Optimal Control of Double Probing in Delay-Tolerant Network

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***Abstract*—Nodes in delay-tolerant network exchange messages only when they meet each other, so detecting the contact efficiently is an important problem. At present, some works explore this problem, but they often assume that only the nodes without messages probe the contacts. In this paper, we study the double probing problem. In particular, at time t, if a node has the message, it detects the contact with probability $p(t)$. Otherwise, the probability is $u(t)$. Then, this paper defines this problem as an optimization problem and solves it by Pontryagin's maximal principle. In addition, this paper proves that the both optimal policies conform to the *threshold* form. This result means that exploring the double probing policy is necessary.**

***Index Terms*—Delay-tolerant network (DTN), optimal control, Pontryagin's maximal principle, probing problem.**

I. Introduction

WITH the popularity of the mobile terminals, such as the smartphones and Personal Digital Assistant (PDA), more and more messages can be exchanged directly between the devices in a peer-to-peer way [1]. In particular, these mobile devices form an ad hoc network. However, due to the mobility, it is hard to keep the connection of the links in such a network. In other words, the network is disconnected, and it is a specific application of the popular delay-tolerant network (DTN). Different from the traditional mobile ad hoc network, the links in DTN may be disrupted. To overcome this problem, nodes in DTN adopt the *store-carry-forward* communication mode [2]. In particular, when a node gets a message, it will first *store* the message in its buffer and *carry* it along its movement. Then, when it *meets* another node, it will *forward* the message to this new node according to a certain routing protocol (e.g., two-hop protocol [3], social analysis-based protocol [4], etc.).

Once a node meets another one, we say that they have a contact. It is easy to see that the routing protocol in DTN closely depends on the contacts. Furthermore, it is necessary for the nodes to probe the environment for the presence of new contacts. However, this is an extremely energy-consuming process. The work in [5] measures the energy consumption in the probing process based on the Nokia 6600 mobile phone, and it finds that the energy consumption nearly equals the consumption in the phone call. In addition, the energy is precious in the wireless application, which has significant impact on the lifetime of the whole network [6]. Therefore, how to probe the contacts efficiently is an important problem.

At present, some works have explored this interesting problem. For example, the works in [5] and [7] study the tradeoff between the energy consumption and the detecting probability and get the optimal probing policy. However, these works assume that the optimal policy has a constant probing interval, so it may not be the global optimal policy. The works in [8] and [9] study the probing policy without the aforementioned assumption based on the two-hop protocol and epidemic routing (ER) protocol [10], respectively. The work in [11] explores a similar problem when there are multiple destinations. However, they simply assume that only the node without messages carries out the probing process, and they do not prove that such an assumption is rational. For this reason, we study the optimal double probing policy in this paper, in which no matter whether a node has the message, it may probe the contacts. The main contributions can be summarized as follows.

1) This paper studies the tradeoff between the energy consumption in the probing process and the message spreading performance based on the ER protocol. First, it defines this problem as an optimization problem. In particular, at time t, if a node has the message, it detects the contact with probability $p(t)$. Otherwise, the probability is $u(t)$. The problem is to get the best values of $p(t)$ and $u(t)$ at any time at the same time.
2) This paper solves this optimization problem through Pontryagin's maximal principle and proves that the optimal probing policies conform to the *threshold* form. Simply speaking, the optimal policy $p(t)$ has at most one *jump* and changes from 1 to 0. A *jump* means that the probing probability can be any value in [0, 1]. The optimal policy $u(t)$ has at most two *jumps*, and it first changes from 0 to 1 and then changes from 1 to 0.
3) The accuracy of the theoretical model is checked by simulations. In addition, numerical results show that the optimal policies are really better and conform to the *threshold* form, which is consistent with our theoretical results.

II. Related Works

At present, most of the works about DTN study how to select the proper relay nodes to improve the performance of the message transmission, which can be seen as the *forwarding problem*. At the initial stage, the researchers explore the forwarding

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problem through simulation, such as the works in [4] and [12]. Although these works can be used in many scenarios, they cannot get the *optimal forwarding policy* based on given settings. For this reason, some works begin to study this problem through the theoretical method. For example, Groenevelt *et al.* propose a theoretical model to evaluate the message delay of the ER and two-hop forwarding algorithms [13]. The work is based on the Markov chain model, which is quite complex. To overcome this problem, Zhang *et al.* present a new model based on ordinary differential equations (ODEs) [14]. However, none of these works consider the optimal control problem. The work in [15] studies the optimal forwarding policy based on the two-hop algorithm for the first time. Then, Li *et al.* studies the optimal forwarding problem based on the ER algorithm [16].

The message transmission in DTN closely depends on the contacts of nodes, so how to find the contacts efficiently is important, which can be seen as the probing problem. At present, there are some good works about this problem. The work in [5] considers the tradeoff between the probability of missing a contact and the contact probing frequency and proposes an adaptive contact probing policy. The work in [7] studies the tradeoff between the energy efficiency and the contact opportunities. However, these works assume that the optimal policy has a constant probing interval, so it may not be the global optimal policy. Altman *et al.* explores the global optimal forwarding and probing policies at the same time based on the two-hop algorithm [8]. Then, Li *et al.* studies the optimal probing policy based on the ER algorithm through the continuous Markov process [9]. The work in [11] studies the probing mechanism when there are multiple destinations. Although the probing problem has been explored deeply, but the existing works often simply assume that only the node without messages carries out the probing process. In fact, a contact can be detected when one of the two nodes probes its environment during their contact with each other [7]. Therefore, the aforementioned assumption may not be rational in some cases. For this reason, we will explore the doubling probing mechanism in this paper.

III. Network Model

A. Mobility Model

In this paper, we consider a DTN with N common nodes and one destination D. Among these common nodes, there is a source node S. At the initial time, only node S has a message *mes*, and the message is useless after time T, so S tries to forward the message to D before T. These nodes move according to the exponential model. In this mobility model, the intermeeting time between two consecutive contacts conforms to the exponential distribution with parameter λ. At present, some works show that this assumption may not be rational in certain motion traces. For example, the authors in [17] find that the intermeeting time conforms to a power-law and exponential decay distribution. Then, the work in [18] proposes a Home-MEG model to fit such distribution. However, this model is too complex to use for the theoretical analysis. In addition, the exponential model is still correct in many applications. For example, the works in [19] and [20] find that the intermeeting time of human or vehicles can be described by the exponential model and validate their model experimentally on real motion traces. For this reason, many works still use the simple exponential distribution, such as [8], [9], [11], [21], [22], etc. Therefore, the assumption is rational in some environments.

B. Routing Protocol

In the store-carry-forward communication mode, two nodes can exchange messages only when they encounter each other. In particular, at time t, if node m meets node n, m may forward a message to n with probability $p_{mn}(t)$. This probability may be different in different routing protocols. The main objective of most routing protocols is to decide the proper value of $p_{mn}(t)$. However, in this paper, we mainly consider the probing policy, so we do not consider the forwarding probability and assume $p_{mn}(t) = 1$. That is, a node carrying a message will forward the message to any new encountered node. It is easy to see that this is the classic ER algorithm. In fact, through some simple extension, our work can be used with many other routing protocols, such as the k-hop protocol [23], spray and wait protocol [24], etc.

IV. Optimization Formulation

A. Problem Description

The common nodes in the network are divided into two classes. The first class is the nodes that have got the message, and they probe for the contact with probability $p(t)$ at time t. The nodes that do not have the message belong to the second class, and they probe with probability $u(t)$ at time t. For simplicity, we assume that the destination D probes all the time. Our work can be extended to the case without the assumption easily.

Let $X(t)$ denote the number of the common nodes carrying the message at time t, and let $F(t)$ denote the probability that the destination D has the message at time t. At the initial time, only the source has the message, so we have $X(0) = 1, F(0) = 0$. As shown in [7], when two nodes meet, the contact can be detected if one of them probes its environment. Therefore, if two nodes belonging to different classes encounter each other at time t, the contact can be found with probability $(1 - p(t))(1 - u(t))$. Then, similar to [9] and [11], we can get the ODEs for $X(t)$ and $F(t)$. In particular, we have

$$\begin{cases} E(\dot{X}(t)) = \lambda E(X(t))(N - E(X(t)))(1 - (1 - p(t))(1 - u(t))) \\ E(\dot{F}(t)) = \lambda E(X(t))(1 - E(F(t))). \end{cases} \tag{1}$$

For a stochastic variable $*$, we let $E(*)$ denote its expectation.

In this paper, we just consider the energy consumption in the transmission and probing processes similar to [9], [11]. In particular, the energy consumption in each transmission includes the sending energy at the forwarding node and the reception energy at the receiving node. We assume that each transmission is successful, so the energy consumption in the transmission process has a linear relation with the number of nodes that get the message from others [9], [11]. Therefore, the energy consumption for message transmission can be denoted as $\alpha(E(X(t)) - 1)$ [9], [11]. α is a positive multiplication, which denotes the energy consumption in each transmission. For the probing process, the energy consumption is proportional to the *probing* probability. According to the authors of [9] and [11], we can denote the energy consumption as

$$\beta \int_0^T \left(p(t)E\left(X(t)\right) + \mu(t)\left(N - E\left(X(t)\right)\right)\right) dt. \tag{2}$$

Then, the total energy consumption $E(C(T))$ is $E(C(T)) = \alpha(E(X(t) - 1)$

$$+ \beta \int_0^T \left(p(t)E\left(X(t)\right) + \mu(t)\left(N - E\left(X(t)\right)\right)\right) dt. \tag{3}$$

Note that, when getting the energy consumption $E(C(T))$, we ignore the destination. This is because there is only one destination D and its energy consumption is too small. A similar assumption has been used in [9] and [11]. However, our work can be used in the case without this assumption, too. Then, the objective is to solve the following optimization problem:

$$\begin{cases} Max\ E\left(F(T)\right) + \varepsilon E\left(C(T)\right) \\ p(t), u(t) \in [0,1], t \in [0,T], \varepsilon < 0 \end{cases} \tag{4}$$

$p(t)$ and $u(t)$ are the controlling parameters. ε is a normalized constant. It is easy to see that, if the energy consumption is big, it is not good for the network, so we can set $\varepsilon < 0$ in this paper.

B. Optimal Control

For any stochastic variable $*$, we simply use $*$ to denote its expectation in the rest of this paper. For example, $E(X(t))$ is simply denoted as $X(t)$. Furthermore, without confusion, we may also ignore t. In this case, we can let $((X,Y),p,u)$ be an optimal solution. Then, based on (4), we can get the Hamiltonian H according to [25, P.108]

$$\begin{aligned} H &= \dot{F} + \varepsilon \dot{C} + \lambda_F \dot{F} + \lambda_X \dot{X} \\ &= (1+\lambda_F)\dot{F} + \varepsilon\left(\alpha \dot{X} + \beta\left(pX + u(N-X)\right)\right) + \lambda_X \dot{X} \\ &= (1+\lambda_F)\lambda X(1-F) + \varepsilon\beta\left(pX + u(N-X)\right) \\ &\quad + (\alpha\varepsilon + \lambda_X)\lambda X(N-X)\left(1-(1-p)(1-u)\right). \end{aligned} \tag{5}$$

From (5), the costate or adjoint functions λ_X, λ_Y, and λ_U follow:

$$\begin{cases} \dot{\lambda}_F = -\frac{\partial H}{\partial F} = (1+\lambda_F)\lambda X \\ \dot{\lambda}_X = -\frac{\partial H}{\partial X} = -(1+\lambda_F)\lambda(1-F) - \varepsilon\beta(p-u) \\ \qquad\qquad - (\lambda_X + \alpha\varepsilon)\lambda(N-2X)\left(1-(1-p)(1-u)\right) \end{cases}. \tag{6}$$

Then, we can get the *transversality* condition, which is shown in

$$\lambda_X(T) = \lambda_F(T) = 0. \tag{7}$$

Furthermore, according to Pontryagin's maximum principle [19, P.108], the continuous or piecewise continuously differentiable state and costate functions exist, and they satisfy

$$(p,u) \in \arg \max_{0 \le (p^*, u^*) \le 1} H\left(\lambda_X, \lambda_F, (X,F), (p^*, u^*)\right). \tag{8}$$

This equation means that solving the optimization problem in (4) equals to maximizing the *Hamiltonian* H. In particular, at given time t, the state (X,F) and *costate* (λ_X, λ_F) can be seen as constants, and (p,u) can maximize H at this time.

Based on the *Hamiltonian* H, we have

$$\begin{cases} \frac{\partial H}{\partial p} = (\lambda_X + \alpha\varepsilon)\lambda X(N-X)(1-u) + \varepsilon\beta X \\ \frac{\partial H}{\partial u} = (\lambda_X + \alpha\varepsilon)\lambda X(N-X)(1-p) + \varepsilon\beta(N-X) \end{cases}. \tag{9}$$

From (9), we can obtain Lemma 1.

Lemma 1: The values of p and u in the optimal probing policies cannot be 1 at the same time.

Proof: First, we assume $p=1$. In this case, because $\varepsilon < 0$, we have $\partial H/\partial u = \varepsilon\beta(N-X) \le 0$. In fact, $\partial H/\partial u = 0$ only when $N - X = 0$. This means that all the common nodes have the message. In other words, the common node without a message does not exist. Therefore, none of the nodes probes according to the policy of u, and u is meaningless. Therefore, we just need to consider the case of $N - X > 0$, and we have $\partial H/\partial u = \varepsilon\beta(N-X) < 0$. In this case, H decreases with u, and u should be 0. Similarly, if $u = 1, \partial H/\partial p = \varepsilon\beta X < 0$, and p should be 0. Therefore, in the optimal policies, p and u cannot be 1 at the same time. ■

Then, we define the functions f and g as follows:

$$\begin{cases} f = \frac{\partial H}{\partial p}/X = (\lambda_X + \alpha\varepsilon)\lambda(N-X)(1-u) + \varepsilon\beta \\ g = \frac{\partial H}{\partial u}/(N-X) = (\lambda_X + \alpha\varepsilon)\lambda X(1-p) + \varepsilon\beta \end{cases}. \tag{10}$$

From (10), we can easily obtain $f_{u=0} = (\lambda_X + \alpha\varepsilon)\lambda(N - X) + \varepsilon\beta$ and $g_{p=0} = (\lambda_X + \alpha\varepsilon)\lambda X + \varepsilon\beta$. Then, the optimal values of p satisfy

$$p = \begin{cases} 1, f_{u=0} > 0 \quad g_{p=0} \le 0 \\ 1, f_{u=0} > 0 \quad g_{p=0} > 0 \quad X \le N/2 \\ 0, \text{others.} \end{cases}. \tag{11}$$

Equation (11) is proved by the following four theorems (Theorems 1–4). First, we introduce Theorem 1 as follows.

Theorem 1: When $f_{u=0} > 0, g_{p=0} \le 0$, the best value of (p,u) is $(1,0)$.

Proof: When $g_{p=0} < 0$, we can easily obtain $g < 0$. Therefore, the *Hamiltonian* H decreases with u. In this case, the best value of u is 0. Furthermore, we have $f = f_{u=0}$. Therefore, if $f_{u=0} > 0$, the *Hamiltonian* H increases with p, and we have $p = 1$. Similarly, when $g_{p=0} = 0$, we can also set $u = 0$ and $p = 1$. That is, when $f_{u=0} > 0$ and $g_{p=0} \le 0$, we have $p = 1$. ■

For the case of $f_{u=0} > 0, g_{p=0} > 0$, and $X < N/2$, we can get Theorem 2.

Theorem 2: When $f_{u=0} > 0, g_{p=0} > 0$, and $X < N/2$, the best value of (p,u) is $(1,0)$.

Proof: First, we define $p = (\lambda_X + \alpha\varepsilon)\lambda(N - X)X$, and then, we have

$$\begin{aligned} H_{new}(p,u) &= \rho\left(1-(1-p)(1-u)\right) + \varepsilon\beta\left(pX + u(N-X)\right) \\ &= \rho(1-u)p + \rho u + \varepsilon\beta pX + \varepsilon\beta u(N-X) \\ &= p\left(\rho(1-u) + \varepsilon\beta X\right) + u\left(\rho + \varepsilon\beta(N-X)\right) \\ &= p(\rho + \varepsilon\beta X) + u\left(\rho(1-p) + \varepsilon\beta(N-X)\right). \end{aligned} \tag{12}$$

We can easily find that maximizing H_{new} equals to maximizing the *Hamiltonian H*. To prove that (1, 0) is the best policy, we just need to get $H_{new}(p,u) - H_{new}(1,0) < 0$ for any policy (p,u). In particular, we have

$$H_{new}(p,u) - H_{new}(1,0) = (p-1)(\rho + \varepsilon\beta X) + u\left(\rho(1-p) + \varepsilon\beta(N-X)\right). \tag{13}$$

When p is fixed, if $\rho(1-p) + \varepsilon\beta(N-X) < 0$, we can find that the best value of u is 0 according to (12). In this case, (13) can be converted to $H_{new}(p,u) - H_{new}(1,0) = (p-1)(\rho + \varepsilon\beta X) \leq 0$. However, the equation $H_{new}(p,u) - H_{new}(1,0) = 0$ holds only when $p = 1$, so we can say that (1, 0) is the best policy. If $\rho(1-p) + \varepsilon\beta(N-X) \geq 0$, according to (13), we have

$$\begin{aligned} H_{new}(p,u) - H_{new}(1,0) &= (p-1)(\rho + \varepsilon\beta X) \\ &\quad + u\left(\rho(1-p) + \varepsilon\beta(N-X)\right) \\ &\leq (p-1)(\rho + \varepsilon\beta X) + \rho(1-p) \\ &\quad + \varepsilon\beta(N-X) \\ &= (p-1)\varepsilon\beta X + \varepsilon\beta(N-X) \\ &= p\varepsilon\beta + \varepsilon\beta(N-2X) \leq 0. \end{aligned} \tag{14}$$

This means that we cannot find a better policy than (1, 0), so (1, 0) is still the best value. ■

Theorem 3: When $f_{u=0} > 0, g_{p=0} > 0$, and $X \leq N/2$, the best value of (p,u) is (0, 1).

Proof: The proving process is similar to that of Theorem 2. In particular, we have

$$H_{new}(p,u) - H_{new}(0,1) = p\left(\rho(1-u) + \varepsilon\beta X\right) + (u-1)\left(\rho + \varepsilon\beta(N0-X)\right). \tag{15}$$

When u is fixed, if $\rho(1-u) + \varepsilon\beta X < 0$, according to (12), we can easily find that the best value of p is 0. In this case, (13) can be converted to $H_{new}(p,u) - H_{new}(0,1) = (u-1)(\rho + \varepsilon\beta(N-X)) \leq 0$. $H_{new}(p,u) - H_{new}(0,1) = 0$ only when $u=1$. Therefore, (0, 1) is the best value. If $\rho(1-u) + \varepsilon\beta X \geq 0$, we have

$$\begin{aligned} H_{new}(p,u) - H_{new}(0,1) &= p\left(\rho(1-u) + \varepsilon\beta X\right) + (u-1) \\ &\quad \times \left(\rho + \varepsilon\beta(N-X)\right) \\ &< \rho(1-u) + \varepsilon\beta X + (u-1) \\ &\quad \times \left(\rho + \varepsilon\beta(N-X)\right) \\ &= \varepsilon\beta X + (u-1)\varepsilon\beta(N-X) \\ &= \varepsilon\beta(2X-N) + u\varepsilon\beta(N-X) \leq 0. \end{aligned} \tag{16}$$

Equation (16) means that we cannot find a policy to get a bigger value for H_{new} than (0, 1). Therefore, the best policy is (0, 1). ■

Theorem 4: When $f_{u=0} \leq 0$, the best value of p is 0.

Proof: First, when $f_{u=0} < 0$, we can easily obtain $f < 0$, so the *Hamiltonian H* decreases with p, and the best value of p is 0. Then, when $f_{u=0} = 0$, as shown in the proving process of Theorem 3, we can get $\rho + \varepsilon\beta X = fX = 0$. Furthermore, we have

$$H_{new}(p,u) = u\left(\rho(1-p) + \varepsilon\beta(N-X)\right). \tag{17}$$

Because $\rho + \varepsilon\beta X = 0$ and $\varepsilon\beta X < 0$, we know $\rho > 0$. Therefore, $\rho(1-p) + \varepsilon\beta(N-X)$ decreases with p. This means that the value of p in the optimal probing policy is 0, too. ■

Remark: Note that, when $p = 0$, if $\rho(1-p) + \varepsilon\beta(N-X) < 0$, we have $g_{p=0} < 0$. From (17), we can find that H_{new} decreases with u, so we have $u = 0$.

Combining with Theorems 1–4, we can get (11). Based on these theorems, we can also get the optimal values of u, which is shown as follows:

$$u = \begin{cases} 1, f_{u=0} \leq 0 & g_{p=0} > 0 \\ 1, f_{u=0} > 0 & g_{p=0} > 0 \quad X > N/2 \\ 0, \text{others} & \end{cases}. \tag{18}$$

For example, as shown in Theorem 4, if $f_{u=0} \leq 0, p = 0$. Then, we have $g = g_{p=0} > 0$, so $u = 1$. Similar to the proving process of Theorem 2, we can also get $u = 1$ when $f_{u=0} > 0$, $g_{p=0} > 0$, and $X > N/2$. When $f_{u=0} > 0$, $g_{p=0} > 0$, and $X \geq N/2$, we know that $p = 1$. According to Lemma 1, we can get $u = 0$. Similar to Theorem 4, when $g_{p=0} \leq 0$, we can also get $u = 0$. Therefore, (18) follows.

C. Structure of the Optimal Policies

Before exploring the structure of the optimal policies, we first introduce the following lemma.

Lemma 2: $1 + \lambda_F > 0$.

Proof: Suppose that $1 + \lambda_F \leq 0$ at time h. Then, we have $\lambda_F(h) \leq -1 < 0$. From (6), we can see that λ_F cannot increase at the time h. Furthermore, we can find that λ_F cannot increase after time h, too. This means that $\lambda_F(T) \leq -1 < 0$, and it is inconsistent with (7). Therefore, $1 + \lambda_F > 0$ is correct all the time. ■

Lemma 3: The function $f_{u=0}$ decreases all the time with the optimal policies.

Proof: From (11), we can find that the value of p closely depends on the function $f_{u=0}$. Let $\psi = f_{u=0}$, and we have

$$\begin{aligned} \frac{\dot{\psi}}{\lambda(N-X)} &= \dot{\lambda}_F - (\lambda_X + \alpha\varepsilon)\lambda X\left(1-(1-p)(1-u)\right) \\ &= -(1+\lambda_F)\lambda(1-F) - (\lambda_X + \alpha\varepsilon)\lambda(N-2X) \\ &\quad \times \left(1-(1-p)(1-u)\right) \\ &\quad - \varepsilon\beta(p-u) - (\lambda_X + \alpha\varepsilon)\lambda X\left(1-(1-p)(1-u)\right) \\ &= -(1+\lambda_F)\lambda(1-F) - f_{u=0} + (1-p)f + u\varepsilon\beta. \end{aligned} \tag{19}$$

Then, we consider three different cases.

Case 1: $f_{u=0} \leq 0$. From Theorem 4, we know that $p = 0$. Then, (19) can be converted to

$$\frac{\overset{\bullet}{\psi}}{\lambda(N-X)} = -(1+\lambda_F)\lambda(1-F) + u\varepsilon\beta - (\lambda_X+\alpha\varepsilon)\lambda(N-X)u. \quad (20)$$

If $(\lambda_X+\alpha\varepsilon)\lambda(N-X)X<0$, we can see $g_{p=0}<0$, so $u=0$. Then, the above equation equals to $-(1+\lambda_F)\lambda(1-F)$. According to Lemma 2, we have $1+\lambda_F>0$. Therefore, we get $-(1+\lambda_F)\lambda(1-F)\leq 0$. Note that $-(1+\lambda_F)\lambda(1-F)=0$ only when $F=1$, i.e., the destination has got the message. In this case, it is not necessary to explore the probing policy anymore, so we just consider the case of $F<1$ in this paper. Therefore, (19) is negative when $f_{u=0}\leq 0$, and this function decreases.

Case 2: $f_{u=0}>0, g_{p=0}<0$ or $g_{p=0}>0$, and $X\leq N/2$. In this case, we have $p=1$ and $u=0$. In this case, we can also find that (19) is negative and $f_{u=0}$ decreases.

Case 3: $f_{u=0}>0, g_{p=0}>0, X>N/2$. In this case, we have $p=0$ and $u=1$. Then, we have

$$\frac{\overset{\bullet}{\psi}}{\lambda(N-X)} = -(1+\lambda_F)\lambda(1-F) - f_{u=0} + 2\varepsilon\beta < 0. \quad (21)$$

Combining the aforementioned three cases, we can find that $f_{u=0}$ decreases all the time. ■

Lemma 4: Under the optimal policies, when $X > N/2$, if $f_{u=0} > 0$ or $g_{p=0} > 0$, we have $f_{u=0} < g_{p=0}$.

Proof: Because $\varepsilon<0$, if $f_{u=0}>0$ or $g_{p=0}>0$, we can easily get $\lambda_X+\alpha\varepsilon>0$. Then, when $X > N/2$, we have $(\lambda_X+\alpha\varepsilon)\lambda(N-X) < (\lambda_X+\alpha\varepsilon)\lambda X$. Furthermore, we have $f_{u=0} = (\lambda_X+\alpha\varepsilon)\lambda(N-X)+\varepsilon\beta < g_{p=0} = (\lambda_X+\alpha\varepsilon)\lambda X+\varepsilon\beta$. ■

Similarly, we have Lemma 5.

Lemma 5: Under the optimal policies, when $X < N/2$, if $f_{u=0} > 0$ or $g_{p=0} > 0$, we have $f_{u=0} > g_{p=0}$.

Now, for the optimal probing policy p, we have the following.

Theorem 5: The optimal policy p conforms to the *threshold* form, and it has at most one *jump*. A *jump* is a point, where the probing policy can be any value. Suppose that the *jump* is time s, the optimal policy p satisfies the following: $p(t) = 1,\ 0 \leq t < s;\ p(t) = 1,\ s < t < T$.

Proof: We consider the following two cases.

Case 1: $f_{u=0}(0) \leq 0$. In this case, we can get $f_{u=0}(h) < 0, 0 < h \leq T$. Therefore, p equals to 0 all the time, and p has no *jump*.

Case 2: $f_{u=0}(0) > 0$. Then, at a specific time s, if $f_{u=0}(s) = 0$ and $X(s) \leq N/2$, we have $f_{u=0}(t) > 0, 0 < t < s; f_{u=0}(t) < 0, s < t \leq T$. According to (11), we can obtain $p(t) = 1, 0 \leq t < s; p(t) = 1, s < t < T$. Time s is the *jump*. If $f_{u=0}(s) = 0$ and $X(s) > N/2$, we can get $p(t) = 1, 0 \leq t \leq s_1, X(s_1) = N/2$. It is easy to prove that X increases with t, so $s_1 < s$. In addition, in time interval (s_1, s), we have $X > N/2$ and $f_{u=0} > 0$. According to Lemma 4, we have $f_{u=0} > 0$ in (s_1, s). Then, according to (11), we have $p(t) = 0, s_1 < t < s$. Similarly, in time interval $[s, T], f_{u=0} \leq 0$. According to Theorem 4, we have $p(t) = 0, s \leq t \leq T$. Therefore, in Case 2, p also conforms to the threshold form, and s_1 is the *jump*.

Combining with the results in Cases 1 and 2, Theorem 5 follows. ■

From the aforementioned proving process, we can easily find that, when $X > N/2$, we have $p = 0$ all the time. Then, we can get the following lemma.

Lemma 6: When $X > N/2$, if $g_{p=0}(s) \leq 0$, we have $g_{p=0}(t) < 0, s < t \leq T$.

Proof: Define $y = g_{p=0}$, and then, we have

$$\frac{\overset{\bullet}{y}}{\lambda X} = -(1+\lambda_F)\lambda(1-F) - \varepsilon\beta(p-\mu) + (\lambda_X+\alpha\varepsilon)\lambda X(1-(1-p)(1-u)). \quad (22)$$

When $X > N/2$, we have $p = 0$, so the above equation equals to

$$\begin{aligned}\frac{\overset{\bullet}{y}}{\lambda X} &= -(1+\lambda_F)\lambda(1-F) + u\varepsilon\beta + (\lambda_X+\alpha\varepsilon)\lambda X u \\ &= -(1+\lambda_F)\lambda(1-F) + uy. \quad (23)\end{aligned}$$

According to Lemma 2, we have $-(1+\lambda_F)\lambda(1-F) < 0$. Therefore, if $y \leq 0, y$ will decrease and remain negative all the time. Because $y = g_{p=0}$, Lemma 6 follows. ■

Based on the aforementioned results, for the optimal policy u, we have the following.

Theorem 6: The optimal policy u conforms to the *threshold* form, and it has at most two *jump*. A *jump* is a point, where the probing policy can be any value. In particular, u may have three forms. If there is no *jump*, we have $u(t) = 0, 0 \leq t \leq T$. If there is only one *jump* s_1, we have $u(t) = 0,\ 0 \leq t < s_1$; $u(t) = 1,\ s_1 < t \leq T$. If there are two *jumps* s_1 and s_2, the optimal policy u satisfies the following: $u(t) = 0,\ 0 \leq t < s_1$; $u(t) = 1,\ s_1 < t < s_2$; $u(t) = 0,\ s_2 < t \leq T$.

Proof: First, when $X \leq N/2$, we have $u = 0$. Otherwise, suppose that, at time h, we have $X(h) \leq N/2$ and $u(h) = 1$. According to (18), we have $g_{p=0}(h) = (\lambda_X(h)+\alpha\varepsilon)\lambda X(h) + \varepsilon\beta > 0$ and $(\lambda_X(h)+\alpha\varepsilon)\lambda > 0$. Because $X(h) \leq N/2$, we can get $f_{u=0}(h) \geq g_{p=0}(h) > 0$. Then, according to (11), we can get $p(h) = 1$. This result is a contradiction with Lemma 1. Therefore, when $X \leq N/2$, we have $u = 0$.

Second, when $X > N/2$, according to Lemma 6, if $g_{p=0}(h) \leq 0$, we have $g_{p=0}(t) < 0, h < t \leq T$. Furthermore, we have $u(t) < 0,\ h < t \leq T$.

Suppose that, at time h_1, we have $X(h_1) = N/2$. Based on the aforementioned analysis, we consider two different cases.

Case 1: $g_{p=0}(T) > 0$. We can easily get $g_{p=0}(t) > 0, h_1 < t < T$. Then, we have $u(t) = 1,\ h_1 < t < T$. In this case, the optimal policy only has one *jump*, which is time h_1. In particular, u satisfies the following: $u(t) = 0,\ 0 \leq t < s_1;\ u(t) = 1,\ s_1 < t \leq T$.

Case 2: $g_{p=0}(T) \leq 0$. If $g_{p=0}(t) \leq 0, h_1 < t \leq T$, we have $u(t) = 0,\ h_1 < t \leq T$. Therefore, there is no *jump*, and we have $u(t) = 0,\ 0 \leq t \leq T$. Otherwise, there is a time h_2, where we have $g_{p=0}(h_2) > 0$. According to Lemma 6, we can get $g_{p=0}(t) > 0, h_1 < t \leq h_2$. Furthermore, because $g_{p=0}(T) \leq 0$ and $g_{p=0}$ is a continuous function, we can find a time $h_3 > h_2, g_{p=0}(h_3) = 0$. Then, according to Lemma 6, we can get $g_{p=0}(t) > 0, h_1 < t < h_3; g_{p=0}(t) \leq 0, h_1 < t \leq T$. From (18), we have $s_1 = h_1, s_2 = h_3$, and u satisfies the following: $u(t) = 0,\ 0 \leq t < s_1;\ u(t) = 1,\ s_1 < t < s_2;\ u(t) = 0,\ s_2 < t \leq T$.

Combining the aforementioned cases, Theorem 6 follows. ■

V. Simulation and Numerical Results

A. Simulations

It is easy to see that the core of the optimization problem is (3), which includes the variables $X(t)$ and $F(t)$. The optimal policies are obtained based on these variables, so it is necessary to check the accuracy of (3) through simulations. In particular, we consider three different scenarios based on the Opportunistic Network Environment simulator [26]. The first scenario is based on the random waypoint (RWP) mobility model [27], which is commonly used in many mobile wireless networks. In this scenario, we generate 500 common nodes and 1 destination. These nodes move within a 10 000 m × 10 000 m terrain according to a scale speed chosen from a uniform distribution from 4 to 10 m/s. The transmission range of these nodes is 50 m. In the second scenario, we use the Poisson-contact process with $\lambda = 3.71 \times 10^{-6} s^{-1}$ to generate the node contact events. The value of λ comes from the Shanghai city motion trace with about 2100 operational taxis [28]. In this scenario, we also generate 501 nodes. In the third scenario, we use the famous Infocom'05 motion trace [29], which is a real data trace and includes 41 nodes. In these nodes, we randomly select 1 node as the destination, and the other nodes are the common nodes. Note that the objective of the simulations is to check the accuracy of the theoretical model, and the model supports any values of $p(t)$ and $u(t)$, so we consider two different probing policies (may not be optimal). In the first case, we set $p(t) = 0,\ u(t) = 1, 0 \leq t \leq T$. In the second policy, we set $p(t) = 0.2,\ u(t) = 0.2,\ 0 \leq t \leq T$. The maximal lifetime of the message T increases from 0 to 5000 s, and the interval is 100 s. Therefore, we carry out $5000/100 = 50$ simulations in each scenario. In the nth simulation of any scenario, the maximal lifetime T equals to $100n$ s. In each simulation, just one message is generated, but each simulation repeats 30 times. Based on these settings, we can get the results shown in Figs. 1–3,respectively.

From the results in Figs. 1 and 2, we can find that the deviation between the theoretical and simulation results is very small. For example, the average deviation in the RWP model is about 4.071%. In the Poisson-contact model, the average deviation is only 3.892%. This shows the accuracy of our theoretical model denoted by (3). The deviation in Fig. 3 is a little bigger and is about 7.922%. In fact, the Infocom 2005 motion trace used in Fig. 3 does not conform to the exponential mobility model accurately, and the work in [17] demonstrates that the intermeeting time in this trace conforms to the power-law and

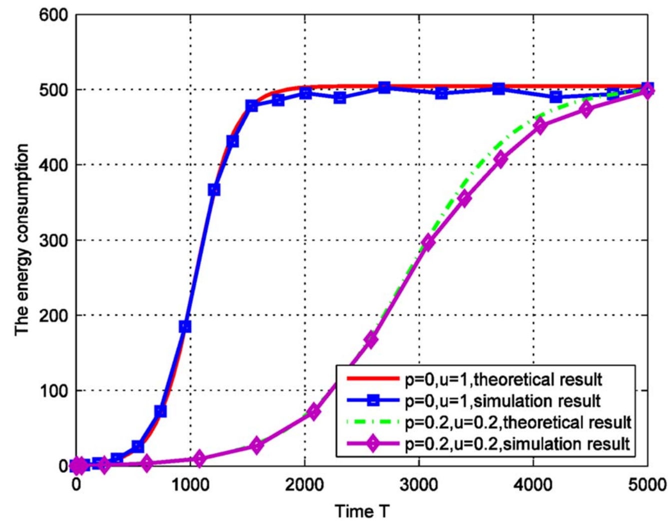

Fig. 1. Simulation and theoretical result comparison based on RWP mobility model.

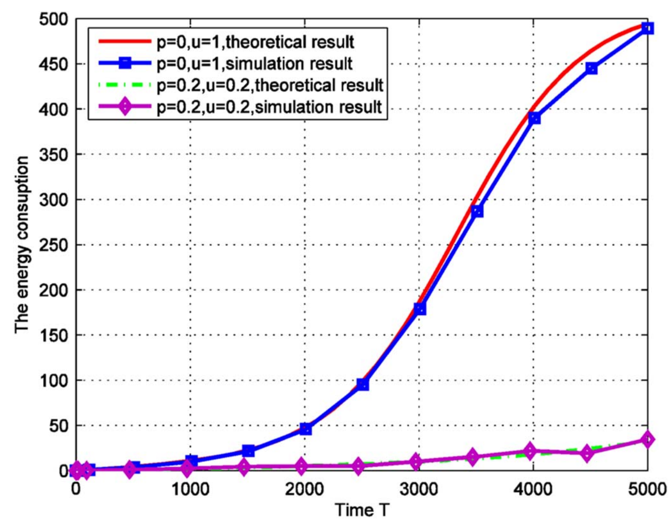

Fig. 2. Simulation and theoretical result comparison based on Poisson-contact mobility model.

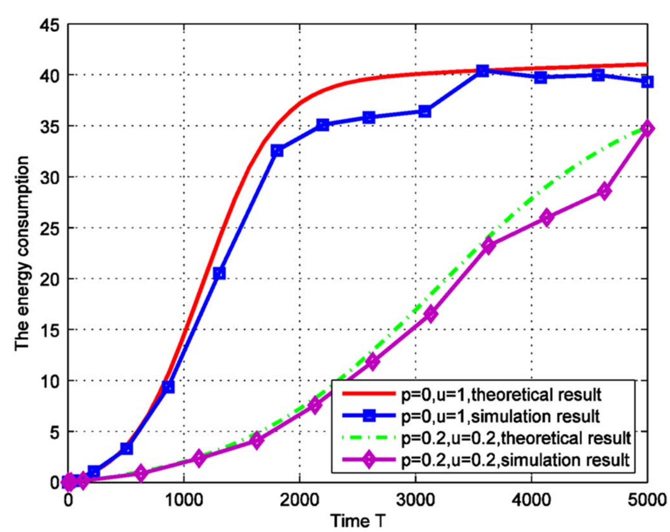

Fig. 3. Simulation and theoretical result comparison based on Infocom 2005 motion trace.

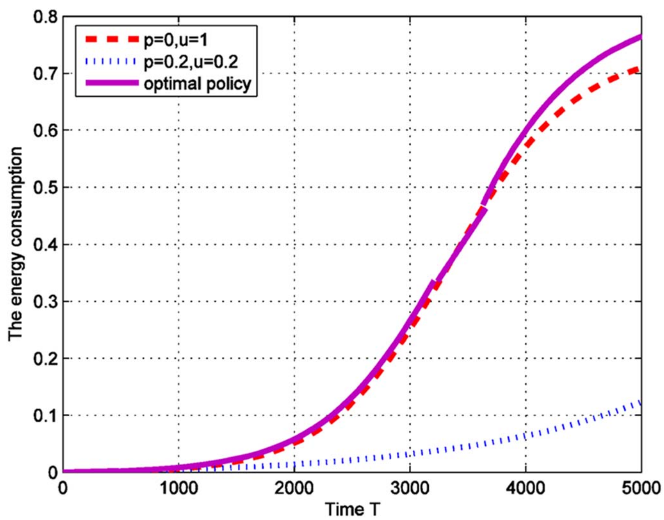

Fig. 4. Performance comparison with different probing policies.

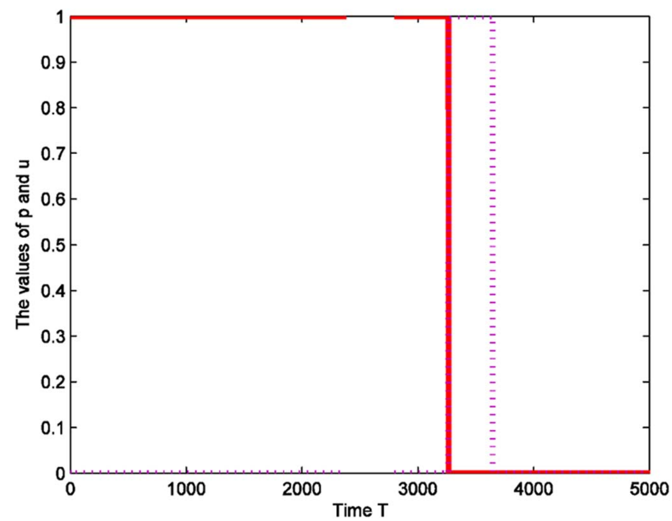

Fig. 5. Optimal policies p and u when $T = 5000$ s and $\varepsilon = -0.0005$.

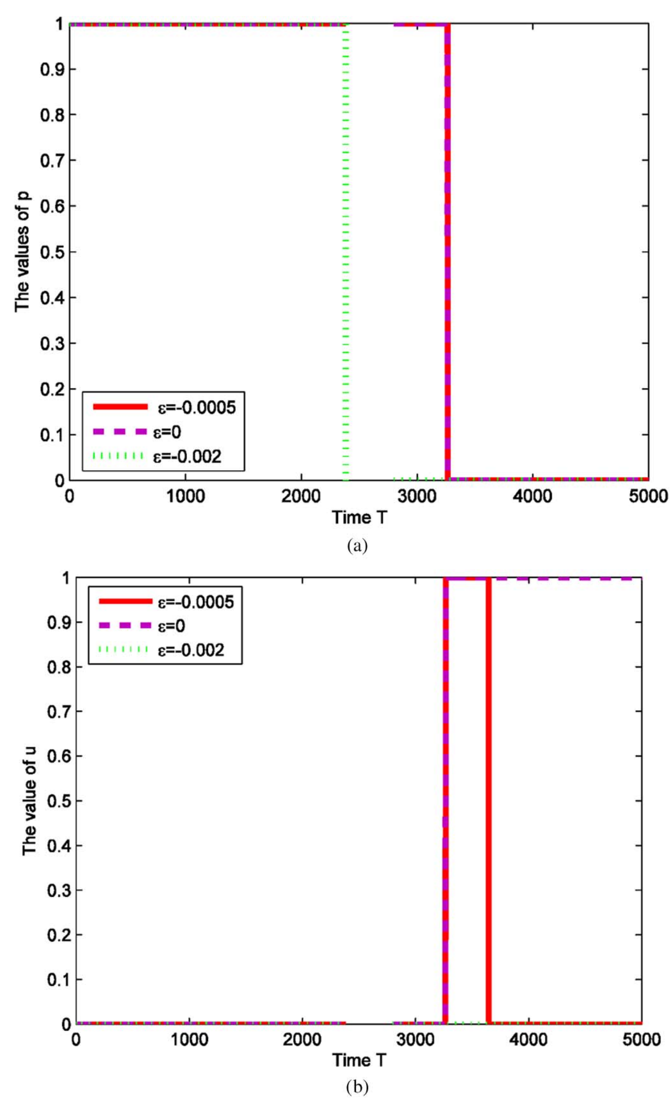

Fig. 6. Optimal policies p and u when ε has different values. (a) Values of p. (b) Values of u.

exponential decay distribution. Therefore, if we use the exponential distribution to fit this trace, we will bring a bigger deviation. Fortunately, the average deviation is not too big, so this means that our model may be used in certain cases, where nodes do not move according to the exponential mobility model.

B. Numerical Results

In this section, we only use the numerical results obtained by our theoretical model to evaluate the performance of different probing policies, and the results are obtained when $\lambda = 3.71 \times 10^{-6}s^{-1}$. Note that, at any time t, $\mu(t)$ and $p(t)$ are stochastic variables whose values belong to [0, 1]. Therefore, the range of the beaconing rate at any time is infinite, and we cannot consider all of the cases. For this reason, we just consider the optimal policies obtained by our model and the two policies in the aforementioned section.

Here, we let the normalized constant ε be -0.0005. This parameter may equal other values, and $\varepsilon = -0.0005$ is just an example. Note that, in the first policy, we have $p = 0$ and $u = 1$ all the time, but in the second policy, we have $p(t) = 0.2,\ u(t) = 0.2,\ 0 \le t \le T$. Based on these settings, we can get Fig. 4. This result shows that the optimal policy obtained in this paper is really better. It is easy to see that, in the first policy, only the nodes without a message probe to find the contact, which is similar to the works in [9] and [11]. However, this policy is worse than the optimal policy, so this result means that exploring the double probing policies is necessary. The optimal policies when $T = 5000$ s are shown in Fig. 5. From this figure, we can find that the optimal policies really conform to the *threshold* form, and it is consistent with Theorems 5 and 6. For example, in Fig. 5, the policy p has one *jump*, and u has two *jumps*.

From (4), we can see that the parameter ε may have an important impact. Now, we will explore the optimal policies when it has different values. Aside from $\varepsilon = -0.0005$, we consider two other cases, i.e., $\varepsilon = 0$ and $\varepsilon = -0.002$, respectively. The results are shown in Fig. 6. From this result, we can find that, when ε has different values, the optimal policies are different.

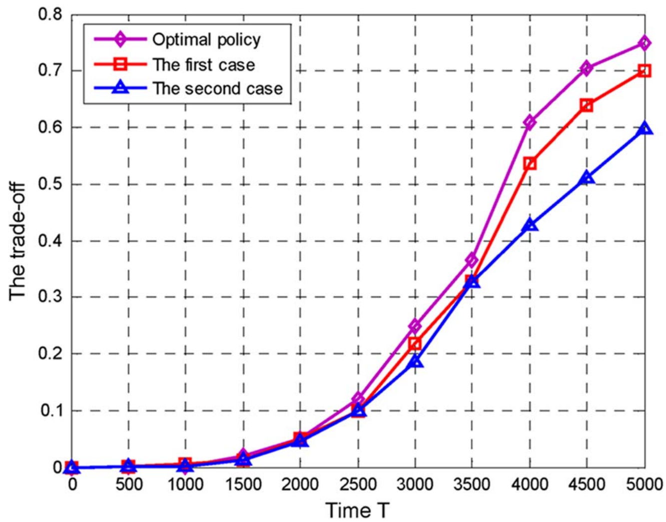

Fig. 7. Performance comparison in the real application based on the taxi drivers in the Shanghai city.

This result also shows that the optimal policies conform to the *threshold* form. For example, the values of p in all the cases have one *jump*, but the *jumps* may be different [see Fig. 6(a)]. In particular, the *jump* when $\varepsilon = -0.002$ is smaller. This is because $-\varepsilon$ is bigger, and the energy consumption has a bigger impact, so the nodes will stop beaconing at an earlier time to save the energy. As shown earlier, when $\varepsilon = -0.0005$, the optimal policy u has two *jumps*, but it only has one jump when $\varepsilon = 0$ [see Fig. 6(b)]. In addition, when $\varepsilon = -0.002$, u equals to 0 all the time. This means that nodes without a message never probe. This is also because $-\varepsilon$ is too big, and the nodes have to save the energy.

Now, we introduce an application in the real world. In particular, we consider the taxi drivers with smartphones in the Shanghai city with the same settings as mentioned earlier. The drivers can download messages from the server (e.g., 2G, 3G network) directly, but it will increase the burden and may bring congestion. To offload the traffic, they exchange messages through Bluetooth, and the server just tells the driver the corresponding actions (e.g., the probing policies) in the message exchanging process. This scenario is commonly studied and is often called as the mobile data offloading application [30]. Here, we will compare the optimal policies obtained in this paper with certain other policies. As shown in [20], the intermeeting time between two consecutive contacts of the taxis in the Shanghai city can be well fitted by the exponential distribution with parameter $3.71 \times 10^{-6} s^{-1}$, so the server can get the optimal policies easily. Note that the optimal policies conform to the *threshold* form, so the server just needs to get the time when the actions are changed. In this case, the server can tell each driver the probing policies through the Internet without increasing the burden significantly. Then, we consider two other policies. In the first case, we have $p = 1, u = 0$. In the second time, we have $p(t) = 1,\ u(t) = 0,\ t \leq T/2$; $p(t) = 0,\ u(t) = 0,\ t > T/2$. Based on these settings, we can get Fig. 7. This result shows that the optimal policies can be easily used in the real applications and can get the best performance, too.

VI. Conclusion

This paper explores the optimal probing problem in DTN. First, a theoretical model about the evolving process of the network states and the energy consumption is proposed. Then, this paper defines an optimization problem, which is solved through Pontryagin's maximal principle. Simulations based on both synthetic and real motion traces show the accuracy of the theoretical model. Numerical results show that the optimal policies obtained in this paper are really better. This means that the traditional policies, in which only the nodes without a message probe to find the contact, may not be rational in some applications.

In this paper, we assume that the nodes are well mixing. Such assumption has been used in many theoretical works, such as [8], [9], and [11]. Although the simulation results show that the theoretical models obtained based on this assumption are very accurate, it is hard to prove that the simulation results are correct for all applications. Therefore, we want to study the optimal probing policy without the assumption in the future.

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