Question 1:

A. Convert the following numbers to their decimal representation. Show your work.

1. 100110112 = 155

1x20 + 1x21 + 0x22 + 1x23 + 1x24 + 0x25 + 0x26 +1x27

=1+ 2 + 8 +16 +128

=155

2. 4567 = 237

6x70 +5x71 + 4x72

=6 + 35 +196

=237

3. 38A16 = 906

10x160 +8x161 +3x162

=10 + 128 + 768

=906

4. 22145 = 309

4x50 +1x51 + 2x52 + 2x53

=4 + 5 +50 +250

=309

Question 2:

Solve the following, do all calculation in the given base. Show your work.

1.

75668 + 45158 = 143038

7566

+

14303

2. 101100112 + 11012 = 110000002

10110011

+

11000000

3. 7A6616 + 45C516 = C02B16

7A66

+ 

C02B

4. 30225 - 24335 = 345

3022

-

34

Question 3:

A. Convert the following numbers to their 8-bits two’s complement representation. Show

your work.

1.12410 = (01111100) 8 bit two’s complement

First convert 124 into binary 1111100, then the 8- bits completing bits = 01111100

2. -12410 = (10000100) 8 bit two’s complement

First find out the 124 8 bit binary

And then flip 0 and 1

01111100 (12410)

+ (-12410)

100000000

3. 10910 =(01101101) 8 bit two’s complement

First turn 109 into binary which is 1101101, the 8 bit completing bit is 01101101

4. -7910 = (10110001) 8 bit two’s complement

First convert 7910 to binary, which is 1001111, 8 bit completing bit is 01001111

01001111 (7910)

+ (-7910)

100000000

B. Convert the following numbers (represented as 8-bit two’s complement) to their

decimal representation. Show your work.

1.000111108 bit 2’s comp =(30)10

the sign is 0 so the binary is positive binary number

0x20+1x21+1x22+1x23+1x24

=30

2. 111001108 bit 2’s comp = (-26)10

the sign is 1 so the binary is negative binary number

First find out the 8 bit 2’s complement

11100110

+

100000000

Turn 00011010 into decimal 0x20+1x21+0x22+1x23+1x24 = 26

Then 111001108 bit 2’s comp is -26

3. 001011018 bit 2’s comp = (45)10

the sign is 0 so the binary is positive binary number

1x20+0x21+1x22+1x23+0x24+1x25=45

4. 100111108 bit 2’s comp =(-98)10

the sign is 1 so the binary is negative binary number, First find out the 8 bit 2’s complement

10011110

+

100000000

Turn 01100010 into decimal 0x20+1x21+0x22+0x23+0x24+1x25 +1x26 = 98

Then 100111108 bit 2’s comp is -98

Question 4:

Solve the following questions from the Discrete Math zyBook:

1.Exercise 1.2.4, sections b, c

Write a truth table for each expression.

(b) ¬(p ∨ q)

|  |  |  |
| --- | --- | --- |
| p | q | ¬(p ∨ q) |
| T | T | F |
| T | F | F |
| F | T | F |
| F | F | T |

(c) r ∨ (p ∧ ¬q)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| p | q | r | p ∧ ¬q | r ∨ (p ∧ ¬q) |
| T | T | T | F | T |
| T | T | F | F | F |
| T | F | T | T | T |
| T | F | F | T | T |
| F | T | T | F | T |
| F | T | F | F | F |
| F | F | T | F | T |
| F | F | F | F | F |

2. Exercise 1.3.4, sections b, d

Give a truth table for each expression.

(b). (p → q) → (q → p)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| p | q | p → q | q → p | (p → q) → (q → p) |
| T | T | T | T | T |
| T | F | F | T | T |
| F | T | T | F | F |
| F | F | T | T | T |

(d). (p ↔ q) ⊕ (p ↔ ¬q)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| p | q | ¬q | p ↔ q | p ↔ ¬q | (p ↔ q) ⊕ (p ↔ ¬q) |
| T | T | F | T | F | T |
| T | F | T | F | T | T |
| F | T | F | F | T | T |
| F | F | T | T | F | T |

Question 5:

Solve the following questions from the Discrete Math zyBook:

1.Exercise 1.2.7, sections b, c

Consider the following pieces of identification a person might have in order to apply for a credit card:

B: Applicant presents a birth certificate.

D: Applicant presents a driver's license.

M: Applicant presents a marriage license.

Write a logical expression for the requirements under the following conditions:

(b)The applicant must present at least two of the following forms of identification: birth certificate, driver's license, marriage license.

Answer: (B∧D) ∨ (B∧M) ∨ (D∧M) ∨( B∧D∧M)

(c)Applicant must present either a birth certificate or both a driver's license and a marriage license.

Answer : B ∨ (D ∧ M)

2. Exercise 1.3.7, sections b – e

Define the following propositions:

s: a person is a senior

y: a person is at least 17 years of age

p: a person is allowed to park in the school parking lot

Express each of the following English sentences with a logical expression:

(b)A person can park in the school parking lot if they are a senior or at least seventeen years of age.

Answer: (s ∨ y )→p

(c)Being 17 years of age is a necessary condition for being able to park in the school parking lot.

Answer: p→y

(d)A person can park in the school parking lot if and only if the person is a senior and at least 17 years of age.

Answer: p↔(s ∧ y )

(e)

Being able to park in the school parking lot implies that the person is either a senior or at least 17 years old.

Answer: p→ (s ∨ y )

3. Exercise 1.3.9, sections c, d

Use the definitions of the variables below to translate each English statement into an equivalent logical expression.

y: the applicant is at least eighteen years old

p: the applicant has parental permission

c: the applicant can enroll in the course

(c)The applicant can enroll in the course only if the applicant has parental permission.

Answer: c → p

(d)Having parental permission is a necessary condition for enrolling in the course.

Answer: c → p

Question 6:

Solve the following questions from the Discrete Math zyBook:

1.

Exercise 1.3.6, sections b – d

Give an English sentence in the form "If...then...." that is equivalent to each sentence.

(b)Maintaining a B average is necessary for Joe to be eligible for the honors program.

Answer:

B: Maintaining a B average

H: Joe is eligible for the honors program

H→B

(c)Rajiv can go on the roller coaster only if he is at least four feet tall.

Answer:

R : Rajiv can go on the roller coaster

F : Rajiv is at least four feet tall

R→F

(d)Rajiv can go on the roller coaster if he is at least four feet tall.

Answer:

R : Rajiv can go on the roller coaster

F : Rajiv is at least four feet tall

F→R

2. Exercise 1.3.10, sections c – f

The variable p is true, q is false, and the truth value for variable r is unknown. Indicate whether the truth value of each logical expression is true, false, or unknown.

(c). (p ∨ r) ↔ (q ∧ r)

(true ∨ unknown) ↔ (false ∧ unknown)

True ↔ false

False

(d). (p ∧ r) ↔ (q ∧ r)

(true ∧ unknown) ↔ (false ∧ unknown)

unknown↔false

Unknown

(e). p → (r ∨ q)

True →(unknown ∨ false)

unknown

(f). (p ∧ q) → r

(true ∧ false) → unknown

False → unknown

true

Question 7:Solve Exercise 1.4.5, sections b – d, from the Discrete Math zyBook:

j: Sally got the job.

l: Sally was late for her interview

r: Sally updated her resume.

Express each pair of sentences using logical expressions. Then prove whether the two expressions are logically equivalent.

(b)If Sally did not get the job, then she was late for her interview or did not update her resume.

¬j → (l ∨¬r)

If Sally updated her resume and was not late for her interview, then she got the job.

(r ∧ ¬l )→ j

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| j | l | r | ¬j | ¬l | l ∨¬r | r ∧ ¬l | ¬j → (l ∨¬r) | (r ∧ ¬l )→ j |
| T | T | T | F | F | T | F | T | T |
| T | T | F | F | F | T | F | T | T |
| T | F | T | F | T | F | T | T | T |
| T | F | F | F | T | T | F | T | T |
| F | T | T | T | F | T | F | T | T |
| F | T | F | T | F | T | F | T | T |
| F | F | T | T | T | F | T | F | F |
| F | F | F | T | T | T | F | T | T |

Answer: according to the truth table ¬j → (l ∨¬r) ≡ (r ∧ ¬l )→ j, so the two expressions are logically equivalent

(c)If Sally got the job then she was not late for her interview. j→ ¬l

If Sally did not get the job, then she was late for her interview. ¬j→ l

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| j | ¬j | l | ¬l | j→ ¬l | ¬j→ l |
| T | F | T | F | F | T |
| T | F | F | T | T | T |
| F | T | T | F | T | T |
| F | T | F | T | T | F |

Answer: according to the truth table j→ ¬l is not logically equivalent with ¬j→ l

(d)If Sally updated her resume or she was not late for her interview, then she got the job. (r ∨ ¬l )→ j

If Sally got the job, then she updated her resume and was not late for her interview. j→( r ∧ ¬l)

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| j | l | r | ¬l | r ∧ ¬l | r ∨ ¬l | (r ∨ ¬l )→ j | j→( r ∧ ¬l) |
| T | T | T | F | F | T | T | F |
| T | T | F | F | F | F | T | F |
| T | F | T | T | T | T | T | T |
| T | F | F | T | F | T | T | F |
| F | T | T | F | F | T | F | T |
| F | T | F | F | F | F | T | T |
| F | F | T | T | T | T | F | T |
| F | F | F | T | F | T | F | T |

Answer: according to the truth table the (r ∨ ¬l )→ j and j→( r ∧ ¬l) is not logically equivalent with j→( r ∧ ¬l)

Question 8:

Solve the following questions from the Discrete Math zyBook:

1.Exercise 1.5.2, sections c, f, i

Use the laws of propositional logic to prove the following:

(c) (p → q) ∧ (p → r) ≡ p → (q ∧ r)

(p → q) ∧ (p → r)

=( ¬p ∨ q) ∧ (¬p ∨ r).…. Conditional identities.

=¬p ∨ (q ∧ r)…… Distributive laws

= p → (q ∧ r) …. Conditional identities.

(f)

¬(p ∨ (¬p ∧ q)) ≡ ¬p ∧ ¬q

¬(p ∨ (¬p ∧ q))

=¬p ∧ ¬(¬p ∧ q) ….De Morgan's laws

=¬p ∧ (p ∨ ¬q) …. De Morgan's laws

=(¬p ∧ p) ∨ (¬p ∧ ¬q)…. Distributive laws

=F ∨ (¬p ∧ ¬q) …. Complement laws

=¬p ∧ ¬q… Identity laws

(i)

(p ∧ q) → r ≡ (p ∧ ¬r) → ¬q

(p ∧ q) → r

=¬ (p ∧ q) ∨ r

=¬p ∨ ¬q∨ r

(p ∧ ¬r) → ¬q

=¬ (p ∧ ¬r) ∨ ¬q

=¬p ∨ ¬¬r ∨ ¬q

=¬p ∨ r ∨ ¬q

=¬p ∨ ¬q ∨ r

So (p ∧ q) → r ≡ (p ∧ ¬r) → ¬q

2. Exercise 1.5.3, sections c, d

Use the laws of propositional logic to prove that each statement is a tautology.

(c)¬r ∨ (¬r → p)

=¬r ∨ (¬¬r ∨p)

=¬r ∨(r ∨ p)

=(¬r ∨r )∨ p

=T ∨ p

=T

(d)

¬(p → q) → ¬q

=¬(¬p ∨ q ) → ¬q

=(¬¬p ∧ ¬q ) → ¬q

=(p∧ ¬q ) → ¬q

=¬(p∧ ¬q ) ∨ ¬q

=¬p ∨ q ∨ ¬q

=¬p ∨ T

=T

Question 9:

Solve the following questions from the Discrete Math zyBook:

1.Exercise 1.6.3, sections c, d

Consider the following statements in English. Write a logical expression with the same meaning. The domain is the set of all real numbers.

(c)There is a number that is equal to its square.

Answer : ∃x(x = x2)

(d)Every number is less than or equal to its square plus 1.

Answer: ∀x(x <= x2+1)

2. Exercise 1.7.4, sections b – d

In the following question, the domain is a set of employees who work at a company. Ingrid is one of the employees at the company. Define the following predicates:

S(x): x was sick yesterday

W(x): x went to work yesterday

V(x): x was on vacation yesterday

Translate the following English statements into a logical expression with the same meaning.

(b)Everyone was well and went to work yesterday.

Answer: ∀x(¬S(x) ∧ W(x))

(c)Everyone who was sick yesterday did not go to work.

Answer: ∀x(S(x) →¬W(x))

(d)Yesterday someone was sick and went to work.

Answer: ∃x (S(x) ∧ W(x))

Question 10:

Solve the following questions from the Discrete Math zyBook:

1.Exercise 1.7.9, sections c – i

The domain for this question is the set {a, b, c, d, e}. The following table gives the value of predicates P, Q, and R for each element in the domain. For example, Q(c) = T because the truth value in the row labeled c and the column Q is T. Using these values, determine whether each quantified expression evaluates to true or false.

(c) ∃x((x = c) → P(x)) Answer: False

(d)∃x(Q(x) ∧ R(x)) Answer: True

(e)Q(a) ∧ P(d) Answer: True

(f)∀x ((x ≠ b) → Q(x)) Answer: True

(g)∀x (P(x) ∨ R(x)) Answer: False

(h)∀x (R(x) → P(x)) Answer: True

(i)∃x(Q(x) ∨ R(x)) Answer: True

2. Exercise 1.9.2, sections b – i

(b)∃x ∀y Q(x, y). Answer: True

(c)∃y ∀x P(x, y) Answer: True

(d)∃x ∃y S(x, y) Answer: False

(e)∀x ∃y Q(x, y) Answer: False

(f)∀x ∃y P(x, y) Answer: True

(g)∀x ∀y P(x, y) Answer: False

(h)∃x ∃y Q(x, y) Answer: True

(i)∀x ∀y ¬S(x, y) Answer: True

Question 11:

Solve the following questions from the Discrete Math zyBook:

1.Exercise 1.10.4, sections c – g

(c)There are two numbers whose sum is equal to their product.

Answer: ∃x ∃y( x+y= xy)

(d)The ratio of every two positive numbers is also positive.

Answer: ∀x ∀y( (x>0 ∧ y>0)→  > 0)

(e)The reciprocal of every positive number less than one is greater than one.

Answer: ∀x( (x>0 ∧ x<1)→(  > 1) )

(f)There is no smallest number.

Answer: ¬∃𝑥∀𝑦(x<=y)

(g)Every number other than 0 has a multiplicative inverse.

∀x∃y ( → (xy = 1))

2. Exercise 1.10.7, sections c – f

P(x, y): x knows y's phone number. (A person may or may not know their own phone number.)

D(x): x missed the deadline.

N(x): x is a new employee.

(c)There is at least one new employee who missed the deadline.

∃x(N(x) ∧ D(x))

(d)Sam knows the phone number of everyone who missed the deadline.

∀x (D(x) ∧ P(sam, y))

(e)There is a new employee who knows everyone's phone number.

∃x∀y(N(x) ∧ P(x, y))

(f)Exactly one new employee missed the deadline.

∃x∀y ((N(x) ∧ )→ D(x))

3. Exercise 1.10.10, sections c – f

(c)Every student has taken at least one class other than Math 101.

∀x ∃ y(() ∧T(x,y))

(d)There is a student who has taken every math class other than Math 101.

∃x ∀y ( →T(x,y))

(e)Everyone other than Sam has taken at least two different math classes.

∀x ∃y∃z (  → (( ) ∧ T(x,y) ∧ T(x,z)))

(f)Sam has taken exactly two math classes.

∃y∃z ∀ w(  ∧ T(sam,y) ∧ T(sam,z) ∧ (()→¬T(sam,w)))

Question 12:

Solve the following questions from the Discrete Math zyBook:

1.Exercise 1.8.2, sections b – e

P(x): x was given the placebo

D(x): x was given the medication

M(x): x had migraines

(b)Every patient was given the medication or the placebo or both.

∀x (D(x) ∨ P(x))

Negation: ¬∀x (D(x) ∨ P(x))

Applying De Morgan's law: ∃x (¬D(x) ∧ ¬P(x))

English: Some patient was either not given the medication or not given the placebo.

(c)There is a patient who took the medication and had migraines.

∃x (D(x) ∧ M(x))

Negation: ¬∃x (D(x) ∧ M(x))

Applying De Morgan's law: ∀x(¬D(x) ∨¬ M(x))

English: Every patient was not given the Medicine or not had migraines( or both not)

(d)Every patient who took the placebo had migraines. (Hint: you will need to apply the conditional identity, p → q ≡ ¬p ∨ q.)

∀x(P(x) → M(x))

Negation: ¬∀x(P(x) → M(x))

Applying De Morgan's law: ∃x(P(x) ∧ ¬ M(x))

English: some patient was given the placebo and not given the Medicine.

(e)There is a patient who had migraines and was given the placebo.

∃x(M(x) ∧ P(x))

Negation: ¬∃x(M(x) ∧ P(x))

Applying De Morgan's law: ∀x(¬M(x) ∨ ¬P(x))

English: Every patient was not give the Medicine or was not give the placebo or both not

2. Exercise 1.9.4, sections c – e

Write the negation of each of the following logical expressions so that all negations immediately precede predicates. In some cases, it may be necessary to apply one or more laws of propositional logic.

(c)∃x ∀y (P(x, y) → Q(x, y))

¬∃x ∀y (P(x, y) → Q(x, y))

=∀x∃y (¬(P(x, y) → Q(x, y)))

=∀x∃y (¬(¬P(x, y) ∨ Q(x, y)))

=∀x∃y (P(x, y) ∧ ¬ Q(x, y))

|  |  |
| --- | --- |
| (d)  ∃x ∀y (P(x, y) ↔ P(y, x))  ¬∃x ∀y (P(x, y) ↔ P(y, x))  =∀x∃y(¬(P(x, y) ↔ P(y, x))  =∀x∃y(¬((P(x, y) → P(y, x)) ∧ (P(y, x) → P(x, y)))  =∀x∃y(¬ (¬P(x, y) ∨ P(y, x) ∧ (¬P(y, x) ∨ P(x, y)))  =∀x∃y((P(x, y) ∧ ¬ P(y, x)) ∨ (P(y, x) ∧ ¬ P(x, y))) | (e)∃x ∃y P(x, y) ∧ ∀x ∀y Q(x, y)  ¬(∃x ∃y P(x, y) ∧ ∀x ∀y Q(x, y))  = ¬∃x ∃y P(x, y) ∨ ¬∀x ∀y Q(x, y)  =∀x∀y¬ P(x, y) ∨ ∃x ∃y¬ Q(x, y) |

Q13 extra credit for typing

Q14 Penalty for not tagging