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Energy efficient beaconing control in delay tolerant networks with multiple destinations

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Abstract: The store-carry-forward communication mode in delay tolerant networks closely depends on the opportunistic contacts between the nodes. In addition, there is a higher probability that a node can detect others if the beaconing rate is higher (although this consumes more energy). This study studies the optimal beaconing policy to obtain a trade-off between the performance and the energy consumption. In particular, the authors consider the case with multiple destinations. According to whether the destinations forward the message to others, they study the problem in non-replicative and replicative cases, respectively. First, a theoretical framework that can evaluate the trade-off under different beaconing policies is introduced for each case. Then, they formulate an optimisation problem based on these frameworks. Through Pontryagin's maximal principle, they obtain the optimal policies and prove that the optimal policies conform to the threshold form in both cases. The imulations show the accuracy of the theoretical frameworks. The numerical results show that they can obtain a better trade-off in the replicative case.

1 Introduction

The concept of delay tolerant networks (DTN) is proposed to support applications where the links between the nodes are uncertain [1]. The uncertainty means that the end-to-end path for a given nodes pair may not exist at a given time, and may be caused by many factors, such as the unpredictable mobility, sparse density and so on [1]. For example, in the military ad-hoc networks, the disconnection of the paths may be caused by the environmental factors or intentional jamming and so on. At present, the DTN is very popular and is used in many emerging applications, such as deep-space exploration [2], mobile social networks [3] and peer-to-peer networks [4]. In fact, the DTN can be seen as an extension of the traditional mobile ad-hoc networks (MANET), and can be used without the assumption of a full connection. The routing protocols in traditional MANET make the assumption that the network is fully connected and fail to route the messages if there is no complete path from the source to the destination at the time of sending [5]. Therefore those protocols cannot work in the DTN efficiently.

To overcome the partitions, the nodes in the DTN adopt the store-carry-forward communication mode. In this case, the nodes can exchange messages only when they meet each other. When they do not encounter others, they will store the messages in their buffer and wait for the next contact opportunity. The most typical routing algorithm is the epidemic routing (ER) protocol, in which the nodes carrying a message will forward the message to all the neighbours [6]. Although the ER can improve the message spreading speed, it uses more energy. This means that if we

forward the message all the time, the energy consumption is high. In addition, the store-carry-forward communication mode closely depends on the opportunistic contact between the nodes, which is further related to the beaconing rate. Therefore another method to improve the message spreading speed is to increase the beaconing rate of the nodes [7], which denotes the probability that the nodes beacon with the maximal ability. The nodes have a higher probability to obtain the message, although more energy is consumed. Therefore better performance often corresponds to more energy consumption and how to obtain the trade-off between the performance and the energy consumption is an important problem. This problem can be divided into many different cases, for example, the optimal forwarding problem and the optimal beaconing problem and so on. In the optimal forwarding problem, the core is to find the optimal probability that nodes forward the message to others at every instant. However, in the optimal beaconing problem, we have to find the optimal beaconing

At present, many papers study the optimal forwarding problem. For example, the paper in [8] studies the trade-off between the message delay and the energy consumption and it proves that the optimal policy conforms to the threshold structure. The paper in [9] explores the optimal forwarding policy to obtain the best trade-off based on the two-hop protocol. On the other hand, there are some papers, which explore the optimal forwarding problem when the total energy is constrained. For example, the paper in [10] explores this problem for the first time and it introduces a discrete-time model to describe the message spreading process. Based on the discrete-time model, it

obtains the optimal forwarding policy. The paper in [11] studies a similar problem, but it adopts the continuous-time model. The paper in [12] explores the problem when the nodes are selfish. Compared with the optimal forwarding problem, there are fewer papers about the optimal beaconing problem. The paper in [13] studies the joint beaconing and the forwarding problem for the first time, but it only uses the two-hop routing protocol. The paper in [14] explores the optimal beaconing policy based on the ER protocol. However, it assumes that there is only one destination. In many applications, there may be multiple destinations. In this case, the energy consumption for the destinations cannot be ignored, hence optimal control with multiple destinations is necessary. The paper in [15] considers the optimal controlling problem in the DTN with multiple destinations. However, this paper just focuses on the optimal forwarding control, which ignores the energy consumption in the beaconing process.

In this paper, we study the trade-off between the performance and the energy consumption by controlling the beaconing rate. In particular, we focus on the case with multiple destinations. According to whether the destinations forward the message to others, we consider two cases, which are the non-replicative and the replicative cases, respectively. In the first one, the destinations just receive the message and never forward towards the others. In the replicative case, the destinations also act as the relay nodes and forward the message to others. The main contributions of this paper are summarised as follows:

- We present two theoretical frameworks to describe the message spreading process for the above two cases, which can be used to evaluate the trade-off under different beaconing rates. Then, we formulate the corresponding optimisation problem, in which the controlling parameter is the beaconing rate.
- Through Pontryagin's maximal principle, we obtain the optimal beaconing policies and prove that the optimal policies conform to the threshold form in both cases.

2 Network model

There are N relay nodes and M destinations. Among these relay nodes, there is a source S, and only S has the message at time 0. The maximal lifetime of the message is T. We use the ER protocol. The main symbols used in this paper are in Table 1.

In the DTN, the messages can be transmitted between two nodes only when they meet each other, which can be seen as a

Table 1 Variables summary

$\mu(t)$	beaconing rate of the relay nodes at time t
$\sigma(t)$	beaconing rate of the destinations at time t
Ν	the number of relay nodes
Μ	the number of destinations
X(t)	the number of relay nodes carrying the message at time t
<i>Y</i> (<i>t</i>)	the number of destinations carrying the message at time t
U(t)	energy consumption up to time t
ε	the parameter that relates the energy consumption and
	the performance
F(T)	objective function
λ_X	co-state corresponding to $X(t)$
λ_Y	co-state corresponding to Y(t)
λυ	co-state corresponding to $U(t)$

contact [1]. Therefore, the mobility of the nodes is critical, and a proper model that can fit the mobility law is necessary. At present, there are many models, such as the home-MEG model [16], the edge-Markov model [17] and so on. Although these models are accurate to fit some motion traces, they are too complex, so many papers still use the exponential model (the inter-meeting time between the two contacts conforms to the exponential distribution) [9–14]. Some papers find that the exponential model is very accurate in certain applications. For example, the paper in [18] finds that the tail of the distribution is exponential. The paper in [19] shows that the inter-meeting time can be shaped to be exponential by choosing an approximate domain size. Moreover, some papers describe the inter-meeting time of humans or vehicles by exponential distribution and validate their model experimentally on the real motion traces [20, 21]. Furthermore, the mobility patterns of the nodes are very sophisticated and cannot be captured well by one mobility model. For example, although the home-MEG model can describe the power-law and the exponential decay distribution, which has been found in certain motion traces (e.g. Infocom'05, MIT BT etc.) [16], it is not accurate in many other applications. For example, the paper in [22] finds a three-segmented distribution in the motion trace of Beijing taxis. In other words, there is no general model that can fit all the motion traces now, and each model has certain applications. Therefore, we use the exponential model, and it is rational in many traces (e.g. RWP, Taxis in Shanghai city etc.) [21].

The contact rate depends on the beaconing rate. In this paper, we assume that if the nodes use the highest beaconing rate, the inter-meeting time follows an exponential distribution with the parameter λ . $\mu(t)$ and $\sigma(t)$ denote the beaconing rates of the relay nodes and the destinations at time t, respectively. In addition, we have $0 \le \mu(t)$ and $\sigma(t) \le 1$. The beaconing rate can be seen as the probability that the nodes beacon with the maximal ability. Given the time interval $[t, t + \Delta t]$, a node finds others with probability $1 - e^{-\lambda \Delta t}$ if it uses the maximal ability. At time t, the probability that the relay nodes beacon with the maximal ability is $\mu(t)$, hence a node detect others with probability $\mu(t)(1 - e^{-\lambda \Delta t})$. If we take the limitation for Δt , we can obtain $\lambda \mu(t)$. For this reason, we say that the contact rate of the relay nodes is $\lambda \mu(t)$ at time t.

The existing papers show that the energy consumption in communication mainly includes the transmitting consumption in the communication state as well as the beaconing consumption in the idle state [23]. Therefore we mainly consider the energy consumption in the transmitting and beaconing processes similar to [13, 14].

Both X(t) and Y(t) (see Table 1) are stochastic variables related to $\mu(t)$, so we consider their expectations. For any random variable V, we use E(V) to denote its expectation. In this paper, we assume that each transmission is successful. This assumption has been used in most of the theoretical works in the DTN, such as [13, 14]. In this case, each transmission will make a node obtain the message, hence the number of nodes obtaining the message from others is equal to the number of the transmissions (denoted by the transmission times). Up to time t, the expectation of the number of nodes, which obtain the message from others is E(X(t)) + E(Y(t)) - 1 (the source has the message at time 0, and it does not receive the message from the others), hence the average value of the transmission times is E(X(t)) + E(Y(t)) - 1. In this case, the energy consumption for the

transmission can be expressed as $\alpha(E(X(t)) + E(Y(t)) - 1)$. α is a positive multiplication, which is system specific, and is used to weight the energy consumption in each transmission [13, 14]. In different applications, its value may be different. For example, if the energy consumption of each transmission is 1 J, we have $\alpha = 1$ J. For the beaconing process, we assume that once a node obtains the message, it will stop beaconing to save energy, and a similar assumption has been used in [13, 14]. Therefore the beaconing energy consumption is proportional to the beaconing rate and the number of nodes that do not have a message. On the other hand, we assume that the beaconing rate of the destinations that do not obtain the message is 1 all the time. Similar to [13, 14], we can obtain the average beaconing energy consumption

$$\beta \int_{0}^{T} (\mu(t)(N - E(X(t)) + (M - E(Y(t))))dt$$
 (1)

Symbol β is the system specified positive constant, and it weights the energy consumption of each beaconing [13, 14]. On combining with the energy consumption in the transmission process, we have

$$E(U(T)) = \alpha(E(X(t) + E(Y(t)) - 1) + \beta \int_{0}^{T} (\mu(t)(N - E(X(t)) + (M - E(Y(t))))dt$$
(2)

Now, our object is to solve the following optimisation problem

Maximise
$$E[F(T)] = E(Y(T)) + \varepsilon E(U(T)), \ \varepsilon \le 0$$
 (3)

 ε is the parameter that relates the energy consumption and the performance [9]. E(Y(T)) denotes the average number of the destinations that have the message up to time T, which can be obtained easily by the ODEs (ordinary differential equations) defined in the next sections. For example, in the non-replicative case, the value of E(Y(T)) can be obtained by solving the ODEs constituted with (7), (10), (12) and (16). We know the initial values of the states (X, Y), and we have (E(X(0)) = 1, E(Y(0)) = 0). In addition, we know the final values of the states (λ_X, λ_Y) [see (13)]. Therefore the numerical methods for solving the boundary value non-linear differential equation problems may now be used to solve for the state and the adjoint functions. Then, we can obtain the value of E(Y(T)). In the replicative case, the value of E(Y(T)) can be obtained by the corresponding ODEs defined in Section 4. If the value of E(Y(T)) is bigger, more destinations obtain the message, and we can say that the performance is better. However, if E(U(T)) is bigger, the energy consumption is higher, and it is not good for the network. Therefore, ε is a non-positive constant, that is, we have $\varepsilon \le 0$. If E(Y(T)) and E(U(T)) can be seen as the income and the cost, respectively, E[F(T)] can be seen as the total income, and it is the objective function. As shown above, better performance often corresponds to more energy consumption. In many applications, more energy consumption may not be rational, hence it is necessary to

explore the trade-off between the energy consumption and the performance [9]. That is why we present (3). By maximising (3), we can know how to use the energy efficiently to obtain the best trade-off, hence this can be seen as the energy efficient beaconing control problem. This objective function is meaningful in many applications. For example, in the wildlife monitor network, some sensors are deployed in certain animals [24]. Owing to the animals' mobility, the network may be disconnected and belongs to the DTN. When a sensor detects an interesting phenomenon, they will forward it to the (destinations). To increase the transmission speed, the other sensors may act as relay nodes and help to forward the message. If the sensors beacon with a higher rate, they contact others frequently, and the message may be transmitted quickly. Then, more sinks may obtain the message when the message is still useful. However, this will consume more energy, hence the trade-off between the performance and energy consumption is necessary.

3 Non-replicative case

3.1 Theoretical model

In this section, we mainly obtain the values of X(t) and Y(t), which are defined in Table 1. For any contiguous time instants t_{n+1} and t_n , the value of $X(t_{n+1})$ just depends on $X(t_n)$, and is independent of the values before time t_n , we can see the result in (5) more clearly. Similarly, Y(t) also has the property. Therefore both X(t) and Y(t) satisfy the following equation (see(4))

In (4), we have $0 \le t_0 < t_1 < \dots < t_{n+1}$, $0 \le i_k$, $i_{n+1} \le N$, $0 \le j_k$, $j_{n+1} \le M$. This means that both X(t) and Y(t) are continuous-time Markov chains.

In the non-replicative case, the destinations never forward the message to others. Given a minor interval Δt , we have [11]

$$X(t + \Delta t) = X(t) + \sum_{j \in \{ \setminus X(t) \}} \phi_j(t, t + \Delta t)$$
 (5)

Symbol $\{ \backslash X(t) \}$ denotes the set of relay nodes that do not have the message at time t. $\varphi_j(t, t + \Delta t)$ denotes the event whether the node j obtains the message in the time interval $[t, t + \Delta t]$. If $\varphi_j(t, t + \Delta t) = 1$, we say that this event happens, or we have $\varphi_j(t, t + \Delta t) = 0$. The nodes meet each other according to the exponential distribution. In particular, at time t, the beaconing rate of the node j is $\mu(t)$. Therefore the node j meets a specific node that has the message (e.g. k) with the probability $\mu(t)(1 - e^{-\lambda \Delta t})$. Furthermore, we have

其实类似于 07 ODE建模方法
$$p(\phi_j(t, t + \Delta t) = 1) = 1 - \left(1 - \mu(t)\left(1 - e^{-\lambda \Delta t}\right)\right)^{X(t)}$$
 (6)

By a similar method in [3], we can obtain the closed-form expression of (6)

$$E(X(t)) = (N - E(X(t))) \lim_{\Delta t \to 0} \frac{E(\phi_j(t, t + \Delta t))}{\Delta t}$$
$$= \lambda (N - E(X(t))) E(X(t)) \mu(t) \tag{7}$$

$$P\{(X(t_{n+1}) = i_{n+1}, Y(t_{n+1}) = j_{n+1}) | (X(t_k) = i_k, Y(t_k) = j_k), 0 \le k \le n\}$$

$$= P\{(X(t_{n+1}) = i_{n+1}, Y(t_{n+1}) = j_{n+1}) | (X(t_n) = i_n, Y(t_n) = j_n)\}$$
(4)

IET Commun., 2014, Vol. 8, Iss. 5, pp. 730–739 doi: 10.1049/iet-com.2013.0237 For the destination nodes, we have

$$X(t + \Delta t) = X(t) + \sum_{j \in \{\backslash Y(t)\}} \rho_j(t, t + \Delta t)$$
 (8)

Symbol $\{Y(t)\}$ denotes the set of the destinations that do not have the message at time t. $\rho_j(t, t + \Delta t)$ denotes the event where the node j obtains the message in the time interval $[t, t + \Delta t]$. Note that the destinations belonging to the set $\{Y(t)\}$ beacon with the maximum rate 1 all the time, hence we have

$$p(\rho_j(t, t + \Delta t) = 1) = 1 - \left(1 - \left(1 - e^{-\lambda \Delta t}\right)\right)^{X(t)}$$
 (9)

Then, we obtain

$$E(Y(t)) = \lambda (M - E(Y(t)))E(X(t)) \tag{10}$$

3.2 Optimal control

Now, we solve the optimisation problem in (3). Let $((X, Y), \mu)$ be an optimal solution. Consider the Hamiltonian H, and the co-state or the adjoint functions λ_X , λ_Y and λ_U defined as follows

$$H = \overset{\bullet}{F} + \lambda_X \overset{\bullet}{X} + \lambda_Y \overset{\bullet}{Y} = \overset{\bullet}{Y} + \varepsilon \overset{\bullet}{U} + \lambda_X \overset{\bullet}{X} + \lambda_Y \overset{\bullet}{Y}$$

$$= (1 + \lambda_Y) \overset{\bullet}{Y} + \lambda_X \overset{\bullet}{X} + \varepsilon (\alpha (\overset{\bullet}{X} + \overset{\bullet}{Y})$$

$$+ \beta (\mu (N - X) + M - Y))$$

$$= (1 + \lambda_Y + \alpha \varepsilon) \overset{\bullet}{Y} + (\lambda_X + \alpha \varepsilon) \overset{\bullet}{X}$$

$$+ \varepsilon \beta (\mu (N - X) + M - Y)$$

$$= \lambda (1 + \lambda_Y + \alpha \varepsilon) X (M - \overset{\bullet}{Y}) + \varepsilon \beta (M - \overset{\bullet}{Y})$$

$$+ \lambda (\lambda_X + \alpha \varepsilon) X (N - X) \mu + \varepsilon \beta \mu (N - X)$$

$$(11)$$

(see (12))

The transversality conditions satisfy

$$\lambda_{V}(T) = \lambda_{V}(T) = 0 \tag{13}$$

Then, according to Pontryagin's maximum principle [25, P.109, Theorem 3.14], we have

$$\mu \in \arg\max_{0 \le \mu^* \le 1} H(\lambda_X, \lambda_Y, (X, Y), \mu^*)$$
 (14)

This equation means that maximising E[F(T)] equals to maximising the corresponding Hamiltonian H. Based on (11), we can obtain the optimal beaconing policy as follows

$$\mu = \begin{cases} 1, \ \lambda(\lambda_X + \alpha \varepsilon)X(N - X) + \varepsilon \beta(N - X) > 0 \\ 0, \ \lambda(\lambda_X + \alpha \varepsilon)X(N - X) + \varepsilon \beta(N - X) < 0 \end{cases}$$
 (15)

As $1 \le X \le N$, N-X equals to 0 only when X=N. However, when X=N, all the relay nodes have the message, hence it is meaningless to explore the optimal beaconing rate for the relay nodes now. Therefore we only consider the case $X \le N$. In addition, we also only consider the case $Y \le M$. In these cases, (15) can be changed to the following expression

$$\mu = \begin{cases} 1, \lambda(\lambda_X + \alpha \varepsilon)X + \varepsilon \beta > 0 \\ 0, \lambda(\lambda_X + \alpha \varepsilon)X + \varepsilon \beta < 0 \end{cases}$$
 (16)

According to (16), we have Lemma 1.

Lemma 1: The optimal policy μ has at most one jump and it satisfies: $\mu(t) = 1$, t < h, and $\mu(t) = 0$, t > h, $0 \le h \le T$. Time h can be seen as the stopping time.

Proof: Here, we use X(t) and Y(t) to denote E(X(t)) and E(Y(t)), respectively. First, we define a new function f as: $f = \lambda(\lambda_X + \alpha \varepsilon)X + \varepsilon \beta$. Then, we have

$$\dot{f} = \lambda(\lambda_X + \alpha\varepsilon)\dot{X} + \lambda(-\lambda(1 + \lambda_Y + \alpha\varepsilon)(M - Y)
- \lambda(\lambda_X + \alpha\varepsilon)(N - 2X)\mu + \varepsilon\beta\mu)X$$

$$= \lambda(\lambda_X + \alpha\varepsilon)\dot{X} + \lambda(-\lambda(1 + \lambda_Y + \alpha\varepsilon)(M - Y)
- \lambda(\lambda_X + \alpha\varepsilon)(N - X)\mu + \lambda(\lambda_X + \alpha\varepsilon)X\mu + \varepsilon\beta\mu)X$$

$$= \lambda(-\lambda(1 + \lambda_Y + \alpha\varepsilon)(M - Y) + f\mu)X$$
(17)

The function g is defined as: $g = 1 + \lambda_Y + \alpha \varepsilon$. From Lemma 3, we can see that if f(s) = 0, we have $g(s) = 1 + \lambda_Y(s) + \alpha \lambda_U(s) > 0$. Since we only consider the case Y < M, we have

$$f(s) = \lambda(-\lambda(1+\lambda_Y(s)+\alpha\varepsilon)(M-Y(s)))X(s)$$

$$= \lambda(-\lambda g(s)(M-Y(s)))X(s) < 0$$
(18)

Therefore the function f is decreasing at t | me s.

Now, we prove that $\inf f(s) < 0$, it will remain negative all the time. Otherwise, if there exist certain times, at which f is non-negative (e.g. $f(v) \ge 0$), according to the property of the continuous function (f is a continuous function based on our continuous-time model), we can say that there exists at least one time between s and v, at which f equals to 0. In other words, if $f(v) \ge 0$, we can find at least one time k in (s, v], and we have f(k) = 0. Furthermore, we assume that k is the first time, at which f equals to 0 in time interval [s, v]. That is, we have f(t) < 0, s < t < k. Time k— is just smaller than k, that is, there is no other time between k— and k. Obviously, we have f(k) = 0. Since the next time of k— is k,

$$\begin{cases} \lambda_X^{\bullet} = -\frac{\partial H}{\partial X} = -\lambda(1 + \lambda_Y + \alpha\varepsilon)(M - Y) - \lambda(\lambda_X + \alpha\varepsilon)(N - 2X)\mu + \varepsilon\beta\mu \\ \lambda_Y^{\bullet} = -\frac{\partial H}{\partial Y} = \lambda(1 + \lambda_Y + \alpha\varepsilon)X + \varepsilon\beta \end{cases}$$
(12)

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and f(k) = 0, f must increase at time k—. Therefore we have

$$f(k-) = \lambda(-\lambda(1+\lambda_Y(k-)+\alpha\varepsilon)(M-Y(k-)))X(k-)$$

$$= \lambda(-\lambda g(k-)(M-Y(k-)))X(k-) > 0$$
(19)

Therefore we have $g(k-) = 1 + \lambda_Y(k-) + \alpha \varepsilon < 0$. Since $g(k) = 1 + \lambda(k)_Y + \alpha \varepsilon > 0$, according to the properties of the continuous functions (g is a continuous function based on our continuous-time model), there should exist at least one time between k- and k, at which the function $g=1+\lambda_Y+\alpha\varepsilon=0$. However, we have assumed that there is no other time between k- and k, hence this is contradiction. Therefore, if f(s) < 0, it will remain negative all the time.

In summary, if f(s) = 0, f will become negative in the next time, and if f(s) < 0, it will remain negative all the time. Therefore, if s is the first time that satisfies $f(s) \le 0$, we have f(t) > 0, t < s and f(t) < 0, $s < t \le T$. Furthermore, according to (16), we have $\mu(t) = 1$, t < s and $\mu(t) = 0$, t > s, $0 \le s \le T$. In this case, the stopping time h is equal to s.

Lemma 2: For the function $g(t) = 1 + \lambda_{Y}(t) + \alpha \varepsilon$, once it is non-positive at any time, it will remain non-positive all the time.

Proof: First, we have

$$g(t) = \lambda_Y(t) = \lambda g(t)X(t) + \varepsilon \beta$$
 (20)

Note that $\varepsilon \le 0$. For any time s, if $g(s) \le 0$, we have

$$g(s) = \lambda_Y(s) = \lambda g(s)X(s) + \varepsilon \beta \le 0$$
 (21)

Therefore the function g decreases at time s. Furthermore, we can see that the function g cannot increase anymore and it will remain non-positive all the time.

Lemma 3: For the functions $f(t) = \lambda(\lambda_X(t) + \alpha \varepsilon)X(t) + \varepsilon\beta$ and $g(t) = 1 + \lambda_Y(t) + \alpha \varepsilon$, if f(s) = 0, we have g(s) > 0.

Proof: Otherwise, we assume $g(s) \le 0$. From Lemma 2, we know $g(t) \le 0$, $s < t \le T$. The function z is defined as $z(t) = 1 + \lambda_Y(t) - \lambda_X(t)$. When f(s) = 0, we have $\lambda(\lambda_X(s) + \alpha \varepsilon)X(s) + \varepsilon\beta = 0$. Then, we have $\lambda(\lambda_X(s) + \alpha \varepsilon)X(s) = -\varepsilon\beta$.

Furthermore, we have $\lambda(s)g(s)X(s) = \lambda(1+\lambda_Y(s)+\alpha\varepsilon)X(s) = \lambda(1+\lambda_Y(s)-\lambda_X(s))X(s)+\lambda(\lambda_X(s)+\alpha\varepsilon)X(s)$. In other words, we have $\lambda(s)g(s)X(s) = \lambda(s)z(s)X(s)-\varepsilon\beta$. When $g(s) \leq 0$, we have $\lambda(s)g(s)X(s) = \lambda(s)z(s)X(s)-\varepsilon\beta \leq 0$. Therefore we obtain $\lambda(s)z(s)X(s) \leq \varepsilon\beta \leq 0$. This means $z(s) \leq 0$. This is in contradiction to Lemma 4, which has proved that when $g(s) \leq 0$, we have z(s) > 0. Therefore the assumption that $g(s) \leq 0$ is not correct. In other words, if f(s) = 0, we have g(s) > 0.

Lemma 4: For the functions $g(t) = 1 + \lambda_Y(t) + \alpha \varepsilon$ and $z(t) = 1 + \lambda_Y(t) - \lambda_X(t)$, when $g(s) \le 0$, we have z(s) > 0.

Proof: Otherwise, we assume that $z(s) \le 0$.

As $g(s) \le 0$, from Lemma 2, we know $g(t) \le 0$, $s < t \le T$. For the function z, we have

$$\dot{z} = \dot{\lambda}_{Y} - \dot{\lambda}_{X} = \lambda(1 + \lambda_{Y} + \alpha \varepsilon)X + \varepsilon \beta + \lambda(1 + \lambda_{Y} + \alpha \varepsilon)(M - Y) + \lambda(\lambda_{X} + \alpha \varepsilon)(N - X)\mu - f\mu = \lambda gX + \varepsilon \beta + \lambda g(M - Y) + \lambda(\lambda_{X} + \alpha \varepsilon)(N - X)\mu - f\mu$$
(22)

From (16), we can obtain $\mu f \ge 0$ all the time. Therefore, if $\lambda_X(s) + \alpha \varepsilon \le 0$, we have

$$z(s) = \lambda g(s)X(s) + \varepsilon \beta + \lambda g(s)(M - Y(s)) + \lambda(\lambda_X(s) + \alpha \varepsilon)(N - X(s))\mu(s) - f(s)\mu(s) \le 0$$
(23)

Therefore z decreases at time s. This means that for the time s+, which is just bigger than s (meaning that there is no other time between s and s+), we have $z(s+) \le z(s) \le 0$.

However, if $\lambda_X(s) + \alpha \varepsilon > 0$, z may not decrease at time s. Now, we explore the value of $\lambda_X + \alpha \varepsilon$ at time s+, which is just bigger than s. If $\lambda_X(s+) + \alpha \varepsilon \geq 0$, we have $z(s+) = 1 + \lambda_Y(s+) - \lambda_X(s+) = 1 + \lambda_Y(s+) + \alpha \varepsilon - \lambda_X(s+) - \alpha \varepsilon = g(s+) - \lambda_X(s+) - \alpha \varepsilon$. We have preved that $g(s+) \leq 0$, hence if $\lambda_X(s+) + \alpha \varepsilon \geq 0$, we have $z(s+) = g(s+) - \lambda_X(s+) - \alpha \varepsilon \leq 0$. If $\lambda_X(s+) + \alpha \varepsilon < 0$, $\lambda_X + \alpha \varepsilon$ decreases from positive value to negative value directly from time s to s+. Since our model is a continuous-time mode, $\lambda_X + \alpha \varepsilon$ is a continuous function. Therefore the transition from the positive value to the negative value directly is impossible. Therefore, if $\lambda_X(s) + \alpha \varepsilon > 0$, we have $\lambda_X(s+) + \alpha \varepsilon \geq 0$. In this case, $z(s+) \leq 0$. This means that if $\lambda_X(s) + \alpha \lambda_U(s) > 0$, we still have $z(s+) \leq 0$.

In summary, when $g(s) \le 0$, if $z(s) \le 0$, we have $z(s+) \le 0$. We can easily see that at time s+, the conditions are the same as that at time s. In other words, at time s+, we still have $g(s+) \le 0$ and $z(s+) \le 0$. Therefore z is still not increasing at time s+. Furthermore, we obtain $z(T) = 1 + \lambda_Y(T) - \lambda_X(T) \le 0$. From (13), we have $\lambda_Y(T) = \lambda_X(T) = 0$, so $z(T) = 1 + \lambda_Y(T) - \lambda_X(T) = 1 > 0$. This is a contradiction. Therefore the assumption that $z(s) \le 0$ is not correct and we have z(s) > 0.

4. Replicative case

4.1. Theoretical model

In this case, the destinations also forward the message to the others. In addition, the continuous-time Markov chain denoted by (4) is also correct. Then, we have [11]

$$E(X(t)) = \lambda \mu(t) (E(X(t)) + E(Y(t))) (N - E(X(t)))$$
 (24)

Similarly, we have

$$E(Y(t)) = \lambda(E(X(t)) + E(Y(t)))(M - E(Y(t)))$$
 (25)

4.2. Optimal control

In this case, the Hamiltonian H, and the co-state functions λ_X , and λ_Y are defined as follows

$$H = \overset{\bullet}{Y} + \varepsilon \overset{\bullet}{U} + \lambda_X \overset{\bullet}{X} + \lambda_Y \overset{\bullet}{Y}$$

$$= (1 + \lambda_Y) \overset{\bullet}{Y} + \lambda_X \overset{\bullet}{X} + \varepsilon (\alpha(\overset{\bullet}{X} + \overset{\bullet}{Y}) + \beta(\mu(N - X) + M - Y))$$

$$= (1 + \lambda_Y + \alpha \varepsilon) \overset{\bullet}{Y} + (\lambda_X + \alpha \varepsilon) \overset{\bullet}{X} + \varepsilon \beta(\mu(N - X) + M - Y)$$

$$= \lambda(1 + \lambda_Y + \alpha \varepsilon)(X + Y)(M - Y) + \varepsilon \beta(M - Y)$$

$$+ \lambda(\lambda_X + \alpha \varepsilon)(X + Y)(N - X)\mu + \varepsilon \beta\mu(N - X)$$
(26)

(see (27))

The transversality conditions are the same as that in (13). Then, we have

$$\mu = \begin{cases} 1, \lambda(\lambda_X + \alpha\varepsilon)(X + Y) + \varepsilon\beta > 0\\ 0, \lambda(\lambda_X + \alpha\varepsilon)(X + Y) + \varepsilon\beta < 0 \end{cases}$$
 (28)

This optimal policy also conforms to the threshold form and we have Lemma 5.

Lemma 5: The optimal policy μ in the replicative case has at most one jump and it satisfies: $\mu(t) = 1$, t < h, and $\mu(t) = 0$, t > h, $0 \le h \le T$. Time h can be seen as the stopping time.

Proof: First, we define a new function f as: $f = \lambda(\lambda_X + \alpha \varepsilon)$ $(X + Y) + \varepsilon \beta$, and we have (see (29))

The function z is defined as: $z = 1 + \lambda_Y - \lambda_X$, and we have

$$\dot{z} = \dot{\lambda}_{Y} - \dot{\lambda}_{X} = \lambda(1 + \lambda_{Y} + \alpha \varepsilon)(X + Y)
- \lambda(\lambda_{X} + \alpha \varepsilon)(X + Y)\mu - \varepsilon\beta\mu + \varepsilon\beta
= \lambda(1 + \lambda_{Y} + \alpha \varepsilon)(X + Y) - \mu f + \varepsilon\beta
= \lambda(1 + \lambda_{Y} - \lambda_{X} + \lambda_{X} + \alpha \varepsilon)(X + Y) - \mu f + \varepsilon\beta
= \lambda(1 + \lambda_{Y} - \lambda_{X})(X + Y) + (1 - \mu)f
= \lambda z(X + Y) + (1 - \mu)f$$
(30)

From (28), we know $(1 - \mu)f \le 0$. Therefore, at any time k, if $z(k) \le 0$, we have $z(t) \le 0$, $k < t \le T$. Now, we assume $f(s) \le 0$. If $z(s) \le 0$, we have $z(t) \le 0$, $s < t \le T$. Therefore z(T) = 1 + t

 $\lambda_Y(T) - \lambda_X(T) \le 0$. From (13), we know $\lambda_Y(T) = \lambda_X(T) = 0$, so $z(T) = 1 + \lambda_Y(T) - \lambda_X(T) = 1 > 0$. This is a contradiction. Therefore, $z(s) \le 0$ is impossible, and we have $z(s) = 1 + \lambda_Y(s) - \lambda_X(s) > 0$.

In addition, when $f(s) \le 0$, we have $\mu(s)f(s) = 0$. Note that we just consider the case Y < M, that is, at least one destination does not have the message. Therefore we have

$$f(s) = -\lambda(1 + \lambda_Y(s) - \lambda_X(s)) Y(s)$$

$$+ \lambda(X(s) + Y(s))\mu(s)f(s) = -\lambda z(s) Y(s) < 0 \quad (31)$$

This means that at any time s, once $f(s) \le 0$, f decreases at time s. Therefore, if $f(s) \le 0$, it will remain negative all the time. Suppose that s is the first time that satisfies $f(s) \le 0$, we have f(t) > 0, t < s and f(t) < 0, $s < t \le T$. Therefore, according to (28), we have $\mu(t) = 1$, t < s and $\mu(t) = 0$, t > s, $0 \le s \le T$. In this case, the stopping time h equals to s.

5 Simulation and numerical results

5.1 Simulation results

We will check the accuracy of the theoretical models by the Opportunistic Network Environment simulator [26]. The simulation in the first scenario is based on the Random Waypoint (RWP) mobility model [27]. We select 500 relay nodes and 50 destinations. These nodes move within a 10 000 m × 10 000 m terrain according to a scale speed chosen from a uniform distribution from 4 to 10 m/s. The transmission range of these nodes is 50 m. In the second scenario, we use a Poisson-contact process with $\lambda = 3.71 \times$ 10⁻⁶s⁻¹ to generate the node contact events. In particular, the value of λ is obtained from the Shanghai city motion trace [28]. Zhu et al. [20] find that the exponential distribution with $\lambda = 3.71 \times 10^{-6} \,\mathrm{s}^{-1}$ can fit the trace very well. For the Poisson-contact scenario, we also generate 550 nodes. Among these nodes, we randomly select 50 nodes as the destinations and the other nodes are the relay nodes. Note that the beaconing rate denotes the probability that the nodes really beacon. Therefore, in the simulations, the nodes can move and detect others with beaconing rate 1. Then, once two nodes meet each other at time t, the ONE simulator acts as a referee and judges that they really contact with probability $\mu(t)$. For example, in the RWP model, once the distance of the two nodes is smaller than

$$\begin{cases} \lambda_X^{\bullet} = -\frac{\partial H}{\partial X} = -\lambda(1 + \lambda_Y + \alpha \varepsilon)(M - Y) - \lambda(\lambda_X + \alpha \varepsilon)(N - 2X - Y)\mu + \varepsilon \beta \mu \\ \lambda_Y^{\bullet} = -\frac{\partial H}{\partial Y} = -\lambda(1 + \lambda_Y + \alpha \varepsilon)(M - 2Y - X) + \varepsilon \beta - \lambda(\lambda_X + \alpha \varepsilon)(N - X)\mu \end{cases}$$
(27)

$$\dot{f} = \lambda(\lambda_X + \alpha\varepsilon)(\dot{X} + \dot{Y}) + \lambda(-\lambda(1 + \lambda_Y + \alpha\varepsilon)(M - Y) - \lambda(\lambda_X + \alpha\varepsilon)
(N - 2X - Y)\mu + \varepsilon\beta\mu)(X + Y)$$

$$= \lambda(\lambda_X + \alpha\varepsilon)(\dot{X} + \dot{Y}) + \lambda(-(1 + \lambda_Y + \alpha\varepsilon)\dot{Y}) + \lambda(-(\lambda_X + \alpha\varepsilon)\dot{X})
+ \lambda((\lambda_X + \alpha\varepsilon)(X + Y)\mu + \varepsilon\beta\mu)(X + Y)$$

$$= \lambda(\lambda_X + \alpha\varepsilon)\dot{Y} + \lambda(-(1 + \lambda_Y + \alpha\varepsilon)\dot{Y}) + \lambda(X + Y)f\mu$$

$$= -\lambda(1 + \lambda_Y - \lambda_X)\dot{Y} + \lambda(X + Y)f\mu$$
(29)

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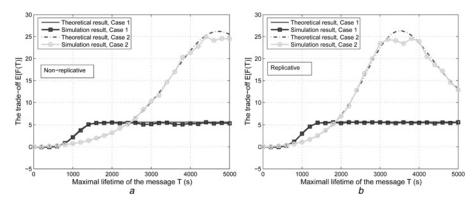


Fig. 1 Simulation and the theoretical results comparison with the RWP mobility model

- a Non-replicative case
- b Replicative case

50 m, we say that they find each other (denoted by false contact). However, they really detect each other in the simulation with probability $\mu(t)$ at time t (denoted by true contact).

For the theoretical related parameters, we consider two cases. In Case 1, we have $\mu(t)=1$, $0 \le t \le T$, but in Case 2, we have $\mu(t)=0.2$, $0 \le t \le T$. Both α and β are system specified positive constants. Their values are related to the system, so they can have different values in different applications. In this section, we let $\alpha=1$, $\beta=10^{-5}$. In addition, ε is the parameter that relates the energy consumption and the performance, which is non-positive. Here, we set $\varepsilon=-0.08$. The maximal lifetime of the message T increases from 0 to 5000 s. Then, through a 50 times simulation, we can obtain the results in Figs. 1 and 2, respectively.

The results show that the deviation between the theoretical and the simulation results is very small. For example, the average deviation in the non-replicative case is about 3.96% for the RWP model and only 3.29% for the Poisson-contact model. This demonstrates the accuracy of our theoretical models. In addition, the theoretical results are the expectations of the real values. Based on the same settings, the theoretical results are the same all the time. However, the results may not be the same in the simulations. Therefore the deviation between the theoretical and the simulation results may be different in different cases. However, the biggest deviation in our simulations is not more than 10%. In this case, the values of Y(T) in the simulations belong to the range [0, 1.1E(Y(T))]. When

E(Y(T)) is bigger, the range is bigger, too. Therefore, the values of Y(T) in the simulations have more selections, and the deviation may be bigger. For example, in the peak of Case 2 in Fig. 1, E(Y(T)) is bigger. Then, the deviation between the theoretical and the simulation results is bigger, too.

5.2 Performance analysis

We use $\lambda = 3.71 \times 10^{-6} \, \mathrm{s}^{-1}$ to obtain all the numerical results, and we compare the performance of the optimal policy with the performance of the other policies. In particular, we mainly consider three other policies: Case 1, $\mu(t) = 1$, $0 \le t \le T$; $\mu(t) = 0$, $0 \le t \le T$; and random. The random policy means that the beaconing rate at any time is randomly selected from the interval [0, 1]. Through letting T increase from 0 to 5000 s, we can obtain Fig. 3.

From Fig. 3, we can see that the performance of the optimal policy obtained by our model is better in both the cases. In addition, by comparing the result in Fig. 3a with the result in Fig. 3b, we find that with the same beaconing rate, E[F(T)] is bigger in the replicative case. This shows that it is necessary to make the destinations cooperative.

The result in Fig. 3 shows that if the destinations forward the message to others, it is good. Therefore the number of the destinations may have a certain effect. Now, we compare the performance of our optimal policy when the number of the destinations is different. In particular, we let M increase from 30 to 300, and set T = 5000 s. The other settings remain unchanged. The above results have shown

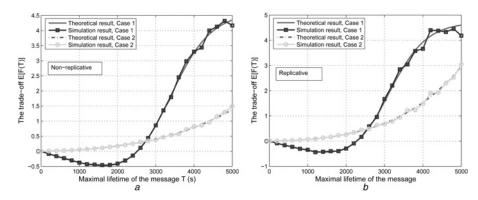
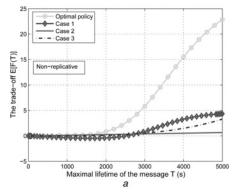


Fig. 2 Simulation and the theoretical results comparison with the Poisson-contact mobility model

- a Non-replicative case
- b Replicative case



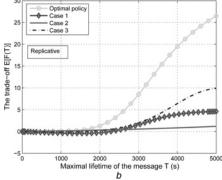


Fig. 3 *Performance comparison when T is different*

a Non-replicative case

b Replicative case

that our optimal policies are much better. Therefore, we ignore the other policies and just give the results of the optimal policies, which are shown in Fig. 4.

The result in Fig. 4 shows that E[F(T)] is bigger in the case, and the deviation between non-replicative and the replicative cases increases with M. As shown in Lemmas 1 and 5, the optimal policies conform to the 'threshold' form and the stopping time is denoted by h. We can see the value of h in Fig. 5. This result shows that h increases with M in the non-replicative case. This means that if there are more destinations, the relay nodes will beacon at the maximal rate for a longer time. However, h first increases with M, and then decreases in the replicative case. This means that when M is small, the increasing of M will make the relay nodes beacon at the maximal rate for a longer time. However, when M is big enough, the increasing of M will make the relay nodes beacon at the maximal rate for less time. In fact, the object to increase the beaconing rate of the relay nodes is to make them obtain the message, and then make the destinations obtain the message with higher probability. When the destinations are cooperative and the value of M is big enough, the destinations can transmit the message faster without the help of the relay nodes. Therefore the stopping time is smaller in this case. From the stopping time in Fig. 5, we can obtain the optimal policy. For example,

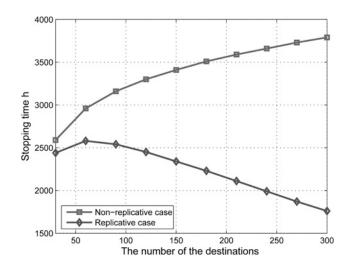


Fig. 5 Stopping time h in both cases when the number of the destinations is different

when M=150, the optimal policies in both the cases are shown in Fig. 6.

Note that ε is a non-positive constant, which is used to regulate the effect of the income (performance, E(U(T))) and 'cost' (energy consumption). Therefore ε may have a

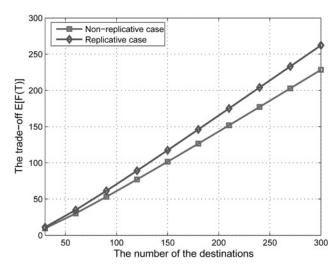


Fig. 4 Performance comparison when the number of the destinations is different

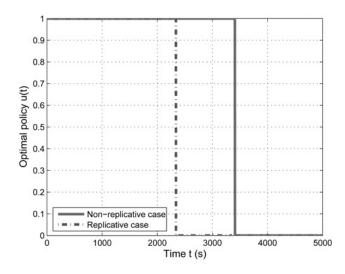


Fig. 6 Optimal policy in both cases when 150 destinations

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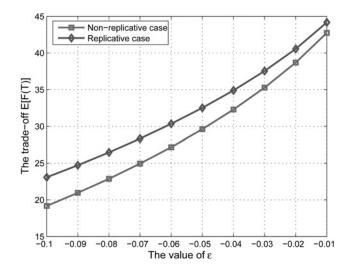


Fig. 7 Performance when the value of ε is different

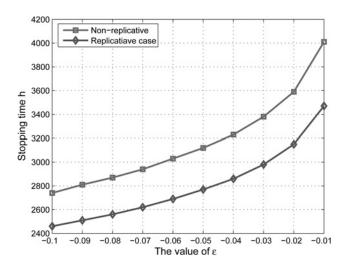


Fig. 8 Stopping time h in both cases when the value of ε is different

certain effect. Here, we set T = 5000 s, M = 50 and let ε increase from -0.1 to -0.01. The other settings remain unchanged and we obtain Fig. 7.

From Fig. 7, we can see that if ε is bigger, E[F(T)] is bigger. In fact, when ε is bigger, the effect of the energy consumption is smaller. In other words, the effect of the cost is smaller, hence the total income (trade-off, E[F(T)] is bigger. On the other hand, the result shows that if the destinations are cooperative, it is good. For example, E[F(T)] is bigger in the replicative case. With the increasing of ε , the stopping time h is shown in Fig. 8.

From Fig. 8, we can see that the stopping time in both the cases increases with ε . When ε is bigger, the effect of the energy consumption is smaller, hence it is cost-effective to make the relay nodes beacon at the maximal rate for a longer time. On the other hand, the stopping time h in the replicative case is smaller. This means that if the destinations are cooperative, they need less help from the relay nodes and the relay nodes will stop beaconing earlier.

6 Conclusions

This paper studies the optimal beaconing problem in the DTN with multiple destinations, and can be used in many cases, such as the sensor networks with multiple sinks and so on.

In this paper, we assume that if the nodes obtain the message, they will stop the beacon to save energy, and such an assumption is used in most of the works in the DTN. However, such assumption may not be proper. In the future, we will relax the assumption, and explore the joint beaconing policies for the nodes with and without the message at the same time.

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