Homework 3

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Question 7:

A. Exercise 3.1.1

(a) $27 \in A$: **True**

(b) $27 \in B$: **False**

(c) $100 \in B$: **True**

(d) $E \subseteq C$ *or* $C \subseteq E$: **False**

(e) $E \subseteq A$: **True**

(f) $A \subset E$: **False**

(g) $E \in A$: **False**

B. Exercise 3.1.2

(a) $15 \subset A$: **False**

(b) $15 \subset B$: **True**

(c) $\varnothing \subset B$: **True**

(d) $A \subseteq A$: **True**

(e) $\varnothing \subseteq B$: **False**

C. Exercise 3.1.5

(b) $\{3, 6, 9, 12, \ldots\}$

$$\text{Let } B = \{3, 6, 9, 12, \ldots\}$$
$$B = \{x \in \mathbb{Z} : x \text{ is an integer multiple of 3 and } x \geq 3\}$$

(d) $\{0, 10, 20, 30 \ldots\ 1000\}$

$$\text{Let } D = \{0, 10, 20, 30 \ldots\ 1000\}$$
$$D = \{x \in \mathbb{Z} : x \text{ is an integer multiple of 10 and } 0 \leq x \leq 1000\}$$
$$\text{The set is finite. Hence } |D| = 101$$

D. Exercise 3.2.1

(a) $2 \in X$: **True**

(b) $\{2\} \subseteq X$: **True**

(c) $\{2\} \in X$: **False**

(d) $3 \in X$: **False**

(e) $\{1, 2\} \in X$: **True**

(f) $\{1, 2\} \subseteq X$: **True**

(g) $\{2, 4\} \subseteq X$: **True**

(h) $\{2, 4\} \in X$: **False**

(i) $\{2, 3\} \subseteq X$: **False**

(j) $\{2, 3\} \in X$: **False**

(k) $|X| = 7$: **False**

Question 8:

Exercise 3.2.4

(b) Let $A = \{1, 2, 3\}$. What is $\{X \in P(A) : 2 \in X\}$

$$P(A) = \{\varnothing, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$$

Let $B = \{X \in P(A) : 2 \in X\}$

$$B = \{\{2\}, \{1, 2\}, \{2, 3\}, \{1, 2, 3\}\}$$

Question 9:

A. Exercise 3.3.1

(c) $A \cap C$

$$A \cap C = \{-3, 1, 17\}$$

(d) $A \cup (B \cap C)$

$$B \cap C = \{-5, 1\}$$

$$A \cup (B \cap C) = \{-5, -3, 0, 1, 4, 17\}$$

(e) $A \cap B \cap C$

$$B \cap C = \{-5, 1\}$$

$$A \cap (B \cap C) = \{1\}$$

B. Exercise 3.3.3

(a)

$$\bigcap_{i=2}^{5} A_i = A_2 \cap A_3 \cap A_4 \cap A_5$$

$$A_2 = \{1, 2, 4\}$$
$$A_3 = \{1, 3, 9\}$$
$$A_4 = \{1, 4, 16\}$$
$$A_5 = \{1, 5, 25\}$$

$$A_2 \cap A_3 \cap A_4 \cap A_5 = \{1\}$$

(b)

$$\bigcup_{i=2}^{5} A_i = A_2 \cup A_3 \cup A_4 \cup A_5$$

$$A_2 = \{1, 2, 4\}$$
$$A_3 = \{1, 3, 9\}$$
$$A_4 = \{1, 4, 16\}$$
$$A_5 = \{1, 5, 25\}$$

$$A_2 \cup A_3 \cup A_4 \cup A_5 = \{1, 2, 3, 4, 5, 9, 16, 25\}$$

(e)

$$\bigcap_{i=1}^{100} C_i = C_1 \cap C_2 \cap C_3 \cap \cdots \cap C_{100}$$

$$C_1 = \{x \in \mathbb{R} : -1 \leq x \leq 1\}$$

$$C_2 = \{x \in \mathbb{R} : \frac{-1}{2} \leq x \leq \frac{1}{2}\}$$

$$C_3 = \{x \in \mathbb{R} : \frac{-1}{3} \leq x \leq \frac{1}{3}\}$$

$$\vdots$$

$$C_{100} = \{x \in \mathbb{R} : \frac{-1}{100} \leq x \leq \frac{1}{100}\}$$

$$C_1 \cap C_2 \cap C_3 \cap \cdots \cap C_{100} = \{x \in \mathbb{R} : \frac{-1}{100} \leq x \leq \frac{1}{100}\}$$

Because the x values between $\frac{-1}{100}$ and $\frac{1}{100}$ is part of all the C in $C_1, C_2, C_3, \ldots, C_{100}$

(f)

$$\bigcup_{i=1}^{100} C_i = C_1 \cup C_2 \cup C_3 \cup \cdots \cup C_{100}$$

$$C_1 = \{x \in \mathbb{R} : -1 \leq x \leq 1\}$$

$$C_2 = \{x \in \mathbb{R} : \frac{-1}{2} \leq x \leq \frac{1}{2}\}$$

$$C_3 = \{x \in \mathbb{R} : \frac{-1}{3} \leq x \leq \frac{1}{3}\}$$

$$\vdots$$

$$C_{100} = \{x \in \mathbb{R} : \frac{-1}{100} \leq x \leq \frac{1}{100}\}$$

$$C_1 \cup C_2 \cup C_3 \cup \cdots \cup C_{100} = \{x \in \mathbb{R} : -1 \leq x \leq 1\}$$

Because the x values between -1 and 1 covers all the values in $C_1, C_2, C_3, \ldots, C_{100}$

C. Exercise 3.3.4

(b) $P(A \cup B)$

$$A \cup B = \{a, b, c\}$$

$$P(A \cup B) = \{\varnothing, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$$

(d) $P(A) \cup P(B)$

$$P(A) = \{\varnothing, \{a\}, \{b\}, \{a, b\}\}$$

$$P(B) = \{\varnothing, \{b\}, \{c\}, \{b, c\}\}$$

$$P(A) \cup P(B) = \{\varnothing, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$$

Question 10:

A. Exercise 3.5.1

(b) Write an element from the set $B \times A \times C$.

$$B \times A \times C = (\text{foam, tall, non-fat})$$

(c) Write the set $B \times C$ using roster notation.

$$B \times C = \{(\text{foam, non-fat}), (\text{foam, whole}), (\text{no-foam, non-fat}), (\text{no-foam, whole})\}$$

B. Exercise 3.5.3

(b) $\mathbb{Z}^2 \subseteq \mathbb{R}^2$: **True**

(c) $\mathbb{Z}^2 \cap \mathbb{Z}^3 = \varnothing$: **True**

(e) For any three sets, A, B, and C, if $A \subseteq B$, then $A \times C \subseteq B \times C$: **True**

C. Exercise 3.5.6

(d) $\{xy$: where $x \in \{0\} \cup \{0\}^2$ and $y \in \{1\} \cup \{1\}^2\}$

$$\{0\} \cup \{0\}^2 = \{0, 00\}$$

$$\{1\} \cup \{1\}^2 = \{1, 11\}$$

$$\text{Let } D = \{xy : \text{where } x \in \{0\} \cup \{0\}^2 \text{ and } y \in \{1\} \cup \{1\}^2\}$$

$$D = \{01, 011, 001, 0011\}$$

(e) $\{xy$: where $x \in \{aa, ab\}$ and $y \in \{a\} \cup \{a\}^2\}$

$$\{a\} \cup \{a\}^2 = \{a, aa\}$$

$$\text{Let } E = \{xy : \text{where } x \in \{aa, ab\} \text{ and } y \in \{a\} \cup \{a\}^2\}$$

$$E = \{aaa, aaaa, aba, abaa\}$$

D. Exercise 3.5.7

(c) $(A \times B) \cup (A \times C)$

$$A \times B = \{ab, ac\}$$

$$A \times C = \{aa, ab, ad\}$$

$$(A \times B) \cup (A \times C) = \{aa, ab, ac, ad\}$$

(f) $P(A \times B)$

$$A \times B = \{ab, ac\}$$

$$P(A \times B) = \{\varnothing, \{ab\}, \{ac\}, \{ab, ac\}\}$$

(g) $P(A) \times P(B)$. Use ordered pair notation for elements of the Cartesian product.

$$P(A) = \{\varnothing, \{a\}\}$$

$$P(B) = \{\varnothing, \{b\}, \{c\}, \{b, c\}\}$$

$$\begin{aligned} P(A) \times P(B) = \{&(\varnothing, \varnothing), (\varnothing, \{b\}), (\varnothing, \{c\}), (\varnothing, \{b, c\}), (\{a\}, \varnothing), (\{a\}, \{b\}), \\ &(\{a\}, \{c\}), (\{a\}, \{b, c\})\} \end{aligned}$$

Question 11:

A. Exercise 3.6.2

(b) $(B \cup A) \cap (\overline{B} \cup A) = A$

$(B \cup A) \cap (\overline{B} \cup A)$	Hypothesis
$(A \cup B) \cap (A \cup \overline{B})$	Commutative Law
$A \cup (B \cap \overline{B})$	Distribution Law
$A \cup \varnothing$	Complement Law
A	Identity Law

(c) $\overline{A \cap \overline{B}} = \overline{A} \cup B$

$\overline{A \cap \overline{B}}$	Hypothesis
$\overline{A} \cup \overline{\overline{B}}$	De Morgan's Law
$\overline{A} \cup B$	Double Complement Law

B. Exercise 3.6.3

(b) $A - (B \cap A) = A$

$$\text{Let } A = \{1, 2, 3, 4\}$$

$$\text{Let } B = \{3, 4, 5\}$$

$$(B \cap A) = \{3, 4\}$$

$$A - (B \cap A) = \{1, 2\} \neq \{1, 2, 3, 4\}$$

$$\therefore A - (B \cap A) \neq A$$

$$\therefore A - (B \cap A) = A \text{ is not a set identity.}$$

(d) $(B - A) \cup A = A$

$$\text{Let } A = \{1, 2, 3, 4\}$$

$$\text{Let } B = \{3, 4, 5, 6\}$$

$$B - A = \{5, 6\}$$

$$(B - A) \cup A = \{1, 2, 3, 4, 5, 6\} \neq \{1, 2, 3, 4\}$$

$$\therefore (B - A) \cup A \neq A$$

$$\therefore (B - A) \cup A = A \text{ is not a set identity.}$$

C. Exercise 3.6.4

(b) $A \cap (B - A) = \varnothing$

$A \cap (B - A)$	Hypothesis
$A \cap (B \cap \overline{A})$	Subtraction Law
$A \cap (\overline{A} \cap B)$	Commutative Law
$(A \cap \overline{A}) \cap B$	Associative Law
$\varnothing \cap B$	Complement Law
$B \cap \varnothing$	Commutative Law
$\varnothing$	Domination Law

(c) $A \cup (B - A) = A \cup B$

$A \cup (B - A)$	Hypothesis
$A \cup (B \cap \overline{A})$	Subtraction Law
$(A \cup B) \cap (A \cup \overline{A})$	Distribution Law
$(A \cup B) \cap U$	Complement Law
$(A \cup B)$	Identity Law
