UM–SJTU Joint Institute

Probabilistic Methods in Engineering
(VE401)

# Term Project Report

Photolithography Overlay for Patterning of Integrated Circuitry

**Instructed by**

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# 1 Abstract

Photolithography is one of the key technologies that are used to create integrated circuits. Whether the different layers are perfectly aligned is crucial to the lithography process. Therefore, the machine alignment should be modified and the overlay error from the nominal coordinate should be precisely predicted and adjusted accordingly. There are several models to conclude the relation between the coordinate of the die $(X, Y)$, the nominal position within each die $(x, y)$ and the deviation of real coordinates with the nominal ones $(o_x, o_y)$. Based on the collected data, this study mainly looks into one given multivariate polynomial model by calculating its coefficients and further explores other regression models that might be better fitting the data, such as the cubic polynomial model, the bi-quadratic polynomial model and the quadratic mixed polynomial model. Measures, such as the $R^2$ values, the t statistic, the Significance F, and the $PRESS$ measure are used for testing the goodness of the models.

**Key Words:** Photolithography, Overlay error, Multiple regression model; Determination of coefficients; Improving model

# Contents

## 2 Introduction

Photolithography is the method that can create extremely small patterns, down to a few tens of nanometers in size. Also, it's an efficient and relatively economical method to provide precise control of the shape and size of the objects it creates; sometimes it creates the patterns over an entire wafer in just one single step. Photolithography is often used in integrated circuit manufacturing. Its general working principle is to use light to produce minutely patterned thin films of suitable materials over a substrate, such as a silicon wafer; this can well protect its selected areas during the operations of subsequent etching, deposition, and implantation. Typically, ultraviolet light is used to transfer a geometric design from an optical mask to a light-sensitive chemical, which we call photoresist, coated on the substrate. The photoresist either breaks down or hardens where it is exposed to light; then, we can create the patterned film by removing the softer parts of the coating with appropriate solvents.[3]

The process of Photolithography seems clear, but multiple factors may influence its precision. One most common factor is whether wafers are perfectly aligned during the whole photolithographing process. There are many reasons for this alignment deviation: it can be introduced by mask deformation or abnormal proportion, deformation of the wafer itself, distortion of the projection lens system of the lithography machine, or by the uneven movement of the wafer workpiece stage.[4]

Due to the multi-layer imprinting of the dies, it is crucial that we take this alignment deviation into consideration, so that we can get our expected final products.[2] Often, we use the overlay errors to describe the displacement errors between a present exposure layer and a preceding exposure layer; the overlay errors must be controlled to within the tolerance incorporated in the manufacturing to ensure the yield of products.

It is now of interest to model the overlay errors so that by adjusting the process machinery, the errors may be corrected. In this report, we firstly construct a full multivariate quadratic model and use the multi-linear regression method to calculate its coefficients. Then, we improve the model by checking whether some terms can be reduced so as to reach a greater $R^2$.

## 2.1 Program Objectives

- Fit the coefficients of the given quadratic polynomial model using the raw data.
- Analyze the result of regression using qualities such as $R^2$, $PRESS$, Variance Inflation Factor($VIF$), Significance F, and t statistic.
- Simplify the resulting regression functions; Discover whether some terms may be eliminated from the full model.
- Improve the fit by adding mixed terms or high-order terms.

# 3 Coefficients for the Given Model

Since Photolithography is a two-dimensional problem, there will be errors occurring on both the X and Y coordinates. Therefore, the following table summarizes the notations for the factors that we have considered in this project.

| Notation | Definition |
|---|---|
| (X,Y) | Position of the die |
| (x,y) | Nominal position within each die |
| ($o_x$, $o_y$) | Deviation of real coordinates with the nominal ones |

Table 1: Notations for the multivariate quadratic model

## 3.1 Verification of the distribution for the response variables

To make sure that the response variables $o_x$ and $o_y$ are useful variables for our data analysis, we make the assumption that they should follow the normal distribution. First we look at the data of $o_x$. According to the histogram (Figure 1) and the boxplot (Figure 2), we can see that generally $o_x$ follows a normal distribution. Although there are few outliers according to the boxplot, the number of data, which is 250, is large enough to accept these outliers.

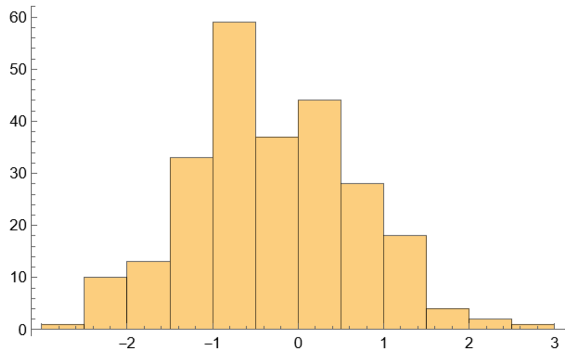


Figure 1: histogram of $o_x$

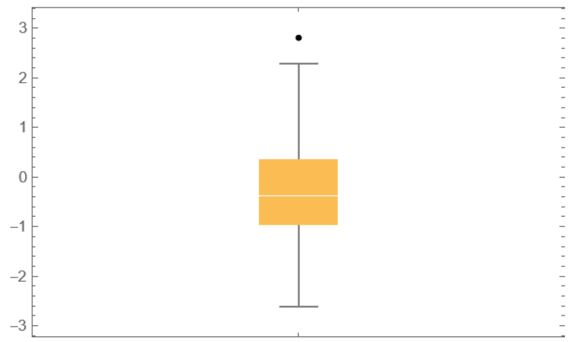


Figure 2: boxplot of $o_x$

Similarly, we can do this analysis to see whether $o_y$ follows a normal distribution. From Figure 3 and Figure 4, This time, we can see there is no outlier according to the boxplot; also, the symmetric median line and equally long whiskers give more information that there is no evidence that $o_y$ does not follow a normal distribution.

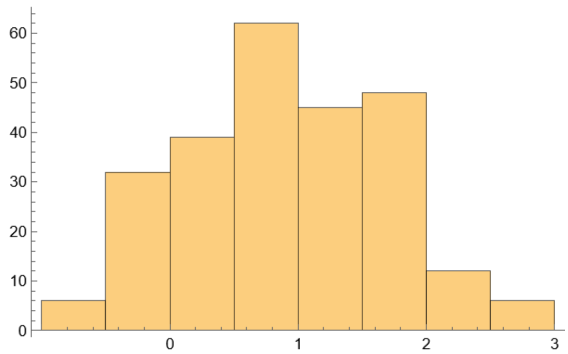


Figure 3: histogram of $o_y$

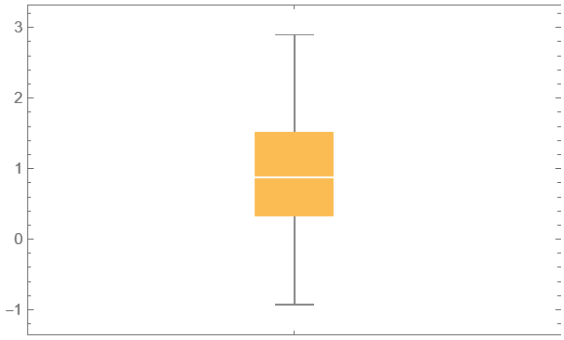

Figure 4: boxplot of $o_y$

## 3.2 Verification of using multilinear regression model

To fit the data to an appropriate model, we decide to use the multilinear model that is provided in the manual. To discuss these two models[2]

$$o_x(X,Y,x,y) = \alpha_0 + \alpha_1 X + \alpha_2 Y + \alpha_3 XY + \alpha_4 X^2 + \alpha_5 Y^2 + \alpha_6 x + \alpha_7 y + \alpha_8 xy + \alpha_9 x^2 + \alpha_{10} y^2 \tag{1}$$

$$o_y(X,Y,x,y) = \beta_0 + \beta_1 X + \beta_2 Y + \beta_3 XY + \beta_4 X^2 + \beta_5 Y^2 + \beta_6 x + \beta_7 y + \beta_8 xy + \beta_9 x^2 + \beta_{10} y^2 \tag{2}$$

we find an appropriate matrix formalism to define the variables.

$$\boldsymbol{Y} = \begin{pmatrix} Y_1 \\ \vdots \\ Y_n \end{pmatrix} \quad \boldsymbol{X} = \begin{pmatrix} 1 & X & \cdots & x^2 & y^2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & X_n & \cdots & x_n^2 & y_n^2 \end{pmatrix} \quad \boldsymbol{\alpha} = \begin{pmatrix} \alpha_0 \\ \vdots \\ \alpha_{10} \end{pmatrix} \quad \boldsymbol{E} = \begin{pmatrix} E_1 \\ \vdots \\ E_n \end{pmatrix}$$

Then we can write these two models as

$$\boldsymbol{Y} = \mathrm{X}\boldsymbol{\alpha} + \boldsymbol{E} \tag{3}$$

Defining

$$\hat{\alpha} = \boldsymbol{b} := \begin{pmatrix} b_0 \\ \vdots \\ b_{10} \end{pmatrix}$$

By minimizing the $SS_E$, we can get the formula to calculate the regression coefficients, which is given by

$$\boldsymbol{b} = (\mathrm{X}^T \mathrm{X})^{-1} \mathrm{X}^T \boldsymbol{Y} \tag{4}$$

## 3.3 Calculation for coefficients of $o_x$

In this project, we use the regression method implemented in excel to get the results of all the terms' coefficients and analysis quantities, such as $R^2$ and $p-value$. The following Table summarized the coefficients and $p-value$ of Equation (1).

| | Coefficients | Standard deviation | t Stat | P-value | Lower 95% | Upper 95% | Lower 95.0% | Upper 95.0% |
|---|---|---|---|---|---|---|---|---|
| Intercept | -0.2624229 | 0.120665768 | -2.17479 | 0.030627 | -0.50013 | -0.02472 | -0.500127 | -0.024719 |
| x | 0.06659785 | 0.00644279 | 10.3368 | 6.09E-21 | 0.053906 | 0.07929 | 0.0539059 | 0.0792898 |
| X | 0.00065211 | 0.000552347 | 1.180623 | 0.238926 | -0.00044 | 0.00174 | -0.000436 | 0.0017402 |
| y | -0.0345959 | 0.003736687 | -9.25845 | 1.23E-17 | -0.04196 | -0.02723 | -0.041957 | -0.027235 |
| Y | 0.00127669 | 0.000563458 | 2.265814 | 0.024358 | 0.000167 | 0.002387 | 0.0001667 | 0.0023867 |
| $X^2$ | -4.621E-05 | 7.86828E-06 | -5.87297 | 1.43E-08 | -6.2E-05 | -3.1E-05 | -6.17E-05 | -3.07E-05 |
| $Y^2$ | -1.311E-05 | 7.71105E-06 | -1.7007 | 0.090301 | -2.8E-05 | 2.08E-06 | -2.83E-05 | 2.076E-06 |
| XY | -5.614E-05 | 8.54462E-06 | -6.56986 | 3.12E-10 | -7.3E-05 | -3.9E-05 | -7.3E-05 | -3.93E-05 |
| $x^2$ | 0.0029393 | 0.001264872 | 2.323789 | 0.020976 | 0.000448 | 0.005431 | 0.0004476 | 0.005431 |
| $y^2$ | 0.00105152 | 0.000358667 | 2.931728 | 0.003698 | 0.000345 | 0.001758 | 0.000345 | 0.0017581 |
| xy | -0.0003529 | 0.000564235 | -0.62544 | 0.532278 | -0.00146 | 0.000759 | -0.001464 | 0.0007586 |

Figure 5: coefficients of $o_x$

Therefore, after plugging these coefficients to Equation (1), we can get the full equation of our quadratic model (the confidence 95% intervals for each coefficients are also in Figure 5.) :

$$\begin{aligned} o_x(X, Y, x, y) = &- 0.262 + 0.001X + 0.001Y - 5.614 \times 10^{-5}XY - 4.621 \times 10^{-5}X^2 \\ &- 1.311 \times 10^{-5}Y^2 + 0.067x - 0.035y - 3.529 \times 10^{-4}xy + 0.003x^2 \\ &+ 0.001y^2 \end{aligned} \tag{5}$$

## 3.4 Calculation for coefficients of $o_y$

Using the same methods as what we did in section 2.3, we get the coefficients and $p-value$ for $o_y$.

| | Coefficients | Standard deviation | t Stat | P-value | Lower 95% | Upper 95% | Lower 95.0% | Upper 95.0% |
|---|---|---|---|---|---|---|---|---|
| Intercept | 0.70648086 | 0.116706149 | 6.053502 | 5.45E-09 | 0.476577 | 0.936385 | 0.4765768 | 0.9363849 |
| x | 0.01631079 | 0.006231372 | 2.617529 | 0.009422 | 0.004035 | 0.028586 | 0.0040354 | 0.02858622 |
| X | 0.00264355 | 0.000534222 | 4.948417 | 1.41E-06 | 0.001591 | 0.003696 | 0.0015912 | 0.00369594 |
| y | 0.01313946 | 0.003614069 | 3.63564 | 0.00034 | 0.00602 | 0.020259 | 0.00602 | 0.02025895 |
| Y | 0.00358417 | 0.000544968 | 6.576836 | 3E-10 | 0.002511 | 0.004658 | 0.0025106 | 0.00465772 |
| $X^2$ | 5.5177E-06 | 7.61008E-06 | 0.725048 | 0.469132 | -9.5E-06 | 2.05E-05 | -9.47E-06 | 2.0509E-05 |
| $Y^2$ | -9.488E-06 | 7.45802E-06 | -1.27214 | 0.204561 | -2.4E-05 | 5.2E-06 | -2.42E-05 | 5.2042E-06 |
| XY | 3.0456E-06 | 8.26423E-06 | 0.368525 | 0.712808 | -1.3E-05 | 1.93E-05 | -1.32E-05 | 1.9326E-05 |
| $x^2$ | -0.0033504 | 0.001223366 | -2.73865 | 0.006634 | -0.00576 | -0.00094 | -0.00576 | -0.00094042 |
| $y^2$ | 0.00265074 | 0.000346898 | 7.641262 | 5.19E-13 | 0.001967 | 0.003334 | 0.0019674 | 0.00333411 |
| xy | 0.00078755 | 0.000545719 | 1.443146 | 0.150289 | -0.00029 | 0.001863 | -0.000287 | 0.00186259 |

Figure 6: coefficients of $o_y$

Equation (2) can now be written as the following (the confidence 95% intervals for each coefficients are also in Figure 6.):

$$\begin{aligned} o_y(X,Y,x,y) = &0.706 + 0.003X + 0.004Y + 3.046\times10^{-6}XY + 5.518\times10^{-6}X^2 \\ &- 9.489\times10^{-6}Y^2 + 0.016x - 0.013y + 0.001xy - 0.003x^2 + 0.003y^2 \end{aligned} \tag{6}$$

## 3.5 Analysis for the given model

To analyze the given model, we would like to use five important coefficients $R^2$, $PRESS$, $VIF$, Significance F, and t statistic to see the performance of the model.

- $R^2$ is the coefficient of determination, which expresses the proportion of the total variation in Y that is explained by the linear model. Generally speaking, we expect to have a $R^2$ value that is close to 1, because this shows our built model has considered much of the variance and there are less differences between the model and the real situation. However, a close-to-one $R^2$ won't necessarily prove that our model is helpful, since it can also lead to an overfitting situation. The calculation formula of $R^2$ is listed below,[1]

$$R^2 := \frac{\mathrm{SS_T} - \mathrm{SS_E}}{\mathrm{SS_T}} \tag{7}$$

  where $\mathrm{SS_T}$ is the total response rate of Y and $\mathrm{SS_E}$ is the Error Sum of Squares, which is the variation of Y after we have applied the model.

- Significance F is used to determine whether the regression can be accepted from a macroscopic perspective. The calculation formula of significance F is listed below,[1]

$$F_{p,n-p-1} = \frac{\mathrm{SS_R}/\mathrm{p}}{\mathrm{SS_E}/(\mathrm{n}-\mathrm{p}-1)} = \frac{\mathrm{SS_R}/\mathrm{p}}{\mathrm{S}^2} \tag{8}$$

  where $\mathrm{SS_E}$ and $\mathrm{SS_T}$ is shown above and

$$\mathrm{SS_R} = \mathrm{SS_T} - \mathrm{SS_E} \tag{9}$$

  In this project, if the significance F is smaller than 0.05, then we can say that our regression test is valid and acceptable.

- The t statistic is another inflection of the validity of the coefficients. It can be combined with the $R^2$ to better prove if our coefficients are acceptable. Let $0 < \alpha \leq 1$ and $\gamma > 0$. We define $t_{\alpha/2,\gamma} \geq 0$ by[1]

$$\int_{t_{\alpha/2,\gamma}}^{\infty} f_{T_\gamma}(t)dt = \alpha/2; \tag{10}$$

where $f_{T_\gamma}$ is the density of the T-distribution with n degrees of freedom. For t statistic with different degrees of freedom, there will be a table for all the boundary value. In this project, if the t statistic for each coefficient is greater than than the boundary values, then we can say that this item should be eliminated.

- $PRESS$ is the abbreviation of the $\boldsymbol{prediction\ sum\ of\ squares}$ , which is a coefficient to see if the model has been overfitted or not. That is to say, if the $R^2$ value is too large, we need to calculate the $PRESS$ value to prevent overfitting. Its calculation formula is given below.[1]

$$PRESS := \sum_{i=1}^{n}(y_i - \hat{y}_i)^2 \tag{11}$$

- As we haven't done the data analysis of the original dependent variables, we don't know whether they are linearly related, which is so called Multicollinearity. Multicollinearity may make the whole model completely different from the actual situation. Therefore, to prevent this situation, we use the $\boldsymbol{Variance\ inflation\ factor}$(VIF) to determine if there is multicollinearity among these variables. The formula of VIF is given below.[5]

$$VIF := \frac{1}{1 - R_i^2} \tag{12}$$

  where $R_i$ is the negative correlation coefficient of the variable on the remaining variables for regression analysis.

The significance F for the above two multilinear regressions are the following:

$$F_{o_x,full} = 1.14683 \times 10^{-44} \tag{13}$$

$$F_{o_y,full} = 3.5136 \times 10^{-34} \tag{14}$$

We can see that the significance F is much smaller than 0.05, so our regression for the model is acceptable.

The $R^2$ for the above two multi-linear regressions are the following:

$$R^2_{o_x,full} = 0.577611 \tag{15}$$

$$R^2_{o_y,full} = 0.389161 \tag{16}$$

We can see that they are far from satisfaction. Therefore, there is no need to calculate the $PRESS$ value. And the $VIF$ value is given in the table below.

| | X | Y | XY | $X^2$ | $Y^2$ | x | y | xy | $x^2$ | $y^2$ |
|---|---|---|---|---|---|---|---|---|---|---|
| VIF | 2.35 | 2.31 | 2.01 | 2.07 | 2.34 | 1.64 | 1.74 | 2.36 | 2.31 | 2.29 |

Table 2: VIF of the $o_x$ model

| | X | Y | XY | $X^2$ | $Y^2$ | x | y | xy | $x^2$ | $y^2$ |
|---|---|---|---|---|---|---|---|---|---|---|
| VIF | 1.48 | 1.39 | 1.64 | 1.63 | 1.63 | 1.60 | 1.55 | 1.62 | 1.59 | 1.31 |

Table 3: VIF of the $o_y$ model

From the $VIF$ value shown above, we can see that they are all much less than 10. Therefore, we can conclude that these variables are not linearly related; the model we choose is relatively reliable.

Also, after we throw away the items whose P-value is greater than 0.05, we found that the t statistic of the remaining items are all smaller than the boundary values that are gained from the t-value table.

# 4 Improvement for the Given Model

$R^2$ is definitely not the only quality to measure whether our model is helpful or not, but the above unsatisfied $R^2$ for our first models has indeed brought us to think whether we can improve them.

## 4.1 Simplification of models based on the $p - value$

Before finding ways to improve the model, the first step is to eliminate some unnecessary terms in the original equations. This can not only help us get a clearer and more simplified regression formula, but can also save much time in our later process of improving the model, which means we don't need to consider those terms that have already been discarded in previous steps. We can simplify models by considering the $p - value$ of each of those coefficients.

$P - value$ is the probability of obtaining the current measured value or a more extreme value.[1] For a $T - test$, which we have used in the above regressions, $p < 0.05$ means the significance level of this test is up to 95%. Therefore, we calculate the $p - value$ of each coefficient and decide to eliminate all those terms whose $p - value$ is greater than 0.05.

In the following sections, we will apply this model simplification method in the process of finding an improved model. We will first calculate the full model, which include all the terms, then using this method to get a simplified, or in other word, reduced version.

## 4.2 Eliminations of coefficients for $o_x$ and $o_y$

We now go back to the Figure 5 and draw out all those terms whose $p - value$ is greater than 0.05. We can see that the $p - value$ of $\alpha_1$, $\alpha_5$ and $\alpha_8$ are greater than 0.05, thus enabling us to eliminate these three coefficients. After that, we get a new model:

$$\begin{aligned} o_x(X, Y, x, y) = &- 0.384 + 0.066x + 1 \times 10^{-4}X - 0.035y + 0.001Y - 4.1 \times 10^{-5}X^2 \\ &- 5.6 \times 10^{-5}XY + 0.003X^2 + 0.001y^2 \end{aligned} \tag{17}$$

To test the whether the model is suitable to use, we again calculate the value of $R^2$ and we get

$$R^2_{ox,reduced} = 0.571795 \tag{18}$$

which is a bit lower than the previous value shown in equation (15), but its value doesn't vary much, so that we can simplify our model in this way.

Similarly, according to the Figure 6, we may eliminate coefficients $\beta_3$, $\beta_4$, $\beta_5$ and $\beta_8$. Then we get the new model:

$$o_y(X, Y, x, y) = 0.670 + 0.016x + 0.003X + 0.014y + 0.004Y - 0.003x^2 + 0.003y^2 \tag{19}$$

The new $R^2$ equals to

$$R^2_{oy,reduced} = 0.375834 \tag{20}$$

and compared to what we got in equation (16), it is also an acceptable value. Hence we can simplify our model by eliminating some of their coefficients. Since the $R^2$ value is not that high, it is unnecessary to consider the value of $PRESS$ as there is no evidence that the model overfits the data. At the same time, from the small $R^2$ value, we can know that this model is not good enough to fit the data. Therefore, we need to find a more powerful model.

## 4.3 Improved model: Cubic polynomial model

Considering cubic polynomial model, we can add elements like $X^3$, $X^2Y$,$XY^2$,$Y^3$, $x^3$,$x^2y$,$xy^2$ and $y^3$. We calculate the coefficients for each term and summarize their values in the following figure.

| | Coefficients | Standard deviation | t Stat | P-value | Lower 95% | Upper 95% | Lower 95.0% | Upper 95.0% |
|---|---|---|---|---|---|---|---|---|
| Intercept | -0.380269 | 0.093287604 | -4.07631 | 6.28E-05 | -0.56406 | -0.19647 | -0.56406426 | -0.1964742 |
| x | 0.0166285 | 0.019057021 | 0.872565 | 0.383799 | -0.02092 | 0.054175 | -0.02091761 | 0.05417459 |
| X | -0.000308 | 0.001527506 | -0.20154 | 0.84045 | -0.00332 | 0.002702 | -0.00331735 | 0.00270163 |
| y | -0.040036 | 0.012637836 | -3.16791 | 0.001741 | -0.06493 | -0.01514 | -0.0649346 | -0.0151365 |
| Y | 0.0030229 | 0.001556749 | 1.941829 | 0.053363 | -4.4E-05 | 0.00609 | -4.4163E-05 | 0.00609004 |
| $X^2$ | -4.12E-05 | 7.07409E-06 | -5.82268 | 1.91E-08 | -5.5E-05 | -2.7E-05 | -5.5128E-05 | -2.725E-05 |
| XY | -5.84E-05 | 8.2011E-06 | -7.12574 | 1.28E-11 | -7.5E-05 | -4.2E-05 | -7.4597E-05 | -4.228E-05 |
| $x^2$ | 0.0025958 | 0.001209144 | 2.146768 | 0.032845 | 0.000213 | 0.004978 | 0.000213499 | 0.004978 |
| $y^2$ | 0.0012318 | 0.0003456 | 3.564115 | 0.000443 | 0.000551 | 0.001913 | 0.000550857 | 0.00191266 |
| $X^3$ | 7.748E-09 | 1.03351E-07 | 0.074969 | 0.940304 | -2E-07 | 2.11E-07 | -1.9587E-07 | 2.1137E-07 |
| $X^2Y$ | 2.176E-07 | 1.37212E-07 | 1.585547 | 0.114198 | -5.3E-08 | 4.88E-07 | -5.2779E-08 | 4.8789E-07 |
| $XY^2$ | 2.565E-07 | 1.30115E-07 | 1.971035 | 0.049904 | 1.09E-10 | 5.13E-07 | 1.0895E-10 | 5.1281E-07 |
| $Y^3$ | -2.37E-07 | 1.03946E-07 | -2.27939 | 0.023549 | -4.4E-07 | -3.2E-08 | -4.4173E-07 | -3.214E-08 |
| $x^3$ | 0.0008605 | 0.000253825 | 3.390206 | 0.00082 | 0.00036 | 0.001361 | 0.000360433 | 0.0013606 |
| $x^2y$ | -6.68E-05 | 0.000100942 | -0.66163 | 0.508865 | -0.00027 | 0.000132 | -0.00026566 | 0.00013209 |
| $xy^2$ | -6.89E-05 | 5.39898E-05 | -1.27572 | 0.203325 | -0.00018 | 3.75E-05 | -0.00017525 | 3.7495E-05 |
| $y^3$ | 3.133E-05 | 4.71734E-05 | 0.6641 | 0.507283 | -6.2E-05 | 0.000124 | -6.1613E-05 | 0.00012427 |

Figure 7: coefficients of cubic $o_x$ model

However, according to Figure 7, we can observe that p-values of coefficient of $x$,$X$,$X^3$,$X^2Y$,$x^2y$,$xy^2$ and $y^3$ are greater than 0.05, hence we need to eliminate these elements. After modification, we can get the cubic polynomial model for $o_x$

$$\begin{aligned} o_{x,cubic}(X,Y,x,y) = & -0.380 + 0.003Y - 5.84 \times 10^{-5}XY - 4.12 \times 10^{-5}X^2 \\ & - 0.040y + 0.003x^2 + 0.001y^2 + 2.565 \times 10^{-7}XY^2 \\ & - 2.37 \times 10^{-7}Y^3 + 0.001x^3 \end{aligned} \tag{21}$$

After calculation, the $R^2_{ox,cubic}$ of this model is 0.61277, which is greater than what we have calculated in equation (18) and is a satisfying result.

Similar process are conducted for $o_y$, and we get the regression results shown in Figure 8.

| | Coefficients | Standard deviation | t Stat | P-value | Lower 95% | Upper 95% | Lower 95.0% | Upper 95.0% |
|---|---|---|---|---|---|---|---|---|
| Intercept | 0.6731746 | 0.071109752 | 9.466698 | 3.2E-18 | 0.533081 | 0.813269 | 0.5330805 | 0.8132686 |
| x | 0.0764202 | 0.016500907 | 4.63127 | 6.02E-06 | 0.043912 | 0.108929 | 0.0439116 | 0.1089288 |
| X | 0.0027608 | 0.001323672 | 2.085711 | 0.038084 | 0.000153 | 0.005369 | 0.000153 | 0.0053686 |
| y | 0.0308628 | 0.010955136 | 2.817198 | 0.005257 | 0.00928 | 0.052446 | 0.00928 | 0.0524456 |
| Y | 0.0052374 | 0.001349681 | 3.880453 | 0.000135 | 0.002578 | 0.007896 | 0.0025784 | 0.0078964 |
| $x^2$ | -0.002652 | 0.001047536 | -2.53131 | 0.012017 | -0.00472 | -0.00059 | -0.004715 | -0.0005879 |
| $y^2$ | 0.0026368 | 0.00029947 | 8.804914 | 2.91E-16 | 0.002047 | 0.003227 | 0.0020468 | 0.0032268 |
| $X^3$ | -5.71E-08 | 8.95666E-08 | -0.63801 | 0.524091 | -2.3E-07 | 1.19E-07 | -2.34E-07 | 1.193E-07 |
| $X^2Y$ | 5.614E-07 | 1.18922E-07 | 4.721043 | 4.03E-06 | 3.27E-07 | 7.96E-07 | 3.271E-07 | 7.957E-07 |
| $XY^2$ | 1.283E-07 | 1.12725E-07 | 1.137762 | 0.256378 | -9.4E-08 | 3.5E-07 | -9.38E-08 | 3.503E-07 |
| $Y^3$ | -3.45E-07 | 9.0104E-08 | -3.83098 | 0.000164 | -5.2E-07 | -1.7E-07 | -5.23E-07 | -1.677E-07 |
| $x^3$ | -0.001016 | 0.000219828 | -4.61954 | 6.34E-06 | -0.00145 | -0.00058 | -0.001449 | -0.0005824 |
| $x^2y$ | 8.639E-06 | 8.7348E-05 | 0.098903 | 0.9213 | -0.00016 | 0.000181 | -0.000163 | 0.0001807 |
| $xy^2$ | 6.577E-05 | 4.67455E-05 | 1.407049 | 0.160734 | -2.6E-05 | 0.000158 | -2.63E-05 | 0.0001579 |
| $y^3$ | -7.52E-05 | 4.08981E-05 | -1.83759 | 0.067386 | -0.00016 | 5.42E-06 | -0.000156 | 5.42E-06 |

Figure 8: coefficients of cubic $o_y$ model

After deleting $X^3$,$XY^2$,$x^2y$,$xy^2$ and $y^3$ whose p-value of their coefficient are greater than 0.05, we get the following model:

$$\begin{aligned} o_{y,cubic}(X,Y,x,y) =& 0.673 + 0.003X + 0.005Y + 0.076x - 0.031y - 0.003x^2 \\ & + 0.003y^2 + 5.614 * 10^{-7}X^2Y - 3.45 * 10^{-7}Y^3 - 0.001x^3 \end{aligned} \tag{22}$$

Again, we can calculate $R^2_{oy,cubic}$ to be 0.541273, which is already much higher than what we have calculated in equation (20).

## 4.4 Improved model: Bi-quadratic polynomial model

As we try to increase the degree of the elements to 4, which means we need to consider elements like $X^4$, $X^3Y$,$X^2Y^2$,$XY^3$,$Y^4$,$x^4$,$x^3y$ and so on, we find that almost all the $p-value$ of the coefficients of these elements are greater than 0.05 (shown in Figure 9).

| | Coefficients | Standard deviation | t Stat | P-value | Lower 95% | Upper 95% | Lower 95.0% | Upper 95.0% |
|---|---|---|---|---|---|---|---|---|
| Intercept | -0.113137 | 0.193451253 | -0.58484 | 0.559222 | -0.49427 | 0.267992 | -0.494266 | 0.26799155 |
| y | -0.033723 | 0.006652692 | -5.06905 | 8.12E-07 | -0.04683 | -0.02062 | -0.04683 | -0.020616 |
| Y | 0.0046783 | 0.001460376 | 3.203477 | 0.001547 | 0.001801 | 0.007555 | 0.0018011 | 0.00755544 |
| $X^2$ | -4.1E-05 | 3.72891E-05 | -1.09976 | 0.272568 | -0.00011 | 3.25E-05 | -0.000114 | 3.2456E-05 |
| XY | -2.37E-05 | 2.07781E-05 | -1.13857 | 0.256048 | -6.5E-05 | 1.73E-05 | -6.46E-05 | 1.7279E-05 |
| $x^2$ | -0.018705 | 0.010332253 | -1.81031 | 0.071531 | -0.03906 | 0.001652 | -0.039061 | 0.00165157 |
| $y^2$ | 0.0009538 | 0.000442357 | 2.156214 | 0.032087 | 8.23E-05 | 0.001825 | 8.231E-05 | 0.00182533 |
| $XY^2$ | 5.433E-08 | 1.0676E-07 | 0.508867 | 0.611325 | -1.6E-07 | 2.65E-07 | -1.56E-07 | 2.6466E-07 |
| $Y^3$ | -3.17E-07 | 1.17814E-07 | -2.69177 | 0.007621 | -5.5E-07 | -8.5E-08 | -5.49E-07 | -8.502E-08 |
| $x^2y$ | -6.98E-05 | 0.000123799 | -0.56379 | 0.57344 | -0.00031 | 0.000174 | -0.000314 | 0.00017411 |
| $X^4$ | -1.64E-10 | 1.87878E-09 | -0.08704 | 0.930711 | -3.9E-09 | 3.54E-09 | -3.87E-09 | 3.5379E-09 |
| $X^3Y$ | -4.27E-09 | 2.1933E-09 | -1.94513 | 0.052958 | -8.6E-09 | 5.49E-11 | -8.59E-09 | 5.4895E-11 |
| $X^2Y^2$ | 8.612E-10 | 2.42209E-09 | 0.35557 | 0.722483 | -3.9E-09 | 5.63E-09 | -3.91E-09 | 5.6331E-09 |
| $XY^3$ | -3.97E-10 | 6.36736E-10 | -0.62289 | 0.533961 | -1.7E-09 | 8.58E-10 | -1.65E-09 | 8.5785E-10 |
| $Y^4$ | 0 | 0 | 65535 | #NUM! | 0 | 0 | 0 | 0 |
| $x^4$ | 0.000251 | 0.000111841 | 2.244278 | #NUM! | 3.07E-05 | 0.000471 | 3.066E-05 | 0.00047135 |
| $x^3y$ | -5.23E-06 | 9.28049E-06 | -0.56373 | 0.573475 | -2.4E-05 | 1.31E-05 | -2.35E-05 | 1.3052E-05 |

Figure 9: coefficients of bi-quadratic $o_x$ model

Also, this time the $R^2_{ox,bi-quadratic}$ for drops to 0.426302, which shows that the bi-quadratic polynomial model does not fit the real data of $o_x$ well.

For $o_y$, its coefficients are summarized in Figure 10.

| | Coefficients | Standard deviation | t Stat | P-value | Lower 95% | Upper 95% | Lower 95.0% | Upper 95.0% |
|---|---|---|---|---|---|---|---|---|
| Intercept | 0.1963565 | 0.169242904 | 1.160205 | 0.247148 | -0.13708 | 0.529791 | -0.137078 | 0.529791 |
| x | 0.0270416 | 0.02311993 | 1.169621 | 0.243343 | -0.01851 | 0.072591 | -0.018508 | 0.0725914 |
| X | 0.0026548 | 0.000453019 | 5.860216 | 1.56E-08 | 0.001762 | 0.003547 | 0.0017623 | 0.0035473 |
| y | 0.0142698 | 0.003136313 | 4.549876 | 8.62E-06 | 0.008091 | 0.020449 | 0.0080908 | 0.0204489 |
| Y | 0.0050237 | 0.001343533 | 3.739171 | 0.000232 | 0.002377 | 0.007671 | 0.0023767 | 0.0076707 |
| $x^2$ | 0.0371185 | 0.01175339 | 3.158112 | 0.001797 | 0.013963 | 0.060275 | 0.0139625 | 0.0602745 |
| $y^2$ | 0.0032355 | 0.000341973 | 9.461348 | 3.4E-18 | 0.002562 | 0.003909 | 0.0025618 | 0.0039093 |
| $X^2Y$ | 5.861E-07 | 1.17561E-07 | 4.985659 | 1.2E-06 | 3.55E-07 | 8.18E-07 | 3.545E-07 | 8.177E-07 |
| $Y^3$ | -3.19E-07 | 9.08794E-08 | -3.50873 | 0.00054 | -5E-07 | -1.4E-07 | -4.98E-07 | -1.4E-07 |
| $x^3$ | -7.01E-05 | 0.000355161 | -0.19739 | 0.843696 | -0.00077 | 0.00063 | -0.00077 | 0.0006296 |
| $X^4$ | 1.148E-10 | 3.79546E-10 | 0.302473 | 0.76256 | -6.3E-10 | 8.63E-10 | -6.33E-10 | 8.626E-10 |
| $X^3Y$ | 7.37E-10 | 7.4162E-10 | 0.993779 | 0.321357 | -7.2E-10 | 2.2E-09 | -7.24E-10 | 2.198E-09 |
| $X^2Y^2$ | -1.24E-09 | 1.29793E-09 | -0.9558 | 0.340161 | -3.8E-09 | 1.32E-09 | -3.8E-09 | 1.317E-09 |
| $XY^3$ | 4.06E-11 | 3.92999E-10 | 0.103317 | 0.9178 | -7.3E-10 | 8.15E-10 | -7.34E-10 | 8.149E-10 |
| $Y^4$ | 0 | 0 | 65535 | #NUM! | 0 | 0 | 0 | 0 |
| $x^4$ | -0.000445 | 0.00013027 | -3.41787 | #NUM! | -0.0007 | -0.00019 | -0.000702 | -0.000189 |
| $x^3y$ | 7.884E-06 | 6.34955E-06 | 1.241735 | 0.215577 | -4.6E-06 | 2.04E-05 | -4.63E-06 | 2.039E-05 |

Figure 10: coefficients of bi-quadratic $o_y$ model

This time, $R^2_{oy,bi-quadratic}$ increases a little to 0.571302, compared to the previous one. However, since the $p - value$ for these new-added elements' coefficients are still too large, then we consider this model unhelpful.

In conclusion, the biquadratic polynomial model does not fit the real data better than cubic polynomial model.

## 4.5 Improved model: Quadratic mixed polynomial model

We can also consider mixed terms, such as $xX, yY, xY,$ and $yX$. To see whether including these terms increase the $R^2$ of our model, we use multilinear regression method to calculate the coefficients of all the terms. They are listed in Figure 11.

| | Coefficients | Standard deviation | t Stat | P-value | Lower 95% | Upper 95% | Lower 95.0% | Upper 95.0% |
|---|---|---|---|---|---|---|---|---|
| Intercept | -0.2106217 | 0.130788251 | -1.6104 | 0.108652 | -0.46829 | 0.047046 | -0.468289 | 0.0470456 |
| x | 0.066969 | 0.006443834 | 10.39273 | 4.62E-21 | 0.054274 | 0.079664 | 0.0542739 | 0.0796641 |
| X | 0.00071072 | 0.000553822 | 1.283301 | 0.200651 | -0.00038 | 0.001802 | -0.00038 | 0.0018018 |
| y | -0.0341741 | 0.003725383 | -9.17332 | 2.41E-17 | -0.04151 | -0.02683 | -0.041514 | -0.026835 |
| Y | 0.00128213 | 0.000562012 | 2.281319 | 0.023425 | 0.000175 | 0.002389 | 0.0001749 | 0.0023894 |
| $X^2$ | -5.111E-05 | 8.68454E-06 | -5.88542 | 1.36E-08 | -6.8E-05 | -3.4E-05 | -6.82E-05 | -3.4E-05 |
| $Y^2$ | -1.718E-05 | 9.00924E-06 | -1.90704 | 0.057735 | -3.5E-05 | 5.68E-07 | -3.49E-05 | 5.683E-07 |
| XY | -6.367E-05 | 9.31955E-06 | -6.83173 | 7.13E-11 | -8.2E-05 | -4.5E-05 | -8.2E-05 | -4.53E-05 |
| $x^2$ | 0.00272647 | 0.001314685 | 2.073861 | 0.039182 | 0.000136 | 0.005317 | 0.0001364 | 0.0053165 |
| $y^2$ | 0.00093979 | 0.000385324 | 2.438954 | 0.015471 | 0.000181 | 0.001699 | 0.0001807 | 0.0016989 |
| xy | -0.0007154 | 0.000605318 | -1.18192 | 0.238431 | -0.00191 | 0.000477 | -0.001908 | 0.0004771 |
| xX | -9.964E-05 | 9.62181E-05 | -1.03559 | 0.301459 | -0.00029 | 8.99E-05 | -0.000289 | 8.992E-05 |
| xY | -4.623E-05 | 9.22642E-05 | -0.50102 | 0.616826 | -0.00023 | 0.000136 | -0.000228 | 0.0001355 |
| yX | -0.0001136 | 5.15675E-05 | -2.20389 | 0.028503 | -0.00022 | -1.2E-05 | -0.000215 | -1.21E-05 |
| yY | -4.356E-05 | 6.06473E-05 | -0.71827 | 0.473303 | -0.00016 | 7.59E-05 | -0.000163 | 7.592E-05 |

Figure 11: coefficients of $o_x$ model with mixed terms

After extracting terms with a greater-than-0.05 $p - value$, we get the following model.

$$\begin{aligned} o_{x,mixed}(X,Y,x,y) = & -0.391 - 0.037y + 0.064x - 4.158 \times 10^{-5}x^2 \\ & + 0.003x^2 + 0.001y^2 - 6.182 \times 10^{-5}XY - 9.487 \times 10^{-5}yX \end{aligned} \tag{23}$$

The $R^2_{ox,mixed} = 0.567215$ is smaller than the $R^2$ for the reduced model, which is shown in Equation (18); it's also smaller than the $R^2$ for cubic model. We conclude that it's not as effective as representing our data as the cubic model does.

For $o_y$, we repeat the above steps and get the coefficients for all terms listed in Figure 12.

| | Coefficients | Standard deviation | t Stat | P-value | Lower 95% | Upper 95% | Lower 95.0% | Upper 95.0% |
|---|---|---|---|---|---|---|---|---|
| Intercept | 0.7819555 | 0.12654991 | 6.179029 | 2.82E-09 | 0.532638 | 1.031273 | 0.5326383 | 1.0312728 |
| x | 0.015826 | 0.006235014 | 2.538253 | 0.011788 | 0.003542 | 0.02811 | 0.0035424 | 0.0281097 |
| X | 0.0026515 | 0.000535875 | 4.948002 | 1.43E-06 | 0.001596 | 0.003707 | 0.0015958 | 0.0037072 |
| y | 0.0131301 | 0.003604658 | 3.642525 | 0.000332 | 0.006028 | 0.020232 | 0.0060285 | 0.0202316 |
| Y | 0.0035186 | 0.000543799 | 6.470334 | 5.63E-10 | 0.002447 | 0.00459 | 0.0024472 | 0.0045899 |
| $X^2$ | 3.911E-06 | 8.4031E-06 | 0.465404 | 0.642073 | -1.3E-05 | 2.05E-05 | -1.26E-05 | 2.047E-05 |
| $Y^2$ | -1.85E-05 | 8.71729E-06 | -2.12483 | 0.034645 | -3.6E-05 | -1.3E-06 | -3.57E-05 | -1.35E-06 |
| XY | 7.436E-06 | 9.01754E-06 | 0.824659 | 0.410402 | -1E-05 | 2.52E-05 | -1.03E-05 | 2.52E-05 |
| $x^2$ | -0.003504 | 0.001272081 | -2.75421 | 0.006344 | -0.00601 | -0.001 | -0.00601 | -0.000997 |
| $y^2$ | 0.0023669 | 0.000372837 | 6.348234 | 1.11E-09 | 0.001632 | 0.003101 | 0.0016323 | 0.0031014 |
| xy | 0.0009753 | 0.000585702 | 1.665223 | 0.097201 | -0.00018 | 0.002129 | -0.000179 | 0.0021292 |
| xX | 2.654E-05 | 9.31001E-05 | 0.285102 | 0.775817 | -0.00016 | 0.00021 | -0.000157 | 0.00021 |
| xY | 7.504E-05 | 8.92743E-05 | 0.840548 | 0.401455 | -0.0001 | 0.000251 | -0.000101 | 0.0002509 |
| yX | 4.161E-05 | 4.98964E-05 | 0.833891 | 0.405189 | -5.7E-05 | 0.00014 | -5.67E-05 | 0.0001399 |
| yY | -0.000124 | 5.8682E-05 | -2.1162 | 0.035379 | -0.00024 | -8.6E-06 | -0.00024 | -8.57E-06 |

Figure 12: coefficients of $o_y$ model with mixed terms

The formula for our model is:

$$
\begin{aligned}
o_{y,mixed}(X,Y,x,y) =& 0.802 + 0.002X + 0.013y + 0.004Y - 2 \times 10^{-5}Y^2 - 0.003x^2 \\
& + 0.002y^2 - 1 \times 10^{-4}yY
\end{aligned} \tag{24}
$$

In this case, $R^2_{oy,mixed} = 0.378573$, which is greater than the $R^2$ of reduced model (Equation (20)), but is still smaller than that of the cubic model.

# 5 Model Two Variables Simultaneously

As a practical problem, we shouldn't just focus on $o_x$ and $o_y$ separately, but to consider the practical meaning behind it.

## 5.1 Methods Clarification

During the production in real life, we would like to focus on the the distance between the actual point to ideal point. Therefore, we introduce a new concept $o_d$

$$
o_d = \sqrt{o_x^2 + o_y^2} \tag{25}
$$

According to the given model of $o_x$ and $o_y$ whose highest degree term is the quadratic term, we can just assume that the model of $o_d$ as follows

$$
\begin{aligned}
o_d(X,Y,x,y) =& \gamma_0 + \gamma_1 X + \gamma_2 Y +_3 XY + \gamma_4 X^2 + \gamma_5 Y^2 + \gamma_6 x + \gamma_7 y + \gamma_8 xy \\
& + \gamma_9 x^2 + \gamma_{10} y^2
\end{aligned} \tag{26}
$$

## 5.2 Analysis for $o_d$

Using the same methods as what we did in section 2.3, we get the coefficients and $p - value$ for $o_d$. Then we eliminate all those terms whose $p - value$ are smaller than 0.05, we now get the following reduced model.

| | Coefficients | Stand deviation | t Stat | P-value | Lower 95% | Upper 95% | Lower 95% | Upper 95% |
|---|---|---|---|---|---|---|---|---|
| Intercept | 0.655173093 | 0.018428213 | 35.5527189 | 6.536E-99 | 0.6188759 | 0.6914703 | 0.6188759 | 0.6914703 |
| y | 0.001971682 | 0.000914179 | 2.15677941 | 0.03199207 | 0.0001711 | 0.0037723 | 0.0001711 | 0.0037723 |
| X*X | 0.299677484 | 0.005426928 | 55.2204618 | 1.278E-140 | 0.2889883 | 0.31036666 | 0.2889883 | 0.31036666 |
| y*y | 0.000269524 | 9.6508E-05 | 2.79276536 | 0.00563679 | 7.944E-05 | 0.00045961 | 7.944E-05 | 0.00045961 |

Figure 13: coefficients of $o_d$

The equation for $o_d$ can now be written as the following (the confidence 95% intervals for each coefficients are also in Figure 6.):

$$o_d(X, Y, x, y) = 0.00197X^2 + 0.29968y + 0.00027y^2 \tag{27}$$

Also,we can find the significance F and $R^2$ as following:

$$R^2_{o_d,reduced} = 0.936035 \tag{28}$$

$$F_{o_d,reduced} = 1.6423 \times 10^{-146} \tag{29}$$

Since the $R^2$ value may be too large and Significance F is much smaller than 0.05, we need to judge whether the model has been overfitted by calculating the $PRESS$ value. According to [6], we can find the $PRESS$ value:

$$PRESS = 6.9964 \tag{30}$$

According to the definition of $PRESS$, the average error of every data is about 0.167, which is sufficiently small. Therefore, the model is relatively acceptable.

# 6 Conclusion

In this project, we dive a little bit deeper into the topic of Photolithography, and try to model the overlay errors using the actual experimental data. In the first step, we aim to fit the coefficients of the given multivariate quadratic equations. Since their constructions are quite similar to the multilinear regression model we have learnt in class, we decide to try using this approach; we calculate the Variance Inflation Factor($VIF$) for each term and find that they are actually not linearly related. Therefore, we prove that the multilinear regression model will be an appropriate approach. This means that we will treat each term as independent new variables containing the original variables $X, Y, x$ and $y$.

By using multilinear regression tools built in excel, we get the coefficients for both the $o_x$ and $o_y$ equations.

$$\begin{aligned} o_{x,full}(X,Y,x,y) = & -0.262 + 0.001X + 0.001Y - 5.614 \times 10^{-5}XY \\ & - 4.621 \times 10^{-5}X^2 - 1.311 \times 10^{-5}Y^2 + 0.067x - 0.035y \\ & - 3.529 \times 10^{-4}xy + 0.003x^2 + 0.001y^2 \end{aligned} \tag{31}$$

$$\begin{aligned} o_{y,full}(X,Y,x,y) = & 0.706 + 0.003X + 0.004Y + 3.046 \times 10^{-6}XY + 5.518 \times 10^{-6}X^2 \\ & - 9.489 \times 10^{-6}Y^2 + 0.016x - 0.013y + 0.001xy - 0.003x^2 \\ & + 0.003y^2 \end{aligned} \tag{32}$$

In the second step, we check which terms can be eliminated by calculating their $p-value$. If the $p-value$ for the coefficient is greater than 0.05, we say that it's not significant and can be eliminated. We also calculate the $R^2$ for both the full and the reduced model of $o_x$ and $o_y$ and find that they don't vary much; therefore, our elimination of these terms is appropriate. The reduced equation for $o_x$ and $o_y$ is listed below.

$$\begin{aligned} o_{x,reduced}(X,Y,x,y) = & -0.384 + 0.066x + 1 \times 10^{-4}X - 0.035y + 0.001Y \\ & - 4.1 \times 10^{-5}X^2 - 5.6 \times 10^{-5}XY + 0.003X^2 + 0.001y^2 \end{aligned} \tag{33}$$

$$\begin{aligned} o_{y,reduced}(X,Y,x,y) = & 0.670 + 0.016x + 0.003X + 0.014y + 0.004Y - 0.003x^2 \\ & + 0.003y^2 \end{aligned} \tag{34}$$

Since the $R^2$ for both the full and reduced model are not high enough to reach our expectation (please refer to Equation (18) and Equation (20)), we try to improve the current model by adding cubic, bi-quadratic or mixed terms to the original ones. We can see from the above equations that only in the cubic model, the $R^2$ value is greater than the original-reduced value for $o_x$, while in both the models with bi-quadratic terms or mixed terms, their $R^2$ value seems to present that they are no better than the original one. For $o_y$, we can see an increase of $R^2$ in both the cubic and mixed-term models, but still the cubic model has a higher $R^2$. Therefore, we conclude that the cubic model is the most helpful model. The $o_x$ and $o_y$ equations can be presented as below. From Figure 17 and Figure 21 in Appendices, we can also see that both their t statistic and Significance F is quite small, then we conclude it's an appropriate model.

$$\begin{aligned} o_{x,cubic}(X,Y,x,y) =& -0.380 + 0.003Y - 5.84 \times 10^{-5}XY - 4.12 \times 10^{-5}X^2 - 0.04y \\ &+ 0.003x^2 + 0.001y^2 + 2.565 \times 10^{-7}XY^2 - 2.37 \times 10^{-7}Y^3 \\ &+ 0.001x^3 \end{aligned} \tag{35}$$

$$\begin{aligned} o_{y,cubic}(X,Y,x,y) =& 0.673 + 0.003X + 0.005Y + 0.076x - 0.031y \\ &- 0.003x^2 + 0.003y^2 + 5.614 * 10^{-7}X^2Y - 3.45 * 10^{-7}Y^3 \\ &- 0.001x^3 \end{aligned} \tag{36}$$

In the third step, we consider $o_x$ and $o_y$ together, because from the practical meaning of Photolithography, we would like to know the overlay error of $o_x$ and $o_y$ as a whole. Therefore, we introduce the concept $o_d$, which represents the distance between the actual point to the ideal point of the die. Using similar approach, we get the reduced model for $o_d$.

$$o_d(X,Y,x,y) = 0.00197X^2 + 0.29968y + 0.00027y^2 \tag{37}$$

From Equation (28), its $R^2$ reaches 0.936035, while its $PRESS$ value still remain low (Equation (30), therefore, we say our regression for this model is acceptable; it's a helpful model without over-fitting.

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# 7 Appendices

This section summarizes all the regression results we got from the excel tool.

SUMMARY OUTPUT

| 回归统计 | |
|---|---|
| Multiple R | 0.562675302 |
| R Square | 0.316603495 |
| Adjusted R Square | 0.288009499 |
| Standard Deviation | 0.548112701 |
| 观测值 | 250 |

方差分析

| | df | SS | MS | F | Significance F |
|---|---|---|---|---|---|
| 回归分析 | 10 | 33.26446811 | 3.32644681 | 11.0723767 | 1.7469E-15 |
| 残差 | 239 | 71.80218028 | 0.30042753 | | |
| 总计 | 249 | 105.0666484 | | | |

| | Coefficients | Standard Deviation | t Stat | P-value | Lower 95% | Upper 95% | Lower 95% | Upper 95% |
|---|---|---|---|---|---|---|---|---|
| Intercept | 0.96036064 | 0.102803024 | 9.3417548 | 6.92118E-18 | 0.75784491 | 1.16287637 | 0.75784491 | 1.16287637 |
| x | 0.005398628 | 0.005489033 | 0.98352991 | 0.326341294 | -0.0054144 | 0.01621169 | -0.0054144 | 0.01621169 |
| X | 0.000780456 | 0.000470581 | 1.65849618 | 0.098529138 | -0.0001466 | 0.00170747 | -0.0001466 | 0.00170747 |
| y | 0.012590543 | 0.003183527 | 3.95490353 | 0.00010093 | 0.00631919 | 0.0188619 | 0.00631919 | 0.0188619 |
| Y | 0.002054906 | 0.000480047 | 4.28063765 | 2.69919E-05 | 0.00110924 | 0.00300057 | 0.00110924 | 0.00300057 |
| X*X | 2.73486E-05 | 6.7035E-06 | 4.07975578 | 6.14649E-05 | 1.4143E-05 | 4.0554E-05 | 1.4143E-05 | 4.0554E-05 |
| Y*Y | 6.6154E-06 | 6.56955E-06 | 1.00697883 | 0.314963346 | -6.326E-06 | 1.9557E-05 | -6.326E-06 | 1.9557E-05 |
| X*Y | 2.76496E-05 | 7.27972E-06 | 3.79817179 | 0.000184927 | 1.3309E-05 | 4.199E-05 | 1.3309E-05 | 4.199E-05 |
| x*x | -2.1475E-05 | 0.001077627 | -0.019928 | 0.984117435 | -0.0021443 | 0.00210138 | -0.0021443 | 0.00210138 |
| y*y | 0.001997748 | 0.000305572 | 6.53772762 | 3.74573E-10 | 0.00139579 | 0.00259971 | 0.00139579 | 0.00259971 |
| x*y | -0.00125973 | 0.000480708 | -2.6205618 | 0.009341118 | -0.0022067 | -0.0003128 | -0.0022067 | -0.0003128 |

Figure 14: results for full model of $o_d$

SUMMARY OUTPUT

| 回归统计 | |
|---|---|
| Multiple R | 0.967488996 |
| R Square | 0.936034956 |
| Adjusted R Square | 0.935254895 |
| 标准误差 | 0.165286063 |
| 观测值 | 250 |

方差分析

| | df | SS | MS | F | Significance F |
|---|---|---|---|---|---|
| 回归分析 | 3 | 98.34605566 | 32.7820186 | 1199.95019 | 1.64E-146 |
| 残差 | 246 | 6.72059274 | 0.02731948 | | |
| 总计 | 249 | 105.0666484 | | | |

| | Coefficients | Stand deviation | t Stat | P-value | Lower 95% | Upper 95% | Lower 95% | Upper 95% |
|---|---|---|---|---|---|---|---|---|
| Intercept | 0.655173093 | 0.018428213 | 35.5527189 | 6.536E-99 | 0.6188759 | 0.6914703 | 0.6188759 | 0.6914703 |
| y | 0.001971682 | 0.000914179 | 2.15677941 | 0.03199207 | 0.0001711 | 0.0037723 | 0.0001711 | 0.0037723 |
| X*X | 0.299677484 | 0.005426928 | 55.2204618 | 1.278E-140 | 0.2889883 | 0.31036666 | 0.2889883 | 0.31036666 |
| y*y | 0.000269524 | 9.6508E-05 | 2.79276536 | 0.00563679 | 7.944E-05 | 0.00045961 | 7.944E-05 | 0.00045961 |

Figure 15: results for $o_d$ after reducing unnecessary terms

SUMMARY OUTPUT

| 回归统计 | |
|---|---|
| Multiple R | 0.756171 |
| R Square | 0.571795 |
| Adjusted R Squa | 0.557581 |
| 标准误差 | 0.645072 |
| 观测值 | 250 |

方差分析

| | df | SS | MS | F | gnificance F |
|---|---|---|---|---|---|
| 回归分析 | 8 | 133.9129 | 16.73911 | 40.22686 | 2.4E-40 |
| 残差 | 241 | 100.2844 | 0.416118 | | |
| 总计 | 249 | 234.1972 | | | |

| | Coefficient: | 标准误差 | t Stat | P-value | Lower 95% | Upper 95% | 下限 95.0% | 上限 95.0% |
|---|---|---|---|---|---|---|---|---|
| Intercept | -0.38431 | 0.097486 | -3.94225 | 0.000106 | -0.57635 | -0.19228 | -0.57635 | -0.19228 |
| x | 0.066385 | 0.006459 | 10.27852 | 8.73E-21 | 0.053663 | 0.079108 | 0.053663 | 0.079108 |
| X | 0.000646 | 0.000554 | 1.16566 | 0.244904 | -0.00045 | 0.001736 | -0.00045 | 0.001736 |
| y | -0.03451 | 0.003736 | -9.23791 | 1.36E-17 | -0.04187 | -0.02715 | -0.04187 | -0.02715 |
| Y | 0.001237 | 0.000564 | 2.193933 | 0.029196 | 0.000126 | 0.002348 | 0.000126 | 0.002348 |
| X*X | -4.1E-05 | 7.38E-06 | -5.58908 | 6.16E-08 | -5.6E-05 | -2.7E-05 | -5.6E-05 | -2.7E-05 |
| X*Y | -5.6E-05 | 8.55E-06 | -6.60943 | 2.46E-10 | -7.3E-05 | -4E-05 | -7.3E-05 | -4E-05 |
| x*x | 0.00325 | 0.001254 | 2.591594 | 0.010136 | 0.00078 | 0.00572 | 0.00078 | 0.00572 |
| y*y | 0.001083 | 0.000359 | 3.018996 | 0.002809 | 0.000376 | 0.00179 | 0.000376 | 0.00179 |

Figure 16: results for quadratic $o_x$ model after reducing unnecessary terms

SUMMARY OUTPUT

| 回归统计 | |
|---|---|
| Multiple R | 0.782796131 |
| R Square | 0.612769782 |
| Adjusted R Square | 0.598248649 |
| 标准误差 | 0.614709416 |
| 观测值 | 250 |

方差分析

| | df | SS | MS | F | Significance F |
|---|---|---|---|---|---|
| 回归分析 | 9 | 143.508978 | 15.94544205 | 42.1984824 | 1.14683E-44 |
| 残差 | 240 | 90.6882399 | 0.377867666 | | |
| 总计 | 249 | 234.197218 | | | |

| | Coefficients | andard deviati | t Stat | P-value | Lower 95% | Upper 95% | Lower 95% | 上限 95.0% |
|---|---|---|---|---|---|---|---|---|
| Intercept | -0.382677719 | 0.09292863 | -4.11797425 | 5.2614E-05 | -0.565737614 | -0.1996178 | -0.565738 | -0.199618 |
| y | -0.036564024 | 0.00358967 | -10.1859029 | 1.751E-20 | -0.043635305 | -0.0294927 | -0.043635 | -0.029493 |
| Y | 0.004676112 | 0.00117176 | 3.990683876 | 8.7564E-05 | 0.00236787 | 0.00698435 | 0.0023679 | 0.0069844 |
| X*X | -4.07415E-05 | 7.0404E-06 | -5.78679421 | 2.2301E-08 | -5.46104E-05 | -2.687E-05 | -5.46E-05 | -2.69E-05 |
| X*Y | -5.74678E-05 | 8.149E-06 | -7.05213872 | 1.8683E-11 | -7.35205E-05 | -4.142E-05 | -7.35E-05 | -4.14E-05 |
| x*x | 0.002595083 | 0.00120037 | 2.161895182 | 0.03161513 | 0.000230469 | 0.0049597 | 0.0002305 | 0.0049597 |
| y*y | 0.001216868 | 0.00034188 | 3.55937807 | 0.00044801 | 0.000543406 | 0.00189033 | 0.0005434 | 0.0018903 |
| X*Y*Y | 2.28752E-07 | 8.769E-08 | 2.608646364 | 0.00966071 | 5.60118E-08 | 4.0149E-07 | 5.601E-08 | 4.015E-07 |
| Y*Y*Y | -3.10407E-07 | 9.3457E-08 | -3.32137433 | 0.00103541 | -4.94508E-07 | -1.263E-07 | -4.95E-07 | -1.26E-07 |
| x*x*y | 0.000969028 | 8.4714E-05 | 11.43885598 | 1.8042E-24 | 0.000802151 | 0.00113591 | 0.0008022 | 0.0011359 |

Figure 17: results for cubic $o_x$ model after reducing unnecessary terms

SUMMARY OUTPUT

| 回归统计 | |
|---|---|
| Multiple R | 0.652918 |
| R Square | 0.426302 |
| Adjusted R Squ | 0.385253 |
| 标准误差 | 0.757748 |
| 观测值 | 250 |

方差分析

| | df | SS | MS | F | gnificance F |
|---|---|---|---|---|---|
| 回归分析 | 16 | 99.83875 | 6.239922 | 11.59201 | 6.26E-22 |
| 残差 | 234 | 134.3585 | 0.574181 | | |
| 总计 | 250 | 234.1972 | | | |

| | Coefficient | 标准误差 | t Stat | P-value | Lower 95% | Upper 95% | 下限 95.0% | 上限 95.0% |
|---|---|---|---|---|---|---|---|---|
| Intercept | -0.11314 | 0.193451 | -0.58484 | 0.559222 | -0.49427 | 0.267992 | -0.49427 | 0.267992 |
| y | -0.03372 | 0.006653 | -5.06905 | 8.12E-07 | -0.04683 | -0.02062 | -0.04683 | -0.02062 |
| Y | 0.004678 | 0.00146 | 3.203477 | 0.001547 | 0.001801 | 0.007555 | 0.001801 | 0.007555 |
| X*X | -4.1E-05 | 3.73E-05 | -1.09976 | 0.272568 | -0.00011 | 3.25E-05 | -0.00011 | 3.25E-05 |
| X*Y | -2.4E-05 | 2.08E-05 | -1.13857 | 0.256048 | -6.5E-05 | 1.73E-05 | -6.5E-05 | 1.73E-05 |
| x*x | -0.0187 | 0.010332 | -1.81031 | 0.071531 | -0.03906 | 0.001652 | -0.03906 | 0.001652 |
| y*y | 0.000954 | 0.000442 | 2.156214 | 0.032087 | 8.23E-05 | 0.001825 | 8.23E-05 | 0.001825 |
| X*Y*Y | 5.43E-08 | 1.07E-07 | 0.508867 | 0.611325 | -1.6E-07 | 2.65E-07 | -1.6E-07 | 2.65E-07 |
| Y*Y*Y | -3.2E-07 | 1.18E-07 | -2.69177 | 0.007621 | -5.5E-07 | -8.5E-08 | -5.5E-07 | -8.5E-08 |
| x*x*y | -7E-05 | 0.000124 | -0.56379 | 0.57344 | -0.00031 | 0.000174 | -0.00031 | 0.000174 |
| X*X*X*X | -1.6E-10 | 1.88E-09 | -0.08704 | 0.930711 | -3.9E-09 | 3.54E-09 | -3.9E-09 | 3.54E-09 |
| X*X*X*Y | -4.3E-09 | 2.19E-09 | -1.94513 | 0.052958 | -8.6E-09 | 5.49E-11 | -8.6E-09 | 5.49E-11 |
| X*X*Y*Y | 8.61E-10 | 2.42E-09 | 0.35557 | 0.722483 | -3.9E-09 | 5.63E-09 | -3.9E-09 | 5.63E-09 |
| X*Y*Y*Y | -4E-10 | 6.37E-10 | -0.62289 | 0.533961 | -1.7E-09 | 8.58E-10 | -1.7E-09 | 8.58E-10 |
| Y*Y*Y*Y | 0 | 0 | 65535 | #NUM! | 0 | 0 | 0 | 0 |
| x*x*x*x | 0.000251 | 0.000112 | 2.244278 | #NUM! | 3.07E-05 | 0.000471 | 3.07E-05 | 0.000471 |
| x*x*x*y | -5.2E-06 | 9.28E-06 | -0.56373 | 0.573475 | -2.4E-05 | 1.31E-05 | -2.4E-05 | 1.31E-05 |

Figure 18: results for bi-quadratic $o_x$ model after reducing unnecessary terms

SUMMARY OUTPUT

| 回归统计 | |
|---|---|
| Multiple R | 0.74893757 |
| R Square | 0.560907484 |
| Adjusted R Square | 0.550065694 |
| 标准误差 | 0.65052763 |
| 观测值 | 250 |

方差分析

| | df | SS | MS | F | Significance F |
|---|---|---|---|---|---|
| 回归分析 | 6 | 131.3629726 | 21.89382876 | 51.73568733 | 8.96735E-41 |
| 残差 | 243 | 102.8342458 | 0.423186197 | | |
| 总计 | 249 | 234.1972184 | | | |

| | Coefficients | 标准误差 | t Stat | P-value | Lower 95% | Upper 95% | 下限 95.0% | 上限 95.0% |
|---|---|---|---|---|---|---|---|---|
| Intercept | -0.384689902 | 0.098303927 | -3.913270951 | 0.000118265 | -0.57832646 | -0.19105334 | -0.57832646 | -0.19105334 |
| y | -0.03706987 | 0.003578084 | -10.36025742 | 4.55051E-21 | -0.044117888 | -0.03002185 | -0.04411789 | -0.03002185 |
| x | 0.064533146 | 0.00632487 | 10.20307909 | 1.41789E-20 | 0.05207458 | 0.076991713 | 0.05207458 | 0.076991713 |
| X*X | -4.13883E-05 | 7.44584E-06 | -5.558577318 | 7.14008E-08 | -5.60549E-05 | -2.6722E-05 | -5.6055E-05 | -2.6722E-05 |
| x*x | 0.003271268 | 0.001264289 | 2.587436073 | 0.010251186 | 0.000780903 | 0.005761633 | 0.000780903 | 0.005761633 |
| y*y | 0.001111962 | 0.0003617 | 3.074267434 | 0.002351151 | 0.000399495 | 0.00182443 | 0.000399495 | 0.00182443 |
| X*Y | -5.59467E-05 | 8.61673E-06 | -6.492801854 | 4.70616E-10 | -7.29197E-05 | -3.8974E-05 | -7.292E-05 | -3.8974E-05 |

Figure 19: results for reduced quadratic $o_x$ model with mixed terms

SUMMARY OUTPUT

| 回归统计 | |
|---|---|
| Multiple R | 0.613053037 |
| R Square | 0.375834026 |
| Adjusted R Square | 0.36042252 |
| 标准误差 | 0.623792553 |
| 观测值 | 250 |

方差分析

| | df | SS | MS | F | Significance F |
|---|---|---|---|---|---|
| 回归分析 | 6 | 56.93544 | 9.489239 | 24.38659 | 1.4774E-22 |
| 残差 | 243 | 94.55547 | 0.389117 | | |
| 总计 | 249 | 151.4909 | | | |

| | Coefficients | 标准误差 | t Stat | P-value | Lower 95% | Upper 95% | 下限 95.0% | 上限 95.0% |
|---|---|---|---|---|---|---|---|---|
| Intercept | 0.670235722 | 0.082928 | 8.082125 | 3E-14 | 0.506885959 | 0.83358549 | 0.50688596 | 0.8335855 |
| x | 0.01643364 | 0.006241 | 2.633191 | 0.009001 | 0.004140357 | 0.02872692 | 0.00414036 | 0.0287269 |
| X | 0.00265161 | 0.000536 | 4.951574 | 1.38E-06 | 0.001596779 | 0.00370644 | 0.00159678 | 0.0037064 |
| y | 0.013708422 | 0.003612 | 3.795115 | 0.000186 | 0.006593354 | 0.02082349 | 0.00659335 | 0.0208235 |
| Y | 0.003520077 | 0.000545 | 6.458484 | 5.71E-10 | 0.002446488 | 0.00459367 | 0.00244649 | 0.0045937 |
| x*x | -0.003154508 | 0.001211 | -2.60431 | 0.009773 | -0.005540425 | -0.0007686 | -0.0055404 | -0.000769 |
| y*y | 0.00268 | 0.000347 | 7.726972 | 2.89E-13 | 0.001996809 | 0.00336319 | 0.00199681 | 0.0033632 |

Figure 20: results for quadratic $o_y$ model after reducing unnecessary terms

SUMMARY OUTPUT

| 回归统计 | |
|---|---|
| Multiple R | 0.73571225 |
| R Square | 0.54127252 |
| Adjusted R Square | 0.52407024 |
| 标准误差 | 0.53810253 |
| 观测值 | 250 |

方差分析

| | df | SS | MS | F | Significance F |
|---|---|---|---|---|---|
| 回归分析 | 9 | 81.99786262 | 9.11087362 | 31.46516 | 5.05716E-36 |
| 残差 | 240 | 69.49304086 | 0.28955434 | | |
| 总计 | 249 | 151.4909035 | | | |

| | Coefficients | Standard deviation | t Stat | P-value | Lower 95% | Upper 95% | Lower 95% | Upper 95% |
|---|---|---|---|---|---|---|---|---|
| Intercept | 0.67777393 | 0.071559745 | 9.47144139 | 2.749E-18 | 0.536808557 | 0.8187393 | 0.53680856 | 0.8187393 |
| x | 0.08371022 | 0.015859102 | 5.27837093 | 2.916E-07 | 0.052469415 | 0.11495103 | 0.05246941 | 0.11495103 |
| X | 0.00266039 | 0.000462481 | 5.75243527 | 2.667E-08 | 0.001749351 | 0.00357143 | 0.00174935 | 0.00357143 |
| y | 0.01399957 | 0.003186221 | 4.39378565 | 1.673E-05 | 0.007723043 | 0.0202761 | 0.00772304 | 0.0202761 |
| Y | 0.0052257 | 0.001359329 | 3.84431987 | 0.0001549 | 0.002547957 | 0.00790344 | 0.00254796 | 0.00790344 |
| x*x | -0.0027383 | 0.001053712 | -2.5987353 | 0.0099361 | -0.004814023 | -0.0006626 | -0.004814 | -0.0006626 |
| y*y | 0.00263486 | 0.000300031 | 8.78193304 | 3.098E-16 | 0.002043824 | 0.00322589 | 0.00204382 | 0.00322589 |
| X*X*Y | 5.6472E-07 | 1.19649E-07 | 4.71982338 | 4.013E-06 | 3.29025E-07 | 8.0042E-07 | 3.2903E-07 | 8.0042E-07 |
| Y*Y*Y | -3.442E-07 | 9.07012E-08 | -3.7950594 | 0.0001869 | -5.22889E-07 | -1.655E-07 | -5.229E-07 | -1.655E-07 |
| x*x*x | -0.0010059 | 0.000220371 | -4.5647145 | 7.991E-06 | -0.001440042 | -0.0005718 | -0.00144 | -0.0005718 |

Figure 21: results for cubic $o_y$ model after reducing unnecessary terms

SUMMARY OUTPUT

| 回归统计 | |
|---|---|
| Multiple R | 0.755845 |
| R Square | 0.571302 |
| Adjusted R Squa | 0.539547 |
| 标准误差 | 0.526819 |
| 观测值 | 250 |

方差分析

| | df | SS | MS | F | gnificance F |
|---|---|---|---|---|---|
| 回归分析 | 16 | 86.547 | 5.409187 | 20.78922 | 2.49E-36 |
| 残差 | 234 | 64.9439 | 0.277538 | | |
| 总计 | 250 | 151.4909 | | | |

| | Coefficient | 标准误差 | t Stat | P-value | Lower 95% | Upper 95% | 下限 95.0% | 上限 95.0% |
|---|---|---|---|---|---|---|---|---|
| Intercept | 0.196356 | 0.169243 | 1.160205 | 0.247148 | -0.13708 | 0.529791 | -0.13708 | 0.529791 |
| x | 0.027042 | 0.02312 | 1.169621 | 0.243343 | -0.01851 | 0.072591 | -0.01851 | 0.072591 |
| X | 0.002655 | 0.000453 | 5.860216 | 1.56E-08 | 0.001762 | 0.003547 | 0.001762 | 0.003547 |
| y | 0.01427 | 0.003136 | 4.549876 | 8.62E-06 | 0.008091 | 0.020449 | 0.008091 | 0.020449 |
| Y | 0.005024 | 0.001344 | 3.739171 | 0.000232 | 0.002377 | 0.007671 | 0.002377 | 0.007671 |
| x*x | 0.037119 | 0.011753 | 3.158112 | 0.001797 | 0.013963 | 0.060275 | 0.013963 | 0.060275 |
| y*y | 0.003236 | 0.000342 | 9.461348 | 3.4E-18 | 0.002562 | 0.003909 | 0.002562 | 0.003909 |
| X*X*Y | 5.86E-07 | 1.18E-07 | 4.985659 | 1.2E-06 | 3.55E-07 | 8.18E-07 | 3.55E-07 | 8.18E-07 |
| Y*Y*Y | -3.2E-07 | 9.09E-08 | -3.50873 | 0.00054 | -5E-07 | -1.4E-07 | -5E-07 | -1.4E-07 |
| x*x*x | -7E-05 | 0.000355 | -0.19739 | 0.843696 | -0.00077 | 0.00063 | -0.00077 | 0.00063 |
| X*X*X*X | 1.15E-10 | 3.8E-10 | 0.302473 | 0.76256 | -6.3E-10 | 8.63E-10 | -6.3E-10 | 8.63E-10 |
| X*X*X*Y | 7.37E-10 | 7.42E-10 | 0.993779 | 0.321357 | -7.2E-10 | 2.2E-09 | -7.2E-10 | 2.2E-09 |
| X*X*Y*Y | -1.2E-09 | 1.3E-09 | -0.9558 | 0.340161 | -3.8E-09 | 1.32E-09 | -3.8E-09 | 1.32E-09 |
| X*Y*Y*Y | 4.06E-11 | 3.93E-10 | 0.103317 | 0.9178 | -7.3E-10 | 8.15E-10 | -7.3E-10 | 8.15E-10 |
| Y*Y*Y*Y | 0 | 0 | 65535 | #NUM! | 0 | 0 | 0 | 0 |
| x*x*x*x | -0.00045 | 0.00013 | -3.41787 | #NUM! | -0.0007 | -0.00019 | -0.0007 | -0.00019 |
| x*x*x*y | 7.88E-06 | 6.35E-06 | 1.241735 | 0.215577 | -4.6E-06 | 2.04E-05 | -4.6E-06 | 2.04E-05 |

Figure 22: results for bi-quadratic $o_y$ model after reducing unnecessary terms

SUMMARY OUTPUT

| 回归统计 | |
|---|---|
| Multiple R | 0.61528278 |
| R Square | 0.3785729 |
| Adjusted R Square | 0.36059774 |
| 标准误差 | 0.6237071 |
| 观测值 | 250 |

方差分析

| | df | SS | MS | F | Significance F |
|---|---|---|---|---|---|
| 回归分析 | 7 | 57.35035 | 8.192907 | 21.0608874 | 4.58354E-22 |
| 残差 | 242 | 94.14055 | 0.389011 | | |
| 总计 | 249 | 151.4909 | | | |

| | Coefficients | 标准误差 | t Stat | P-value | Lower 95% | Upper 95% | 下限 95.0% | 上限 95.0% |
|---|---|---|---|---|---|---|---|---|
| Intercept | 0.80162692 | 0.100284 | 7.993567 | 5.3724E-14 | 0.604085957 | 0.9991679 | 0.604086 | 0.99916788 |
| X | 0.00233779 | 0.00052 | 4.495714 | 1.0758E-05 | 0.001313476 | 0.0033621 | 0.0013135 | 0.0033621 |
| y | 0.01345494 | 0.003614 | 3.7229 | 0.00024502 | 0.006335829 | 0.0205741 | 0.0063358 | 0.02057406 |
| Y | 0.00351104 | 0.000546 | 6.429407 | 6.7703E-10 | 0.002435342 | 0.0045867 | 0.0024353 | 0.00458674 |
| Y*Y | -2.024E-05 | 8.03E-06 | -2.52009 | 0.01237722 | -3.60637E-05 | -4.42E-06 | -3.61E-05 | -4.42E-06 |
| x*x | -0.0032209 | 0.001216 | -2.64887 | 0.00860766 | -0.005616172 | -0.000826 | -0.005616 | -0.0008257 |
| y*y | 0.0023679 | 0.000372 | 6.359511 | 1.0012E-09 | 0.001634459 | 0.0031013 | 0.0016345 | 0.00310134 |
| y*Y | -0.0001364 | 5.8E-05 | -2.35242 | 0.01945345 | -0.000250672 | -2.22E-05 | -0.000251 | -2.219E-05 |

Figure 23: results for reduced quadratic $o_y$ model with mixed terms