Simplicial surfaces

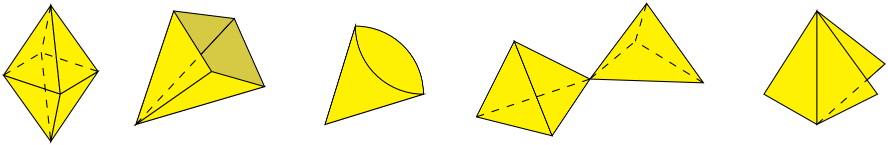
Markus Baumeister
(j/w Alice Niemeyer, Wilhelm Plesken, Ansgar Strzelczyk)

Lehrstuhl B für Mathematik
RWTH Aachen University

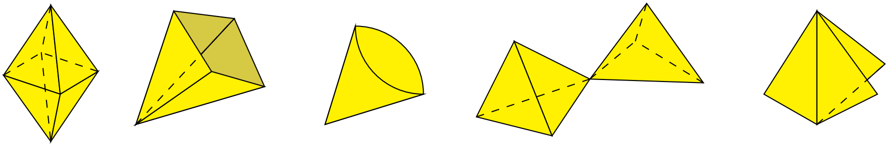
27.09.2017

Simplicial surfaces

Simplicial surfaces

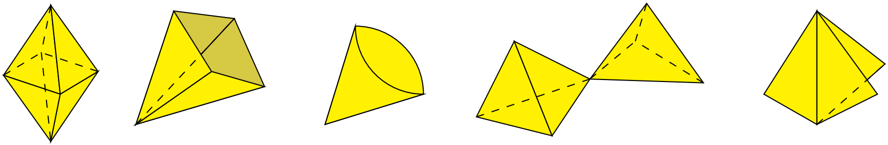


Simplicial surfaces



$\rightsquigarrow$ structures build from triangles

Simplicial surfaces



$\rightsquigarrow$ structures build from triangles (or arbitrary polygons)

Classification

Classification

Goal: Classify all closed simplicial surfaces.

Classification

Goal: Classify all closed simplicial surfaces.
Plesken/Strzelczyk classified the building blocks up to 20 triangles.

Classification

Goal: Classify all closed simplicial surfaces.
Plesken/Strzelczyk classified the building blocks up to 20 triangles.

- e. g. exactly 87 non–isomorphic surfaces with 20 triangles

Classification

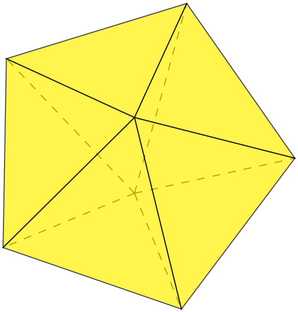
Goal: Classify all closed simplicial surfaces.
Plesken/Strzelczyk classified the building blocks up to 20 triangles.

- e. g. exactly 87 non–isomorphic surfaces with 20 triangles
- e. g. only one surface with 10 triangles:

Classification

Goal: Classify all closed simplicial surfaces.
Plesken/Strzelczyk classified the building blocks up to 20 triangles.

- e. g. exactly 87 non–isomorphic surfaces with 20 triangles
- e. g. only one surface with 10 triangles:



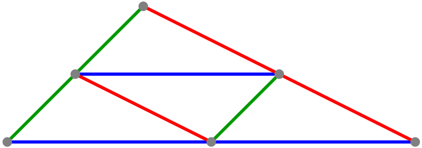
One type of triangle

One type of triangle

We are interested in surfaces that are built from one type of triangle.

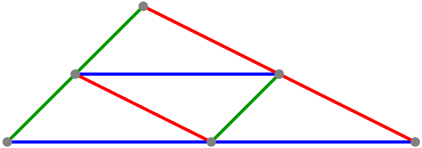
One type of triangle

We are interested in surfaces that are built from one type of triangle.



One type of triangle

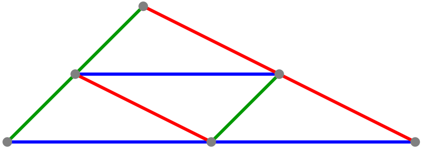
We are interested in surfaces that are built from one type of triangle.



$\rightsquigarrow$ edge–colouring encodes lengths

One type of triangle

We are interested in surfaces that are built from one type of triangle.

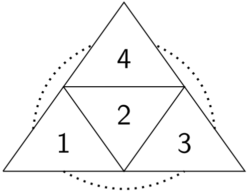


$\rightsquigarrow$ edge–colouring encodes lengths
$\rightsquigarrow$ analyse edge–coloured surfaces

Colouring as permutation

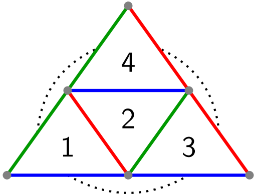
Colouring as permutation

Consider a tetrahedron



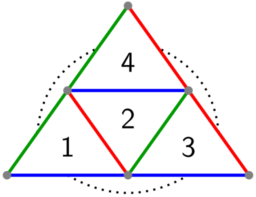
Colouring as permutation

Consider a tetrahedron with an edge colouring



Colouring as permutation

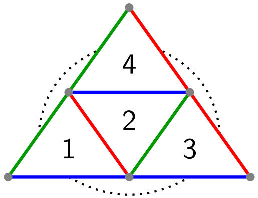
Consider a tetrahedron with an edge colouring



simplicial surface $\Rightarrow$

Colouring as permutation

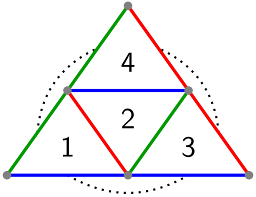
Consider a tetrahedron with an edge colouring



simplicial surface $\Rightarrow$ two faces at each edge

Colouring as permutation

Consider a tetrahedron with an edge colouring

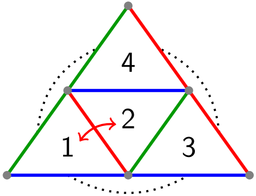


simplicial surface $\Rightarrow$ two faces at each edge

$\rightsquigarrow$ every edge defines a transposition of incident faces

Colouring as permutation

Consider a tetrahedron with an edge colouring



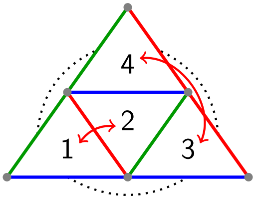
simplicial surface $\Rightarrow$ two faces at each edge

$\rightsquigarrow$ every edge defines a transposition of incident faces

- $(1,2)$

Colouring as permutation

Consider a tetrahedron with an edge colouring



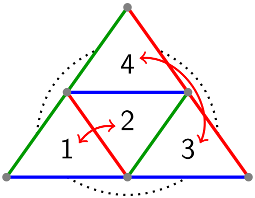
simplicial surface $\Rightarrow$ two faces at each edge

$\rightsquigarrow$ every edge defines a transposition of incident faces

- (1,2)(3,4)

Colouring as permutation

Consider a tetrahedron with an edge colouring

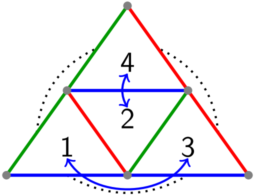


simplicial surface $\Rightarrow$ two faces at each edge

- $\rightsquigarrow$ every edge defines a transposition of incident faces
- $\rightsquigarrow$ every colour class defines a permutation of the faces
- (1,2)(3,4)

Colouring as permutation

Consider a tetrahedron with an edge colouring

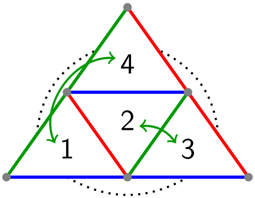


simplicial surface ⇒ two faces at each edge

- ⇝ every edge defines a transposition of incident faces
- ⇝ every colour class defines a permutation of the faces
- (1,2)(3,4) , (1,3)(2,4)

Colouring as permutation

Consider a tetrahedron with an edge colouring

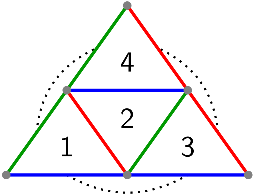


simplicial surface $\Rightarrow$ two faces at each edge

- $\rightsquigarrow$ every edge defines a transposition of incident faces
- $\rightsquigarrow$ every colour class defines a permutation of the faces
  - $(1,2)(3,4)$, $(1,3)(2,4)$, $(1,4)(2,3)$

Colouring as permutation

Consider a tetrahedron with an edge colouring



simplicial surface $\Rightarrow$ two faces at each edge

- $\rightsquigarrow$ every edge defines a transposition of incident faces
- $\rightsquigarrow$ every colour class defines a permutation of the faces
  - (1,2)(3,4) , (1,3)(2,4) , (1,4)(2,3)
- $\rightsquigarrow$ group theoretic considerations

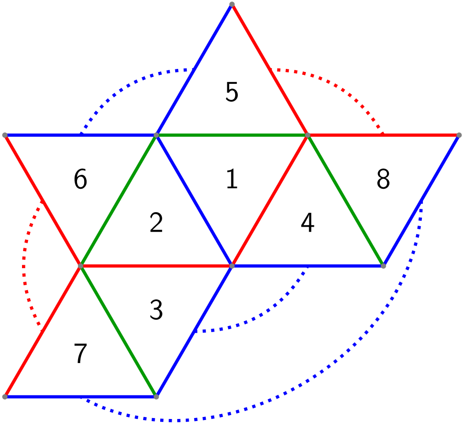
Construction example

Construction example

$\sigma_a = (1, 2)(3, 4)(5, 6)(7, 8)$
$\sigma_b = (1, 4)(2, 3)(5, 8)(6, 7)$
$\sigma_c = (1, 5)(2, 6)(3, 7)(4, 8)$

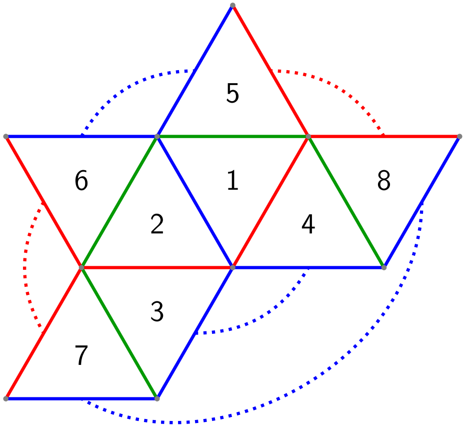
Construction example

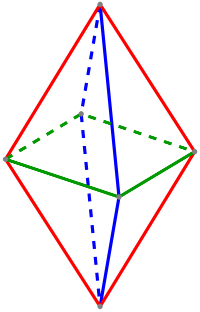
$\sigma_a = (1,2)(3,4)(5,6)(7,8)$
$\sigma_b = (1,4)(2,3)(5,8)(6,7)$
$\sigma_c = (1,5)(2,6)(3,7)(4,8)$



Construction example

$\sigma_a = (1,2)(3,4)(5,6)(7,8)$
$\sigma_b = (1,4)(2,3)(5,8)(6,7)$
$\sigma_c = (1,5)(2,6)(3,7)(4,8)$





Net of an icosahedron

Net of an icosahedron

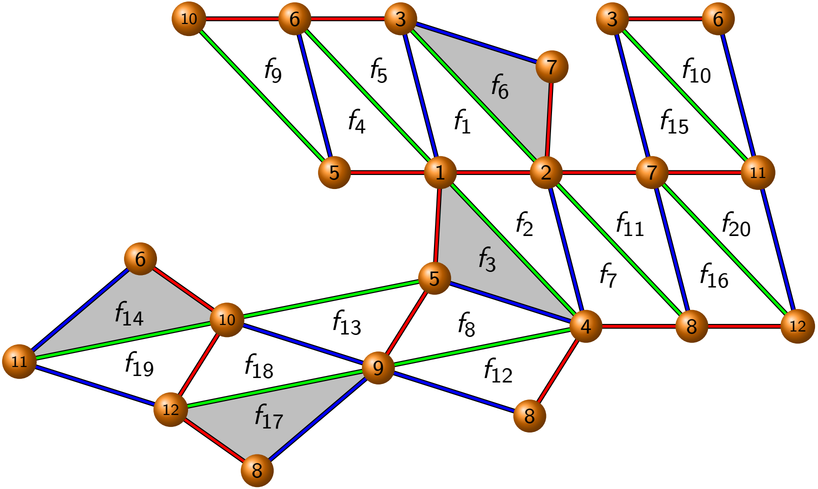
`iko`: coloured icosahedron

```
gap> DrawSurfaceToTikZ(iko,"NetIko.tex");
```

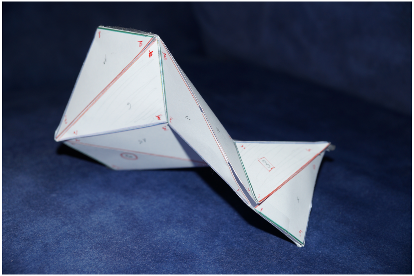
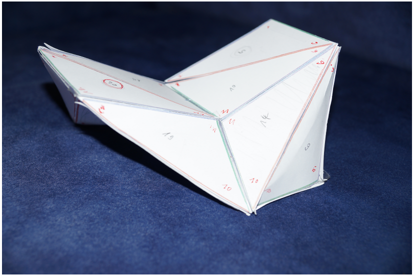
Net of an icosahedron

`iko`: coloured icosahedron

```
gap> DrawSurfaceToTikZ(iko,"NetIko.tex");
```



Embedded icosahedron

Further research

Further research

- Structural properties of coloured surfaces

Further research

- Structural properties of coloured surfaces
- Embedding of surfaces into $\mathbb{R}^3$

Further research

- Structural properties of coloured surfaces
- Embedding of surfaces into $\mathbb{R}^3$
- Folding of surfaces (description and classification)

Further research

- Structural properties of coloured surfaces
- Embedding of surfaces into $\mathbb{R}^3$
- Folding of surfaces (description and classification)
- GAP–implementation of this theory

Further research

- Structural properties of coloured surfaces
- Embedding of surfaces into $\mathbb{R}^3$
- Folding of surfaces (description and classification)
- GAP–implementation of this theory

Thanks for your attention

Alice Niemeyer
Wilhelm Plesken

Markus Baumeister
Ansgar Strzelczyk