\lambda t} \int \rho \frac{\lambda N}{\lambda+\rho}(1-e^{-(\lambda+\rho)t}) e^{\lambda t} dt \\ &= C_D e^{-\lambda t} + \frac{\lambda N}{\lambda+\rho} e^{-(\lambda+\rho)t} + \frac{\rho N}{\lambda+\rho} \end{aligned} \tag{4}$$

Because of $D(0) = 0$,

$$D(t) = -N e^{-\lambda t} + \frac{\lambda N}{\lambda+\rho} e^{-(\lambda+\rho)t} + \frac{\rho N}{\lambda+\rho}.$$

Since $I(t) + D(t) + R(t) = N$, $0 \leq t \leq T$, $R(t)$ can be computed based on the solved solution of $I(t)$ and $D(t)$. Thus the solution of (3) can be derived as

$$\begin{aligned} I(t) &= \frac{\lambda N}{\lambda+\rho}(1-e^{-(\lambda+\rho)t}), \\ D(t) &= N(\frac{\lambda e^{-(\lambda+\rho)t} + \rho}{\lambda+\rho} - e^{-\lambda t}), \\ R(t) &= N e^{-\lambda t}, \end{aligned} \tag{5}$$

which depicts the change of the states when the time ranges from 0 to T. And $I(t)$, $D(t)$, $R(t) \geq 0$ always hold when $t \leq 0$. From the solutions of $I(t)$, $D(t)$ and $R(t)$, we can find that $I(t) \to \frac{\lambda N}{\lambda+\rho}$, $D(t) \to \frac{\rho N}{\lambda+\rho}$, and $R(t) \to 0$ when $t \to +\infty$. To verify the validity of the ODE model (3), we conduct the simulations with randomly settings. The corresponding results are presented in Section. VI-A.

Note that $U(t) = 0$, $\forall t$, in the situation without detection. The total cost J in (1) is determined by $D(t)$, $0 \leq t \leq T$, which is the total wasted reward by the selfish behaviors. Based on the calculated result in (5), we can compute J as

$$\begin{aligned} J &= \int_0^T (1-\alpha) D(t) dt, \\ &= \int_0^T (1-\alpha) N(\frac{\lambda e^{-(\lambda+\rho)t} + \rho}{\lambda+\rho} - e^{-\lambda t}) dt, \\ &= N(1-\alpha)\left(\frac{\lambda(1-e^{-(\lambda+\rho)T})}{(\lambda+\rho)^2} + \frac{\rho T}{\lambda+\rho} - \frac{1-e^{-\lambda T}}{\lambda}\right). \end{aligned} \tag{6}$$

The total paid reward can be calculated as

$$P = \beta \int_0^T I(t) + D(t) dt = N\beta(T - \frac{1-e^{-\lambda T}}{\lambda}).$$

Furthermore, the fraction between the wasted reward and the total paid reward is

$$p = \frac{\int_0^T D(t)dt}{\int_0^T I(t) + D(t)dt} = \frac{\frac{\lambda(1-e^{-(\lambda+\rho)T})}{(\lambda+\rho)^2} + \frac{\rho T}{\lambda+\rho} - \frac{1-e^{-\lambda T}}{\lambda}}{T - \frac{1-e^{-\lambda T}}{\lambda}}.$$

B. Case 2: with full detection

In the case with full detection, src conducts the selfish detection in the whole time-to-live. Note that when checking a selfish relay node n_i (state D), which means that n_i has discards the message and pretends as a node with message, src will let the node state change from state D to state R. When checking a normal node, i.e., state R and state I the number of nodes in each state will not change. Considering

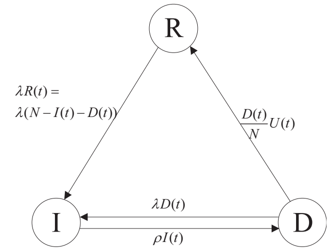

Fig. 3. State transition of the relay nodes.

that the checked relay node is randomly selected from the N node set, we calculate the probability of checking a selfish node as $\frac{D(t)}{N}$. Since the detection rate is constrained by $U(t)$, we let $\frac{D(t)}{N}U(t)$ denote the change rate with time from state D to state R. Thus the state transition of the fully detection case is constructed as Fig. 3. The ODE model in (3) will be redefined as

$$\begin{aligned} \frac{\mathrm{d}I(t)}{\mathrm{d}t} &= \lambda(N - I(t)) - \rho I(t), \\ \frac{\mathrm{d}D(t)}{\mathrm{d}t} &= -\lambda D(t) + \rho I(t) - \frac{D(t)}{N}U(t), \\ \frac{\mathrm{d}R(t)}{\mathrm{d}t} &= -\lambda(N - I(t) - D(t)) + \frac{D(t)}{N}U(t), \end{aligned} \tag{7}$$

where $U(t) = U_m$, $\forall t$, $0 \leq t \leq T$. The initial state is that $I(0) = D(0) = 0$ and $R(0) = N$. So the solution of $I(t)$, which does not change from (5), is that $I(t) = \frac{\lambda N}{\lambda+\rho}(1 - e^{-(\lambda+\rho)t})$. From $\frac{\mathrm{d}D(t)}{\mathrm{d}t} + (\lambda + \frac{U_m}{N})D(t) = \rho I(t)$, we can get that

$$\begin{aligned} &D(t) \\ =&C_{2D} e^{\int -(\lambda+\frac{U_m}{N})dt} + e^{\int -(\lambda+\frac{U_m}{N})dt} \int \rho I(t) e^{\int (\lambda+\frac{U_m}{N})dt} dt \\ =&C_{2D} e^{-(\lambda+\frac{U_m}{N})t} - \frac{\rho\lambda N}{\lambda+\rho}\frac{1}{\frac{U_m}{N}-\rho} e^{-(\lambda+\rho)t} + \frac{\rho\lambda N}{\lambda+\rho}\frac{1}{\lambda+\frac{U_m}{N}}. \end{aligned}$$

Since $D(0) = 0$ and $I(t) + D(t) + R(t) = N$, the solution of (7) is that

$$\begin{aligned} I(t) =& \frac{\lambda N}{\lambda+\rho}(1-e^{-(\lambda+\rho)t}), \\ D(t) =& \frac{\rho\lambda N}{(\lambda+\rho)(\lambda+\frac{U_m}{N})} + \frac{\rho\lambda N}{(\lambda+\frac{U_m}{N})(\frac{U_m}{N}-\rho)} e^{-(\lambda+\frac{U_m}{N})t} \\ &- \frac{\rho\lambda N}{(\lambda+\rho)(\frac{U_m}{N}-\rho)} e^{-(\lambda+\rho)t}, \\ R(t) =& N - \frac{\lambda N}{\lambda+\rho}\left(\frac{\rho}{\lambda+\frac{U_m}{N}}+1\right) + \frac{\lambda U_m}{(\lambda+\rho)(\frac{U_m}{N}-\rho)} e^{-(\lambda+\rho)t} \\ &- \frac{\rho\lambda N}{(\lambda+\frac{U_m}{N})(\frac{U_m}{N}-\rho)} e^{-(\lambda+\frac{U_m}{N})t}. \end{aligned} \tag{8}$$

We can find that $I(t) \to \frac{\lambda N}{\lambda+\rho}$, $D(t) \to \frac{\rho\lambda N}{(\lambda+\rho)(\lambda+\frac{U_m}{N})}$, and $R(t) \to N - \frac{\lambda N}{\lambda+\rho}\left(\frac{\rho}{\lambda+\frac{U_m}{N}}+1\right)$ when $t \to +\infty$ according to (8). Here $R(+\infty) \neq 0$ in the steady state is caused by the selfish detection. Based on the formulation (7) and the corresponding solutions (8), the estimation of the total cost $\hat{J}$ in (1) can be computed as

$$\begin{aligned}\hat{J} =& \int_0^T (1-\alpha)D(t) + \alpha U(t)dt,\\ =&\frac{(1-\alpha)\rho\lambda NT}{(\lambda+\rho)(\lambda+\frac{U_m}{N})} - \frac{(1-\alpha)\rho\lambda N}{(\lambda+\frac{U_m}{N})^2(\frac{U_m}{N}-\rho)}(e^{-(\lambda+\frac{U_m}{N})T}-1)\\ &+ \frac{(1-\alpha)\rho\lambda N}{(\lambda+\rho)^2(\frac{U_m}{N}-\rho)}(e^{-(\lambda+\rho)T}-1) + \alpha T U_m.\end{aligned} \tag{9}$$

The reason why (9) is the estimation of the cost is that the decrement of $D(t)$ actually occurs in the end of the detection period. However, the change rate of $D(t)$ in (7) is denoted by $\frac{D(t)}{N}U(t)$ in the above analysis. So there exists a deviation between the true cost J and the estimated cost $\hat{J}$ in the case with fully detection.

Lemma 1. *Let $D(t)$ When $t \to +\infty$, $T_m \ll T$, a deviation between $D(t)$ (8) and the real world scenario is limited.*

Proof. At first we discuss the real world scenario. Without loss of generality, assume that $(0, T)$ can be divided into k periods and a following duration t_{k+1}. Here the i-th period is denoted by $(t_{i-1}, t_i]$, where $t_i - t_{i-1} = T_m$ and $t_{k+1} < T_m$. $D(t)$ increases from $D(t_{i-1})$ to $D(t_i^-)$ in the period (t_0, t_1^-). Since the detection occurs at t_i, $D(t_i^+) = \frac{N-1}{N}D(t_i^-)$. Thus when $t \to +\infty$, we can obtain from (4) that

$$\begin{aligned}D(t_{i-1}^+) =& C_i e^{-\lambda t_{i-1}^+} + \frac{\lambda N}{\lambda+\rho}e^{-(\lambda+\rho)t_{i-1}^+} + \frac{\rho N}{\lambda+\rho},\\ D(t_i^-) =& C_i e^{-\lambda t_i^+} + \frac{\lambda N}{\lambda+\rho}e^{-(\lambda+\rho)t_i^+} + \frac{\rho N}{\lambda+\rho}.\end{aligned}$$

Then, when $i \to +\infty$,

$$\begin{aligned}D(t_i^+) =& \frac{N-1}{N}D(t_i^-)\\ =& \frac{N-1}{N}D(t_{i-1}^+)e^{-\lambda T_m} + \frac{\rho(N-1)}{\lambda+\rho}(1-e^{-\lambda T_m})\end{aligned}$$

Thus considering that $D(t_{i-1}^+) = D(t_i^+)$, we can get that

$$\begin{aligned}\lim_{i\to+\infty} D(t_i^+) &= \frac{\rho(N-1)}{\lambda+\rho}\frac{1-e^{-\lambda T_m}}{(1-\frac{N-1}{N}e^{-\lambda T_m})}\\ \lim_{i\to+\infty} D(t_i^-) &= \frac{\rho N}{\lambda+\rho}\frac{1-e^{-\lambda T_m}}{(1-\frac{N-1}{N}e^{-\lambda T_m})}\end{aligned}$$

According to (8), $D(+\infty) = \frac{\rho\lambda N}{(\lambda+\rho)(\lambda+\frac{U_m}{N})}$. Since these limitations are the limited values related to ρ, λ, N, U_m. The deviation is limited. □

Lemma 2. *In the case with fully detection, $|J - \hat{J}|$ is less than $(1-\alpha)TN$.*

Proof. Considering that $U(t) = \hat{U(t)} = U_m$, we can derive that $\int_0^T U(t)dt = TU_m = \int_0^T \hat{U(t)}dt$.

$$\begin{aligned}|J-\hat{J}| =& |\int_0^T (1-\alpha)D(t)dt - \int_0^T (1-\alpha)\hat{D(t)}dt|\\ \le& (1-\alpha)\int_0^T |(D(t) - \hat{D(t)})|dt\\ \le& (1-\alpha)TN\end{aligned} \tag{10}$$

where $0 \le D(t), \hat{D(t)} \le N$. □

We also can compute the approximate total reward is

$$\hat{P} = \beta\int_0^T I(t) + D(t)dt,$$

The utilization ratio of the reward is that

$$\hat{p} = \frac{\int_0^T D(t)dt}{\int_0^T I(t)+D(t)dt}$$

Here we find that p% reward is wasted in the selfish node. Although the wasted reward is reduced because of the detection, the additional cost, which is caused by the detection behavior, i.e., energy, bandwidth and wireless communication charge. is introduced.

V. Optimal Detection

A. Problem Formulation

Assume that the detection can be conducted. The detection rate is $U(t)$, $0 \le U(t) \le U_m$. U_m is the limitation of the detection rate, which is the constraint from the hardware and the time sequences. Then, the ODEs can be reformulated as

$$\begin{aligned}\frac{dI(t)}{dt} &= \lambda(N - I(t)) - \rho I(t),\\ \frac{dD(t)}{dt} &= \rho I(t) - \lambda D(t) - \frac{D(t)}{N}U(t),\\ \frac{dR(t)}{dt} &= -\beta(N - I(t) - D(t)) + \frac{D(t)}{N}U(t).\end{aligned} \tag{11}$$

Meanwhile,

$$\begin{aligned}I(0) &= 0,\\ D(0) &= 0,\\ R(0) &= N.\end{aligned} \tag{12}$$

Thus $I(t)$ is the same with that in the situation without detection, which is

$$I(t) = \frac{\lambda N}{\lambda+\rho}(1-e^{-(\lambda+\rho)t}). \tag{13}$$

Considering that the detection is also the cost, the object function will be

$$J = \int_0^T (1-\alpha)D + \alpha U dt.$$

Here α is the weight, which can control the importance between the cost of selfish relay nodes and detections. Thus $0 < \alpha < 1$. Similar with the previous section, $I(t)$ and $D(t)$ is the state functions. $U(t)$ is the controllable variable, $0 \le U(t) \le U_m$.

B. Optimal Control by Pontryagin's Maximum Principle

Now we utilize the Pontryagin's maximal principle [34] to find the optimal $U(t)$, which will minimize the total cost. First, the Hamilton function is

$$\begin{aligned}H =& (1-\alpha)D + \alpha U + \lambda_I(\lambda(N-I) - \rho I)\\ &+ \lambda_D(\rho I - \lambda D - \frac{D}{N}U)\\ =& (1-\alpha)D + \lambda_I(\lambda(N-I) - \rho I)\\ &+ \lambda_D(\rho I - \lambda D) + (\alpha - \lambda_D\frac{D}{N})U.\end{aligned}$$

Note that λ_I and λ_D denote two co-state functions. Without the final constraint, the terminal condition is $\lambda_I(T) = 0$ and $\lambda_D(T) = 0$. Then the adjoint function is

$$\dot{\lambda_D} = -\frac{\partial H}{\partial D} = \lambda_D(\lambda + \frac{U}{N}) - (1-\alpha).$$

Thus

$$U^*(t) = \begin{cases} 0, & \text{if } \alpha - \lambda_D \frac{D}{N} \geq 0 \\ U_m, & \text{if } \alpha - \lambda_D \frac{D}{N} < 0 \end{cases} \tag{14}$$

In summary, we have the ODE functions $\dot{D}$, $\dot{\lambda_2}$, the initial condition $D(0) = 0$ and the boundary condition $\lambda_D(T) = 0$, Thus the problem is to solve a BVP problem, which is

$$\begin{aligned} \dot{D} &= -(\lambda + \frac{U^*}{N})D + \rho I, \\ \dot{\lambda_2} &= (\lambda + \frac{U^*}{N})\lambda_D - (1-\alpha), \end{aligned} \tag{15}$$

where $D(0) = 0$ and $\lambda_D(T) = 0$. We can solve the BVP problem with the shooting method by the bvpSolve package of R. Then we analyze the properties of the optimal control variable.

Lemma 3. *At the beginning and the end of the whole duration, the optimal control stop the selfish detection, which is* $U(0) = U(T) = 0$.

Proof. At the beginning of the duration, $M(0) = 0$, which is the initial condition of 15. Then $\alpha - \lambda_2(0)\frac{M(0)}{N} = \alpha > 0$. Following (14), the optimal $U(0) = 0$.

At the end of the duration, $\lambda_2(T) = 0$, which is the boundary condition of 15. Then $\alpha - \lambda_2(T)\frac{M(T)}{N} = \alpha > 0$. Based on (14), the optimal $U(T) = 0$. ☐

Based on the differential function $\dot{I}$, the equilibrium point of I can be obtained from $\dot{I} = 0$, which is $I^* = \frac{\beta N}{\beta+\rho}$. When $I(t) < I^*$, $I(t)$ will increase with t and approach to $\frac{\beta N}{\beta+\rho}$. Meanwhile, in this paper $I(0) = 0$ at the beginning of time.

Based on the differential function $\dot{M}$, the equilibrium point is obtained from $\dot{M} = 0$, which is $M^* = \frac{\rho I}{\beta + \frac{1}{N}U}$. In the situation without detection, the equilibrium point is $M^* = \frac{\rho I^*}{\beta} = \frac{\rho N}{\beta+\rho}$. In the situation with full detection, the equilibrium point is $M^* = \frac{\rho I^*}{\beta + \frac{1}{N}U_m} = \frac{\rho}{\beta + \frac{1}{N}U_m}\frac{\beta N}{\beta+\rho}$.

Since α is the weight of detecting the selfish nodes, we can assume that if α is enough high, the detection will not perform according to the optimal control strategy.

Lemma 4. *If* $\alpha \geq \alpha_{th}$*, the optimal control let the detection stop in the whole duration, namely* $U(t) = 0$, $0 \leq t \leq T$.

Proof. Assume that ρ, N, β is given. Let $W(t) = \lambda_2(t)M(t)$.

$$\begin{aligned} W^{'}(t) =& M^{'}(t)\lambda_2(t) + M(t)\lambda_2^{'}(t) \\ =& (\rho I(t) - \beta M(t) - \frac{M(t)}{N}U(t))\lambda_2(t) \\ &+ M(t)(\lambda_2(t)(\beta + \frac{U(t)}{N}) - (1-\alpha)) \\ =& \rho\lambda_2(t)I(t) - (1-\alpha)M(t). \end{aligned} \tag{16}$$

Since $M(0) = 0$ and $\lambda_2(T) = 0$, $W(0) = W(T) = 0 < \alpha N$.

Now we focus on the poles of $W(t)$, namely t^*, where $W^{'}(t^*) = \rho\lambda_2(t^*)I(t^*) - (1-\alpha)M(t^*) = 0$. Then $M(t^*) = \frac{\rho\lambda_2(t^*)I(t^*)}{1-\alpha}$.

$$W(t^*) = \lambda_2(t^*)M(t^*) = \frac{\rho I(t^*)\lambda_2(t^*)^2}{1-\alpha}. \tag{17}$$

According to $\dot{\lambda_2}$ in (15), the equilibrium point of λ_2 is that $\lambda_2^* = \frac{1-\alpha}{\beta+\frac{U}{N}}$. Since $0 \leq U \leq U_m$, $0 < \frac{1-\alpha}{\beta+\frac{U_m}{N}} \leq \lambda_2^* \leq \frac{1-\alpha}{\beta}$. Note $\lambda_2(T) = 0$. Based on the phase line in ODE for $\dot{\lambda_2}$, $\lambda_2(t)$ decreases with t when $\lambda_2(t) < \lambda_2^*$. Conversely, $\lambda_2(t)$ increases with t when $\lambda_2(t) > \lambda_2^*$. Thus $0 \leq \lambda_2(t) \leq \lambda_2^* \leq \frac{1-\alpha}{\beta}$ when $0 \leq t \leq T$. Additionally, $0 \leq I(t) \leq \frac{\beta N}{\beta+\rho}$. From (17), we can derive that the upper boundary of $W(t)$, W_{up}, which is

$$W(t) \leq W(t^*) \leq \frac{\rho}{1-\alpha}\frac{\beta N}{\beta+\rho}(\frac{1-\alpha}{\beta})^2 = \frac{\rho N(1-\alpha)}{\beta(\beta+\rho)} = W_{up}.$$

Assume that α can satisfy that $W_{up} \leq \alpha N$, which means that $\alpha \geq \frac{\rho}{\beta(\beta+\rho)+\rho} = \alpha_{th}$. Then $W(t) \leq \alpha N$, when $0 \leq t \leq T$. Therefore the optimal control $U^*(t) \equiv 0$, when $0 \leq t \leq T$. ☐

VI. Performance Evaluation

We consider a $5000 \times 5000m^2$ sparse sensing field with 100 relay nodes. The Poisson-contact mobility model is quasi-synthetic, in which the parameter λ is set to 0.004. The source node is fixed at the center of the network scenario. The speed of nodes is randomly selected in a uniform distribution changing from 4 to 10 m/s, and the communication range of these nodes is set to be $20m$. The parameter α is limited, i.e., $\alpha \in [0, 1]$. We consider two cases in the simulations. In the first case (Section IV-A), we set $U(t) = 0$, which means there is no selfish detection. In the second case (Section IV-B), we adopt the selfish detection method and keep detecting during whole lifetime of network. In each simulation, M messages are created, whose maximal lifetime T increases from $0s$ to $2000s$. Note that, all statistical results of our scheme are obtained by repeating 50 times.

A. Accuracy Analysis Based on Simulations

As shown in Sections IV and V, we mathematically model the state transition of nodes by the ODEs, based on which we analyze the optimal control through the Pontryagin's Maximum Principle. It is critical to verity the accurate of the proposed model. Therefore, in the first experiment, we compare the the simulation and the analytical results to check the accuracy of models. Fig. 4 shows the comparison between the simulation and the analytical result in Case 1, in which $D(t)$, $I(t)$ and $R(t)$ with time t are computed from prediction and simulations when $\lambda = 0.004$, $\rho = 0.01$, $N = 100$ and $T = 2,500$, and the dotted lines represent the analytical results. As can be seen, the analytical results match the simulation results reasonably, which validates the proposed analytical model.

The result in Fig. 5 shows that the accuracy of the proposed ODE models in Case 2. In this figure, $I(t)$, $D(t)$ and $R(t)$ with

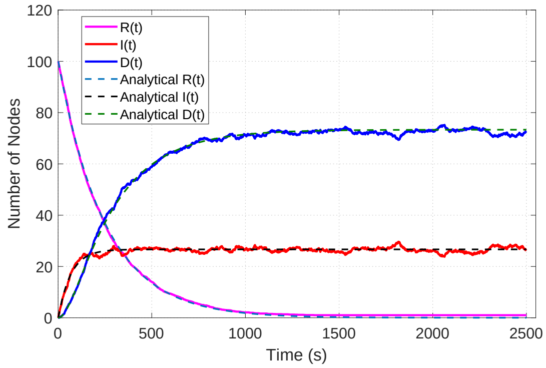

Fig. 4. Comparison of the theoretical and simulation results of the proposed ODE model in case 1.

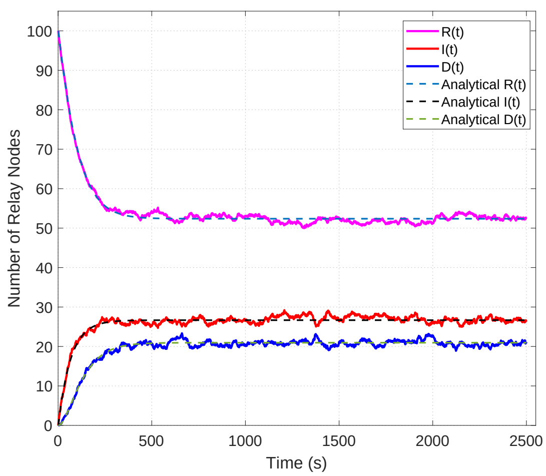

Fig. 5. Comparison of the theoretical an simulation results or the proposed ODE model in case 2.

time t are computed from prediction and simulations when $\lambda = 0.004$, $\rho = 0.011$, $N = 100$, $T_m = 1$ and $T = 2,500$. According to Equation (8), we can know that the analytical results about total number of $D(t)$ is in a monotone increasing situation when the growing T. The blue line in Fig. 5 proves this conclusion. However, the number of selfish number reduce to 20 when comparing with Fig. 4. This is because adopting detection methods will mitigate the selfish behaviour of nodes. Fig 5 verifies the accuracy of the proposed ODE model again.

B. Efficacy of the approximate method

In the second experiment, we analysis efficacy of the approximate method. Fig. 6 shows the state transition of nodes with the messages maximal lifetime T, in which $M(t)$ represent , $I(t)$ represent ?, $S(t)$ represent ? and λ_2 represent ?. As we can see, the value of $M(t)$ and the value of λ_2 decrease with T. In contrast, both $S(t)$ and $I(t)$ have growing trend with increasing T. This verify the sate transition is balanced.

Fig. 7 shows the control variable $U(t)$ with increasing time. From this figure, we can easily obtain the optimal control policy to minimize the J. For example, when T equals to 100, the complete detection is on an 'on state'. When time equals to 450, the network will switch from the 'on state' to an 'off state'.

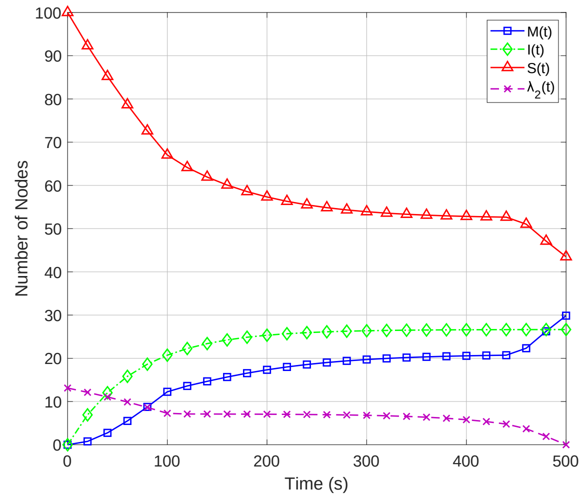

Fig. 6. State variable of analysis with time.

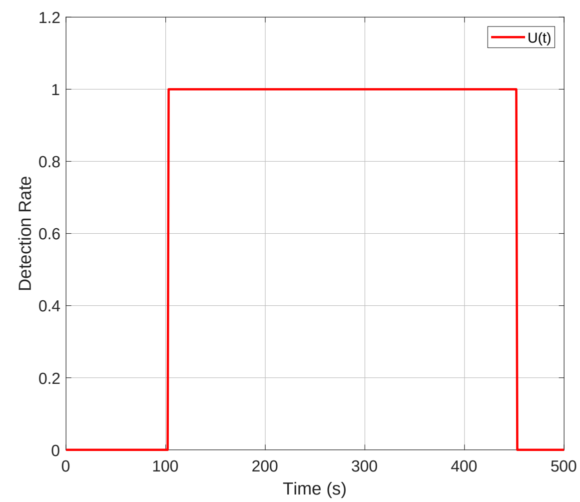

Fig. 7. The optimal control policy of $U(t)$.

C. Optimal solution of selfish detection

Lemma 1 introduce that the parameter t_0 and t_1 ?. Therefore, in the third experiment, we analysis the impact of different t_0 and t_1. As shown in Fig. 8, the total cost is calculated by adjusting t_0 and t_1 form 0 to 400 and from 0 to 500, respectively. We can observe that the total cost of system has a minimal value. For example, when $t_0 = 80$ and $t_1 = 430$, the cost is ?. It is clear that the optimal cost can guide the design of OppNets, where the selfish nodes exist.

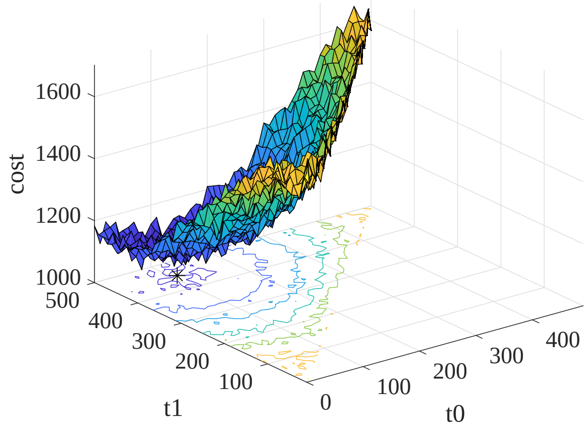

Fig. 8. Different choices of $t0$ and $t1$.

VII. Conclusion

In this paper, we have analytically investigated the state transition of nodes in the opportunistic networks. The ordinary differential equation models have been constructed to capture the message dissemination with complete detection, which can suppress the increment of selfish node number. To achieve the tradeoff the reward and the detection cost in the message lifetime, we propose the optimal solution of the selfish node detection based on the Pontryagin's maximum principle. The soundness of the models and the accuracy of the analysis have been verified via extensive simulation.

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