

Home Work 1

Com S 435X/535X

Due Feb 2: 12:10pm

1. For each of the following experiments, what is the sample space?
 - (a) Toss a biased coin (probability of head $2/3$) n times.
 - (b) Throw a balanced dice n times.
 - (c) Toss a fair coin. If the outcome is Head throw a balanced dice; otherwise toss another fair coin.
 - (d) Let $S = \{1, 2, \dots, n\}$, and $T = \phi$. Repeat the following step as long as $|T| < n$: Uniformly at random pick a number n from S ; if $n \notin T$, then $T = T \cup \{n\}$.
2. Suppose that we toss a balanced dice n times. For each of the following random variables compute the expectation.
 - (a) Let X denote the number of times the outcome is even.
 - (b) Y is the sum of all outcomes.
3. Suppose that there are n balls numbered 1 through n and there are n bins numbered 1 through n . For each of the following random variables compute the expectation.
 - X is number of bins with exactly 1 ball.
 - Y is the cardinality of the following set
$$\{\langle i, j, k \rangle \mid i < j < k, \text{ Balls numbered } i, j \text{ and } k \text{ are in the same bin}\}.$$
 - Z is the cardinality of the following set
$$\{i \mid \text{Ball numbered } i \text{ is not in Bin numbered } i\}.$$
4. Given an undirected graph G , we say that three vertices i, j and k form a *wedge* if the number of edges in the graph induced by i, j and k (Graph obtained by removing all vertices and incident edges other than i, j and k) is exactly 2. Suppose that we created a n -vertex graph G via the following random process: For every pair of vertices place an undirected edge with probability $1/4$. What is the expected number of edges in this graph? What is the expected number of wedges in this graph?

5. Suppose the we toss a fair coin n times. What is the probability that we see less than $\frac{n}{2} - 5 \log n$ heads? Compute and upper bound on the is probability using each of Markov, Chebyshev and Chernoff bounds. You may use the fact that if two random variables X and Y are independent then $Var(X + Y) = Var(X) + Var(Y)$.

Guidelines:

- You are allowed to discuss with your classmates, while doing your homework. However, I strongly suggest that you think about the problems on your own before discussing.
- Definition of *classmates*: Students who are taking CS 435X/535X in Spring 15.
- However, You should write the final solutions alone, without consulting your classmates. Your writing should demonstrate that you understand the proofs completely. If I suspect that you wrote the proofs without understanding, I may ask you to explain the proofs to me in person. In such scenarios, failure to explain proofs will be taken as evidence of *academic dishonesty*.
- For each problem, you should acknowledge the students with whom you discussed. This will not affect your grade. Failure to acknowledge is considered *academic dishonesty*, and it will affect your grade.
- Any student found guilty of academic dishonesty will receive “F” in the. course.
- When proofs are required, make them both clear and rigorous. Do not hand wave. Even when proofs are not required, you should justify your answers and explain your work.
- Late homeworks are not accepted.