AutomataGroups

Package for computations in groups generated by finite automata

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Abstract

This is a manual for an

`AutomataGroups`

package, implementing in GAP basic functions and algorithms for groups generated by finite automat a, contracting groups.

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Colophon

The project was originally started in 2000 mostly for personal use. It was gradually expadnig during consequent years, including both addition of new algorithms and simplification of user interface. It was used in the process of classification of groups generated by 3-state automata over 2-letter alphabet.

Contents

Chapter 1

Introduction

1.1 Mathematical Background

1.2 Notations and Agreements

1.3 Quick Example

Chapter 2

Automata groups

2.1 Trees

These functions allow to construct and operate with vertices of the trees.

2.1.1 VertexNumber

◊ `VertexNumber( num, lev, deg )` (function)

One can naturally enumerate all the vertices of the *lev*-th level of the `deg`-ary tree by n umbers $1, \ldots, \deg^n$. This function returns the vertex of this level, which has number `num`.

Example

```
gap> VertexNumber(1,3,2);
[ 1, 1, 1 ]
gap> VertexNumber(4,4,3);
[ 1, 1, 2, 1 ]
```

2.1.2 NumberOfVertex

◊ `NumberOfVertex( ver, deg )` (function)

Let `ver` belong to *n*-th level of the `deg`-ary tree. One can naturally enume rate all the vertices of this level by numbers $1, \ldots, \deg^n$. This function returns the number, which corresponds to the vertex `ver`.

Example

```
gap> NumberOfVertex([1,2,1,2],2);
6
gap> NumberOfVertex("333",3);
27
```

2.2 Tree Automorphisms

These functions allow to construct orbit of vertex.

2.2.1 OrbitOfVertex

◊ OrbitOfVertex(ver, g[, n]) (operation)

Returns the list of vertics in the orbit of vertex ver under the action of a semigroup gene rated by an automorphism g. If n is specified returns only first n elements of the orbit. Vertices are defined either as lists with entries from [1..d], or as strings containing characters 1,...,d, where d is the degree of the tree.

Example

```
gap> g:=AutomGroup("t=(1,t)(1,2)");;
gap> OrbitOfVertex([1,1,1],t);
[ [ 1, 1, 1 ], [ 2, 1, 1 ], [ 1, 2, 1 ], [ 2, 2, 1 ], [ 1, 1, 2 ], [ 2, 1, 2 ], [ 1, 2, 2 ], [ 2, 2
gap> OrbitOfVertex("111111111111",t,6);
[ [ 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1 ], [ 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1 ],
[ 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1 ], [ 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1 ],
[ 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1 ], [ 2, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1 ] ]
```

2.2.2 PrintOrbitOfVertex

◊ PrintOrbitOfVertex(ver, g[, n]) (operation)

Prints the orbit of vertex ver under the action of a semigroup generated by an automorphism g. Each vertex is printed as a string containing characters 1,...,d, wher e d is the degree of the tree. In case of binary tree the symbols ' ' and 'x' are used to represent 1 and 2. If n is specified only first n elements of the orbit are printed. Vertices are defined either as lists with entries from [1..d], or as strings

Example

```
gap> g:=AutomGroup("a=(b,a)(1,2),b=(b,a)");;
gap> PrintOrbitOfVertex("22222222222222222222222222222",a*b^-2,6);
xxxxxxxxxxxxxxxxxxxxxxxxxxxxx

x x x x x x x x x x x x x x x x
 xx  xx  xx  xx  xx  xx  xx  xx
xxx xxx xxx xxx xxx xxx xxx xxx
   xxxx    xxxx    xxxx    xxxx
gap> h:=AutomGroup("a=(b,1,1)(1,2,3),b=(a,b,a)(1,2)");;
gap> PrintOrbitOfVertex([1,2,1],b^2);
121
132
123
131
122
133
```

Chapter 3

Contracting groups