Translations of a completely 0-simple semigroup

This section is based on [1], which describes the left and right translations of a completely 0-simple semigroup, as well as the translational hull. The notation of this section is the same as that paper. Define $W_G(I)$ and G_G to be generating sets for $W(I)$ and G. Define $\phi_{i,a} : I \to G$ by

$$\phi_{i,a}(j) = \begin{cases} a & i = j \\ 1_G & i \neq j \end{cases}$$

Define $\Phi : I \to G$ as mapping every element to the identity of G.

Claim: $L(I,G)$ is generated by $X = \{(\mathrm{id}, \phi_{i,a}) \,|i \in I, a \in G_G\} \cup \{(\alpha, \Phi) \,|\alpha \in W_G(I)\}$.

Proof:

It is easy to see how to generate all elements of the form (id, ϕ) from the first term in the union. An element of the form (α, Φ) can be generated from elements in the second term of the union. An element of the form (α, ϕ) can then be expressed as $(\alpha, \phi) = (\alpha, \Phi)\,(\mathrm{id}, \phi)$.

Size

$|L(I,G)| = (|I||G| + 1)^{|I|}$

Proof:

$(|I||G| + 1)^{|I|} = \sum_{n=0}^{|I|} \binom{|I|}{n} |I|^n |G|^n$, and this sum can be seen as choosing a domain for the partial transformation, choosing values for the partial transformation, and choosing a function from the domain to the group.

A smaller generating set

Instead of including functions mapping each element of the index set to each generator of the group, we can choose a single element of the index set - denoted by 1 - and use only functions from this element to each generator, together with transformations on the index set - i.e.

$$X' = \{(\mathrm{id}, \phi_{1,a}) \,|a \in G_G\} \cup \{(\alpha, \Phi) \,|\alpha \in W_G(I)\}$$

is a generating set for $L(I,G)$. To see that this is true, note that
$(\mathrm{id}, \phi_{i,a}) = ((1\ i), \Phi)\,((\mathrm{id}, \phi_{1,a})\,((1\ i), \Phi))$

References

[1] Mario Petrich. The translational hull of a completely 0-simple semigroup. *Glasgow Mathematical Journal*, 9:1–11, 1 1968.