Optimal detection to improve the performance of two-hop routing in selfish OppNets

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***Index Terms*—OppNet, Selfish, Black (Gray) Hole, ODE, Pontryagain's maximal principle**

I. Introduction

II. Related Work

A. Selfish OppNets

Observing the importance of OppNets over traditional networks for exchanging information, much efforts has been made to explore the OppNets with selfish nodes in the past few years. The majority of existing studies focus on selfish node detection(or misbehavior detection), for the behavior of selfish nodes may cause vulnerability and decrease the performance of OppNets, which was verified and numerical results were given in [1]. Most selfish node detection approaches can be broadly classified into two groups-the first groups relies on watchdog systems and the other depends on social trust-based communications [2].

For example, [3] [4] both proposed a collaborative watchdog approach based on the diffusion of selfish nodes awareness. [5] proposed a social-based watchdog system(SoWatch) for selfish nodes detection. Compared to most of existing detection schemes that primariliy rely on the nodes' contact records, SoWatch takes nodes' individual and social preferences into account. Zhu et al. proposed a probabilistic misbehavior detection scheme(iTrust) to judge a node's behavior, based on the collected routing evidences and probabilistically checking in [6]. A metric of misbehavior(MoM) for mathematically evaluating the extent of misbehavior of a node was introduced in [7], in which the misbehaving nodes were considered as the voting alternatives and the normally behaving nodes as the voters based on the Theory of Social Choice. A "Friendship and Selfishness Forwarding"(FSF) algorithm for accessing the relay node's selfishness was presented in [8], with the consideration of the friendship strength among a pair of nodes by using a machine learning algorithm. [9] proposed a provenance-based trust framework that aims to achieve accurate peer-to-peer trust assessment. Except approaches mentioned above that belong to the two classes, researchers also investigated other methods to improve selfish node detection. Basu et al. combined watchdog technique with trust-based communications and integrated with PRoPHET to build a global perception of forwarding behavior for detection of selfish nodes in [10]. Devi V. et al. introduced Semi Markov process for quantifying and future forecasting the probability with which the node could turn into selfish in WSN in [11].

Routing is a critical bottleneck after selfish nodes are detected and many literatures designed their routing algorithms for selfish OppNets with incentive mechanisms [12] [13] [14] [15] [16]. For instance, Li et al. proposed an incentive aware routing for selfish OppNets from a game theoretic perspective, which jointly considered individual selfishness and social selfishness to improve the performance of OppNets in [14]. What's more, Energy-aware routing schemes were presented in [17] [18]. Mao et al. investigated the energy-aware routing problem in MANETs with nodes' selfishness and solution based on game theory was given in [17]. Wu et al. proposed an energy-efficient copy-limit-optimized algorithm for epidemic routing in multi-community scenarios with social selfishness considerations using the Ordinary Differential Equations(ODEs) in [18]. [19] proposed a routing algorithm based on Geographic Information and Node Selfishness, which combines nodes' willingness to forward and their geographic information to maximize the possibility of contacting the destination.

B. Optimizations for Selfish OppNets

Optimization schemes for selfish OppNets can be classified into several types, the most typical one tries to explore how to control the transmitting process to get a trade-off between the energy consumption and the transmission performance. Since message in selfish OppNets is often transmitted in the store/carry/forward mode that includes beaconing and forwarding process, the optimal control of the message transmission in OppNets contains two parts. For instance, Yang et al. proposed optimal energy-efficient neighbor discovery schemes in OppNets in [20] [21] [22]. [23] applied Markov Chain Monte Carlo (MCMC) optimization to solve the problem of optimal relay selection for group communication, based on node contact patterns. After neighborhoods are found in beaconing process, forwarding process transmits the message. An optimal replication algorithm(QCR) for impatient content requesters was developed in [24], which drives the global cache in OppNets towards the optimal allocation. [25]introduced a model(ORBOPH) aims for specifying the message copy rule to optimize the distribution of message in OppNets. [26] proposed a multi-objectives based technique for optimized routing(MOTOR) in OppNets, which involves the use of a

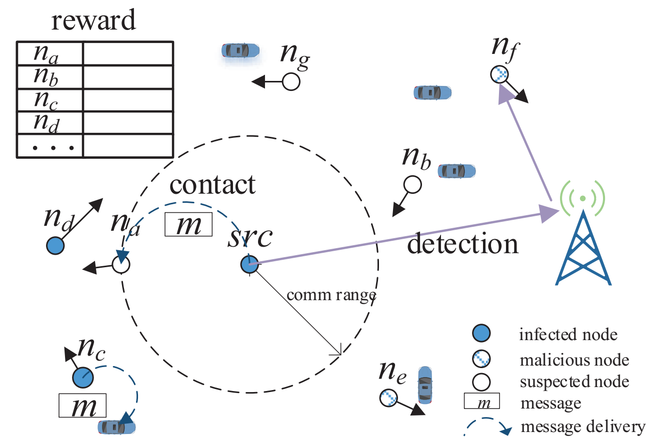

Fig. 1: Reward and Detection of the selfish nodes in OppNets.

weighted function to decide on the next hop selection of a node based on a combination of objectives. [27] modeled the OppIoT environment as a Markov decision process(MDP) and proposed a routing protocol RLProph for routing process optimization, which seeks to fully automate the OppIoT routing process by using the Policy Iteration algorithm to maximize the possibility of message delivery. [28] proposed both centralized and decentralized single-copy message forwarding algorithms to minimize the expected latencies from any node in the Opportunistic DTNs. All of the above works just consider one part of the optimal control of the message transmission in OppNets, Wu et al. considered the optimal forwarding and beaconing control problem at the same time in DTN and gave solution based on Pontryagin's Maximum Principle in [29], where multiple destinations exist.

Another one is optimal mobile data offloading schemes. Li et al. established a mathematical framework to study the problem of coding-based mobile data offloading in opportunistic vehicular networks in [30], they formulated the problem as a users' interest satisfaction maximization problem with multiple linear constraints of limited storage and proposed an efficient scheme to solve it. Wang et al. tried to find an optimal traffic offloading scheme through data partition to minimize the data delivery latency in opportunistic mobile networks in [**?**], they formulated the optimal cellular traffic offloading problem and proposed an approach to generate forwarding paths with possible heterogeneous data chunks.

What's more, a formal description of optimal communication topologies was introduced for the non-cooperative and cooperative settings in OppNets in [31], based on a game theoretic approach. An optimal data collection scheme for mobile crowdsensing was proposed in [32], which utilized integrated cellular and OppNets to implement data collection.

From above description, we can find that none of the existing works studies optimal control schemes for selfish nodes detection in OppNets, but this is the main objective of our work in this paper. (More info later)

III. Preliminaries

The source node src needs to disseminate its message m to vehicles or pedestrians. The N relay nodes can replicate m and send it to the vehicles, which is shown in Fig. 1. Thus the potential coverage area of the message is broadened by the opportunistic network. To encourage the collaboration of relay nodes, src should reward the relay node n_i $(1 \leq i \leq N)$ based on the time, when the message are carried by n_i. The time ranges from the replication time (τ_i) to the time-to-live of the message (T). τ_i can be recorded by src when n_i contacts src and replicates m. However, n_i may discard m immediately after the contact to earn the reward without carrying m, which is the selfish behavior. So src can check the checksum of m's specific part, which is store in the randomly selected relay node n_i. If the check failed, n_i will be identified as the selfish node and can not receive the reward. In this paper, we propose the optimal randomly detection strategy to achieve the tradeoff between the cost of the random detections and the wasted reward of the selfish behaviors.

$E(R(t))$ denotes the expected number of the relay nodes, which have not contacted src before time t. $E(I(t))$ denotes the expected number of infected relay nodes, which still carry the message at time t. $E(D(t))$ denotes the expected number of selfish relay nodes, which have discarded the message but are not known by src at time t. Similar to [33] and [34], the contacts between each pair of nodes including src are assumed to occur according to the Poisson process, in which the contact rate is λ. The total number of relay nodes is N, and $N = R(t) + I(t) + D(t)$, $\forall t$, $0 \leq t \leq T$. We also assume the change rate of becoming the selfish node is a constant value ρ. The detection rate is $U(t)$, $0 \leq U(t) \leq U_m$, $\forall t$, $0 \leq t \leq T$. which is the control function. For example, if the minimal circle of once detection T_m is that 2 seconds, the maximal detection rate is that $U_m = \frac{1}{T_m} = 0.5$ times per second. To simplify the denotations, we use $R(t)$, $I(t)$ and $D(t)$ to replace $E(R(t))$, $E(I(t))$ and $E(D(t))$, respectively. Then the main objective of our work is to to solve the following problem,

$$Min : J = \int_0^T (1-\alpha)D(t) + \alpha U(t)dt, \tag{1}$$

which minimizes the linear combination of the wasted reward and the detection cost through the weight α, $0 \leq \alpha \leq 1$. We can also get the total paid reward is

$$P = \int_0^T \beta(I(t) + D(t))dt, \tag{2}$$

where β is the reward paid for the one node's message carrying in a unit of time.

IV. Construction of ODE Model

We investigate the selfish detection in this and the following sections. Specifically, in this section, the ordinary differential equation model is constructed to capture the state change with time.

A. Case 1: without detection

In the case without detection, the relay node with message can become the selfish node, but the selfish detection is not conducted. Then the state transition is shown in Fig. 2 with the following rules. The nodes change from state R to state I if they contact src. The corresponding incremental rate of state I is $\lambda R(t)$ at time t. The selfish node also may contact

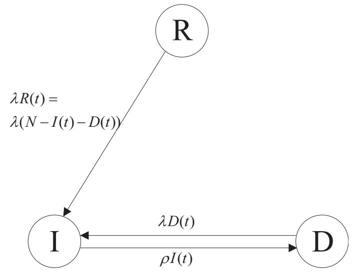

Fig. 2: State transition of the relay nodes without detection.

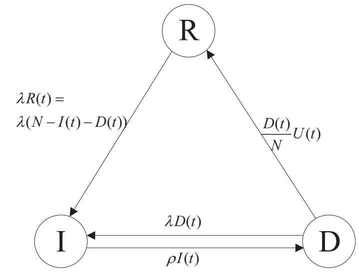

Fig. 3: State transition of the relay nodes.

src in the opportunistic network. Then the total incremental rate of I is $\lambda(R(t) + D(t)) = \lambda(N - I(t))$. Additionally, the infected node may become the selfish node with rate ρ. Thus we can obtain the derivative of $I(t)$ with respect to t,

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} = \lambda(N - I(t)) - \rho I(t).$$

where λ and ρ are constants. Similar to $\frac{\mathrm{d}I(t)}{\mathrm{d}t}$, we can get the change rate of state D and state R, i.e. $\frac{\mathrm{d}D(t)}{\mathrm{d}t}$ and $\frac{\mathrm{d}R(t)}{\mathrm{d}t}$, and obtain the model,

$$\begin{aligned}\frac{\mathrm{d}I(t)}{\mathrm{d}t} &= \lambda(N - I(t)) - \rho I(t),\\ \frac{\mathrm{d}D(t)}{\mathrm{d}t} &= -\lambda D(t) + \rho I(t),\\ \frac{\mathrm{d}R(t)}{\mathrm{d}t} &= -\lambda(N - I(t) - D(t)).\end{aligned} \tag{3}$$

Since $I(t)$ in (3) is formed by the first-order first-power ordinary differential equations (ODE) [34], we can calculate the general solutions of $I(t)$, that is,

$$I(t) = C_I e^{-(\lambda+\rho)t} + \frac{\lambda N}{\lambda + \rho}.$$

Note that $I(0) = 0$, $D(0) = 0$ and $R(0) = N$, which means only src carries the message. Thus $C_I = \frac{-\lambda N}{\lambda+\rho}$, and

$$I(t) = \frac{\lambda N}{\lambda + \rho}(1 - e^{-(\lambda+\rho)t}),$$

where $0 \leq t \leq T$. Similarly, we can calculate the general solution of the first-order ODE $D(t)$ from $\frac{\mathrm{d}D(t)}{\mathrm{d}t} + \lambda D(t) = \rho I(t)$,

$$\begin{aligned}D(t) &= C_D e^{-\int \lambda dt} + e^{-\int \lambda dt} \int \rho I(t) e^{\int \lambda dt} dt\\ &= C_D e^{-\lambda t} + e^{-\lambda t} \int \rho \frac{\lambda N}{\lambda + \rho}(1 - e^{-(\lambda+\rho)t}) e^{\lambda t} dt\\ &= C_D e^{-\lambda t} + \frac{\lambda N}{\lambda + \rho} e^{-(\lambda+\rho)t} + \frac{\rho N}{\lambda + \rho}\end{aligned}$$

Because of $D(0) = 0$,

$$D(t) = -Ne^{-\lambda t} + \frac{\lambda N}{\lambda + \rho} e^{-(\lambda+\rho)t} + \frac{\rho N}{\lambda + \rho}.$$

Since $I(t) + D(t) + R(t) = N$, $0 \leq t \leq T$, $R(t)$ can be computed based on the solved solution of $I(t)$ and $D(t)$. Thus the solution of (3) can be derived as

$$\begin{aligned}I(t) &= \frac{\lambda N}{\lambda + \rho}(1 - e^{-(\lambda+\rho)t}),\\ D(t) &= N(\frac{\lambda e^{-(\lambda+\rho)t} + \rho}{\lambda + \rho} - e^{-\lambda t}),\\ R(t) &= Ne^{-\lambda t},\end{aligned} \tag{4}$$

which depicts the change of the states when the time ranges from 0 to T. And $I(t)$, $D(t)$, $R(t) \geq 0$ always hold when $t \leq 0$. From the solutions of $I(t)$, $D(t)$ and $R(t)$, we can find that $I(t) \rightarrow \frac{\lambda N}{\lambda+\rho}$, $D(t) \rightarrow \frac{\rho N}{\lambda+\rho}$, and $R(t) \rightarrow 0$ when $t \rightarrow +\infty$. To verify the validity of the ODE model (3), we conduct the simulations with randomly settings. The corresponding results are presented in Section. VI-A.

Note that $U(t) = 0$, $\forall t$, in the situation without detection. The total cost J in (1) is determined by $D(t)$, $0 \leq t \leq T$, which is the total wasted reward by the selfish behaviors. Based on the calculated result in (4), we can compute J as

$$\begin{aligned}J &= \int_0^T (1 - \alpha) D(t) dt,\\ &= \int_0^T (1 - \alpha) N(\frac{\lambda e^{-(\lambda+\rho)t} + \rho}{\lambda + \rho} - e^{-\lambda t}) dt,\\ &= N(1 - \alpha)\left(\frac{\lambda(1 - e^{-(\lambda+\rho)T})}{(\lambda + \rho)^2} + \frac{\rho T}{\lambda + \rho} - \frac{1 - e^{-\lambda T}}{\lambda}\right).\end{aligned} \tag{5}$$

The total paid reward can be calculated as

$$\begin{aligned}P &= \beta \int_0^T I(t) + D(t) dt,\\ &= \beta \int_0^T (N - Ne^{-\lambda t}) dt,\\ &= N\beta(T - \frac{1 - e^{-\lambda T}}{\lambda}).\end{aligned}$$

Furthermore, the fraction between the wasted reward and the total paid reward is

$$p = \frac{\int_0^T D(t) dt}{\int_0^T I(t) + D(t) dt} = \frac{\frac{\lambda(1-e^{-(\lambda+\rho)T})}{(\lambda+\rho)^2} + \frac{\rho T}{\lambda+\rho} - \frac{1-e^{-\lambda T}}{\lambda}}{T - \frac{1-e^{-\lambda T}}{\lambda}}.$$

B. Case 2: with full detection

In the case with full detection, src conducts the selfish detection in the whole time-to-live. Note that when checking a selfish relay node n_i (state D), which means that n_i has discards the message and pretends as a node with message, src will let the node state change from state D to state R. When checking a normal node, i.e., state R and state I the number of nodes in each state will not change. Considering that the checked relay node is randomly selected from the N node set, we calculate the probability of checking a selfish node as $\frac{D(t)}{N}$. Since the detection rate is constrained by $U(t)$, we let $\frac{D(t)}{N}U(t)$ denote the change rate with time from state D to state R. Thus the state transition of the fully detection case is constructed as Fig. 3. The ODE model in (3) will be redefined as

$$\begin{aligned}\frac{\mathrm{d}I(t)}{\mathrm{d}t} &= \lambda(N-I(t))-\rho I(t),\\ \frac{\mathrm{d}D(t)}{\mathrm{d}t} &= -\lambda D(t)+\rho I(t)-\frac{D(t)}{N}U(t),\\ \frac{\mathrm{d}R(t)}{\mathrm{d}t} &= -\lambda(N-I(t)-D(t))+\frac{D(t)}{N}U(t),\end{aligned} \tag{6}$$

where $U(t)=U_m$, $\forall t$, $0\le t\le T$. The initial state is that $I(0)=D(0)=0$ and $R(0)=N$. So the solution of $I(t)$, which does not change from (4), is that $I(t)=\frac{\lambda N}{\lambda+\rho}(1-e^{-(\lambda+\rho)t})$. From $\frac{\mathrm{d}D(t)}{\mathrm{d}t}+(\lambda+\frac{U_m}{N})D(t)=\rho I(t)$, we can get that

$$\begin{aligned}&D(t)\\ =&C_{2D}e^{\int-(\lambda+\frac{U_m}{N})dt}+e^{\int-(\lambda+\frac{U_m}{N})dt}\int\rho I(t)e^{\int(\lambda+\frac{U_m}{N})dt}dt\\ =&C_{2D}e^{-(\lambda+\frac{U_m}{N})t}-\frac{\rho\lambda N}{\lambda+\rho}\frac{1}{\frac{U_m}{N}-\rho}e^{-(\lambda+\rho)t}+\frac{\rho\lambda N}{\lambda+\rho}\frac{1}{\lambda+\frac{U_m}{N}}.\end{aligned}$$

Since $D(0)=0$ and $I(t)+D(t)+R(t)=N$, the solution of (6) is that

$$\begin{aligned}I(t)=&\frac{\lambda N}{\lambda+\rho}(1-e^{-(\lambda+\rho)t}),\\ D(t)=&\frac{\rho\lambda N}{\lambda+\rho}\frac{1}{\lambda+\frac{U_m}{N}}-\frac{\rho\lambda N}{(\lambda+\frac{U_m}{N})(\frac{U_m}{N}-\rho)}e^{-(\lambda+\frac{U_m}{N})t}\\ &-\frac{\rho\lambda N}{\lambda+\rho}\frac{1}{\frac{U_m}{N}-\rho}e^{-(\lambda+\rho)t},\\ R(t)=&N-\frac{\lambda N}{\lambda+\rho}\left(\frac{\rho}{\lambda+\frac{U_m}{N}}+1\right)+\frac{\rho\lambda N}{\lambda+\rho}\frac{1}{\frac{U_m}{N}-\rho}e^{-(\lambda+\rho)t}\\ &+\frac{\lambda N}{\lambda+\rho}\left(\frac{\rho}{\frac{U_m}{N}-\rho}+1\right)e^{-(\lambda+\rho)t}.\end{aligned} \tag{7}$$

We can find that $I(t)\to\frac{\lambda N}{\lambda+\rho}$, $D(t)\to\frac{\rho\lambda N}{(\lambda+\rho)(\lambda+\frac{U_m}{N})}$, and $R(t)\to N-\frac{\lambda N}{\lambda+\rho}\left(\frac{\rho}{\lambda+\frac{U_m}{N}}+1\right)$ when $t\to+\infty$. Here $R(+\infty)\ne 0$ in the steady state is caused by the selfish detection. Thus the estimation of the total cost $\hat{J}$ in (1) can be computed as

$$\begin{aligned}\hat{J}=&\int_0^T(1-\alpha)D(t)+\alpha U(t)dt,\\ =&\frac{\rho\lambda N}{\lambda+\rho}\frac{(1-\alpha)T}{\lambda+\frac{U_m}{N}}+\frac{(1-\alpha)\rho\lambda N}{(\lambda+\frac{U_m}{N})^2(\frac{U_m}{N}-\rho)}(e^{-(\lambda+\frac{U_m}{N})T}-1)\\ &+\frac{(1-\alpha)\rho\lambda N}{(\lambda+\rho)^2(\frac{U_m}{N}-\rho)}(e^{-(\lambda+\rho)T}-1)+\alpha TU_m.\end{aligned} \tag{8}$$

The reason why (8) is the estimation of the cost is that the decrement of $D(t)$ actually occurs in the end of the detection period. However, the change rate of $D(t)$ caused by the detection is denoted by $\frac{D(t)}{N}U(t)$ in the above analysis. So there exists a deviation between the true cost J and the estimated cost $\hat{J}$ in the case with fully detection.

Lemma 1. *In the case with fully detection, $|J-\hat{J}|$ is less than $***$.*

Proof: Without loss generality, assume that T can be divided into k periods (T_m) and a duration $t^{'}$, where $t^{'}<T_m$. The i-th detection period in $[0,T]$ is denoted by $(t_{i-1},t_i]$, where $t_i-t_{i-1}=T_m$. In the first detection period $(t_0,t_1]$, $D(t)$ increases from $D(t_0)$ and approximately approaches to $D(t_1)$.

In the real word scenario,

■

We also can compute the approximate total reward is

$$P=\beta\int_0^T I(t)+D(t)dt,$$

The utilization ratio of the reward is that

$$p=\frac{\int_0^T D(t)dt}{\int_0^T I(t)+D(t)dt}$$

Here we find that p% reward is wasted in the selfish node. Although the wasted reward is reduced because of the detection, the additional cost, which is caused by the detection behavior, i.e., energy, bandwidth and wireless communication charge. is introduced.

Although we can get the change of state in the case of fully detection, there exists a difference between the true state change and (6), i.e. error ratio.......

V. Optimal Detection

A. Problem Formulation

Assume that the detection can be conducted. The detection rate is $U(t)$, $0\le U(t)\le U_m$. U_m is the limitation of the detection rate, which is the constraint from the hardware and the time sequences. We also use $\dot{U}$ to denote $U(t)$. Then, the ODEs can be reformed as

$$\begin{aligned}\dot{I}&=\beta(N-I)-\rho I,\\ \dot{M}&=\rho I-\beta M-\frac{M}{N}U,\\ \dot{S}&=-\beta(N-I-M)+\frac{M}{N}U.\end{aligned} \tag{9}$$

Meanwhile,

$$\begin{aligned}I(0)&=0,\\ M(0)&=0,\\ S(0)&=N.\end{aligned} \tag{10}$$

Thus $I(t)$ is the same with that in the situation without detection, which is

$$I(t)=\frac{\beta N}{\beta+\rho}-\frac{\beta N}{\beta+\rho}e^{-(\beta+\rho)t}. \tag{11}$$

Considering that the detection is also the cost, the object function will be

$$J=\int_0^T(1-\alpha)M+\alpha U dt.$$

Here α is the weight, which can control the importance between the cost of selfish relay nodes and detections. Thus $0<\alpha<1$. Similar with the previous section, $I(t)$ and $M(t)$ is the state variable. $U(t)$ is the controllable variable, $0\le U(t)\ \le U_m$.

B. Optimal Control by Pontryagin's Maximal Principle

Now we utilize the Pontryagin's maximal principle to find the optimal $U(t)$, which will minimize the total cost. First, the

Hamilton function is

$$\begin{aligned}H =&(1-\alpha)M + \alpha U + \lambda_1(\beta(N-I)-\rho I)\\&+\lambda_2(\rho I - \beta M - \frac{M}{N}U)\\=&(1-\alpha)M + \alpha U + \lambda_1(\beta(N-I)-\rho I) + \lambda_2 \rho I\\&-\beta\lambda_2 M - \lambda_2\frac{1}{N}UM\\=&(1-\alpha)M + \lambda_1(\beta(N-I)-\rho I)\\&+\lambda_2(\rho I - \beta M) + (\alpha - \lambda_2\frac{M}{N})U.\end{aligned}$$

Note that λ_1 and λ_2 denote $\lambda_1(t)$ and $\lambda_2(t)$, respectively. Without the final constraint, the terminal condition is $\lambda_2(T) = 0$ and $\lambda_3(T) = 0$. The adjoint function is

$$\dot{\lambda_2} = -\frac{\partial H}{\partial M} = \lambda_2(\beta + \frac{U}{N}) - (1-\alpha).$$

Thus

$$U^*(t) = \begin{cases} 0, & \text{if } \alpha - \lambda_2\frac{M}{N} \geq 0 \\ U_m, & \text{if } \alpha - \lambda_2\frac{M}{N} < 0 \end{cases} \tag{12}$$

In summary, we have the ODE functions $\dot{M}$, $\dot{\lambda_2}$, the initial condition $M(0) = 0$ and the boundary condition $\lambda_2(T) = 0$, Thus the problem is to solve a BVP problem, which is

$$\begin{aligned}&\dot{M} = \rho I - \beta M - \frac{M}{N}U,\\&\dot{\lambda_2} = -\frac{\partial H}{\partial M} = \lambda_2(\beta + \frac{U}{N}) - (1-\alpha),\\&M(0) = 0,\\&\lambda_2(T) = 0.\end{aligned} \tag{13}$$

We can solve the BVP problem with the shooting method by the bvpSolve package of R. Then we analyze the properties of the optimal control variable.

Lemma 2. *There exists a unique solution of (13).*

Proof: (13) comforts to Lipschitz condition. Then the solution exists and the solution is unique. ... (ODE). ∎

Lemma 3. *At the beginning and the end of the whole duration, the optimal control stop the selfish detection, which is $U(0) = U(T) = 0$.*

Proof: At the beginning of the duration, $M(0) = 0$, which is the initial condition of 13. Then $\alpha - \lambda_2(0)\frac{M(0)}{N} = \alpha > 0$. Following (12), the optimal $U(0) = 0$.

At the end of the duration, $\lambda_2(T) = 0$, which is the boundary condition of 13. Then $\alpha - \lambda_2(T)\frac{M(T)}{N} = \alpha > 0$. Based on (12), the optimal $U(T) = 0$. ∎

Based on the differential function $\dot{I}$, the equilibrium point of I can be obtained from $\dot{I} = 0$, which is $I^* = \frac{\beta N}{\beta+\rho}$. When $I(t) < I^*$, $I(t)$ will increase with t and approach to $\frac{\beta N}{\beta+\rho}$. Meanwhile, in this paper $I(0) = 0$ at the beginning of time.

Based on the differential function $\dot{M}$, the equilibrium point is obtained from $\dot{M} = 0$, which is $M^* = \frac{\rho I}{\beta + \frac{1}{N}U}$. In the situation without detection, the equilibrium point is $M^* = \frac{\rho I^*}{\beta} = \frac{\rho N}{\beta+\rho}$. In the situation with full detection, the equilibrium point is $M^* = \frac{\rho I^*}{\beta+\frac{1}{N}U_m} = \frac{\rho}{\beta+\frac{1}{N}U_m}\frac{\beta N}{\beta+\rho}$.

Since α is the weight of detecting the selfish nodes, we can assume that if α is enough high, the detection will not perform according to the optimal control strategy.

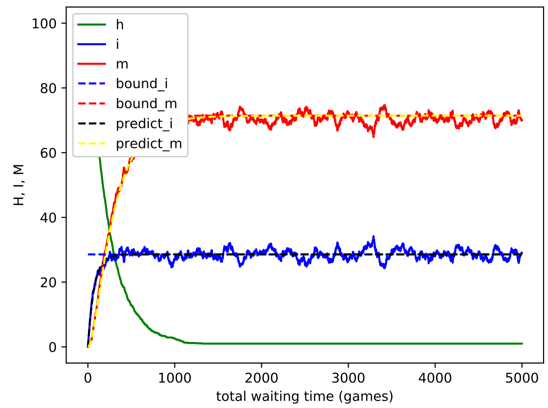

Fig. 4: $I(t)$ and $M(t)$ with time t obtained from prediction and simulations when $\beta = 0.004$, $\rho = 0.01$ and $N = 100$. Here h and i is the mean value of 20 simulations.

Lemma 4. *If $\alpha \geq \alpha_{th}$, the optimal control let the detection stop in the whole duration, namely $U(t) = 0$, $0 \leq t \leq T$.*

Proof: Assume that ρ, N, β is given. Let $W(t) = \lambda_2(t)M(t)$.

$$\begin{aligned}W^{'}(t) =&M^{'}(t)\lambda_2(t) + M(t)\lambda_2^{'}(t)\\=&(\rho I(t) - \beta M(t) - \frac{M(t)}{N}U(t))\lambda_2(t)\\&+M(t)(\lambda_2(t)(\beta + \frac{U(t)}{N}) - (1-\alpha))\\=&\rho\lambda_2(t)I(t) - (1-\alpha)M(t).\end{aligned} \tag{14}$$

Since $M(0) = 0$ and $\lambda_2(T) = 0$, $W(0) = W(T) = 0 < \alpha N$.

Now we focus on the poles of $W(t)$, namely t^*, where $W^{'}(t^*) = \rho\lambda_2(t^*)I(t^*) - (1-\alpha)M(t^*) = 0$. Then $M(t^*) = \frac{\rho\lambda_2(t^*)I(t^*)}{1-\alpha}$.

$$W(t^*) = \lambda_2(t^*)M(t^*) = \frac{\rho I(t^*)\lambda_2(t^*)^2}{1-\alpha}. \tag{15}$$

According to $\dot{\lambda_2}$ in (13), the equilibrium point of λ_2 is that $\lambda_2^* = \frac{1-\alpha}{\beta+\frac{U}{N}}$. Since $0 \leq U \leq U_m$, $0 < \frac{1-\alpha}{\beta+\frac{U_m}{N}} \leq \lambda_2^* \leq \frac{1-\alpha}{\beta}$. Note $\lambda_2(T) = 0$. Based on the phase line in ODE for $\dot{\lambda_2}$, $\lambda_2(t)$ decreases with t when $\lambda_2(t) < \lambda_2^*$. Conversely, $\lambda_2(t)$ increases with t when $\lambda_2(t) > \lambda_2^*$. Thus $0 \leq \lambda_2(t) \leq \lambda_2^* \leq \frac{1-\alpha}{\beta}$ when $0 \leq t \leq T$. Additionally, $0 \leq I(t) \leq \frac{\beta N}{\beta+\rho}$. From (15), we can derive that the upper boundary of $W(t)$, W_{up}, which is

$$W(t) \leq W(t^*) \leq \frac{\rho}{1-\alpha}\frac{\beta N}{\beta+\rho}(\frac{1-\alpha}{\beta})^2 = \frac{\rho N(1-\alpha)}{\beta(\beta+\rho)} = W_{up}.$$

Assume that α can satisfy that $W_{up} \leq \alpha N$, which means that $\alpha \geq \frac{\rho}{\beta(\beta+\rho)+\rho} = \alpha_{th}$. Then $W(t) \leq \alpha N$, when $0 \leq t \leq T$. Therefore the optimal control $U^*(t) \equiv 0$, when $0 \leq t \leq T$. ∎

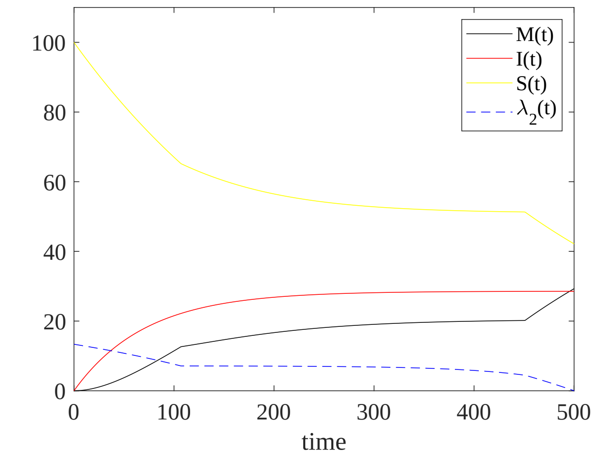

Fig. 5: State variable of analysis with time.

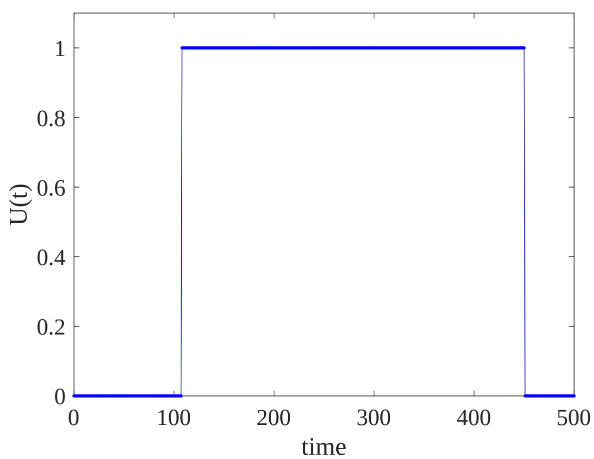

Fig. 6: Control variable of analysis with time.

VI. Performance Evaluation

A. Efficacy of the ODE model

Fig. 4 shows that the change of the states in the experiments conforms to the solved solutions.

Fig. XX for (6) and error ratio.

B. Optimal solution of selfish detection

VII. Conclusion

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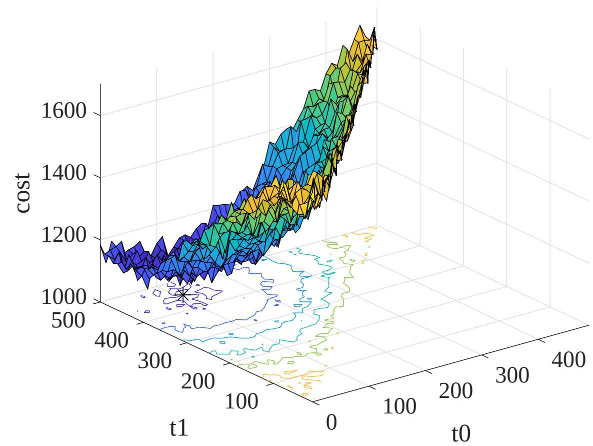

Fig. 7: Different choices of $t0$ and $t1$.

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