Simplicial surfaces in GAP

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Lehrstuhl B für Mathematik
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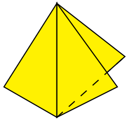

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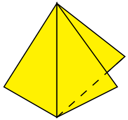

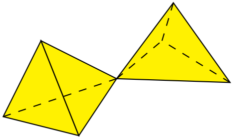

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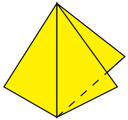

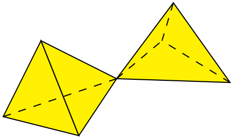

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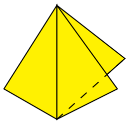

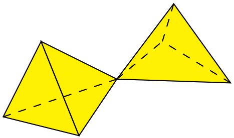

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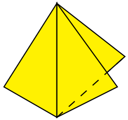

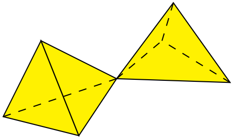

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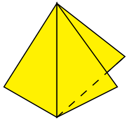

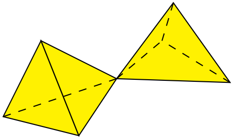

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$\rightsquigarrow$ incidence geometry

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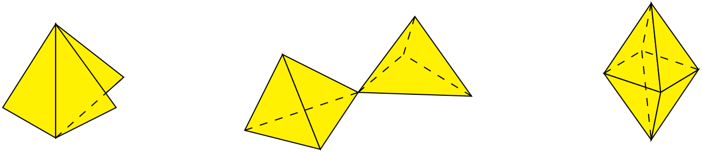

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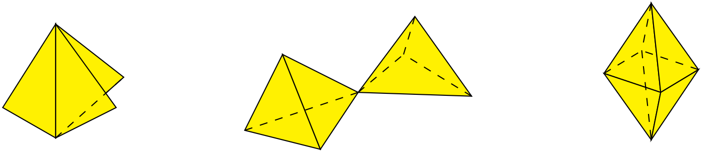

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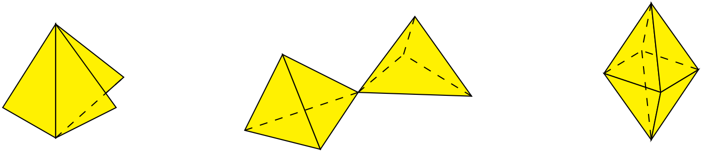

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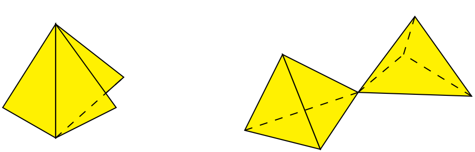

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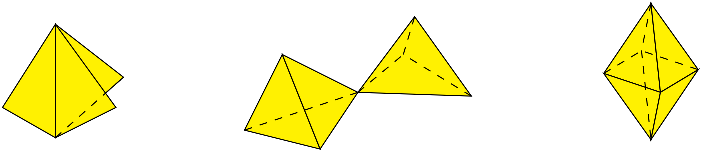

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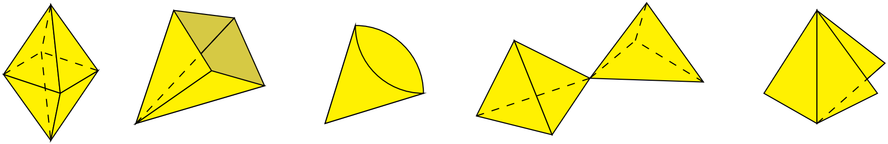

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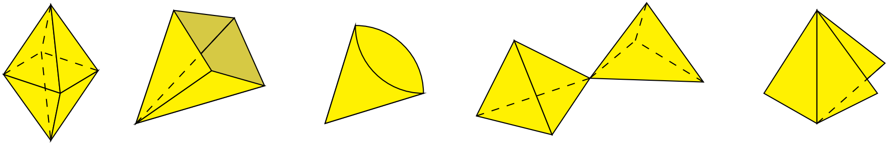

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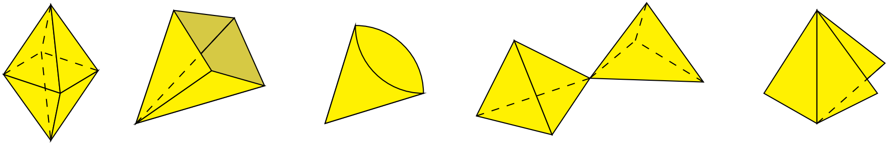

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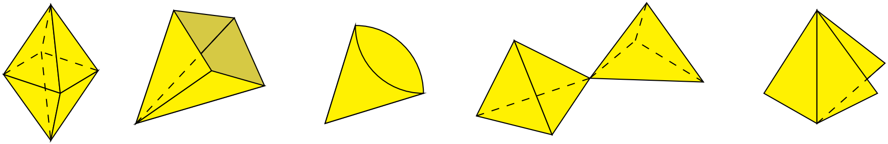

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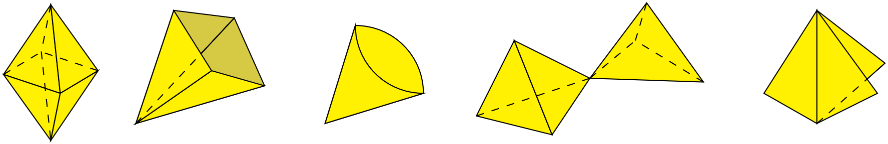

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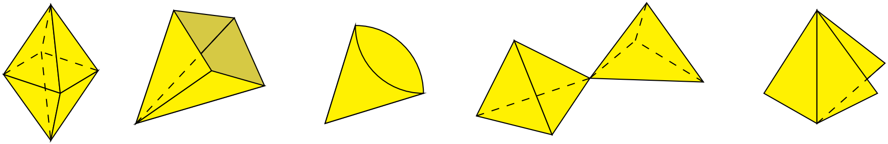

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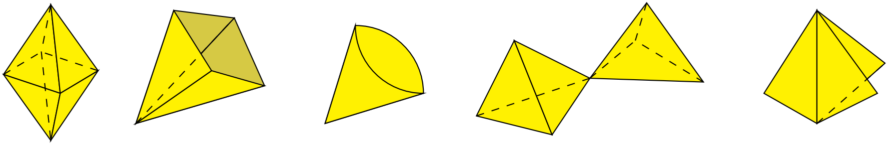

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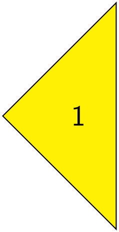

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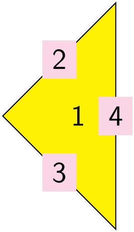

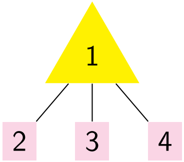

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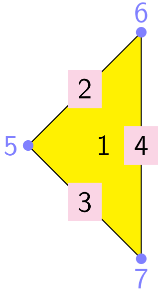

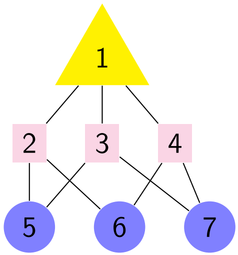

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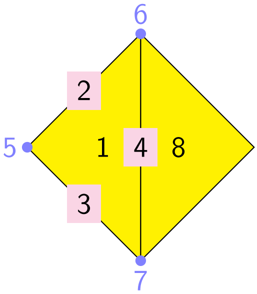

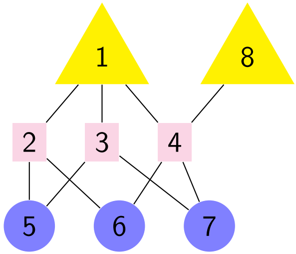

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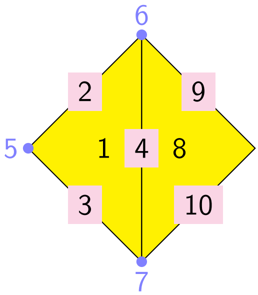

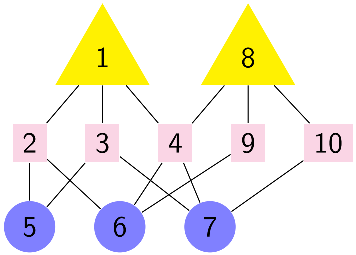

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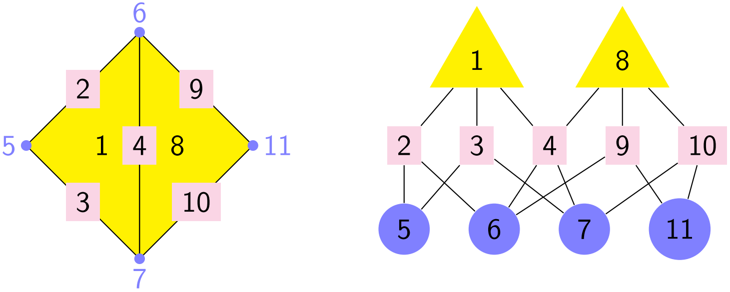

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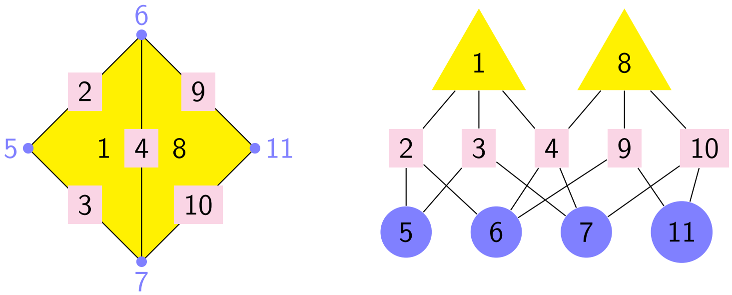

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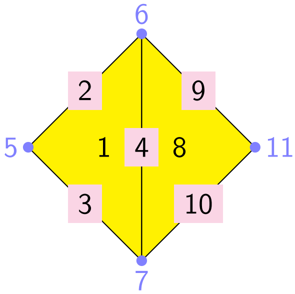

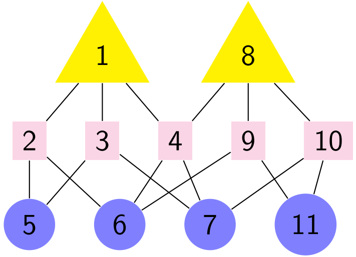

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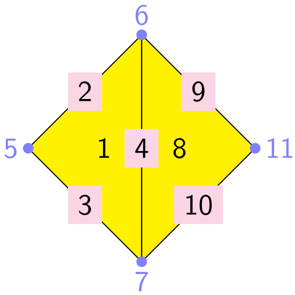

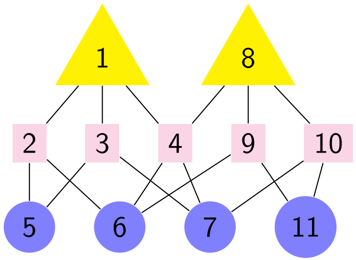

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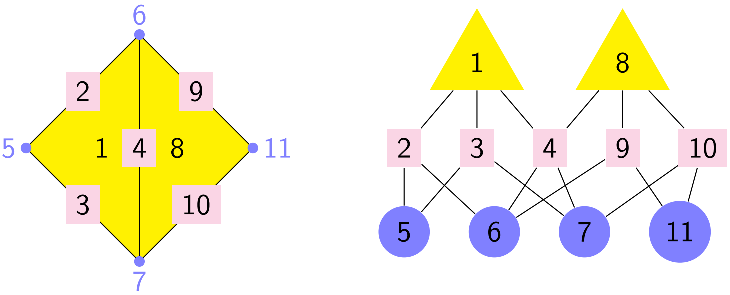

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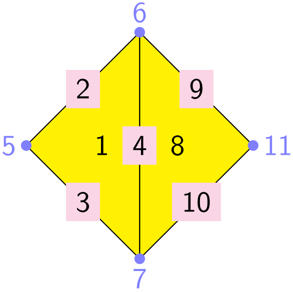

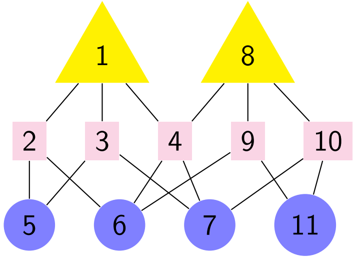

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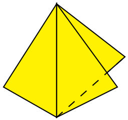

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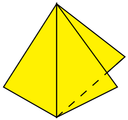

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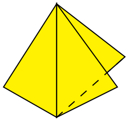

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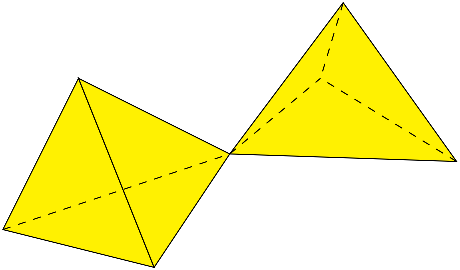

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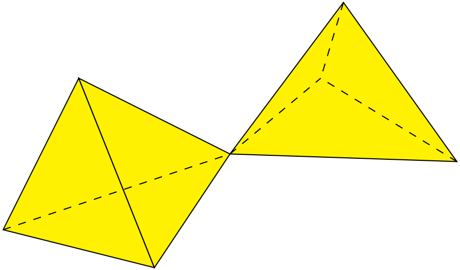

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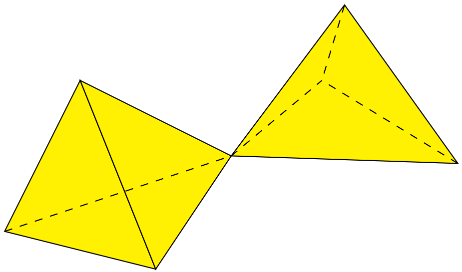

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A **simplicial surface** does not have these ramifications.

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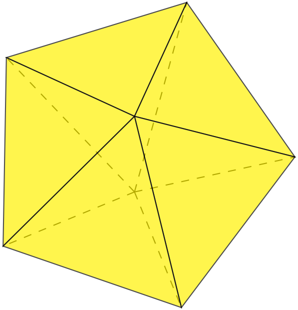

Progress report of triangular complexes

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- advanced properties (any wishes?)

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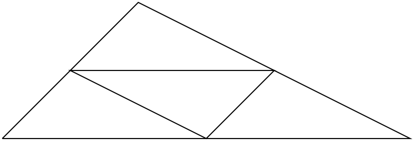

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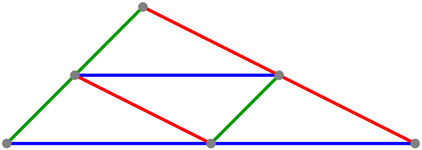

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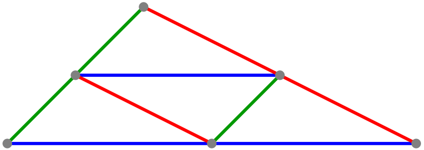

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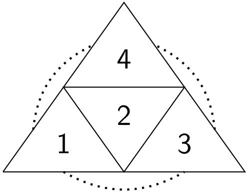

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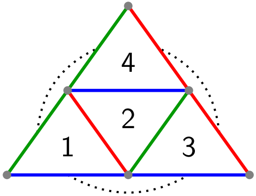
Consider a tetrahedron with an edge colouring



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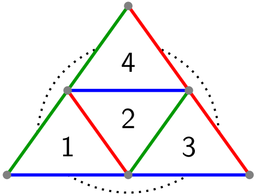

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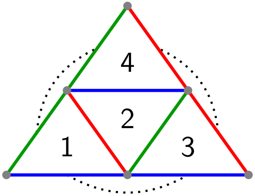

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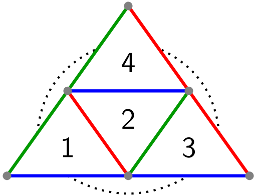

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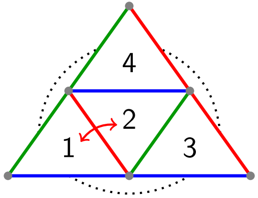

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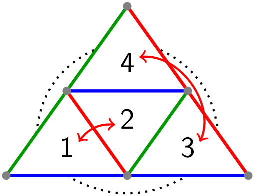

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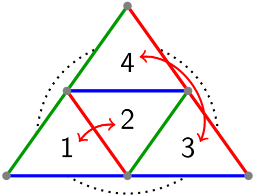

simplicial surface $\Rightarrow$ at most two faces at each edge

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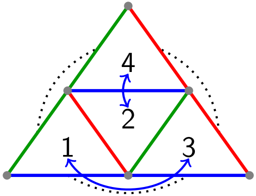

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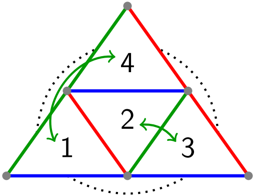

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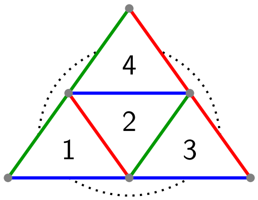

simplicial surface ⇒ at most two faces at each edge

- ⇝ every edge defines a transposition of incident faces
- ⇝ every colour class defines a permutation of the faces
  - $(1,2)(3,4)$, $(1,3)(2,4)$, $(1,4)(2,3)$
- ⇝ group theoretic considerations

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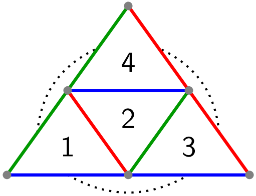

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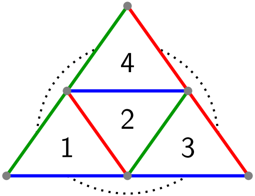

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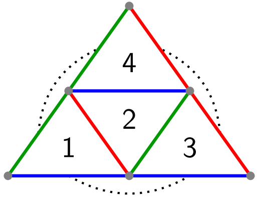

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- (1,2)(3,4) , (1,3)(2,4) , (1,4)(2,3)

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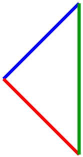
- The connected components of the surface correspond to the orbits of $\langle \sigma_a, \sigma_b, \sigma_c \rangle$ on the faces (fast computation for permutation groups)

How do faces fit together?

How do faces fit together?

Consider a face of the surface



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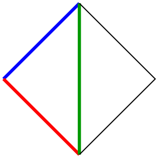

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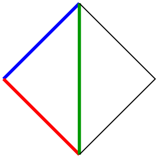
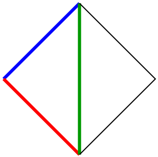
Consider a face of the surface and a neighbouring face
The neighbour can be coloured in two ways:





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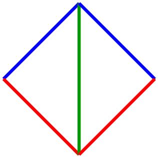

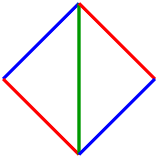

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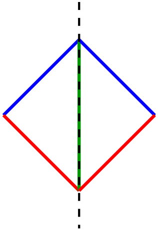
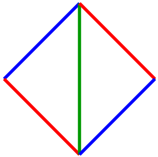


mirror (m)



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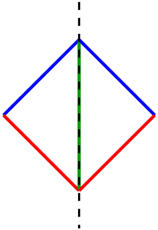

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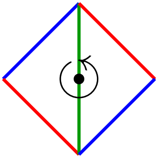

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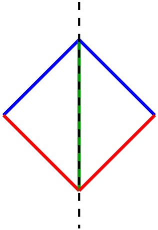

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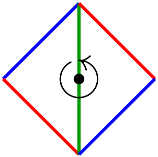

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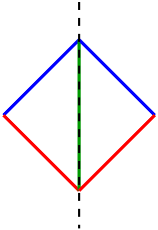

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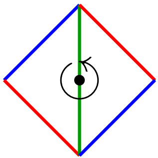

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$\rightsquigarrow$ **mr–assignment** for the edges of the surface

Theorem

Permutations and mr–assignment uniquely determine the surface.

Constructing surfaces from groups

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A general mr–assignment leads to complicated surfaces.

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Simplification: edges of same colour have the same mr–type

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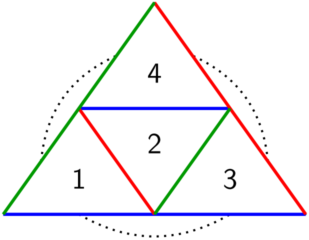

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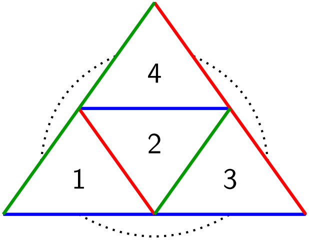

has only r–edges.

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If all edges are mirrors, the situation is simple

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A simplicial surface has only mirror–edges iff it covers a single triangle, i. e. there is a surjective incidence–preserving map to the simplicial surface consisting of exactly one face.

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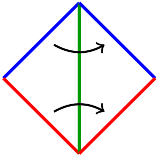

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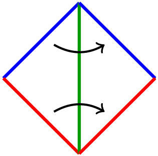

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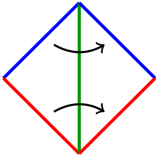

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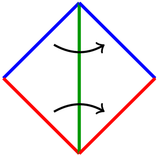

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- Mirror–colouring defines a map to the triangle

Construction from permutations

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Start with three involutions σ_a, σ_b, σ_c in permutation representation

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There exists a coloured surface with the given involutions where all edges are mirror edges.

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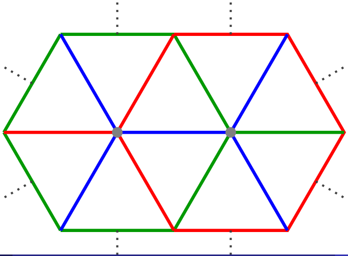

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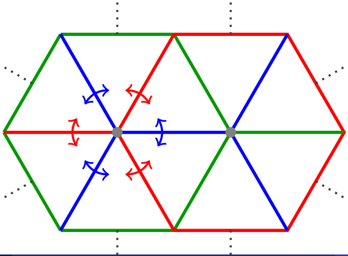

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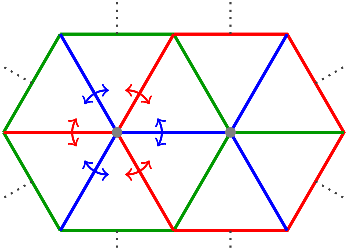

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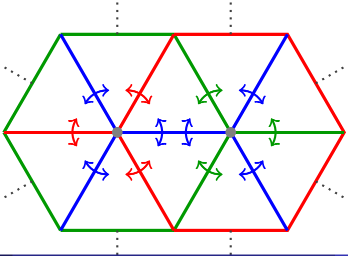

Construction example

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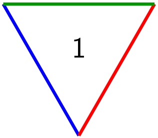

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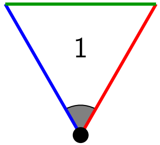

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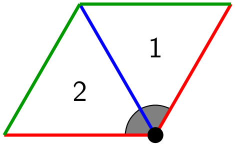

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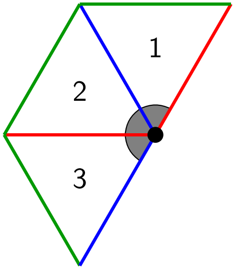

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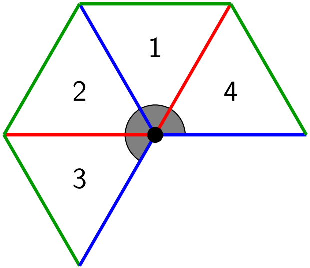

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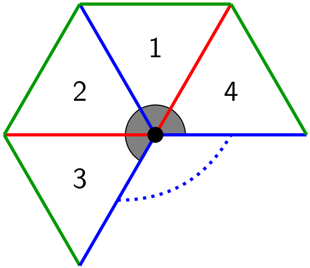

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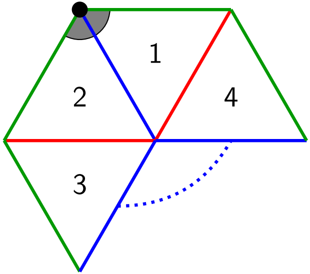

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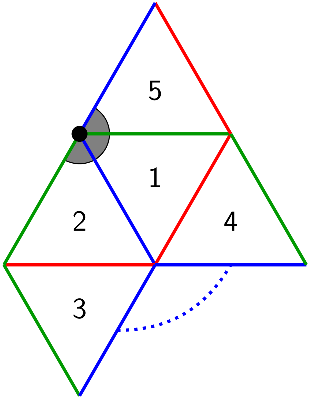

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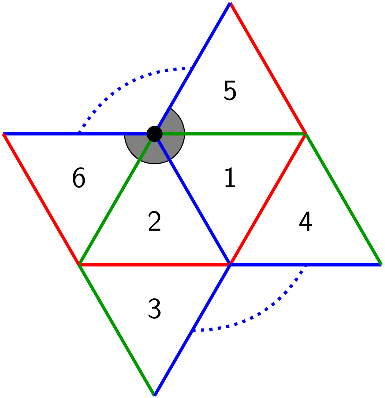

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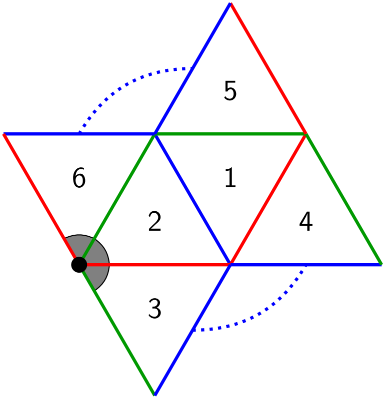

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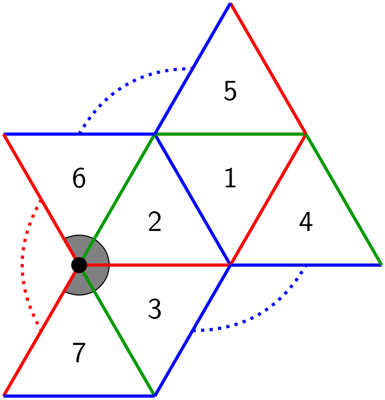

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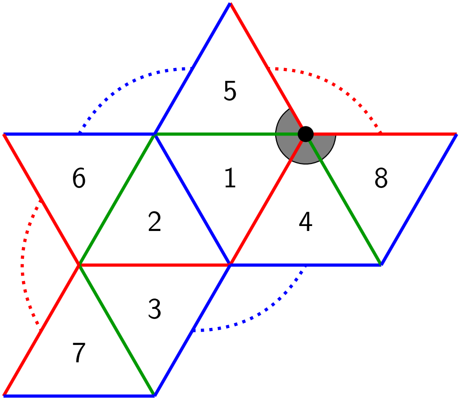

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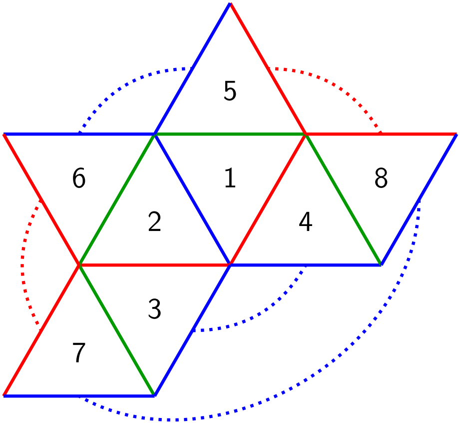

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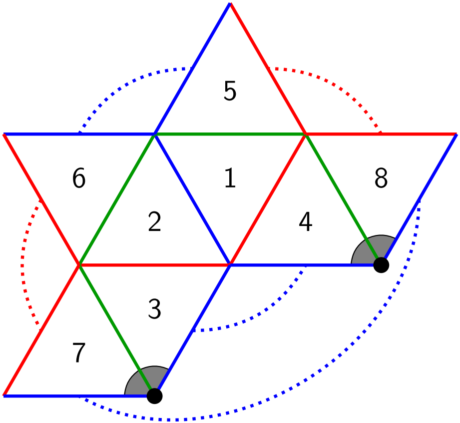

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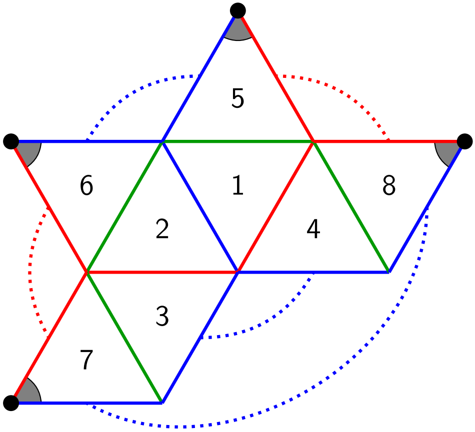

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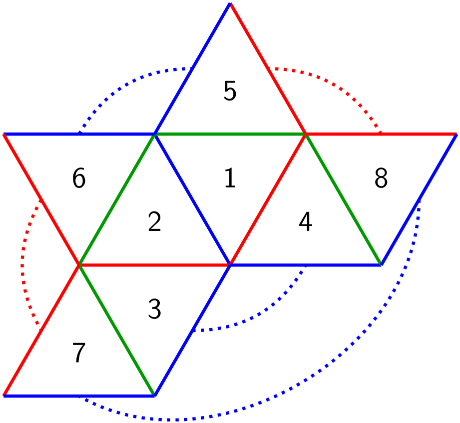

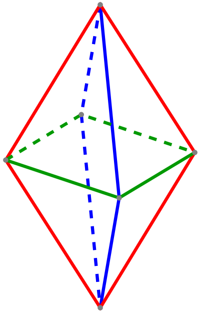

Net of an icosahedron

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`iko`: coloured icosahedron

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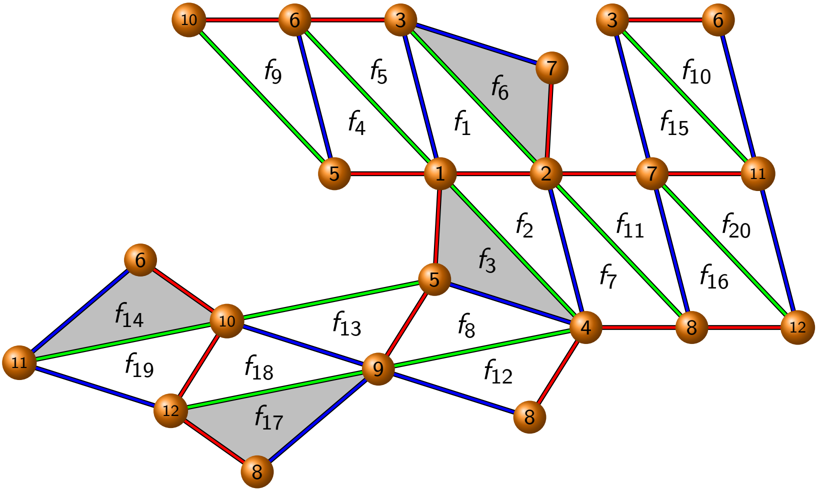

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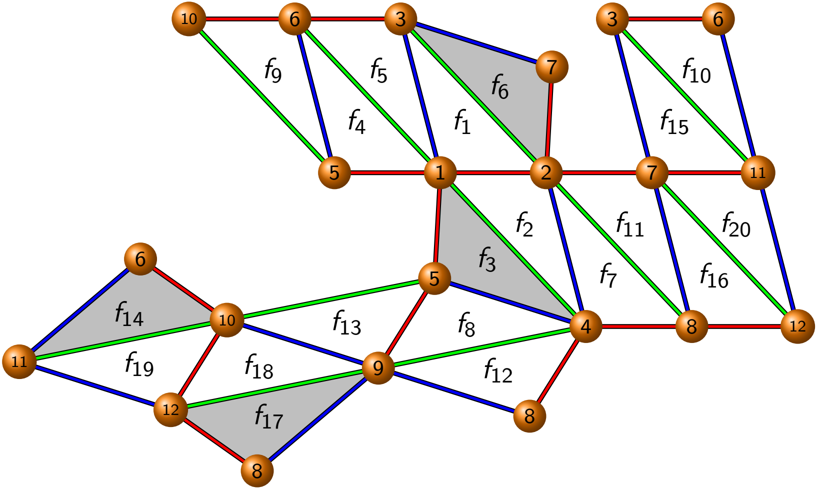
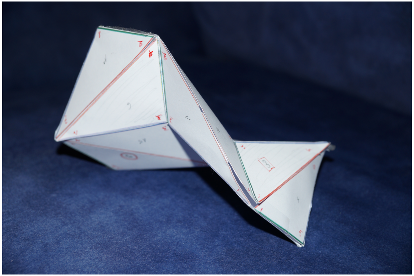
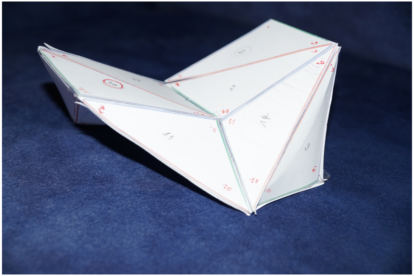


Embedded icosahedron





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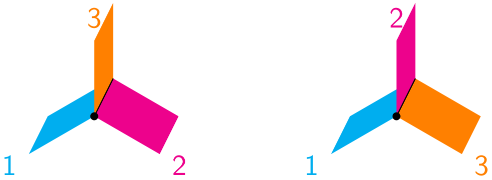

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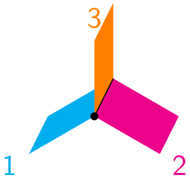

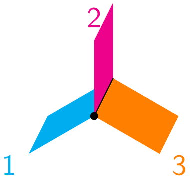

$\rightsquigarrow$ **folding complex**

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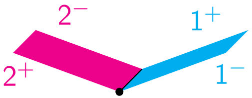

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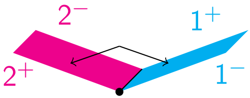

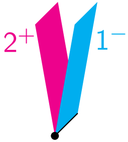

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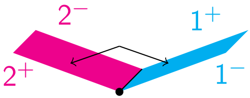

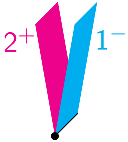

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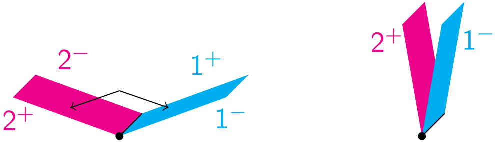

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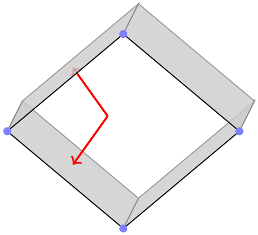

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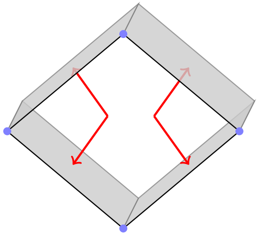

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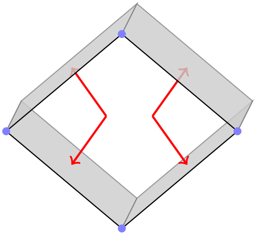

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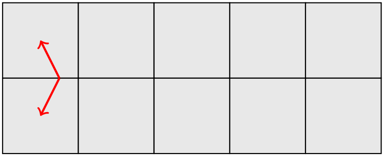

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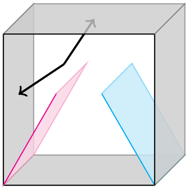

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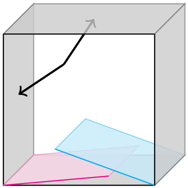

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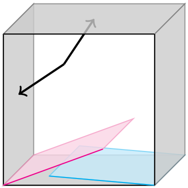

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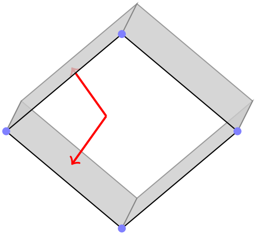

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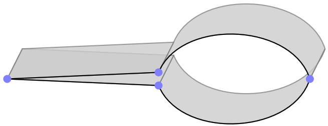

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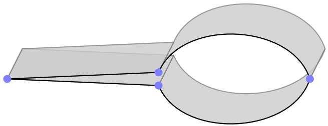

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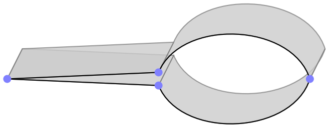
  $\rightsquigarrow$ Relax the rigidity–constraint:

  - Allow non–rigid configurations as transitional states



Structure of multiple foldings

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With folding plans we can perform the same folding in different folding complexes

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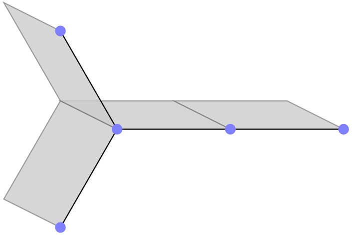

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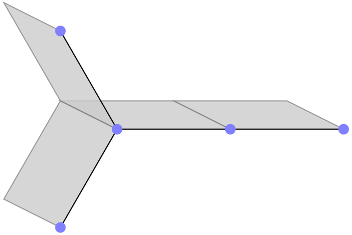

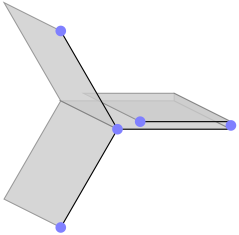

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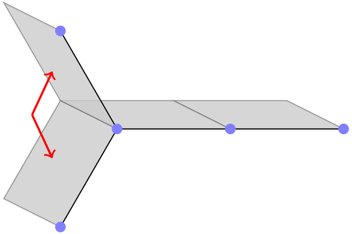

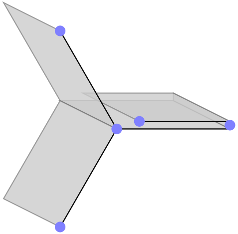

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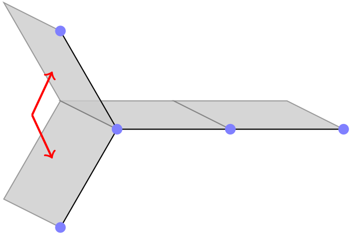

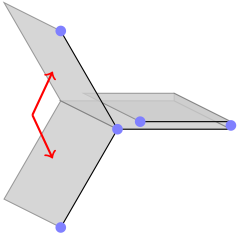

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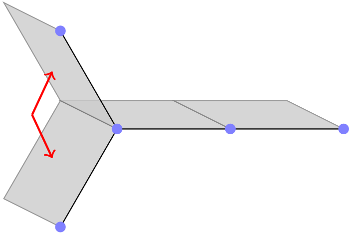

$\rightsquigarrow$ more structure on the set of possible foldings

Folding graph

Folding graph

- Vertices are folding complexes (modelling folding states)

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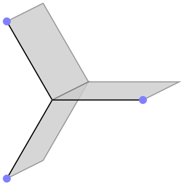

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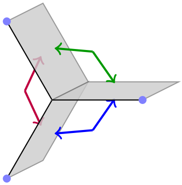

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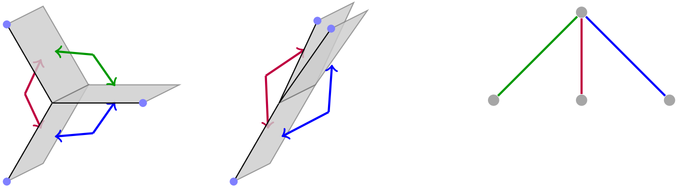

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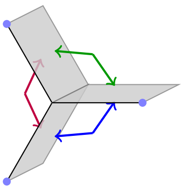

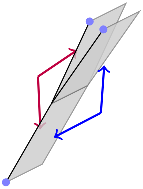

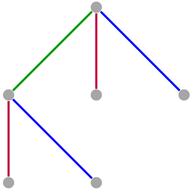

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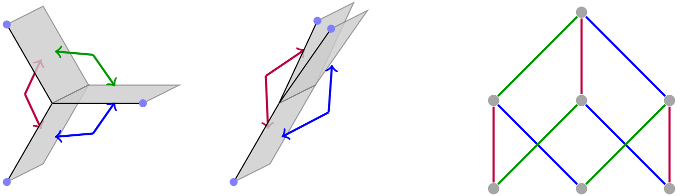

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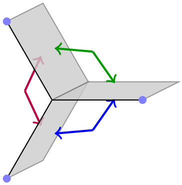

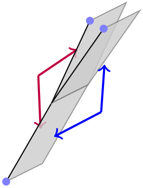

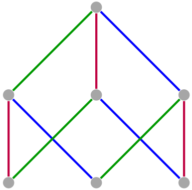

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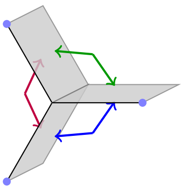

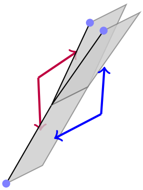

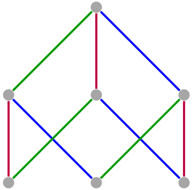

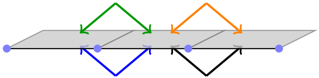

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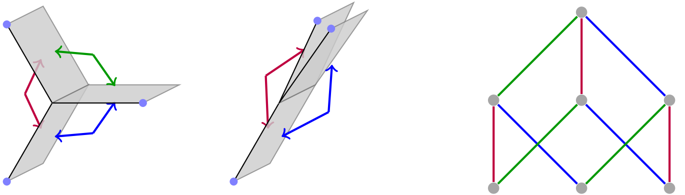

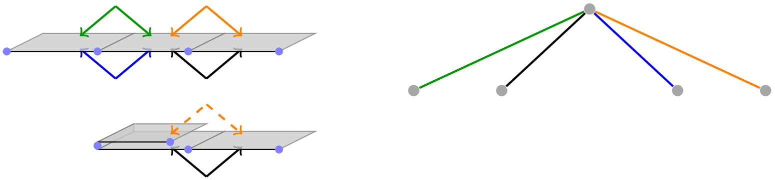

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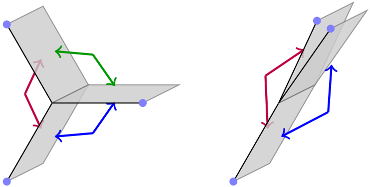

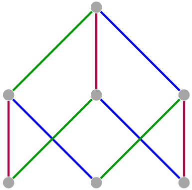

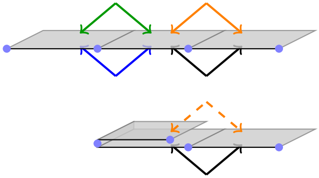

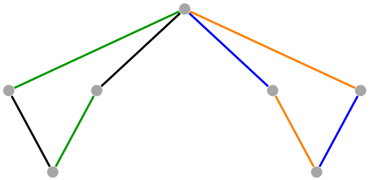

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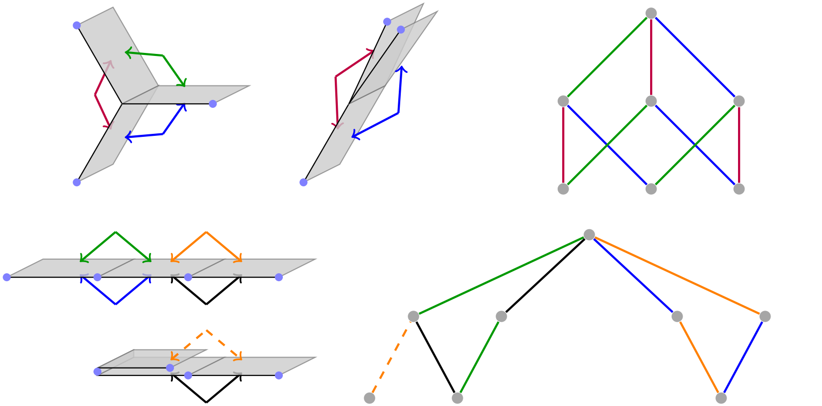

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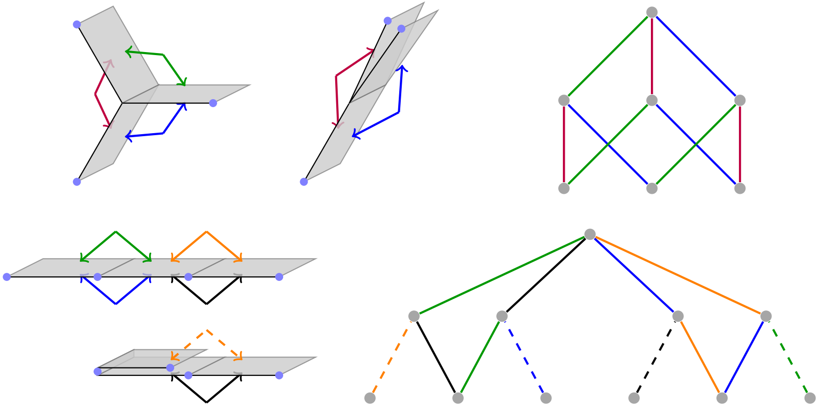

Larger graph

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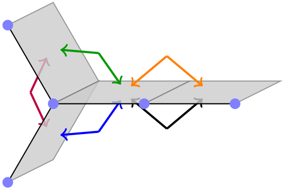

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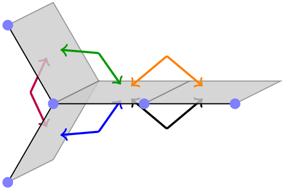

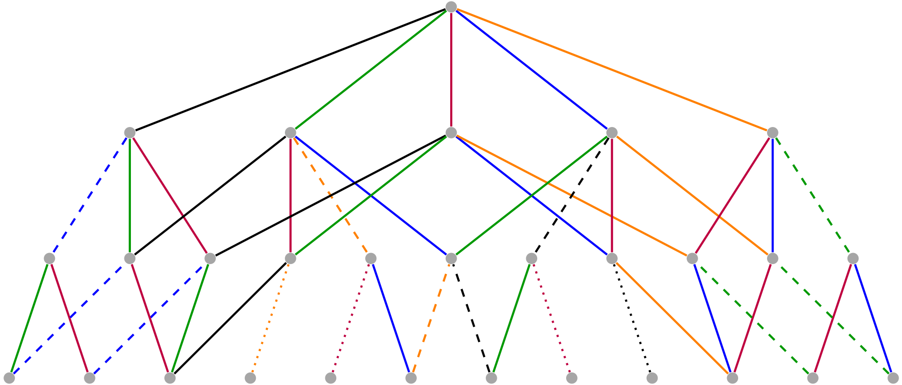

Drawback of folding plans

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Some foldings that "should" be the same, aren't:

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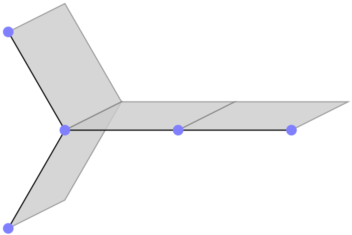

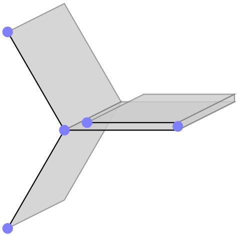

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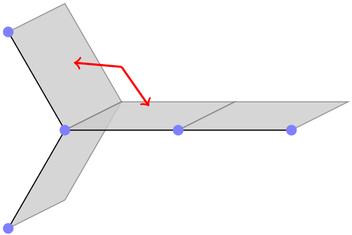

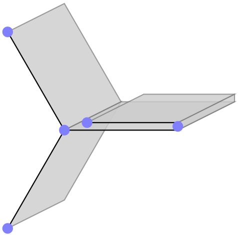

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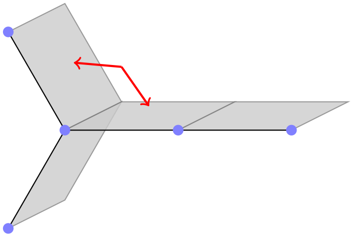

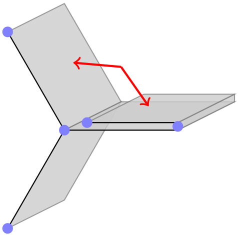

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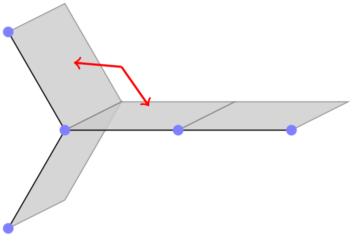

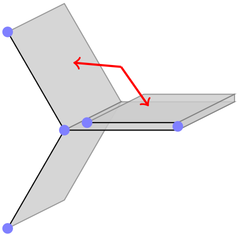

$\Rightarrow$ If you know the folding structure of a small complex,

Drawback of folding plans

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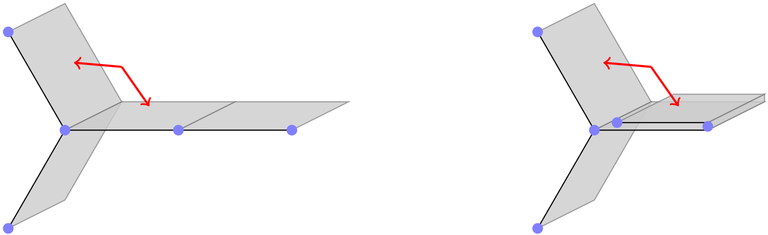

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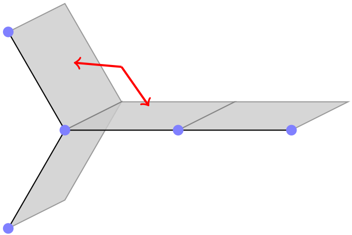

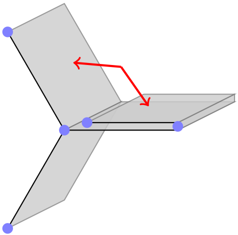

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$\rightsquigarrow$ Folding plans are not optimal to model folding

Progress report of abstract folding

Progress report of abstract folding

In development:

Progress report of abstract folding

In development:

- folding complex

Progress report of abstract folding

In development:

- folding complex
- folding plans

Progress report of abstract folding

In development:

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- folding graph

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Missing:

Progress report of abstract folding

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Missing:

- better folding description

Progress report of abstract folding

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Progress report of abstract folding

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Summary: `SimplicialSurfaces`

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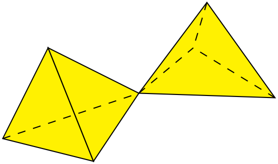
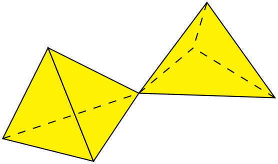
Triangular complexes



Summary: SimplicialSurfaces

Triangular complexes

- mostly complete



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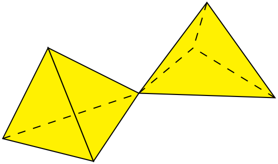
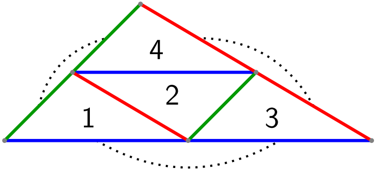


Edge colouring



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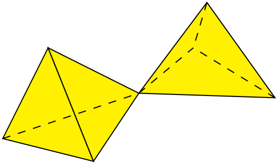
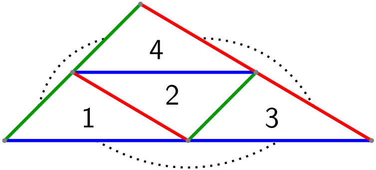


Edge colouring

- current theory implemented



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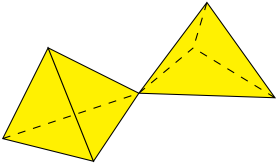

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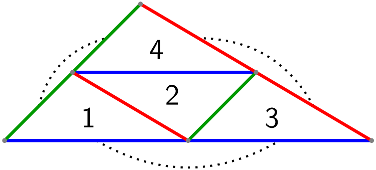

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- current theory implemented
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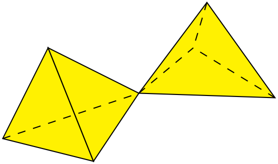

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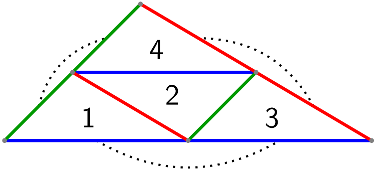
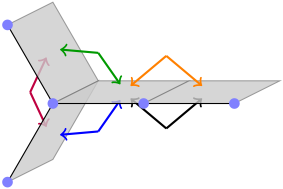


Abstract folding



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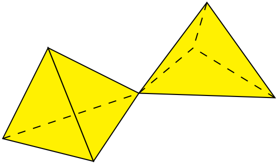

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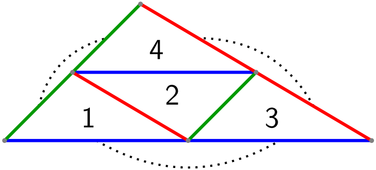
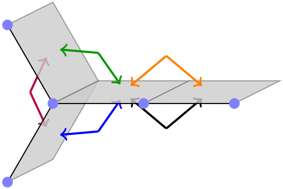


Abstract folding

- framework exists



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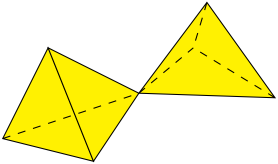

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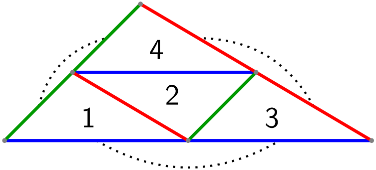

Abstract folding

- framework exists
- needs proper implementation