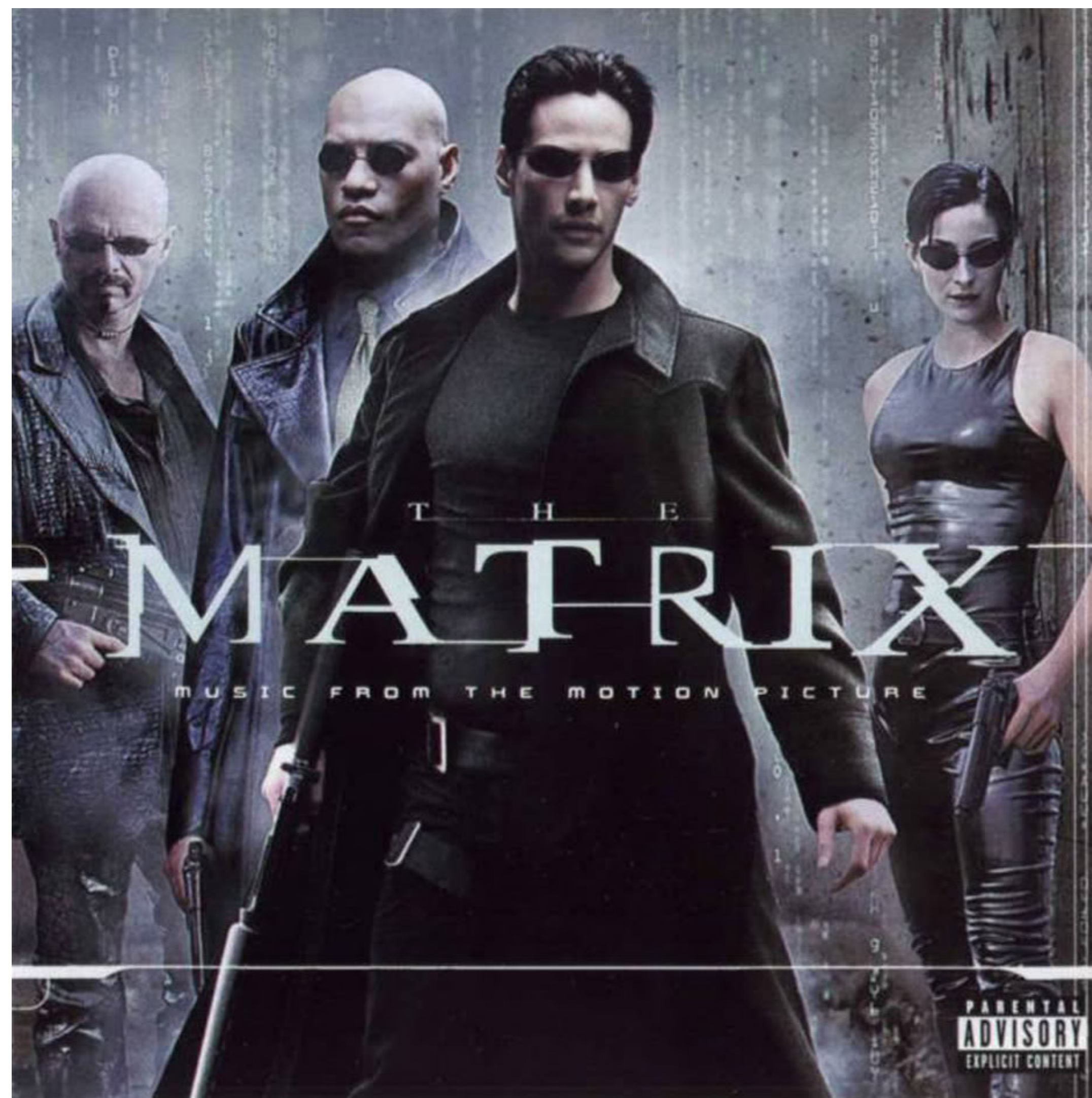


矩阵不是简单的 $m*n$ 个数

什么是矩阵

矩阵 Matrix



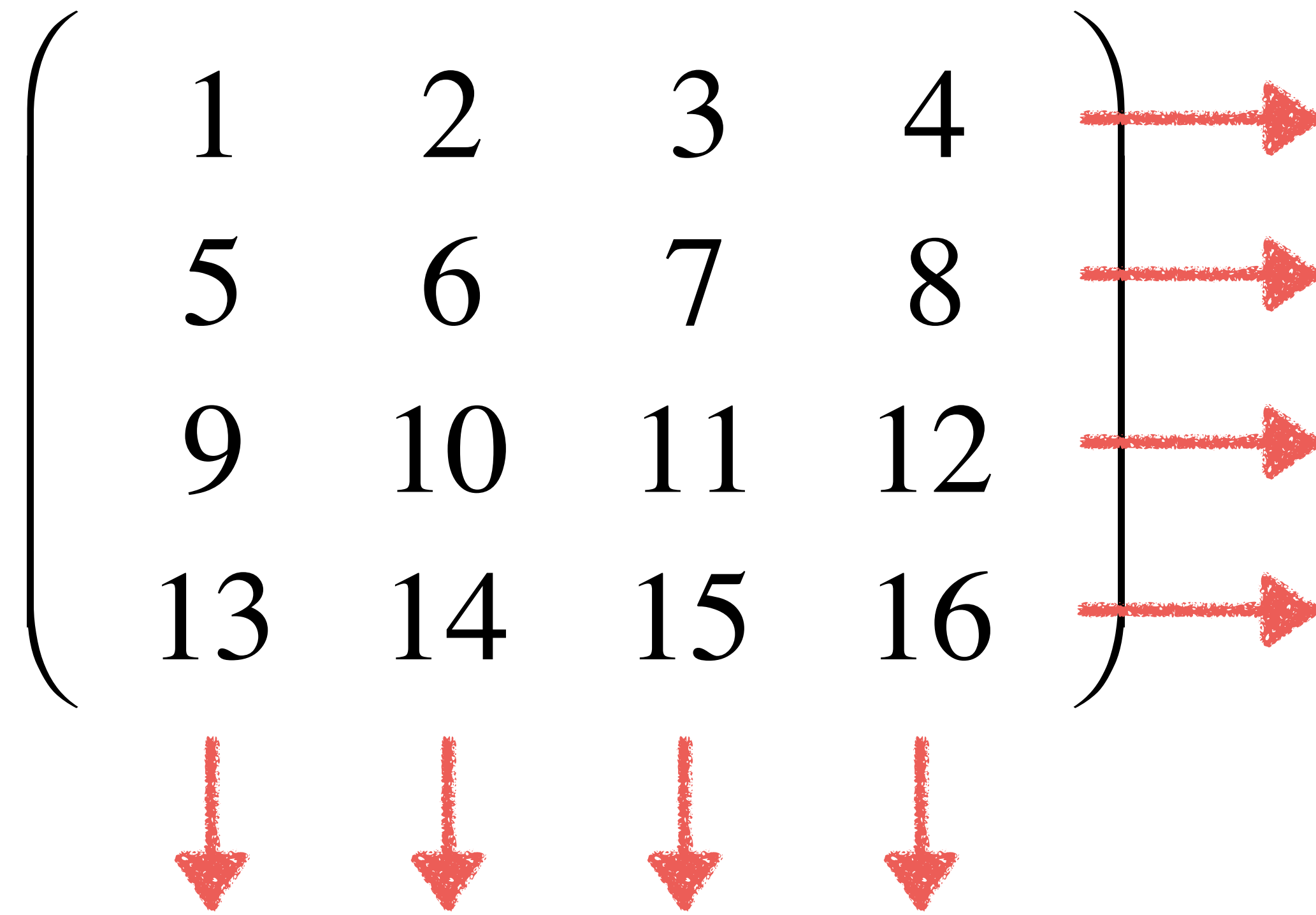
矩阵 Matrix

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{pmatrix}$$

向量是对数的拓展，
一个向量表示一组数

矩阵是对向量的拓展，
一个矩阵表示一组向量

矩阵 Matrix



A 4x4 matrix is shown with elements arranged in four rows and four columns. The elements are: Row 1: 1, 2, 3, 4; Row 2: 5, 6, 7, 8; Row 3: 9, 10, 11, 12; Row 4: 13, 14, 15, 16. The matrix is enclosed in large parentheses. To the right of the matrix, four horizontal red arrows point to the right, one for each row. Below the matrix, four vertical red arrows point downwards, one for each column.

| | | | |
|----|----|----|----|
| 1 | 2 | 3 | 4 |
| 5 | 6 | 7 | 8 |
| 9 | 10 | 11 | 12 |
| 13 | 14 | 15 | 16 |

行向量

4 * 4 矩阵

行数为4，列数为4

列向量

矩阵 Matrix

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{pmatrix}$$

3 * 4 矩阵

行数为3，列数为4

矩阵 Matrix

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{pmatrix}$$

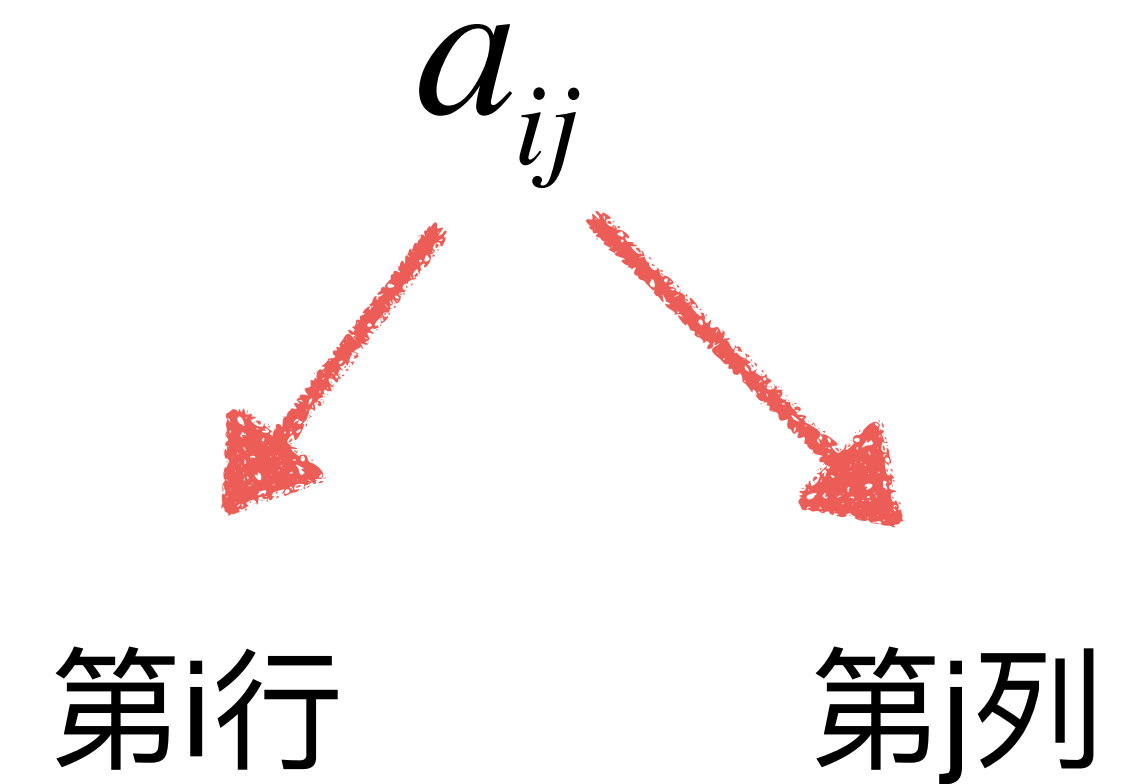
行数 = 列数  方阵

方阵有很多特殊的性质

有很多特殊的矩阵是方阵

矩阵 Matrix

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{pmatrix}$$



和计算机中的二维数组的表示一样！

矩阵 Matrix

| | 语文 | 数学 | 英语 | 物理 | 化学 | |
|-------|----|----|----|----|----|----|
| $A =$ | 90 | 76 | 88 | 92 | 90 | 张三 |
| | 88 | 82 | 98 | 95 | 92 | 李四 |
| | 86 | 68 | 70 | 80 | 77 | 王五 |

实现属于我们自己的矩阵类

实践： 实现属于我们自己的矩阵类

矩阵的基本运算

矩阵的基本运算

回忆向量的基本运算

$$\vec{u} + \vec{v}$$

$$k \cdot \vec{u}$$

矩阵的基本运算

$$A + B$$

$$k \cdot A$$

矩阵的基本运算

矩阵加法:

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1c} \\ a_{21} & a_{22} & \cdots & a_{2c} \\ \cdots & \cdots & \cdots & \cdots \\ a_{r1} & a_{r2} & \cdots & a_{rc} \end{pmatrix} \quad B = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1c} \\ b_{21} & b_{22} & \cdots & b_{2c} \\ \cdots & \cdots & \cdots & \cdots \\ b_{r1} & b_{r2} & \cdots & b_{rc} \end{pmatrix}$$

$A + B$

$$A + B = \begin{pmatrix} a_{11} + b_{11} & a_{12} + b_{12} & \cdots & a_{1c} + b_{1c} \\ a_{21} + b_{21} & a_{22} + b_{22} & \cdots & a_{2c} + b_{2c} \\ \cdots & \cdots & \cdots & \cdots \\ a_{r1} + b_{r1} & a_{r2} + b_{r2} & \cdots & a_{rc} + b_{rc} \end{pmatrix}$$

矩阵的基本运算

矩阵加法： $A + B$

| | 语文 | 数学 | 英语 | 物理 | 化学 | |
|-------|----|----|----|----|----|----|
| $A =$ | 90 | 76 | 88 | 92 | 90 | 张三 |
| | 88 | 82 | 98 | 95 | 92 | 李四 |
| | 86 | 68 | 70 | 80 | 77 | 王五 |

上学期成绩

| | 语文 | 数学 | 英语 | 物理 | 化学 | |
|-------|----|----|----|----|----|----|
| $B =$ | 92 | 74 | 96 | 92 | 95 | 张三 |
| | 88 | 92 | 94 | 86 | 78 | 李四 |
| | 82 | 74 | 80 | 88 | 80 | 王五 |

下学期成绩

矩阵的基本运算

矩阵数量乘法:

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1c} \\ a_{21} & a_{22} & \cdots & a_{2c} \\ \cdots & \cdots & \cdots & \cdots \\ a_{r1} & a_{r2} & \cdots & a_{rc} \end{pmatrix}$$

$k \cdot A$

$$k \cdot A = \begin{pmatrix} k \cdot a_{11} & k \cdot a_{12} & \cdots & k \cdot a_{1c} \\ k \cdot a_{21} & k \cdot a_{22} & \cdots & k \cdot a_{2c} \\ \cdots & \cdots & \cdots & \cdots \\ k \cdot a_{r1} & k \cdot a_{r2} & \cdots & k \cdot a_{rc} \end{pmatrix}$$

矩阵的基本运算

矩阵数量乘法： $k \cdot A$

| | 语文 | 数学 | 英语 | 物理 | 化学 | |
|-------|----|----|----|----|----|----|
| $A =$ | 90 | 76 | 88 | 92 | 90 | 张三 |
| | 88 | 82 | 98 | 95 | 92 | 李四 |
| | 86 | 68 | 70 | 80 | 77 | 王五 |

上学期成绩

| | 语文 | 数学 | 英语 | 物理 | 化学 | |
|-------|----|----|----|----|----|----|
| $B =$ | 92 | 74 | 96 | 92 | 95 | 张三 |
| | 88 | 92 | 94 | 86 | 78 | 李四 |
| | 82 | 74 | 80 | 88 | 80 | 王五 |

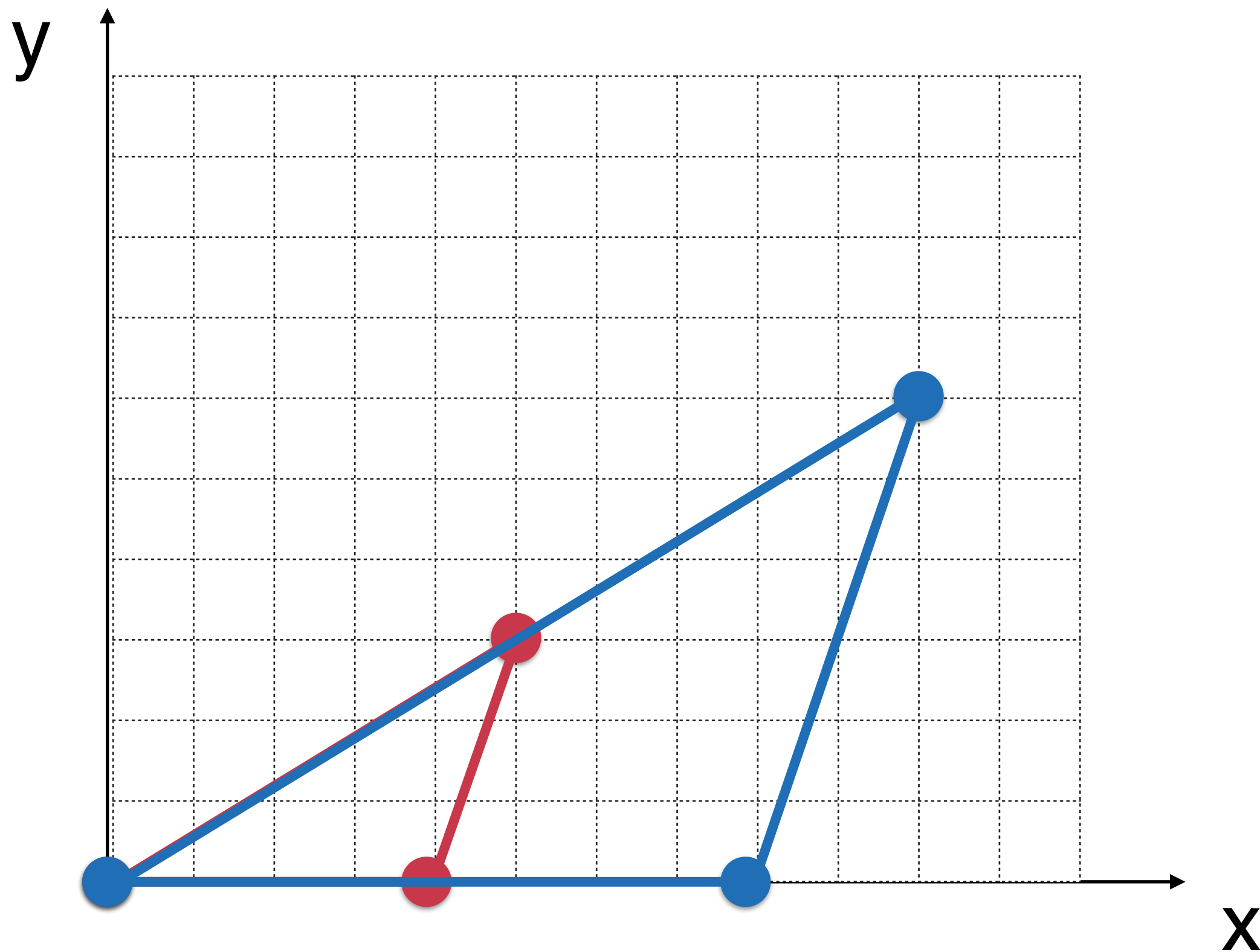
下学期成绩

$$\frac{1}{2} \cdot (A + B)$$

两学期成绩的平均分

矩阵的基本运算

矩阵数量乘法: $k \cdot A$



$$P = \begin{pmatrix} 0 & 0 \\ 4 & 0 \\ 5 & 3 \end{pmatrix}$$

$$2 \cdot P = \begin{pmatrix} 0 & 0 \\ 8 & 0 \\ 10 & 6 \end{pmatrix}$$

矩阵的基本运算性质

矩阵的基本运算

$$A + B \quad k \cdot A$$

矩阵的基本运算性质

$$A + B = B + A$$

$$(A + B) + C = A + (B + C)$$

存在矩阵 O ，满足： $A + O = A$

存在矩阵 $-A$ ，满足： $A + (-A) = O$

$-A$ 唯一； $-A = -1 \cdot A$

矩阵的基本运算性质

矩阵的基本运算性质

$$(ck)A = c(kA)$$

$$k \cdot (A + B) = k \cdot A + k \cdot B$$

$$(c + k) \cdot A = c \cdot A + k \cdot A$$

矩阵的基本运算性质

$$k \cdot (A + B) = k \cdot A + k \cdot B$$

基本证明思路：

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1c} \\ a_{21} & a_{22} & \dots & a_{2c} \\ \dots & \dots & & \dots \\ a_{r1} & a_{r2} & \dots & a_{rc} \end{pmatrix} \quad B = \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1c} \\ b_{21} & b_{22} & \dots & b_{2c} \\ \dots & \dots & & \dots \\ b_{r1} & b_{r2} & \dots & b_{rc} \end{pmatrix}$$

矩阵的基本运算性质

$$k \cdot (A + B) = k \cdot A + k \cdot B$$

基本证明思路:

$$k \cdot (A + B) = \begin{pmatrix} k(a_{11} + b_{11}) & k(a_{12} + b_{12}) & \dots & k(a_{1c} + b_{1c}) \\ k(a_{21} + b_{21}) & k(a_{22} + b_{22}) & \dots & k(a_{2c} + b_{2c}) \\ \dots & \dots & \dots & \dots \\ k(a_{r1} + b_{r1}) & k(a_{r2} + b_{r2}) & \dots & k(a_{rc} + b_{rc}) \end{pmatrix}$$

$$k \cdot A + k \cdot B = \begin{pmatrix} ka_{11} + kb_{11} & ka_{12} + kb_{12} & \dots & ka_{1c} + kb_{1c} \\ ka_{21} + kb_{21} & ka_{22} + kb_{22} & \dots & ka_{2c} + kb_{2c} \\ \dots & \dots & \dots & \dots \\ ka_{r1} + kb_{r1} & ka_{r2} + kb_{r2} & \dots & ka_{rc} + kb_{rc} \end{pmatrix}$$

矩阵的基本运算性质

$$A + B = B + A$$

$$(A + B) + C = A + (B + C)$$

存在矩阵 O ，满足： $A + O = A$

存在矩阵 $-A$ ，满足： $A + (-A) = O$

$-A$ 唯一； $-A = -1 \cdot A$

$$(ck)A = c(kA)$$

$$(c + k) \cdot A = c \cdot A + k \cdot A$$

$$k \cdot (A + B) = k \cdot A + k \cdot B$$

实现矩阵的基本运算

实践：实现矩阵的基本运算

看待矩阵的另一个视角：系统

矩阵 Matrix

| | 语文 | 数学 | 英语 | 物理 | 化学 | |
|-------|----|----|----|----|----|----|
| $A =$ | 90 | 76 | 88 | 92 | 90 | 张三 |
| | 88 | 82 | 98 | 95 | 92 | 李四 |
| | 86 | 68 | 70 | 80 | 77 | 王五 |

之前，我们的例子中，矩阵式一个数据表格

矩阵还可以表示一个系统

矩阵 Matrix

矩阵还可以表示一个系统

经济系统中，对IT，电子，矿产，房产的投入 x_{it} x_e x_m x_h

$$x_{it} = 100 + 0.2x_e + 0.1x_m + 0.5x_h$$

$$x_e = 50 + 0.5x_{it} + 0.2x_m + 0.1x_h$$

$$x_m = 20 + 0.4x_e + 0.3x_h$$

$$x_h = 666 + 0.2x_{it}$$

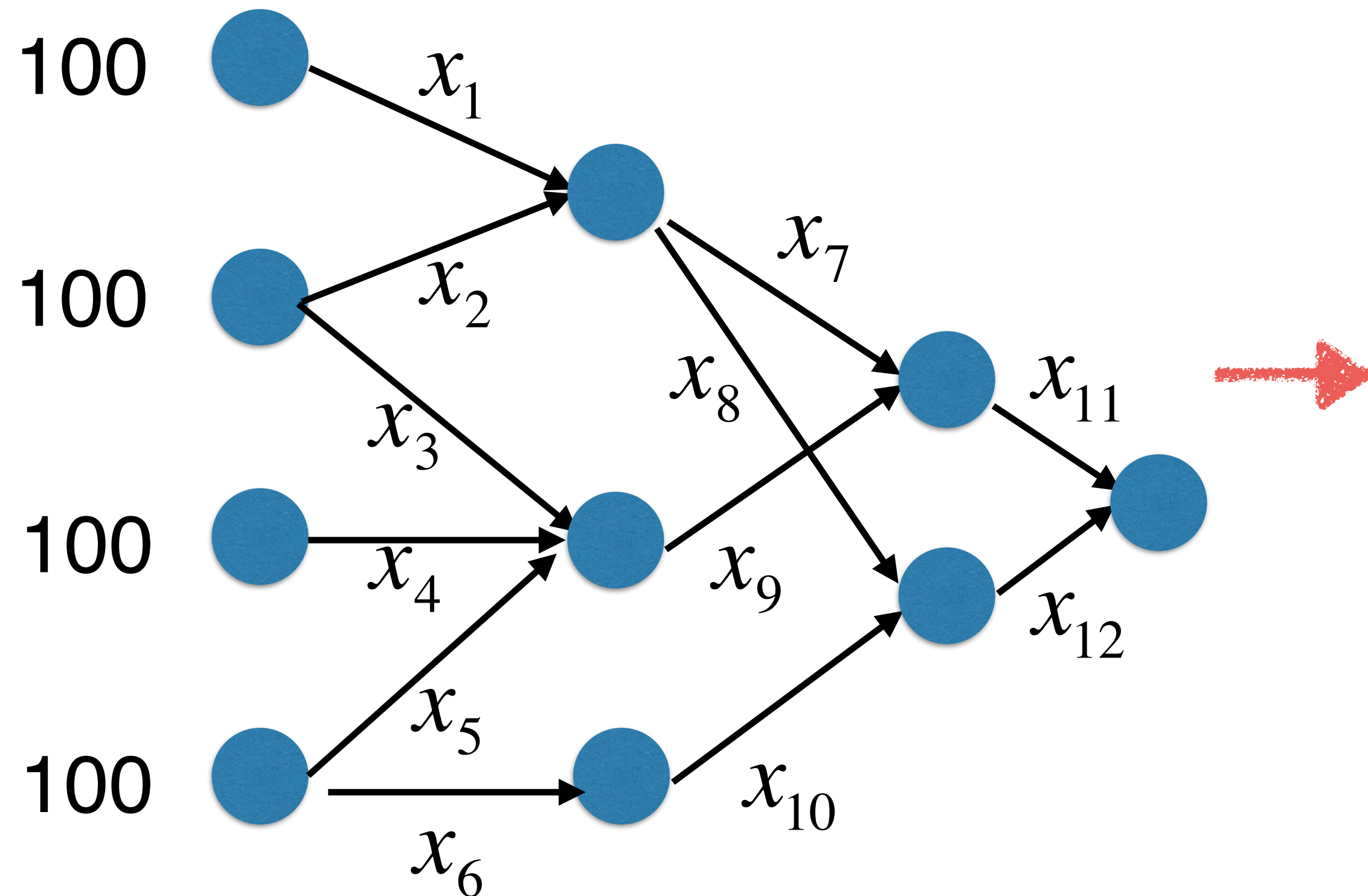
矩阵 Matrix

经济系统中，对IT，电子，矿产，房产的投入 x_{it} x_e x_m x_h

$$\begin{aligned} x_{it} &= 100 + 0.2x_e + 0.1x_m + 0.5x_h \\ x_e &= 50 + 0.5x_{it} + 0.2x_m + 0.1x_h \\ x_m &= 20 + 0.4x_e + 0.3x_h \\ x_h &= 666 + 0.2x_{it} \end{aligned} \quad \rightarrow \quad \left\{ \begin{aligned} x_{it} - 0.2x_e + 0.1x_m + 0.5x_h &= 100 \\ -0.5x_{it} - x_e + 0.2x_m + 0.1x_h &= 50 \\ -0.4x_e - x_m + 0.3x_h &= 20 \\ -0.2x_{it} + x_h &= 666 \end{aligned} \right.$$

矩阵 Matrix

网络中 (交通网络, 信息网络...)



$$x_1 = 100$$

$$x_2 + x_3 = 100$$

$$x_4 = 100$$

$$x_5 + x_6 = 100$$

$$x_7 + x_8 = x_1 + x_2$$

$$x_9 = x_3 + x_4 + x_5$$

$$x_{10} = x_6$$

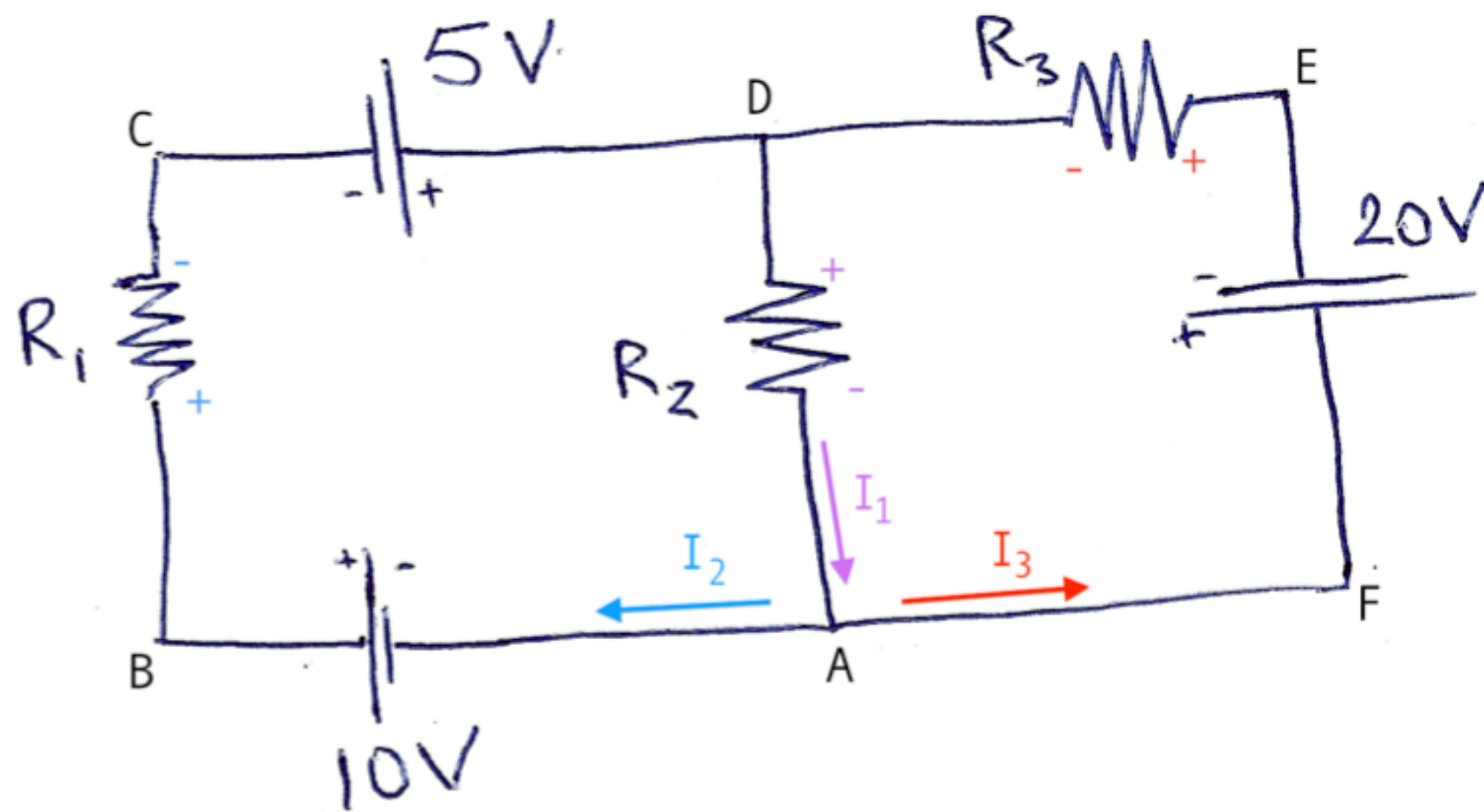
$$x_{11} = x_7 + x_9$$

$$x_{12} = x_8 + x_{10}$$

$$x_{11} + x_{12} = 400$$

矩阵 Matrix

电路系统中



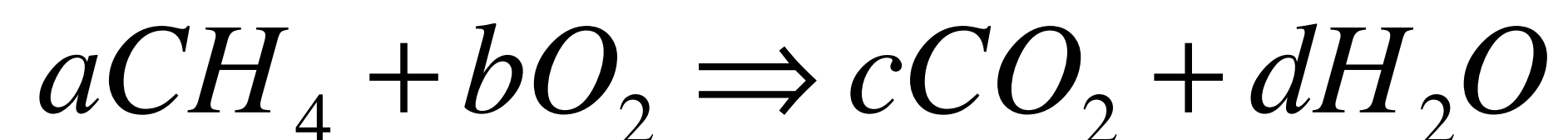
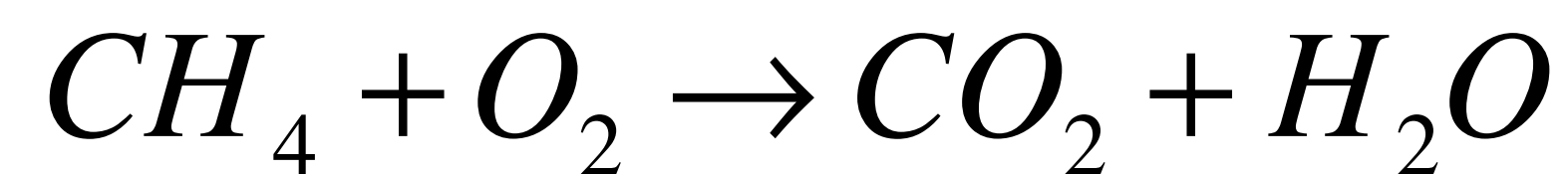
$$10 - R_1 I_2 + 5 - R_2 I_1 = 0$$

$$-20 - R_3 I_3 - R_2 I_1 = 0$$

$$I_1 = I_2 + I_3$$

矩阵 Matrix

化学方程式中



$$a = c$$

$$4a = 2d$$

$$2b = 2c + d$$

矩阵 Matrix

线性方程组在各个领域，有着重要的应用

在线性代数中，称为线性系统

矩阵 Matrix

$$\left\{ \begin{array}{l} x_{it} - 0.2x_e + 0.1x_m + 0.5x_h = 100 \\ -0.5x_{it} - x_e + 0.2x_m + 0.1x_h = 50 \\ -0.4x_e - x_m + 0.3x_h = 20 \\ -0.2x_{it} + x_h = 666 \end{array} \right.$$

矩阵 Matrix

$$\left\{ \begin{array}{l} x_{it} - 0.2x_e + 0.1x_m + 0.5x_h = 100 \\ -0.5x_{it} - x_e + 0.2x_m + 0.1x_h = 50 \\ -0.4x_e - x_m + 0.3x_h = 20 \\ -0.2x_{it} + x_h = 666 \end{array} \right.$$

$$\begin{pmatrix} 1 & -0.2 & 0.1 & 0.5 \\ -0.5 & -1 & 0.2 & 0.1 \\ 0 & -0.4 & -1 & 0.3 \\ -0.2 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 100 \\ 50 \\ 20 \\ 666 \end{pmatrix}$$

列向量!

矩阵和向量的乘法

矩阵 Matrix

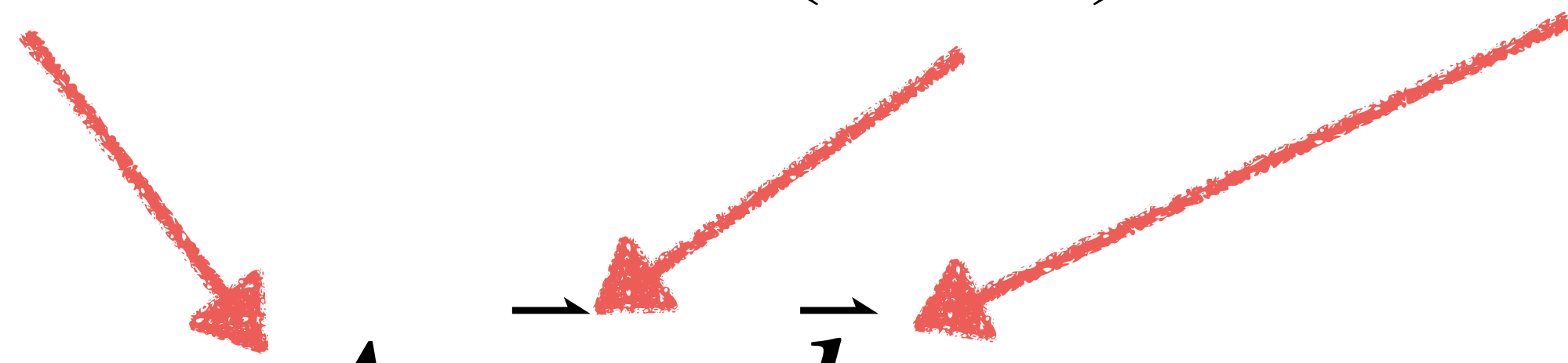
$$\left\{ \begin{array}{l} x_{it} - 0.2x_e + 0.1x_m + 0.5x_h = 100 \\ -0.5x_{it} - x_e + 0.2x_m + 0.1x_h = 50 \\ -0.4x_e - x_m + 0.3x_h = 20 \\ -0.2x_{it} + x_h = 666 \end{array} \right.$$

$$\begin{pmatrix} 1 & -0.2 & 0.1 & 0.5 \\ -0.5 & -1 & 0.2 & 0.1 \\ 0 & -0.4 & -1 & 0.3 \\ -0.2 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_{it} \\ x_e \\ x_m \\ x_h \end{pmatrix} = \begin{pmatrix} 100 \\ 50 \\ 20 \\ 666 \end{pmatrix}$$

列向量!

矩阵和向量相乘

$$\begin{pmatrix} 1 & -0.2 & 0.1 & 0.5 \\ -0.5 & -1 & 0.2 & 0.1 \\ 0 & -0.4 & -1 & 0.3 \\ -0.2 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_{it} \\ x_e \\ x_m \\ x_h \end{pmatrix} = \begin{pmatrix} 100 \\ 50 \\ 20 \\ 666 \end{pmatrix}$$


$$A \cdot \vec{x} = \vec{b}$$

矩阵和向量相乘

$$\begin{pmatrix} 1 & -0.2 & 0.1 & 0.5 \\ -0.5 & -1 & 0.2 & 0.1 \\ 0 & -0.4 & -1 & 0.3 \\ -0.2 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_{it} \\ x_e \\ x_m \\ x_h \end{pmatrix} = \begin{pmatrix} 100 \\ 50 \\ 20 \\ 666 \end{pmatrix}$$

$$\begin{pmatrix} x_{it} - 0.2x_e + 0.1x_m + 0.5x_h \\ -0.5x_{it} - x_e + 0.2x_m + 0.1x_h \\ -0.4x_e - x_m + 0.3x_h \\ -0.2x_{it} + x_h \end{pmatrix}$$

矩阵和向量相乘

$$\begin{pmatrix} 1 & -0.2 & 0.1 & 0.5 \\ -0.5 & -1 & 0.2 & 0.1 \\ 0 & -0.4 & -1 & 0.3 \\ -0.2 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_{it} \\ x_e \\ x_m \\ x_h \end{pmatrix} = \begin{pmatrix} x_{it} - 0.2x_e + 0.1x_m + 0.5x_h \\ -0.5x_{it} - x_e + 0.2x_m + 0.1x_h \\ -0.4x_e - x_m + 0.3x_h \\ -0.2x_{it} + x_h \end{pmatrix}$$

矩阵和向量相乘

$$\begin{pmatrix} 1 & -0.2 & 0.1 & 0.5 \\ -0.5 & -1 & 0.2 & 0.1 \\ 0 & -0.4 & -1 & 0.3 \\ -0.2 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_{it} \\ x_e \\ x_m \\ x_h \end{pmatrix} = \begin{pmatrix} x_{it} - 0.2x_e + 0.1x_m + 0.5x_h \\ -0.5x_{it} - x_e + 0.2x_m + 0.1x_h \\ -0.4x_e - x_m + 0.3x_h \\ -0.2x_{it} + x_h \end{pmatrix}$$

矩阵和向量相乘

$$\begin{pmatrix} 1 & -0.2 & 0.1 & 0.5 \\ -0.5 & -1 & 0.2 & 0.1 \\ 0 & -0.4 & -1 & 0.3 \\ -0.2 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_{it} \\ x_e \\ x_m \\ x_h \end{pmatrix} = \begin{pmatrix} x_{it} - 0.2x_e + 0.1x_m + 0.5x_h \\ -0.5x_{it} - x_e + 0.2x_m + 0.1x_h \\ -0.4x_e - x_m + 0.3x_h \\ -0.2x_{it} + x_h \end{pmatrix}$$

矩阵和向量相乘

$$\begin{pmatrix} 1 & -0.2 & 0.1 & 0.5 \\ -0.5 & -1 & 0.2 & 0.1 \\ 0 & -0.4 & -1 & 0.3 \\ -0.2 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_{it} \\ x_e \\ x_m \\ x_h \end{pmatrix} = \begin{pmatrix} x_{it} - 0.2x_e + 0.1x_m + 0.5x_h \\ -0.5x_{it} - x_e + 0.2x_m + 0.1x_h \\ -0.4x_e - x_m + 0.3x_h \\ -0.2x_{it} + x_h \end{pmatrix}$$

矩阵和向量相乘

$$\begin{pmatrix} 1 & -0.2 & 0.1 & 0.5 \\ -0.5 & -1 & 0.2 & 0.1 \\ 0 & -0.4 & -1 & 0.3 \\ -0.2 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_{it} \\ x_e \\ x_m \\ x_h \end{pmatrix} = \begin{pmatrix} x_{it} - 0.2x_e + 0.1x_m + 0.5x_h \\ -0.5x_{it} - x_e + 0.2x_m + 0.1x_h \\ -0.4x_e - x_m + 0.3x_h \\ -0.2x_{it} + x_h \end{pmatrix}$$

矩阵和向量相乘

$$\begin{pmatrix} \text{red bar} \\ \text{red bar} \\ \text{red bar} \\ \text{red bar} \end{pmatrix} \cdot \begin{pmatrix} \text{blue bar} \end{pmatrix} = \begin{pmatrix} \text{red bar} \cdot \text{blue bar} \\ \text{red bar} \cdot \text{blue bar} \\ \text{red bar} \cdot \text{blue bar} \\ \text{red bar} \cdot \text{blue bar} \end{pmatrix}$$

矩阵和向量相乘

$$\begin{pmatrix} \text{red bar} \\ \text{red bar} \\ \text{red bar} \\ \text{red bar} \end{pmatrix} \cdot \begin{pmatrix} \text{blue bar} \end{pmatrix} = \begin{pmatrix} \text{red bar} \cdot \text{blue bar} \\ \text{red bar} \cdot \text{blue bar} \\ \text{red bar} \cdot \text{blue bar} \\ \text{red bar} \cdot \text{blue bar} \end{pmatrix}$$

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \cdot \begin{pmatrix} u_1 \\ u_2 \\ \dots \\ u_n \end{pmatrix} = \begin{pmatrix} a_{11}u_1 + a_{12}u_2 + \dots + a_{1n}u_n \\ a_{21}u_1 + a_{21}u_2 + \dots + a_{2n}u_n \\ \dots \\ a_{m1}u_1 + a_{m2}u_2 + \dots + a_{mn}u_n \end{pmatrix}$$

$$\begin{pmatrix} \text{red bar} \\ \text{red bar} \\ \text{red bar} \\ \text{red bar} \end{pmatrix} \cdot \begin{pmatrix} \text{blue bar} \end{pmatrix} = \begin{pmatrix} \text{red bar} \cdot \text{blue bar} \\ \text{red bar} \cdot \text{blue bar} \\ \text{red bar} \cdot \text{blue bar} \\ \text{red bar} \cdot \text{blue bar} \end{pmatrix}$$

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \cdot \begin{pmatrix} u_1 \\ u_2 \\ \dots \\ u_n \end{pmatrix} = \begin{pmatrix} a_{11}u_1 + a_{12}u_2 + \dots + a_{1n}u_n \\ a_{21}u_1 + a_{21}u_2 + \dots + a_{2n}u_n \\ \dots \\ a_{m1}u_1 + a_{m2}u_2 + \dots + a_{mn}u_n \end{pmatrix}$$

矩阵A的列数必须和向量u的元素个数一致！

矩阵A的行数没有限制。

$$\begin{pmatrix} \text{red bar} \\ \text{red bar} \\ \text{red bar} \\ \text{red bar} \end{pmatrix} \cdot \begin{pmatrix} \text{blue bar} \end{pmatrix} = \begin{pmatrix} \text{red bar} \cdot \text{blue bar} \\ \text{red bar} \cdot \text{blue bar} \\ \text{red bar} \cdot \text{blue bar} \\ \text{red bar} \cdot \text{blue bar} \end{pmatrix}$$

$$\begin{pmatrix} \text{---} \vec{r_1} \text{---} \\ \text{---} \vec{r_2} \text{---} \\ \dots \\ \text{---} \vec{r_m} \text{---} \end{pmatrix} \cdot \vec{u} = \begin{pmatrix} \vec{r_1} \cdot \vec{u} \\ \vec{r_2} \cdot \vec{u} \\ \dots \\ \vec{r_m} \cdot \vec{u} \end{pmatrix}$$

行视角

矩阵A的列数必须和向量u的元素个数一致！

矩阵A的行数没有限制。

再看向量的点乘

The diagram illustrates the dot product of a matrix row and a vector. On the left, a matrix A is represented by a large left parenthesis followed by four horizontal red bars, each representing a row, and a large right parenthesis. This is followed by a dot operator \cdot and a vector u , represented by a large left parenthesis followed by a single vertical blue bar and a large right parenthesis. An equals sign $=$ follows. To the right of the equals sign is a large left parenthesis followed by four rows. Each row consists of a horizontal red bar, a dot operator \cdot , and a horizontal blue bar, all enclosed within the large right parenthesis. This represents the resulting vector where each element is the dot product of a row from A and the vector u .

矩阵A的列数必须和向量u的元素个数一致！

矩阵A的行数没有限制。

A的行数为1

再看向量的点乘

The diagram illustrates the dot product of a row vector and a column vector. On the left, a row vector is represented by a red horizontal bar inside large parentheses, followed by a dot operator. Next to it is a column vector represented by a blue vertical bar inside large parentheses. An equals sign follows, leading to a single large parentheses containing a red horizontal bar, a dot operator, and a blue horizontal bar, representing the resulting scalar value.

矩阵A的列数必须和向量u的元素个数一致！

矩阵A的行数没有限制。

A的行数为1

再看向量的点乘

The diagram shows the dot product of a row vector and a column vector. On the left, a red horizontal bar representing a row vector is enclosed in large parentheses. This is followed by a black dot representing the dot product operation. Next is a blue vertical bar representing a column vector, also enclosed in large parentheses. An equals sign follows. To the right of the equals sign is a large set of parentheses containing a red horizontal bar (the row vector) followed by a black dot and then a blue horizontal bar (the column vector), representing the element-wise multiplication of the two vectors.

矩阵A的列数必须和向量u的元素数一致！

矩阵A的行数没有限制。

A的行数为1

$$\vec{u} \cdot \vec{v} = u_1 \cdot v_1 + u_2 \cdot v_2 + \dots + u_n \cdot v_n$$

矩阵和向量的乘法

$$\begin{pmatrix} \text{red bar} \\ \text{red bar} \\ \text{red bar} \\ \text{red bar} \end{pmatrix} \cdot \begin{pmatrix} \text{blue bar} \end{pmatrix} = \begin{pmatrix} \text{red bar} \cdot \text{blue bar} \\ \text{red bar} \cdot \text{blue bar} \\ \text{red bar} \cdot \text{blue bar} \\ \text{red bar} \cdot \text{blue bar} \end{pmatrix}$$

$$T \cdot \vec{a} = \vec{b}$$

矩阵T实际上将向量a转换成了向量b!

可以把矩阵理解成向量的函数!

矩阵和矩阵的乘法

矩阵和向量的乘法

$$\begin{pmatrix} \text{red bar} \\ \text{red bar} \\ \text{red bar} \\ \text{red bar} \end{pmatrix} \cdot \begin{pmatrix} \text{blue bar} \end{pmatrix} = \begin{pmatrix} \text{red bar} \cdot \text{blue bar} \\ \text{red bar} \cdot \text{blue bar} \\ \text{red bar} \cdot \text{blue bar} \\ \text{red bar} \cdot \text{blue bar} \end{pmatrix}$$

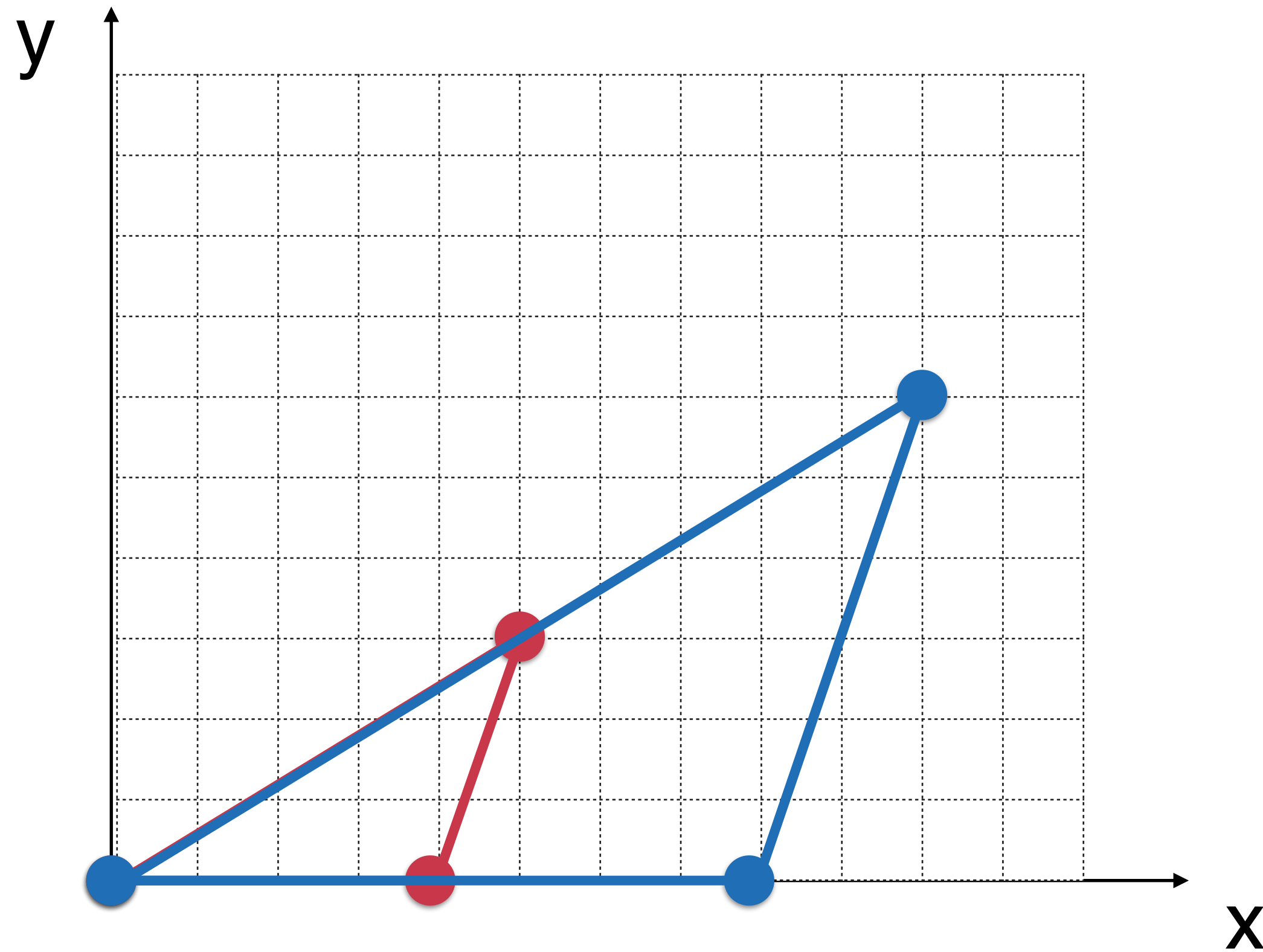
$$T \cdot \vec{a} = \vec{b}$$

矩阵T实际上将向量a转换成了向量b!

可以把矩阵理解成向量的函数!

矩阵在图形变换中的应用

让每个点的横坐标扩大1.5倍，纵坐标扩大2倍

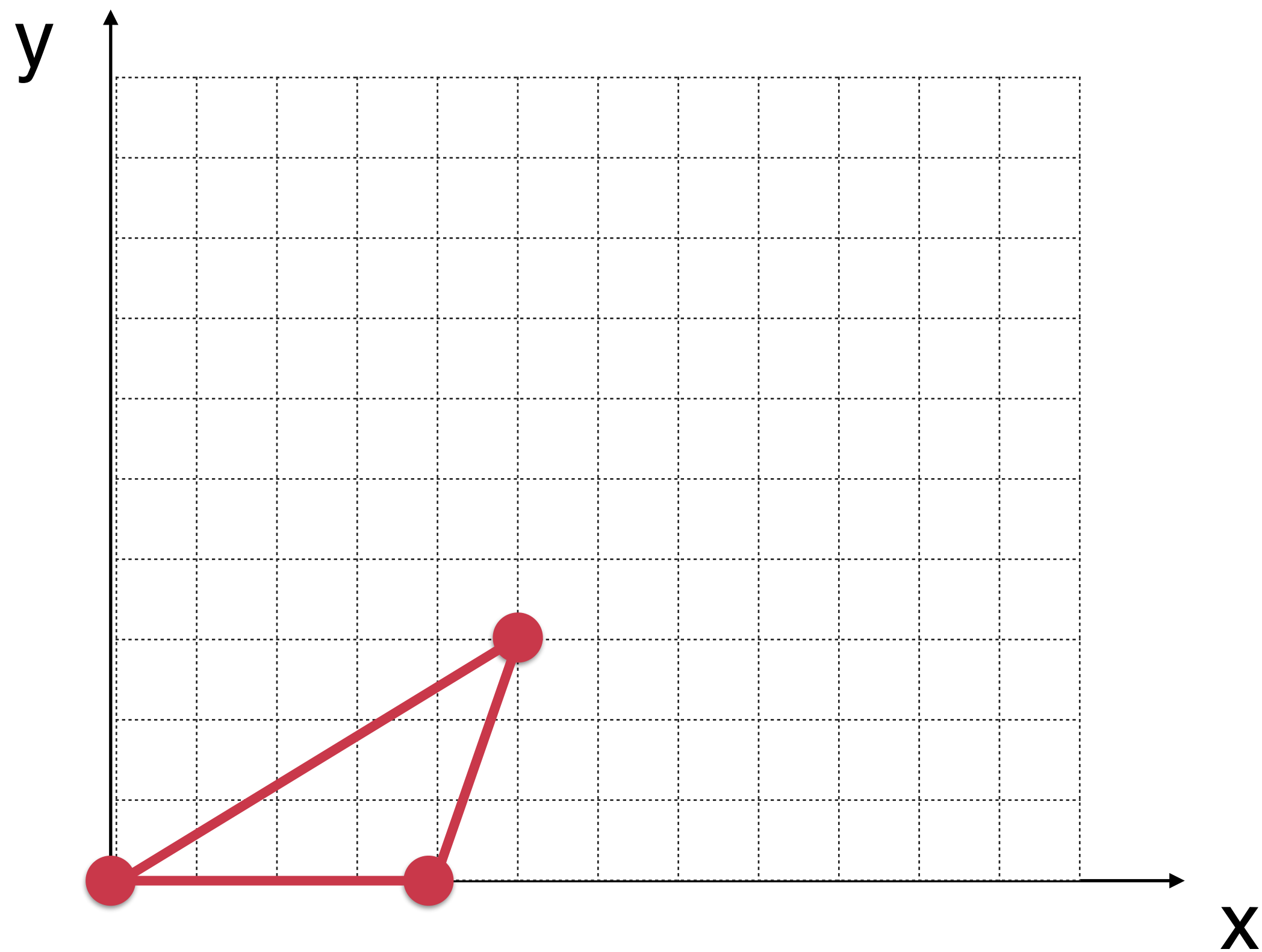


$$P = \begin{pmatrix} 0 & 0 \\ 4 & 0 \\ 5 & 3 \end{pmatrix}$$

$$2 \cdot P = \begin{pmatrix} 0 & 0 \\ 8 & 0 \\ 10 & 6 \end{pmatrix}$$

矩阵在图形变换中的应用

让每个点的横坐标扩大1.5倍，纵坐标扩大2倍



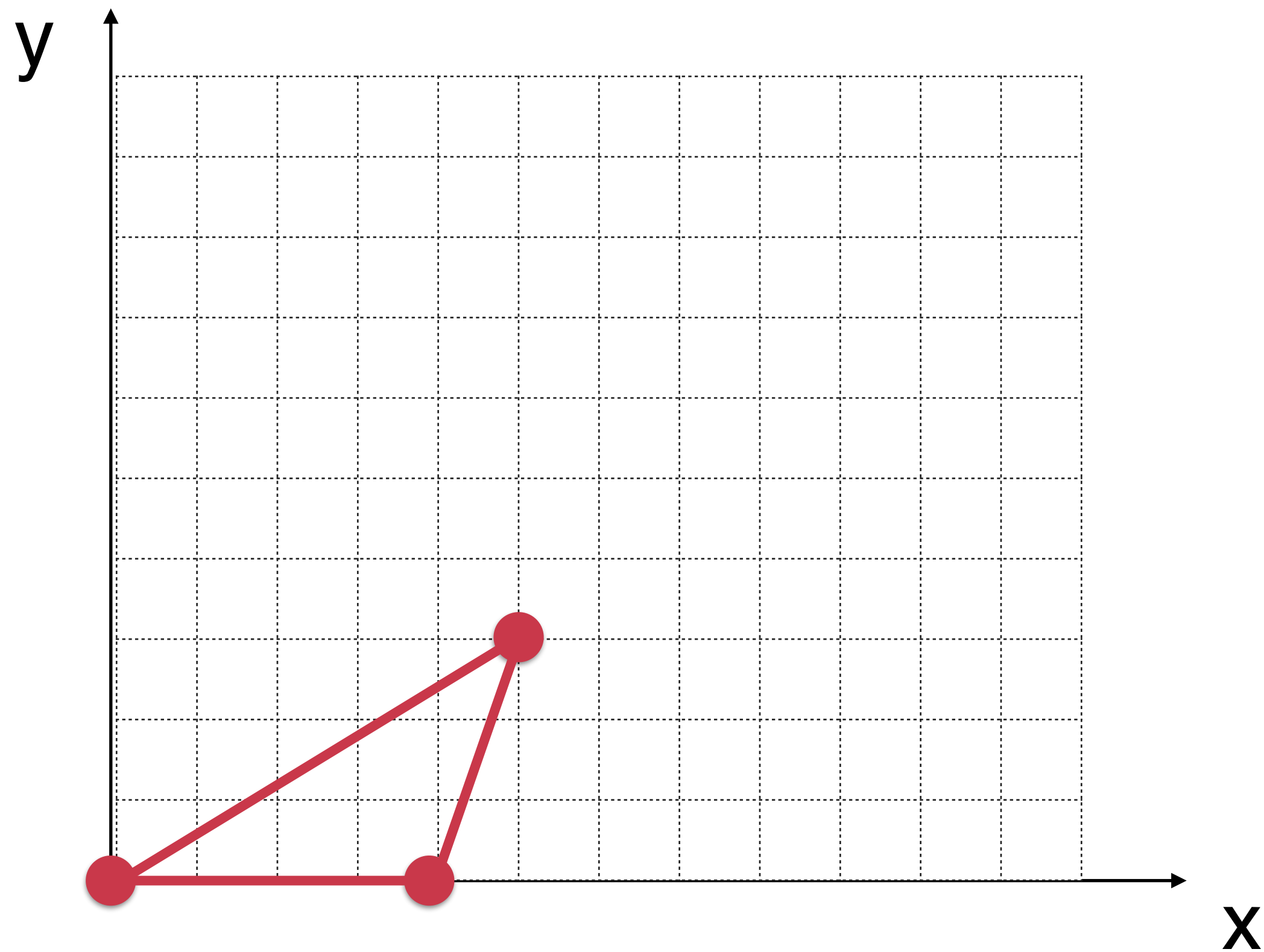
$$T \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1.5x \\ 2y \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$$

$$T = \begin{pmatrix} 1.5 & 0 \\ 0 & 2 \end{pmatrix}$$

矩阵在图形变换中的应用

让每个点的横坐标扩大1.5倍，纵坐标扩大2倍



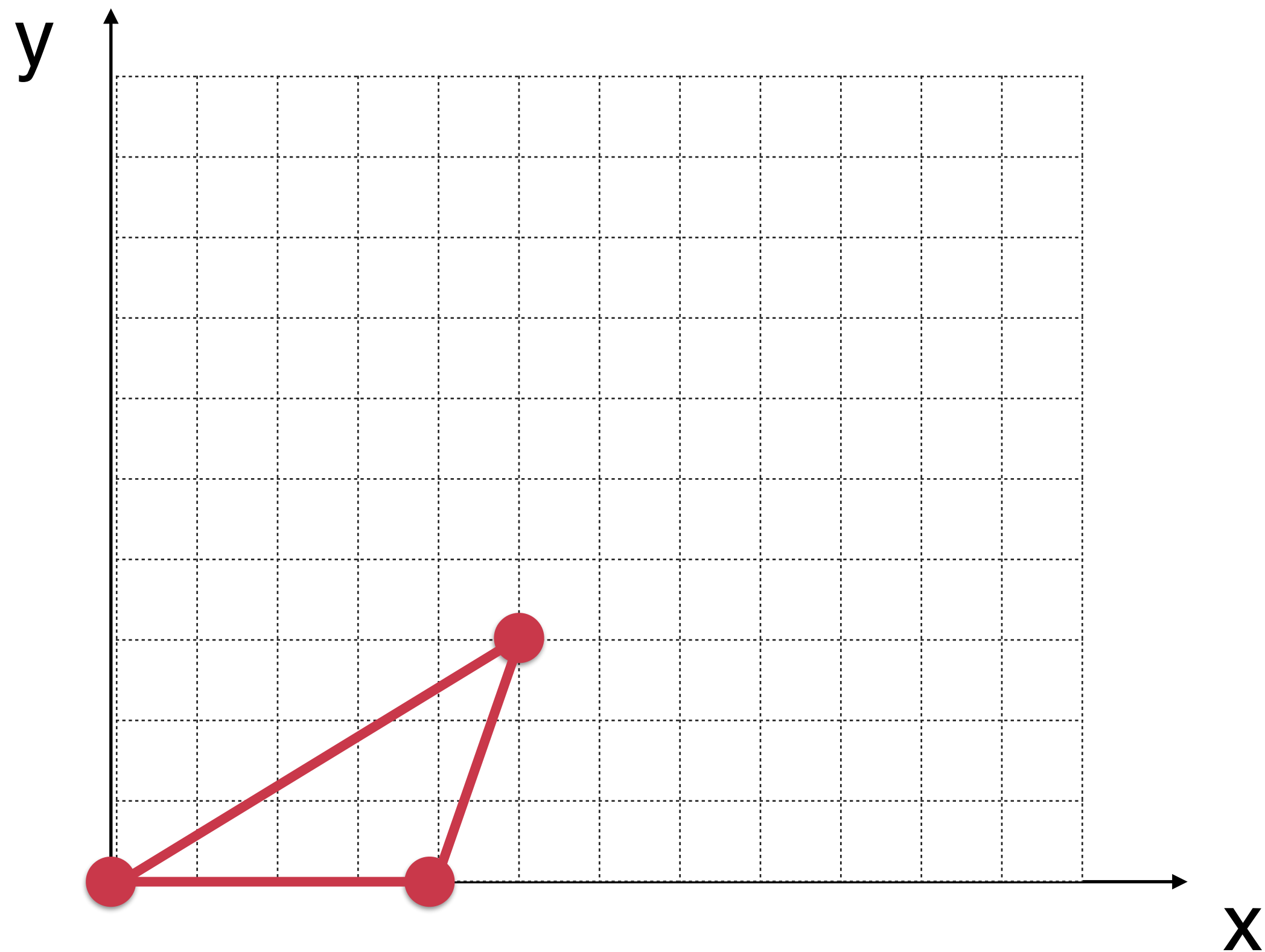
$$T \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1.5x \\ 2y \end{pmatrix}$$

$$T = \begin{pmatrix} 1.5 & 0 \\ 0 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1.5 & 0 \\ 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1.5x \\ 2y \end{pmatrix}$$

矩阵在图形变换中的应用

让每个点的横坐标扩大1.5倍，纵坐标扩大2倍



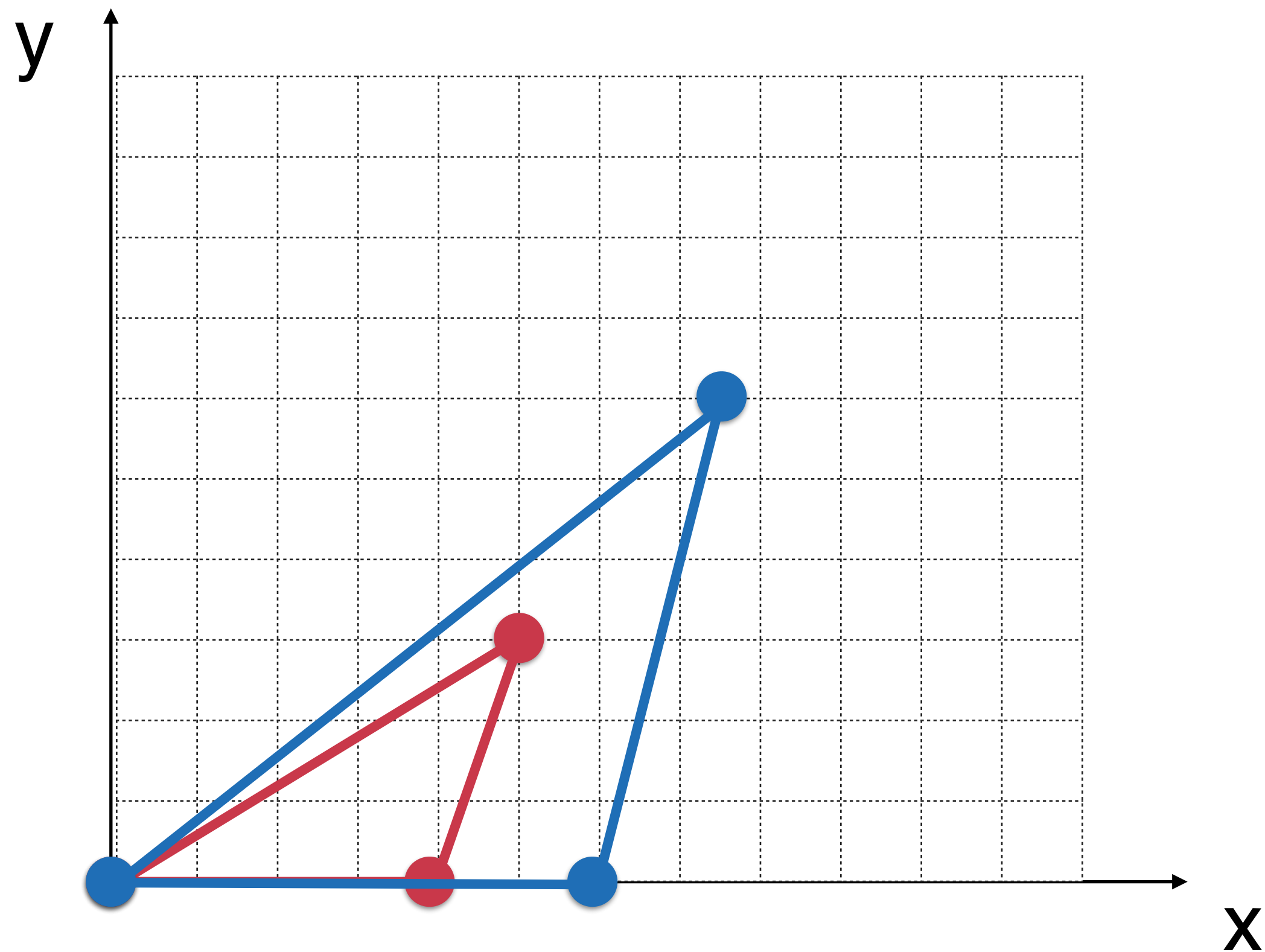
$$T = \begin{pmatrix} 1.5 & 0 \\ 0 & 2 \end{pmatrix}$$

$$P = \begin{pmatrix} 0 & 4 & 5 \\ 0 & 0 & 3 \end{pmatrix}$$

\downarrow \downarrow \downarrow
 p_1 p_2 p_3

矩阵在图形变换中的应用

让每个点的横坐标扩大1.5倍，纵坐标扩大2倍

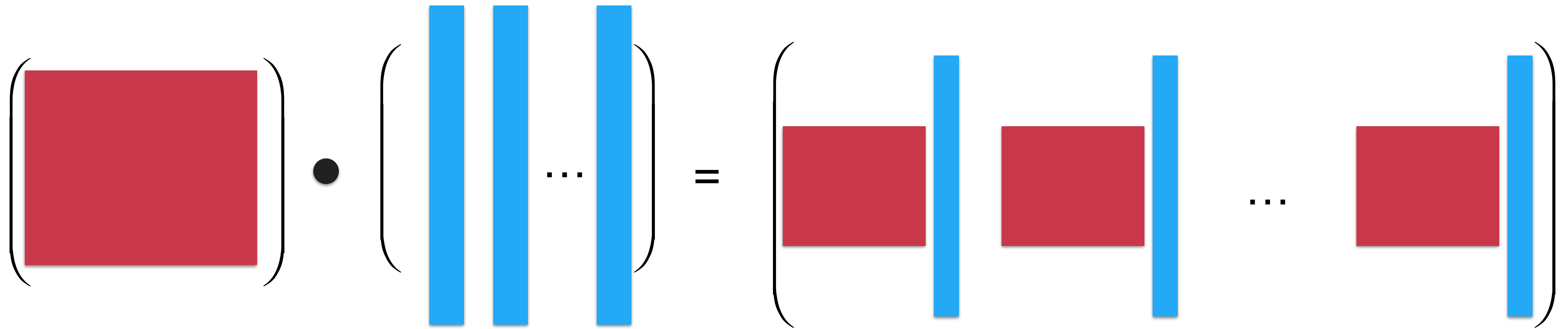


$$T = \begin{pmatrix} 1.5 & 0 \\ 0 & 2 \end{pmatrix} \quad P = \begin{pmatrix} 0 & 4 & 5 \\ 0 & 0 & 3 \end{pmatrix}$$

$$T \cdot P = \begin{pmatrix} 1.5 & 0 \\ 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} 0 & 4 & 5 \\ 0 & 0 & 3 \end{pmatrix} \\ = \begin{pmatrix} 0 & 6 & 7.5 \\ 0 & 0 & 6 \end{pmatrix}$$

矩阵乘法

$$T \cdot P = \begin{pmatrix} 1.5 & 0 \\ 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} 0 & 4 & 5 \\ 0 & 0 & 3 \end{pmatrix} = \begin{pmatrix} 0 & 6 & 7.5 \\ 0 & 0 & 6 \end{pmatrix}$$



矩阵A的列数必须和矩阵B的行数一致！

矩阵乘法

$$A \cdot B = A \cdot \begin{pmatrix} | & | & \dots & | \\ \overrightarrow{c_1} & \overrightarrow{c_2} & \dots & \overrightarrow{c_n} \\ | & | & & | \end{pmatrix} = \begin{pmatrix} | & | & \dots & | \\ A \cdot \overrightarrow{c_1} & A \cdot \overrightarrow{c_2} & \dots & A \cdot \overrightarrow{c_n} \\ | & | & & | \end{pmatrix}$$

$$\begin{pmatrix} - & \overrightarrow{r_1} & - \\ - & \overrightarrow{r_2} & - \\ & \dots & \\ - & \overrightarrow{r_m} & - \end{pmatrix} \cdot \begin{pmatrix} | & | & \dots & | \\ \overrightarrow{c_1} & \overrightarrow{c_2} & \dots & \overrightarrow{c_n} \\ | & | & & | \end{pmatrix} = \begin{pmatrix} \overrightarrow{r_1} \cdot \overrightarrow{c_1} & \overrightarrow{r_1} \cdot \overrightarrow{c_2} & \dots & \overrightarrow{r_1} \cdot \overrightarrow{c_n} \\ \overrightarrow{r_2} \cdot \overrightarrow{c_1} & \overrightarrow{r_2} \cdot \overrightarrow{c_2} & \dots & \overrightarrow{r_2} \cdot \overrightarrow{c_n} \\ \dots & \dots & & \dots \\ \overrightarrow{r_m} \cdot \overrightarrow{c_1} & \overrightarrow{r_m} \cdot \overrightarrow{c_2} & \dots & \overrightarrow{r_m} \cdot \overrightarrow{c_n} \end{pmatrix}$$

矩阵乘法

$$\begin{pmatrix} - & \overrightarrow{r_1} & - \\ - & \overrightarrow{r_2} & - \\ & \dots & \\ - & \overrightarrow{r_m} & - \end{pmatrix} \cdot \begin{pmatrix} | & | & & | \\ \overleftarrow{c_1} & \overleftarrow{c_2} & \dots & \overleftarrow{c_n} \\ | & | & & | \end{pmatrix} = \begin{pmatrix} \overrightarrow{r_1} \cdot \overleftarrow{c_1} & \overrightarrow{r_1} \cdot \overleftarrow{c_2} & \dots & \overrightarrow{r_1} \cdot \overleftarrow{c_n} \\ \overrightarrow{r_2} \cdot \overleftarrow{c_1} & \overrightarrow{r_2} \cdot \overleftarrow{c_2} & \dots & \overrightarrow{r_2} \cdot \overleftarrow{c_n} \\ \dots & \dots & & \dots \\ \overrightarrow{r_m} \cdot \overleftarrow{c_1} & \overrightarrow{r_m} \cdot \overleftarrow{c_2} & \dots & \overrightarrow{r_m} \cdot \overleftarrow{c_n} \end{pmatrix}$$

矩阵A的列数必须和矩阵B的行数一致！

A是m*k的矩阵； B是k*n的矩阵， 则结果矩阵为m*n的矩阵

矩阵乘法

A是m*k的矩阵； B是k*n的矩阵， 则结果矩阵为m*n的矩阵

矩阵乘法不遵守交换律！

$$A \cdot B \neq B \cdot A$$

很有可能根本不能相乘！

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \cdot \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$$

即使可以相乘， 结果也不一样

$$\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

实现矩阵的乘法

实践：实现矩阵和向量的乘法

实践：实现矩阵和矩阵的乘法

矩阵乘法的更多性质和矩阵的幂

矩阵乘法的性质

矩阵乘法不遵守交换律！ $A \cdot B \neq B \cdot A$

矩阵乘法遵守： $(A \cdot B) \cdot C = A \cdot (B \cdot C)$

$$A \cdot (B + C) = A \cdot B + A \cdot C$$

$$(B + C) \cdot A = B \cdot A + C \cdot A$$

对任意 $r \times c$ 的矩阵 A ，存在 $c \times x$ 的矩阵 O ，满足： $A \cdot O_{cx} = O_{rx}$

对任意 $r \times c$ 的矩阵 A ，存在 $x \times r$ 的矩阵 O ，满足： $O_{xr} \cdot A = O_{xc}$

矩阵乘法的性质

证明思路: $(A \cdot B) \cdot C = A \cdot (B \cdot C)$

假设A, B, C是任意矩阵:

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & \dots & & \dots \\ a_{m1} & a_{m2} & \dots & a_{mk} \end{pmatrix} \quad B = \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \dots & \dots & & \dots \\ b_{k1} & b_{k2} & \dots & b_{kn} \end{pmatrix} \quad C = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1l} \\ c_{21} & c_{22} & \dots & c_{2l} \\ \dots & \dots & & \dots \\ c_{n1} & c_{n2} & \dots & c_{nl} \end{pmatrix}$$

矩阵乘法的性质

矩阵的幂：
$$A^k = \underbrace{A \cdot A \cdot \dots \cdot A}_k$$

只有方阵才可以进行矩阵的幂运算！

$$A^0 ? \quad A^{-1} ? \quad A^{-2} ?$$

下一章见分晓：)

矩阵乘法的性质

$$(A + B)^2 \neq A^2 + 2AB + B^2$$

$$(A + B)^2 = (A + B) \cdot (A + B)$$

$$= A \cdot (A + B) + B \cdot (A + B)$$

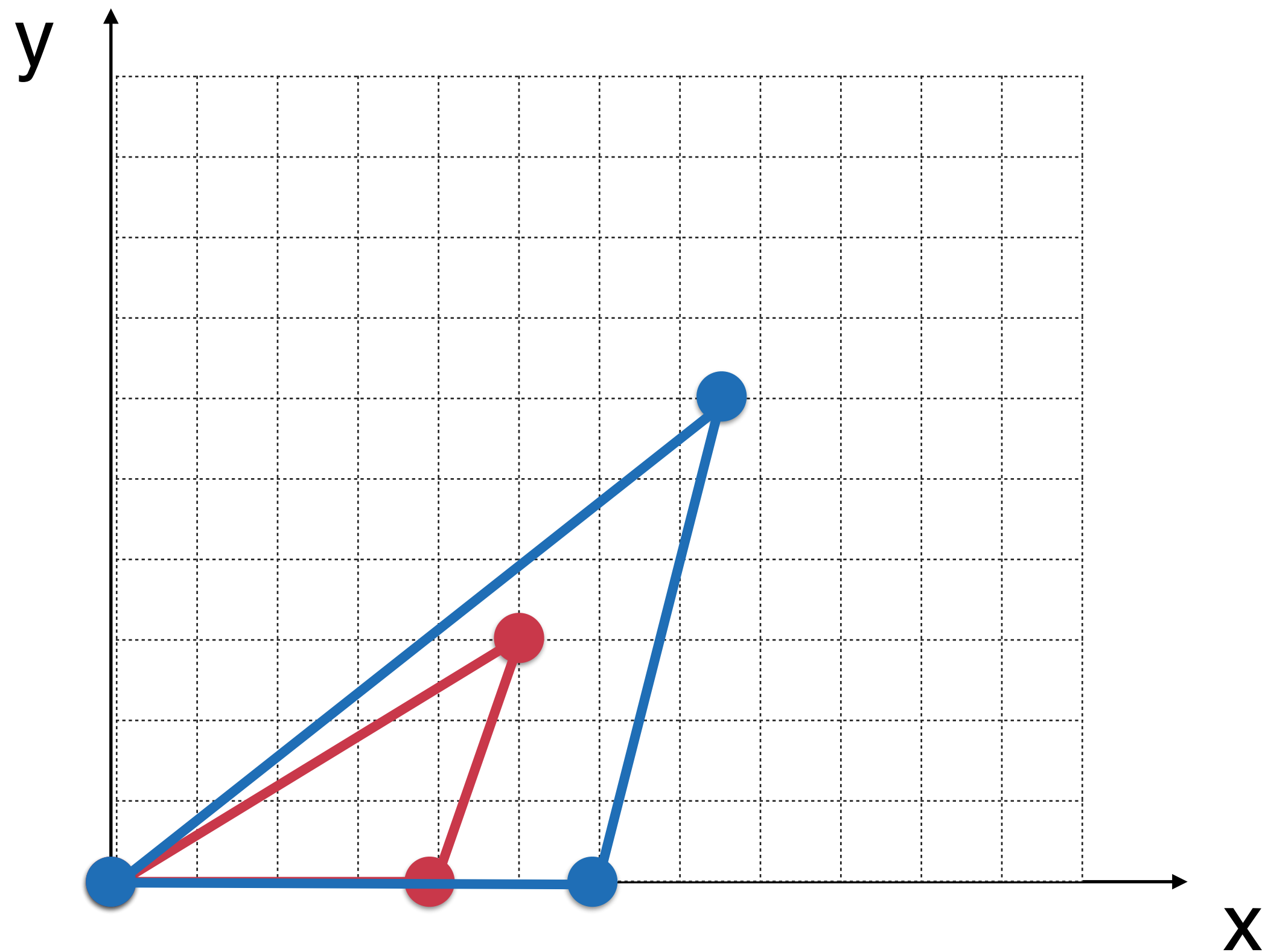
$$= A \cdot A + A \cdot B + B \cdot A + B \cdot B$$

$$= A^2 + A \cdot B + B \cdot A + B^2 \neq A^2 + 2AB + B^2$$

矩阵的转置

矩阵的转置

让每个点的横坐标扩大1.5倍，纵坐标扩大2倍



$$T = \begin{pmatrix} 1.5 & 0 \\ 0 & 2 \end{pmatrix} \quad P = \begin{pmatrix} 0 & 4 & 5 \\ 0 & 0 & 3 \end{pmatrix}$$

$$T \cdot P = \begin{pmatrix} 1.5 & 0 \\ 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} 0 & 4 & 5 \\ 0 & 0 & 3 \end{pmatrix} \\ = \begin{pmatrix} 0 & 6 & 7.5 \\ 0 & 0 & 6 \end{pmatrix}$$

矩阵的转置

$$T = \begin{pmatrix} 1.5 & 0 \\ 0 & 2 \end{pmatrix}$$

矩阵的转置：行变成列；列变成行

$$P = \begin{pmatrix} 0 & 0 \\ 4 & 0 \\ 5 & 3 \end{pmatrix}$$

$$P^T = \begin{pmatrix} 0 & 4 & 5 \\ 0 & 0 & 3 \end{pmatrix}$$

$$A = (a_{ij})$$

$$A^T = (a_{ji})$$

矩阵的转置

回忆：行向量和列向量 $(3,4)$ $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

由于横版印刷原因，使用符号： $(3,4)^T$

矩阵转置的性质

$$(A^T)^T = A$$

$$(A + B)^T = A^T + B^T$$

$$(k \cdot A)^T = k \cdot A^T$$

矩阵转置的性质

证明思路: $(A + B)^T = A^T + B^T$

假设A, B是任意矩阵:

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \quad B = \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \dots & \dots & & \dots \\ b_{m1} & b_{m2} & \dots & b_{mn} \end{pmatrix}$$

矩阵转置的性质

$$(A^T)^T = A$$

$$(A + B)^T = A^T + B^T$$

$$(k \cdot A)^T = k \cdot A^T$$

$$(A \cdot B)^T = B^T \cdot A^T$$

矩阵转置的性质

证明思路: $(A \cdot B)^T = B^T \cdot A^T$

A是m*k的矩阵, B是k*n的矩阵

AB是m*n的矩阵, AB的转置是n*m的矩阵

A的转置是k*m的矩阵, B的转置是n*k的矩阵

(B的转置)(A的转置)是n*m的矩阵