AMATH 581 Autumn Quarter 2011

Homework 5: Reaction-Diffusion Systems

DUE: Tuesday, November 22, 2011 (actually Wednesday, 11/23 at 3 a.m.)

Consider the $\lambda - \omega$ reaction-diffusion system

$$U_t = \lambda(A)U - \omega(A)V + D_1 \nabla^2 U$$
$$V_t = \omega(A)U + \lambda(A)V + D_2 \nabla^2 V$$

where $A^2 = U^2 + V^2$ and $\nabla^2 = \partial_x^2 + \partial_y^2$.

Boundary Conditions: Consider the two boundary conditions in the $x-$ and $y-$directions:

- Periodic
- No flux: $\partial U/\partial n = \partial V/\partial n = 0$ on the boundaries

Numerical Integration Procedure: The following numerical integration procedures are to be investigated and compared.

- For the periodic boundaries, transform the right-hand with FFTs
- For the no flux boundaries, use the Chebychev polynomials

You can advance the solution in time using ode45.

Initial Conditions Start with spiral initial conditions in U and V.

```
[X,Y]=meshgrid(x,y);
m=1; % number of spirals
u=tanh(sqrt(X.^2+Y.^2)).*cos(m*angle(X+i*Y)-(sqrt(X.^2+Y.^2)));
v=tanh(sqrt(X.^2+Y.^2)).*sin(m*angle(X+i*Y)-(sqrt(X.^2+Y.^2)));
```

Consider the specific $\lambda - \omega$ system:

$$\lambda(A) = 1 - A^2$$
$$\omega(A) = -\beta A^2$$

Look to construct one- and two-armed spirals for this system. Also investigate when the solutions become unstable and "chaotic" in nature. Investigate the system for all three boundary conditions. Note $\beta > 0$ and further consider the diffusion to be not too large, but big enough to kill off Gibbs phenomena at the boundary, i.e. $D_1 = D_2 = 0.1$.

ANSWERS: With $x, y \in [-10, 10]$, $n = 64$, $\beta = 1$, $D_1 = D_2 = 0.1$, $m = 1$, $tspan = 0 : 0.5 : 4$ and u stacked on top of v, write out the solution of your numerical evolution from ode45 with periodic boundaries as A1.dat. (NOTE: your solution will be in the Fourier domain when your write it out.)

ANSWERS: With $x, y \in [-10, 10]$, $n = 30$, $\beta = 1$, $D_1 = D_2 = 0.1$, $m = 1$, $tspan = 0 : 0.5 : 4$ and u stacked on top of v, write out the solution of your numerical evolution from ode45 with no-flux boundaries as A2.dat. (NOTE: be sure to remember that you have to rescale the problem to -1 to 1 for cheb.m.)