Homework 1

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Question 1:

A.1) 10011011_2

$$
\begin{aligned}
&= 1 \cdot 2^0 + 1 \cdot 2^1 + 0 \cdot 2^2 + 1 \cdot 2^3 + 1 \cdot 2^4 + 0 \cdot 2^5 + 0 \cdot 2^6 + 1 \cdot 2^7 \\
&= 1 + 2 + 0 + 8 + 16 + 0 + 0 + 128 \\
&= 155_{10}
\end{aligned}
$$

A.2) 456_7

$$
\begin{aligned}
&= 6 \cdot 7^0 + 5 \cdot 7^1 + 4 \cdot 7^2 \\
&= 6 + 35 + 196 \\
&= 237_{10}
\end{aligned}
$$

A.3) $38A_{16}$

$$
\begin{aligned}
&= 10 \cdot 16^0 + 8 \cdot 16^1 + 3 \cdot 16^2 \\
&= 10 + 128 + 768 \\
&= 906_{10}
\end{aligned}
$$

A.4) 2214_5

$$
\begin{aligned}
&= 4 \cdot 5^0 + 1 \cdot 5^1 + 2 \cdot 5^2 + 2 \cdot 5^3 \\
&= 4 + 5 + 50 + 250 \\
&= 309_{10}
\end{aligned}
$$

B.1) 69_{10}

$$\begin{array}{cccccccc} \underline{0} & \underline{1} & \underline{0} & \underline{0} & \underline{0} & \underline{1} & \underline{0} & \underline{1} \\ 2^7 & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \\ 128 & 64 & 32 & 16 & 8 & 4 & 2 & 1 \end{array}$$

$$\begin{aligned} &= 64 + 4 + 1 \\ &= 69 \\ \therefore 69_{10} &= 1000101_2 \end{aligned}$$

B.2) 485_{10}

$$\begin{array}{cccccccccc} \underline{0} & \underline{1} & \underline{1} & \underline{1} & \underline{1} & \underline{0} & \underline{0} & \underline{1} & \underline{0} & \underline{1} \\ 2^9 & 2^8 & 2^7 & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \\ 512 & 256 & 128 & 64 & 32 & 16 & 8 & 4 & 2 & 1 \end{array}$$

$$\begin{aligned} &= 256 + 128 + 64 + 32 + 4 + 1 \\ &= 485 \\ \therefore 485_{10} &= 111100101_2 \end{aligned}$$

B.3) $6D1A_{16}$

$$\begin{aligned} &= 10 \cdot 16^0 + 1 \cdot 16^1 + 13 \cdot 16^2 + 6 \cdot 6^3 \\ &= 10 + 16 + 3328 + 24576 \\ &= 27930_{10} \end{aligned}$$

$$\begin{array}{cccccccccccccccc} \underline{0} & \underline{1} & \underline{1} & \underline{0} & \underline{1} & \underline{1} & \underline{0} & \underline{1} & \underline{0} & \underline{0} & \underline{0} & \underline{1} & \underline{1} & \underline{0} & \underline{1} & \underline{0} \\ 2^{15} & 2^{14} & 2^{13} & 2^{12} & 2^{11} & 2^{10} & 2^9 & 2^8 & 2^7 & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \\ 32768 & 16384 & 8192 & 4096 & 2048 & 1024 & 512 & 256 & 128 & 64 & 32 & 16 & 8 & 4 & 2 & 1 \end{array}$$

$$\begin{aligned} &= 16384 + 8192 + 2048 + 1024 + 256 + 16 + 8 + 2 \\ &= 27930_{10} \\ \therefore 6D1A_{16} &= 110110100011010_2 \end{aligned}$$

C.1) 1101011_2

HEXADECIMAL	BINARY
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

$$1101011_2 = \underline{0110} \;\; \underline{1011}$$

From the Table above,

$$0110 = 6$$

$$1011 = B$$

$$\therefore 1101011_2 = 6B_{16}$$

C.2) 895_{10}

$$\begin{array}{cccc} \underline{0} & \underline{3} & \underline{7} & \underline{15} \\ 16^3 & 16^2 & 16^1 & 16^0 \\ 4096 & 256 & 16 & 1 \end{array}$$

$$= 15 \cdot 16^0 + 7 \cdot 16^1 + 3 \cdot 16^2$$

$$= 15 + 112 + 768$$

$$= 895$$

$$\therefore 895_{10} = 37F_{16} \quad (15 = F)$$

Question 2:

1) $7566_8 + 4515_8$

$$\begin{array}{r} {\scriptstyle 1\ 11\ \ } \\ 7\,566_8 \\ +\,4\,515_8 \\ \hline 14\,303_8 \\ \hline \end{array}$$

$$\therefore 7566_8 + 4515_8 = 14303_8$$

---

2) $10110011_2 + 1101_2$

$$\begin{array}{r} {\scriptstyle 111\ 111\ \ \ } \\ 1011\,0011_2 \\ +\,0000\,1101_2 \\ \hline 1100\,0000_2 \\ \hline \end{array}$$

$$\therefore 10110011_2 + 1101_2 = 11000000_2$$

---

3) $7A66_{16} + 45C5_{16}$

$$\begin{array}{rrrrr} & \overset{1}{7} & \overset{1}{10} & 06 & 06_{16} \\ + & 4 & 05 & 12 & 05_{16} \\ \hline & 12 & 0 & 2 & 11_{16} \\ \hline \end{array}$$

$$\therefore 7A66_{16} + 45C5_{16} = C02B_{16} \quad (12 = C, 11 = B)$$

---

4) $3022_5 - 2433_5$

$$\begin{array}{rrrrr} & {}^{2}\cancel{3} & {}^{4}\cancel{0} & {}^{11}\cancel{2} & {}^{12}\cancel{2}_5 \\ - & 2 & 4 & 3 & 3_5 \\ \hline & 0 & 0 & 3 & 4_5 \\ \hline \end{array}$$

$$\therefore 3022_5 - 2433_5 = 34_5$$

Question 3:

A.1) 124_{10}

$\underline{0}$	$\underline{1}$	$\underline{1}$	$\underline{1}$	$\underline{1}$	$\underline{1}$	$\underline{0}$	$\underline{0}$
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
128	64	32	16	8	4	2	1

$$\therefore 124_{10} = 01111100_{8-bit\ 2's\ comp}$$

A.2) -124_{10}

$$\begin{array}{r} {\scriptstyle 1\,1\,1\,1\;1} \\ 0111\,1100 \\ +\,1000\,0100 \\ \hline 10000\,0000 \\ \hline \end{array}$$

$$\therefore -124_{10} = 10000100_{8-bit\ 2's\ comp}$$

A.3) 109_{10}

$\underline{0}$	$\underline{1}$	$\underline{1}$	$\underline{0}$	$\underline{1}$	$\underline{1}$	$\underline{0}$	$\underline{1}$
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
128	64	32	16	8	4	2	1

$$\therefore 109_{10} = 01101101_{8-bit\ 2's\ comp}$$

A.4) -79_{10}

$$79_{10} = 01001111_{8-bit\ 2's\ comp}$$

$$\begin{array}{r} {\scriptstyle 1\,1\,1\,1\;1\,1\,1} \\ 0100\,1111 \\ +\,1011\,0001 \\ \hline 10000\,0000 \\ \hline \end{array}$$

$$\therefore -79_{10} = 10110001_{8-bit\ 2's\ comp}$$

B.1) $00011110_{\text{8-bit 2's comp}}$

$$
\begin{aligned}
&= 0 \cdot 2^0 + 1 \cdot 2^1 + 1 \cdot 2^2 + 1 \cdot 2^3 + 1 \cdot 2^4 \\
&= 2 + 4 + 8 + 16 \\
&= 30_{10}
\end{aligned}
$$

B.2) $11100110_{\text{8-bit 2's comp}}$

$$
\begin{array}{r}
{\scriptstyle 1\,1\,1\,1\;1\,1\;\;\;} \\
1110\,0110 \\
+\,0001\,1010 \\
\hline
10000\,0000 \\
\hline
\end{array}
$$

$$
\begin{aligned}
00011010_{\text{8-bit 2's comp}} &= 0 \cdot 2^0 + 1 \cdot 2^1 + 0 \cdot 2^2 + 1 \cdot 2^3 + 1 \cdot 2^4 \\
&= 0 + 2 + 0 + 8 + 16 \\
&= 26_{10}
\end{aligned}
$$

$$
\therefore 11100110_{\text{8-bit 2's comp}} = -26_{10}
$$

B.3) $00101101_{\text{8-bit 2's comp}}$

$$
\begin{aligned}
&= 1 \cdot 2^0 + 0 \cdot 2^1 + 1 \cdot 2^2 + 1 \cdot 2^3 + 0 \cdot 2^4 + 1 \cdot 2^5 \\
&= 1 + 0 + 4 + 8 + 0 + 32 \\
&= 45_{10}
\end{aligned}
$$

B.4) $10011110_{\text{8-bit 2's comp}}$

$$
\begin{array}{r}
{\scriptstyle 1\,1\,1\,1\;1\,1\;\;\;} \\
1001\,1110 \\
+\,0110\,0010 \\
\hline
10000\,0000 \\
\hline
\end{array}
$$

$$
\begin{aligned}
01100010_{\text{8-bit 2's comp}} &= 0 \cdot 2^0 + 1 \cdot 2^1 + 0 \cdot 2^2 + 0 \cdot 2^3 + 0 \cdot 2^4 + 1 \cdot 2^5 + 1 \cdot 2^6 \\
&= 0 + 2 + 0 + 0 + 0 + 32 + 64 \\
&= 98_{10}
\end{aligned}
$$

$$
\therefore 10011110_{\text{8-bit 2's comp}} = -98_{10}
$$

Question 4:

1. Exercise 1.2.4

(b) $\neg(p \vee q)$

p	q	$\neg(p \vee q)$
T	T	F
T	F	F
F	T	F
F	F	T

(c) $r \vee (p \wedge \neg q)$

p	q	r	$(p \wedge \neg q)$	$r \vee (p \wedge \neg q)$
T	T	T	F	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	F	T
F	T	F	F	F
F	F	T	F	T
F	F	F	F	F

---

2. Exercise 1.3.4

(b) $(p \to q) \to (q \to p)$

p	q	$(p \to q)$	$(q \to p)$	$(p \to q) \to (q \to p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

(d) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

p	q	$(p \leftrightarrow q)$	$(p \leftrightarrow \neg q)$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	T	F	T

Question 5:

1. Exercise 1.2.7

(b) The applicant must present at least two of the following forms of identification: birth certificate, driver's license, marriage license.

$$(B \land D) \lor (B \land M) \lor (D \land M)$$

(c) Applicant must present either a birth certificate or both a driver's license and a marriage license.

$$B \lor (D \land M)$$

---

2. Exercise 1.3.7

(b) A person can park in the school parking lot if they are a senior or at least seventeen years of age.

$$(s \lor y) \to p$$

(c) Being 17 years of age is a necessary condition for being able to park in the school parking lot.

$$p \to y$$

(d) A person can park in the school parking lot if and only if the person is a senior and at least 17 years of age.

$$p \leftrightarrow (s \land y)$$

(e) Being able to park in the school parking lot implies that the person is either a senior or at least 17 years old.

$$p \to (s \lor y)$$

---

3. Exercise 1.3.9

(c) The applicant can enroll in the course only if the applicant has parental permission.

$$c \to p$$

(d) Having parental permission is a necessary condition for enrolling in the course.

$$c \to p$$

Question 6:

1. Exercise 1.3.6

(b) Maintaining a B average is necessary for Joe to be eligible for the honors program.

If Joe wants to be eligible for the program, then he must maintain a B average.

(c) Rajiv can go on the roller coaster only if he is at least four feet tall.

If Rajiv goes on the roller coaster, then he is atleast four feet tall.

(d) Rajiv can go on the roller coaster if he is at least four feet tall.

If Rajiv is atleast four feet tall, then he can go on the roller coaster.

---

2. Exercise 1.3.10

(c) $(p \vee r) \leftrightarrow (q \wedge r)$

False. Truth values are not same. $(p \vee r)$ is True and $(q \wedge r)$ is False.

(d) $(p \wedge r) \leftrightarrow (q \wedge r)$

Unknown. If r is True, then the expression is False; If r is False, then the expression is True.

(e) $p \rightarrow (r \vee q)$

Unknown. If r is True, then the expression is True; If r is False, then the expression is False.

(f) $(p \wedge q) \rightarrow r$

True. The hypothesis is False. Therefore, irrespective of the r value, the expression is True.

Question 7:

Exercise 1.4.5

(b) If Sally did not get the job, then she was late for interview or did not update her resume.
If Sally updated her resume and was not late for her interview, then she got the job.

$$\neg j \to (l \lor \neg r) \equiv (r \land \neg l) \to j$$

j	l	r	$\neg j \to (l \lor \neg r)$	$(r \land \neg l) \to j$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	F
F	F	F	T	T

(c) If Sally got the job then she was not late for her interview.
If Sally did not get the job, then she was late for her interview.

$$j \to \neg l \not\equiv \neg j \to l$$

j	l	$j \to \neg l$	$\neg j \to l$
T	T	F	T
T	F	T	T
F	T	T	T
F	F	T	F

(d) If Sally updated her resume or she was not late for her interview, then she got the job.
If Sally got the job, then she updated her resume and was not late for her interview.

$$(r \lor \neg l) \to j \not\equiv j \to (r \land \neg l)$$

j	l	r	$(r \lor \neg l) \to j$	$j \to (r \land \neg l)$
T	T	T	T	F
T	T	F	F	T
T	F	T	T	T
T	F	F	F	T
F	T	T	T	F
F	T	F	F	T
F	F	T	T	F
F	F	F	T	T

Question 8:

1. Exercise 1.5.2

(c) $(p \to q) \land (p \to r) \equiv p \to (q \land r)$

$(p \to q) \land (p \to r)$	
$(\neg p \lor q) \land (\neg p \lor r)$	Conditional Identity
$\neg p \lor (q \land r)$	Distributive Law
$p \to (q \land r)$	Conditional Identity

(f) $\neg(p \lor (\neg p \land q)) \equiv \neg p \land \neg q$

$\neg(p \lor (\neg p \land q))$	
$\neg p \land \neg(\neg p \land q)$	De Morgan's Law
$\neg p \land (\neg\neg p \lor \neg q)$	De Morgan's Law
$\neg p \land (p \lor \neg q)$	Double Negation Law
$(\neg p \land p) \lor (\neg p \land \neg q)$	Distributive Law
$\text{F} \lor (\neg p \land \neg q)$	Complement Law
$(\neg p \land \neg q) \lor \text{F}$	Commutative Law
$(\neg p \land \neg q)$	Identity Law

(i) $(p \land q) \to r \equiv (p \land \neg r) \to \neg q$

$(p \land q) \to r$	
$\neg(p \land q) \lor r$	Conditional Identity
$(\neg p \lor \neg q) \lor r$	De Morgan's Law
$r \lor (\neg p \lor \neg q)$	Commutative Law
$(r \lor \neg p) \lor \neg q$	Associative Law
$\neg\neg(r \lor \neg p) \lor \neg q$	Double Negation Law
$\neg(r \lor \neg p) \to \neg q$	Conditional Identity
$\neg(\neg p \lor r) \to \neg q$	Commutative Law
$(\neg\neg p \land \neg r) \to \neg q$	De Morgan's Law
$(p \land \neg r) \to \neg q$	Double Negation Law

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2. Exercise 1.5.3

(c) $\neg r \vee (\neg r \to p)$

$\neg r \vee (\neg r \to p)$	
$\neg r \vee (\neg\neg r \vee p)$	Conditional Identity
$\neg r \vee (r \vee p)$	Double Negation Law
$(\neg r \vee r) \vee p$	Associative Law
$(r \vee \neg r) \vee p$	Commutative Law
$T \vee p$	Complement Law
$p \vee T$	Commutative Law
T	Domination Law

(d) $\neg(p \to q) \to \neg q$

$\neg(p \to q) \to \neg q$	
$\neg(\neg p \vee q) \to \neg q$	Conditional Identity
$\neg\neg(\neg p \vee q) \vee \neg q$	Conditional Identity
$(\neg p \vee q) \vee \neg q$	Double Negation Law
$\neg p \vee (q \vee \neg q)$	Associative Law
$\neg p \vee T$	Complement Law
T	Domination Law

Question 9:

1. Exercise 1.6.3

(c) There is a number that is equal to its square.

$$\exists x(x = x^2)$$

(d) Every number is less than or equal to its square.

$$\forall x(x \leq x^2)$$

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2. Exercise 1.7.4

(b) Everyone was well and went to work yesterday.

$$\forall x(\neg S(x) \land W(x))$$

(c) Everyone who was sick yesterday did not go to work.

$$\forall x(S(x) \to \neg W(x))$$

(d) Yesterday someone was sick and went to work.

$$\exists x(S(x) \land W(x))$$

Question 10:

1. Exercise 1.7.9

(c) $\exists x((x = c) \to P(x))$

False

(d) $\exists x(Q(x) \wedge R(x))$

True

(e) $Q(a) \wedge P(d)$

True

(f) $\forall x((x \neq b) \to Q(x))$

True

(g) $\forall x(P(x) \vee R(x))$

False. Counter-Example is c

(h) $\forall x(P(x) \to R(x))$

True

(i) $\exists x(Q(x) \vee R(x))$

True

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2. Exercise 1.9.2

(b) $\exists x \forall y Q(x, y)$

True. When x = 2, it's True for all y.

(c) $\exists x \forall y P(y, x)$

True. For every y, there is a x which makes P(y,x) True.

(d) $\exists x \exists y S(x, y)$

False. There is no such x or y where P(x,y) is True.

(e) $\forall x \exists y Q(x, y)$

False. When x = 1 there is no y which makes Q(x,y) True.

(f) $\forall x \exists y P(x, y)$

True. For every x, there is a y which makes P(x,y) True.

(g) $\forall x \forall y P(x, y)$

False. There are x and y which makes P(x,y) False.

(h) $\exists x \exists y Q(x, y)$

True. For Q(2,y), Q(3,y) there exists atleast 1 y which is True.

(i) $\forall x \forall y \neg S(x, y)$

True. For all combinations of x and y, ¬S(x,y) is True.

Question 11:

1. Exercise 1.10.4

(c) There are two numbers whose sum is equal to their product.

$$\exists x \exists y (x + y = xy)$$

(d) The ratio of every two positive numbers is also positive.

$$\forall x \forall y (((x > 0) \wedge (y > 0)) \rightarrow (x/y > 0))$$

(e) The reciprocal of every positive number less than one is greater than one.

$$\forall x (((x > 0) \wedge (x < 1)) \rightarrow (1/x > 1))$$

(f) There is no smallest number.

$$\neg \exists x \forall y (x < y)$$

(g) Every number besides 0 has a multiplicative inverse.

$$\forall x \exists y ((x \neq 0) \rightarrow (xy = 1))$$

---

2. Exercise 1.10.7

(c) There is at least one new employee who missed the deadline.

$$\exists x(N(x) \land D(x))$$

(d) Sam knows the phone number of everyone who missed the deadline.

$$\forall y(D(y) \to P(Sam, y))$$

(e) There is a new employee who knows everyone's phone number.

$$\exists x \forall y(N(x) \land P(x, y))$$

(f) Exactly one new employee missed the deadline.

$$\exists x \forall y((N(x) \land D(x)) \land (((x \neq y) \land N(y)) \to \neg D(y)))$$

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3. Exercise 1.10.10

(c) Every student has taken at least one class besides Math 101.

$$\forall x \exists y(T(x, y) \land (y \neq Math101))$$

(d) There is a student who has taken every math class besides Math 101.

$$\exists x \forall y((y \neq Math101) \to T(x, y))$$

(e) Everyone besides Sam has taken at least two different math classes.

$$\forall x \exists y \exists z(((x \neq Sam)) \to ((y \neq z) \land T(x, y) \land T(x, z)))$$

(f) Sam has taken exactly two math classes.

$$\exists y \exists z \forall w(((y \neq z) \land T(Sam, y) \land T(Sam, z)) \land (((w \neq y) \land (w \neq z)) \to \neg T(Sam, w)))$$

Question 12:

1. Exercise 1.8.2

(b) Every patient was given the medication or the placebo or both.

$$\begin{aligned} & \forall x(D(x) \vee P(x)) \\ \text{Negation:}\quad & \neg\forall x(D(x) \vee P(x)) \\ \text{Applying De Morgan's Law:}\quad & \exists x(\neg D(x) \wedge \neg P(x)) \end{aligned}$$

There is a patient who is not given medication and not given placebo.

(c) There is a patient who took the medication and had migraines.

$$\begin{aligned} & \exists x(D(x) \wedge M(x)) \\ \text{Negation:}\quad & \neg\exists x(D(x) \wedge M(x)) \\ \text{Applying De Morgan's Law:}\quad & \forall x(\neg D(x) \vee \neg M(x)) \end{aligned}$$

Every patient was not given medication or not had migraines or both.

(d) Every patient who took the placebo had migraines.

$$\begin{aligned} & \forall x(P(x) \rightarrow M(x)) \\ \text{Negation:}\quad & \neg\forall x(P(x) \rightarrow M(x)) \\ \text{Applying De Morgan's Law:}\quad & \exists x(P(x) \wedge \neg M(x)) \end{aligned}$$

There is a patient who took the placebo and not had migraines.

(e) There is a patient who had migraines and was given the placebo.

$$\begin{aligned} & \exists x(M(x) \wedge P(x)) \\ \text{Negation:}\quad & \neg\exists x(M(x) \wedge P(x)) \\ \text{Applying De Morgan's Law:}\quad & \forall x(\neg M(x) \vee \neg P(x)) \end{aligned}$$

Every patient either not had migraines or not given placebo or both.

---

2. Exercise 1.9.4

(c) $\exists x \forall y (P(x, y) \to Q(x, y))$

$$\forall x \exists y (P(x, y) \land \neg Q(x, y))$$

(d) $\exists x \forall y (P(x, y) \leftrightarrow P(y, x))$

$$\forall x \exists y ((P(x, y) \land \neg P(y, x)) \lor (P(y, x) \land \neg P(x, y)))$$

(e) $\exists x \exists y P(x, y) \land \forall x \forall y Q(x, y)$

$$\forall x \forall y \neg P(x, y) \lor \exists x \exists y \neg Q(x, y)$$
