Simplicial surfaces in GAP

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(j/w Alice Niemeyer)

Lehrstuhl B für Mathematik
RWTH Aachen University

30.08.2017

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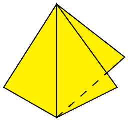

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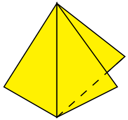

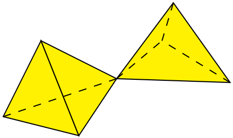

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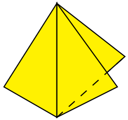

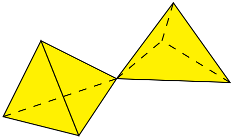

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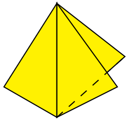

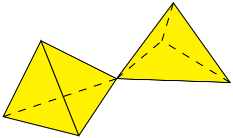

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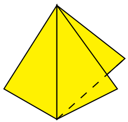

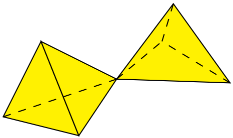

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$\rightsquigarrow$ incidence geometry

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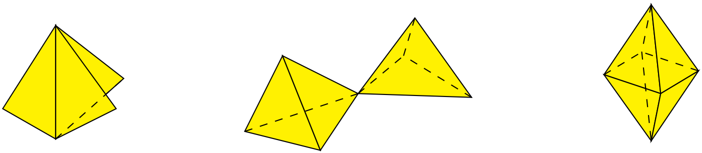

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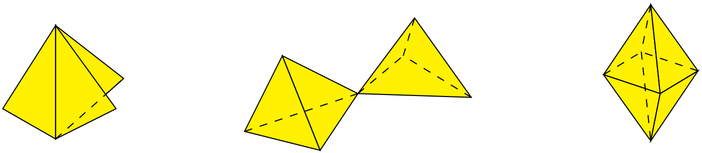

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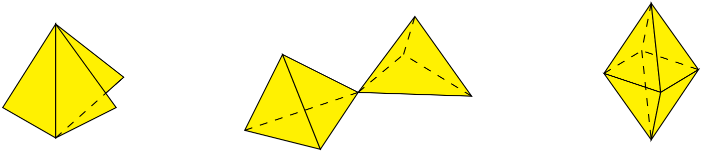

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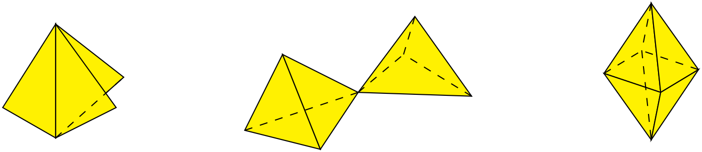

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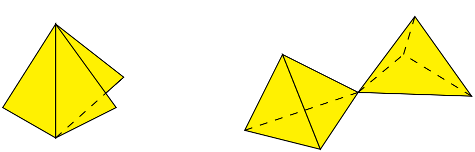

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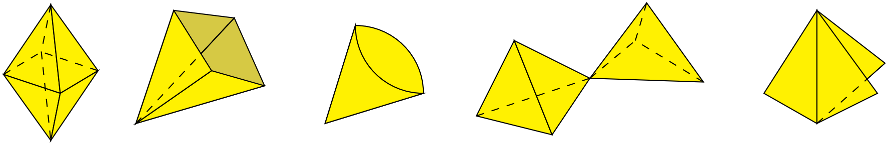

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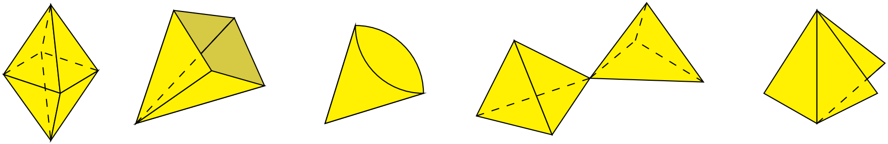

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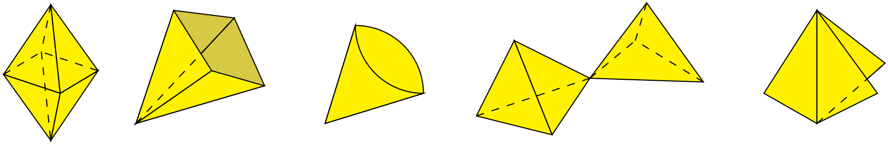

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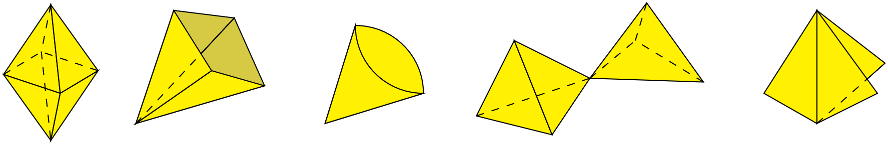

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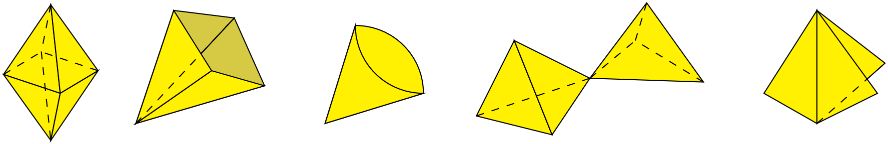

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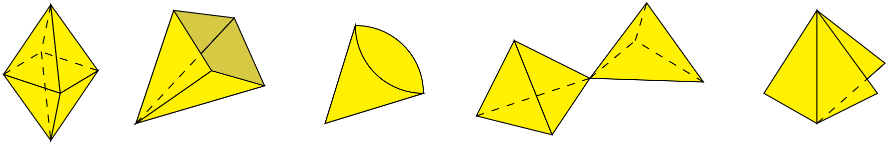

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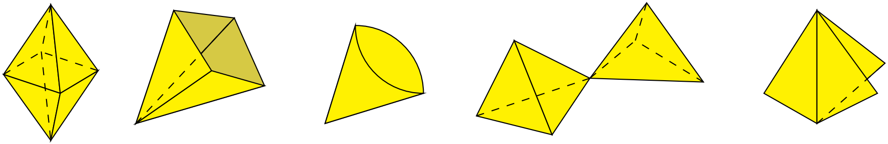

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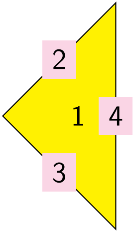

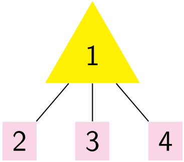

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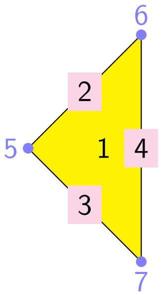

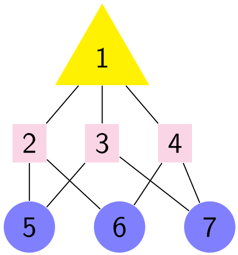

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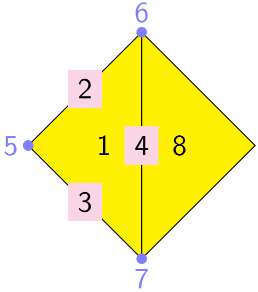

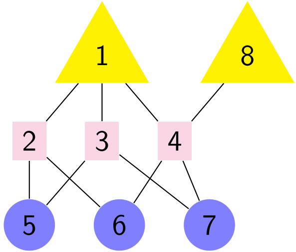

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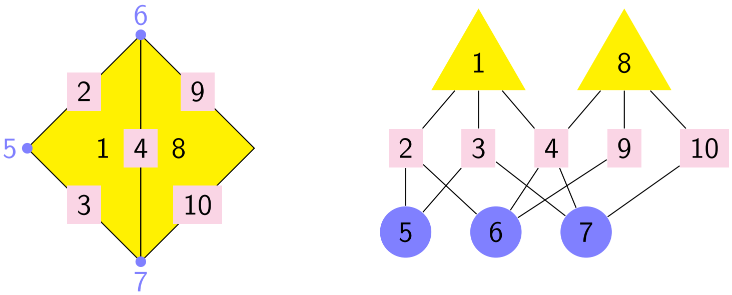

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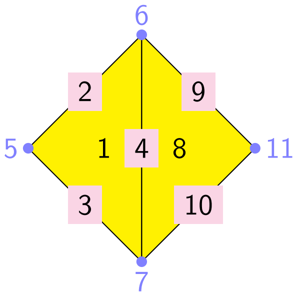

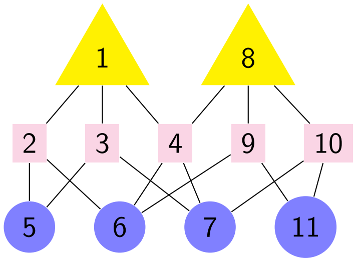

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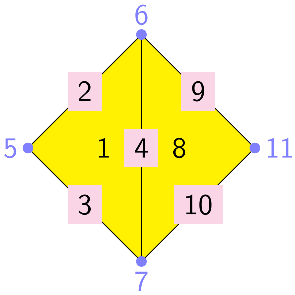

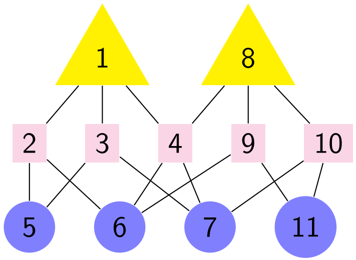

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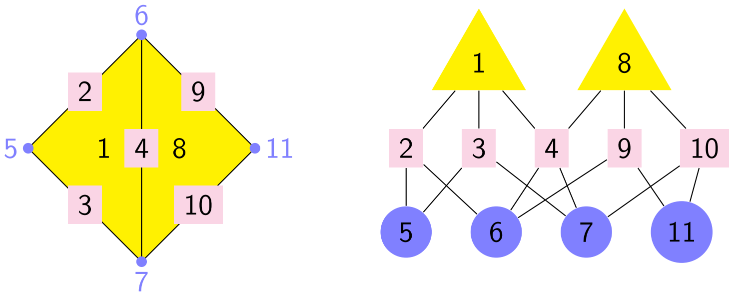

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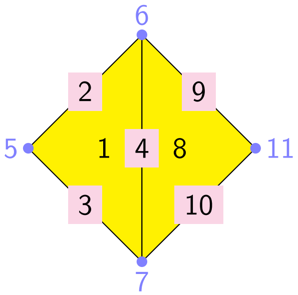

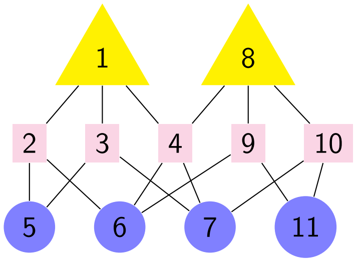

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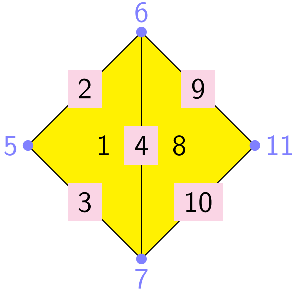

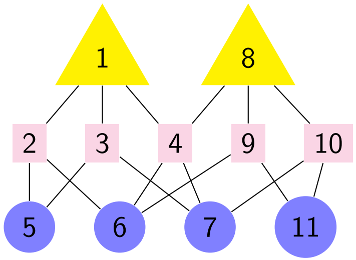

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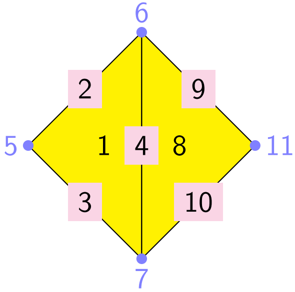

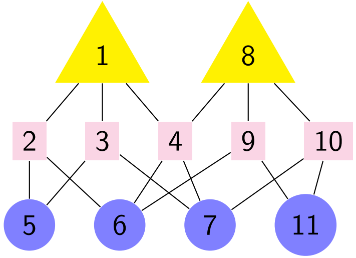

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Some properties can be computed for all triangular complexes:

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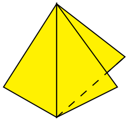

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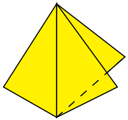

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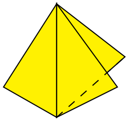

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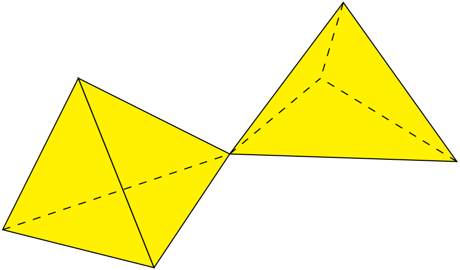

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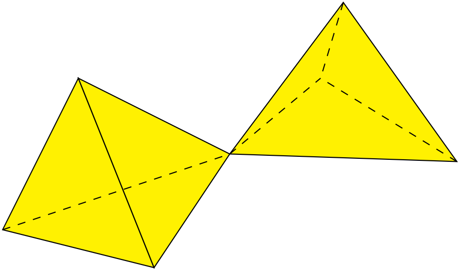

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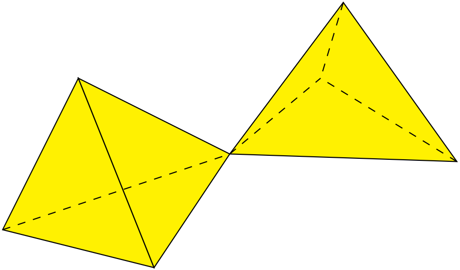

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A **simplicial surface** does not have these ramifications.

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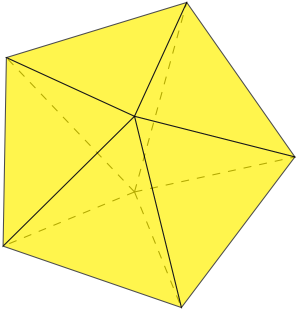

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Progress report of triangular complexes

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- surface hierarchy
- elementary properties (e. g. connectivity, orientability [if defined])
- isomorphism testing
- classification of small surfaces (as a data base)

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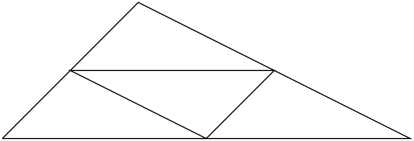

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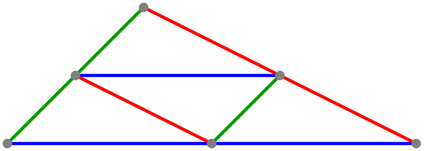

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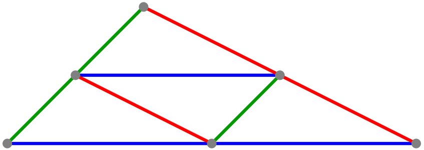

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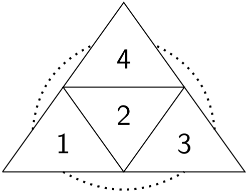

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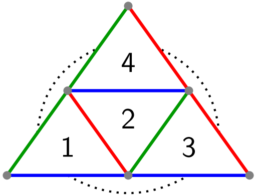

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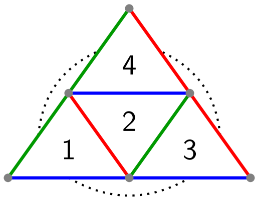

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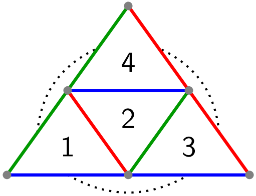

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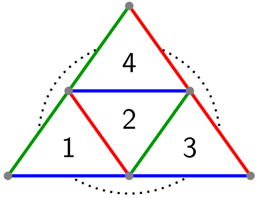

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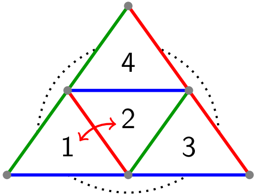

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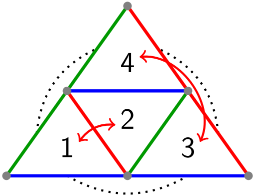

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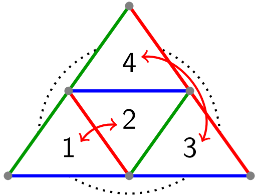

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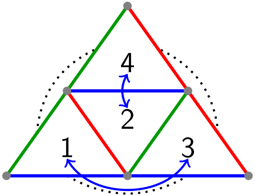

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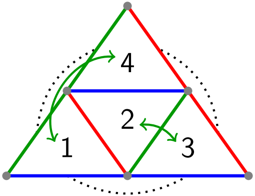

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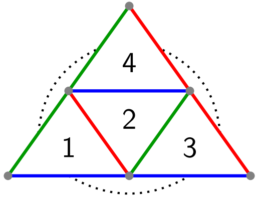

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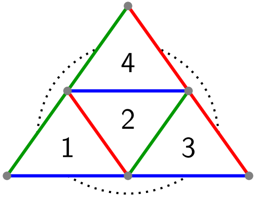

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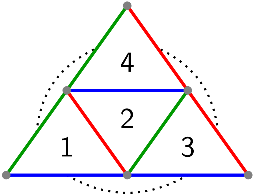

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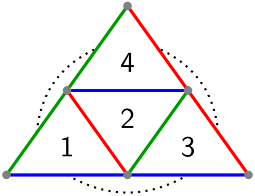

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- The connected components of the surface correspond to the orbits of $\langle \sigma_a, \sigma_b, \sigma_c \rangle$ on the faces (fast computation for permutation groups)

How do faces fit together?

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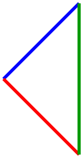
Consider a face of the surface



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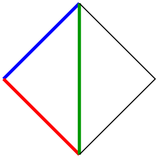

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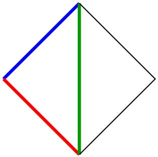

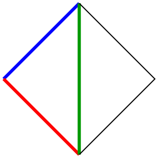

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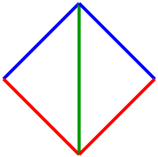

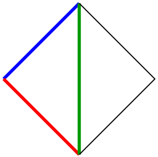

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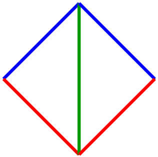

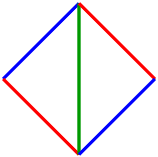

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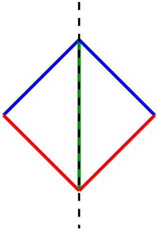
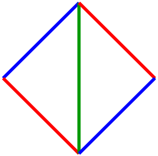


mirror (m)



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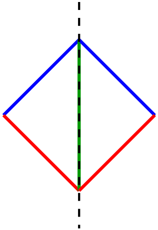

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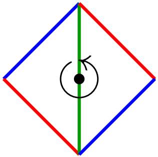

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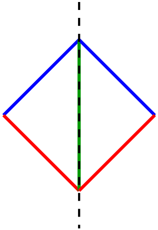

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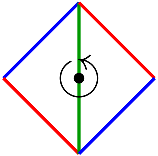

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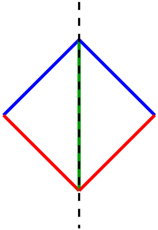

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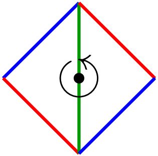

rotation (r)

$\rightsquigarrow$ **mr–assignment** for the edges of the surface

Satz

Permutations and mr–assignment uniquely determine the surface.

Constructing surfaces from groups

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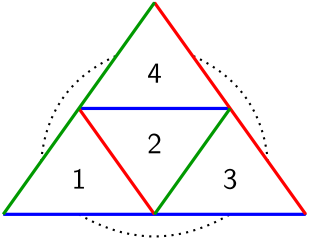

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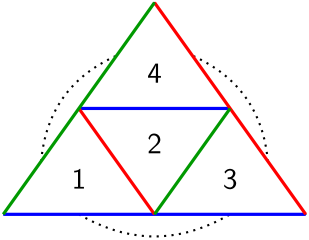

has only r–edges.

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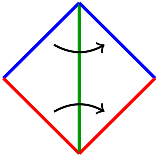

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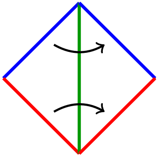

$\Rightarrow$ Unique map that preserves incidence

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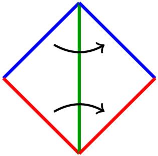

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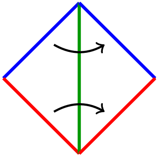

$\Rightarrow$ Unique map that preserves incidence

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- Mirror–colouring defines a map to the triangle

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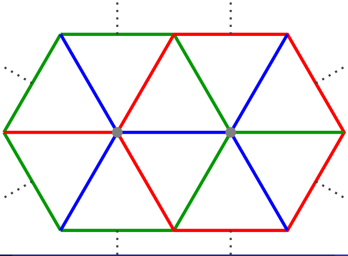

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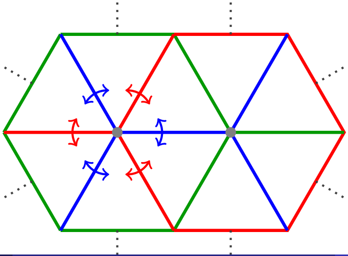

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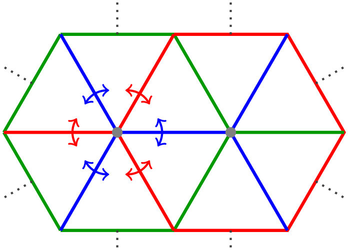

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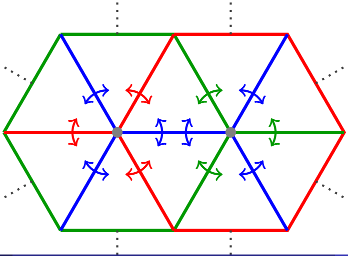

Construction example

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$\sigma_a = (1,2)(3,4)(5,6)(7,8)$

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$\sigma_a = (1,2)(3,4)(5,6)(7,8)$
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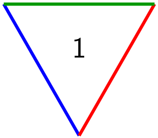

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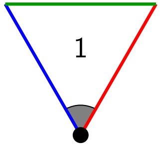

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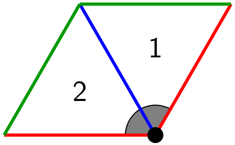

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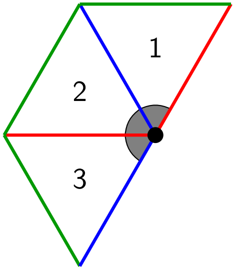

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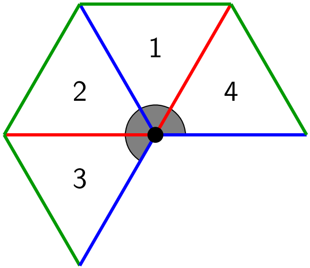

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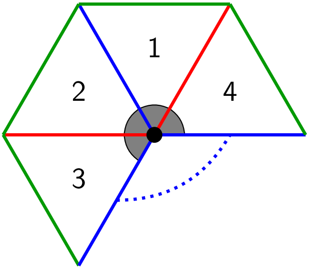

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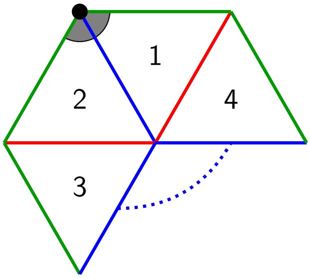

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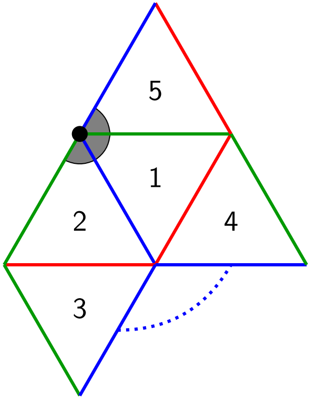

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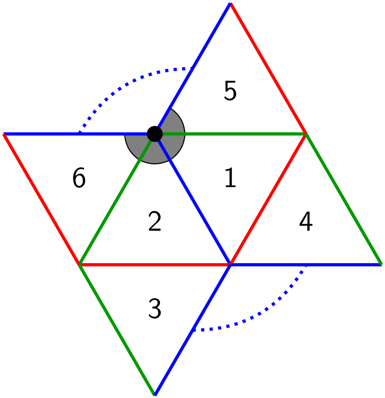

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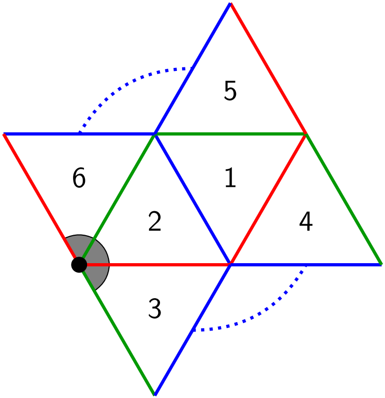

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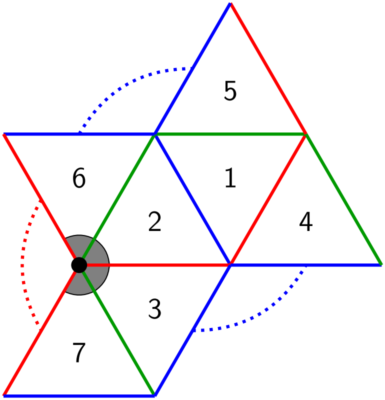

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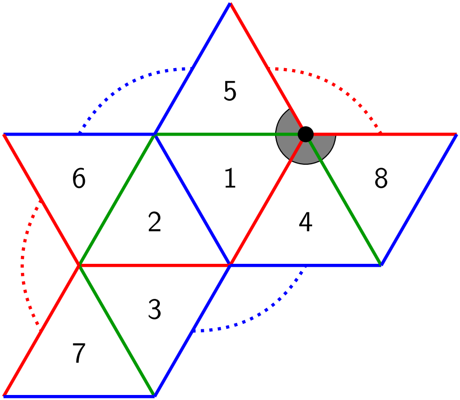

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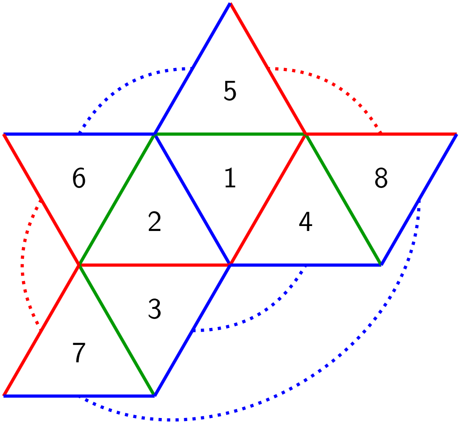

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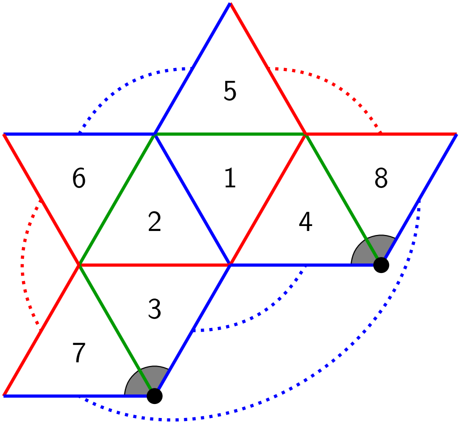

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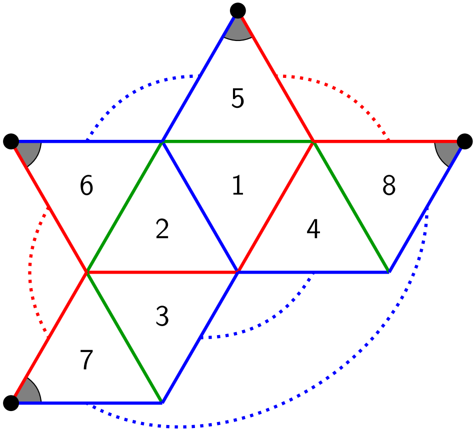

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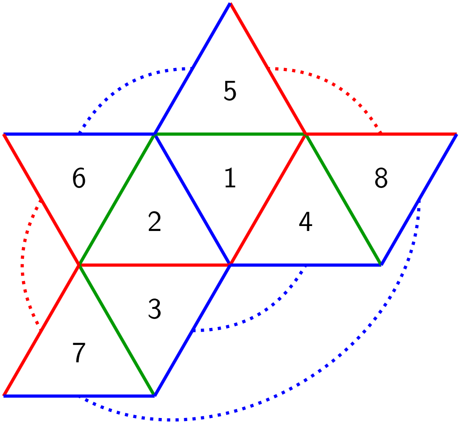

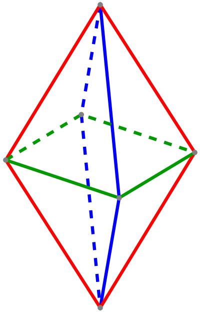

Net of an icosahedron

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`iko`: coloured icosahedron

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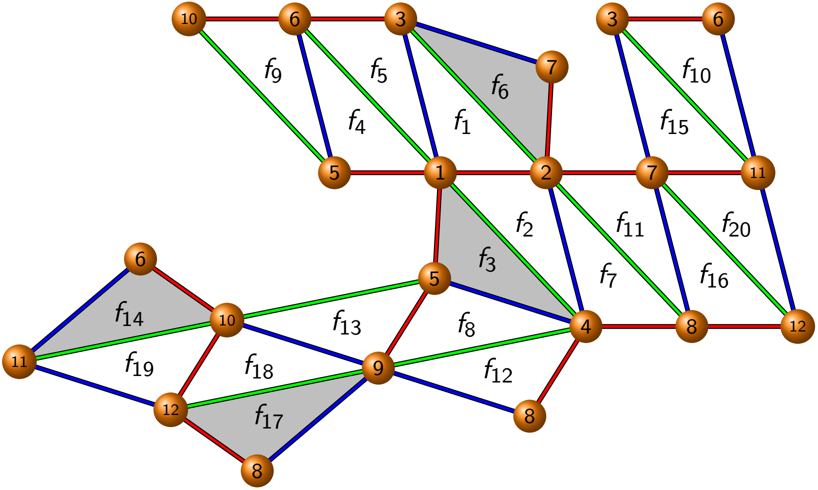

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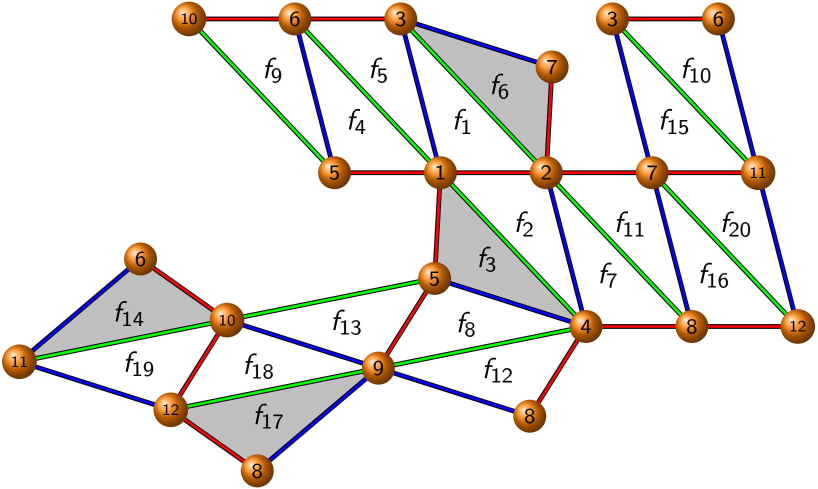

Embedded icosahedron

Progress report of edge colouring

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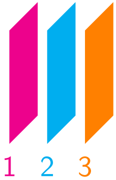

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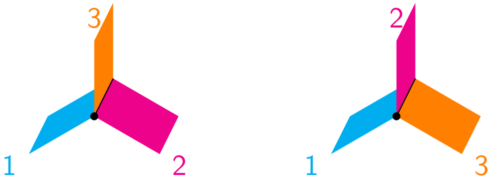

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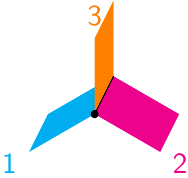

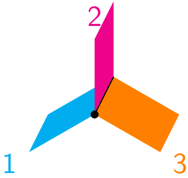

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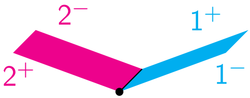

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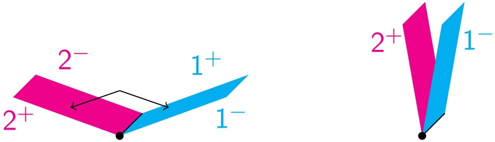

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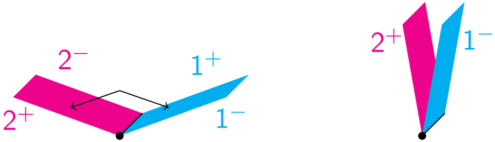

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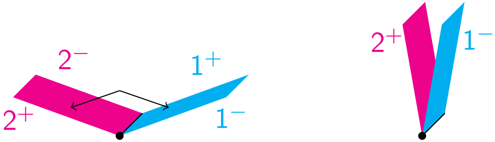

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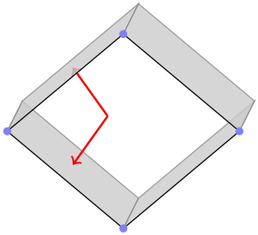

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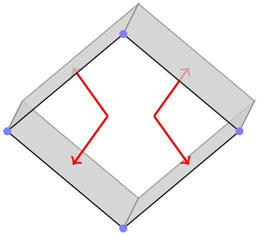

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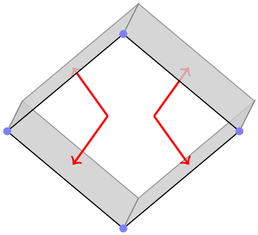

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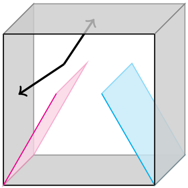

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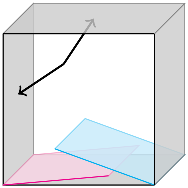

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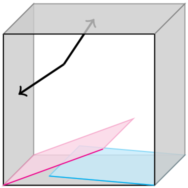

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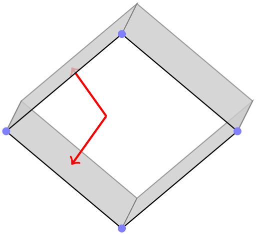

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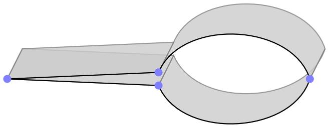

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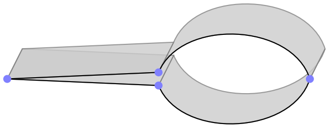

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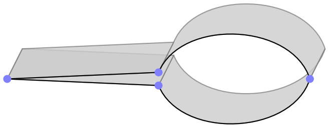

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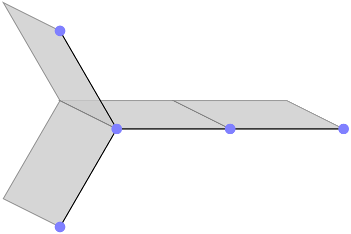

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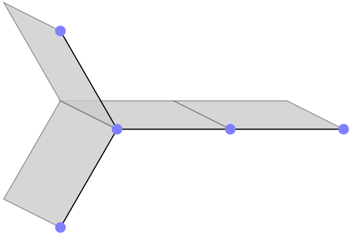

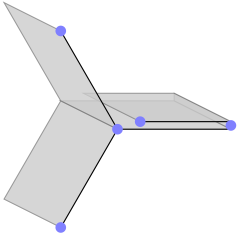

Structure of multiple foldings

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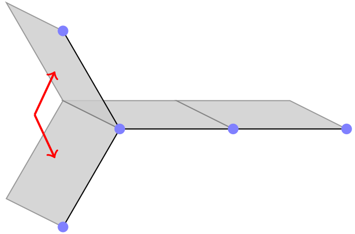

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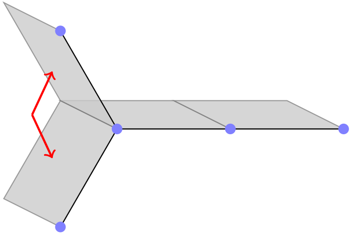

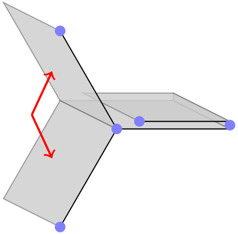

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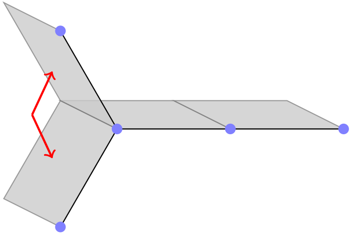

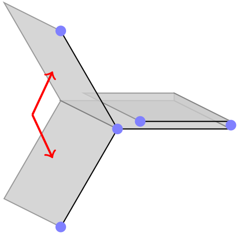

$\rightsquigarrow$ more structure on the set of possible foldings

Folding graph

Folding graph

- Vertices are folding complexes (modelling folding states)

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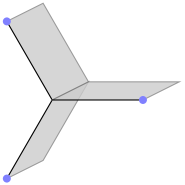

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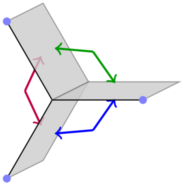

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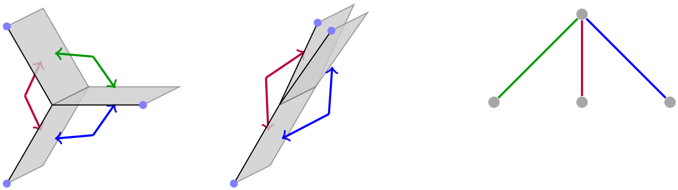

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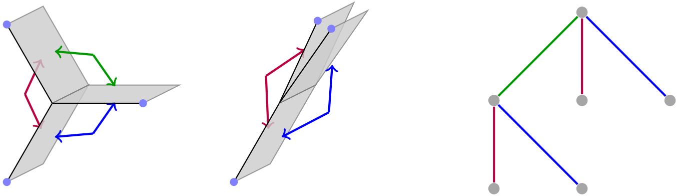

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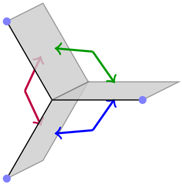

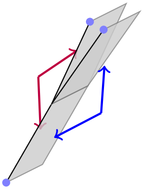

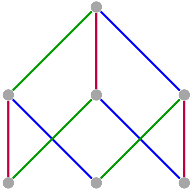

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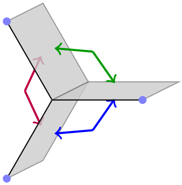

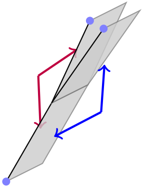

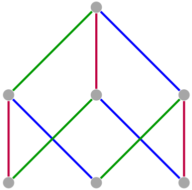

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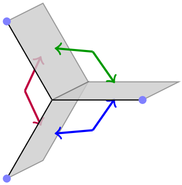

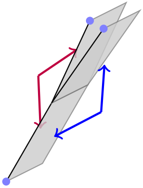

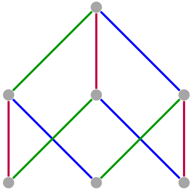

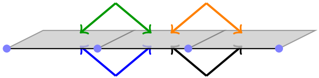

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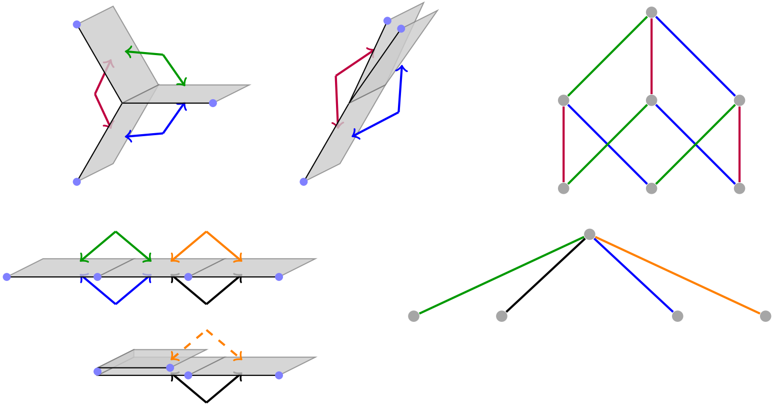

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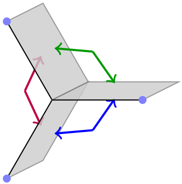

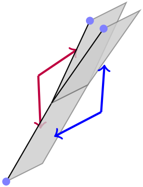

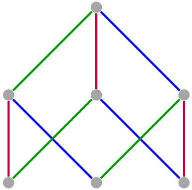

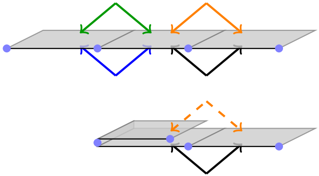

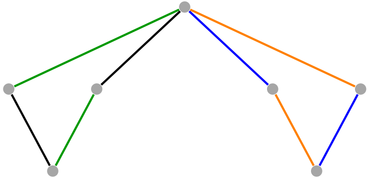

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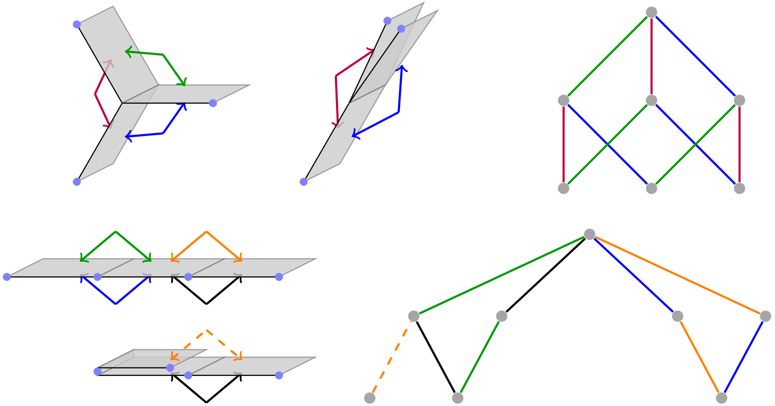

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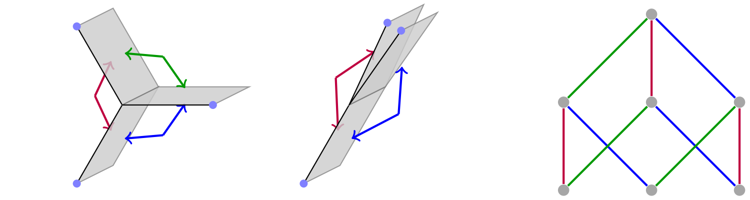

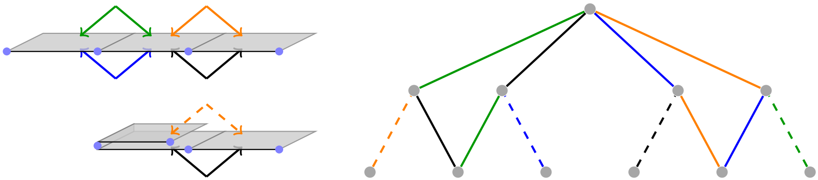

Larger graph

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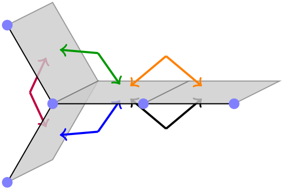

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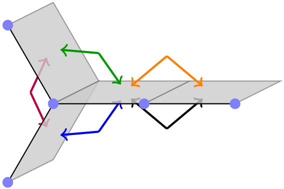

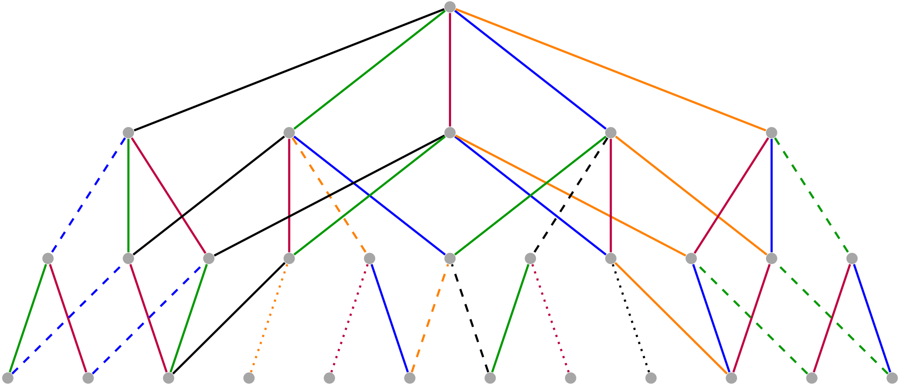

Drawback of folding plans

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Some foldings that "should" be the same, aren't:

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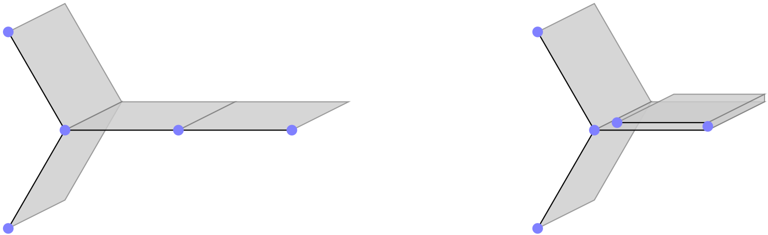

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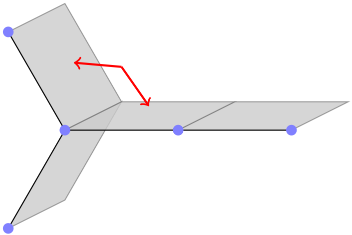

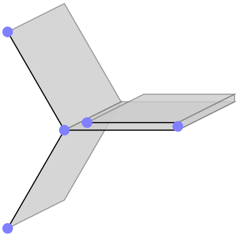

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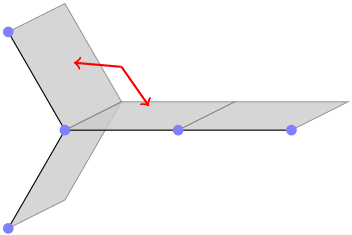

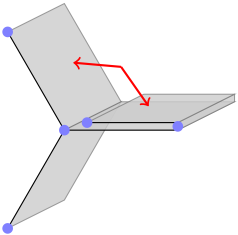

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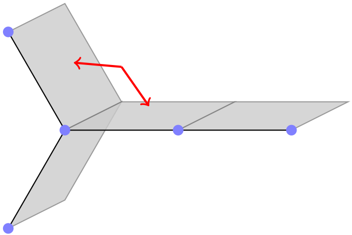

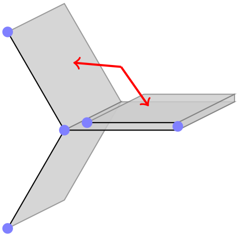

$\Rightarrow$ If you know the folding structure of a small complex,

Drawback of folding plans

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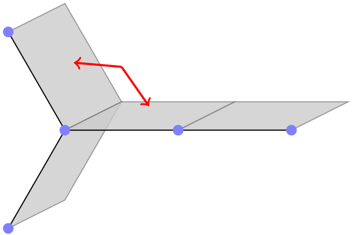

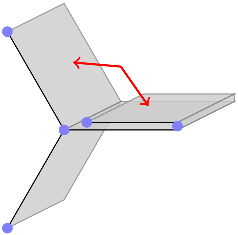

⇒ If you know the folding structure of a small complex, you can’t easily find the folding structure of an extended complex

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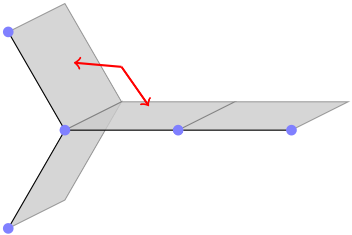

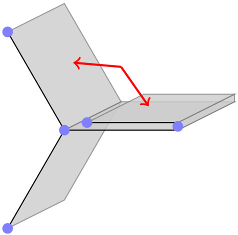

⇒ If you know the folding structure of a small complex, you can't easily find the folding structure of an extended complex

⇝ Folding plans are not optimal to model folding

Progress report of abstract folding

Progress report of abstract folding

In development:

Progress report of abstract folding

In development:

- folding complex

Progress report of abstract folding

In development:

- folding complex
- folding plans

Progress report of abstract folding

In development:

- folding complex
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- folding graph

Progress report of abstract folding

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Missing:

Progress report of abstract folding

In development:

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Missing:

- better folding description

Progress report of abstract folding

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Progress report of abstract folding

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- properties of folding graphs

Summary: SimplicialSurfaces

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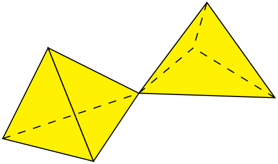
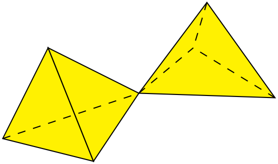
Triangular complexes



Summary: `SimplicialSurfaces`

Triangular complexes

- mostly complete



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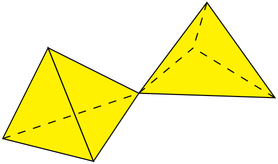
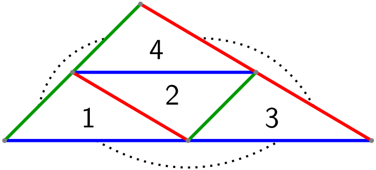


Edge colouring



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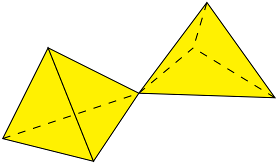

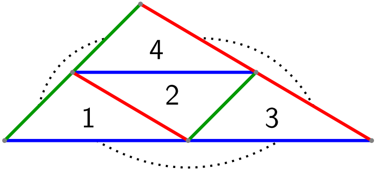

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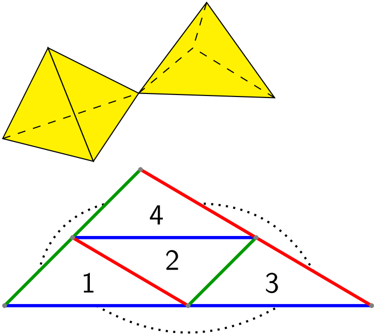

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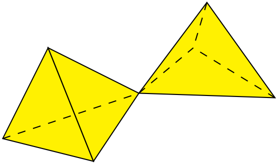

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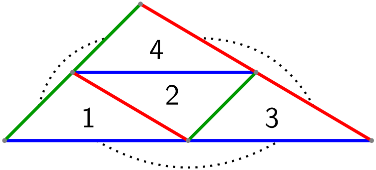
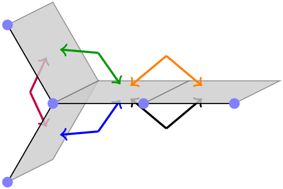


Abstract folding



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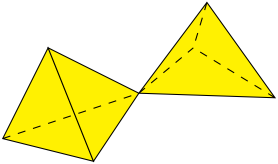

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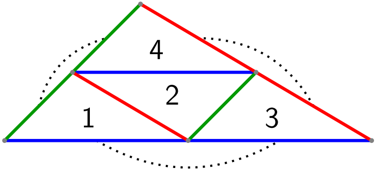
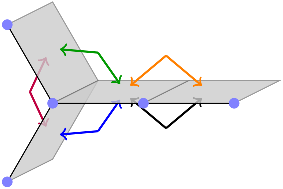


Abstract folding

- framework exists



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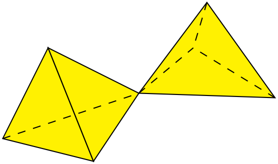

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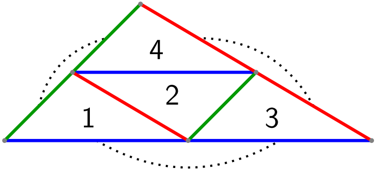

Abstract folding

- framework exists
- needs proper implementation