Analytical Optimal Solution of Selfish Node Detection with 2-hop Constraints in OppNets: A Pontryagin's Maximum Principle Approach

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***Index Terms*—OppNet, Selfish, Black (Gray) Hole, ODE, Pontryagin's maximal principle**

I. Introduction

Our main contributions are as follows:
1)
2)
3)
Section.

II. Related Work

A. Message Transmission in Selfish OppNets

In order to mitigate the performance degradation caused by the selfish behaviors in OppNets, much effort has been made to explore the methods of selfish node detection [1], [2]. An early investigation on the selfish behaviour detection is [3], where the watchdog nodes were proposed to analyze the traffic received from their encountered nodes. This work was extended for applications with the elimination of the limited knowledge on node detection by single watchdog, and the cooperative systems with multiple watchdogs were proposed in [4]–[6]. [4] proposed a collaborative approach (CoCoWa, Collaborative Contact-based Watchdog), which considered the diffusion of local selfish nodes awareness, to conduct the selfish node detection in MANETs. Through accelerating the information propagation, the method improved the performance of selfish node detection in terms of the time and the precision. [6] proposed a social-based watchdog system (SoWatch), with a watchdog module to protect SoWatch against the wrong watchdogs manipulated by malicious nodes.

The other approach tries to establish social trust relationships between mobile nodes in OppNets by leveraging their online social information (explicit trust) as well as their interactions or mobility properties (implicit trust). In [7], a probabilistic misbehavior detection scheme(iTrust), which introduced a periodically available Trusted Authority(TA), was presented to judge a node's behaviour . Another trust framework PROVEST(PROVEnance-baSed Trust model) that aimed to achieve accurate peer-to-peer trust assessment was presented in [8]. The partial selfishness was investigated and credit-based algorithm to measure the degree of selfishness was designed in [9].

Except approaches mentioned above, [10] combined watchdog technique with trust-based communications and integrated with PRoPHET to build a global perception of forwarding behavior for detection of selfish nodes. [11] introduced ensemble learning for environment-adaptive malicious node detection. [12] integrated buffer-aided full-duplex/half-duplex relaying with non-orthogonal multiple access(BAHyNOMA) for relay selection.

As the next step after selfish node detection, Routing is a critical bottleneck for message transmission in selfish OppNets. Incentive-based protocals, such as SEIR [13], Multicent [14], were devised to increase node participation in message forwarding by opting for mechanisms that reward active participation of nodes in the forwarding of messages and penalize them otherwise. To balance the tradeoff between the delivery rate and forwarding cost, game theory was introduced to optimize the configuration in MANET for more efficient energy-aware routing in [15]. While geo-casting routing protocols like LoSeRo [16] exploited the location data to enhance the message routing performance, onion-based anonymous routing approach [17] and ePRIVO [18] were proposed to keep users' information private. For MOSNs, which exhibits a nested core-periphery hierarchy(NCPH), [19] presented an up-and down routing protocol to upload message from source node to the network core and then download to the destination. [20] proposed a context-aware self-adaptive routing protocol that is able to adapt to different scenarios.

B. Optimizations for Selfish OppNets

Optimization schemes for selfish OppNets can be classified into several types, the most typical one tries to explore how to control the transmitting process to get a trade-off between the energy consumption and the transmission performance. For instance, [**?**] modeled the OppIoT environment as a Markov decision process(MDP) and proposed a routing protocol RL-Proph for routing process optimization, which seeks to fully automate the OppIoT routing process by using the Policy Iteration algorithm to maximize the possibility of message delivery. [21] proposed both centralized and decentralized

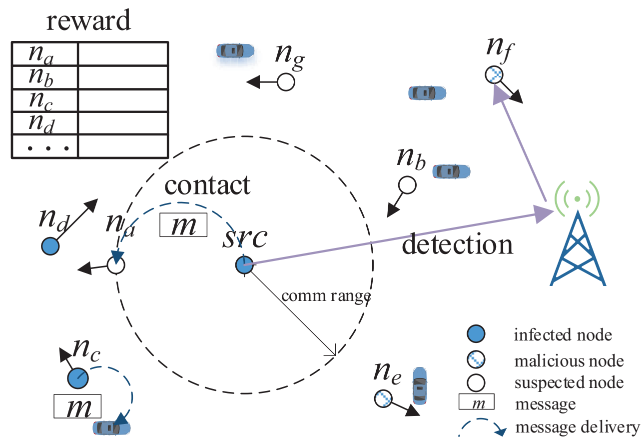

Fig. 1: Reward and Detection of the selfish nodes in OppNets.

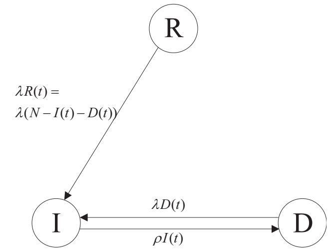

Fig. 2: State transition of the relay nodes without detection.

single-copy message forwarding algorithms to minimize the expected latencies from any node in the Opportunistic DTNs. All of the above works just consider one part of the optimal control of the message transmission in OppNets, [22] mathematically characterized message transmission of the selfish and altruistic cases as an optimal control problem, whose controlling parameters were chosen according to the forwarding rate and beaconing rate, respectively. Then the Pontryagin's Maximum Principle was exploited to search the problem solution in multiple destinations scenario and the optimal control policies were proved to satisfy the threshold form.

Another one is optimal mobile data offloading schemes. Li et al. established a mathematical framework to study the problem of coding-based mobile data offloading in opportunistic vehicular networks in [23], they formulated the problem as a users' interest satisfaction maximization problem with multiple linear constraints of limited storage and proposed an efficient scheme to solve it. Wang et al. tried to find an optimal traffic offloading scheme through data partition to minimize the data delivery latency in opportunistic mobile networks in [24], they formulated the optimal cellular traffic offloading problem and proposed an approach to generate forwarding paths with possible heterogeneous data chunks.

From above description, we can find that none of the existing works studies optimal control schemes for selfish nodes detection in OppNets, but this is the main objective of our work in this paper. (More info later)

III. Preliminaries

The source node src needs to disseminate its message m to vehicles or pedestrians. The N relay nodes can replicate m and send it to the vehicles, which is shown in Fig. 1. Thus the potential coverage area of the message is broadened by the opportunistic network. To encourage the collaboration of relay nodes, src should reward the relay node n_i $(1 \leq i \leq N)$ based on the time, when the message are carried by n_i. The time ranges from the replication time (τ_i) to the time-to-live of the message (T). τ_i can be recorded by src when n_i contacts src and replicates m. However, n_i may discard m immediately after the contact to earn the reward without carrying m, which is the selfish behavior. So src can check the checksum of m's specific part, which is store in the randomly selected relay node n_i. If the check failed, n_i will be identified as the selfish node and can not receive the reward. In this paper, we propose the optimal randomly detection strategy to achieve the tradeoff between the cost of the random detections and the wasted reward of the selfish behaviors.

$E(R(t))$ denotes the expected number of the relay nodes, which have not contacted src before time t. $E(I(t))$ denotes the expected number of infected relay nodes, which still carry the message at time t. $E(D(t))$ denotes the expected number of selfish relay nodes, which have discarded the message but are not known by src at time t. Similar to [25] and [26], the contacts between each pair of nodes including src are assumed to occur according to the Poisson process, in which the contact rate is λ. The total number of relay nodes is N, and $N = R(t) + I(t) + D(t)$, $\forall t$, $0 \leq t \leq T$. We also assume the change rate of becoming the selfish node is a constant value ρ. The detection rate is $U(t)$, $0 \leq U(t) \leq U_m$, $\forall t$, $0 \leq t \leq T$. which is the control function. For example, if the minimal circle of once detection T_m is that 2 seconds, the maximal detection rate is that $U_m = \frac{1}{T_m} = 0.5$ times per second. To simplify the denotations, we use $R(t)$, $I(t)$ and $D(t)$ to replace $E(R(t))$, $E(I(t))$ and $E(D(t))$, respectively. Then the main objective of our work is to to solve the following problem,

$$Min : J = \int_0^T (1-\alpha)D(t) + \alpha U(t)dt, \tag{1}$$

which minimizes the linear combination of the wasted reward and the detection cost through the weight α, $0 \leq \alpha \leq 1$. We can also get the total paid reward is

$$P = \int_0^T \beta(I(t) + D(t))dt, \tag{2}$$

where β is the reward paid for the one node's message carrying in a unit of time.

IV. Construction of ODE Model

We investigate the selfish detection in this and the following sections. Specifically, in this section, the ordinary differential equation model is constructed to capture the state change with time.

A. Case 1: without detection

In the case without detection, the relay node with message can become the selfish node, but the selfish detection is not conducted. Then the state transition is shown in Fig. 2 with the following rules. The nodes change from state R to state I if they contact src. The corresponding incremental rate of state I is $\lambda R(t)$ at time t. The selfish node also may contact src in the opportunistic network. Then the total incremental rate of I is $\lambda(R(t)+D(t)) = \lambda(N-I(t))$. Additionally, the infected node may become the selfish node with rate ρ. Thus we can obtain the derivative of $I(t)$ with respect to t,

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} = \lambda(N-I(t)) - \rho I(t).$$

where λ and ρ are constants. Similar to $\frac{\mathrm{d}I(t)}{\mathrm{d}t}$, we can get the change rate of state D and state R, i.e. $\frac{\mathrm{d}D(t)}{\mathrm{d}t}$ and $\frac{\mathrm{d}R(t)}{\mathrm{d}t}$, and obtain the model,

$$\begin{aligned} \frac{\mathrm{d}I(t)}{\mathrm{d}t} &= \lambda(N-I(t)) - \rho I(t), \\ \frac{\mathrm{d}D(t)}{\mathrm{d}t} &= -\lambda D(t) + \rho I(t), \\ \frac{\mathrm{d}R(t)}{\mathrm{d}t} &= -\lambda(N-I(t)-D(t)). \end{aligned} \tag{3}$$

Since $I(t)$ in (3) is formed by the first-order first-power ordinary differential equations (ODE) [26], we can calculate the general solutions of $I(t)$, that is,

$$I(t) = C_I e^{-(\lambda+\rho)t} + \frac{\lambda N}{\lambda+\rho}.$$

Note that $I(0)=0$, $D(0)=0$ and $R(0)=N$, which means only src carries the message. Thus $C_I = \frac{-\lambda N}{\lambda+\rho}$, and

$$I(t) = \frac{\lambda N}{\lambda+\rho}(1-e^{-(\lambda+\rho)t}),$$

where $0 \le t \le T$. Similarly, we can calculate the general solution of the first-order ODE $D(t)$ from $\frac{\mathrm{d}D(t)}{\mathrm{d}t} + \lambda D(t) = \rho I(t)$,

$$\begin{aligned} D(t) &= C_D e^{-\int \lambda dt} + e^{-\int \lambda dt}\int \rho I(t) e^{\int \lambda dt} dt \\ &= C_D e^{-\lambda t} + e^{-\lambda t}\int \rho \frac{\lambda N}{\lambda+\rho}(1-e^{-(\lambda+\rho)t})e^{\lambda t} dt \\ &= C_D e^{-\lambda t} + \frac{\lambda N}{\lambda+\rho}e^{-(\lambda+\rho)t} + \frac{\rho N}{\lambda+\rho} \end{aligned} \tag{4}$$

Because of $D(0)=0$,

$$D(t) = -Ne^{-\lambda t} + \frac{\lambda N}{\lambda+\rho}e^{-(\lambda+\rho)t} + \frac{\rho N}{\lambda+\rho}.$$

Since $I(t)+D(t)+R(t)=N$, $0 \le t \le T$, $R(t)$ can be computed based on the solved solution of $I(t)$ and $D(t)$. Thus the solution of (3) can be derived as

$$\begin{aligned} I(t) &= \frac{\lambda N}{\lambda+\rho}(1-e^{-(\lambda+\rho)t}), \\ D(t) &= N(\frac{\lambda e^{-(\lambda+\rho)t}+\rho}{\lambda+\rho} - e^{-\lambda t}), \\ R(t) &= Ne^{-\lambda t}, \end{aligned} \tag{5}$$

which depicts the change of the states when the time ranges from 0 to T. And $I(t)$, $D(t)$, $R(t) \ge 0$ always hold when $t \le 0$. From the solutions of $I(t)$, $D(t)$ and $R(t)$, we can find that $I(t) \to \frac{\lambda N}{\lambda+\rho}$, $D(t) \to \frac{\rho N}{\lambda+\rho}$, and $R(t) \to 0$ when $t \to +\infty$. To verify the validity of the ODE model (3), we conduct the simulations with randomly settings. The corresponding results are presented in Section. VI-A.

Note that $U(t)=0$, $\forall t$, in the situation without detection. The total cost J in (1) is determined by $D(t)$, $0 \le t \le T$, which is the total wasted reward by the selfish behaviors.

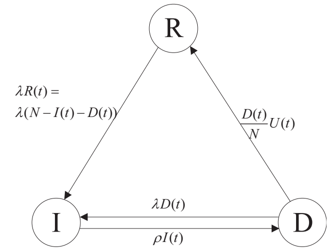

Fig. 3: State transition of the relay nodes.

Based on the calculated result in (5), we can compute J as

$$\begin{aligned} J &= \int_0^T (1-\alpha)D(t)dt, \\ &= \int_0^T (1-\alpha)N(\frac{\lambda e^{-(\lambda+\rho)t}+\rho}{\lambda+\rho} - e^{-\lambda t})dt, \\ &= N(1-\alpha)\left(\frac{\lambda(1-e^{-(\lambda+\rho)T})}{(\lambda+\rho)^2} + \frac{\rho T}{\lambda+\rho} - \frac{1-e^{-\lambda T}}{\lambda}\right). \end{aligned} \tag{6}$$

The total paid reward can be calculated as

$$P = \beta\int_0^T I(t)+D(t)dt = N\beta(T - \frac{1-e^{-\lambda T}}{\lambda}).$$

Furthermore, the fraction between the wasted reward and the total paid reward is

$$p = \frac{\int_0^T D(t)dt}{\int_0^T I(t)+D(t)dt} = \frac{\frac{\lambda(1-e^{-(\lambda+\rho)T})}{(\lambda+\rho)^2} + \frac{\rho T}{\lambda+\rho} - \frac{1-e^{-\lambda T}}{\lambda}}{T - \frac{1-e^{-\lambda T}}{\lambda}}.$$

B. Case 2: with full detection

In the case with full detection, src conducts the selfish detection in the whole time-to-live. Note that when checking a selfish relay node n_i (state D), which means that n_i has discards the message and pretends as a node with message, src will let the node state change from state D to state R. When checking a normal node, i.e., state R and state I the number of nodes in each state will not change. Considering that the checked relay node is randomly selected from the N node set, we calculate the probability of checking a selfish node as $\frac{D(t)}{N}$. Since the detection rate is constrained by $U(t)$, we let $\frac{D(t)}{N}U(t)$ denote the change rate with time from state D to state R. Thus the state transition of the fully detection case is constructed as Fig. 3. The ODE model in (3) will be redefined as

$$\begin{aligned} \frac{\mathrm{d}I(t)}{\mathrm{d}t} &= \lambda(N-I(t)) - \rho I(t), \\ \frac{\mathrm{d}D(t)}{\mathrm{d}t} &= -\lambda D(t) + \rho I(t) - \frac{D(t)}{N}U(t), \\ \frac{\mathrm{d}R(t)}{\mathrm{d}t} &= -\lambda(N-I(t)-D(t)) + \frac{D(t)}{N}U(t), \end{aligned} \tag{7}$$

where $U(t) = U_m$, $\forall t$, $0 \le t \le T$. The initial state is that $I(0)=D(0)=0$ and $R(0)=N$. So the solution of $I(t)$, which does not change from (5), is that $I(t) = \frac{\lambda N}{\lambda+\rho}(1-e^{-(\lambda+\rho)t})$. From $\frac{\mathrm{d}D(t)}{\mathrm{d}t} + (\lambda + \frac{U_m}{N})D(t) = \rho I(t)$, we can get that

$$\begin{aligned} &D(t) \\ =&C_{2D}e^{\int -(\lambda+\frac{U_m}{N})dt} + e^{\int -(\lambda+\frac{U_m}{N})dt}\int \rho I(t)e^{\int(\lambda+\frac{U_m}{N})dt}dt \\ =&C_{2D}e^{-(\lambda+\frac{U_m}{N})t} - \frac{\rho\lambda N}{\lambda+\rho}\frac{1}{\frac{U_m}{N}-\rho}e^{-(\lambda+\rho)t} + \frac{\rho\lambda N}{\lambda+\rho}\frac{1}{\lambda+\frac{U_m}{N}}. \end{aligned}$$

Since $D(0)=0$ and $I(t)+D(t)+R(t)=N$, the solution of (7) is that

$$
\begin{aligned}
I(t)=&\frac{\lambda N}{\lambda+\rho}(1-e^{-(\lambda+\rho)t}),\\
D(t)=&\frac{\rho\lambda N}{(\lambda+\rho)(\lambda+\frac{U_m}{N})}+\frac{\rho\lambda N}{(\lambda+\frac{U_m}{N})(\frac{U_m}{N}-\rho)}e^{-(\lambda+\frac{U_m}{N})t}\\
&-\frac{\rho\lambda N}{(\lambda+\rho)(\frac{U_m}{N}-\rho)}e^{-(\lambda+\rho)t},\\
R(t)=&N-\frac{\lambda N}{\lambda+\rho}\left(\frac{\rho}{\lambda+\frac{U_m}{N}}+1\right)+\frac{\lambda U_m}{(\lambda+\rho)(\frac{U_m}{N}-\rho)}e^{-(\lambda+\rho)t}\\
&-\frac{\rho\lambda N}{(\lambda+\frac{U_m}{N})(\frac{U_m}{N}-\rho)}e^{-(\lambda+\frac{U_m}{N})t}.
\end{aligned}
\tag{8}
$$

We can find that $I(t)\to\frac{\lambda N}{\lambda+\rho}$, $D(t)\to\frac{\rho\lambda N}{(\lambda+\rho)(\lambda+\frac{U_m}{N})}$, and $R(t)\to N-\frac{\lambda N}{\lambda+\rho}\left(\frac{\rho}{\lambda+\frac{U_m}{N}}+1\right)$ when $t\to+\infty$ according to (8). Here $R(+\infty)\neq 0$ in the steady state is caused by the selfish detection. Based on the formulation (7) and the corresponding solutions (8), the estimation of the total cost $\hat{J}$ in (1) can be computed as

$$
\begin{aligned}
\hat{J}=&\int_0^T(1-\alpha)D(t)+\alpha U(t)dt,\\
=&\frac{(1-\alpha)\rho\lambda NT}{(\lambda+\rho)(\lambda+\frac{U_m}{N})}-\frac{(1-\alpha)\rho\lambda N}{(\lambda+\frac{U_m}{N})^2(\frac{U_m}{N}-\rho)}(e^{-(\lambda+\frac{U_m}{N})T}-1)\\
&+\frac{(1-\alpha)\rho\lambda N}{(\lambda+\rho)^2(\frac{U_m}{N}-\rho)}(e^{-(\lambda+\rho)T}-1)+\alpha TU_m.
\end{aligned}
\tag{9}
$$

The reason why (9) is the estimation of the cost is that the decrement of $D(t)$ actually occurs in the end of the detection period. However, the change rate of $D(t)$ in (7) is denoted by $\frac{D(t)}{N}U(t)$ in the above analysis. So there exists a deviation between the true cost J and the estimated cost $\hat{J}$ in the case with fully detection.

Lemma 1. *Let $D(t)$ When $t\to+\infty$, $T_m\ll T$, a deviation between $D(t)$ (8) and the real world scenario is limited.*

Proof. At first we discuss the real world scenario. Without loss of generality, assume that $(0,T)$ can be divided into k periods and a following duration t_{k+1}. Here the i-th period is denoted by $(t_{i-1},t_i]$, where $t_i-t_{i-1}=T_m$ and $t_{k+1}<T_m$. $D(t)$ increases from $D(t_{i-1})$ to $D(t_i^-)$ in the period (t_0,t_1^-). Since the detection occurs at t_i, $D(t_i^+)=\frac{N-1}{N}D(t_i^-)$. Thus when $t\to+\infty$, we can obtain from (4) that

$$
\begin{aligned}
D(t_{i-1}^+)=&C_ie^{-\lambda t_{i-1}^+}+\frac{\lambda N}{\lambda+\rho}e^{-(\lambda+\rho)t_{i-1}^+}+\frac{\rho N}{\lambda+\rho},\\
D(t_i^-)=&C_ie^{-\lambda t_i^+}+\frac{\lambda N}{\lambda+\rho}e^{-(\lambda+\rho)t_i^+}+\frac{\rho N}{\lambda+\rho}.
\end{aligned}
$$

Then, when $i\to+\infty$,

$$
\begin{aligned}
D(t_i^+)=&\frac{N-1}{N}D(t_i^-)\\
=&\frac{N-1}{N}D(t_{i-1}^+)e^{-\lambda T_m}+\frac{\rho(N-1)}{\lambda+\rho}(1-e^{-\lambda T_m})
\end{aligned}
$$

Thus considering that $D(t_{i-1}^+)=D(t_i^+)$, we can get that

$$
\lim_{i\to+\infty}D(t_i^+)=\frac{\rho(N-1)}{\lambda+\rho}\frac{1-e^{-\lambda T_m}}{(1-\frac{N-1}{N}e^{-\lambda T_m})}
$$

$$
\lim_{i\to+\infty}D(t_i^-)=\frac{\rho N}{\lambda+\rho}\frac{1-e^{-\lambda T_m}}{(1-\frac{N-1}{N}e^{-\lambda T_m})}
$$

According to (8), $D(+\infty)=\frac{\rho\lambda N}{(\lambda+\rho)(\lambda+\frac{U_m}{N})}$. Since these limitations are the limited values related to ρ, λ, N, U_m. The deviation is limited. □

Lemma 2. *In the case with fully detection, $|J-\hat{J}|$ is less than $(1-\alpha)TN$.*

Proof. Considering that $U(t)=\hat{U(t)}=U_m$, we can derive that $\int_0^T U(t)dt=TU_m=\int_0^T\hat{U(t)}dt$.

$$
\begin{aligned}
|J-\hat{J}|=&|\int_0^T(1-\alpha)D(t)dt-\int_0^T(1-\alpha)\hat{D(t)}dt|\\
\leq&(1-\alpha)\int_0^T|(D(t)-\hat{D(t)})|dt\\
\leq&(1-\alpha)TN
\end{aligned}
\tag{10}
$$

where $0\leq D(t),\hat{D(t)}\leq N$.

□

We also can compute the approximate total reward is

$$\hat{P}=\beta\int_0^T I(t)+D(t)dt,$$

The utilization ratio of the reward is that

$$\hat{p}=\frac{\int_0^T D(t)dt}{\int_0^T I(t)+D(t)dt}$$

Here we find that p% reward is wasted in the selfish node. Although the wasted reward is reduced because of the detection, the additional cost, which is caused by the detection behavior, i.e., energy, bandwidth and wireless communication charge. is introduced.

V. Optimal Detection

A. Problem Formulation

Assume that the detection can be conducted. The detection rate is $U(t)$, $0\leq U(t)\leq U_m$. U_m is the limitation of the detection rate, which is the constraint from the hardware and the time sequences. Then, the ODEs can be reformulated as

$$
\begin{aligned}
\frac{dI(t)}{dt}&=\lambda(N-I(t))-\rho I(t),\\
\frac{dD(t)}{dt}&=\rho I(t)-\lambda D(t)-\frac{D(t)}{N}U(t),\\
\frac{dR(t)}{dt}&=-\beta(N-I(t)-D(t))+\frac{D(t)}{N}U(t).
\end{aligned}
\tag{11}
$$

Meanwhile,

$$
\begin{aligned}
I(0)&=0,\\
D(0)&=0,\\
R(0)&=N.
\end{aligned}
\tag{12}
$$

Thus $I(t)$ is the same with that in the situation without detection, which is

$$I(t)=\frac{\lambda N}{\lambda+\rho}(1-e^{-(\lambda+\rho)t}).\tag{13}$$

Considering that the detection is also the cost, the object function will be

$$J=\int_0^T(1-\alpha)D+\alpha Udt.$$

Here α is the weight, which can control the importance between the cost of selfish relay nodes and detections. Thus $0<\alpha<1$. Similar with the previous section, $I(t)$ and

$D(t)$ is the state functions. $U(t)$ is the controllable variable, $0 \leq U(t) \ \leq U_m$.

B. Optimal Control by Pontryagin's Maximal Principle

Now we utilize the Pontryagin's maximal principle to find the optimal $U(t)$, which will minimize the total cost. First, the Hamilton function is

$$\begin{aligned}H =&(1-\alpha)D + \alpha U + \lambda_1(\lambda(N-I) - \rho I) \\ &+ \lambda_2(\rho I - \lambda D - \frac{D}{N}U) \\ =&(1-\alpha)D + \lambda_1(\lambda(N-I) - \rho I) \\ &+ \lambda_2(\rho I - \lambda D) + (\alpha - \lambda_2 \frac{D}{N})U.\end{aligned}$$

Note that λ_1 and λ_2 denote two co-state functions. Without the final constraint, the terminal condition is $\lambda_2(T) = 0$ and $\lambda_3(T) = 0$. Then the adjoint function is

$$\dot{\lambda_2} = -\frac{\partial H}{\partial D} = \lambda_2(\lambda + \frac{U}{N}) - (1-\alpha).$$

Thus

$$U^*(t) = \begin{cases} 0, & \text{if } \alpha - \lambda_2 \frac{D}{N} \geq 0 \\ U_m, & \text{if } \alpha - \lambda_2 \frac{D}{N} < 0 \end{cases} \tag{14}$$

In summary, we have the ODE functions $\dot{D}$, $\dot{\lambda_2}$, the initial condition $D(0) = 0$ and the boundary condition $\lambda_2(T) = 0$, Thus the problem is to solve a BVP problem, which is

$$\begin{aligned}&\dot{D} = -(\lambda + \frac{U^*}{N})D + \rho I, \\ &\dot{\lambda_2} = -\frac{\partial H}{\partial D} = (\lambda + \frac{U^*}{N})\lambda_2 - (1-\alpha), \\ &D(0) = 0, \\ &\lambda_2(T) = 0.\end{aligned} \tag{15}$$

We can solve the BVP problem with the shooting method by the bvpSolve package of R. Then we analyze the properties of the optimal control variable.

Lemma 3. *At the beginning and the end of the whole duration, the optimal control stop the selfish detection, which is $U(0) = U(T) = 0$.*

Proof. At the beginning of the duration, $M(0) = 0$, which is the initial condition of 15. Then $\alpha - \lambda_2(0)\frac{M(0)}{N} = \alpha > 0$. Following (14), the optimal $U(0) = 0$.

At the end of the duration, $\lambda_2(T) = 0$, which is the boundary condition of 15. Then $\alpha - \lambda_2(T)\frac{M(T)}{N} = \alpha > 0$. Based on (14), the optimal $U(T) = 0$. □

Based on the differential function $\dot{I}$, the equilibrium point of I can be obtained from $\dot{I} = 0$, which is $I^* = \frac{\beta N}{\beta+\rho}$. When $I(t) < I^*$, $I(t)$ will increase with t and approach to $\frac{\beta N}{\beta+\rho}$. Meanwhile, in this paper $I(0) = 0$ at the beginning of time.

Based on the differential function $\dot{M}$, the equilibrium point is obtained from $\dot{M} = 0$, which is $M^* = \frac{\rho I}{\beta + \frac{1}{N}U}$. In the situation without detection, the equilibrium point is $M^* = \frac{\rho I^*}{\beta} = \frac{\rho N}{\beta+\rho}$. In the situation with full detection, the equilibrium point is $M^* = \frac{\rho I^*}{\beta+\frac{1}{N}U_m} = \frac{\rho}{\beta+\frac{1}{N}U_m}\frac{\beta N}{\beta+\rho}$.

Since α is the weight of detecting the selfish nodes, we can assume that if α is enough high, the detection will not perform according to the optimal control strategy.

Lemma 4. *If $\alpha \geq \alpha_{th}$, the optimal control let the detection stop in the whole duration, namely $U(t) = 0$, $0 \leq t \leq T$.*

Proof. Assume that ρ, N, β is given. Let $W(t) = \lambda_2(t)M(t)$.

$$\begin{aligned}W'(t) =& M'(t)\lambda_2(t) + M(t)\lambda_2'(t) \\ =&(\rho I(t) - \beta M(t) - \frac{M(t)}{N}U(t))\lambda_2(t) \\ &+ M(t)(\lambda_2(t)(\beta + \frac{U(t)}{N}) - (1-\alpha)) \\ =&\rho\lambda_2(t)I(t) - (1-\alpha)M(t).\end{aligned} \tag{16}$$

Since $M(0) = 0$ and $\lambda_2(T) = 0$, $W(0) = W(T) = 0 < \alpha N$.

Now we focus on the poles of $W(t)$, namely t^*, where $W'(t^*) = \rho\lambda_2(t^*)I(t^*) - (1-\alpha)M(t^*) = 0$. Then $M(t^*) = \frac{\rho\lambda_2(t^*)I(t^*)}{1-\alpha}$.

$$W(t^*) = \lambda_2(t^*)M(t^*) = \frac{\rho I(t^*)\lambda_2(t^*)^2}{1-\alpha}. \tag{17}$$

According to $\dot{\lambda_2}$ in (15), the equilibrium point of λ_2 is that $\lambda_2^* = \frac{1-\alpha}{\beta+\frac{U}{N}}$. Since $0 \leq U \leq U_m$, $0 < \frac{1-\alpha}{\beta+\frac{U_m}{N}} \leq \lambda_2^* \leq \frac{1-\alpha}{\beta}$. Note $\lambda_2(T) = 0$. Based on the phase line in ODE for $\dot{\lambda_2}$, $\lambda_2(t)$ decreases with t when $\lambda_2(t) < \lambda_2^*$. Conversely, $\lambda_2(t)$ increases with t when $\lambda_2(t) > \lambda_2^*$. Thus $0 \leq \lambda_2(t) \leq \lambda_2^* \leq \frac{1-\alpha}{\beta}$ when $0 \leq t \leq T$. Additionally, $0 \leq I(t) \leq \frac{\beta N}{\beta+\rho}$. From (17), we can derive that the upper boundary of $W(t)$, W_{up}, which is

$$W(t) \leq W(t^*) \leq \frac{\rho}{1-\alpha}\frac{\beta N}{\beta+\rho}(\frac{1-\alpha}{\beta})^2 = \frac{\rho N(1-\alpha)}{\beta(\beta+\rho)} = W_{up}.$$

Assume that α can satisfy that $W_{up} \leq \alpha N$, which means that $\alpha \geq \frac{\rho}{\beta(\beta+\rho)+\rho} = \alpha_{th}$. Then $W(t) \leq \alpha N$, when $0 \leq t \leq T$. Therefore the optimal control $U^*(t) \equiv 0$, when $0 \leq t \leq T$. □

VI. Performance Evaluation

We consider a $500 \times 500 m^2$ sparse sensing field with $50-100$ relay nodes. The Poisson-contact mobility model is quasi-synthetic, in which the parameter λ is set to ?. The speed of nodes is randomly selected in a uniform distribution changing from ? to ? m/s, and the communication range of these nodes is set to be ? m. The parameter α is limited, i.e., $\alpha \in [0, 1]$. we consider two cases in the simulations. In the first case (Case 1), we set ?. in Case 2, the contact rate is ?. In each simulation, M messages are created, whose maximal lifetime T_m increases from 0 to ? s. Note that, all statistical results of our scheme are obtained by repeating 50 times.

A. Efficacy of the ODE model

Fig. 4 shows that the change of the states in the experiments conforms to the solved solutions (5). Here $D(t)$, $I(t)$ and $R(t)$ are the mean values at the specific time t of 20 simulations. We can see from the figure that the R, I, D with The change of the states in the experiments conforms to the solved solutions.

[*** add $R(t)$ ***] [*** add J P p ***]

B. Efficacy of the approximate method

Fig. 5 shows (7).

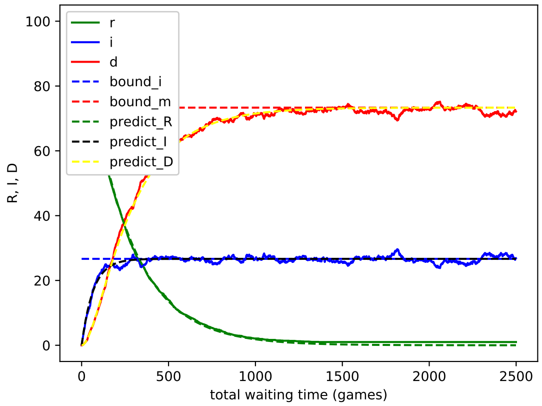

Fig. 4: $I(t)$, $D(t)$ and $R(t)$ with time t computed from prediction and simulations when $\lambda = 0.004$, $\rho = 0.01$, $N = 100$ and $T = 2,500$.

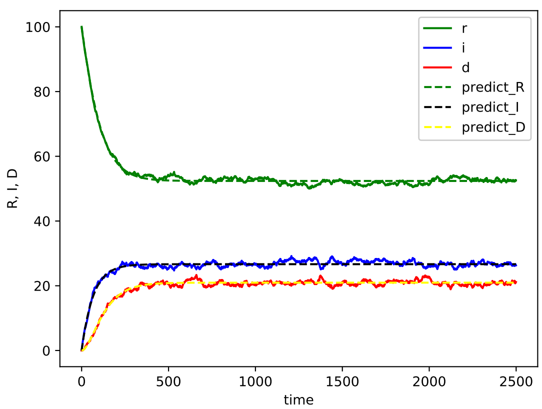

Fig. 5: $I(t)$, $D(t)$ and $R(t)$ with time computed from prediction and simulations when $\lambda = 0.004$, $\rho = 0.011$, $N = 100$, $T_m = 1$ and $T = 2,500$.

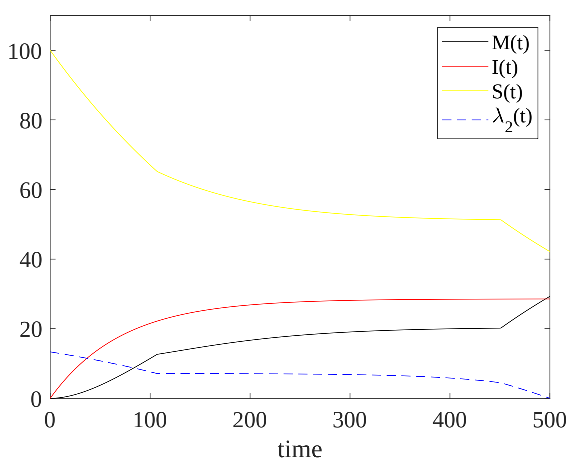

Fig. 6: State variable of analysis with time.

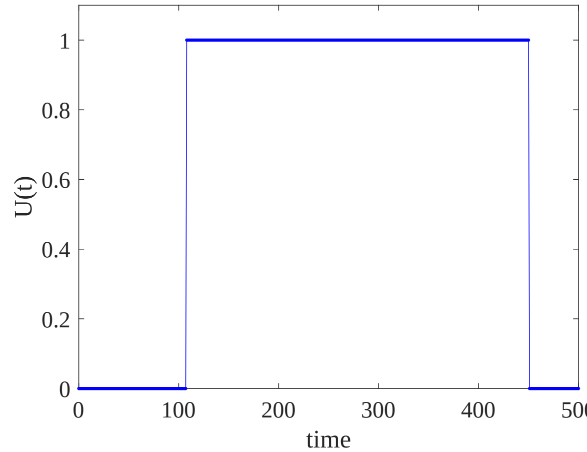

Fig. 7: Control variable of analysis with time.

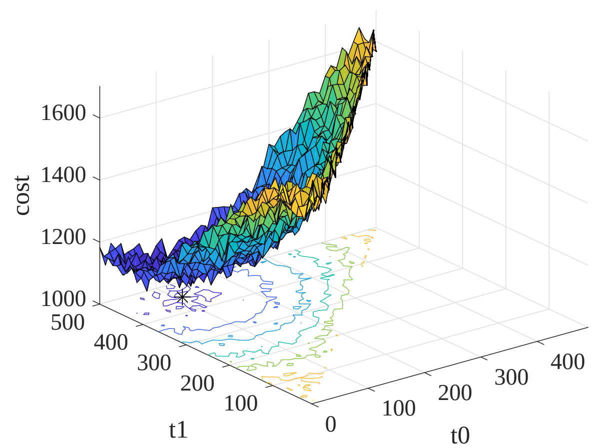

Fig. 8: Different choices of $t0$ and $t1$.

C. *Optimal solution of selfish detection*

VII. Conclusion

References

[1] Y. Li, G. Su, D. O. Wu, D. Jin, L. Su, and L. Zeng, "The impact of node selfishness on multicasting in delay tolerant networks," *IEEE Trans. Veh. Technol.*, vol. 60, no. 5, pp. 2224–2238, 2011. [Online]. Available: https://doi.org/10.1109/TVT.2011.2149552

[2] B. Jedari, F. Xia, and Z. Ning, "A survey on human-centric communications in non-cooperative wireless relay networks," *IEEE Commun. Surv. Tutorials*, vol. 20, no. 2, pp. 914–944, 2018. [Online]. Available: https://doi.org/10.1109/COMST.2018.2791428

[3] S. Marti, T. J. Giuli, K. Lai, and M. Baker, "Mitigating routing misbehavior in mobile ad hoc networks," in *MOBICOM 2000, Proceedings of the sixth annual international conference on Mobile computing and networking, Boston, MA, USA, August 6-11, 2000*, 2000, pp. 255–265.

[4] E. Hernández-Orallo, M. D. S. Olmos, J. Cano, C. T. Calafate, and P. Manzoni, "Cocowa: A collaborative contact-based watchdog for detecting selfish nodes," *IEEE Trans. Mob. Comput.*, vol. 14, no. 6, pp. 1162–1175, 2015. [Online]. Available: https://doi.org/10.1109/TMC.2014.2343627

[5] J. A. F. F. Dias, J. J. P. C. Rodrigues, F. Xia, and C. X. Mavromoustakis, "A cooperative watchdog system to detect misbehavior nodes in vehicular delay-tolerant networks," *IEEE Trans. Ind.*

Electron., vol. 62, no. 12, pp. 7929–7937, 2015. [Online]. Available: https://doi.org/10.1109/TIE.2015.2425357

[6] B. Jedari, F. Xia, H. Chen, S. K. Das, A. Tolba, and Z. Al-Makhadmeh, "A social-based watchdog system to detect selfish nodes in opportunistic mobile networks," *Future Gener. Comput. Syst.*, vol. 92, pp. 777–788, 2019. [Online]. Available: https://doi.org/10.1016/j.future.2017.10.049

[7] H. Zhu, S. Du, Z. Gao, M. Dong, and Z. Cao, "A probabilistic misbehavior detection scheme toward efficient trust establishment in delay-tolerant networks," *IEEE Trans. Parallel Distributed Syst.*, vol. 25, no. 1, pp. 22–32, 2014. [Online]. Available: https://doi.org/10.1109/TPDS.2013.36

[8] J. Cho and I. Chen, "PROVEST: provenance-based trust model for delay tolerant networks," *IEEE Trans. Dependable Secur. Comput.*, vol. 15, no. 1, pp. 151–165, 2018. [Online]. Available: https://doi.org/10.1109/TDSC.2016.2530705

[9] J. Choi, K. Shim, S. Lee, and K. Wu, "Handling selfishness in replica allocation over a mobile ad hoc network," *IEEE Trans. Mob. Comput.*, vol. 11, no. 2, pp. 278–291, 2012. [Online]. Available: https://doi.org/10.1109/TMC.2011.57

[10] S. Basu, A. Biswas, S. Roy, and S. D. Bit, "Wise-prophet: A watchdog supervised prophet for reliable dissemination of post disaster situational information over smartphone based DTN," *J. Netw. Comput. Appl.*, vol. 109, pp. 11–23, 2018. [Online]. Available: https://doi.org/10.1016/j.jnca.2018.02.012

[11] B. Gao, T. Maekawa, D. Amagata, and T. Hara, "Environment-adaptive malicious node detection in manets with ensemble learning," in *38th IEEE International Conference on Distributed Computing Systems, ICDCS 2018, Vienna, Austria, July 2-6, 2018*. IEEE Computer Society, 2018, pp. 556–566. [Online]. Available: https://doi.org/10.1109/ICDCS.2018.00061

[12] N. Nomikos, T. Charalambous, D. Vouyioukas, R. Wichman, and G. K. Karagiannidis, "Integrating broadcasting and NOMA in full-duplex buffer-aided opportunistic relay networks," *IEEE Trans. Veh. Technol.*, vol. 69, no. 8, pp. 9157–9162, 2020. [Online]. Available: https://doi.org/10.1109/TVT.2020.2999655

[13] A. Chhabra, V. Vashishth, and D. K. Sharma, "SEIR: A stackelberg game based approach for energy-aware and incentivized routing in selfish opportunistic networks," in *51st Annual Conference on Information Sciences and Systems, CISS 2017, Baltimore, MD, USA, March 22-24, 2017*. IEEE, 2017, pp. 1–6. [Online]. Available: https://doi.org/10.1109/CISS.2017.7926113

[14] K. Chen, H. Shen, and L. Yan, "Multicent: A multifunctional incentive scheme adaptive to diverse performance objectives for DTN routing," *IEEE Trans. Parallel Distributed Syst.*, vol. 26, no. 6, pp. 1643–1653, 2015. [Online]. Available: https://doi.org/10.1109/TPDS.2014.2323057

[15] Y. Mao and P. Zhu, "A game theoretical model for energy-aware DTN routing in manets with nodes' selfishness," *Mob. Networks Appl.*, vol. 20, no. 5, pp. 593–603, 2015. [Online]. Available: https://doi.org/10.1007/s11036-015-0610-7

[16] G. Costantino, R. R. Maiti, F. Martinelli, and P. Santi, "Losero: A locality sensitive routing protocol in opportunistic networks with contact profiles," *IEEE Trans. Mob. Comput.*, vol. 19, no. 10, pp. 2392–2408, 2020. [Online]. Available: https://doi.org/10.1109/TMC.2019.2923224

[17] K. Sakai, M. Sun, W. Ku, J. Wu, and F. S. Alanazi, "An analysis of onion-based anonymous routing for delay tolerant networks," in *36th IEEE International Conference on Distributed Computing Systems, ICDCS 2016, Nara, Japan, June 27-30, 2016*. IEEE Computer Society, 2016, pp. 609–618. [Online]. Available: https://doi.org/10.1109/ICDCS.2016.9

[18] N. Magaia, C. Borrego, P. R. Pereira, and M. Correia, "eprivo: An enhanced privacy-preserving opportunistic routing protocol for vehicular delay-tolerant networks," *IEEE Trans. Veh. Technol.*, vol. 67, no. 11, pp. 11 154–11 168, 2018. [Online]. Available: https://doi.org/10.1109/TVT.2018.2870113

[19] H. Zheng and J. Wu, "Up-and-down routing through nested core-periphery hierarchy in mobile opportunistic social networks," *IEEE Trans. Veh. Technol.*, vol. 66, no. 5, pp. 4300–4314, 2017. [Online]. Available: https://doi.org/10.1109/TVT.2016.2604369

[20] E. Rosas, F. Garay, and N. Hidalgo, "Context-aware self-adaptive routing for delay tolerant network in disaster scenarios," *Ad Hoc Networks*, vol. 102, p. 102095, 2020. [Online]. Available: https://doi.org/10.1016/j.adhoc.2020.102095

[21] S. Shaghaghian and M. Coates, "Optimal forwarding in opportunistic delay tolerant networks with meeting rate estimations," *IEEE Trans. Signal Inf. Process. over Networks*, vol. 1, no. 2, pp. 104–116, 2015. [Online]. Available: https://doi.org/10.1109/TSIPN.2015.2452811

[22] Y. Wu, S. Deng, and H. Huang, "Control of message transmission in delay/disruption tolerant network," *IEEE Trans. Comput. Soc. Syst.*, vol. 5, no. 1, pp. 132–143, 2018. [Online]. Available: https://doi.org/10.1109/TCSS.2017.2776322

[23] Y. Li, D. Jin, Z. Wang, L. Zeng, and S. Chen, "Coding or not: Optimal mobile data offloading in opportunistic vehicular networks," *IEEE Trans. Intell. Transp. Syst.*, vol. 15, no. 1, pp. 318–333, 2014. [Online]. Available: https://doi.org/10.1109/TITS.2013.2281104

[24] N. Wang and J. Wu, "Optimal cellular traffic offloading through opportunistic mobile networks by data partitioning," in *2018 IEEE International Conference on Communications, ICC 2018, Kansas City, MO, USA, May 20-24, 2018*. IEEE, 2018, pp. 1–6. [Online]. Available: https://doi.org/10.1109/ICC.2018.8422387

[25] Y. Wu, S. Deng, and H. Huang, "Control of message transmission in delay/disruption tolerant network," *IEEE Trans. Comput. Soc. Syst.*, vol. 5, no. 1, pp. 132–143, 2018.

[26] X. Zhang, G. Neglia, J. F. Kurose, and D. F. Towsley, "Performance modeling of epidemic routing," *Comput. Networks*, vol. 51, no. 10, pp. 2867–2891, 2007.