CreditMetrics Revisited: A Lévy alpha-stable distribution modeling of loans for standalone exposure

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ABSTRACT

This paper deals with a modified Credit Metrics approach by considering all possible credit migration paths and their associated probabilities of state. This leads to the development of a realistic assumption on the distribution of loan values as a stable distribution. We estimate year-ahead future loan values and compute credit risk by employing the statistical technique, CVaR as a risk measure. We subject our model to a numerical experiment and provide financial insights.

KEYWORDS

CreditMetrics; stable distribution; value of loans; credit risk

1. INTRODUCTION

Exposure to credit risk has over the years become one of the valuable tasks undertaken by financial corporations (Li et al. 2014; Noman et al. 2015). Andersson et al. (2001) defines credit risk as the risk of one party not meeting their obligations in full on the due date or at any time thereafter. Credit risk arises because of variability in the value of the portfolio as a result of credit quality changes. We therefore expect that any credit risk measure should have a reflection of this variability. Undoubtedly, the quality of credit can alter over a time period. Migration in terms of credit is determined by the likelihood of moving from one rating to another, including payment defaults within a risk horizon, which is mostly one year. A well-known approach for credit measurement that applies the technique of migration analysis is CreditMetrics. J.P Morgan and its associates introduced CreditMetrics in 1997 to estimate forward distribution of values for nontradable assets like loans and bonds. Specifically, Gupton et al. (1997) define CreditMetrics as “a tool for assessing portfolio risk due to changes in debt value caused by changes in obligor credit quality”.

Unfortunately, there exist some restrictions with the CreditMetrics approach, which include but not limited to the following. The first limitation is that credit returns are highly skewed and fat tailed therefore not normally distributed (Smithson 2003; Tavakoli 2004) as shown in Figure 1. Saunders & Allen (2010) found that loans have acutely truncated upside returns and long downside risks and that the actual loan portfolio value distribution shows negative skew or a long-tailed downside. Thus, the mean and standard deviation measure from normal or Gaussian distribution are not enough to fully provide insights into a credit's return distribution. Secondly, CreditMetrics approach does not consider all possible credit migration paths and their associated probabilities. It only considers a sample out of the many possibilities. It assumes that once an asset experiences an upgrade or a downgrade in its credit rating, it remains in that state till the end of its maturity which is not true in real life situations.

Gabriel & Lau (2014) suggested the skewed student's *t* or the stable distribution to describe bond return distributions of debt capital markets. Acıtaş et al. (2013) used a skew extension of the student's *t* distribution to model data sets that have skewness and heavy-tails. Hoechstoetter et al. (2005) found out that skewed student's *t* distribution is outperformed by stable distribution in fitting returns of stocks. Stable distribution was used in fitting US Corporate bond returns by Rachev et al. (2003). We make an informed decision on loan returns distribution based on existing studies that credit returns are skewed, or lopsided, with a long fat tail and assume that loan returns can be described by a stable distribution.

In this paper, we propose an extension of the CreditMetrics approach by considering all possible credit migration paths and their associated probabilities of state. This leads to the development of a realistic assumption on loan values distribution. We deal with the valuation and credit risk estimation of loans in CreditMetrics

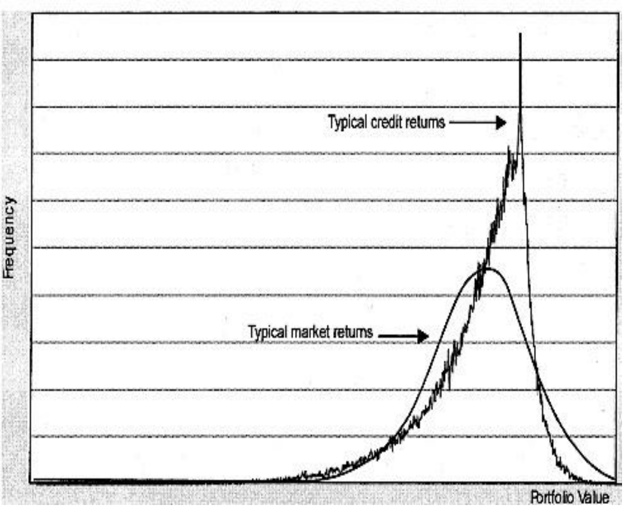

Figure 1. Comparison of credit returns distribution and Market returns (Source: CIBC)

framework assuming that year-ahead loan values follow a stable distribution. We proceed as follows: first, we evaluate real world probabilities of default using transition matrix, second, we specify credit risk period, typically 1 year, third, we revalue the loan based on all possible credit migration paths and in case of default, the introduction of recovery rates, and lastly, an estimation of credit risk is performed using a statistical technique known as Conditional Value at Risk (CVaR) approach (see Section 3 to deal with the skewed return-loss distribution. It is interesting to note that CreditMetrics produces two VaR (Value at Risk) measures based on the basis of normal distribution of loan values and actual distribution of loan values.

We subject the valuation and credit risk estimation of loans under a theoretical perspective as well as a thorough simulation analysis. Specifically, we first come up with a mathematical formalization of the valuation and credit risk estimation of loans considering all possible migration paths including default and then study it by applying numerical experiments to provide more in-depth details.

The paper is organized as follows. Next section presents finance related properties of stable distribution and its parameterizations. Section 3 describes our modified CreditMetrics approach used for valuation of loans. Results and concluding remarks are in section 5.

2. PROBABILITY DISTRIBUTION OF CREDIT RETURNS

In this section, we describe some finance related properties of stable distributions and its parameterizations.

2.1 Definition, properties and parameters of stable distributions

According to (Nolan 2009), a stable distribution satisfies the following property: Let V_1 and V_2 be independent copies of V, then a random variable V is said to be stable if for any constants $a > 0$ and $b > 0$ the random variable $aV_1 + bV_2$ has the same distribution as $cV + m$ for some constants $c > 0$ and $m \in \mathbb{R}$. The distribution is said to be strictly stable if this holds with $m = 0$. Thus

$$aV_1 + bV_2 \overset{\text{m}}{=} cV + m \qquad (1)$$

Stable distributions ($V \sim S_\alpha(\beta, \gamma, \lambda)$) constitute four parameters; characteristic component or index of stability, $0 < \alpha \leq 2$, a skewness parameter, $-1 \leq \beta \leq 1$, a scale parameter, $0 < \gamma < \infty$ and $\lambda \in \mathbb{R}$ for the location parameter. A stable random variable (V) can be described by its characteristic function as (Nolan 2009; Voit 2013)

$$\varphi_V(t; \alpha, \beta, \gamma, \lambda) = \exp[it\lambda - |\lambda t|^\alpha (1 - i\beta\, sign(t)\Phi)] \qquad (2)$$

where $\Phi = \tan(\pi\alpha/2)$, if $\alpha \neq 1$ and $\Phi = \left(-\frac{2}{\pi}\right) \log t$, if $\alpha = 1$.

When the index of stability, $\alpha = 2$, then the stable distribution becomes a Gaussian distribution. However, empirical evidence have revealed that most financial return data are modeled by stable distribution with $\alpha \in (1,2)$. The four parameters in stable distributions have the following dynamics. The smaller α is, the higher the peak of the density and tails are heavier. The location parameter is a measure of mean (μ) of the distribution when $\alpha > 1$. If $\beta < 0$, the stable distribution is skewed to the left. If $\beta > 0$, then the stable distribution is skewed to the right. If $\beta = 0$, the distribution of V is symmetric. The heavy tail characteristic causes the variance of stable (non-Gaussian) distribution to be infinite for all $\alpha < 2$, but when $\alpha = 2$ (Gaussian distribution), the variance becomes finite ($2\lambda^2$).

Over the year's estimation of the parameters of the stable distribution have been extensively studied (Gamrowski & Rachev 1994; Rachev & SenGupta 1993; Chobanov et al. 1996; Peng 2001 among others). Using skewed-t distribution as an auxiliary model, Garcia et al. (2011) estimated the parameters of an α-stable distribution with indirect inference. Meraghni & Necir (2007) via extreme value approach estimated the scale parameter, γ of a L$\acute{e}$vy-stable distribution.

The additivity property of stable distributions is a principal advantage for the use of stable laws for credit returns. In summary, if $\alpha > 1$, then μ is the mean vector, $\mu = E(V)$, otherwise infinite and the variance of any non-Gaussian stable distribution is infinite.

We introduce the concept of percentile levels of loss distributions and suggest Conditional Value-at-Risk (CVaR) as a measure of credit risk in this study. CVaR has properties that solve the deficiencies of variance and Value-at-Risk (VaR).

3. CREDITMETRICS APPROACH

This section presents the methodology employed by our modified version of CreditMetrics to compute the forward distribution of loan values for a single or stand alone exposure and also measure credit risk. We develop a framework to include all feasible migrations and scenarios.

3.1 Credit Rating Transition

Credit rating transition is the migration of loan/bond from one rating to another (default inclusive) over a risk horizon. In our study, credit risk does not only emanate from default but including changes in loan value as a result of upgrades and downgrades. Thus, it is imperative to estimate not only the probability of default but also the likelihood of migration to other ratings. From historical data, a transition matrix is constructed by CRISIL as shown in Table 1.

Table 1

CRISIL's one-year average transition rates (1993-2014)(%)

Rating	AAA	AA	A	BBB	BB	B	C	D
AAA	98.23	1.54	0.23	0.00	0.00	0.00	0.00	0.00
AA	17.04	78.52	3.70	0.74	0.00	0.00	0.00	0.00
A	9.59	15.07	71.23	4.11	0.00	0.00	0.00	0.00
BBB	4.02	3.29	15.69	76.28	0.00	0.37	0.37	0.00
BB	11.11	22.22	22.22	22.22	22.22	0.00	0.00	0.00
B	0.00	0.00	0.00	0.00	0.00	50.00	0.00	50.00
C	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100

Source: CRISIL Default Study 2014

The probability that a B borrower will remain at B over the next year is 50%. There exist a probability that the borrower will upgrade (to BBB), downgrade (to C) or at worse, default (D).

3.2 Valuation of loans

In this section, we compute the year-ahead forward value of loans and also estimate the credit risk. In the worst case of loan default, two inputs are important, the collateral status of the loan and recovery rate.

The value of the loan is dependent on the recovery rate by activating securities proposed for the loan that is also dependent on these parameters; collateral status, mean recovery rate and volatility of recovery rate.

Calabrese (2008) defines recovery rate as the payback quota of the loan. Empirical research works of recovery rates have mostly focused on bonds. The difficulty in obtaining loan recovery data given the privacy nature of debt contacts can be one of the attributes. Asarnow & Edwards (1995) and Schuermann (2004) studies defaulted loans and found out that the recovery rates distribution is that a of bimodal one. Acharya et al. (2007) suggests 81.12% for bank loans after analyzing data on defaulted firms in the USA from 1982 to 1999. Using Moody’s Ultimate Recovery Database , Khieu et al. (2012) estimates a model for bank loan recoveries by employing factors like loan characteristics, borrower characteristics and so on. They report that recovery rates of 84.14% for all bank loans. For the purpose of our study, we consider the loan recovery rates for loans secured by all firm assets and unsecured loans obtained from Khieu et al.'s (2012) paper.

Table 2

Loan recovery rates by collateral status (%)

Collateral status	Mean	Standard deviation
Secured by all firm assets	62.61	48.40
Unsecured	7.70	26.67

Source: Khieu et al. (2012)

We now revalue the loan on the basis of considering all the possible migration paths, from the one-year forward rates and cash flows for the suitable rating category. Changes in the value of loans are as a result of upgrade or downgrade in credit rating. The magnitude of these changes can be evaluated by estimating forward zero curves for each rating.

Consider $f_r^{1,k}$ as $k-1$ year forward rate with credit rating r so for year one; we have $f_r^{1,2}$ and so on. By this logic, the entries in Table 3 i.e. $f_r^{1,k}$, are spot rates from today. Consider $\tilde{f}_r^{j,j+1}$ as one-year forward rate with credit rating r_i from the end of the jth year to the end of the $j+1$th year. Then, the one-year forward rate can be computed by

$$\tilde{f}_{r_1}^{1,2} = f_{r_1}^{1,2}, j = 1$$

$$\left(1 + f_{r_j}^{j,j+1}\right)^j = \left(1 + \tilde{f}_{r_j}^{j,j+1}\right)\left(1 + f_{r_j}^{1,j}\right)^{j-1}, \; j = 2, \dots, z-1 \qquad (3)$$

Table 3

One-year forward zero curve for credit ratings (%)

Ratings	$f_r^{1,2}$	$f_r^{1,3}$	$f_r^{1,4}$	$f_r^{1,5}$
AAA	3.60	4.17	4.73	5.12
AA	3.65	4.22	4.78	5.17
A	3.72	4.32	4.93	5.32
BBB	4.10	4.67	5.25	5.63
BB	5.55	6.02	6.78	7.27
B	6.05	7.02	8.03	8.52
C	15.05	15.02	14.03	13.52

Source: CreditMetrics, JP Morgan

The discount factor (d_{ik}) of cash flow from the end of the kth year to the end of the first year for loan type i can be estimated by

$$d_{i1}^{-1} = 1, \qquad k = 1$$

$$d_{ik}^{-1} = \prod_{j=1}^{k-1}\left(1 + \tilde{f}_{r_j}^{j,j+1}\right) = \prod_{j=1}^{k-1}\frac{\left(1+f_{cr_i}^{j,j+1}\right)^{j}}{\left(1+f_{r_j}^{1,j}\right)^{i-1}}, \quad k = 2,\dots,z \qquad (4)$$

The year-ahead loan value, V_{im}, of a unit capital invested in the ith loan is the summation of discounted cash flows of each year in relation to feasible rating migration paths m till maturity z. Thus the year-ahead loan value, V_{im}, of a unit capital invested in ith with a non-default rating migration path m is

$$V_{im} = \sum_{k-1}^{z-1} \theta_i d_{ik} + (1 + \theta_i)\, d_{iz} \tag{5}$$

where θ_i is interest rate of the ith loan.

When the ith loan defaults before or at maturity, $s = 1,2, \dots, z$ then the year-ahead loan value, V_{idm}, of a unit capital invested in ith with a default rating migration path, dm, is defined by

$$V_{idm} = \sum_{k-1}^{z-1} \theta_i d_{ik} + R_i d_{iz} \tag{6}$$

where R_i is recovery rate of the ith loan as shown in Table 2.

The ith loan with maturity z has 7^z non-default feasible rating migration paths and the ith loan with maturity z, has $Q = \sum_{s=1}^{z} 7^{s-1}$ default feasible rating migration paths.

Considering rating migration paths satisfying Markov property, then the migration probability of the ith loan with m non-default path is

$$P_{im=} \prod_{j=0}^{z-1} P_{r_j, r_{j+1}}, \quad m = 1, \dots, 7^z \tag{7}$$

and for default path

$$P_{idm} = \prod_{j=0}^{s-1} P_{r_j, r_{j+1}}, \quad m = 1, \dots, \sum_{s=1}^{z} 7^{s-1} \tag{8}$$

where $P_{r_j, r_{j+1}}$, signify the probability of migration from r_j credit rating at the end of the jth year to r_{j+1} credit rating at the end of $j + 1$th year. Following the assumption of year-ahead future loan value of unit capital invested in the ith loan following α-stable distribution (non -

Gaussian distribution) with $\in (1,2)$, then expected year-ahead future loan value of unit capital invested in ith loan is

$$E(V_i) = \sum_{m=1}^{7^z} V_{im} P_{im} + \sum_{m=1}^{Q} V_{idm} P_{idm} \qquad (9)$$

3.3 Credit Risk Estimation

Now that we know the likelihood of all feasible outcomes - all upgrades, downgrades including default and not a sample as used by Gupton et al. (2007) - and the distribution of loan value within each outcome, we can estimate the credit risk. As already established, loan values are not symmetrically distributed and heavily tailed. We cope with the skewness factor by using a statistical technique known as Conditional Value-at- Risk (CVaR) as the risk measure. Also, with the assumption of loan value data fitting an α-stable (non-Gaussian) model results in $\alpha \in (1,2)$, which suggests existence of the first moment and infinite second moment (variance) leads us to employ percentile levels as an appropriate risk measure.

This section describes the concept of Conditional Value-at-Risk (CVaR). VaR has several limitations as a risk measure. First, its non-subadditivity property i.e. the risk measured VaR of a portfolio of two assets might be greater than the sum of the risks of the two individual portfolios (Arzner 1997; Heath et al. 1999). Second, from generated scenarios, VaR is a function of nonsmooth and conconvex property. In this regard, the VaR function tends to have multi-stationary points making it hard to control and optimize. Last, the computation of VaR does not take into account the magnitude of losses above the VaR value (Grootveld & Hallerbach 2000).

A set of properties for a risk measure were proposed by Heath et al. (1999) to solve the limitations of Value-at-risk. The risk measures satisfying the desirable properties are termed coherent risk measures. For a risk measure ρ to be termed a coherent measure of risk, it must satisfy the monotonicity property (If $A \geq 0$, then $\rho(A) \leq 0$), the subadditivity property ($\rho(A+B) \leq \rho(A) + \rho(B)$), the positive homogeneity property ($\rho(cA) = c\rho(A),\ c > 0$) and the translational invariance ($\rho(A+c) \leq \rho(A) - c,\ c\epsilon\mathbb{R}$) where A and B are random variables.

If $V \in L^P(\mathcal{F})$ is the year-ahead future loan value and 0$< 1-\varepsilon < 1$, then we define the CVaR as

$$CVaR_{1-\varepsilon} = \frac{1}{1-\varepsilon}\int_{0}^{1-\varepsilon} VaR_{\eta}(V)d\eta$$

where $VaR_{\eta}(V) = \min\{(v|P(-V \leq v) \geq \eta\}$ is the Value-at-risk.

Equivalently, the above equation can be written as

$$CVaR_{1-\varepsilon} = -\frac{1}{1-\varepsilon}\left(E\left[V\,1_{\{V\leq v_{1-\varepsilon}\}}\right] + v_{1-\varepsilon}(1-\varepsilon-P[V\leq v_{1-\varepsilon}])\right)$$

where $v_{1-\varepsilon} = \inf\{v \in \mathbb{R}: P(V \leq v) \geq 1-\varepsilon\}$ is the lower $(1-\varepsilon)$ -quantile and $1_A = \begin{cases} 1 & if\ v \in A \\ 0 & else \end{cases}$ represents the indicator function (Rockafellar & Uryasev 2000). Therefore, the dual representation is $CVaR_{1-\varepsilon} = inf_{M\in Q_{1-\varepsilon}} E^M[V]$

where $Q_{1-\varepsilon}$ describes the set of measures of probability that absolutely continuous to the physical measure P. Given the nature of the stable distribution (non-Gaussian) for V, then the CVaR is defined by

$$CVaR_{1-\varepsilon}(V) = E(-V|-V \geq VaR_{1-\varepsilon}(V)) \qquad (10)$$

CVaR measures the expected amount of losses in the tail of the distribution of possible losses, beyond portfolio VaR. VaR gives a range of potential losses and CVaR gives us an average expected loss that is difficult to account for.

4. RESULTS OF THE APPLICATION AND CONCLUDING REMARKS

Using our modified CreditMetrics approach, consider an AA rated loan of unit capital to be repaid in 2 years at an annual interest rate of 7%. We compute expected year-ahead future loan value of the 2-year AA rated loan using our modified CreditMetrics approach and measure its credit risk using CVaR.

Key inputs adopted in the application process can be synthesized as follows:

- CRISIL's one-year average transition rates given in Table 1 will be used.
- All firm assets secure the 2-year AAA rated loan. Therefore, the recovery rate is 62.61%.
- One year forward rates $\tilde{f}_r^{j,j+1}$ and $k-1$ year forward rate $f_r^{1,k}$ shown in (3) and Table 3 respectively will be used.

Table 4
Year-ahead future value of unit capital invested in 2-year AA loan

Non-Default migration path									
	z = 0	z = 0	z = 2	V_{im}		z = 0	z = 1	z = 2	V_{im}
1	AA	AAA	AAA	1.1028	25	AA	BBB	BBB	1.0979

2	AA	AAA	AA	1.1023	26	AA	BBB	BB	1.0837
3	AA	AAA	A	1.1016	27	AA	BBB	B	1.0790
4	AA	AAA	BBB	1.0979	28	AA	BBB	C	1.0000
5	AA	AAA	BB	1.0837	29	AA	BB	AAA	1.1028
6	AA	AAA	B	1.0790	30	AA	BB	AA	1.1023
7	AA	AAA	C	1.0000	31	AA	BB	A	1.1016
8	AA	AA	AAA	1.1028	32	AA	BB	BBB	1.0979
9	AA	AA	AA	1.1023	33	AA	BB	BB	1.0837
10	AA	AA	A	1.1016	34	AA	BB	B	1.0790
11	AA	AA	BBB	1.0979	35	AA	BB	C	1.0000
12	AA	AA	BB	1.0837	36	AA	B	AAA	1.1028
13	AA	AA	B	1.0790	37	AA	B	AA	1.1023
14	AA	AA	C	1.0000	38	AA	B	A	1.1016
15	AA	A	AAA	1.1028	39	AA	B	BBB	1.0979
16	AA	A	AA	1.1023	40	AA	B	BB	1.0837
17	AA	A	A	1.1016	41	AA	B	B	1.0790
18	AA	A	BBB	1.0979	42	AA	B	C	1.0000
19	AA	A	BB	1.0837	43	AA	C	AAA	1.1028
20	AA	A	B	1.0790	44	AA	C	AA	1.1023
21	AA	A	C	1.0000	45	AA	C	A	1.1016
22	AA	BBB	AAA	1.1028	46	AA	C	BBB	1.0979
23	AA	BBB	AA	1.1023	47	AA	C	BB	1.0837
24	AA	BBB	A	1.1016	48	AA	C	B	1.0790
					49	AA	C	C	1.0000

Default migration path

	z=0	z=1	z=2	V_{idm}	
50	AA	D		0.6261	Expected year-ahead value of unit capital invested in the 2-year AA loan is 1.1023
51	AA	AAA	D	0.6743	
52	AA	AA	D	0.6741	
53	AA	A	D	0.6736	
54	AA	BBB	D	0.6714	
55	AA	BB	D	0.6632	
56	AA	B	D	0.6604	
57	AA	C	D	0.6142	

Table 4 shows year-ahead future loan values for all feasible outcomes as proposed by (5) and (6). We now have all the information that we need to estimate the volatility of value due to credit quality changes for this one exposure on a stand-alone basis. Having obtained the year-ahead value of the 2-year AA loan for each feasible outcome (migration path), we can estimate CVaR at 90% and 95% as a risk measure of credit risk.

Table 5

Results of credit risk estimation using CVaR

Confidence Interval	90%	95%
CVaR	0.6858	0.6670

In the worst-case scenario, on the average there is a 10% probability that the loan value will fall below 0.6858. Also, the worst 5% of loan values will fall below 0.6670 on average.

This paper's primary aim is to deal with the limitations offered by CreditMetrics approach by J.P Morgan. However, it did not focus on portfolio risk calculation. Further research may focus on portfolio risk calculation and also use another coherent statistical technique to estimate the portfolio credit risk.

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