Simplicial surfaces

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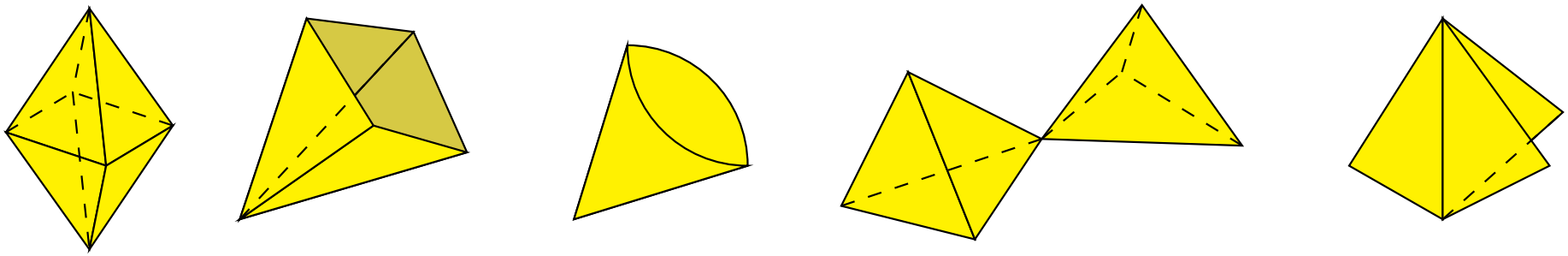
Simplicial surfaces

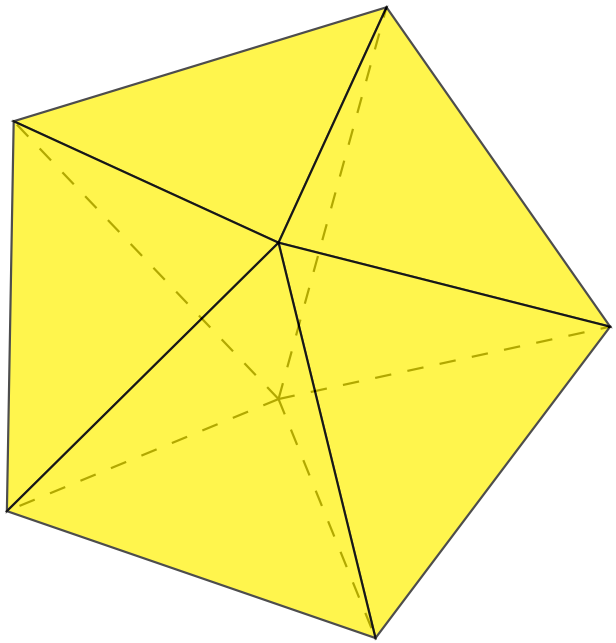
We want to describe different structures built from triangles:



Classification

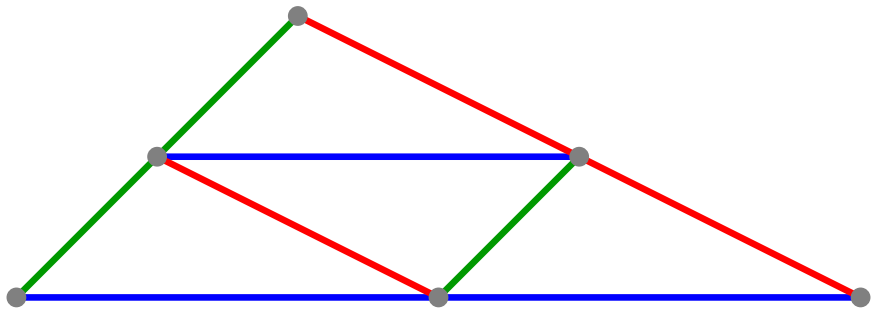
Goal: Classify all closed simplicial surfaces.
Plesken/Strzelczyk classified the building blocks up to 20 triangles.

- e. g. exactly 87 non–isomorphic surfaces with 20 triangles
- e. g. only one surface with 10 triangles:



One type of triangle

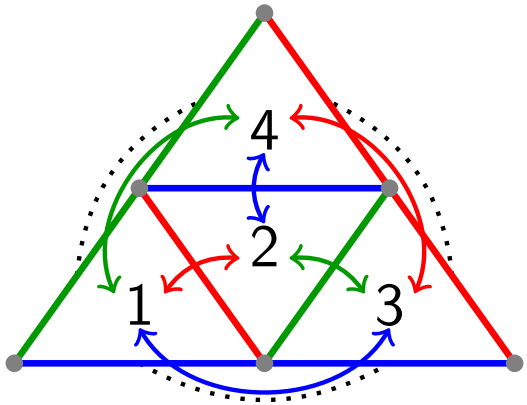
We are interested in surfaces that are built from one type of triangle.



$\rightsquigarrow$ edge–colouring encodes lengths

Colouring as permutation

Consider a tetrahedron with an edge colouring

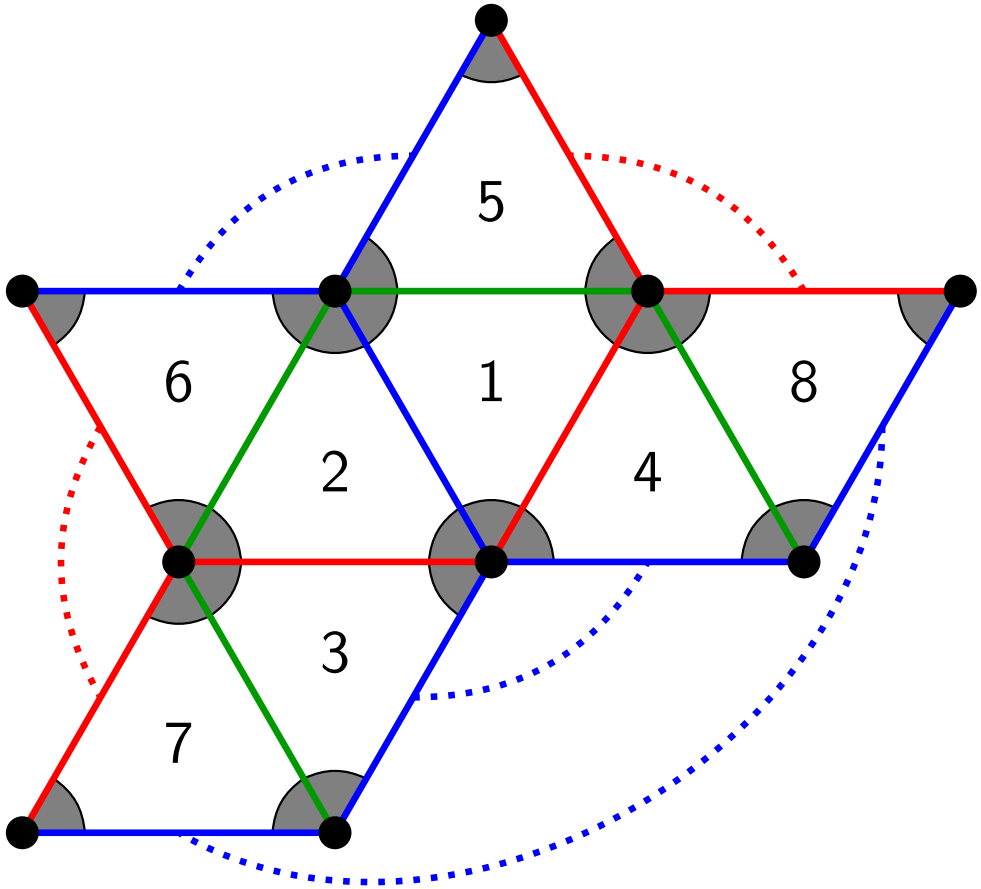


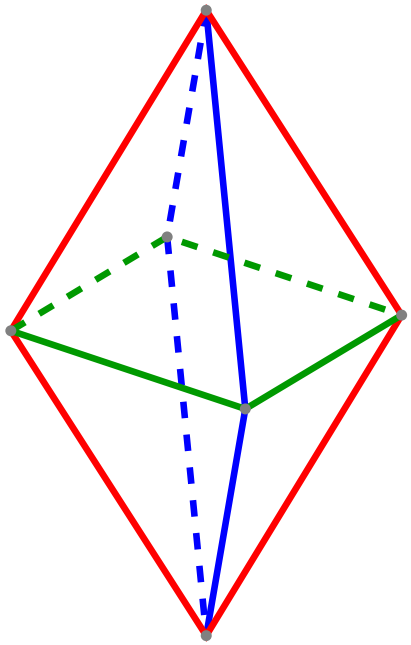
simplicial surface $\Rightarrow$ at most two faces at each edge

- $\rightsquigarrow$ every edge defines a transposition of incident faces
- $\rightsquigarrow$ every colour class defines a permutation of the faces
  - (1,2)(3,4) , (1,3)(2,4) , (1,4)(2,3)
- $\rightsquigarrow$ group theoretic considerations

Construction example

$\sigma_a = (1,2)(3,4)(5,6)(7,8)$
$\sigma_b = (1,4)(2,3)(5,8)(6,7)$
$\sigma_c = (1,5)(2,6)(3,7)(4,8)$

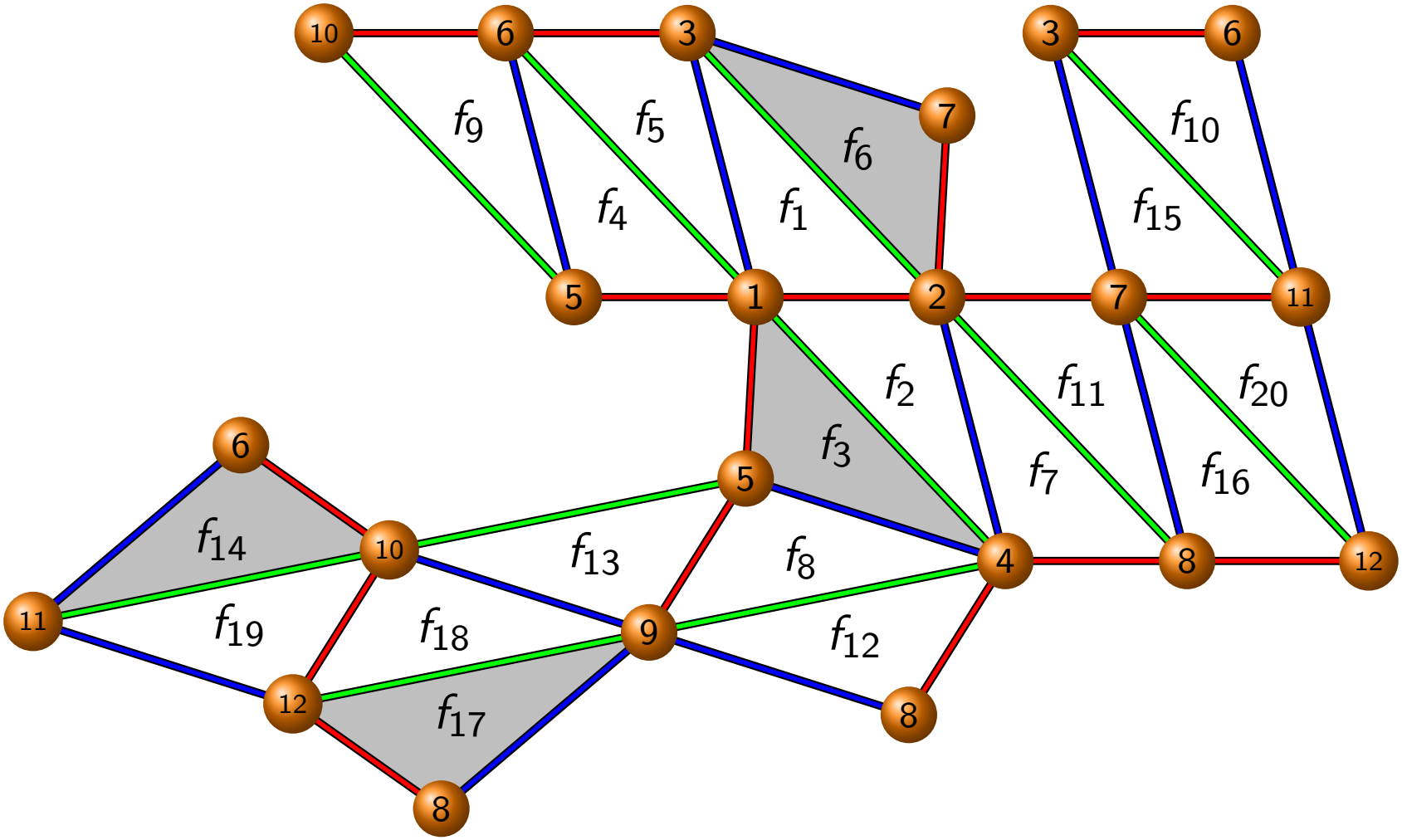




Net of an icosahedron

`iko`: coloured icosahedron

```
gap> DrawSurfaceToTikZ(iko,"NetIko.tex");
```



Embedded icosahedron