Optimal detection to improve the performance of two-hop routing in selfish OppNets

A, B, C
School of Cyber Sci. & Engr., Southeast University, Nanjing, China
Key Lab of CNII, MOE, Southeast University, Nanjing, China
Jiangsu Provincial Key Laboratory of Computer Network Technology, Southeast University, Nanjing, China

***Index Terms*—OppNet, Selfish, Black (Gray) Hole, ODE, Pontryagain's maximal principle**

I. Introduction

II. Related Work

III. Selfish Nodes without Detection in Two-Hop Routing

A. Analysis in temporal space by solving ODE

State I is the state where the node has message m. State M is the state where the node has dropped m as receiving it. $I(t)$ is the number of nodes with m at time t. $M(t)$ is the number of nodes, which had received m and then drop it, at time t. The rate of transforming from state I to state M is ρ. At first, $M(0) = 0$ and $I(0) = 0$, only src has the messages. Note that $0 \leq I(t) \leq N$ and $0 \leq M(t) \leq N$. We use $\dot{I}$ and $\dot{M}$ denote $I(t)$ and $M(t)$. Following the two-hop routing, where only the source node can replicate the message to other nodes, we get that the corresponding ODEs like [1], which are

$$\dot{I} = \beta(N-I) - \rho I,$$
$$\dot{M} = \rho I - \beta M,$$
$$\dot{S} = -\beta(N-I-M),$$

when $0 \leq I(t) \leq N$ and $0 \leq M(t) \leq N$.

And then we find the close-form value of $I(t)$ and $M(t)$ to ensure I_s and M_s. From the first ODE, $\dot{I} + \rho I = \beta(N-I)$, we can obtain

$$I(t) = Ce^{-(\beta+\rho)t} + \frac{\beta N}{\beta+\rho}.$$

Considering that $I(t=0) = 0$, $C = \frac{-\beta N}{\beta+\rho}$. Thus

$$I(t) = \frac{\beta N}{\beta+\rho} - \frac{\beta N}{\beta+\rho}e^{-(\beta+\rho)t}.$$

Similarly, from $\dot{M} + \beta M = \rho I$,

$$\begin{aligned} M(t) &= Ce^{-\int \beta dt} + e^{-\int \beta dt}\int \rho I e^{\int \beta dt} dt \\ &= Ce^{-\beta t} + e^{-\beta t}\int \rho \frac{\beta N}{\beta+\rho}(1-e^{-(\beta+\rho)t})e^{\beta t} dt \\ &= Ce^{-\beta t} + \frac{\beta N}{\beta+\rho}e^{-(\beta+\rho)t} + \frac{\rho N}{\beta+\rho} \end{aligned}$$

Because of $M(0) = 0$,

$$M(t) = -Ne^{-\beta t} + \frac{\beta N}{\beta+\rho}e^{-(\beta+\rho)t} + \frac{\rho N}{\beta+\rho}$$

We can find that when $t \to +\infty$, $I(t) \to \frac{\beta N}{\beta+\rho}$ and $M(t) \to \frac{\rho N}{\beta+\rho}$.

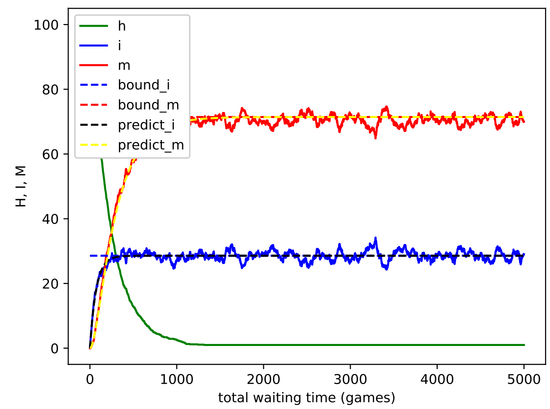

Fig. 1: $I(t)$ and $M(t)$ with time t obtained from prediction and simulations when $\beta = 0.004$, $\rho = 0.01$ and $N = 100$. Here h and i is the mean value of 20 simulations.

B. Object Function

The expected number of nodes, which declare that holding the messages, in the range $t \in (0, T]$ can be viewed as the contribution of the relay nodes, which will be proportional to the reward paid from the message sender. Thus the total paid reward for the selfish behaviors is

$$J = \int_0^T M(t)dt,$$

where T is the Time to Live (TTL) of message m. $M(t)$ is the waste of the reward at the instant time t. which also can be calculated. Based on the calculated result, We can find that $()\%$ reward are paid to the nodes without messages.

IV. Full detection in two-hop routing

The detection rate is U_m. For each detection, a node will be chosen randomly from N nodes. Thus the probability that find the blackhole node is $\frac{M}{N}$.

$$\dot{I} = \beta(N-I) - \rho I,$$
$$\dot{M} = \rho I - \beta M - \frac{M}{N}U_m,$$
$$\dot{S} = -\beta(N-I-M) + \frac{M}{N}U_m,$$

So

$$I(t) = \frac{\beta N}{\beta+\rho} - \frac{\beta N}{\beta+\rho}e^{-(\beta+\rho)t},$$

Then

$$\dot{M}+(\beta+\frac{U_m}{N})M=\rho I$$

$$\begin{aligned}&M(t)\\=&Ce^{\int-(\beta+\frac{U_m}{N})dt}+e^{\int-(\beta+\frac{U_m}{N})dt}\int\rho Ie^{\int(\beta+\frac{U_m}{N})dt}dt\\=&Ce^{-(\beta+\frac{U_m}{N})t}+e^{-(\beta+\frac{U_m}{N})t}\rho\frac{\beta N}{\beta+\rho}\int(1-e^{-(\beta+\rho)t})e^{(\beta+\frac{U_m}{N})t}dt\\=&Ce^{-(\beta+\frac{U_m}{N})t}+\frac{\rho\beta N}{\beta+\rho}\frac{1}{\beta+\frac{U_m}{N}}-\frac{\rho\beta N}{\beta+\rho}\frac{1}{\frac{U_m}{N}-\rho}e^{-(\beta+\rho)t}\end{aligned}$$

Because of $M(0)=0$.

$$\begin{aligned}M(t)=&\frac{\rho\beta N}{\beta+\rho}\frac{1}{\beta+\frac{U_m}{N}}-\frac{\rho\beta N}{(\beta+\frac{U_m}{N})(\frac{U_m}{N}-\rho)}e^{-(\beta+\frac{U_m}{N})t}\\&-\frac{\rho\beta N}{\beta+\rho}\frac{1}{\frac{U_m}{N}-\rho}e^{-(\beta+\rho)t}\end{aligned}$$

Here we find that ()% reward is wasted in the selfish node. Although the wasted reward is reduced because of the detection, the additional cost, which is caused by the detection behavior, i.e., energy, bandwidth and wireless communication charge. is introduced.

V. Optimal Detection

A. Problem Formulation

Assume that the detection can be conducted. The detection rate is $U(t)$, $0\le U(t)\le U_m$. U_m is the limitation of the detection rate, which is the constraint from the hardware and the time sequences. We also use U to denote $U(t)$. Then, the ODEs can be reformed as

$$\begin{aligned}\dot{I}&=\beta(N-I)-\rho I,\\\dot{M}&=\rho I-\beta M-\frac{M}{N}U,\\\dot{S}&=-\beta(N-I-M)+\frac{M}{N}U.\end{aligned}\tag{1}$$

Meanwhile,

$$\begin{aligned}I(0)&=0,\\M(0)&=0,\\S(0)&=N.\end{aligned}\tag{2}$$

Thus $I(t)$ is the same with that in the situation without detection, which is

$$I(t)=\frac{\beta N}{\beta+\rho}-\frac{\beta N}{\beta+\rho}e^{-(\beta+\rho)t}.\tag{3}$$

Considering that the detection is also the cost, the object function will be

$$J=\int_0^T(1-\alpha)M+\alpha U dt.$$

Here α is the weight, which can control the importance between the cost of selfish relay nodes and detections. Thus $0<\alpha<1$. Similar with the previous section, $I(t)$ and $M(t)$ is the state variable. $U(t)$ is the controllable variable, $0\le U(t)\ \le U_m$.

B. Optimal Control by Pontryagin's Maximal Principle

Now we utilize the Pontryagin's maximal principle to find the optimal $U(t)$, which will minimize the total cost. First, the Hamilton function is

$$\begin{aligned}H=&(1-\alpha)M+\alpha U+\lambda_1(\beta(N-I)-\rho I)\\&+\lambda_2(\rho I-\beta M-\frac{M}{N}U)\\=&(1-\alpha)M+\alpha U+\lambda_1(\beta(N-I)-\rho I)+\lambda_2\rho I\\&-\beta\lambda_2M-\lambda_2\frac{1}{N}UM\\=&(1-\alpha)M+\lambda_1(\beta(N-I)-\rho I)\\&+\lambda_2(\rho I-\beta M)+(\alpha-\lambda_2\frac{M}{N})U.\end{aligned}$$

Note that λ_1 and λ_2 denote $\lambda_1(t)$ and $\lambda_2(t)$, respectively. Without the final constraint, the terminal condition is $\lambda_2(T)=0$ and $\lambda_3(T)=0$. The adjoint function is

$$\dot{\lambda_2}=-\frac{\partial H}{\partial M}=\lambda_2(\beta+\frac{U}{N})-(1-\alpha).$$

Thus

$$U^*(t)=\begin{cases}0, & \text{if } \alpha-\lambda_2\frac{M}{N}\ge 0\\U_m, & \text{if } \alpha-\lambda_2\frac{M}{N}<0\end{cases}\tag{4}$$

In summary, we have the ODE functions $\dot{M}$, $\dot{\lambda_2}$, the initial condition $M(0)=0$ and the boundary condition $\lambda_2(T)=0$, Thus the problem is to solve a BVP problem, which is

$$\begin{aligned}\dot{M}&=\rho I-\beta M-\frac{M}{N}U,\\\dot{\lambda_2}&=-\frac{\partial H}{\partial M}=\lambda_2(\beta+\frac{U}{N})-(1-\alpha),\\M(0)&=0,\\\lambda_2(T)&=0.\end{aligned}\tag{5}$$

We can solve the BVP problem with the shooting method by the bvpSolve package of R.

VI. Analysis of Optimal Detection

In this section, we will introduce the properties of the optimal control variable.

Lemma 1. *At the beginning and the end of the whole duration, the optimal control stop the selfish detection, which is $U(0)=U(T)=0$.*

Proof. At the beginning of the duration, $M(0)=0$, which is the initial condition of 5. Then $\alpha-\lambda_2(0)\frac{M(0)}{N}=\alpha>0$. Following (4), the optimal $U(0)=0$.

At the end of the duration, $\lambda_2(T)=0$, which is the boundary condition of 5. Then $\alpha-\lambda_2(T)\frac{M(T)}{N}=\alpha>0$. Based on (4), the optimal $U(T)=0$. □

Based on the differential function $\dot{I}$, the equilibrium point of I can be obtained from $\dot{I}=0$, which is $I^*=\frac{\beta N}{\beta+\rho}$. When $I(t)<I^*$, $I(t)$ will increase with t and approach to $\frac{\beta N}{\beta+\rho}$. Meanwhile, in this paper $I(0)=0$ at the beginning of time.

Based on the differential function $\dot{M}$, the equilibrium point is obtained from $\dot{M}=0$, which is $M^*=\frac{\rho I}{\beta+\frac{1}{N}U}$. In the situation without detection, the equilibrium point is $M^*=\frac{\rho I^*}{\beta}=\frac{\rho N}{\beta+\rho}$. In the situation with full detection, the equilibrium point is $M^*=\frac{\rho I^*}{\beta+\frac{1}{N}U_m}=\frac{\rho}{\beta+\frac{1}{N}U_m}\frac{\beta N}{\beta+\rho}$.

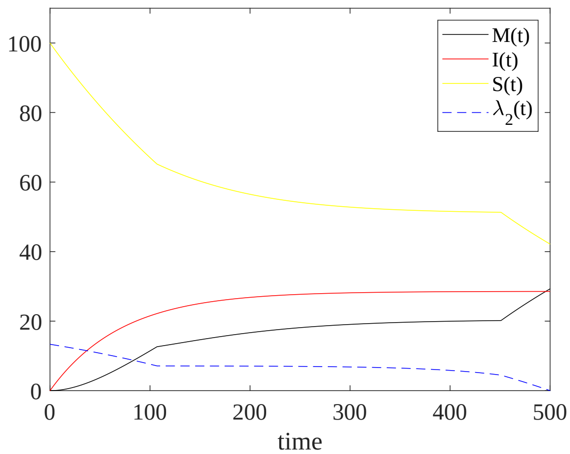

Fig. 2: State variable of analysis with time.

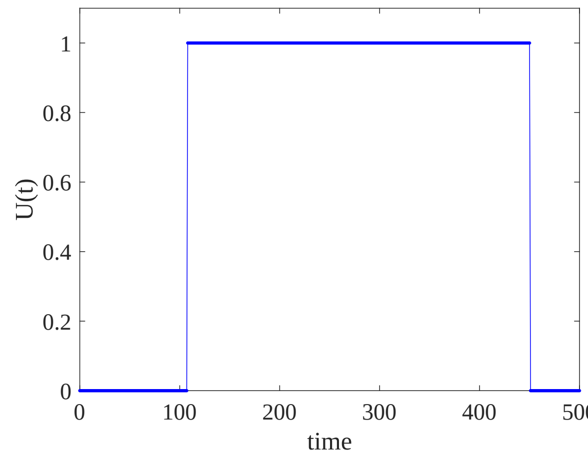

Fig. 3: Control variable of analysis with time.

Since α is the weight of detecting the selfish nodes, we can assume that if α is enough high, the detection will not perform according to the optimal control strategy.

Lemma 2. *If $\alpha \geq \alpha_{th}$, the optimal control let the detection stop in the whole duration, namely $U(t) = 0$, $0 \leq t \leq T$.*

Proof. Assume that ρ, N, β is given. Let $W(t) = \lambda_2(t)M(t)$.

$$\begin{aligned} W^{'}(t) =& M^{'}(t)\lambda_2(t) + M(t)\lambda_2^{'}(t) \\ =& (\rho I(t) - \beta M(t) - \frac{M(t)}{N}U(t))\lambda_2(t) \\ &+ M(t)(\lambda_2(t)(\beta + \frac{U(t)}{N}) - (1-\alpha)) \\ =& \rho\lambda_2(t)I(t) - (1-\alpha)M(t). \end{aligned} \tag{6}$$

Since $M(0) = 0$ and $\lambda_2(T) = 0$, $W(0) = W(T) = 0 < \alpha N$.

Now we focus on the poles of $W(t)$, namely t^*, where $W^{'}(t^*) = \rho\lambda_2(t^*)I(t^*) - (1-\alpha)M(t^*) = 0$. Then $M(t^*) = \frac{\rho\lambda_2(t^*)I(t^*)}{1-\alpha}$.

$$W(t^*) = \lambda_2(t^*)M(t^*) = \frac{\rho I(t^*)\lambda_2(t^*)^2}{1-\alpha}. \tag{7}$$

According to $\dot{\lambda_2}$ in (5), the equilibrium point of λ_2 is that $\lambda_2^* = \frac{1-\alpha}{\beta+\frac{U}{N}}$. Since $0 \leq U \leq U_m$, $0 < \frac{1-\alpha}{\beta+\frac{U_m}{N}} \leq \lambda_2^* \leq \frac{1-\alpha}{\beta}$. Note $\lambda_2(T) = 0$. Based on the phase line in ODE for $\dot{\lambda_2}$, $\lambda_2(t)$ decreases with t when $\lambda_2(t) < \lambda_2^*$. Conversely, $\lambda_2(t)$ increases with t when $\lambda_2(t) > \lambda_2^*$. Thus $0 \leq \lambda_2(t) \leq \lambda_2^* \leq \frac{1-\alpha}{\beta}$ when $0 \leq t \leq T$. Additionally, $0 \leq I(t) \leq \frac{\beta N}{\beta+\rho}$. From (7), we can derive that the upper boundary of $W(t)$, W_{up}, which is

$$W(t) \leq W(t^*) \leq \frac{\rho}{1-\alpha}\frac{\beta N}{\beta+\rho}(\frac{1-\alpha}{\beta})^2 = \frac{\rho N(1-\alpha)}{\beta(\beta+\rho)} = W_{up}.$$

Assume that α can satisfy that $W_{up} \leq \alpha N$, which means that $\alpha \geq \frac{\rho}{\beta(\beta+\rho)+\rho} = \alpha_{th}$. Then $W(t) \leq \alpha N$, when $0 \leq t \leq T$. Therefore the optimal control $U^*(t) \equiv 0$, when $0 \leq t \leq T$. □

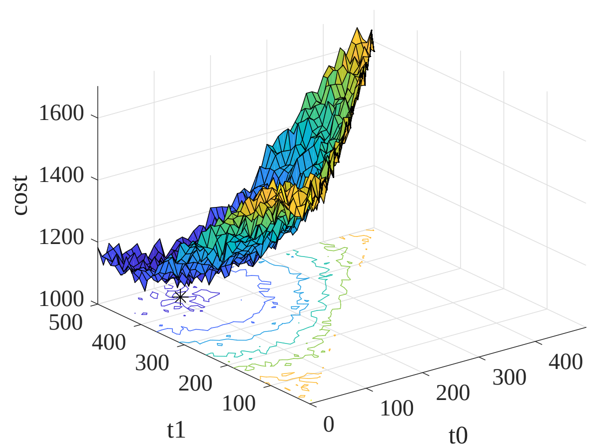

Fig. 4: Different choices of $t0$ and $t1$.

VII. Performance Evaluation

VIII. Conclusion

References

[1] X. Zhang, G. Neglia, J. F. Kurose, and D. F. Towsley, "Performance modeling of epidemic routing," *Comput. Networks*, vol. 51, no. 10, pp. 2867–2891, 2007. [Online]. Available: https://doi.org/10.1016/j.comnet.2006.11.028