Optimal Resource Allocation in a Price Based Opportunistic Cognitive Radio Network

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***Abstract*—In this paper, we consider a cooperative cognitive radio network where the fusion center acts as final decision maker as well as resource allocator. The secondary users access the licensed band only when the primary user is inactive. The fusion center behaves as a profit seeking entity, which charges the secondary users for letting them use the licensed band. However, the fusion center pays a penalty to the primary network for causing interference. Hence to improve detection performance, the fusion center encourages the SUs to perform cooperative spectrum sensing. Moreover, we map the secondary users' average data rate with a payoff. All the secondary users maintain data buffers and pay to the fusion center according to their traffic type. The fusion center tries to maximize it's own utility while maintaining non-negative utilities at the secondary users. Feasible conditions which guarantee non-negative utilities at secondary users are evaluated. We show that the optimal number of observations which is required for spectrum sensing at secondary users strongly depends on the interference penalty paid by the fusion center. The secondary users get time durations for accessing the licensed band according to their stored energies and paid costs to the fusion center.**

***Index Terms*—Cognitive radio, resource allocation.**

I. Introduction

Cognitive radio (CR) has emerged as an efficient paradigm with great potential to improve spectrum utilization. In [1], [2], the researchers have discussed spectrum reuse in televisions (TV) bands and radar bands via CR technology. The licensed user and opportunistic spectrum user are known as primary user (PU) and secondary user (SU) respectively. Opportunistic users adapt communication parameters and use the temporarily unused licensed spectrum. However, while accessing the licensed band, PU's quality of service (QoS) should be maintained. Typically two different approaches for accessing the licensed spectrum, i.e., underlay and opportunistic access, are considered. In opportunistic spectrum access (OSA), secondary network performs spectrum sensing before accessing the licensed band. Erroneous sensing result leads to degraded QoS at the PU. In [3], authors have discussed about several spectrum sensing techniques' merits and demerits. Cooperative spectrum sensing (CSS) may provide substantial improvement in spectrum sensing performance for CR networks, where multiple SUs send their sensing data/decisions to the fusion center (FC). Typically sending decisions is preferred over sending data (e.g. log-likelihood ratio) more for it's low bandwidth and power requirement. The FC takes the final decision about the primary transmitter's activity on the licensed band.

In the context of CR networks, optimal resource allocation has drawn significant attention. The SUs adapt communication parameters based on the sensing result and utilize the temporarily unused licensed resource (i.e., spectrum in this case) in order to increase their own utilities or to increase a global utility. The former can be solved by the means of distributed techniques (e.g., [4], [5]); whereas, the later is solved in a centralized manner (e.g., [6], [7]). Though in a centralized resource allocation technique, network overhead is higher, it is more efficient than the distributed approach [8]. Typically the FC conducts the resource allocation job in a centralized manner. In [6], [7], the FC performs the resource allocation in order to maximize the secondary networks' throughput. The FC may charge the SUs for performing the resource allocation job [9], which brings an economic perspective to the resource allocation problem in CR networks. Related literature includes [10], where the authors have considered auction based allocation in cooperative sensing based CR networks. The FC acts as an auctioneer, which allocates resource among the SUs based on their (i.e. the SUs) bids. We formulate a resource allocation problem in a price based cooperative CR network, where the CR nodes access the licensed band opportunistically.

Associated prices are: (a) interference penalty, i.e., penalty paid by the FC to the PU, (b) SUs' payoff to the FC for their allotted time durations, (c) SUs make profits by transmitting data using the licensed band, (d) incurred costs at the SUs' for performing spectrum sensing, and (e) sensing remunerations to the SUs by the FC. As the interference on the PU is caused due to erroneous sensing, in OSA, the FC encourages the SUs to participate in spectrum sensing by paying the SUs a declared remuneration. To the best of our knowledge, sensing remuneration in resource allocation has not been considered earlier for CR networks. We make our framework more flexible by considering heterogeneous requirements at the SUs. All the SUs maintain data buffers and pay a declared amount to the FC for allotted time, which may not be same for all SUs. The SUs generate revenue by utilizing the licensed band. We construct utility functions at the FC and SUs considering all the associated costs. Based on the data at buffer, an upper limit on allotted time is considered for each ST. The resource allocation is performed by the FC in order to maximize the FC's utility while satisfying SUs' utility constraints and time constraints. We summarize our contributions as follows:

- A centralized resource allocation problem in CR network

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has been considered where we formulate economy based utility functions at the FC and SUs. We impose constraints on SUs' utilities (i.e., non-negative) and find necessary conditions under which these constraints are satisfied. The FC allocates different time durations among the SUs based on SUs' energies and paid costs to the FC.

- Number of observations collected at SUs for spectrum sensing and allocated time durations to SUs are considered as optimization parameters. It is shown that the optimization problem is overall non-convex one. However, for fixed number of sensing observations at SUs the optimization problem becomes convex.

In Section II, we describe our system model and the co-operative sensing procedure. Section III deals with the utility functions (i.e., at the FC and the STs), different constraints, and corresponding resource allocation problem. In Section IV, we present some results and corresponding discussions. Finally, in Section V, we give some concluding remarks.

II. System Model

In Fig. 1, we provide an overview of our system model, where a secondary network co-exist with a primary network. The secondary network tries to use the licensed spectrum of primary network by using the OSA method. We consider single primary transmitter (PT) and primary receiver (PR) in the primary network. Whereas, in the secondary network we denote the secondary transmitters by ST_i, $i = 1, .., M$. The receiver node corresponding to ST_i is denoted by SR_i. All the STs perform spectrum sensing and send their hard decisions (i.e, 0 or 1) to the FC which takes the global decision.

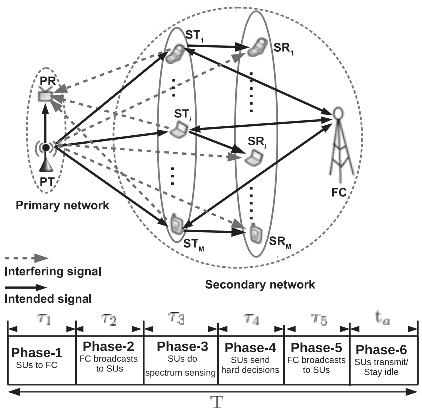


Figure 1: System Model and frame structure

We consider following assumptions in our system model:

- Both the primary and secondary networks follow synchronized frame structure. Communications between the STs and the FC are made using a predefined control channel.
- During a frame duration, an ST possesses a particular traffic type. The STs offer prices to the FC based on their corresponding traffic types. The SUs possess information about the remunerations received from the FC for sensing beforehand.

In Fig. 1, we represent the frame of duration T. We show different phases which are maintained by each ST within a frame duration. A brief summary of the phases within a frame is given below:

1) *Phase-1:* At the beginning of a frame, each ST informs the FC about it's stored data at buffer, reserved energy, and traffic type. The STs use control channel and send their information using TDMA. Required time is denoted by $\tau_1 = M\tau_r$, where τ_r is the required reporting time by a ST.
2) *Phase-2:* The FC broadcasts the number of samples to be collected by each ST for spectrum sensing. We denote the required time for broadcasting by τ_2. Broadcasting is done on the control channel.
3) *Phase-3:* The STs perform spectrum sensing synchronously for $\tau_3 = N\tau_s$, where τ_s is the sampling interval.
4) *Phase-4:* After sensing, all the STs send their hard decisions over the control channel in TDMA fashion. Total reporting time is $\tau_4 = M\tau_r^{'}$, where $\tau_r^{'}$ is the reporting time of each ST.
5) *Phase-5:* The FC takes the final decision with the help of collected hard decisions. If the PT is detected as idle, then the FC broadcasts the allotted time durations to the STs over the control channel. Required broadcasting time is τ_5.
6) *Phase-6:* The rest of the duration over which the STs may access the licensed band becomes $t_a = T - \sum_{k=1}^{5} \tau_k$. As the STs access the licensed channel in TDMA fashion we can write $\sum_{i=1}^{M} t_i \leq t_a$, where t_i is the allotted time for data transmission for ST_i.

A. Cooperative sensing procedure

We denote the observation vector at ST_i for spectrum sensing as $\boldsymbol{x_i} = [x_i(1), ..., x_i(N)]$. We define the presence and absence of the PU by hypotheses H_1 and H_0 respectively, and write the received signal at ST_i during j^{th} sampling instant as:

$$\begin{array}{ll} H_1: & x_i(j) = h_i p(j) + w_i(j) \\ H_0: & x_i(j) = w_i(j) \end{array} \tag{1}$$

where, $j = 1, ..., N$. h_i is the channel coefficient between the PU and i^{th} ST, which remains constant during a frame duration. $p(j)$ and $w_i(j)$ are the PU's signal during j^{th} sampling instant and the additive white Gaussian noise (AWGN) at i^{th} ST. We denote noise and PU signal's variances by σ_w^2 and σ_p^2 respectively.

We choose energy detection at the STs as it is less complex [3]. Test statistic for energy detection at i^{th} ST for taking a hard decision (i.e., 0 or 1) is:

$$y_i = \frac{1}{N} \sum_{j=1}^{N} |x_i(j)|^2 \underset{H_0}{\overset{H_1}{\gtrless}} \epsilon_i \tag{2}$$

where, ϵ_i is the detection threshold at ST_i.

Under the assumption of sufficiently large value of N, the false-alarm and detection probabilities of the i^{th} sensor may be written as [11]:

$$P_{fa}^{i} = Q\left\{\left(\frac{\epsilon_i}{\sigma_w^2} - 1\right)\sqrt{N}\right\} \tag{3a}$$

$$P_{d}^{i} = Q\left\{\frac{1}{\sqrt{2\gamma+1}}\left(Q^{-1}(P_{fa}) - \sqrt{N}\gamma\right)\right\} \tag{3b}$$

where, $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty}\exp\left(-\frac{t^2}{2}\right)dt$ and $\gamma = \frac{\mathbb{E}[|h_i|^2]\sigma_p^2}{\sigma_w^2}$ [1] is the received SNR at each ST. $\mathbb{E}_z[.]$ denotes the expectation is taken on the random variable z.

In [12], authors have shown that if the sensor nodes receive i.i.d. observations in a collaborative sensor network, then choosing identical sensors' threshold is asymptotically optimal. In this paper, we assume identical CR nodes' decision rules (i.e., $\epsilon_i = \epsilon$) and hence identical false-alarm and detection probabilities as denoted by: $P_{fa}^i = P_{fa}$ and $P_d^i = P_d$ respectively. We consider that each ST maintains a particular false-alarm probability, i.e., $P_{fa} = \overline{P}_{fa}$. Threshold ϵ is set in order to achieve $\overline{P}_{fa}$.

The FC collects all the hard decisions (i.e., vector $\{1,0\}^{1\times M}$) and fuse them by using appropriate fusion rule. In this paper, we consider AND and OR fusion rules; corresponding detection and false-alarm probabilities can be found in [13]. We denote the detection and false-alarm probabilities at the FC by P_D and P_{FA} respectively.

III. Optimal resource allocation

In this section, we briefly describe the utility function at the FC and constraints under which the utility function is maximized during the resource allocation. It is to be noted that, the resource allocation is performed on an average manner as we consider sensing error probabilities and take average over the channels (i.e., channels between the STs and their corresponding SRs).

A. Utility function at the FC

We consider that the FC allocates the licensed band among the STs over time only when the licensed band is sensed as idle. In order to maintain the QoS of the primary network we consider the FC pays penalty to the primary network for interference. In Table I, we show associated costs considered for our scenario. It can be observed that the FC gets a payoff from the STs for their each successful bit transmission to the SRs. However, the FC pays to the primary network and the STs for causing interference and performing sensing respectively.

Average number of successfully transmitted bits by ST_i is:

$$R_i = P(H_0)(1-P_{FA})r_0^i + P(H_1)(1-P_D)r_1^i \tag{4}$$

[1]As we have assumed unit variance for the channel coefficient h_i^s, the average SNR at the STs become identical.

where, r_0^i and r_1^i are the received normalized rates at SR_i under hypothesis H_0 and H_1 respectively, which are [2]:

$$r_0^i = \mathbb{E}_{g_i}\left[\log_2\left(1 + \frac{g_i e_i}{t_i N_0}\right)\right]$$
$$r_1^i = \mathbb{E}_{g_i, g_{pi}}\left[\log_2\left(1 + \frac{g_i e_i}{t_i(g_{pi}\sigma_p^2 + N_0)}\right)\right].$$

g_i is the channel power gain between ST_i and SR_i, e_i is the spent energy by ST_i for transmitting data to SR_i, g_{pi} is the channel power gain between the PT and SR_i, and N_0 is the noise power at each SR.

If ST_i pays the amount a_i to the FC for each successfully transmitted bit to SR_i, then the expected payoff at the FC is:

$$U_{payoff} = \sum_{i=1}^{M} R_i t_i a_i. \tag{6}$$

The FC pays a penalty to the PU when the secondary network interferes with the PU's activity. As the FC allows the STs to transmit only when the PT is detected idle, interference occurs when the FC miss-detects the PT. If the FC pays an amount c_c for per unit collision time, then the average cost paid to the PU by the FC becomes:

$$U_{price}^1 = P(H_1)(1-P_D)c_c\sum_{i=1}^{M} t_i. \tag{7}$$

The FC encourages the STs to perform spectrum sensing, such that, the detection performance improves and hence the interference penalty as given in Equation (7) reduces. Here we consider that the FC pays amount c_s and c_t to each ST for collecting an observation for sensing and for transmitting a decision respectively. For N number of observations, the total cost paid to the STs by the FC can be written as:

$$U_{price}^2 = M\left[Nc_s + c_t\right]. \tag{8}$$

From Equation (6), (7), and (8), the utility function at the FC can be written as: $U_{FC}(N, \boldsymbol{t}) = U_{payoff} - U_{price}^1 - U_{price}^2$, which is given in Equation (9), where $\boldsymbol{t} = [t_1, t_2, .., t_M]$. Detection performance improves when we increase the value of N, which effects the utility function of the FC as follows:

- The payoff as given in Equation (6) reduces as increase in N reduces the transmission opportunities of the STs.
- Collision penalty as given in Equation (7) reduces.
- Cost due to sensing increases.

This indicates a possible trade-off between the sensing performance and the utility function at the FC.

Table I: Different associated costs

Actual state	Sensing result	FC to PU	ST_i to FC	FC to ST_i	ST_i's profit from usage	ST_i's cost for sensing and transmission
H_0	H_0	-	$t_i r_0^i a_i$	$\mathcal{S}$	$t_i r_0^i b_i$	$\mathcal{T}$
H_0	H_1	-	-	$\mathcal{S}$	-	$\mathcal{T}$
H_1	H_0	$c_c\sum_{i=1}^{M} t_i$	$t_i r_1^i a_i$	$\mathcal{S}$	$t_i r_1^i b_i$	$\mathcal{T}$
H_1	H_1	-	-	$\mathcal{S}$	-	$\mathcal{T}$

where, $\mathcal{S} = Nc_s + c_t$ and $\mathcal{T} = Na_s + a_t$.

[2]We assume unit bandwidth here.

$$U_{FC}(N,\boldsymbol{t}) = \sum_{i=1}^{M} \left[P(H_0)(1-P_{FA})a_i r_0^i + P(H_1)(1-P_D)(a_i r_1^i - c_c)\right] t_i - M\left[N c_s + c_t\right] \tag{9}$$

B. System constraints

We now define the constraints which are considered during the resource allocation problem.

1) *Positive utility at each ST:* ST_i participates in the cooperative sensing if it's own utility function becomes positive. We consider that for each successful bit transmission, ST_i generates the revenue b_i. Moreover, for collecting observations and sending hard decision, each ST depletes some energy. Here we represent the depleted energy in terms of cost, i.e., for each observation collection and decision transmission, required costs are a_s and a_t respectively as given in Table I. We use the same analogy of forming the utility function (as given in Section III-A) and write the constraint on the utility function at ST_i as:

$$R_i t_i(b_i - a_i) + [N(c_s - a_s) + (c_t - a_t)] \geq 0 \tag{10}$$

where, R_i has been defined in Equation (4).

2) *Total time constraint:* If we consider the total frame duration as T, then we can write:

$$\sum_{k=1}^{5} \tau_k + \sum_{i=1}^{M} t_i \leq T \tag{11}$$

where, $\tau_k, k = 1,2,3,4,5$, have been defined in Section II.

3) *Buffer data constraint:* We consider data buffer at each ST. The FC allocates time to the STs according to their buffer sizes. Allocating more time to ST_i than what it (i.e., ST_i) requires to clear the buffer becomes a wastage. For this reason, we consider following constraint:

$$R_i t_i \leq B_i \tag{12}$$

where, B_i denotes the number of bits stored in ST_i's buffer and R_i is given in Equation (4).
We observe that the left hand side of Equation (12) is a monotonically increasing and concave function of t_i (proof is given in Appendix A). Therefore, we can write:

$$t_i \leq T_{t,i}. \tag{13}$$

where,

$$T_{t,i} = \{t_i | R_i t_i = B_i\}.$$

C. Optimization problem

We can write the optimization problem at the FC as:

$$P1: \quad \underset{N,\boldsymbol{t}}{\text{maximize}} \quad U_{FC}(N,\boldsymbol{t})$$
$$\text{subject to:} \quad \text{Equations (10), (11), (13).}$$

After analysing the optimization problem $P1$, we frame our observations in Proposition III.1.

Proposition III.1. *Optimization problem $P1$ is non-convex. However, for fixed value of N it is convex over $\boldsymbol{t}$.*

Proof. Corresponding proof for the first statement is given in Appendix B. For fixed value of N, the objective function of $P1$ is concave (which can be easily proved with the help of Appendix A). As the constraints are affine functions of $\boldsymbol{t}$, we can conclude that for a fixed value of N, the optimization problem $P1$ is convex. ■

For $N = \overline{N}$, we represent the corresponding optimization problem for as:

$$P2: \quad \underset{\boldsymbol{t}}{\text{maximize}} \quad \sum_{i=1}^{M} U_{FC}^i(\overline{N}, t_i) \tag{14a}$$
$$\text{subject to:} \quad t_i \leq T_{t,i}, \forall i \tag{14b}$$
$$R_i t_i c_1^i + c_2 \geq 0, \forall i \tag{14c}$$
$$\sum_{i=1}^{M} t_i \leq T' \tag{14d}$$

where,

$$U_{FC}^i(\overline{N}, t_i) = R_i t_i a_i - P(H_1)(1-P_D)c_c t_i - \left[\overline{N} c_s + c_t\right],$$
$$c_1^i = b_i - a_i, c_2 = \overline{N}(c_s - a_s) + (c_t - a_t), T' = T - \sum_{k=1}^{5} \tau_k.$$

It is observed that as the constraint (i.e., given in Equation (14c)) is associated with different costs, the optimal feasible set for $t_i, \forall i$, strongly depends on these costs. In Proposition III.2, we give the feasible ranges for c_1^i and c_2 for which the optimization problem $P2$ holds.

We define the following notations which are used for further analysis.

$$t_{i,l}^1 = \left\{t_i | R_i t_i c_1^i + c_2 = 0, c_1^i > 0, c_2 < 0\right\} \tag{15a}$$
$$t_{i,l}^2 = \left\{t_i | R_i t_i c_1^i + c_2 = 0, c_1^i < 0, c_2 > 0\right\}. \tag{15b}$$

Proposition III.2. *Ranges of c_2 and $c_1^i, \forall i$, for which optimization problem $P2$ becomes feasible and the corresponding feasible range of t_i can be represented as given in Table II*

Table II: Feasible costs and corresponding time ranges

Range of c_2	Range of c_1^i	Feasible range of t_i
$c_2 < 0$	$c_1^i < 0$	Not feasible
	$c_1^i = 0$	Not feasible
	$c_1^i > 0$	$t_{i,l}^1 \leq t_i \leq T_{t,i}$ if $t_{i,l}^1 \leq T_{t,i}$ and $\sum_{i=1}^{M} t_{i,l}^1 \leq T'$
$c_2 = 0$	$c_1^i < 0$	$t_i = 0$
	$c_1^i = 0$	$0 \leq t_i \leq T_{t,i}$
	$c_1^i > 0$	$0 \leq t_i \leq T_{t,i}$
$c_2 > 0$	$c_1^i < 0$	$t_i = 0$ or $t_{i,l}^2 \leq t_i \leq T_{t,i}$ if $t_{i,l}^2 \leq T_{t,i}$ and $\sum_{i=1}^{M} t_{i,l}^2 \leq T'$
	$c_1^i = 0$	$0 \leq t_i \leq T_{t,i}$
	$c_1^i > 0$	$0 \leq t_i \leq T_{t,i}$

Proof. The optimization problem $P2$ becomes feasible when all the constraints are satisfied for the solution. We analyse the constraint as given in Equation (10). From the fact that $R_i t_i$ is a positive and monotonically increasing function for $t_i \geq 0$ (proof is given in Appendix A), we get the feasible ranges for c_2 and c_1^i, which are given in Table II. ∎

If we consider that $P2$ is a feasible optimization problem, then from Proposition III.2, we can write an equivalent optimization problem of $P2$ as:

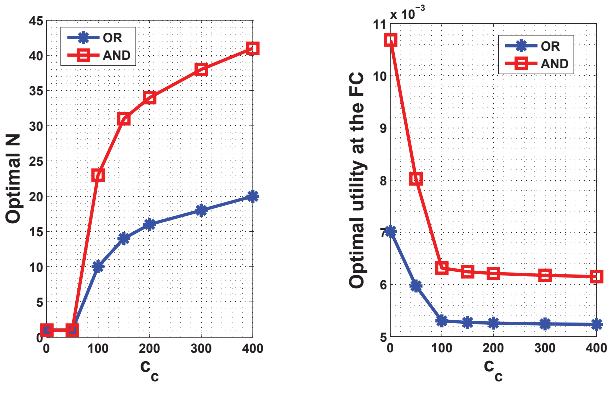
$$\begin{aligned} P3: \quad & \underset{\boldsymbol{t}}{\text{maximize}} \quad \sum_{i=1}^{M} U_{FC}^{i}(\overline{N}, t_i) \\ & \text{subject to:} \quad \text{Equation (14d)} \\ & \qquad\qquad\quad t_{i,l} \leq t_i \leq T_{t,i}. \end{aligned} \tag{16a}$$

It is to be noted that $t_{i,l} \in \{0, t_{i,l}^1, t_{i,l}^2\}$, based on the values of c_1^i and c_2. As the objective function of $P3$ is concave and the constraints are affine with respect to $\boldsymbol{t}$, the optimization problem $P3$ is convex one.

We perform a one dimensional search over $N = 1, .., \left\lceil \frac{T}{\tau_s} \right\rceil$. For each value of N, we solve the convex optimization problem $P3$. The corresponding value of N for which we get maximum utility value at the FC becomes optimal.

IV. Results and Discussions

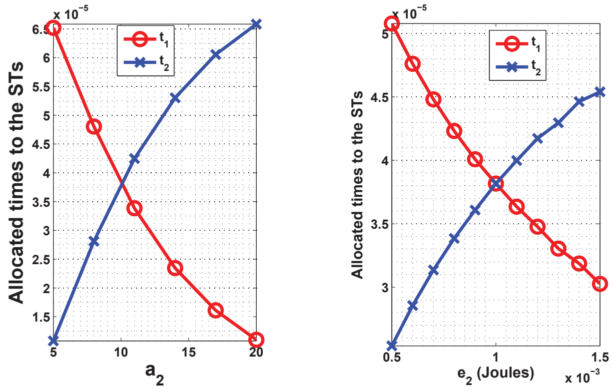
In this section, we show some results received for $P(H_0) = 0.2, \tau_r = 4\mu\text{sec.}, \tau_s = .16\mu\text{sec.}, \gamma = 0$ dB, $\overline{P}_{fa} = 0.1, \sigma_p^2 = 20$ dBm, $N_0 = 1$ watt, $T = 0.0001$sec., $M = 2$, $c_s = a_s = 0.01, c_t = a_t = 0.05, b_1 = b_2 = 20$, and $B_1 = B_2 = 100$ bits. Channel gains, i.e., g_i and $g_{p,i}$, follow unit mean exponential distribution. We neglect the required times for broadcasting by the FC, i.e, τ_2 and τ_5. Also we consider $\tau_1 = \tau_4 = M\tau_r$.



(a) (b)

Figure 2: Optimal number of sensing observations at the STs and optimal utility at the FC for varying collision penalty

In Fig. 2(a) and 2(b), we show how the optimal number of observations for spectrum sensing at the STs and optimal utility at the FC depend on the value of c_c considering $a_1 = a_2 = 20$ and $e_1 = e_2 = 0.001$ Joules. It is observed in Fig. 2(a), that the optimal number of observations for spectrum sensing at the STs increases with the interference penalty cost, which is intuitively correct. As the collision cost increases, the FC asks the STs to collect more number of observations for sensing, such that, the detection performance improves and hence the chance of interfering with the primary network reduces. As the optimal number of observations for spectrum sensing at the STs increases, effective sharing time (i.e., sharing of the licensed spectrum by the STs) reduces. Therefore, the optimal utility at the FC reduces for higher values of collision penalty, which is shown in Fig. 2(b). It is observed that the optimal value for N is higher in case of AND fusion rule, which can be explained from the fusion process. In AND fusion process, the FC infers the hypothesis H_0/H_1 if all the STs infer H_0/H_1. For this reason, the required number of sensing observations at the STs increases to maintain the quality of detection at the STs and hence at the FC. As the AND fusion rule offers better detection performance, it gives better utility at the FC compared to the OR fusion rule.



(a) (b)

Figure 3: Allotted time to the STs for varying costs and energies for ST_2

In Fig. 3(a) and 3(b), we plot allotted times to the two STs for their different reserved energies and paid costs to the FC. We consider AND fusion rule while generating the plots in Fig. 3(a) and 3(b). For ST_1, we consider the energy and paid cost to the FC as $e_1 = 0.001$ Joules and $a_1 = 10$ in both Fig. 3(a) and 3(b). We fix the energy of ST_2 as $e_2 = 0.001$ Joules and vary the paid cost to the FC by ST_2, i.e., a_2, in Fig. 3(a). It is observed that for lower values of a_2, ST_2 gets lesser time than ST_1 to access the licensed band. However, the scenario changes for higher values of a_2. Similar thing is observed in 3(b), where we fix $a_2 = 10$ and vary e_2. From Fig. 3(a) and 3(b), it is observed that the allotted times to the STs depend on their energies and paid costs to the FC.

V. Conclusion

In this paper, we have considered an economy based resource allocation problem in a cooperative CR network. We have formulated cost based utility functions at the FC and the

STs. The FC gets payoff from the STs for allocating different time durations to the STs. However, the FC pays interference penalty to the primary network and remuneration to the STs for participating in sensing. All the STs maintain data buffer and pay to the FC based on their traffic type. The FC allocates different time durations among the STs in order to maximize it's own utility function while maintaining positive utility function at each ST and allocation time constraint (i.e., based on the buffer size) for each ST. We have shown the conditions on different costs for which the resource allocation problem becomes feasible. In result section, we show how the FC's utility function and sensing observations' number at the STs depend on the collision cost. The STs get their corresponding times for accessing the licensed band based on their reserved energies and paid costs to the FC.

Knowing that typical CR nodes are energy constrained, this work may be further extended considering energy constraints at the STs. Moreover, it is also possible to find out the optimal set of the STs, which makes the optimization problem feasible irrespective of any conditions on different costs.

Appendix A
Proof: $R_i t_i$ is a monotonically increasing and concave function of t_i

We can write $R_i t_i$ as:

$$R_i t_i = \mathbb{E}_{g_i, g_{pi}} \left\{ \mathcal{R}_0^i + \mathcal{R}_1^i \right\} \tag{17}$$

where, $\mathcal{R}_0^i = P(H_0)(1 - P_{FA}) \log_2 \left(1 + \frac{g_i e_i}{t_i N_0}\right) t_i$ and $\mathcal{R}_1^i = P(H_1)(1 - P_D) \log_2 \left(1 + \frac{g_i e_i}{t_i (g_{pi} \sigma_p^2 + N_0)}\right) t_i$. It can be observed that both $\mathcal{R}_0^i$ and $\mathcal{R}_1^i$ can be written in the form like $C_1 t_i \log_2 \left(1 + \frac{C_2}{t_i}\right)$, where C_1 and C_2 are positive constants. The first and second order derivatives can be evaluated as:

$$\frac{\partial \left[C_1 t_i \log_2 \left(1 + \frac{C_2}{t_i}\right)\right]}{\partial t_i} = C_1 \log_2(e) . \left\{ \ln \left(1 + \frac{C_2}{t_i}\right) - \frac{C_2}{C_2 + t_i} \right\} \tag{18a}$$

$$\frac{\partial^2 \left[C_1 t_i \log_2 \left(1 + \frac{C_2}{t_i}\right)\right]}{\partial^2 t_i} = -\frac{C_1 C_2^2 \log_2(e)}{t_i (C_2 + t_i)^2} . \tag{18b}$$

From Equation (18b), it can be checked that the second order derivative is always positive, which means that the function is concave. Moreover, from Equation (18a), we get: $\lim_{t_i \to 0} \frac{\partial \left[C_1 t_i \log_2 \left(1 + \frac{C_2}{t_i}\right)\right]}{\partial t_i} = \infty$ and $\lim_{t_i \to \infty} \frac{\partial \left[C_1 t_i \log_2 \left(1 + \frac{C_2}{t_i}\right)\right]}{\partial t_i} = 0$. Therefore, we conclude that $C_1 t_i \log_2 \left(1 + \frac{C_2}{t_i}\right)$ is a monotonically increasing and concave function of t_i.

As both $\mathcal{R}_0^i$ and $\mathcal{R}_1^i$ are monotonically increasing and concave functions of t_i, we can conclude that $R_i t_i$ is a monotonically increasing and concave function of t_i.

Appendix B
Proof: Optimization problem $P1$ is not convex

A function of two variables is convex if and only if it becomes convex for individual variable. We use this property of convexity and show that the objective function in $P1$, i.e., $\sum_{i=1}^{M} U_{FC}^i(N, t_i)$, is non-convex. In [11], authors have considered sensing duration as continuous variable; from that same analogy, we evaluate the second order derivative of P_d with respect to N as follows:

$$\frac{\partial^2 P_d}{\partial^2 N} = \frac{c_2 \exp\left(\frac{-(c_1 - c_2\sqrt{N})^2}{2}\right)}{\sqrt{8\pi}} \left\{ -\frac{1}{2N^{3/2}} + \frac{c_1 - c_2\sqrt{N}}{N} \right\}$$

where, $c_1 = Q^{-1}(P_{fa})/\sqrt{2\gamma + 1}, c_2 = \gamma/\sqrt{2\gamma + 1}$. From the above equation, it can be observed that we get different ranges for N, over which $\partial^2(P_d)/\partial^2 N$ can be either positive or negative. This makes the term of $U_{FC}^i(N, t_i)$, i.e., $P(H_1)(1 - P_D)c_c t_i$, non-convex for both AND and OR fusion rule (proof has been omitted due to brevity). Hence, we can say that $U_{FC}^i(N, t_i)$ is a non-convex function of N. As the summation of non-convex functions is non-convex, we can conclude that $\sum_{i=1}^{M} U_{FC}^i(N, t_i)$ is non-convex.

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