



Super-Resolution Channel Estimation for MmWave Massive MIMO

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Outline

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System Model

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Proposed 2D Unitary ESPRIT Based Channel Estimation Scheme

- a) Design of Training Signals
- b) Modified 2D Unitary ESPRIT Algorithm
- c) Reconstruct High-Dimensional mmWave MIMO Channel

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Simulation Results

System Model

- The received signal \mathbf{y} for the uplink:

$$\begin{aligned} \mathbf{y} &= \mathbf{W}^H \mathbf{H} \mathbf{F} \mathbf{s} + \mathbf{W}^H \mathbf{n} \\ &= \mathbf{W}_{\text{BB}}^H \mathbf{W}_{\text{RF}}^H \mathbf{H} \mathbf{F}_{\text{RF}} \mathbf{F}_{\text{BB}} \mathbf{s} + \mathbf{W}_{\text{BB}}^H \mathbf{W}_{\text{RF}}^H \mathbf{n} \end{aligned} \quad (1) \quad \begin{cases} \mathbf{W} = \mathbf{W}_{\text{RF}} \mathbf{W}_{\text{BB}} \\ \mathbf{F} = \mathbf{F}_{\text{RF}} \mathbf{F}_{\text{BB}} \end{cases}$$

N_S : number of data streams

$N_{\text{BS}}, N_{\text{MS}}$: number of antennas at the BS and MS

$N_{\text{RF}}^{\text{BS}}, N_{\text{RF}}^{\text{MS}}$: number of RF chains at the BS and MS

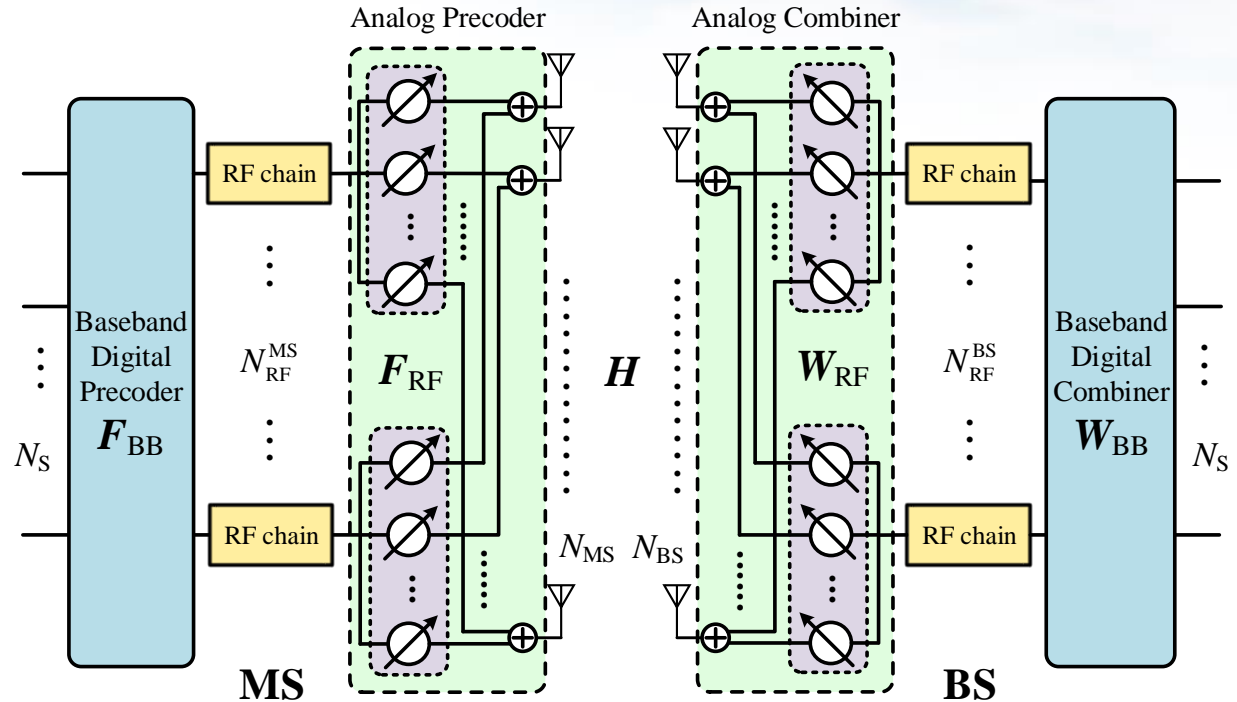


Fig. 1. Block diagram of transceiver for a typical mmWave massive MIMO system with hybrid precoding

System Model

- The geometric mmWave channel model with L dominated paths can be written as

$$\mathbf{H} = \sqrt{\frac{N_{\text{BS}} N_{\text{MS}}}{L}} \sum_{l=1}^L \alpha_l \mathbf{a}_{\text{BS}}(\theta_l) \mathbf{a}_{\text{MS}}^H(\varphi_l) \quad (2) \quad \Rightarrow \quad \mathbf{H} = \mathbf{A}_{\text{BS}} \mathbf{D} \mathbf{A}_{\text{MS}}^H \quad (4)$$

- α_l is the complex gain of the l -th path
- θ and φ are azimuth angles of AoA and AoD

- The steering vectors with ULA:

$$\mathbf{a}_{\text{BS}}(\theta_l) = \frac{1}{\sqrt{N_{\text{BS}}}} \left[1, e^{2\pi \Delta \sin(\theta_l)}, \dots, e^{2\pi (N_{\text{BS}}-1) \Delta \sin(\theta_l)} \right]^T$$

$$\mathbf{a}_{\text{MS}}(\varphi_l) = \frac{1}{\sqrt{N_{\text{MS}}}} \left[1, e^{2\pi \Delta \sin(\varphi_l)}, \dots, e^{2\pi (N_{\text{MS}}-1) \Delta \sin(\varphi_l)} \right]^T \quad (3)$$

$\Delta = d / \lambda$ — the normalized spacing of adjacent antennas

$$\left\{ \begin{array}{l} \mathbf{A}_{\text{BS}} = [\mathbf{a}_{\text{BS}}(\theta_1), \dots, \mathbf{a}_{\text{BS}}(\theta_L)] \\ \mathbf{A}_{\text{MS}} = [\mathbf{a}_{\text{MS}}(\varphi_1), \dots, \mathbf{a}_{\text{MS}}(\varphi_L)] \\ \mathbf{D} = \text{diag}(\mathbf{d}) \\ \mathbf{d} = \sqrt{N_{\text{BS}} N_{\text{MS}} / L} [\alpha_1, \dots, \alpha_L] \end{array} \right.$$

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Simulation Results

Design of Training Signals

- Each baseband observation mixes the signals from different antennas via the RF phase shift network;
- The shift-invariance of array response is destroyed by the RF phase shift network, which is the precondition of using the conventional ESPRIT algorithms;
- Solution:
 - Design the training signals to obtain a low-dimensional effective channel matrix with low pilot overhead;
 - This channel matrix has the same shift-invariance of array response as the high-dimensional MIMO channel matrix;
 - Obtain the paired super-resolution estimates of AoAs and AoDs.

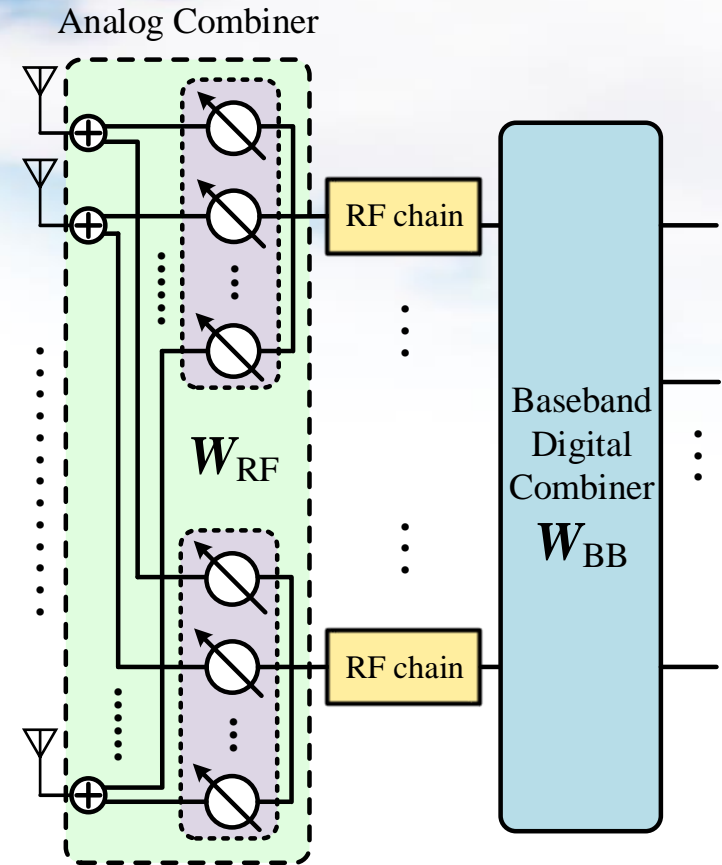


Fig. 2. Receiver with hybrid precoding

Design of Training Signals

- Consider T_{MS} time slots as a time-block, we have

$$\mathbf{y} = \mathbf{W}^H \mathbf{H} \mathbf{F} \mathbf{s} + \mathbf{W}^H \mathbf{n} \xrightarrow{T_{\text{MS}}} \mathbf{Y} = [\mathbf{y}_1, \dots, \mathbf{y}_{T_{\text{MS}}}] = \mathbf{W}^H \mathbf{H} \mathbf{F} \mathbf{S} + \mathbf{W}^H \mathbf{N} \quad (5)$$

$\mathbf{S} = [\mathbf{s}_1, \dots, \mathbf{s}_{T_{\text{MS}}}]$ — the transmitted pilot signal block

- Furthermore, jointly consider $N_b^T N_b^R$ time-blocks, the aggregated received signal is

$$\tilde{\mathbf{Y}} = \begin{bmatrix} \mathbf{Y}_{1,1} & \cdots & \mathbf{Y}_{1,N_b^T} \\ \vdots & \ddots & \vdots \\ \mathbf{Y}_{N_b^R,1} & \cdots & \mathbf{Y}_{N_b^R,N_b^T} \end{bmatrix} = \tilde{\mathbf{W}}^H \mathbf{H} \tilde{\mathbf{F}} \bar{\mathbf{S}} + \bar{\mathbf{W}}^H \tilde{\mathbf{N}} \quad (6)$$

- $\tilde{\mathbf{F}} = [\mathbf{F}_1, \dots, \mathbf{F}_{N_b^T}]$, $\tilde{\mathbf{W}} = [\mathbf{W}_1, \dots, \mathbf{W}_{N_b^R}]$ — the aggregated hybrid combiner/precoder (need to design)
- $\bar{\mathbf{S}} = \text{diag}[\mathbf{S}, \dots, \mathbf{S}]$ — the aggregated pilot signal block (block diagonal matrix)
- $\bar{\mathbf{W}} = \text{diag}[\mathbf{W}_1, \dots, \mathbf{W}_{N_b^R}]$
- Thus, the total number of pilot overhead required for channel estimation is $T_{\text{pilot}} = T_{\text{MS}} N_b^R N_b^T$

Design of Training Signals

- To guarantee the same shift-invariance of array response in $\tilde{\mathbf{Y}}$, we can design the $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{W}}$ as

$$\tilde{\mathbf{F}} = \alpha_f \begin{bmatrix} \mathbf{I}_{N_b^T N_S} \\ \mathbf{O}_{(N_{MS} - N_b^T N_S) \times N_b^T N_S} \end{bmatrix}, \quad \tilde{\mathbf{W}} = \alpha_w \begin{bmatrix} \mathbf{I}_{N_b^R N_S} \\ \mathbf{O}_{(N_{BS} - N_b^R N_S) \times N_b^R N_S} \end{bmatrix} \quad (7)$$

- The low-dimensional effective channel matrix is

$$\bar{\mathbf{H}} = \tilde{\mathbf{W}}^H \mathbf{H} \tilde{\mathbf{F}} = \alpha_f \alpha_w \begin{bmatrix} \mathbf{H}_{1,1} & \cdots & \mathbf{H}_{1,N_b^T N_S} \\ \vdots & \ddots & \vdots \\ \mathbf{H}_{N_b^R N_S,1} & \cdots & \mathbf{H}_{N_b^R N_S, N_b^T N_S} \end{bmatrix} \quad (8)$$

- Next, design the aggregated hybrid combiner/precoder as

$$\left\{ \begin{array}{l} \tilde{\mathbf{F}} = [\mathbf{F}_1, \dots, \mathbf{F}_{N_b^T}] \\ \tilde{\mathbf{W}} = [\mathbf{W}_1, \dots, \mathbf{W}_{N_b^R}] \\ \mathbf{F} = \mathbf{F}_{\text{RF}} \mathbf{F}_{\text{BB}} \\ \mathbf{W} = \mathbf{W}_{\text{RF}} \mathbf{W}_{\text{BB}} \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} \{\mathbf{F}_{\text{RF},j}\}_{j=1}^{N_b^T} \\ \{\mathbf{F}_{\text{BB},j}\}_{j=1}^{N_b^T} \\ \{\mathbf{W}_{\text{RF},i}\}_{i=1}^{N_b^R} \\ \{\mathbf{W}_{\text{BB},i}\}_{i=1}^{N_b^R} \end{array} \right\} \quad (9)$$

Design of Training Signals

- The orthogonality of unitary matrix:

$$\mathbf{U}_{N_{\text{RF}}^{\text{MS}}} = [\mathbf{u}_1, \dots, \mathbf{u}_{N_{\text{RF}}^{\text{MS}}}] \text{ — as the set of the uplink training signals, orthogonality: } \begin{cases} \mathbf{u}_m^H \mathbf{u}_m = N_{\text{RF}}^{\text{MS}} \\ \mathbf{u}_m^H \mathbf{u}_n = 0, m \neq n \end{cases}$$

- For j -th digital/analog precoder,

$$\begin{aligned} \mathbf{F}_{\text{BB},j} &= [\mathbf{u}_1, \dots, \mathbf{u}_{N_s}] \\ \mathbf{F}_{\text{RF},j} &= [\mathbf{F}_{\text{RF},j}^1, \mathbf{F}_{\text{BB},j}, \mathbf{F}_{\text{RF},j}^2]^H \quad (10) \\ \mathbf{F}_{\text{RF},j}^1 &= [\underbrace{\mathbf{u}_{N_{\text{RF}}^{\text{MS}}}, \dots, \mathbf{u}_{N_{\text{RF}}^{\text{MS}}}}_{(j-1)N_s}] \\ \mathbf{F}_{\text{RF},j}^2 &= [\underbrace{\mathbf{u}_{N_{\text{RF}}^{\text{MS}}}, \dots, \mathbf{u}_{N_{\text{RF}}^{\text{MS}}}}_{N_{\text{MS}} - jN_s}] \end{aligned}$$

j -th hybrid precoder: $\mathbf{F}_j = \mathbf{F}_{\text{RF},j} \mathbf{F}_{\text{BB},j}, j = 1, \dots, N_b^T$

Similarly, i -th hybrid combiner: $\mathbf{W}_i = \mathbf{W}_{\text{RF},i} \mathbf{W}_{\text{BB},i}, i = 1, \dots, N_b^R$

$$\tilde{\mathbf{F}} = [\mathbf{F}_1, \dots, \mathbf{F}_{N_b^T}], \tilde{\mathbf{W}} = [\mathbf{W}_1, \dots, \mathbf{W}_{N_b^R}] \quad (11)$$

$$\begin{aligned} & \left[\begin{array}{c} \mathbf{I}_{N_b^T} \\ \mathbf{0}_{(N_{\text{MS}} - N_b^T N_s) \times N_b^T N_s} \end{array} \right] \left[\begin{array}{c} \mathbf{I}_{N_b^R} \\ \mathbf{0}_{(N_{\text{BS}} - N_b^R N_s) \times N_b^R N_s} \end{array} \right] \sim \left[\begin{array}{c} \mathbf{I}_{N_b^T} \\ \mathbf{0}_{(N_{\text{MS}} - N_b^T N_s) \times N_b^T N_s} \end{array} \right] \end{aligned} \quad (12)$$

LS estimator $\rightarrow \bar{\mathbf{H}} = \tilde{\mathbf{Y}} \bar{\mathbf{S}}^\dagger = \tilde{\mathbf{Y}} \bar{\mathbf{S}}^H (\bar{\mathbf{S}} \bar{\mathbf{S}}^H)^{-1} \quad (13)$

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Modified 2D Unitary ESPRIT Algorithm

$$\bar{\mathbf{H}} \in \mathbb{C}^{N_R \times N_T} \quad \begin{cases} R = N_R - m_2 + 1 \\ T = N_T - m_1 + 1 \end{cases}$$

➤ Step1: Spatial Smoothing Preprocessing

m_1 and m_2 are the smoothing parameters

$$\bar{\mathbf{H}}^{(i,j)} = \begin{bmatrix} \bar{\mathbf{H}}_{i,j} & \cdots & \bar{\mathbf{H}}_{i,N_T-m_1+j} \\ \vdots & \ddots & \vdots \\ \bar{\mathbf{H}}_{N_R-m_2+i,j} & \cdots & \bar{\mathbf{H}}_{N_R-m_2+i,N_T-m_1+j} \end{bmatrix} \quad \rightarrow \quad \mathcal{H} = \begin{bmatrix} \bar{\mathbf{H}}^{(1,1)} & \cdots & \bar{\mathbf{H}}^{(m_2,1)} \\ \vdots & \ddots & \vdots \\ \bar{\mathbf{H}}^{(1,m_1)} & \cdots & \bar{\mathbf{H}}^{(m_2,m_1)} \end{bmatrix}$$

➤ Step2: Real Processing

$$\mathcal{H}_R = (\mathbf{Q}_{m_1}^H \otimes \mathbf{Q}_R^H) \begin{bmatrix} \mathcal{H} & \mathbf{J}_{m_1 R} \mathcal{H}^* \mathbf{J}_{m_2 T} \end{bmatrix} \mathbf{Q}_{2m_2 T} \quad \begin{cases} \mathbf{J}_n & \text{--- the exchange matrix} \\ \mathbf{Q}_n & \text{--- the left-real exchange matrix} \end{cases}$$

➤ Step3: Rank Reduction

$$\mathcal{H}_R = \mathbf{U} \Sigma \mathbf{V}^H, \quad \hat{\mathbf{U}} = \mathbf{U}_{\{:, 1:L\}}$$

$$\mathbf{T} \tilde{\boldsymbol{\Theta}} \mathbf{T}^{-1} = (\mathbf{E}_{\theta, R} \hat{\mathbf{U}})^\dagger \mathbf{E}_{\theta, I} \hat{\mathbf{U}}$$

➤ Step4: Joint Diagonalization

$$\mathbf{T} \tilde{\boldsymbol{\Phi}} \mathbf{T}^{-1} = (\mathbf{E}_{\varphi, R} \hat{\mathbf{U}})^\dagger \mathbf{E}_{\varphi, I} \hat{\mathbf{U}}$$

$$\Re\{\mathbf{E}_\theta\} \hat{\mathbf{U}} \mathbf{T} \tilde{\boldsymbol{\Theta}} = \Im\{\mathbf{E}_\theta\} \hat{\mathbf{U}} \mathbf{T}$$

$$\Re\{\mathbf{E}_\varphi\} \hat{\mathbf{U}} \mathbf{T} \tilde{\boldsymbol{\Phi}} = \Im\{\mathbf{E}_\varphi\} \hat{\mathbf{U}} \mathbf{T}$$

LS

$$\mathbf{E}_\theta = \mathbf{I}_{m_1} \otimes (\mathbf{Q}_{R-1}^H [\mathbf{0} \quad \mathbf{I}_{R-1}] \mathbf{Q}_{R-1})$$

$$\mathbf{E}_\varphi = (\mathbf{Q}_{m_1-1}^H [\mathbf{0} \quad \mathbf{I}_{m_1-1}] \mathbf{Q}_{m_1}) \otimes \mathbf{I}_R$$

EVD

$$\boldsymbol{\Theta} = \text{diag}(\tilde{\theta}_{BS,1}, \dots, \tilde{\theta}_{BS,L}), \quad \tilde{\theta}_{BS,l} = \tan(\pi \Delta \sin(\hat{\theta}_{BS,l}))$$

$$\boldsymbol{\Phi} = \text{diag}(\tilde{\varphi}_{MS,1}, \dots, \tilde{\varphi}_{MS,L}), \quad \tilde{\varphi}_{MS,l} = \tan(\pi \Delta \sin(\hat{\varphi}_{MS,l}))$$

$$\{\hat{\theta}_l\}_{l=1}^L, \quad \{\hat{\varphi}_l\}_{l=1}^L$$

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Reconstruct High-Dimensional mmWave MIMO Channel

1. Based on obtained AoAs $\{\hat{\theta}_l\}_{l=1}^L$ and AoDs $\{\hat{\phi}_l\}_{l=1}^L$, we can reconstruct the steering vector matrices

$$\hat{\mathbf{A}}_{\text{BS}} = [\mathbf{a}(\hat{\theta}_1), \dots, \mathbf{a}(\hat{\theta}_L)], \hat{\mathbf{A}}_{\text{MS}} = [\mathbf{a}(\hat{\phi}_1), \dots, \mathbf{a}(\hat{\phi}_L)]$$

2. Vectorizing the low-dimensional effective channel matrix $\bar{\mathbf{H}} = \tilde{\mathbf{W}}^H \hat{\mathbf{A}}_{\text{BS}} \mathbf{D} \hat{\mathbf{A}}_{\text{MS}}^H \tilde{\mathbf{F}} + \bar{\mathbf{N}}$

$$\bar{\mathbf{h}} = \text{vec}(\bar{\mathbf{H}}) = \underbrace{\left[\left(\hat{\mathbf{A}}_{\text{MS}}^H \tilde{\mathbf{F}} \right)^T \odot \left(\tilde{\mathbf{W}}^H \hat{\mathbf{A}}_{\text{BS}} \right) \right]}_{\mathbf{Z}} \mathbf{d} + \bar{\mathbf{n}} = \mathbf{Z} \mathbf{d} + \bar{\mathbf{n}}$$

3. Using the LS estimator, we can obtain the LS solution of associated path gain \mathbf{d}

$$\hat{\mathbf{d}} = \arg \min_{\mathbf{d}} \|\bar{\mathbf{h}} - \mathbf{Z} \mathbf{d}\|_2^2 = \mathbf{Z}^\dagger \bar{\mathbf{h}} = (\mathbf{Z}^H \mathbf{Z})^{-1} \mathbf{Z}^H \bar{\mathbf{h}}$$

4. Finally, we can reconstruct the high-dimensional mmWave MIMO channel as

$$\hat{\mathbf{H}} = \hat{\mathbf{A}}_{\text{BS}} \text{diag}(\hat{\mathbf{d}}) \hat{\mathbf{A}}_{\text{MS}}^H$$

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Simulation Results

Simulation Parameters and Comparison Schemes

Simulation Parameters

- $N_{BS} = N_{MS} = 64$
- $N_{RF} = 4$
- $N_S = T_{MS} = 3$
- $N_b^T = N_b^R = 10$
- $L = 5$
- $m_1 = m_2 = 13$
- $\theta_l, \phi_l \sim \mathcal{U}[-\pi/3, \pi/3]$

Comparison Schemes

- ① Adaptive Compressed Sensing (ACS) based channel estimation scheme:
A. Alkhateeb, O. El Ayach, G. Leus, and R. W. Heath, Jr., “Channel estimation and hybrid precoding for millimeter wave cellular systems,” *IEEE J. Sel. Topics Signal Process.*, vol. 8, no. 5, pp. 831-846, Oct. 2014. ^[7]
- ② Orthogonal Matching Pursuit (OMP) based channel estimation scheme:
J. Lee, G. T. Gil, and Y. H. Lee, “Channel estimation via orthogonal matching pursuit for hybrid MIMO systems in millimeter wave communications,” *IEEE Trans. Commun.*, vol. 64, no. 6, pp. 2370-2386, Jun. 2016. ^[8]

• Pilot Overhead Comparison

$$T_{\text{Proposed}} = T_{MS} N_b^R N_b^T = 300$$

↓ 80%

$$T_{\text{ACS}} = KL^2 (KL / N_{RF}) \log_K (G_{\text{ACS}} / L) = 1500$$

↓ 48%

$$T_{\text{OMP}} = N_T^{\text{Beam}} N_R^{\text{Beam}} / N_{RF} = 576$$

Performance Evaluation Metrics

- normalized mean square error (NMSE): $\text{NMSE} = 10 \log_{10} \left(\mathbb{E} \left[\frac{\|\mathbf{H} - \hat{\mathbf{H}}\|_F^2}{\|\mathbf{H}\|_F^2} \right] \right)$

Simulation Results

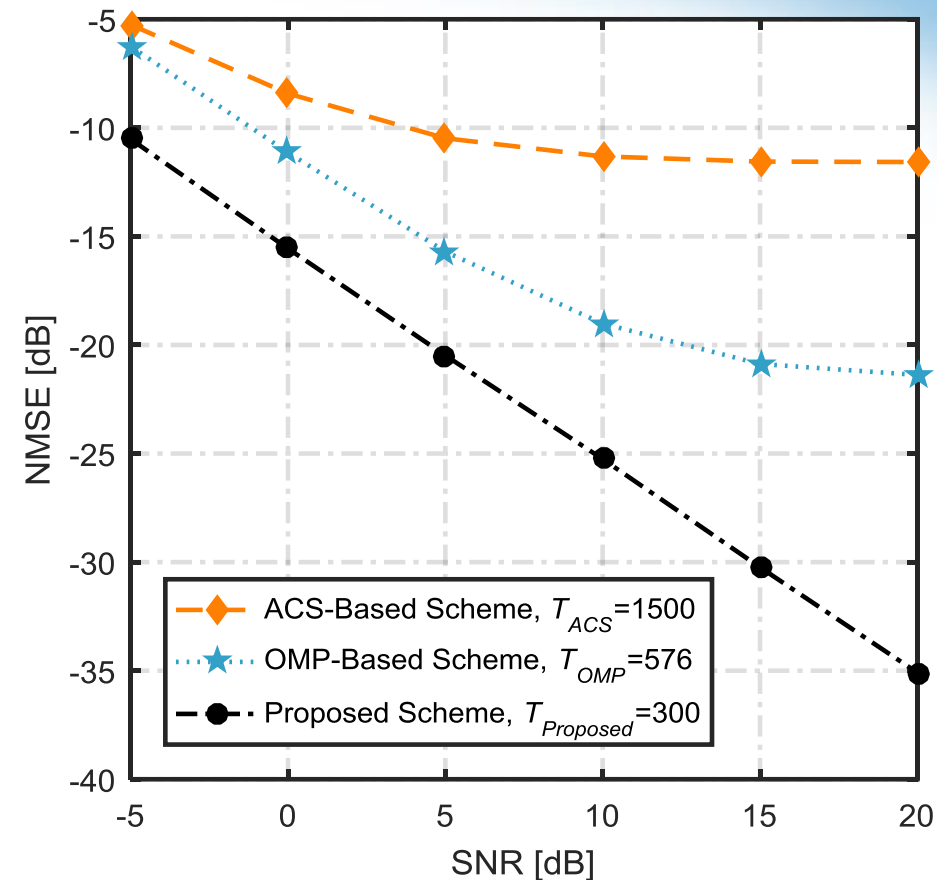


Fig. 1. NMSE performance comparison of different channel estimation schemes versus SNRs for $L = 5$.

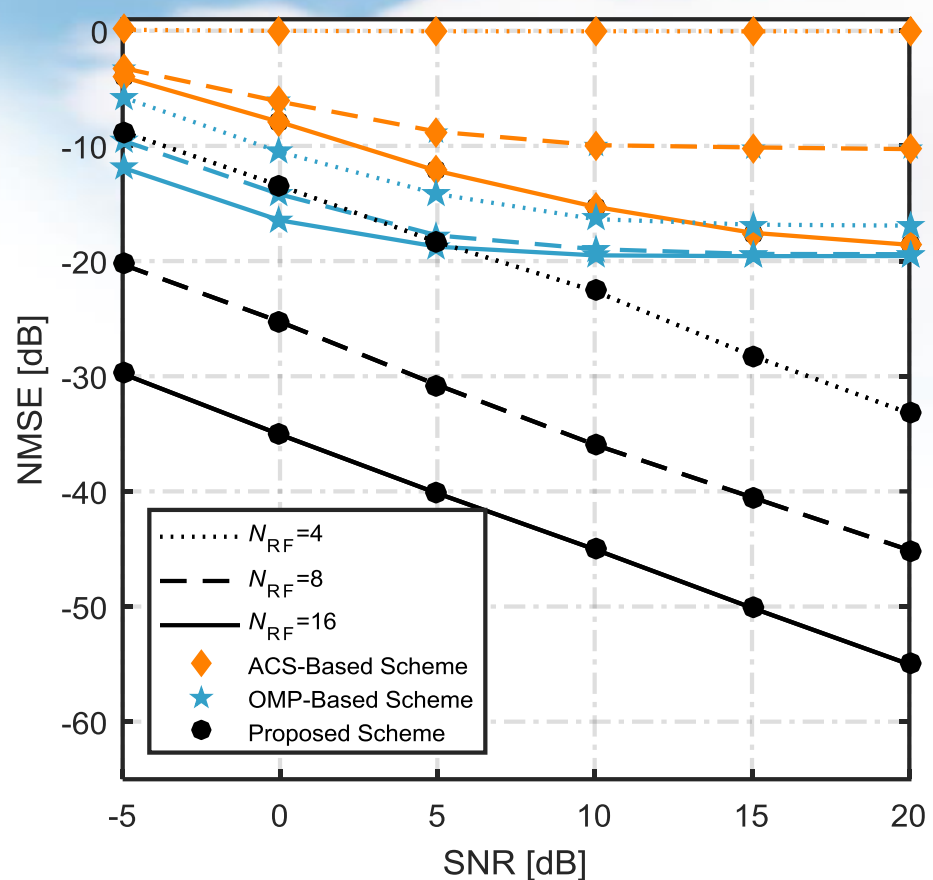


Fig. 2. NMSE performance comparison of different schemes versus SNRs, where $N_{RF} = \{4, 8, 16\}$ are considered, respectively.

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THANKS!

