Derivation of Einstein’s Field Equations

Hello, today I’m going to try to derive Einstein’s Field Equations.

This will be a basic introduction. It won’t be rigorous, it will cover the essence, but it will be still quite complex. If you want more detail and I refer you to Professor Leonard Susskind’s excellent lectures that he gave from October to December 2008 at Stanford. I will put a link to the YouTube version of these lectures in the annotation on this video.

I’m going to follow his approach so that it will be familiar to you. But I shall do it in a much more simplified version. And let’s always remember that it took Einstein ten years to develop these equations, and he had access to many math professors on the way.

As usual, I’m making the assumption that you understand calculus and basic geometry, and you also have seen or aware of information in my general relativity videos that already upon you to, and in particular that you are aware of two key points: the first was called the principle of equivalence, what that says is that if you are in a box with no windows, then you cannot tell the difference between being in the out space and accelerating with an acceleration g, or being in the same box on the surface of the Earth stationary but subjected to the gravitational force which carries an acceleration g. There is no experiment, let’s put you in the box, there is no experiment you can do that will distinguish between those two situations.

Now that’s not quite true. The difference, of course, is if you are now in out space, traveling with an acceleration g, then all parts of this box, including you, are accelerating at the acceleration g. If you are on the Earth, then because the value of g varies according to the height above the Earth. The value of g at this point will be ever so slightly greater than the value of g at this point, so there is slight difference in the value of g in this situation, and if you can measure that, then you can tell the difference. That difference is called the tidal force. But apart from that, Einstein said that was principle of equivalence, you cannot tell the difference whether you are accelerating at g or subjected to a gravitational force which is represented by the acceleration g.

The second principle is the light bends in the gravitational field. This is Einstein’s reasoning. If you take the person who is in the box that is accelerating through space with an acceleration g, and here is the box. And this time we are going to shine a light from this side of the box, to the other side of the box. But of course that will take a finite time or being small. And if the box is accelerating with an acceleration g, by the time the light gets half way across, the box will be accelerated a little bit further forward, so it will now be here, which means the light will reach that position there. By the time the light gets to the right hand side, the box will be, of course, accelerated still further. And now the light, which of course is just going in straight line, will reach here. So if you look at it, it appears within the box, the light goes from here, to a point lower down, to a point even lower down. In other words, the overall