

Control of Message Transmission in Delay/Disruption Tolerant Network

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Abstract—Message in delay/disruption tolerant network (DTN) is often transmitted in the store/carry/forward mode. In this mode, the transmission process includes two important steps (beaconing and forwarding). Both processes consume much energy which is very precious for the wireless application. At present, most of the works considers just one step in the controlling process (some ignore the beaconing control and some ignore the forwarding control). In addition, state-of-the-art works often assume that there is only one destination for a specific message. In this paper, we consider the optimal forwarding and beaconing control problem at the same time in DTN, where multiple destinations exist. In particular, based on whether those destinations are willing to relay message for others, we consider the selfish and altruistic cases, respectively. Based on these settings, we mathematically characterize this problem as an optimal control problem. The controlling parameters are the forwarding rate and beaconing rate, respectively. Then, we solve the problem by Pontryagin's Maximum Principle, and mathematically prove that both the optimal forwarding and beaconing policies conform to the threshold form. Next, we check the accuracy of the theoretical models by simulations based on both synthetic and real motion traces. Finally, we check the result in a real scenario with 100 students, and show that the optimal policy can really improve the performance.

Index Terms—Altruistic case, forwarding-beaconing policy, multiple destinations, selfish case.

I. INTRODUCTION

DELAY/DISRUPTION tolerant network (DTN) has been proposed for several years, and is used to describe the scenario, where the communication link between the nodes may be disrupted [1]. At present, with the rapid increasing of hand held mobile devices, DTN becomes more popular, especially in mobile social networks. The Google loon project uses DTN to provide internet access to rural and remote areas [2]. In this project, many balloons are placed in the stratosphere, and the messages can be relayed between these balloons. Wu *et al.* [3] use DTN to cooperatively transmit photographs in some emergency situations, such as the natural disaster or a battlefield. More examples can be found in the intelligent transport systems, such as [4] and [5].

In DTN, the communication link may be disrupted, which makes the routing algorithms (e.g., AODV [6]) in traditional mobile *ad hoc* network inefficient. Due to this reason,

DTN adopts a new transmission concept, which is called as store/carry/forward (SCF) mode. Under this mode, when a node *A* carrying a message *M* meets a node *B* without *M*, *A* will forward *M* to *B*. Then, *B* stores and carries the message along its movement. When *B* meets another node *C* without the message, *B* will forward the message to *C*. Epidemic routing (ER) protocol is a classic algorithm that conforms to the SCF mode [7]. However, ER tries to forward the message to every neighborhood, which uses too much energy. For this reason, many works begin to explore how to control the transmitting process to get a tradeoff between the energy consumption and the transmission performance. The message transmission in SCF mode includes two critical steps, which are the beaconing and forwarding processes, respectively. The objective of beaconing process is to find neighborhoods, and the forwarding process is to transmit the message. Both steps consume much energy, so the optimal control of the message transmission in DTN includes two parts, too. At present, most of the related works just consider one part. For example, Altman *et al.* [8] are the first to consider the optimal control problem in DTN, but they only study the monotone forwarding policy. Then, based on a discrete time model, Altman *et al.* [9] study the optimal forwarding policy of the two-hop algorithm, which is a simplified version of the ER algorithm. Li *et al.* [10] studies the continuous-time optimal forwarding problem of the ER algorithm. Altman *et al.* [11] study the adaptive forwarding control problem, where the knowledge of some system parameters may be unknown. The forwarding control when two files spread at the same time is presented in [12]. Then, Li *et al.* [13] explore the optimal beaconing policy and proves that the optimal policy conforms to the threshold form. Wu *et al.* [14] consider the optimal beaconing problem when multiple destinations exist at the same time. Wang *et al.* [15] study the optimal beaconing policy based on the time-continuous markov model. However, these works ignore the control of the forwarding process.

Above works get the optimal policy based on mathematical analysis. There are also many simulation-based routing policies. Within them, to decrease the energy consumption, some fix the copies' number, such as the spray and wait routing and its variants [16], [17]. Some other works fix the message transmission hop, such as the hop-limited routing policy [18], [19]. There are also some works, which decrease the energy consumption by reducing the unnecessary forwarding process based on certain factors, such as the network structure and the energy distribution in the nodes [20], [21]. However, these simulation-based routing protocols are checked only

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through simulations. That is, none of them mathematically prove whether their methods are optimal, so they are different from our work. In addition, they ignore the beaconing process, which also consumes much energy.

From above description, we can find that none of the existing works studies the **joint forwarding and beaconing control problem with multiple destinations**, but this is the main objective of our work in this paper. Simply speaking, the main goal of this paper is to maximize the tradeoff between the performance and energy consumption. This is a hard work, because the tradeoff is closely related to time. For example, at the beginning, there is more energy and there are fewer nodes carrying the message, so it may be cost-effective to use more energy to get better performance. However, when the message is transmitted for a longer time, more nodes may get the message and there is less energy. In this case, it may not be cost-effective to use more energy. Therefore, the weight of performance and energy consumption may be varying with time. To our best knowledge, we are the first to study **the optimal controlling problem** with this phenomenon in DTN. According to whether the destinations have **incentive** to relay the message to others after getting a message, we divide the message dissemination process into two cases, which are *selfish* and *altruistic* cases, respectively. In the first case, the destinations are selfish and never forward the message to others. However, in the *altruistic* case, the destinations also act as relay nodes and forward the message to others.

In this paper, we first mathematically formulate our goal as an optimal control problem, in which the controlling parameters are the *forwarding* and *beaconing* rates, respectively. Then, we present the **ordinary differential equations** (ODEs) model to describe the message dissemination process, which can be used to evaluate the tradeoff under different policies. Based on the model, we solve the optimal control problem with **Pontryagin's Maximum Principle**, and mathematically prove that the optimal policies conform to **the threshed form** in both *selfish* and *altruistic* cases. This demonstrates that the optimal policies have **a bang-bang structure**, and are easy to use. For example, nodes beacon with the maximum rate before certain time and then with the minimum rate in the rest time. Finally, we check the accuracy of the model and run the optimal policy in a real scenario based on 100 students in our university. The result shows that the optimal policy is really better.

II. NETWORK MODEL

The network totally has $N - 1$ relay nodes and one source node S . The message generated in S has M destinations. The task of the relay nodes is to help the source to forward message to the destinations. For simplicity, in the rest of this paper, the source also can be called as relay nodes. At the initial time (time 0), the source generates a message denoted as *mes*, whose maximal lifetime is T . Once a relay node encounters a node (other relay nodes or destinations in the *selfish* case) that has the message at time t , the relay node will obtain the message with probability $p(t)$. This means that the *forwarding rate* of the relay nodes at time t is $p(t)$. Without loss of generality, we assume that nodes forward the message to the destinations with probability 1 all the time.

In the SCF transmission mode, the message can be transmitted only when nodes meet each other, so the performance closely depends on the nodes' mobility model. Similar to most of the previous works, we assume that the nodes move according to **the exponential model** [8]–[14]. The model has been checked by many real motion traces. For the exponential model, an important parameter is the contact rate, which equals to the reciprocal of the average time between two consecutive contacts. In this paper, we set the contact rate as θ if the beaconing rate is 1. Then, we define $\mu(t)$ as the real *beaconing rate* at time t , and we have $0 \leq \mu(t) \leq 1$, where 0 is the minimum *beaconing rate*. Therefore, at time t , the real contact rate is $\theta\mu(t)$. Furthermore, we assume that each transmission is successful [8]–[14]. In other words, if node m forward a message to n , n can get the message without errors. In this case, each transmission will make a node get the message, so the energy consumption of the forwarding process is proportional to the number of nodes that get the message from others [8]–[14].

The number of relay nodes carrying the message at time t is defined as $R(t)$. Similarly, $D(t)$ denotes the number of destinations that have message at this time. Then, we can obtain $R(0) = 1$, $D(0) = 0$. Both the symbols $R(t)$ and $D(t)$ are random variables which are related to the **forwarding rate $p(t)$ and beaconing rate $\mu(t)$** . The expectations of them are denoted as $E(R(t))$ and $E(D(t))$, respectively. Then, the average number of nodes which receive message successfully from others can be denoted as $E(R(t)) + E(D(t)) - 1$, which means that the average number of the transmissions is $E(R(t)) + E(D(t)) - 1$. Now, we can find that the energy consumption in the message forwarding step is $\sigma(E(R(t)) + E(D(t)) - 1)$ [13], [14], where σ is a positive constant, which is used for weighting the energy consumption in each transmission.

Every node has two states: *active* and *sleep*. A node in the *active* state will proactively beacon to find its neighbors, and also can be found by other nodes in the *active* state. However, nodes in the *sleep* state do not beacon, so they just can be found by other *active* nodes, but cannot find others proactively. Obviously, when two nodes meet, if one of them is in the *active* state, the connection can be built. This means that it is not necessary for all the nodes stating in the *active* state. For simplicity, we use the same assumption as [13], [14], and suppose that **once a node obtains the message, it will stop beaconing**. In other words, the nodes carrying message are in the *sleep* state. In addition, we suppose that the destinations without message are in the *active* state all the time. Such assumption is rational, because the destinations always try to find and receive messages from others. In fact, through certain extension, our work also can be used without above assumptions.

Then, similar to [13] and [14], the energy consumption in the beaconing process is

$$\tau \int_0^T ((N - E(R(t)))\mu(t) + (M - E(D(t))))dt. \quad (1)$$

Symbol τ is a positive constant, which is the weight of energy consumption in the beaconing process. $N - E(R(t))$ is the average number of relay nodes without message.

The probability that such nodes staying in the *active* state is $\mu(t)$, so $\tau\mu(t)(N - E(R(t)))$ is the energy consumption of the relay nodes at time t in the beaconing process. Similarly, we can get the energy consumption of the destinations, and then (1) follows.

Based on (1), the total energy consumption in the beaconing and forwarding processes is

$$E(U(T)) = \sigma(E(R(T)) + E(D(T)) - 1) + \tau \int_0^T ((N - E(R(t)))\mu(t) + (M - E(D(t))))dt. \quad (2)$$

$E(U(T))$ denotes the average energy consumption up to time T . Then, the main objective of our work is to solve the following optimization problem:

$$\text{Max. } E(F(T)) = \int_0^T (\zeta(t) E(\dot{D}(t)) + \omega(t) E(\dot{U}(t)))dt. \quad (3)$$

In this paper, we just assume that it is better if there are more destinations getting the message before the deadline T , and do not assume that it is better if the destinations get the message at earlier time. In other words, the utility of the event that a destination gets the message at time 0 equals to the utility at time T . In this case, the function $\zeta(t)$ can be seen as a constant. However, the energy decreases with time, so the left energy is different at different times. When there is less energy, the energy may be more important and has bigger weight. Therefore, $\omega(t)$ may be varying with time. Then, through letting $\varepsilon(t) = \omega(t)/\zeta(t)$, the optimization problem can be converted to the following expressions:

$$\text{Max. } E(F(T)) = E(D(T)) + \int_0^T \varepsilon(t) E(\dot{U}(t))dt. \quad (4)$$

$\varepsilon(t)$ is the parameter that adjusts the weight of the energy consumption. If the value of $E(D(T))$ is bigger, more destinations get the message, and we can say that the performance is better. However, if the energy consumption is bigger, it is not good for the network. Therefore, $\varepsilon(t)$ is a nonpositive function, that is, we have $\varepsilon(t) \leq 0$, and $-\varepsilon(t)$ can be seen as the weight of the energy. After each forwarding or beaconing process, certain energy will be used, so the total energy is nonincreasing. In other words, the total energy in the current time may be less than the front time. When there is less energy, the energy may be more valuable, and the weight may be bigger. Therefore, we can see that $-\varepsilon(t)$ is nondecreasing with time, and $\varepsilon(t)$ is nonincreasing. In addition, we assume that $\varepsilon(t)$ is a continuously differentiable function. If $E(D(T))$ and $E(U(T))$ can be seen as the *income* and *expenditure*, respectively, $E(F(T))$ can be seen as the *total income*. The specific form of $\varepsilon(t)$ is system specified, and may be any value, which satisfies above conditions. Through varying the value of $\varepsilon(t)$, one can adjust the scale of the energy consumption to fit the scale of the performance. In fact, (4) can be seen as an extension of (1) in [12], in which the weight of the energy consumption is a constant. In other words, the values of $\varepsilon(t)$ remain unchanged in different times in [12].

In summary, (4) presents an optimal control problem, which is related to $E(R(T))$ and $E(D(T))$. In addition,

both $E(R(T))$ and $E(D(T))$ depend on the values of $\mu(t)$ and $p(t)$, so the controlling parameters are the *beaconing* and *forwarding* rates, respectively.

III. SELFISH CASE

A. Message Transmission

In the selfish case, the destinations are not willing to forward message toward others. Then, based on [13]–[15], we can easily get the ODE for the relay nodes

$$E(\dot{R}(t)) = \theta(N - E(R(t)))E(R(t))p(t)\mu(t). \quad (5)$$

For the destinations, they are beacon all the time and the relay nodes are always willing to forward the message to them, so (6) is as follows:

$$E(\dot{D}(t)) = \theta(M - E(D(t)))E(R(t)). \quad (6)$$

B. Optimal Control

Now, we begin to solve the interesting optimization problem shown in (4) based on the famous Pontryagin's Maximum Principle [22], which presents the necessity condition for the dynamic optimization problem. This means that if the optimal solution of (4) exists, it must be included in the solutions obtained by Pontryagin's Maximum Principle.

Define $[(R, D), \mu, p]$ as an optimal policy. According to [22], the Hamiltonian function can be obtained through getting the derivative of the objective function and the derivation of the corresponding state functions. Then, based on (4)–(6), the Hamiltonian H is shown as follows:

$$\begin{aligned} H &= \dot{D} + \varepsilon \dot{U} + \lambda_R \dot{R} + \lambda_D \dot{D} \\ &= \theta(1 + \lambda_D + \varepsilon\sigma)R(M - D) + \varepsilon\tau(M - D) \\ &\quad + \theta(\lambda_R + \varepsilon\sigma)R(N - R)p\mu + \varepsilon\tau(N - R)\mu. \end{aligned} \quad (7)$$

In (7), the symbols λ_R and λ_D are called as costate or adjoint functions [22]. Their derivations can be got from the function H , which are shown as

$$\begin{cases} \dot{\lambda}_R = -\frac{\partial H}{\partial R} = -\theta(1 + \lambda_D + \varepsilon\sigma)(M - D) \\ \quad -\theta(\lambda_R + \varepsilon\sigma)(N - 2R)p\mu + \varepsilon\tau\mu \\ \dot{\lambda}_D = -\frac{\partial H}{\partial D} = \theta(1 + \lambda_D + \varepsilon\sigma)R + \varepsilon\tau. \end{cases} \quad (8)$$

Moreover, we can get the transversality conditions of the costate functions, which satisfy

$$\lambda_R(T) = \lambda_D(T) = 0. \quad (9)$$

Next, from the [22, Th. 3.14], we can say that there exists continuous (or may be piecewise continuously) differentiable state/costate functions, and they satisfy

$$(p, \mu) \in \arg \max_{0 \leq p^*, \mu^* \leq 1} H(\lambda_R, \lambda_D, (R, D), p^*, \mu^*). \quad (10)$$

As shown in [22], (10) permits us to model the controlling parameters (μ, p) as a function of the co-states (λ_R, λ_D) and states (R, D) . Then, such expression forms a system consisted of several ODEs which involves only the costate and state functions. In this paper, we can get the values of R and D at

the initial time, and we have $R(0) = 1, D(0) = 0$. On the other hand, (9) sets the values for the co-states λ_R and λ_D at the final time. Then, multiple numerical algorithms for calculating boundary value ODEs problem can be adopted here to get the values of (R, D) and (λ_R, λ_D) at any time.

In addition, (10) demonstrates that the optimization problem shown in (4) equals to maximizing the *Hamiltonian* function H [22]. Specifically, the states (R, D) and co-states (λ_R, λ_D) can be regarded as constants at any given time. In this case, we can further find that maximizing the *Hamiltonian* function H equals to maximizing the function $b = \theta(\lambda_R + \varepsilon\sigma)R(N - R)p\mu + \varepsilon\tau(N - R)\mu$. When exploring the optimal policy, we just need to consider the case $R < N$ and $D < M$. This means that not all of the relay nodes and destinations have message. Otherwise, it is not necessary to study the optimal policy at all. Under such condition, we just need to maximize the function $f = b/(N - R) = \theta(\lambda_R + \varepsilon\sigma)Rp\mu + \varepsilon\tau\mu$. Next, we define another function $g = \theta(\lambda_R + \varepsilon\sigma)R + \varepsilon\tau$. When $g \leq 0$, we have $f = \theta(\lambda_R + \varepsilon\sigma)Rp\mu + \varepsilon\tau\mu \leq \max\{\varepsilon\tau\mu, g\mu\}$. Because $g \leq 0$ and $\varepsilon \leq 0$, we get $f = \theta(\lambda_R + \varepsilon\sigma)Rp\mu + \varepsilon\tau\mu \leq 0$. Therefore, $\mu = 0$ is an optimal value. In particular, if $\theta(\lambda_R + \varepsilon\sigma)Rp + \varepsilon\tau < 0$, μ must be 0. However, if $\theta(\lambda_R + \varepsilon\sigma)Rp + \varepsilon\tau = 0$, μ can be any value in $[0, 1]$, and $\mu = 0$ is just one of the optimal value. On the other hand, when $\mu = 0$, f will be 0, so p cannot have any impact on the value of f and p can be any value, too. In this case, we can say that $p = 0$ is one of the optimal values. This means that when $g \leq 0$, $(0, 0)$ is one of the optimal values. When $g > 0$, we have $\theta(\lambda_R + \varepsilon\sigma)R > -\varepsilon\tau \geq 0$. Therefore, if $\mu > 0$, the optimal value of p must be 1. When $p = 1$, $f = g = \theta(\lambda_R + \varepsilon\sigma)R + \varepsilon\tau > 0$, so the optimal value of μ be 1, too. In this case, the maximal value of f is $g = \theta(\lambda_R + \varepsilon\sigma)R + \varepsilon\tau > 0$. If $\mu = 0$, the maximal value of f is 0. Therefore, if $g = \theta(\lambda_R + \varepsilon\sigma)R + \varepsilon\tau > 0$, only $\mu = 1$ can maximize f , and then we have $p = 1$.

In summary, when $g = \theta(\lambda_R + \varepsilon\sigma)R + \varepsilon\tau > 0$, the optimal forwarding and beaconing rates are $(1, 1)$. When $g = \theta(\lambda_R + \varepsilon\sigma)R + \varepsilon\tau \leq 0$, one of the optimal forwarding and beaconing rates are $(0, 0)$. Therefore, we have

$$(p, \mu) = \begin{cases} 0, & g = \theta(\lambda_R + \varepsilon\sigma)R + \varepsilon\tau \leq 0 \\ 1, & g = \theta(\lambda_R + \varepsilon\sigma)R + \varepsilon\tau > 0 \end{cases} \quad (11)$$

This policy is one of the optimal solutions obtained by the Pontryagin's Maximum Principle, and it has the same performance with other optimal solutions. In addition, we can easily check that this is a feasible solution of (4). Therefore, (11) is the optimal policy. On the other hand, the optimal policy denoted by (11) conforms to a *threshold* form, which is easy to carry out. For example, both the forwarding rate and beaconing rate are 1 before certain time (*stopping time*) and then remains 0 all the time. Specifically, we have Lemma 1.

Lemma 1: The optimal forwarding and beaconing policy (p, μ) denoted by (11) jumps at most once, which satisfies: $(p(t), \mu(t)) = 1, t < h$, and $(p(t), \mu(t)) = 0, t \geq h, 0 \leq h \leq T$. The parameter h would be regarded as *stopping time*.

Proof: see Appendix A.

In addition, we can get Lemmas 2–4, respectively.

Lemma 2: For function $l(t) = 1 + \lambda_D(t) + \varepsilon(t)\sigma$, once it is no-positive at a moment (e.g., s), it will keep nonpositive in the rest time, that is, we have $l(t) \leq 0, t > s$.

Proof: see Appendix B.

Lemma 3: For the functions $g(t) = \theta(\lambda_R(t) + \varepsilon(t)\sigma)R(t) + \varepsilon(t)\tau$ and $l(t) = 1 + \lambda_D(t) + \varepsilon(t)\sigma$, if $g(s) = 0$, we have $l(s) > 0$.

Proof: see Appendix C.

Lemma 4: Under the policy denoted by (11), for $l(t) = 1 + \lambda_Y(t) + \alpha\varepsilon(t)$ and $z(t) = 1 + \lambda_D(t) - \lambda_R(t)$, when $l(s) \leq 0$, we can get $z(s) > 0$.

Proof: see Appendix D.

IV. ALTRUISTIC CASE

A. Message Transmission

In such scenario, the destinations are altruistic and act as relay nodes to forward message to other nodes, so we get

$$\dot{E}(R(t)) = \theta(E(R(t)) + E(D(t)))(N - E(R(t)))\mu(t)p(t). \quad (12)$$

Then, we have

$$\dot{E}(D(t)) = \theta(E(R(t)) + E(D(t)))(M - E(D(t))). \quad (13)$$

B. Optimal Control

Similar to (7) and (8), the *Hamiltonian* function H is defined as (14), and the corresponding costate functions λ_R and λ_D satisfy the conditions in (15)

$$\begin{aligned} H &= \dot{D} + \varepsilon \dot{U} + \lambda_R \dot{R} + \lambda_D \dot{D} \\ &= \theta(1 + \lambda_D + \sigma\varepsilon)(R + D)(M - D) + \varepsilon\tau(M - D) \\ &\quad + \theta(\lambda_R + \sigma\varepsilon)(R + D)(N - R)p\mu + \varepsilon\tau(N - R)\mu. \end{aligned} \quad (14)$$

$$\begin{cases} \dot{\lambda}_R = -\frac{\partial H}{\partial R} = -\theta(1 + \lambda_D + \sigma\varepsilon)(M - D) + \varepsilon\tau\mu \\ \quad - \theta(\lambda_R + \sigma\varepsilon)(N - 2R - D)p\mu, \\ \dot{\lambda}_D = -\frac{\partial H}{\partial D} = \theta(1 + \lambda_D + \sigma\varepsilon)(2D + R - M) + \varepsilon\tau \\ \quad - \theta(\lambda_R + \sigma\varepsilon)(N - R)p\mu. \end{cases} \quad (15)$$

The transversality conditions in this scenario also can be denoted by (9).

Based on these settings, according to [22], we can easily find that the optimization problem is equivalent to maximizing the function $f = \theta(\lambda_R + \sigma\varepsilon)(R + D)p\mu + \varepsilon\tau\mu$. Then, we define another function g as: $g = \theta(\lambda_R + \sigma\varepsilon)(R + D) + \varepsilon\tau$.

If $g \leq 0$, we can get $f = \theta(\lambda_R + \sigma\varepsilon)(R + D)p\mu + \varepsilon\tau\mu \leq \max\{\varepsilon\tau\mu, (\theta(\lambda_R + \sigma\varepsilon)(R + D) + \varepsilon\tau)\mu = g\mu\}$. Because $g \leq 0$, $\varepsilon \leq 0$, we further get $f = \theta(\lambda_R + \sigma\varepsilon)(R + D)p\mu + \varepsilon\tau\mu \leq 0$, so $\mu = 0$ is an optimal value. Note that if $g = \theta(\lambda_R + \sigma\varepsilon)(R + D) + \varepsilon\tau = 0$, μ can be any value belonging to $[0, 1]$, and $\mu = 0$ is just one of the optimal value. However, when $\mu = 0$, f will be 0, so p cannot have any impact on the value of f and p also can be any value. Therefore, $p = 0$ is one of the optimal values. This means that when $g = \theta(\lambda_R + \sigma\varepsilon)(R + D) + \varepsilon\tau \leq 0$, $(0, 0)$ is one of the optimal values. When $g = \theta(\lambda_R + \sigma\varepsilon)(R + D) + \varepsilon\tau > 0$,

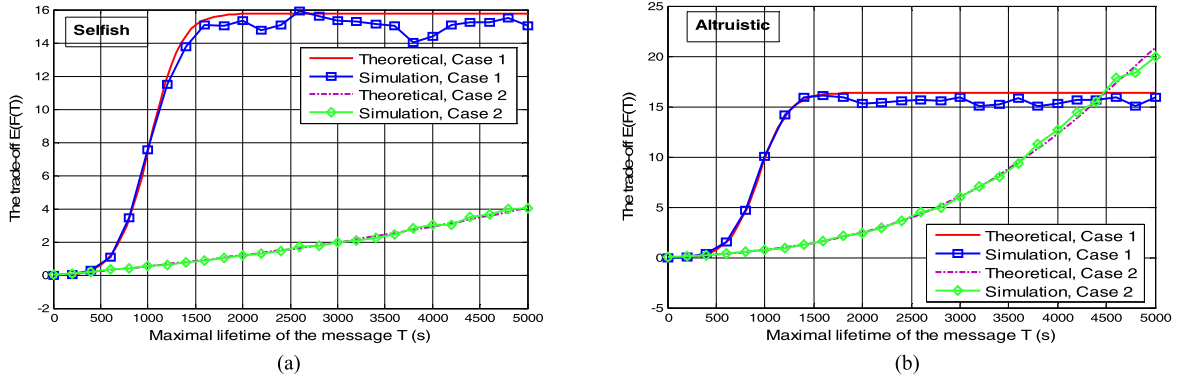


Fig. 1. Comparison of the theoretical and simulation results based on RWP model. (a) Selfish case. (b) Altruistic case.

we have $\theta(\lambda_R + \sigma\epsilon)(R + D) > -\epsilon\tau \geq 0$. Therefore, if $\mu > 0$, the optimal value of p must be 1. When $p = 1$, the derivative of the function f with respect to μ equals to g , which satisfies $g = \theta(\lambda_R + \sigma\epsilon)(R + D) + \epsilon\tau > 0$, so the optimal value of μ be 1, too. In this case, the maximal value of f equals to g . If $\mu = 0$, the maximal value of f is 0. Therefore, if $g > 0$, only $\mu = 1$ can maximize f , and then we have $p = 1$.

In summary, when $g = \theta(\lambda_R + \sigma\epsilon)(R + D) + \epsilon\tau > 0$, both the optimal forwarding rate and beaconing rate are 1. When $g = \theta(\lambda_R + \sigma\epsilon)(R + D) + \epsilon\tau \leq 0$, the forwarding and beaconing policy $(0, 0)$ is one of the optimal policies. Therefore, the optimal policy can be denoted as the following expression:

$$(p, \mu) = \begin{cases} 0, & g = \theta(\lambda_R + \sigma\epsilon)(R + D) + \epsilon\tau \leq 0 \\ 1, & g = \theta(\lambda_R + \sigma\epsilon)(R + D) + \epsilon\tau > 0. \end{cases} \quad (16)$$

Similar to (11), we can easily check that (16) is the optimal policy, which also conforms to the *threshold* form and we have Lemma 5.

Lemma 5: The forwarding and beaconing policy described in (16) jumps at most once, which satisfies: $(p(t), \mu(t)) = 1$, $t < h$, and $(p(t), \mu(t)) = 0$, $t \geq h$, $0 \leq h \leq T$.

Proof: see Appendix E.

V. PERFORMANCE EVALUATION

A. Accuracy Analysis Based on Simulations

As shown in Sections III and IV, the message transmission processes are mathematically modeled by the ODEs, and the optimal control analysis is based on these theoretical models, so whether the models are accurate is critical. To check their accuracy, this paper runs three simulations based on the opportunistic network environment (ONE) simulator [23]. The data sets used here are Random Waypoint (RWP) model [24], Poisson-contact model and a real motion trace, respectively. The RWP trace is a fully synthetic model, which has been commonly used in many works [8]–[15]. The Poisson-contact mobility model is quasi-synthetic, which is also a synthetic model, but the parameter comes from the real motion trace. Here, we set the parameter θ as $3.71 \times 10^{-6} s^{-1}$, and the value comes from the taxis' motion trace in Shanghai city [25], [26]. The third data set used in this paper is the Infocom'05 motion trace, which is a real trace including

41 nodes [27]. For the front two data sets, we generate 550 nodes, where we randomly select 50 nodes as the destinations and the others are the relay nodes. For the RWP trace, these nodes move in an area with size $10000 \text{ m} \times 10000 \text{ m}$. The speed is randomly selected in a uniform distribution changing from 4 to 10 m/s. The communication range of these nodes is set to be 50 m. For the third trace, ten of the 41 nodes are selected as the destinations, and other 31 nodes act as relay nodes. To use this trace in ONE, the original Infocom'05 motion trace is first divided into discrete sequential contact events, and then we let the events be the inputs of the simulator. The beaconing rate $\mu(t)$ can be seen as the probability that nodes beacon at time t . Therefore, if the contact rate is set to be θ , the real contact rate at time t is $\theta\mu(t)$. In other words, in these simulations, at time t , for each contact event, the nodes really meet each other with probability $\mu(t)$. For example, in the discrete sequential contact events of the Infocom'05 trace, if there is a contact event at time t , the contact really happens with probability $\mu(t)$.

For other parameters in the theoretical models, there are infinite settings. For example, the parameter $p(t)$ can be any value in $[0, 1]$. Therefore, we just consider certain specific cases as the examples. In particular, we consider two cases in the simulations. In the first case (Case 1), we set $(p(t), \mu(t)) = (1, 1)$, $0 \leq t \leq T$, so in this case, the contact rate is θ . In the second case (Case 2), we set $(p(t), \mu(t)) = (0.2, 0.2)$, $0 \leq t \leq T$. Obviously, in Case 2, the contact rate is 0.2θ . The parameters σ and τ are positive constants and used for weighting the energy consumption in the transmission and beaconing processes. The values of the two parameters are closely related to the types of the nodes. For simplicity, most of the existing works assumed that the nodes have the same type and assign certain specific values for them. Similar to [13] and [14], in this section, we set $\sigma = 1$, $\tau = 10^{-5}$. In addition, note that ϵ is the parameter that relates the energy consumption and the performance, which is nonpositive and nonincreasing with time. Here, we set $\epsilon(t) = -0.05 \times \exp(t/5000)$.

In each simulation, 200 messages are created, whose maximal lifetime T increases from 0 to 5000 s. Then, each simulation repeats 50 times, and we can get the results in Figs. 1–3, respectively. Figs. 1–3 show that the simulation results can well fit the theoretical results, and this demonstrates that the

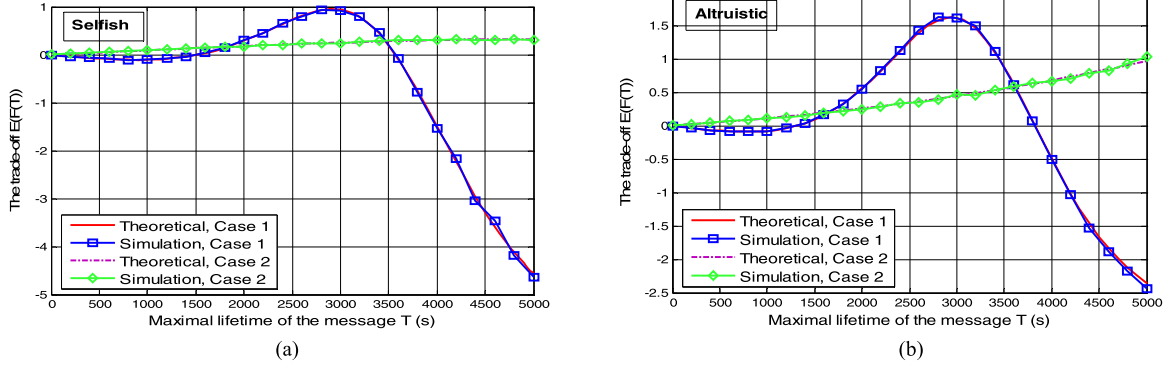


Fig. 2. Comparison of the theoretical and simulation results based on Poisson-contact model. (a) Selfish case. (b) Altruistic case.

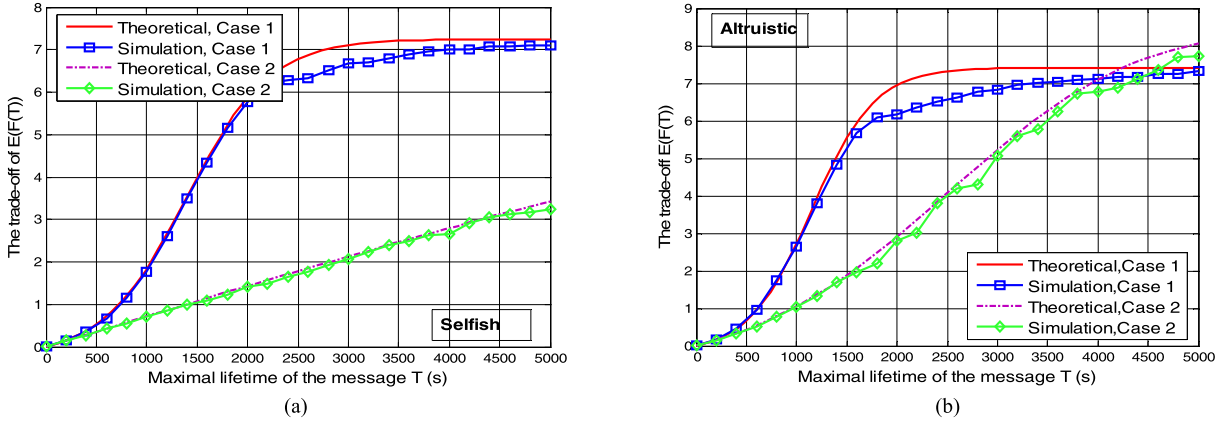


Fig. 3. Comparison of the theoretical and simulation results based on Poisson-contact model. (a) Selfish case. (b) Altruistic case.

theoretical models are accurate. As shown in Figs. 1 and 2, the average deviations in the *selfish* case are about 4.11% and 3.79%, respectively. However, we can also find that the average deviation in Fig. 3 is bigger than that in above two figures, and is about 7.01%. This result comes from the simulation based on the real motion trace (Infocom'05 data set). In fact, at present, some works have shown that the contact times of this trace conform to the Power law and exponential decay distribution [28]. That is, only an exponential model may not fit the trace very well. Therefore, in the simulation, we use the exponential model will bring bigger deviation. On the other hand, we can find that the deviation is not too big, and may be receivable. Therefore, this result further demonstrates that the exponential model is commonly useful as shown in [8]–[15].

In addition, above results also show that the value of $E(F(T))$ is different under different policies. From example, when $(p(t), \mu(t)) = (1, 1)$, the tradeoff (also seen as *total income*) will be negative if T is big enough, but if $(p(t), \mu(t)) = (0.2, 0.2)$, the *total income* is never negative (see Fig. 1). Therefore, we can get better tradeoff in Case 2. Even though the *total income* is not negative, the *total income* with $(p(t), \mu(t)) = (1, 1)$ may not be maximal. For example, when $T = 5000$ s, the value of $E(F(T))$ in Case 2 is bigger [see Fig. 2(b)]. This means that forwarding with probability 1 and beaconing with the maximal rate all the time may not be a good policy. Therefore, the optimal control in the forwarding and beaconing processes is necessary.

B. Performance Analysis Based on Numerical Results

To get the numerical results, we set the contact rate to be $3.71 \times 10^{-6} s^{-1}$, which comes from the Shanghai city motion trace. The performance under the optimal policies denoted by (11) and (16) will be compared with some other policies. Specifically, we introduce four other different policies: Case 1, $(p(t), \mu(t)) = (1, 1)$, $0 \leq t \leq T$; Case 2, $(p(t), \mu(t)) = (1, 0.1)$, $0 \leq t \leq T$; Case 3, $(p(t), \mu(t)) = (0.1, 0.1)$, $0 \leq t \leq T$; Case 4, $\mu(t) = 1$, $p(t)$ is decided by the spray and wait (SW) protocol, $0 \leq t \leq T$. Obviously, Case 1 corresponds to the famous ER protocol [7], and Case 4 is the SW protocol [16]. The other two cases is the probability ER protocol, which is a modification of the ER algorithm. These protocols are commonly used for comparing with other protocols. For the SW protocol, we further consider two subcases with the maximal number of copies equals to 10 and 200, respectively. Moreover, we use the policy in [17] to adjust the message transmission in SW protocol to adapt to the network dynamic. When the message's maximal lifetime T increases from 0 to 5000 s, we can get Fig. 4.

This result shows that our optimal policy is much better than other polices in both *selfish* and *altruistic* cases. The performance of SW protocol with 100 copies is much closer to our optimal policy, but it is still worse than our policy. In addition, the deviation between them increases after certain time. For example, between the times 4000 and 5000 s, the deviation of their performance becomes much

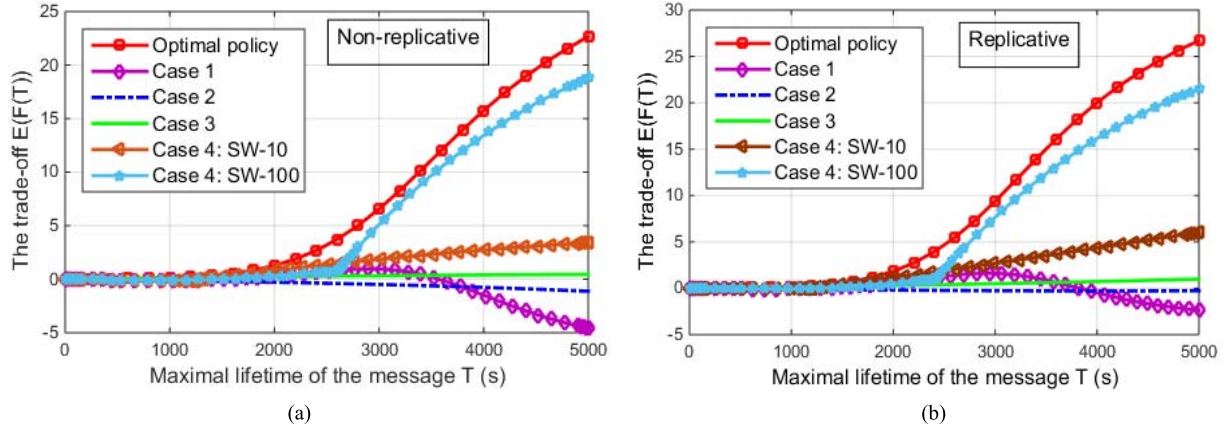


Fig. 4. Comparison when the maximal lifetime T has different values. (a) Selfish case. (b) Altruistic case.

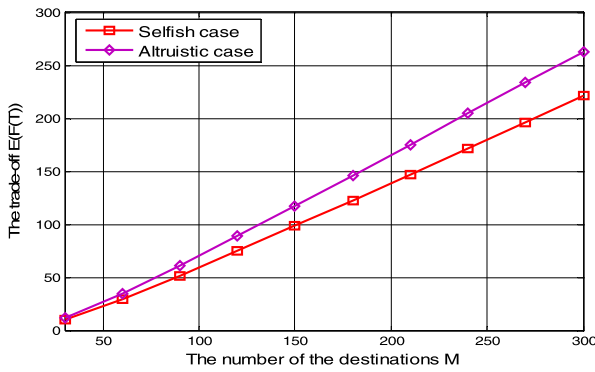


Fig. 5. Performance comparison under different number of destinations.

bigger. On the other hand, through comparing the results of Fig. 4(a) with Fig. 4(b), we find that we can get better tradeoff in the *altruistic* case under the same policy. This means that the cooperation of the destinations is necessary, so the destinations' number M may have certain influence on the performance. Now, supposing M changes between 30 and 300, and then setting the value of T be 5000 s. Any other parameter keeps unchanged. Both Lemmas 1 and 5 and above numerical results have demonstrated that our optimal policy has a better performance. Therefore, here we just present the result for the optimal policy (*threshold form*), which is shown in Fig. 5.

The result in Fig. 5 further shows that the value of $E(F(T))$ is bigger all the time in the *selfish* case. In addition, the deviation between the *selfish* and *altruistic* cases increases with M . As described in our mathematical analysis, the optimal policies denoted by (11) and (16) conform to the *threshold form* with *stopping time* denoted as h . The value of h can be found in Fig. 6(a). From this result, we can find that h in the *selfish* case increases with M . This demonstrates that if the network has more destinations, the relay nodes will beacon and forward with the maximal value lasting for longer time. But the value of h in the *altruistic* case first increases and then begins to decrease with M . This phenomenon demonstrates that when M has smaller value, its increasing will incentive the relay nodes to forward and beacon with the maximal

value for a longer time. However, if M becomes to be big enough, its increasing may make relay nodes stop forward and beacon earlier. In other words, if there are enough destinations, they may need less help from the relay nodes. The objective to increase the *forwarding* and *beaconing* rates is to make the relay nodes has higher rate to get the message, which will increase the probability that the destinations get message timely. When there are enough destinations and they are *altruistic*, the message can be transmitted faster between the destinations, so they may not need the help from relay nodes at all. In this case, the relay nodes will stop to help earlier by ending forward and beacon in earlier time, so the value of h is smaller. From Fig. 6(a), we can easily obtain the optimal forwarding and beaconing policy in different cases. For example, while M equals to 30 and 210, the optimal policies can be found in Fig. 6(b), which really conform to the *threshold form*.

Now, we begin to evaluate the influence of $\varepsilon(t)$, which is a nonincreasing function varying with time. This parameter acts as a regulator to normalize the value of $E(D(T))$ (*income*) and $E(U(T))$ (*cost*). First, we set the values of T and M to be 5000 and 50 s, respectively. Then, we set $\varepsilon(t) = -c \times \exp(t/5000)$. The values of other parameters are the same as that in Fig. 6. Through letting c increase from 0.01 to 0.2, we can obtain Fig. 7. From the result, we can see that the value of $E(F(T))$ decreases with c in both cases. In fact, when c is bigger, $-\varepsilon(t)$ is bigger, so the influence of $E(U(T))$ (the total energy consumption) is bigger. Namely, under the same energy consumption, if $\varepsilon(t)$ is bigger, the *total income* ($E(F(T))$) is smaller. In this case, it may not be good to use more energy to make more destinations get the message. Therefore, the *stopping time* may be smaller, and we can see this phenomenon more clearly in Fig. 8, which shows that the *stopping time* decreases with c . Therefore, if the influence of the energy consumption is bigger, it will stop forwarding the message to the relay nodes and the relay nodes will stop beaconing at earlier time.

C. Usage in Real Scenario

In this paper, we mathematically get the optimal forwarding and beaconing policies at the same time. Therefore, besides the

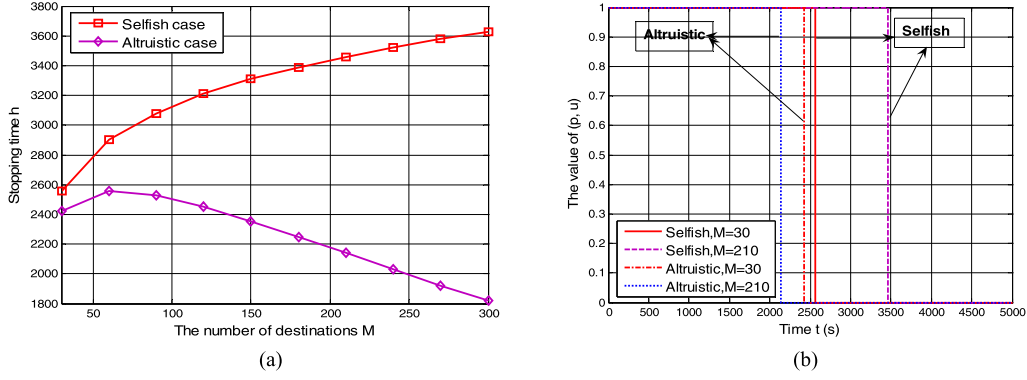


Fig. 6. Presentation of the stopping time and optimal policy in different cases. (a) Value of the stopping time. (b) Optimal policy when $M = 30$ and 210.

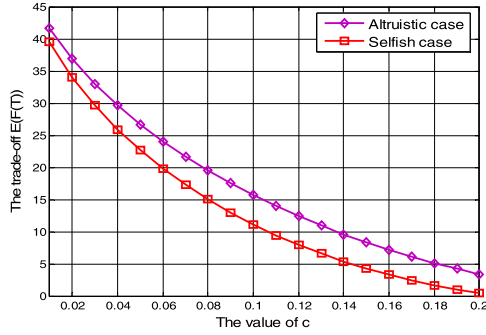


Fig. 7. Comparison when the function of $\varepsilon(t)$ is different.

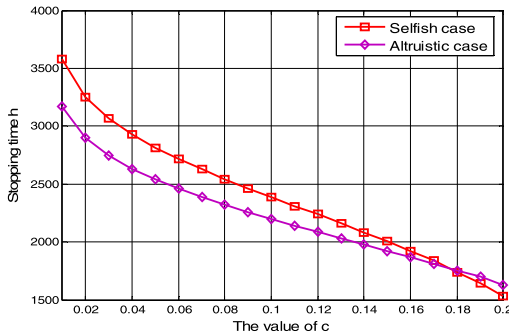


Fig. 8. Values of the *stopping time* in both selfish and altruistic cases when the value of $\varepsilon(t)$ is different.

beaconing policy, we also design the optimal routing protocol. According to the forwarding policy, the nodes can make the routing decision at any time. In addition, we prove that both the optimal forwarding and beaconing policies conform to the *threshold* form. This simple structure means that the optimal policies can be easily used in the real applications.

Now, we will introduce how to use the optimal policies in the real world. To use the optimal policy, we build a platform as Fig. 9. The framework includes two parts: the server and client. The server collects the sources' demands and receives the helpers' (relay nodes are denoted as helpers) registration information. Such information is transmitted through 3G/4G. Based on the collected information, the server will select proper helpers for each source and allocate the task for

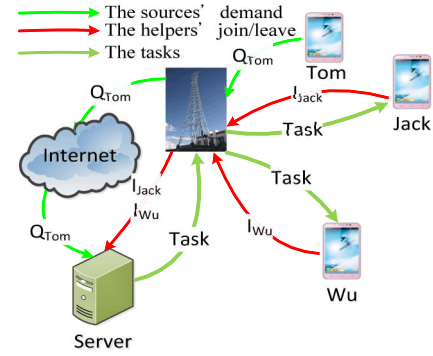


Fig. 9. Workflow of the framework.

the helpers. Then, the tasks are forwarded to these helpers through 3G/4G, too. After accepting a task, the helpers can build connection with other nodes based on WiFi, and then receive/forward messages from/toward these nodes.

As shown in Fig. 9, once the server receives a demand Q_{Tom} from the source *Tom*, it will put the demand in its message queue. The demand includes the size, lifetime, and the corresponding destinations of the message. The queue is sorted according to messages' lifetime. The messages with smaller lifetime will put in front of the queue. The server also has a nodes queue. Once a helper joins in the framework, the registration information (e.g., I_{Jack} , I_{Wu}) about the helper will be stored in this queue. The information mainly includes the time when the helper can help others. For example, $I_{Wu} = (t_1, t_2)$ means that the helper *Wu* can help others in the time interval $[t_1, t_2]$. This is the baseline for the scheduling of the helpers. After a message is transmitted, the server will select a new message m_{head} from the head of the message queue. Then, it will select the helpers which can help others during the lifetime of the message m_{head} . When the helpers are selected, we meet another important problem, that is, how to schedule them optimally. To overcome this problem, we will use the theoretical results obtained in Section V-B.

Based on the framework presented in Fig. 9, we build a real system. The client is developed based on a modular and lightweight implementation of delay tolerant network, which proposes architecture for building the DTN application [29]. The main interface of the client is shown in Fig. 10. In partic-



Fig. 10. Main interface of the client. (a) Interface of basic settings. (b) Neighbors found.

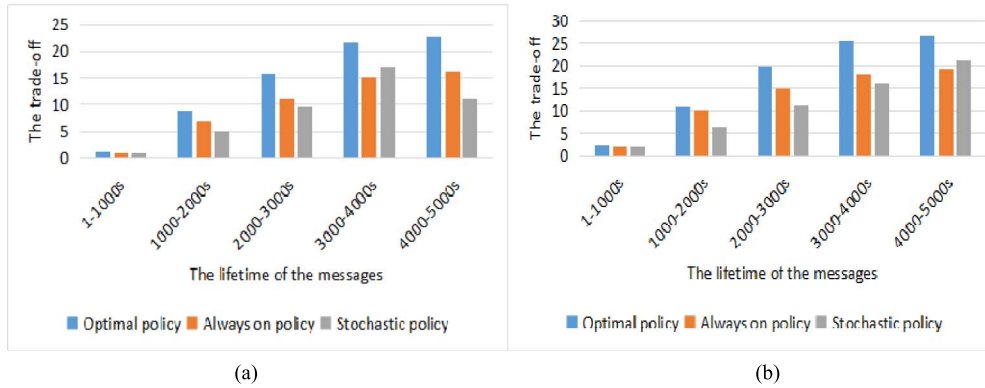


Fig. 11. Performance based on our platform with 100 students. (a) Selfish case. (b) Altruistic case.

ular, Fig. 10(a) introduces an interface for the basic settings. The *EndPoint Identity (ID)* shows the name of a client in the whole network. The *routing* model allows a user to select the forwarding policy designed by the server, and the server can design any routing policy for the client, such as the policy obtained in this paper. The users can set how to store the messages based on the *Storage* model. For example, the messages can be stored in a secure digital card or memory. The *Discovery* model sets the method to find neighbors. There are mainly three methods: *off*, *always on*, and *smart*. In the *off* state, the client does not beacon, but the client always beacon to find others in the *always on* state. The *smart* state allows the client to beacon according to the rule designed by the server. Fig. 10(b) shows the neighbors found by a client. Note that a client may find multiple neighbors at the same time. In this paper, we suppose that the client builds connection with the nearest neighbors. To ensure robustness of the connection, we ignore the neighbors 50 m away.

Once a server receives a message, it first packages the message as bundles according to the RFC 5050 protocol [30], and we set the size of each bundle to 2M. Then, the server will put the bundles into the message queue according to the message's lifetime. Note that a message may be divided into multiple bundles, and these bundles will be transmitted one by one. The optimal policy proposed above can be used for each bundle. Next, the server will select the proper helpers for each bundle.

After the helpers are selected, the server will calculate the optimal policy based on the method proposed in Section V-B. Note that we have proved that both the optimal forwarding and beaconing policies conform to the *threshold* form. Therefore, the server just needs to tell the helpers the time when to stop forwarding and beaconing, so the optimal policy is easy to use in the real application. On the other hand, to get the optimal policy, we need to get the contact rate θ between the users as a premise. The contact rates are different in different scenarios. In this paper, we deploy the clients in 100 students with Android OS smart phones coming from the National University of Defense Technology. Then, we collect their moving trajectories for 60 days. In particular, we only select the data between the times 11:00 am to 14:00 pm. The data is defined as (ID, Location, Time), which is reported to the server by the users periodically, and the period is set to be 6 s. Based on the data, the server can evaluate the value of the contact rate and then obtain the optimal policy. To compare the performance, we consider another two policies: *always on* and *stochastic*. In the *always on* policy, the forwarding and beaconing rates are one all the time. In the *stochastic* policy, the server randomly sets the forwarding and beaconing rates. In the 100 users, we randomly select ten users as the sources and they generate messages with size 2 M periodically. The length of a period is 2 h. The lifetime of these messages belongs to [1, 5000 s]. Each message has several destinations, which is also randomly selected in the 100 users.

We run the platform for five days (October 07–11, 2016), and calculate the average tradeoff. The result is shown in Fig. 11. From Fig. 11, we can see that the optimal policy proposed in this paper is really better.

VI. CONCLUSION

In this paper, we study the joint forwarding and beaconing control in DTNs with multiple destinations. The main objective of this paper is to get better tradeoff between the performance and energy consumption. Specifically, we consider two cases, which are the *selfish* and *altruistic* cases, respectively. Mathematical analysis proves that the optimal policy conforms to the *threshold* from. Extensive simulation and numerical results are carried out to evaluate the performance of our theoretical policy. Finally, we propose the method about how to use our optimal policies in the real world.

APPENDIX

A. The Proof of Lemma 1

For simplicity, we directly use $U(t)$, $R(t)$ and $D(t)$ to denote $E(U(t))$, $E(R(t))$ and $E(D(t))$, respectively. For the function $g = \theta(\lambda_R + \varepsilon\sigma)R + \varepsilon\tau$, we have

$$\begin{aligned}\dot{g} &= \theta(\dot{\lambda}_R + \sigma \dot{\varepsilon})R + \theta(\lambda_R + \sigma\varepsilon)\dot{R} + \dot{\varepsilon}\tau \\ &= -\theta\theta(1 + \lambda_D + \varepsilon\sigma)(M - D)R - \theta\theta(\lambda_R + \varepsilon\sigma)(N - 2R)p\mu R \\ &\quad + \theta\varepsilon\tau\mu R + \theta\sigma\dot{\varepsilon}R + \theta\theta(\lambda_R + \sigma\varepsilon)(N - R)p\mu R + \dot{\varepsilon}\tau \\ &= \theta(-\theta(1 + \lambda_D + \varepsilon\sigma)(M - D) + \theta(\lambda_R + \varepsilon\sigma)Rp\mu + \varepsilon\tau\mu)R \\ &\quad + \theta\sigma\dot{\varepsilon}R + \dot{\varepsilon}\tau \\ &= \theta(-\theta(1 + \lambda_D + \varepsilon\sigma)(M - D) + f)R + \theta\sigma\dot{\varepsilon}R + \dot{\varepsilon}\tau.\end{aligned}$$

Next, we define a function l as: $l = 1 + \lambda_D + \varepsilon\sigma$. From Lemma 3, we can easily find that if $g(s) = 0$, $l(s)$ satisfies $l(s) = 1 + \lambda_D(s) + \varepsilon\sigma(s) > 0$. Because $\varepsilon(t)$ is continuously differentiable and nonincreasing with time, we have $\dot{\varepsilon} \leq 0$. Therefore, when $g(s) = 0$, we have $p(s) = 0$ and $\mu(s) = 0$, so $f(s) = \theta(\lambda_R(s) + \sigma\varepsilon(s))R(s)p(s)\mu(s) + \varepsilon(s)\tau\mu(s) = 0$. In addition, we only consider the case $Y < M$, so we have

$$\begin{aligned}g(s) &= \theta(-\theta(1 + \lambda_D(s) + \varepsilon(s)\sigma)(M - D(s)) + f(s))R(s) \\ &\quad + \theta\sigma\dot{\varepsilon}(s)R(s) + \varepsilon(s)\tau < 0.\end{aligned}$$

Therefore, the function g decreases at the time s , and it will change to be negative at the next time. Below we will prove that once $g(s) < 0$, we have $g(t) < 0$, $s < t \leq T$. In other words, g will keep negative in the rest time.

Otherwise, we can assume that there exists a moment (e.g., v), where g can be nonnegative ($g(v) \geq 0$). Then, based on the famous property of continuous function (g is a continuous function based on our continuous-time model), we can find that there must exist **at least one moment** (e.g., k) between the time interval (s, v) , where the function g equals to 0 [that is, $g(k) = 0$]. Furthermore, according to the Lemma 3, we have $l(k) = 1 + \lambda_D(k) + \varepsilon(k)\sigma > 0$. Without loss of generality, we can suppose that k is the first moment, where g equals to 0 between s and v . In this case, we get $g(t) < 0$, $s < t < k$.

Next, we suppose that the time moment k' is just smaller than k , that is, there is no other time between k' and k . Then, we can get $g(k') = \theta(\lambda_R(k') + \varepsilon\sigma(k'))R(k') + \varepsilon(k')\tau < 0$. Moreover, from (11), we obtain $\mu(k') = 0$ and this means that $f(k') = 0$ is correct. Because the next time of k' is k , and $g(k) = 0$, g must increase at time k' . Therefore, we have

$$\begin{aligned}g(s) &= \theta(-\theta(1 + \lambda_D(k') + \varepsilon(k')\sigma)(M - D(k')) \\ &\quad + f(k'))R(k') + \theta\sigma\dot{\varepsilon}(k')R(s) + \varepsilon(k')\tau \\ &= \theta(-\theta(1 + \lambda_D(k') + \varepsilon(k')\sigma)(M - D(k')) \\ &\quad + \theta\sigma\dot{\varepsilon}(k')R(s) + \varepsilon(k')\tau) > 0.\end{aligned}$$

Then, we can obtain the following expression:

$$\begin{aligned}&\theta(-\theta(1 + \lambda_D(k') + \varepsilon(k')\sigma)(M - D(k')) \\ &> -\theta\sigma\dot{\varepsilon}(k')R(s) - \varepsilon(k')\tau \geq 0 \\ &\Rightarrow l(k') = 1 + \lambda_D(k') + \varepsilon(k')\sigma < 0.\end{aligned}$$

Because $l(k) = 1 + \lambda_D(k) + \varepsilon(k)\sigma > 0$, from the properties of continuous functions (l is a continuous function based on our continuous-time model), there should exist at least one moment between k' and k , where the function l satisfies $l = 1 + \lambda_D + \varepsilon\sigma = 0$. However, we have supposed that there is no other time between k' and k , so this is a contradiction. This proves that **if $g(s) < 0$, it will keep negative in the rest time.**

In short, once $g(s) = 0$, in the following moment, g will change to be negative. If $g(s) < 0$, it will keep negative in all of the rest time. Therefore, if we set s as the first moment, where g satisfies $g(s) \leq 0$, we will obtain $g(t) > 0$, $t < s$ and $g(t) < 0$, $s < t \leq T$. Moreover, from (11), we get $(p(t), \mu(t)) = 1$, $0 \leq t < s$, and $(p(t), \mu(t)) = 0$, $s \leq t \leq T$. The moment s is **stopping time**. This demonstrates that the optimal forwarding and beaconing policy conforms to the *threshold* time and jumps at most once. ■

B. The Proof of Lemma 2

First, we have, $l(s) = 1 + \lambda_D(s) + \varepsilon\sigma(s)$

$$\begin{aligned}l(t) &= \lambda_D(t) + \sigma\varepsilon(t) \\ &= \theta(1 + \lambda_D(t) + \varepsilon(t)\sigma)R(t) + \varepsilon(t)\tau + \sigma\dot{\varepsilon}(t) \\ &= \theta l(t)R(t) + \varepsilon(t)\tau + \sigma\dot{\varepsilon}(t).\end{aligned}$$

Note that we have supposed $\dot{\varepsilon} \leq 0$, $\varepsilon \leq 0$. Therefore, for any time s , if $l(s) \leq 0$, we have

$$l(s) = \theta l(s)R(s) + \varepsilon(s)\tau + \sigma\dot{\varepsilon}(s) \leq 0.$$

This demonstrates that the function l decreases at the moment. Moreover, we can easily find that the function l cannot increase anymore and it will keep nonpositive in the rest time, so this lemma follows. ■

C. The Proof of Lemma 3

Now, we suppose $l(s) \leq 0$. From Lemma 2, we can get $l(t) \leq 0, s < t \leq T$.

Define another function z as $z(t) = \lambda_D(t) - \lambda_R(t) + 1$. When $g(s) = 0$, we have $g(s) = \theta(\lambda_R(s) + \sigma\varepsilon(s))R(s) + \varepsilon(s)\tau = 0$, so we can obtain $\theta(\lambda_R(s) + \sigma\varepsilon(s))R(s) = -\varepsilon(s)\tau$. Then, we can further get $\theta l(s)R(s) = \theta(1 + \lambda_D(s) + \sigma\varepsilon(s))R(s) = \theta(1 + \lambda_D(s) - \lambda_R(s))R(s) + \theta(\lambda_R(s) + \sigma\varepsilon(s))R(s) = \theta z(s)R(s) - \varepsilon(s)\tau$. In this case, if $l(s) \leq 0$, we have $\theta l(s)R(s) = \theta z(s)R(s) - \varepsilon(s)\tau \leq 0$. Moreover, we can get $\theta z(s)R(s) \leq \varepsilon(s)\tau \leq 0$, which demonstrates $z(s) \leq 0$. This is a contradiction with Lemma 4, so $l(s) \leq 0$ is not a rational assumption. Then, the Lemma 3 follows. ■

D. The Proof of Lemma 4

Here, we suppose $z(s) \leq 0$.

Because $l(s) \leq 0$, based on Lemma 2, we get $l(t) \leq 0, s < t \leq T$. The derivative of function z is

$$\begin{aligned} \dot{z} &= \dot{\lambda}_D - \dot{\lambda}_R \\ &= \theta(1 + \lambda_D + \varepsilon\sigma)R + \varepsilon\tau + \theta(1 + \lambda_D + \varepsilon\sigma)(M - D) \\ &\quad + \theta(\lambda_R + \varepsilon\sigma)(N - 2R)p\mu - \varepsilon\tau\mu \\ &= \theta lR + \varepsilon\tau + \theta l(M - D) + \theta(\lambda_R + \varepsilon\sigma)(N - R)p\mu - f. \end{aligned}$$

According to (11), we can easily find $f \geq 0$. Therefore, if $\lambda_R(s) + \sigma\varepsilon(s) \leq 0$, we get

$$\begin{aligned} \dot{z}(s) &= \theta l(s)R(s) + \varepsilon(s)\tau + \theta l(s)(M - D(s)) \\ &\quad + \theta(\lambda_R(s) + \varepsilon(s)\sigma)(N - R(s))p(s)\mu(s) - f(s) \leq 0. \end{aligned}$$

This means that the function z decreases at time s . This demonstrates that for a moment $s+$, which is just bigger than s (there is no other time between s and $s+$), we can obtain $z(s+) \leq z(s) \leq 0$.

However, if $\lambda_R(s) + \sigma\varepsilon(s) > 0$, z may not decrease at time s . Now, we explore the value of $\lambda_R + \sigma\varepsilon$ at time $s+$, which is just bigger than s . If $\lambda_R(s+) + \sigma\varepsilon(s+) \geq 0$, then we have $z(s+) = 1 + \lambda_D(s+) - \lambda_R(s+) = 1 + \lambda_D(s+) + \sigma\varepsilon(s+) - \lambda_R(s+) - \sigma\varepsilon(s+) = l(s+) - \lambda_R(s+) - \sigma\varepsilon(s+)$. We have proved that $l(s+) \leq 0$, so if $\lambda_R(s+) + \sigma\varepsilon(s+) \geq 0$, we get $z(s+) = l(s+) - \lambda_R(s+) - \sigma\varepsilon(s+) \leq 0$. If $\lambda_R(s+) + \sigma\varepsilon(s+) < 0$, $\lambda_R + \sigma\varepsilon$ changes from the positive value to negative value directly from time s to $s+$. Because our model is a continuous-time mode, $\lambda_R + \sigma\varepsilon$ is a continuous function. Therefore, the transition from a positive value to a negative one directly from time s to $s+$ is impossible. Therefore, if $\lambda_R(s) + \sigma\varepsilon(s) > 0$, we have $\lambda_R(s+) + \sigma\varepsilon(s+) \geq 0$. In this case, $z(s+) \leq 0$. This means that if $\lambda_R(s) + \sigma\varepsilon(s) > 0$, we still have $z(s+) \leq 0$.

In summary, when $l(s) \leq 0$ and $z(s) \leq 0$, we can get $z(s+) \leq 0$. This shows that at the time $s+$, we still have $l(s+) \leq 0$ and $z(s+) \leq 0$. Therefore, the conditions at time $s+$ are the same as that at time s . Moreover, we can

get $z(T) = 1 + \lambda_D(T) - \lambda_R(T) \leq 0$. From the transversality conditions in (9), we know $\lambda_D(T) = \lambda_R(T) = 0$, so $z(T) = 1 + \lambda_D(T) - \lambda_R(T) = 1 > 0$. This is a contradiction. Therefore, the assumption that $z(s) \leq 0$ is not rational and we have $z(s) > 0$. ■

E. The Proof of Lemma 5

The derivative of the function $g = \theta(\lambda_R + \sigma\varepsilon)(R + D) + \varepsilon\tau$ is

$$\begin{aligned} \dot{g} &= \theta(\lambda_R + \sigma\varepsilon)(\dot{R} + \dot{D}) + \theta(\dot{\lambda}_R + \sigma\dot{\varepsilon})(R + D) + \dot{\varepsilon}\tau \\ &= \theta(\lambda_R + \sigma\varepsilon)(\dot{R} + \dot{D}) \\ &\quad + \theta(-\theta(1 + \lambda_D + \sigma\varepsilon)(M - D) \\ &\quad \quad - \theta(\lambda_R + \sigma\varepsilon)(N - 2R - D)p\mu + \varepsilon\tau\mu)(R + D) \\ &\quad + \sigma\dot{\varepsilon}(R + D) + \dot{\varepsilon}\tau \\ &= \theta(\lambda_R + \sigma\varepsilon)(\dot{R} + \dot{D}) + \theta(-(1 + \lambda_D + \sigma\varepsilon))\dot{D} \\ &\quad - \lambda(\lambda_R + \sigma\varepsilon)\dot{R} + \theta(R + D)f + \sigma\dot{\varepsilon}(R + D) + \dot{\varepsilon}\tau \\ &= -\theta(1 + \lambda_D - \lambda_R)\dot{D} + \theta(R + D)f + \sigma\dot{\varepsilon}(R + D) + \dot{\varepsilon}\tau. \end{aligned}$$

The function z is defined as $z = 1 + \lambda_D - \lambda_R$, so we have

$$\begin{aligned} \dot{z} &= \dot{\lambda}_D - \dot{\lambda}_R \\ &= \theta(1 + \lambda_D + \varepsilon\sigma)(R + 2D - M) \\ &\quad + \varepsilon\tau - \theta(\lambda_R + \varepsilon\sigma)(N - R)p\mu \\ &\quad + \theta(1 + \lambda_D + \varepsilon\sigma)(M - D) \\ &\quad + \theta(\lambda_R + \varepsilon\sigma)(N - 2R - D)p\mu - \varepsilon\tau\mu \\ &= \theta(1 + \lambda_D - \lambda_R + \lambda_R + \varepsilon\sigma)(R + D) + \varepsilon\tau - f \\ &= \theta z(D + R) + g - f. \end{aligned}$$

If $g \leq 0$, we have $p = \mu = 0$, so $f = \theta(\lambda_R + \sigma\varepsilon)(R + D)p\mu + \varepsilon\tau\mu = 0$. Moreover, we get $g - f = g \leq 0$. If $g > 0$, we have $p = \mu = 1$, so we can obtain $f = \theta(\lambda_R + \sigma\varepsilon)(R + D)p\mu + \varepsilon\tau\mu = \theta(\lambda_R + \sigma\varepsilon)(R + D) + \varepsilon\tau = g$. Therefore, $g - f = 0$. In summary, $g - f \leq 0$ is correct all the time.

Now, we assume $g(s) \leq 0$. Then, if $z(s) \leq 0$, we can easily obtain $z(t) \leq 0, s < t \leq T$, so $z(T) = 1 + \lambda_D(T) - \lambda_R(T) \leq 0$ is correct. According to the transversality conditions in (9), we get $\lambda_D(T) = \lambda_R(T) = 0$, so z satisfies $z(T) = 1 + \lambda_D(T) - \lambda_R(T) = 1 > 0$. This is a contradiction. Therefore, $z(s) \leq 0$ is impossible, and we have $z(s) = 1 + \lambda_D(s) - \lambda_R(s) > 0$.

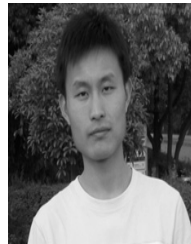
On the other hand, when $g(s) \leq 0$, according to (16), we have $p(s) = \mu(s) = 0$, so $f(s) = 0$. Because $\varepsilon(t)$ is continuously differentiable and nonincreasing, we get $\dot{\varepsilon} \leq 0$. Note that we just consider the case $Y < M$, that is, at least one destination does not have the message. Then, we can get

$$\begin{aligned} \dot{g}(s) &= -\theta(1 + \lambda_D(s) - \lambda_R(s))\dot{D}(s) \\ &\quad + f(s)(R(s) + D(s)) + \sigma\dot{\varepsilon}(s)(R(s) + D(s)) + \dot{\varepsilon}(s)\tau \\ &= -\theta z(s)\dot{D}(s) + \alpha\dot{\varepsilon}(s)(R(s) + D(s)) + \dot{\varepsilon}(s)\tau < 0. \end{aligned}$$

This means that at any moment s , once $g(s) \leq 0$, g will decrease at this time. Therefore, if $g(s) \leq 0$, it will keep negative in the rest time. Without loss of generality, we can suppose that s is the first time which satisfies $g(s) \leq 0$, and then we have $g(t) > 0$, $t < s$; $g(t) < 0$, $s < t \leq T$. Moreover, according to (16), we have $(p(t), \mu(t)) = 1$, $0 \leq t < s$; $(p(t), \mu(t)) = 0$, $s \leq t \leq T$. Then, the lemma follows. ■

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