**Question 5:**

a) Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.12.2, sections b, e

(b)

p → (q ∧ r)  
¬q

\_\_\_\_\_\_\_\_\_  
∴ ¬p

|  |  |  |
| --- | --- | --- |
| 1. | ¬q | Hypothesis |
| 2. | ¬q v ¬r | Addition |
| 3. | ¬(q ∧ r) | De Morgan’s laws,2 |
| 4. | p → (q ∧ r) | Hypothesis |
| 5. | ¬p | Modus tollens, 3, 4 |

(e)

p ∨ q

¬p ∨ r

¬q

\_\_\_\_\_\_

∴ r

|  |  |  |
| --- | --- | --- |
| 1. | p ∨ q | Hypothesis |
| 2. | ¬p ∨ r | Hypothesis |
| 3. | q v r | Resolution, 1,2 |
| 4. | ¬q | Hypothesis |
| 5. | r | Disjunctive syllogism, 3, 4 |

2. Exercise 1.12.3, section c

(c)

One of the rules of inference is Disjunctive syllogism :

p ∨ q

¬p

\_\_\_\_\_

∴ q

Prove that Disjunctive syllogism is valid using the laws of propositional logic and any of the other rules of inference besides Disjunctive syllogism. (Hint: you will need one of the conditional identities from the laws of propositional logic).

|  |  |  |
| --- | --- | --- |
| 1. | p ∨ q | Hypothesis |
| 2. | ¬¬p ∨ q | Double negation,1 |
| 3. | ¬p → q | Conditional identity,2 |
| 4. | ¬p | Hypothesis |
| 5. | q | Modus ponens, 3, 4 |

3. Exercise 1.12.5, sections c, d

(c)

I will buy a new car and a new house only if I get a job.

I am not going to get a job.

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

∴ I will not buy a new car.

Answer:

c : I will buy a new car

h: I will buy a new house

j : I get a job

(c ∧ h) → j

¬j

\_\_\_\_\_\_\_\_

∴ ¬c

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| c | h | j | c ∧ h | (c ∧ h) → j | ¬j | ((c ∧ h) → j) ∧¬j | ¬c | ((c ∧ h) → j) ∧¬j) → ¬c |
| T | T | T | T | T | F | F | F | T |
| T | T | F | T | F | T | F | F | T |
| T | F | T | F | T | F | F | F | T |
| T | F | F | F | T | T | T | F | F |
| F | T | T | F | T | F | F | T | T |
| F | T | F | F | T | T | T | T | T |
| F | F | T | F | T | F | F | T | T |
| F | F | F | F | T | T | T | T | T |

Argument is not a tautology, hence the argument is invalid.

(d)

I will buy a new car and a new house only if I get a job.

I am not going to get a job.

I will buy a new house.

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

∴ I will not buy a new car.

Answer:

c : I will buy a new car

h: I will buy a new house

j : I get a job

(c ∧ h) → j

¬j

h

\_\_\_\_\_\_\_\_\_

∴ ¬c

|  |  |  |
| --- | --- | --- |
| 1. | (c ∧ h) → j | Hypothesis |
| 2. | ¬j | Hypothesis |
| 3. | ¬( c ∧ h) | Modus tollens 1,2 |
| 4. | ¬c ∨ ¬h | De Morgan’s laws, 3 |
| 5. | ¬h ∨¬c | Commutative laws,4 |
| 6. | h | Hypothesis |
| 7. | ¬¬h | Double negation law,5 |
| 8. | ¬c | Disjunctive, 5, 7 |

∴ so the argument is valid.

b) Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.13.3, section b

Show that the given argument is invalid by giving values for the predicates P and Q over the domain {a, b}.

(b)

∃x (P(x) ∨ Q(x))

∃x ¬Q(x)

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

∴ ∃x P(x)

|  |  |  |
| --- | --- | --- |
|  | P | Q |
| a | F | T |
| b | F | F |

The Hypothesis ∃x (P(x) ∨ Q(x)) is true, so P(a) ∨ Q(b) is true

The Hypothesis ∃x ¬Q(x) is true, since ¬Q(b) is true.

However, since P(a) = P(b) = F, ∃x P(x) is false.

Both hypotheses are true and the conclusion is false.

∴the argument is invalid

2. Exercise 1.13.5, sections d, e

(d)

Every student who missed class got a detention.

Penelope is a student in the class.

Penelope did not miss class.

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Penelope did not get a detention.

Answer:

M(x): x missed the class.

D(x): x who got a detention.

∀x( M(x) →D(x))

Penelope, a student in the class.

¬M(Penelope)

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

∴¬D(Penelope)

When hypotheses 3 ¬M(Penelope) is true, then M(Penelope) is false,

thus D(Penelope) can be true or false to make the hypotheses 1 true,

when D(Penelope) is true then ¬D(Penelope) is false, which means the conclusion is false while the three hypothesis are all true, so the argument is invalid.

(e)

Every student who missed class or got a detention did not get an A.

Penelope is a student in the class.

Penelope got an A.

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Penelope did not get a detention.

Answer:

M(x): x missed class

D(x): x got a detention

A(x): x got an A

∀x((M(x) ∨ D(x)) → ¬ A(x))

Penelope, a student in the class

A(Penelope)

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

¬D(Penelope)

|  |  |  |
| --- | --- | --- |
| 1. | ∀x((M(x) ∨ D(x)) → ¬ A(x)) | Hypothesis |
| 2. | Penelope, a student in the class | Hypothesis |
| 3 | (M(Penelope) ∨ D(Penelope)) → ¬ A(Penelope) | Universal Instantiation |
| 4. | A(Penelope) | Hypothesis |
| 5. | ¬ (M(Penelope) ∨ D(Penelope)) | Modus tollens,3,4 |
| 6. | ¬M(Penelope) ∧¬ D(Penelope)) | De Morgan’s laws, 5 |
| 7. | ¬D(Penelope) | Simplification,6 |

∴ the hypothesis are all true and the conclusion is also true, so the argument is valid

Question 6:

Solve Exercise 2.4.1, section d; Each statement below involves odd and even integers. An odd integer is an integer that can be expressed as 2k+1 , where k is an integer. An even integer is an integer that can be expressed as 2k, where k is an integer.

(d)The product of two odd integers is an odd integer.

Let x and y be odd integers, we shall prove that xy is odd.

Since x is odd, there is an integer k such that x= 2k+1

Since y is odd, there is an integer k such that y = 2k+1

xy = (2k+1)2 = 2(2k2 + 2k) +1

since k is an integer, 2k2 + 2k is also an integer

xy is equal to 2m+1, where m = k2 + k is an integer, so xy is odd

Exercise 2.4.3, section b, from the Discrete Math zyBook:

Prove each of the following statements using a direct proof.

(b)If x is a real number and x≤3, then 12−7x+x2≥0.

Assume x is a real number and x≤3, we shall prove that 12−7x+x2≥0

since x≤3, minus 3 to both side of the expression: x-3≤0

since x≤3, minus 4 to both side of the expression: x-4≤-1

Plug in x-3≤0 and x-4≤-1 into the expression (x -3)(x -4)

Double negative is positive, so (x -3)(x -4) ≥0

Thus (x -3)(x -4) = 12−7x+x2 ≥0

Question 7:

Solve Exercise 2.5.1, section d; Prove each statement by contrapositive

(d)For every integer n, if n2−2n+7 is even, then n is odd.

Prove this statement by contrapositive

Assume n is even, we shall prove that n2−2n+7 is odd

Since n is even, therefore n = 2k for some integer k,

Plug the expression for n into n2−2n+7 gives

n2−2n+7 = (2k)2 – 2(2k) +7 =4k2 -4k +7 = 2(2k2 -2k+3)+1

since k is integer, 2k2 -2k+3 is also integer

2(2k2 -2k+3)+1= 2m +1 for some integer m

Therefore 2m +1 is odd, it means n2−2n+7 is odd.

Exercise 2.5.4, sections a, b; Prove each statement by contrapositive

(a)For every pair of real numbers x and y, if x3+xy2≤x2y+y3, then x ≤ y.

Prove this statement by contrapositive

Assume that x >y, we shall prove that x3+xy2>x2y+y3

Since x >y , therefore x – y >0

Since

x3+xy2 - x2y - y3

=x(x2 + y2) – y(x2 + y2)

= (x – y)( x2 + y2)

x and y is a pair of real numbers and x > y , so x2 + y2 > 0

Since x – y >0 and x2 + y2 > 0, (x – y)( x2 + y2) > 0

therefore x3+xy2 - x2y - y3 >0, which means x3+xy2>x2y+y3

(b)For every pair of real numbers x and y, if x+y>20, then x>10 or y>10.

Prove this statement by contrapositive

Assume that x ≤ 10 and y ≤ 10 we shall prove that x+y ≤ 20

Since x ≤ 10 and y ≤ 10, both side minus 10

we will get x – 10 ≤ 0, y – 10 ≤ 0

(x – 10) + (y – 10) ≤ 0

x + y -20 ≤ 0

therefore, x + y ≤ 20

Exercise 2.5.5, section c, proving statements using a direct proof or by contrapositive.

(c) For every non-zero real number x, if x is irrational, then  is also irrational.

I will prove statement by using contrapositive

Assume that   is not irrational, we shall prove that x is also not irrational

Since x is non-zero real number, and  is not irrational, so  is rational

Let  = a/b, for some two integers a and b , since x  0 , so a  0 , b 0

x = b/a, which means that x is equal to the ratio of two integers with a non-zero denominator, so x is rational.

therefore, x is not irrational.

**Question 8:**

Solve Exercise 2.6.6, sections c, d, Proofs by contradiction.

(c)The average of three real numbers is greater than or equal to at least one of the numbers.

Prove the statement by contradiction.

Assume that the average of three real numbers is less than each of the three numbers.

Let a be the average of three real numbers x, y, z, then a =  , x + y + z = 3a,

We assume that a is less than x , y , z , so we know that a < x , a < y, a< z

Therefore, x+ y + z > 3a, so  > a , which is a > a

which makes the assumption incorrect, so the average of three real numbers is not less than all of the three numbers.

Therefore, the statement that the average of three real numbers is greater than or equal to at least one of the numbers is valid

(d)There is no smallest integer.

Prove the statement by contradiction.

Assume that there is a smallest integer named a

Thus a is an integer, a – 1 is also an integer

Since a-1< a , which means a – 1 always smaller than a,

Therefore it is contradiction to the assumption that a is the smallest number

So the statement that there is no a smallest integer is valid.

**Question 9:** Solve Exercise 2.7.2, section b, Proofs by cases - even/odd integers and divisibility.

(b)If integers x and y have the same parity, then x+y is even.

The parity of a number tells whether the number is odd or even. If x and y have the same parity, they are either both even or both odd.

Case 1 x and y are both even. Since x is even , x = 2k for some integer k. since y is even, y = 2j, for some integer j. x+ y = 2k + 2j = 2(k+j), since k and j are both integer, so 2(k+j) must be even number. Therefore, x + y is even

Case 2 x and y are both odd. Since x is odd, x = 2k+1 for some integer k. since y is odd, y = 2j+1 for some integer j. thus, x+y = 2k+1 + 2j+1 = 2(k+j+1), since k, j are some integers so k+j+1 is also some integer, 2(k+j+1) is even. Therefore, x + y is even.