Simplicial surfaces in GAP

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(j/w Alice Niemeyer)

Lehrstuhl B für Mathematik
RWTH Aachen University

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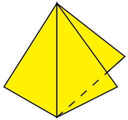

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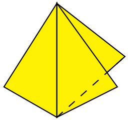

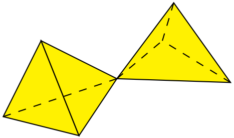

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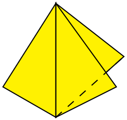

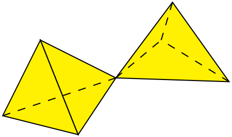

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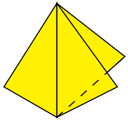

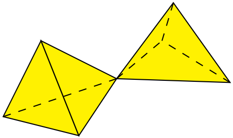

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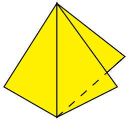

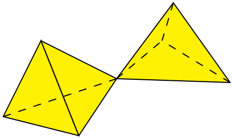

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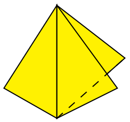

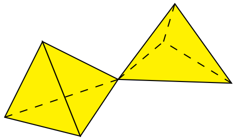

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$\rightsquigarrow$ incidence geometry

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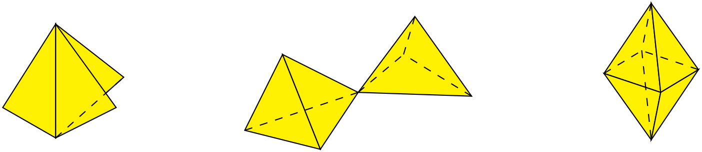

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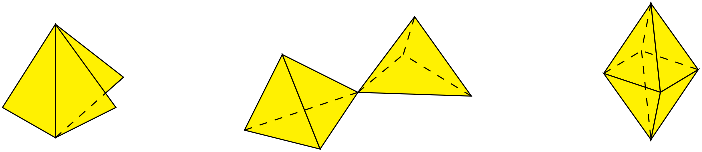

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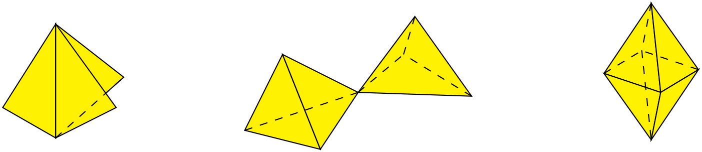

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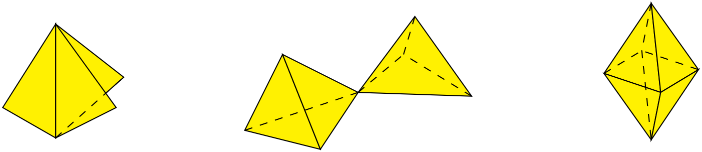

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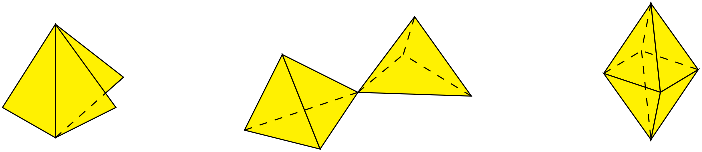

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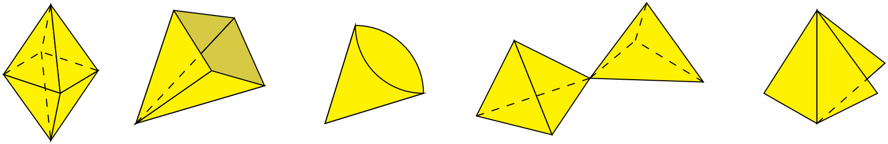

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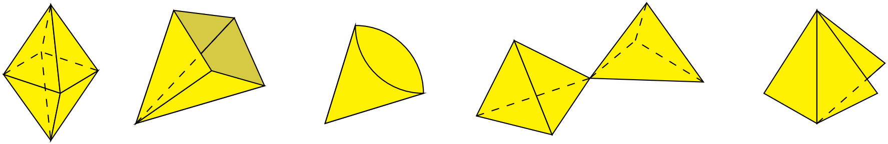

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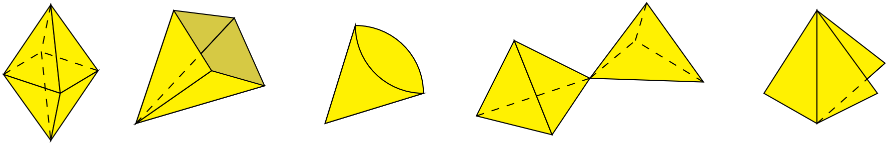

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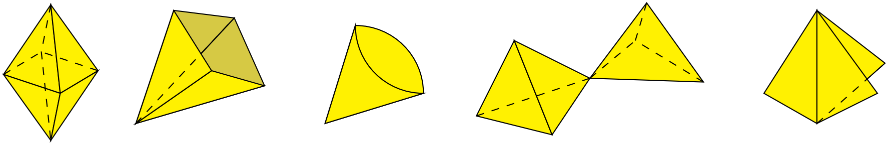

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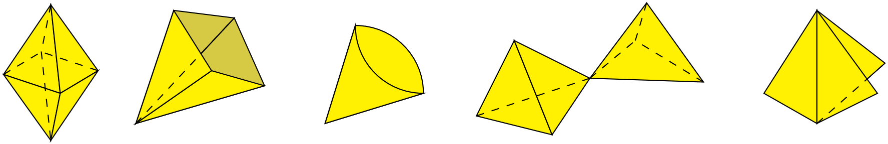

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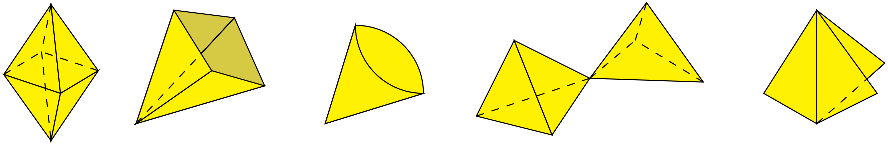

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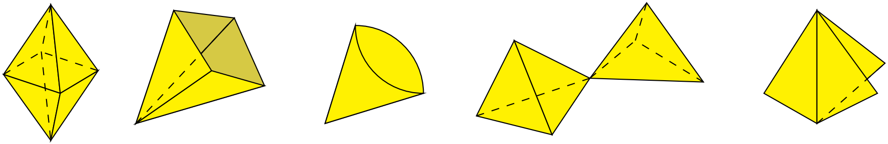

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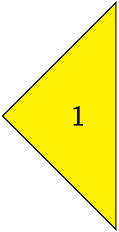

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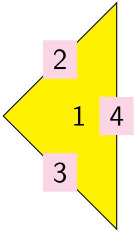

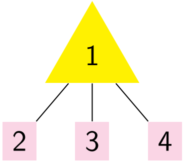

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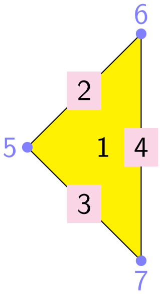

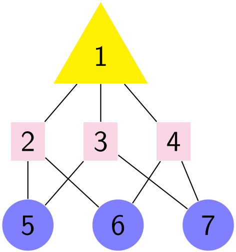

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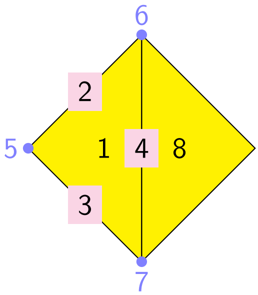

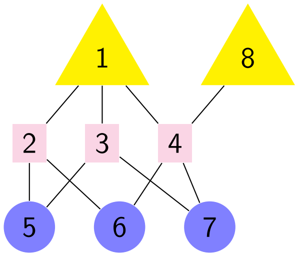

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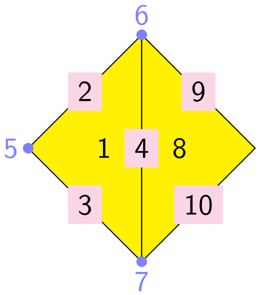

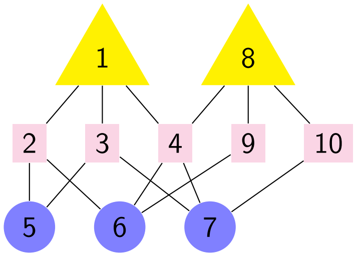

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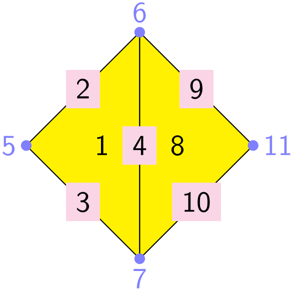

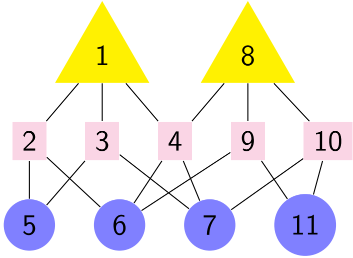

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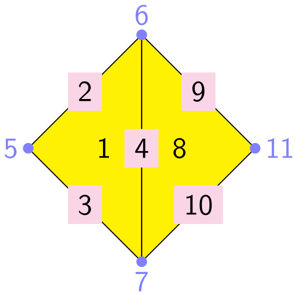

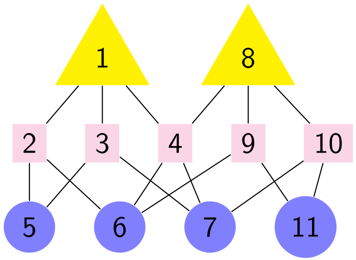

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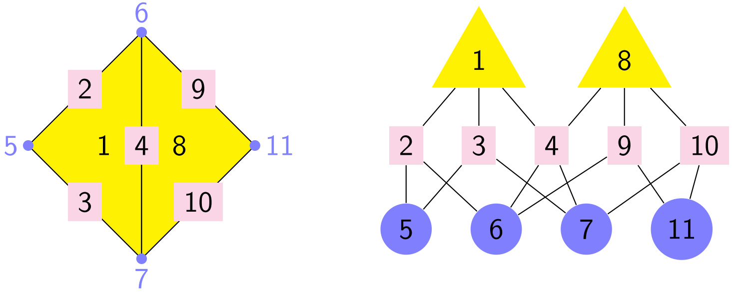

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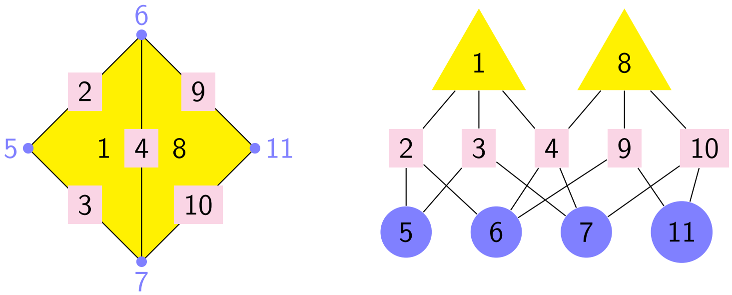

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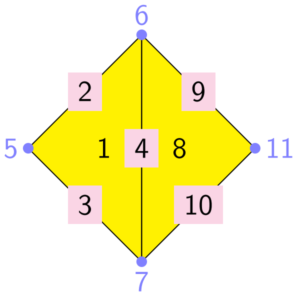

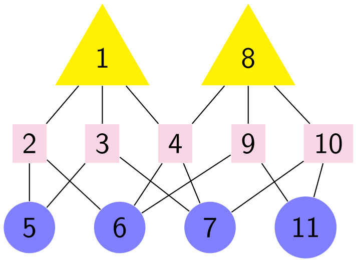

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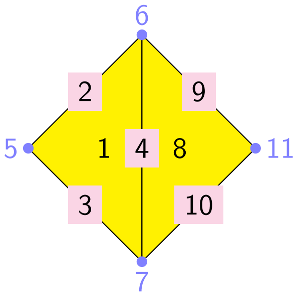

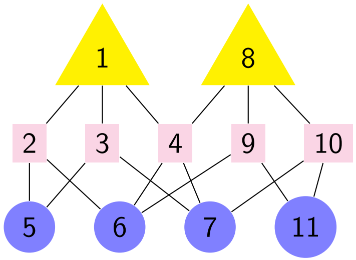

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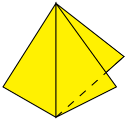

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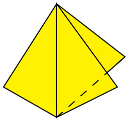

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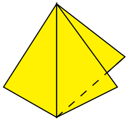

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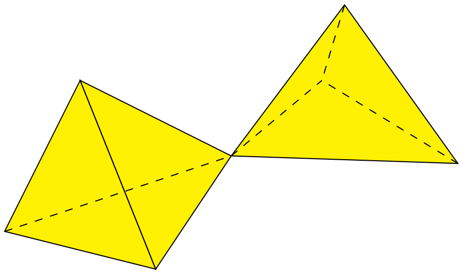

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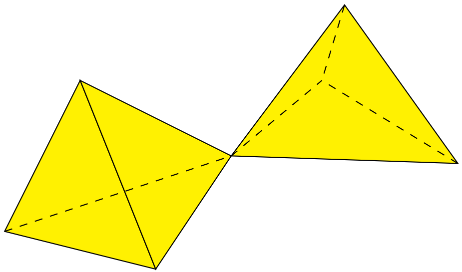

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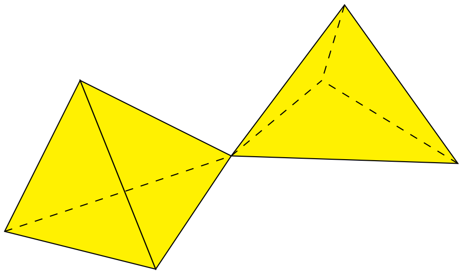

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A **simplicial surface** does not have these ramifications.

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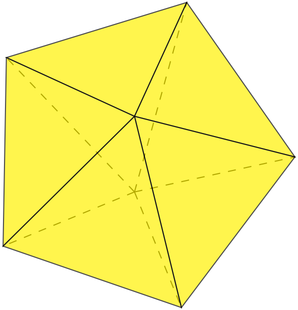
- e. g. only one surface with 10 triangles:



Progress report of triangular complexes

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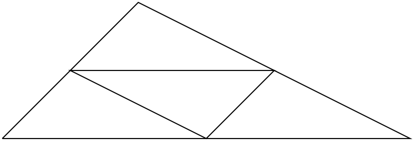

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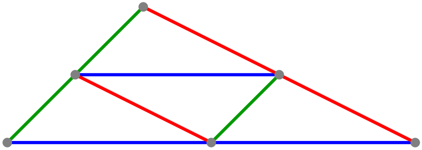

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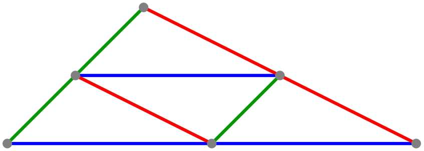

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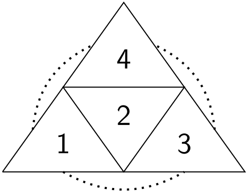

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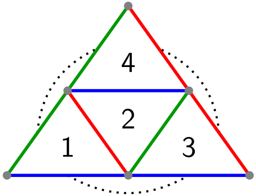
Consider a tetrahedron with an edge colouring



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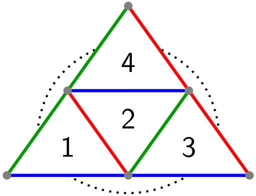

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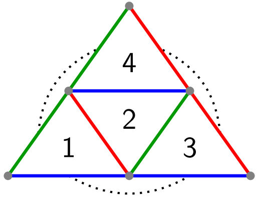

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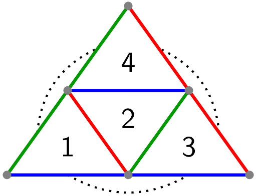

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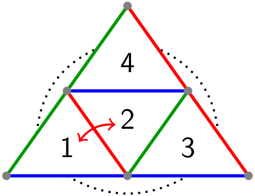

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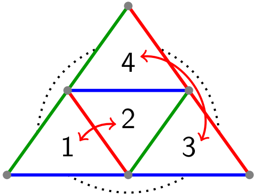

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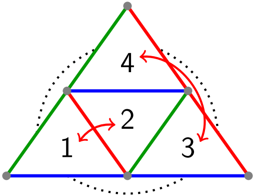

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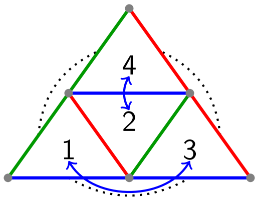

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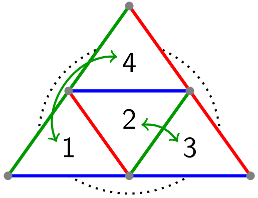

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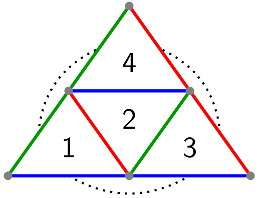

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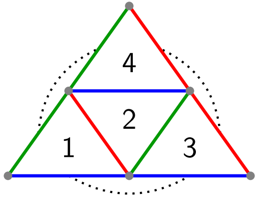

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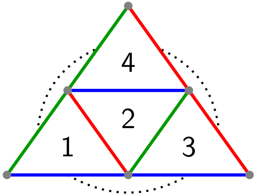

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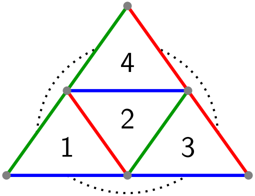

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- The connected components of the surface correspond to the orbits of $\langle \sigma_a, \sigma_b, \sigma_c \rangle$ on the faces (fast computation for permutation groups)

How do faces fit together?

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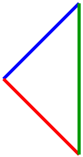
Consider a face of the surface



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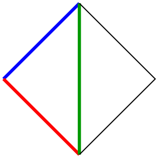

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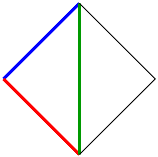

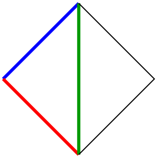

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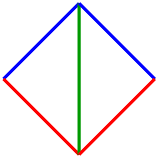

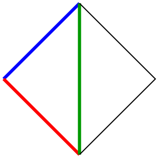

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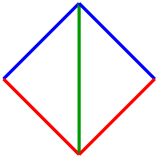

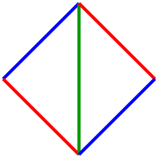

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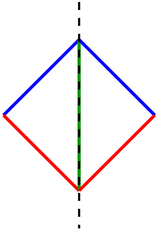
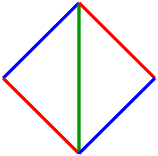


mirror (m)



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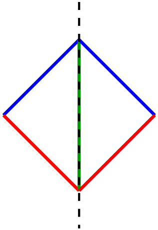

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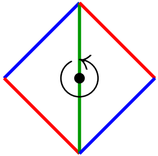

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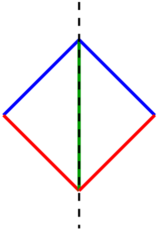

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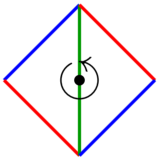

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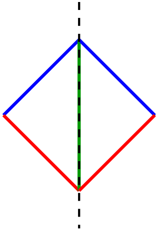

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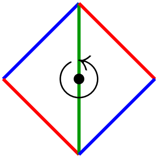

rotation (r)

$\rightsquigarrow$ **mr–assignment** for the edges of the surface

Satz

Permutations and mr–assignment uniquely determine the surface.

Constructing surfaces from groups

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A general mr–assignment leads to complicated surfaces.

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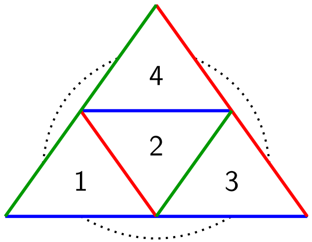

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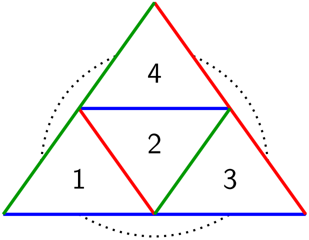

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The mirror–case

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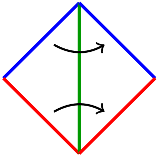

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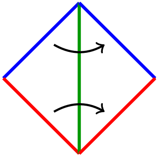

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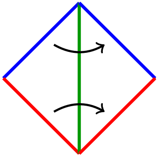

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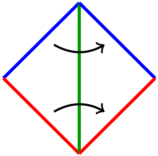

$\Rightarrow$ Unique map that preserves incidence

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- Mirror–colouring defines a map to the triangle

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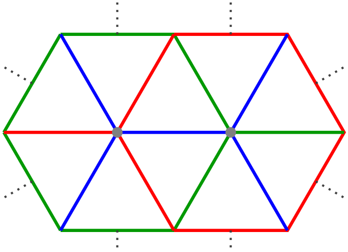

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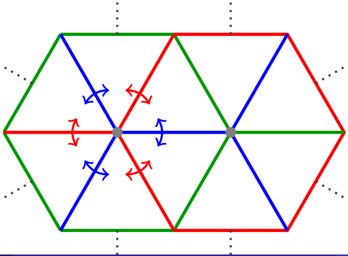

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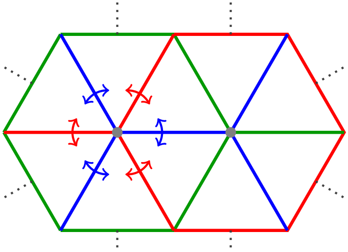

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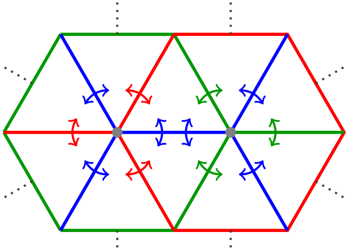
Satz

There exists a coloured surface with the given involutions where all edges are mirror edges.

- The faces are the points moved by the involution triple
- The edges are the cycles of the involutions
- The vertices are the orbits of $\langle \sigma_a, \sigma_b \rangle$ on the faces (for all pairs)



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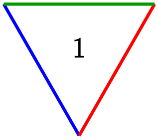

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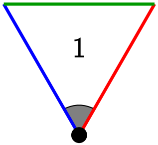

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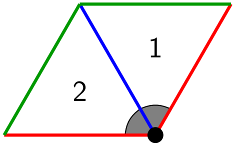

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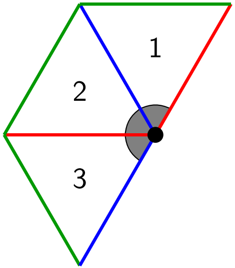

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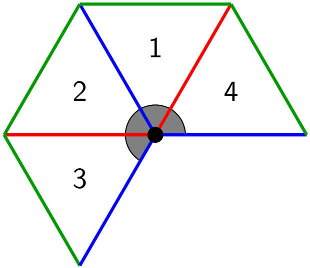

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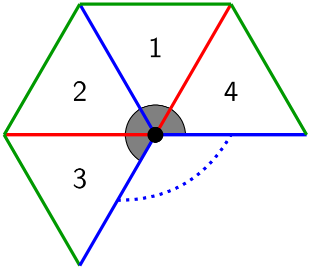

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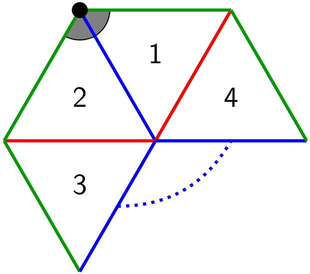

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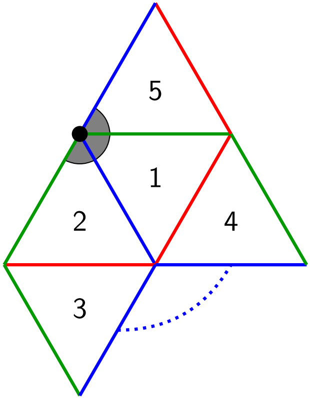

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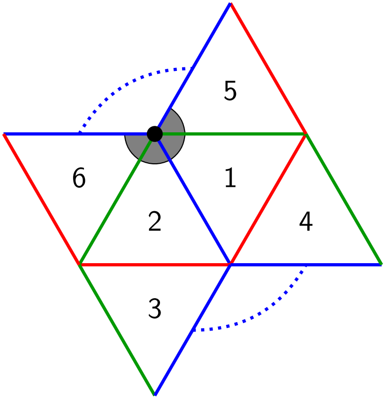

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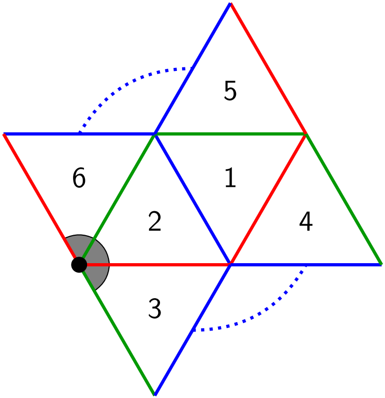

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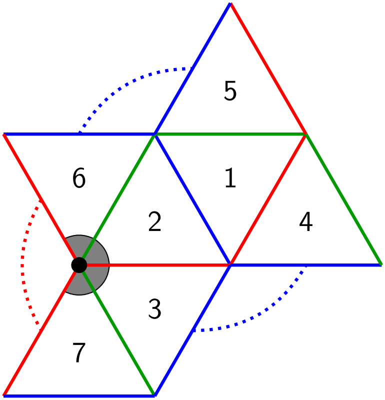

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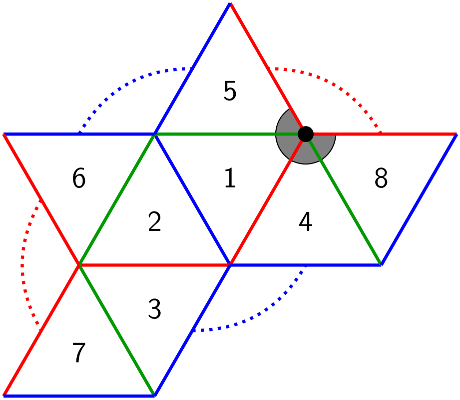

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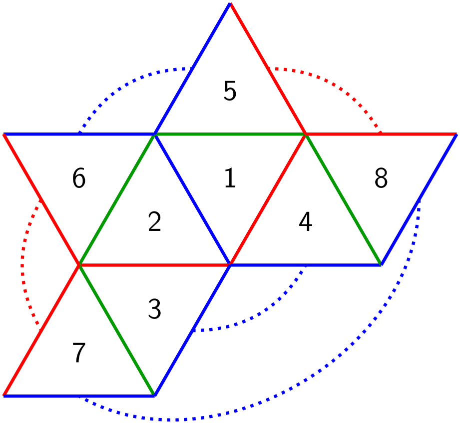

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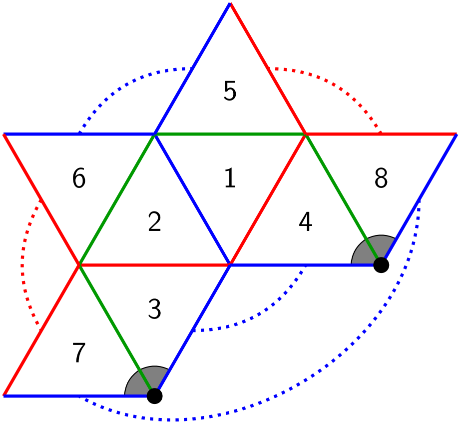

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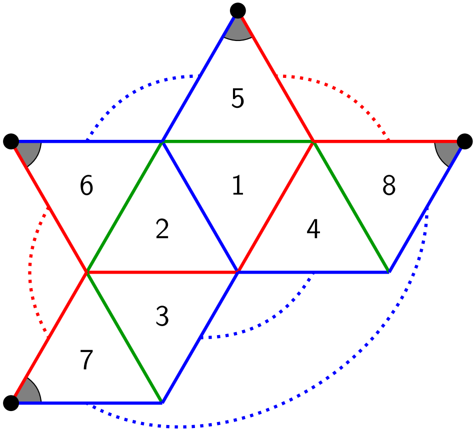

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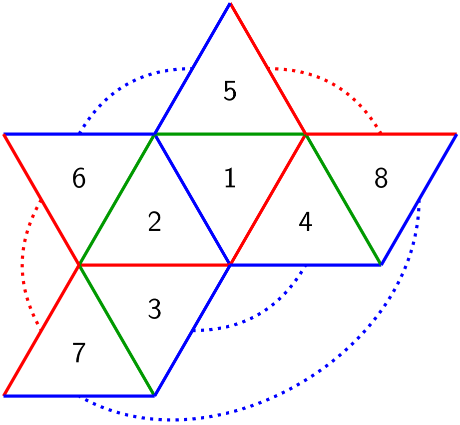

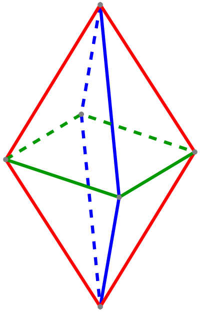

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`iko`: coloured icosahedron

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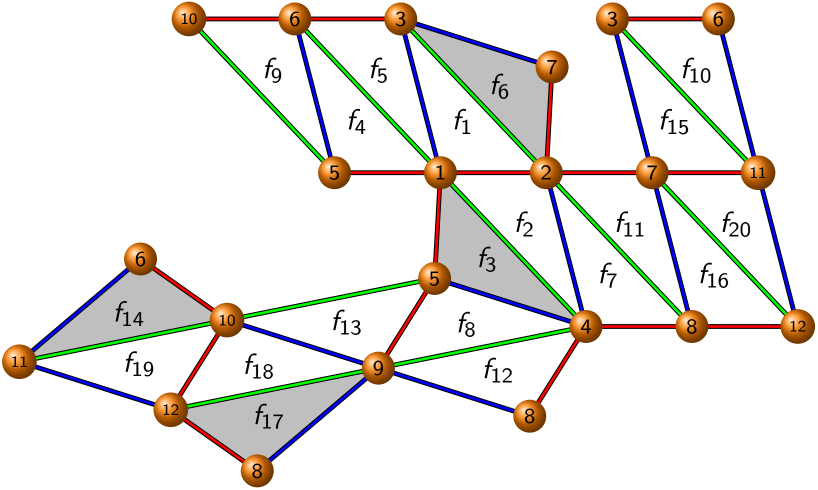

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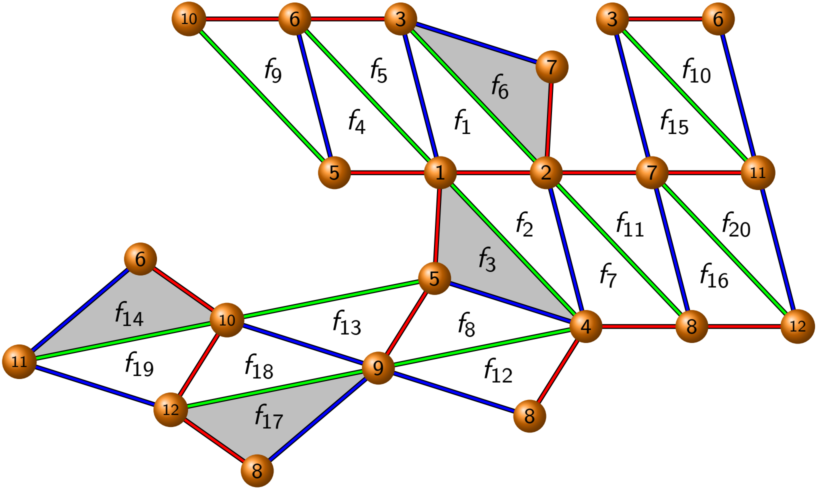
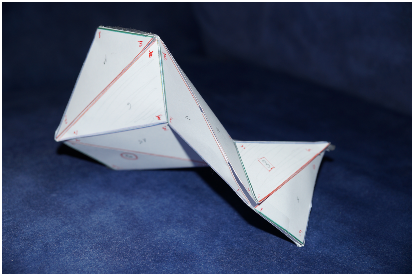
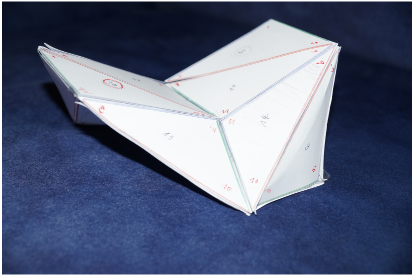


Embedded icosahedron





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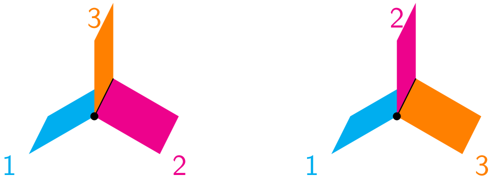

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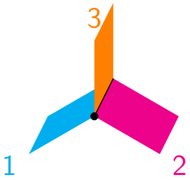

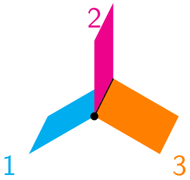

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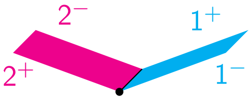

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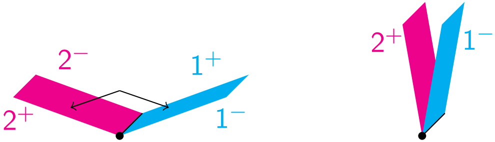

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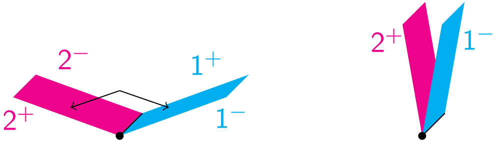

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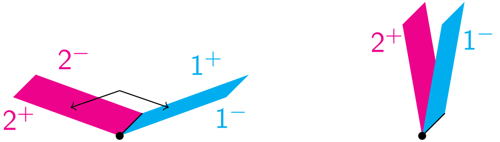

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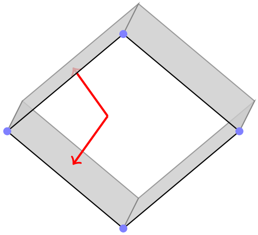

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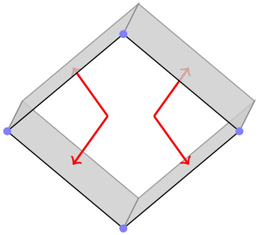

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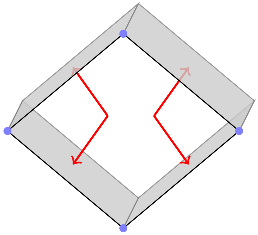

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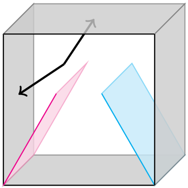

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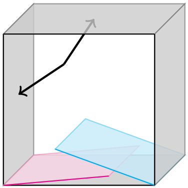

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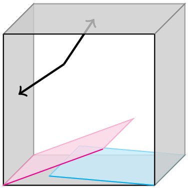

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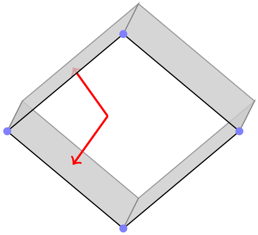

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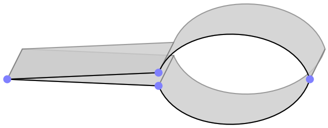

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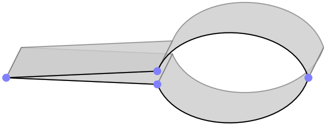

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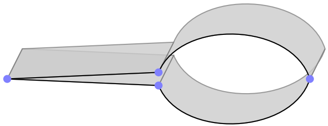
  $\rightsquigarrow$ Relax the rigidity–constraint:
  - Allow non–rigid configurations as transitional states



Structure of multiple foldings

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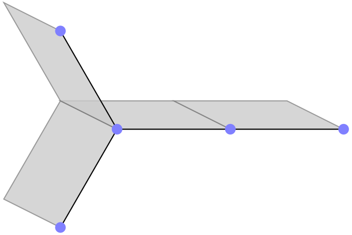

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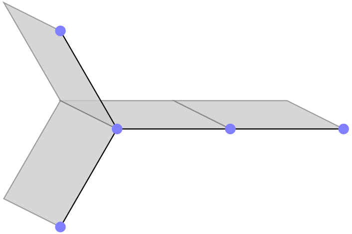

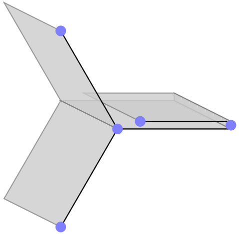

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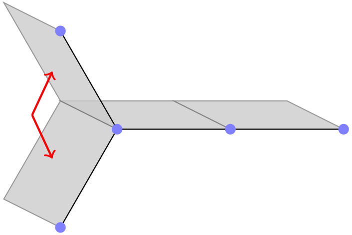

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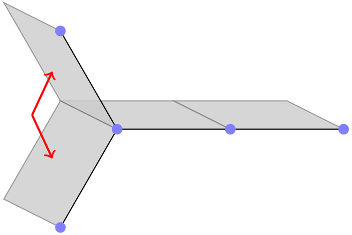

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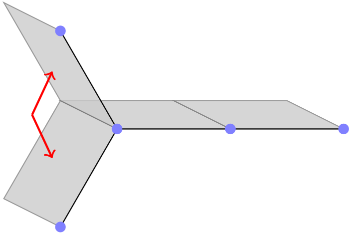

$\rightsquigarrow$ more structure on the set of possible foldings

Folding graph

Folding graph

- Vertices are folding complexes (modelling folding states)

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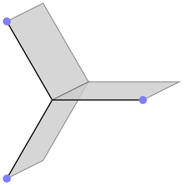

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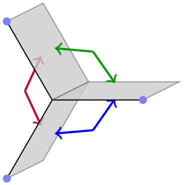

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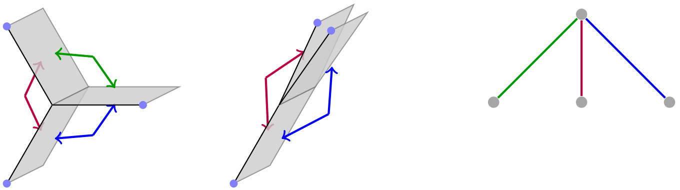

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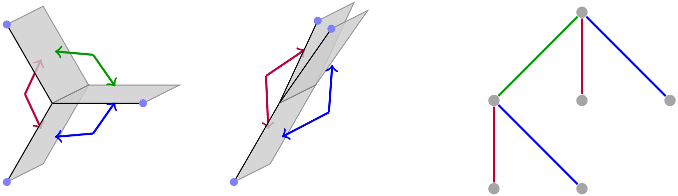

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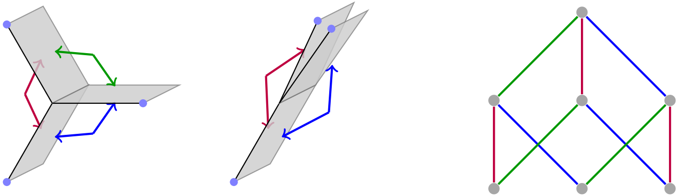

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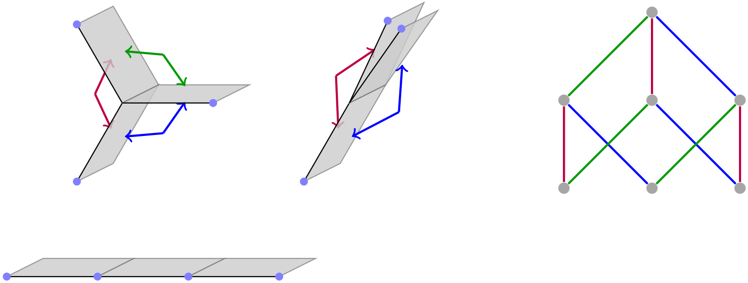

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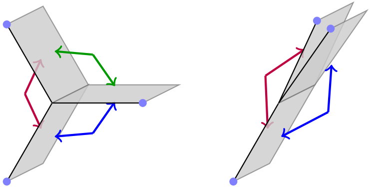

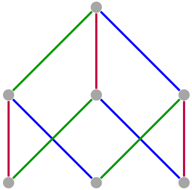

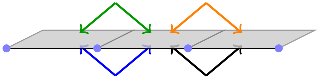

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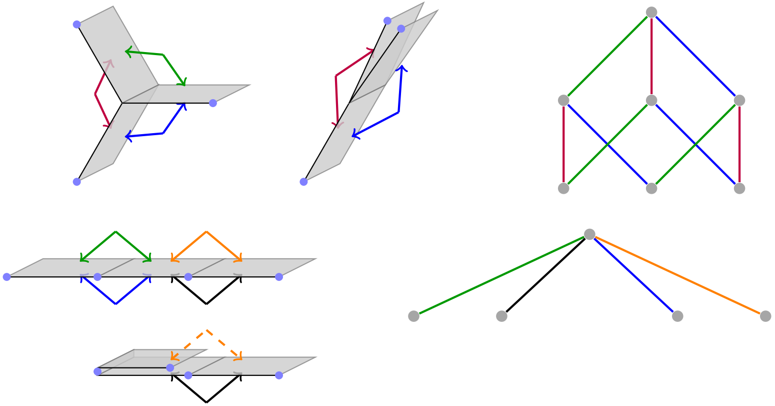

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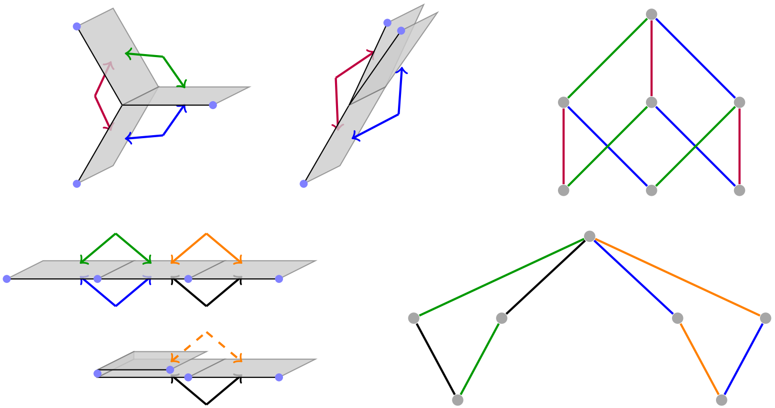

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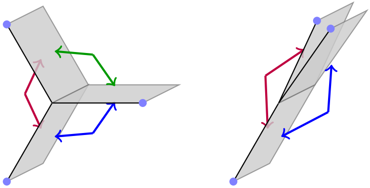

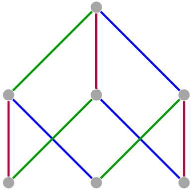

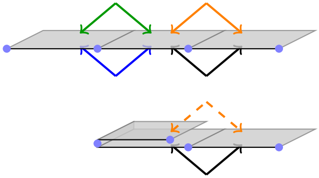

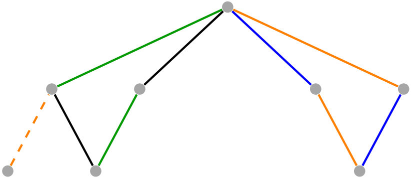

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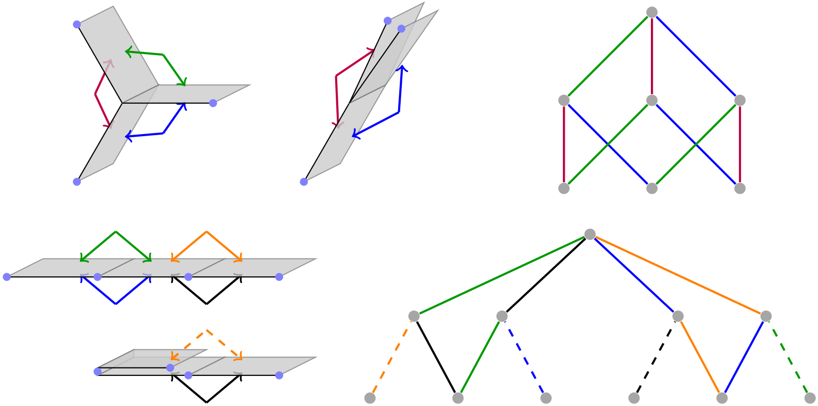
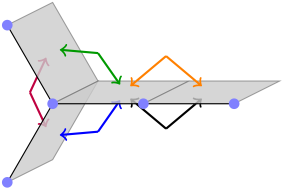


Larger graph

Larger graph



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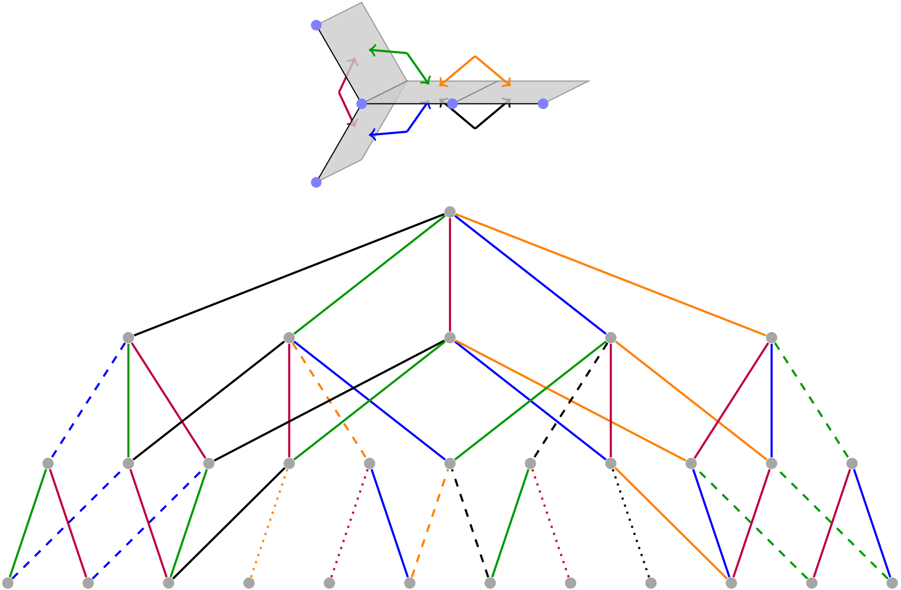

Drawback of folding plans

Drawback of folding plans

Some foldings that "should" be the same, aren't:

Drawback of folding plans

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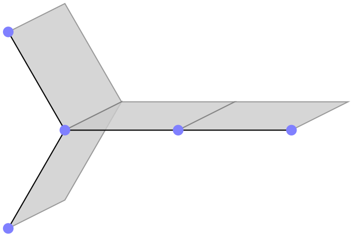

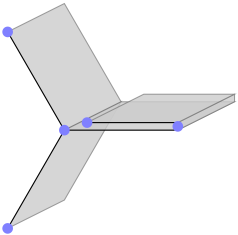

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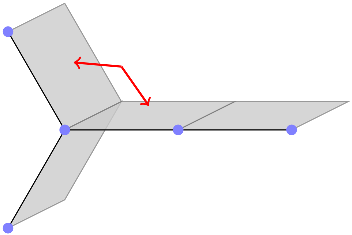

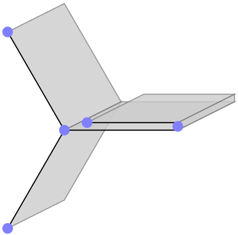

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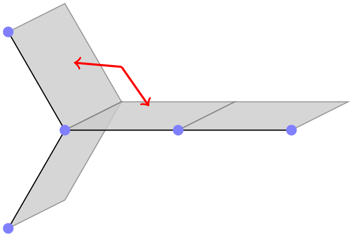

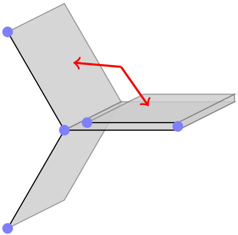

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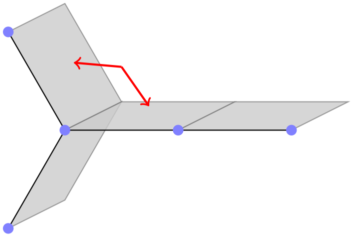

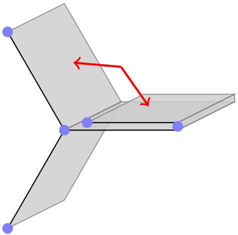

$\Rightarrow$ If you know the folding structure of a small complex,

Drawback of folding plans

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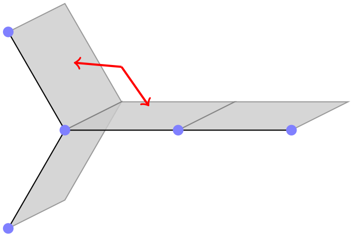

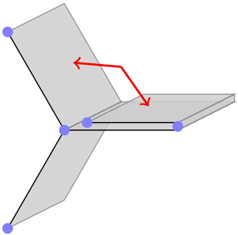

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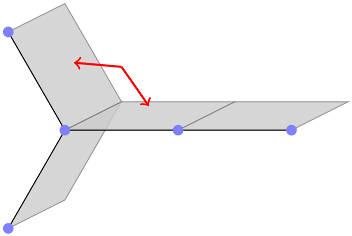

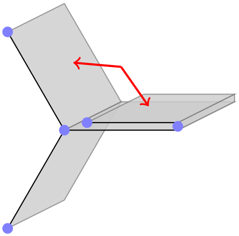

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$\rightsquigarrow$ Folding plans are not optimal to model folding

Progress report of abstract folding

Progress report of abstract folding

In development:

Progress report of abstract folding

In development:

- folding complex

Progress report of abstract folding

In development:

- folding complex
- folding plans

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In development:

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Missing:

Progress report of abstract folding

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Missing:

- better folding description

Progress report of abstract folding

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Progress report of abstract folding

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- properties of folding graphs

Summary: `SimplicialSurfaces`

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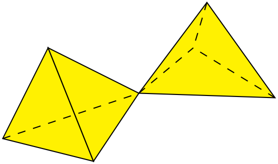
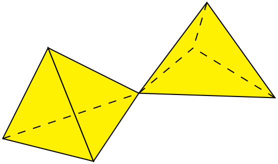
Triangular complexes



Summary: `SimplicialSurfaces`

Triangular complexes

- mostly complete



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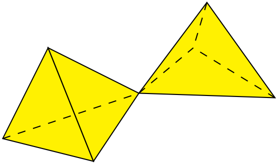

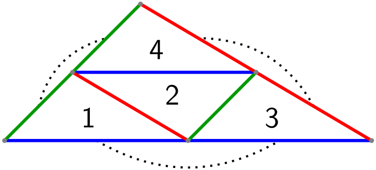

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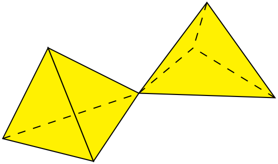
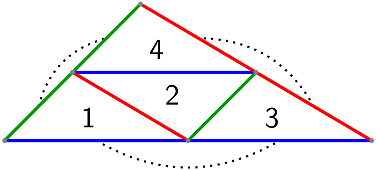


Edge colouring

- current theory implemented



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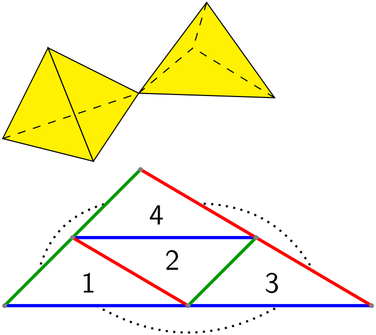

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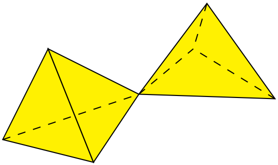

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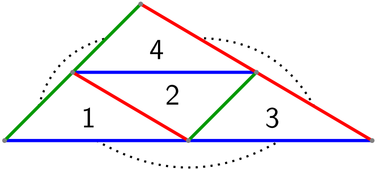
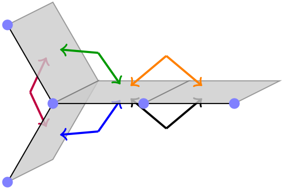


Abstract folding



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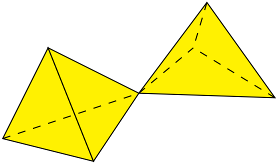

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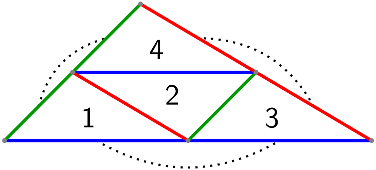
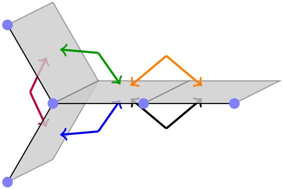


Abstract folding

- framework exists



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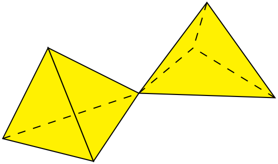

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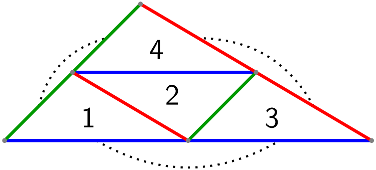

Abstract folding

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- needs proper implementation