

Simplicial surfaces

Markus Baumeister

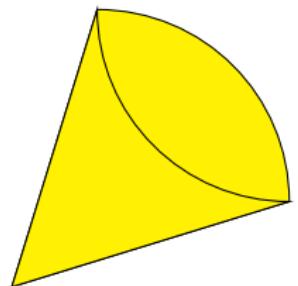
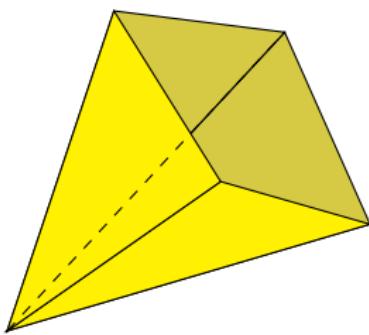
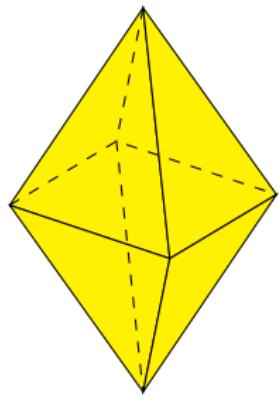
(j/w Alice Niemeyer, Wilhelm Plesken, Ansgar Strzelczyk)

Lehrstuhl B für Mathematik
RWTH Aachen University

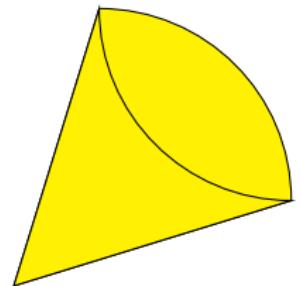
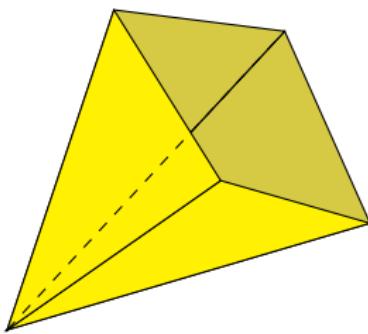
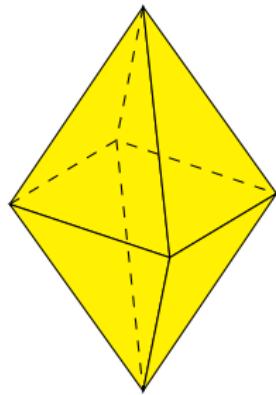
27.09.2017

Simplicial surfaces

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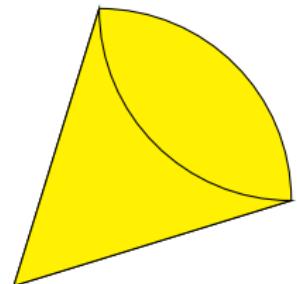
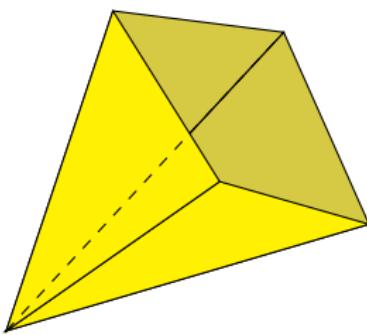
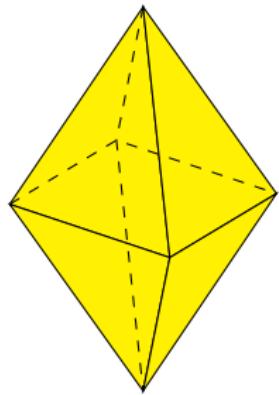


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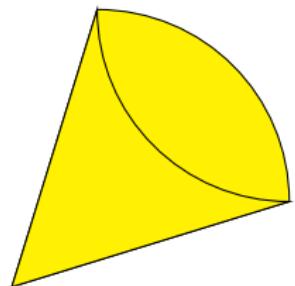
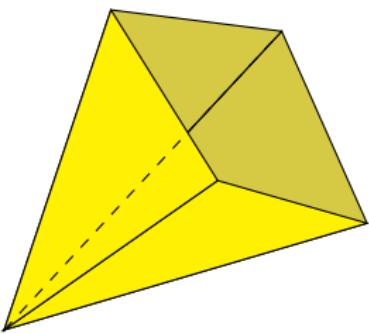
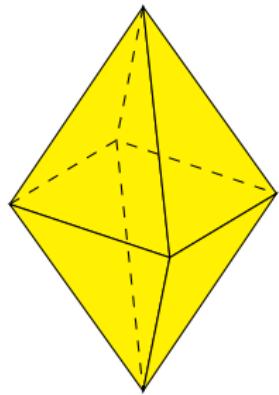
↔ structures build from triangles

Simplicial surfaces



↔ structures build from triangles (or arbitrary polygons)

Simplicial surfaces



↔ structures build from triangles (or arbitrary polygons)
→ focus on the combinatorial/incidence structure (without embedding)

Classification

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Goal: Classify all closed simplicial surfaces.

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Plesken/Strzelczyk classified the basic surfaces (building blocks) up to 20 triangles.

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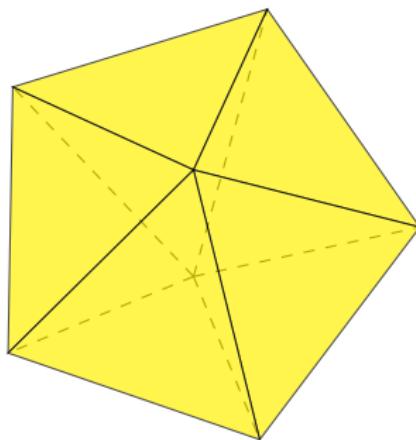
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One type of triangle

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For embedding purposes:

One type of triangle

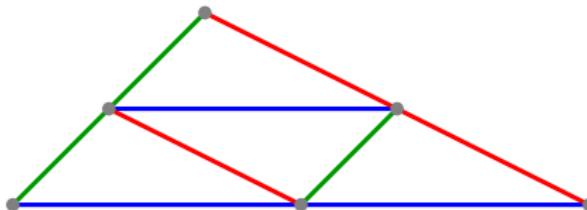
For embedding purposes:

We are interested in surfaces that are built from one type of triangle.

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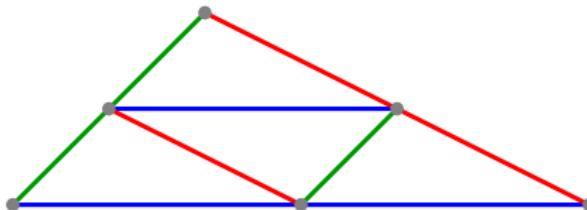
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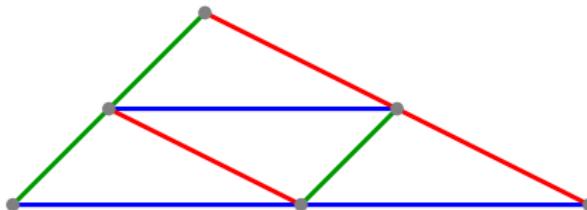


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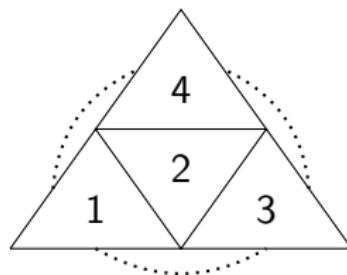


- ~~ edge-colouring encodes lengths
- ~~ analyse edge-coloured surfaces

Colouring as permutation

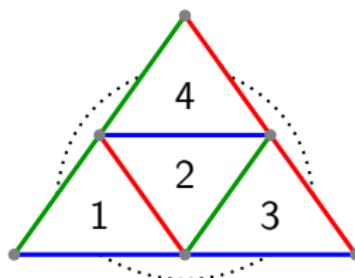
Colouring as permutation

Consider a tetrahedron



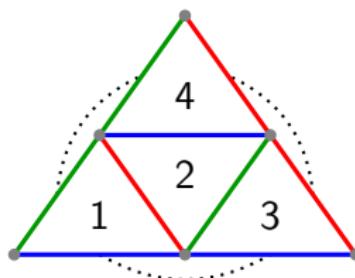
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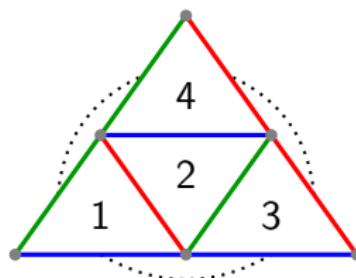
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simplicial surface \Rightarrow

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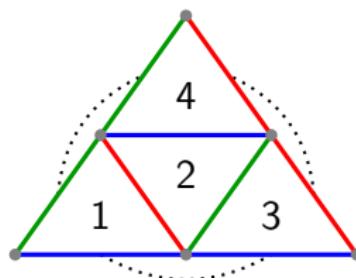
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simplicial surface $\Rightarrow \leq 2$ faces at each edge

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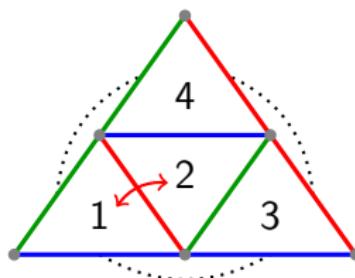


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~ every edge defines a transposition of incident faces

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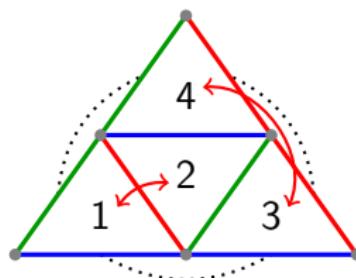
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- (1,2)

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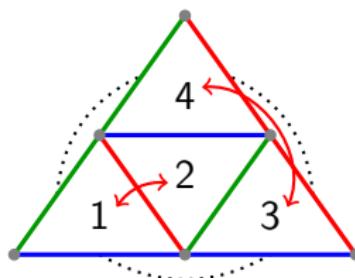
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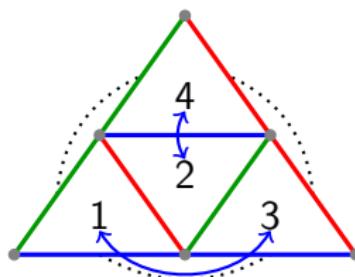


simplicial surface $\Rightarrow \leq 2$ faces at each edge

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Colouring as permutation

Consider a tetrahedron with an edge colouring

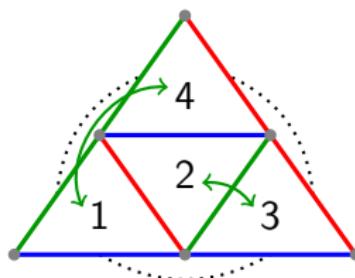


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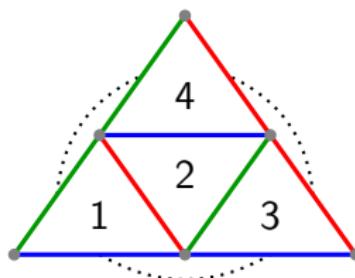


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simplicial surface $\Rightarrow \leq 2$ faces at each edge

- ~ every edge defines a transposition of incident faces
- ~ every colour class defines a permutation of the faces
- $(1,2)(3,4)$, $(1,3)(2,4)$, $(1,4)(2,3)$
- ~ group theoretic considerations

Implementation in GAP

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Implemented in GAP (package SimplicialSurfaces, unpublished)

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Example: define a coloured icosahedron:

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gap> iko := ColouredSimplicialSurface(  
[[ 1, 2 ], [ 1, 3 ], [ 1, 4 ], [ 1, 5 ], [ 1, 6 ], [ 2, 3 ], [ 2, 4 ], [ 2, 7 ],  
[ 2, 8 ], [ 3, 6 ], [ 3, 7 ], [ 3, 11 ], [ 4, 5 ], [ 4, 8 ], [ 4, 9 ], [ 5, 6 ],  
[ 5, 9 ], [ 5, 10 ], [ 6, 10 ], [ 6, 11 ], [ 7, 8 ], [ 7, 11 ], [ 7, 12 ], [ 8, 9 ],  
[ 8, 12 ], [ 9, 10 ], [ 9, 12 ], [ 10, 11 ], [ 10, 12 ], [ 11, 12 ] ],  
  
[[ [ 1, 2, 6 ], [ 1, 3, 7 ], [ 3, 4, 13 ], [ 4, 5, 16 ], [ 2, 5, 10 ], [ 6, 8, 11 ],  
[ 7, 9, 14 ], [ 13, 15, 17 ], [ 16, 18, 19 ], [ 10, 12, 20 ], [ 8, 9, 21 ],  
[ 14, 15, 24 ], [ 17, 18, 26 ], [ 19, 20, 28 ], [ 11, 12, 22 ], [ 21, 23, 25 ],  
[ 24, 25, 27 ], [ 26, 27, 29 ], [ 28, 29, 30 ], [ 22, 23, 30 ] ],  
  
[ 1, 2, 3, 1, 3, 3, 2, 1, 3, 1, 2, 3, 2, 1, 3, 2, 1, 3, 1, 2, 2, 1, 3, 2, 1, 2, 3, 3,  
1, 2 ] );;
```

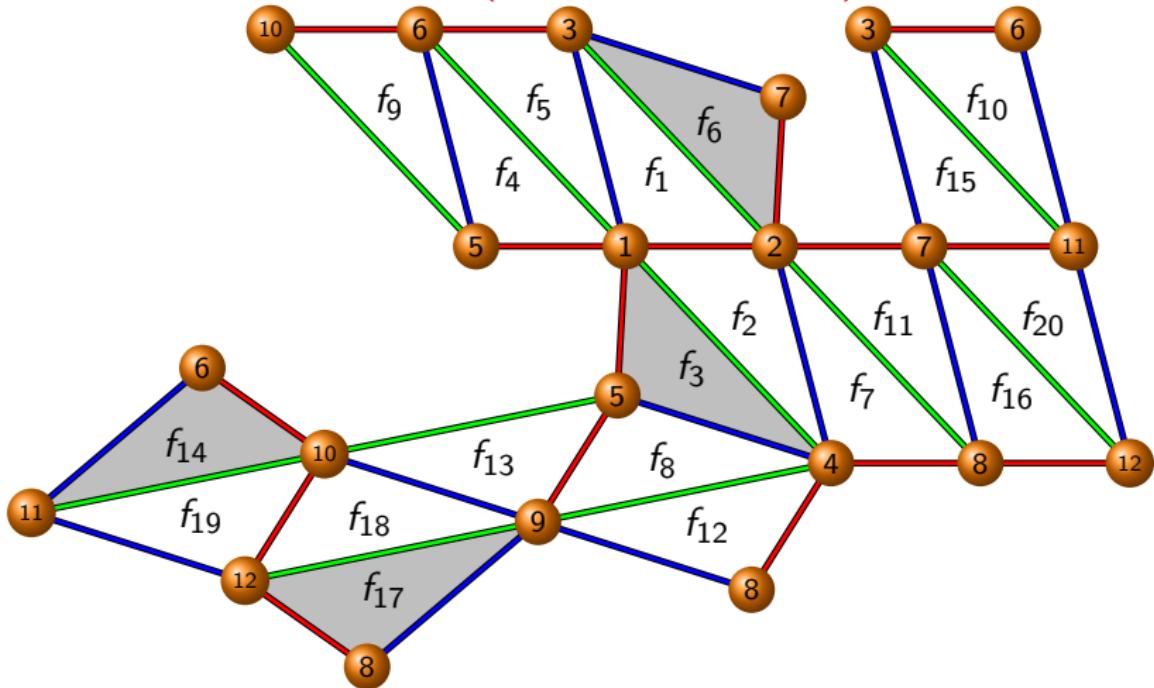
Net of an icosahedron

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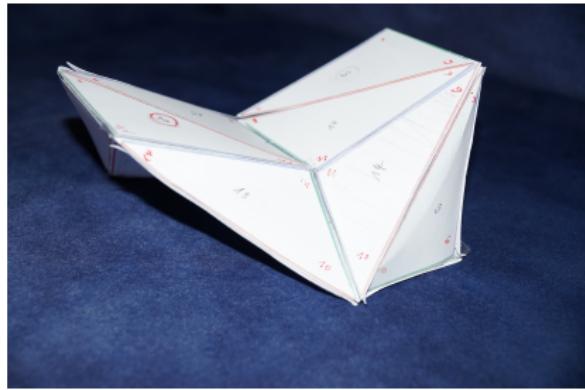
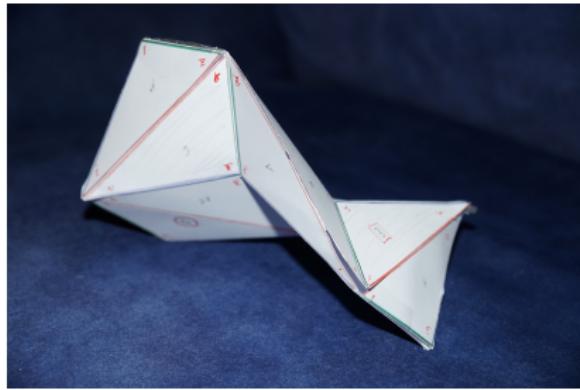
```
gap> DrawSurfaceToTikZ(iko,"NetIko.tex");
```

Net of an icosahedron

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Embedded icosahedron



Further research

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- Structural properties of coloured surfaces

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Thanks for your attention

Alice Niemeyer
Wilhelm Plesken

Markus Baumeister
Ansgar Strzelczyk