Question 7: Solve the following questions from the Discrete Math zyBook:

a)Exercise 3.1.1, sections a-g

(a)27 ∈ A

Answer: True. Because 21 = 9\*3 , which is an integer multiple of 3.

(b)27 ∈ B

Answer: False. Because there is no integer y such that 27 = y2

(c)100 ∈ B.

Answer: True. Because 100 = 102 is a perfect square.

(d) since E ⊆ C is false, C ⊆ E is false, E ⊆ C or C ⊆ E.

Answer: False. Because E  C since 3 ∈ E but 3  C. C E since 5 ∈ C but 5  E.

(e)E ⊆ A.

Answer: True. Because each element of A is an integer multiple of 3.

(f)A ⊂ E

Answer: False. Because such as 18 ∈ A but 18  E.

(g)E ∈ A False

Answer: False. Because E is a set and A is a set of integer elements

b) Exercise 3.1.2, sections a-e

(a)15 ⊂ A

Answer: False. Because 15 is not a set.

(b){15} ⊂ A

Answer: True. Because 15 is an integer multiple of 3. Also, 3 ∈ A but 3  {15}.

(c)∅ ⊂ A True

Answer: True. Because the statement is vacuously true.

(d)A ⊆ A

Answer: True.

(e)∅ ∈ B

Answer: False. Because the empty set is a set while the elements of B are integers.

c) Exercise 3.1.5, sections b, d

(b)

Let A = { 3, 6, 9, 12, .... }

A = {x ∈ Z : x is an integer multiply of 3 and x  3 }

A is an infinite set.

(d)

Let B = { 0, 10, 20, 30, ...., 1000 }

B ={ x ∈Z and x  1000 : x is an integer multiply of 10 and 0  x  1000}

The set is finite

|B|= 101

d) Exercise 3.2.1, sections a-k

(a)2 ∈ X True

(b){2} ⊆ X True

(c){2} ∈ X False

(d)3 ∈ X False

(e){1, 2} ∈ X. True

(f){1, 2} ⊆ X. True. Because both 1 and 2 are elements of X.

(g){2, 4} ⊆ X. True. Because both 2 and 4 are elements of X.

(h){2, 4} ∈ X False

(i){2, 3} ⊆ X False. Because 3  X.

(j){2, 3} ∈ X False

(k)|X| = 7. False. |X | = 6.

Question 8:

Solve Exercise 3.2.4

(b)

Let A = {1, 2, 3}. What is {X ∈ P(A): 2 ∈ X}?

Answer:

Since A = {1, 2, 3},

P(A) = {∅, {1},{2},{3},{1,2},{1,3},{2,3},{1,2,3}}

Let B = { X ∈ P(A): 2 ∈ X }

B = {{2}, {1,2}, {2,3}, {1,2,3}}

Question 9:

1. Exercise 3.3.1, sections c-e

(c)A ∩ C

Answer: {-3, 1, 17}

(d)A ∪ (B ∩ C) = A ∪ ({-5, 1})

Answer: {-5, -3, 0, 1, 4, 17}

(e)A ∩ B ∩ C = {1, 4} ∩ C

Answer :{1}

1. Exercise 3.3.3, sections a, b, e, f

(a)

* 5i = 2Ai = A2 ∩ A3 ∩ A4 ∩A5

A2 = {1, 2, 4}

A3 = {1, 3, 9}

A4 = {1, 4, 16}

A5 = {1, 5, 25}

  5i = 2Ai  = A2 ∩ A3 ∩ A4 ∩A5 = { 1 }

(b)

* 5i = 2Ai = A2 ∪ A3 ∪ A4 ∪ A5

A2 = {1, 2, 4}

A3 = {1, 3, 9}

A4 = {1, 4, 16}

A5 = {1, 5, 25}

  5i = 2Ai  = A2 ∪ A3 ∪ A4 ∪ A5 = {1,2, 3,4,5,9,16,25}

(e)

* 100i = 1Ci = C1 ∩ C2 ∩ C3… ∩ C100

C1 = {X ∈ R: -  x   } = { X ∈ R: -1  x  1 }

C2 = { X ∈ R: -  x   }

C3 = { X ∈ R: -   x   }

.

.

.

C100 = {x∈R: -  x  }

* 100i = 1Ci = C1 ∩ C2 ∩ C3… ∩ C100 = {x∈R: -  x  }

Because the x values between -  and  is part of all the C in C1 ∩ C2 ∩ C3… ∩ C100

(f)

100i = 1C i= C1∪ C2 ∪ C3… ∪ C100

C1 = {X ∈ R: -  x   } = { X ∈ R: -1  x  1 }

C2 = { X ∈ R: -  x   }

C3 = { X ∈ R: -   x   }

.

.

.

C100 = {x∈R: -  x  }

* 100i = 1Ci = C1∪ C2 ∪ C3… ∪ C100 = {x∈R: -1  x  1}

Because the x values between -1 and 1 covers all the values in C1∪ C2 ∪ C3… ∪ C100

1. Exercise 3.3.4, sections b, d : use the set definitions A = {a, b} and B = {b, c} to express each set below. Use roster notation in your solutions.

(b)P(A ∪ B)

A ∪ B ={a, b, c}

 P(A ∪ B) = {∅, { a }, { b }, { c }, { a, b}, { a, c }, { b, c }, { a, b, c }}

(d)P(A) ∪ P(B)

P(A) = {∅, { a }, { b }, { a, b}}

P(B) = {∅, { b }, { c }, { b, c}}

 P(A) ∪ P(B) = {∅, { a }, { b }, { c }, { a, b},{ b, c}}

Question 10:

a)Exercise 3.5.1, sections b, c

(b)Write an element from the set B × A × C.

Answer: one example is (foam, tall, non-fat)

(c)Write the set B × C using roster notation.

Answer:

B × C = {(b , c) | b ∈ B ∧ c ∈ C}

B × C ={(foam, non-fat), (foam, whole), (non-foam, non-fat), (non-foam, whole)}

b) Exercise 3.5.3, sections b, c, e

(b)Z2 ⊆ R2

Answer: True

If (x, y) ∈ Z2, then x and y are both elements of Z. Since Z ⊆ R, then x

and y are also elements of R. Therefore (x, y) ∈ R2.

(c)Z2 ∩ Z3 = ∅

Answer: True

The elements in Z2 are pairs. The elements in Z3 are triples. Therefore the

two sets have no elements in common.

(e)For any three sets, A, B, and C, if A ⊆ B, then A × C ⊆ B × C.

Answer: True

Prove A ⊆ B → A × C ⊆ B × C

A × C = {(a , c): a ∈ A ∧c ∈C }

B × C = {(b , c): b ∈ B ∧c ∈C }

 A ⊆ B  a ∈ A and b ∈ B

A × C ⊆ B × C is True



c)Exercise 3.5.6, sections d, e

(d){xy: where x ∈ {0} ∪ {0}2 and y ∈ {1} ∪ {1}2}

x ∈ {0} ∪ {0}2

{0}2 = {0} x {0} = {00}

{0} ∪ {0}2 = {0, 00}

x ∈ {0, 00}

y ∈ {1} ∪ {1}2

{1}2 = {1} x {1} = {11}

{1} ∪ {1}2 = {1, 11}

y ∈ {1, 11}

let A = {xy: where x ∈ {0} ∪ {0}2 and y ∈ {1} ∪ {1}2}

= {01, 011, 001, 0011}

The answer is {01, 011, 001, 0011}.

(e){ xy: x ∈ {aa, ab} and y ∈ {a} ∪ {a}2}

y ∈ {a} ∪ {a}2

{a}2 = {a} x {a} = {aa}

{a} ∪ {a}2  = {a, aa}

y ∈{a, aa}

let A = { xy: x ∈ {aa, ab} and y ∈ {a} ∪ {a}2}

= {aaa, aaaa, aba, abaa}

The answer is {aaa, aaaa, aba, abaa}

1. Exercise 3.5.7, sections c, f, g

(c)(A × B) ∪ (A × C)

A × B = { ab, ac}

A × C ={aa, ab, ad}

(A × B) ∪ (A × C) = {aa, ab, ac, ad}

Answer is {aa, ab, ac, ad}

(f)P(A × B)

(A × B) = { ab, ac}

P(A × B) = { ∅, { ab}, { ac }, { ab, ac}}

Answer: { ∅, { ab}, { ac }, { ab, ac}}

(g)P(A) × P(B). Use ordered pair notation for elements of the Cartesian product.

P(A) = {∅, {a}}

P(B)={ ∅, {b}, {c}, {b, c}}

P(A) × P(B) = {(∅, ∅), (∅, {b}), (∅, {c}), (∅, {b, c}), ({a}, ∅), ({a}, {b}), ({a}, {c}), ({a}, {b, c})}

Answer is {(∅, ∅), (∅, {b}), (∅, {c}), (∅, {b, c}), ({a}, ∅), ({a}, {b}), ({a}, {c}), ({a}, {b, c})}

Question 11: a)Exercise 3.6.2, sections b, c

(b)(B ∪ A) ∩ ( ∪ A) = A

|  |  |  |
| --- | --- | --- |
| 1. | (B ∪ A) ∩ ( ∪ A) | Hypothesis |
| 2. | (A ∪ B) ∩ (A ∪ ) | Commutative laws, 1,2 |
| 3. | A ∪(B ∩) | Distributive laws, 2,3 |
| 4. | A ∪∅ | Complement laws, 3,4 |
| 5. | A | Identity laws, 4,5 |

(c)  = 

|  |  |  |
| --- | --- | --- |
| 1. |  | Hypothesis |
| 2. |  | De Morgan's laws, 1,2 |
| 3. | U B | Complement laws, 2,3 |

b) Exercise 3.6.3, sections b, d

(b)A - (B ∩ A) = A

Answer:

Let A = {1, 2}

Let B = {1}

then B ∩ A = {1} and A - (B ∩ A) = {2},

which means that A - (B ∩ A) ≠ A.

so the set equation is not identical

(d)(B - A) ∪ A = A

Answer:

Let A = {1}

Let B = {1, 2}

then B - A = {2} and (B - A) ∪ A = {1, 2}

which means that (B - A) ∪ A ≠ A

so the set equation is not identical.

c) Exercise 3.6.4, sections b, c

(b)A ∩ (B - A) = ∅

|  |  |  |
| --- | --- | --- |
| 1 | A ∩ (B - A) | Hypothesis |
| 2. | A ∩ (B ∩) | Set Subtraction law, 1 |
| 3. | A ∩ (A ∩ B) | Commutative law, 2 |
| 4. | (A ∩)∩B | Associative laws, 3 |
| 5. | ∅∩B | Complement laws, 4 |
| 6. | ∅ | Domination laws, 5 |

(c)A ∪ (B - A) = A ∪ B

|  |  |  |
| --- | --- | --- |
| 1. | A ∪ (B - A) | Hypothesis |
| 2. | A ∪(B ∩) | Set Subtraction law, 1 |
| 3. | (A ∪B ) ∩ (A ∪) | Distributive laws, 2 |
| 4. | (A ∪B ) ∩U | Complement laws, 3 |
| 5. | A ∪B | Identity laws, 4 |