582631 Introduction to Machine Learning
Period II, Autumn 2013

Exercise #1 (Matlab/Octave/R practice)
Two alternative sessions:
Tuesday, 29 October, 12-14 in B221
Friday, 1 November, 12-14 in B221.
(No preparation necessary, no handing-in of solutions, no points for exercises.)

Exercise 1. Generate 100 samples from a normal distribution with zero mean and unit variance.

(a) Visualize the data by drawing a histogram.

(b) Sort the data points into increasing order.

(c) Draw a line plot of points (x_i, y_i) for $i = 1 \dots 100$ where x_i is the i:th data point (in the sorted order) and y_i is equal to $i/100$.

(d) Can you tell what you have drawn?

MATLAB/OCTAVE: randn, hist, sort, ':', plot;
R: rnorm, hist, sort, ':', seq, plot, x11;

Exercise 2. Generate a random matrix (dimensions for example 10×5) and store it into a file. Write a function that

- Takes a filename as an argument.
- Reads a data matrix X from the given file.
- Outputs row and column sums of X as bar plots. Try to put both plots into the same window.
- Returns two values: the dimensions of the matrix as a vector and the sum of all elements.

Test your function by calling the function with the name of the file.

MATLAB/OCTAVE: save, load, sum, bar, figure, subplot, size;
R: save, load, rowSums, colSums, barplot, par(mfcol=c(2,1)), dim, sum, list;

Exercise 3. The law of large numbers states that the average of a series of i.i.d. samples approaches the expectation of the generating distribution as the number of samples goes to infinity. More formally, if all x_n are i.i.d. sampled from some random variable X we have

$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} x_n \quad = \quad E(X)$$

with probability 1. Let's verify this theorem empirically.

(a) Generate samples from a normal distribution with mean 0 and variance 1.

(b) Plot N vs. the empirical mean $\frac{1}{N}\sum_{n=1}^{N} x_n$ in logarithmic scale.

(c) How fast does the empirical mean approach the expectation?

MATLAB/OCTAVE: randn, sum, plot;
R: rnorm, sum, plot;

Exercise 4. The expectation of a random variable is a linear operator:

$$\begin{aligned} E(\alpha X) &= \alpha E(X) \\ E(X+Y) &= E(X)+E(Y) \end{aligned}$$

Let F^n be a random variable describing the number of fixed points (elements that don't change place) in a random permutation of n elements.

(a) Write a function that takes n as an argument and returns a sample of lenght k from the distribution of F^n.

(b) Generate samples from F^n with a few different values of n.

(c) Draw histograms of the different samples.

(d) Can you guess what the expected value $E(F^n)$ of the distribution might be?

(e) Use the linearity of the expectation operator to calculate $E(F^n)$. (Hint: Take another random variable F_i^n , such that $F_i^n = 1$ when the i:th element stays fixed, otherwise $F_i^n = 0$. Now we can write $F^n = \sum_{i=1}^{n} F_i^n$.)

MATLAB/OCTAVE: randperm;
R: sample, sum, hist;

Exercise 5.

(a) Write a function that

- takes 6 arguments: a sample size n, a probability π ($0 \leq \pi \leq 1$), and the expectatations and variances of two univariate normal distributions.
- returns two n-element vectors $\mathbf{x}$ and $\mathbf{y}$, such that for each $i \in \{1, \ldots, n\}$ holds:
  With probability π, $\mathbf{y}[i] = 0$ and $\mathbf{x}[i]$ is a point from the first normal distribution
  otherwise $\mathbf{y}[i] = 1$, and $\mathbf{x}[i]$ is a point from the second normal distribution.
  (The distribution of $\mathbf{x}$ is called a "Gaussian mixture".)

(b) Visualize the data generated by the function with different values of the arguments.

(c) Can you come up with any example that follows such a distribution?

MATLAB/OCTAVE: hist, plot, for;
R: runif, rnorm, plot;