Compressive Massive Random Access for Massive Machine-Type Communications (mMTC)

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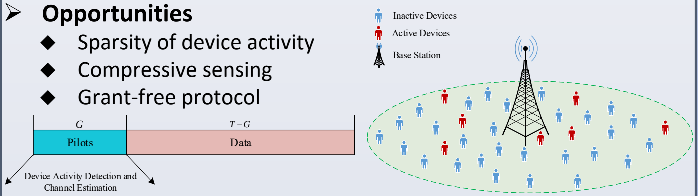
Introduction

- **Conventional Cellular Network**
  - Toward applications for human users, limited number of users
  - The users are assigned with orthogonal pilots to access the BS
  - Grant-based random access protocol
- **Future Wireless Network**
  - Massive connectivity with massive number of machine-type devices to support Internet-of-Things (IoT)
  - It is not possible to assign orthogonal pilots
  - Grant-based protocol suffers from inefficient scheduling
- **Opportunities**
  - Sparsity of device activity
  - Compressive sensing
  - Grant-free protocol



System Model

- **A typical uplink mMTC system:**
  - K single-antenna devices, BS is equipped with M antennas
  - OFDM with N subcarriers is adopted to combat the time dispersive channels
  - P pilots are uniformly allocated across N subcarriers

For pth subcarrier, received signal from kth device at tth OFDM symbol

$$\mathbf{y}_{p,k}^{t}=\mathbf{h}_{p,k}s_{p,k}^{t}+\mathbf{w}_{p}^{t}$$

Received signals from all active devices

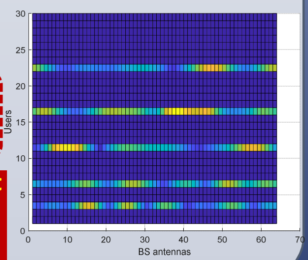
$$\mathbf{y}_{p}^{t}=\sum_{k=1}^{K}\alpha_{k}\mathbf{h}_{p,k}s_{p,k}^{t}+\mathbf{w}_{p}^{t}=\mathbf{H}_{p}\mathbf{s}_{p}^{t}+\mathbf{w}_{p}^{t} \Rightarrow \boxed{\mathbf{Y}_{p}=\mathbf{S}_{p}\mathbf{X}_{p}+\mathbf{W}_{p}}$$

in which $\mathbf{Y}_{p}=\left[\mathbf{y}_{p}^{1},\cdots,\mathbf{y}_{p}^{G}\right]^{T}\in\mathbb{C}^{G\times M}$, $\mathbf{S}_{p}=\left[\mathbf{s}_{p}^{1},\cdots,\mathbf{s}_{p}^{G}\right]^{T}\in\mathbb{C}^{G\times K}$, $\mathbf{x}_{p,k}=\alpha_{k}\mathbf{h}_{p,k}$, $\mathbf{X}_{p}=[\mathbf{x}_{p,1},\cdots,\mathbf{x}_{p,K}]^{T}\in\mathbb{C}^{K\times M}$ and

$$\alpha_{k}=\begin{cases}1, & \text{if the } k\text{th device is active,}\\ 0, & \text{otherwise}\end{cases}\quad\forall k.$$

- Sporadic traffic of devices
- Common support among antennas
- Common sparsity among subcarriers

Distributed multiple measurement vector (DMMV) compressive sensing theory



DMMV-AMP Algorithm

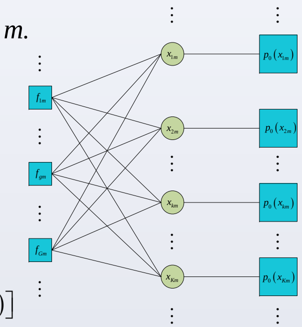
- **MMSE estimation**

$$\hat{x}_{km}=\int x_{km}\,p\left(x_{km}\mid\mathbf{Y}\right)dx_{km},\quad\forall k,m.$$

in which $p(x_{km}|\mathbf{Y})=\int p(\mathbf{X}\mid\mathbf{Y})d_{\mathbf{X}\backslash km}$

$$p(\mathbf{X}\mid\mathbf{Y})=\frac{p(\mathbf{Y}\mid\mathbf{X})p(\mathbf{X})}{p(\mathbf{Y})}=\frac{1}{Z}\prod_{g=1}^{G}\prod_{m=1}^{M}p\left(y_{gm}\mid\mathbf{X}\right)\prod_{k=1}^{K}\prod_{m=1}^{M}p_{0}\left(x_{km}\right)$$

$$p_{0}(\mathbf{X})=\prod_{m=1}^{M}\prod_{k=1}^{K}p_{0}\left(x_{km}\right)=\prod_{m=1}^{M}\prod_{k=1}^{K}\left[\left(1-\lambda_{km}\right)\delta\left(x_{km}\right)+\lambda_{km}f\left(x_{km}\right)\right]$$



Solver: Sum-Product Algorithm

- **Message Passing in SP Algorithm**

Variable node k to factor node g: $\nu_{k\to g}^{t+1}(x_k)\propto p_0(x_k)\prod_{b\neq g}\hat{\nu}_{b\to k}^{t}(x_k)$

Factor node g to variable node k: $\hat{\nu}_{g\to k}^{t}(x_k)\propto\int\prod_{j\neq k}\nu_{j\to g}^{t}(x_j)p(y_g\mid\mathbf{x})d\mathbf{x}_{\backslash k}$

Multi-dimensional integrals, high complexity !!

- **Message approximation in large system limit**

The messages $\hat{\nu}_{g\to k}^{t}(x_k)$ are approximately Gaussian densities :

$$\prod_{g}\hat{\nu}_{g\to k}^{t}(x_k)\propto\mathcal{CN}\left(x_k;R_k^t,\Sigma_k^t\right),\qquad \nu_{k\to g}^{t+1}(x_k)\propto\mathcal{CN}\left(x_k;\hat{x}_{k\to g}^{t+1},\nu_{k\to g}^{t+1}\right)$$

- **Further Approximation of Messages**

The posterior distributions are finally obtained as

$$\boxed{p(x_{km}\mid R_{km}^{t},\Sigma_{km}^{t})=(1-\pi_{km}^{t})\delta(x_{km})+\pi_{km}^{t}\mathcal{N}(x_{km};A_{km},\Delta_{km})}$$

The matrix estimation problem has been decoupled into independent scalar problems, and the posterior mean and posterior variance can now be explicitly calculated as

$$g_a\left(R_{km}^t,\Sigma_{km}^t\right)=\pi_{km}^{t}A_{k,m},\qquad g_c\left(R_{km}^t,S_{km}^t\right)=\pi_{km}^{t}(\left|A_{km}^{t}\right|^2+\Delta_{km}^{t})-\left|g_a\right|^2$$

While assume full knowledge of the prior distribution !!

- **Learning of hyperparameters (EM Algorithm)**

$$Q(\boldsymbol{\theta},\boldsymbol{\theta}^{t})=E\left(\ln p(\mathbf{x},\mathbf{y})\mid\mathbf{y};\boldsymbol{\theta}^{t}\right),\quad \boldsymbol{\theta}^{t+1}=\arg\max_{\boldsymbol{\theta}}Q(\boldsymbol{\theta},\boldsymbol{\theta}^{t})$$

- **Threshold-based Activity Detector**

$$\hat{\alpha}_k=\begin{cases}1, & \sum_p\sum_m r\left(\hat{x}_{km}\right)\geq p_{\mathrm{th}}MP\\ 0, & \sum_p\sum_m r\left(\hat{x}_{km}\right)< p_{\mathrm{th}}MP\end{cases}$$

- **State Evolution**

AMP algorithms allow us to can accurately analyze their performance in asymptotic regime. Define MSE and variance of the estimated signal

$$E^t=\frac{1}{KM}\sum_k\sum_m\left|\hat{x}_{km}^{t}-x_{km}^{t}\right|^2,\quad V^t=\frac{1}{KM}\sum_k\sum_m v_{k,m}^{t}$$

The mean and variance of posterior distribution are expressed as

$$X_0\sim p_0(\mathbf{X}),\ Z\sim\mathcal{CN}(z;0,1)\Rightarrow R^t=x_0+\sqrt{\frac{\sigma_0^2+E^t}{G/K}}z,\quad \Sigma^t=\frac{(\sigma^2)^t+V^t}{G/K}$$

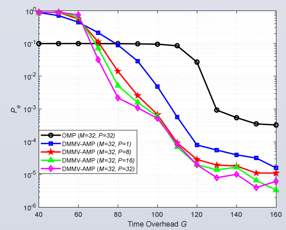
Thus the performance can be predicted as

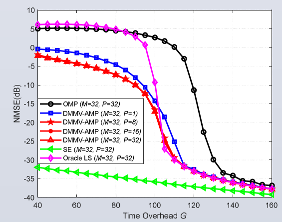
$$\boxed{E^t=\mathbb{E}\left[\left|g_a\left(\mathrm{R}^t,\Sigma^t\right)-x_0\right|^2\right],\quad V^t=\mathbb{E}\left[g_c\left(\mathrm{R}^t,\Sigma^t\right)\right]}$$

Simulation Results

The performance is evaluated as

$$P_e=\frac{\sum_k\left|\hat{\alpha}_k-\alpha_k\right|}{K},\quad \mathrm{NMSE}=10\log\frac{\sum_p\left\|\hat{\mathbf{X}}_p-\mathbf{X}_p\right\|_F^2}{\sum_p\left\|\mathbf{X}_p\right\|_F^2}$$





- The proposed DMMV-AMP algorithm outperforms the conventional state-of-art algorithm.
- Common sparsity observed from different subcarriers can be exploited to improve the performance.
- The performance can no longer be enhanced when P is large enough, since the support is detected exactly in this case.
- The performance of the proposed scheme can be well predicted by SE when time overhead is sufficient.