Simplicial surfaces

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27.09.2017

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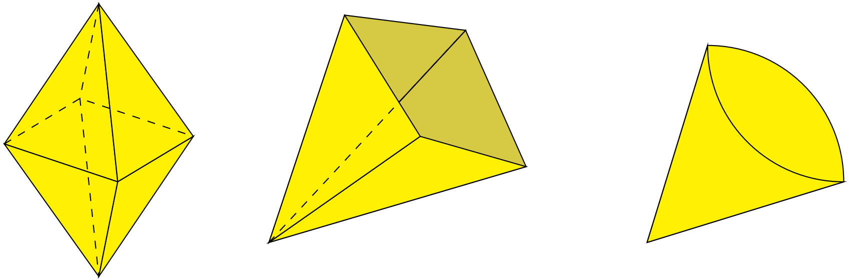
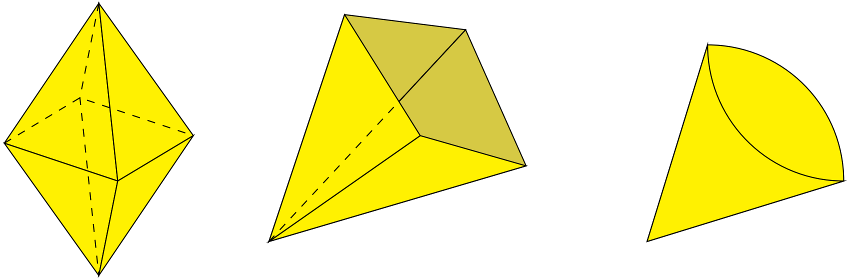
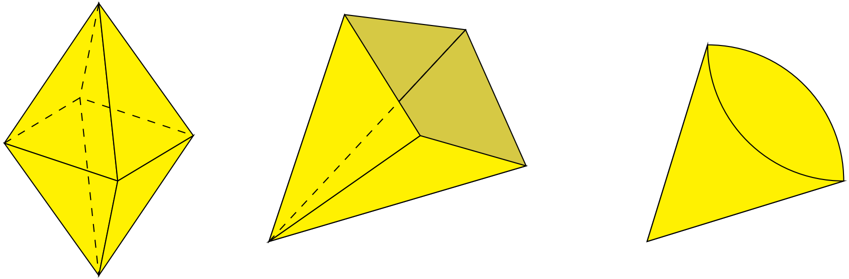


Simplicial surfaces



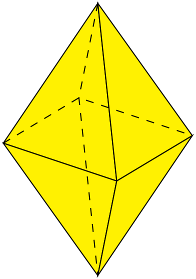
⇝ structures build from triangles

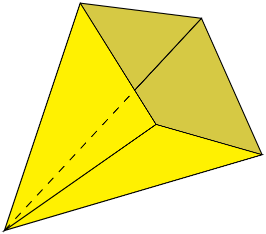
Simplicial surfaces

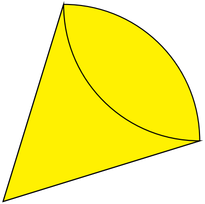


$\rightsquigarrow$ structures build from triangles (or arbitrary polygons)

Simplicial surfaces







$\rightsquigarrow$ structures build from triangles (or arbitrary polygons)

$\rightarrow$ focus on the combinatorial/incidence structure (without embedding)

Classification

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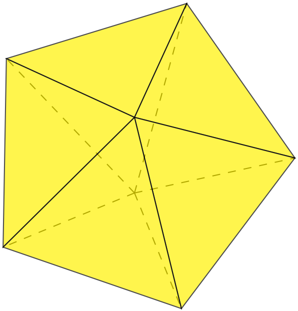

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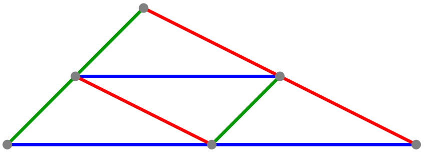

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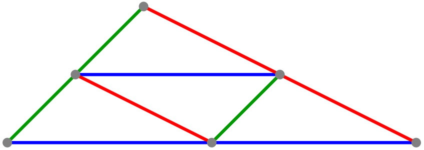

$\rightsquigarrow$ edge–colouring encodes lengths

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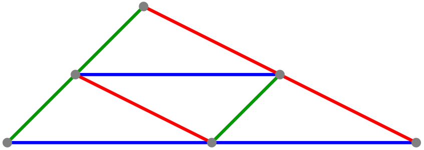

$\rightsquigarrow$ edge–colouring encodes lengths
$\rightsquigarrow$ analyse edge–coloured surfaces

Colouring as permutation

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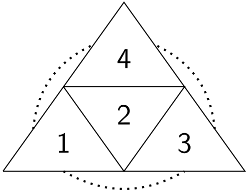
Consider a tetrahedron



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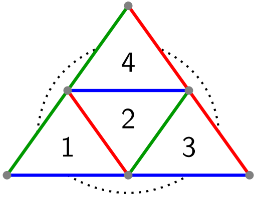
Consider a tetrahedron with an edge colouring



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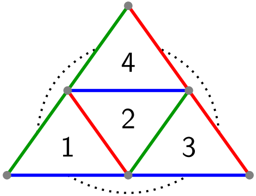

simplicial surface $\Rightarrow$

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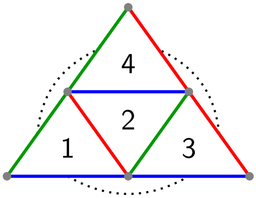

simplicial surface $\Rightarrow \leq 2$ faces at each edge

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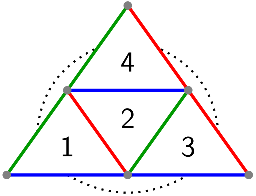
Consider a tetrahedron with an edge colouring



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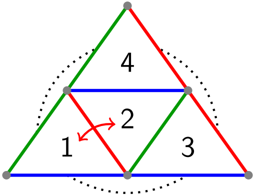

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- $(1,2)$

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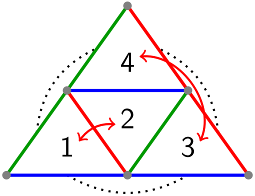

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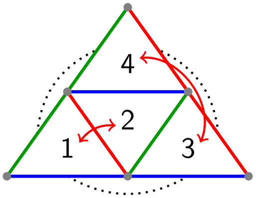

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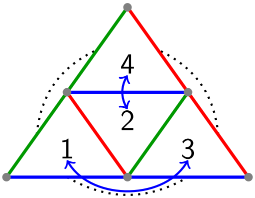

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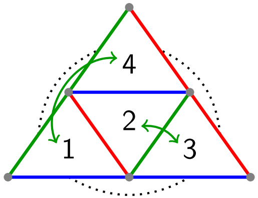

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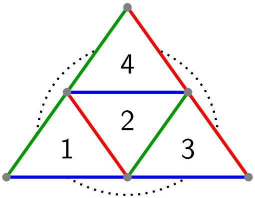

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- $\rightsquigarrow$ group theoretic considerations

Implementation in GAP

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Example: define a coloured icosahedron:

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[ [ 1, 2 ], [ 1, 3 ], [ 1, 4 ], [ 1, 5 ], [ 1, 6 ], [ 2, 3 ], [ 2, 4 ], [ 2, 7 ],
[ 2, 8 ], [ 3, 6 ], [ 3, 7 ], [ 3, 11 ], [ 4, 5 ], [ 4, 8 ], [ 4, 9 ], [ 5, 6 ],
[ 5, 9 ], [ 5, 10 ], [ 6, 10 ], [ 6, 11 ], [ 7, 8 ], [ 7, 11 ], [ 7, 12 ], [ 8, 9 ],
[ 8, 12 ], [ 9, 10 ], [ 9, 12 ], [ 10, 11 ], [ 10, 12 ], [ 11, 12 ] ],

[ [ 1, 2, 6 ], [ 1, 3, 7 ], [ 3, 4, 13 ], [ 4, 5, 16 ], [ 2, 5, 10 ], [ 6, 8, 11 ],
[ 7, 9, 14 ], [ 13, 15, 17 ], [ 16, 18, 19 ], [ 10, 12, 20 ], [ 8, 9, 21 ],
[ 14, 15, 24 ], [ 17, 18, 26 ], [ 19, 20, 28 ], [ 11, 12, 22 ], [ 21, 23, 25 ],
[ 24, 25, 27 ], [ 26, 27, 29 ], [ 28, 29, 30 ], [ 22, 23, 30 ] ],

[ 1, 2, 3, 1, 3, 3, 2, 1, 3, 1, 2, 3, 2, 1, 3, 2, 1, 3, 1, 2, 2, 1, 3, 2, 1, 2, 3, 3,
1, 2 ] );;
```

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gap> DrawSurfaceToTikZ(iko,"NetIko.tex");
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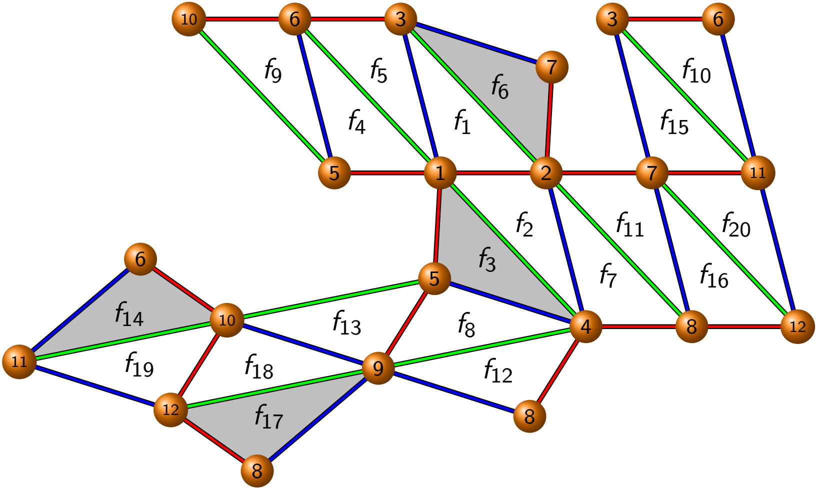
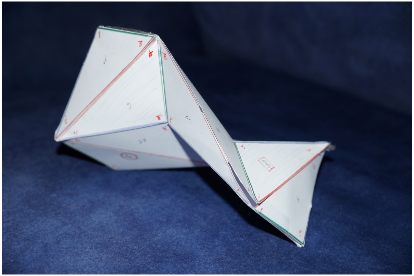
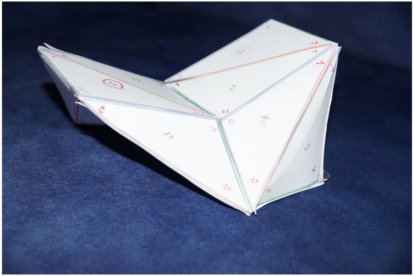


Embedded icosahedron





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Thanks for your attention

Alice Niemeyer
Wilhelm Plesken

Markus Baumeister
Ansgar Strzelczyk