Contents

Chapter 1

Resolutions of the ground ring

`ResolutionAbelianGroup(L,n)` `ResolutionAbelianGroup(G,n)` Inputs a list $L := [m_1, m_2, ..., m_d]$ of nonnegativ
`ResolutionAlmostCrystalGroup(G,n)` Inputs a positive integer n and an almost crystallographic pcp group G. It ret
`ResolutionAlmostCrystalQuotient(G,n,c)` `ResolutionAlmostCrystalQuotient(G,n,c,false)` An almost
`ResolutionArtinGroup(D,n)` Inputs a Coxeter diagram D and an integer $n > 1$. It returns n terms of a free ZG-resolu
`ResolutionAsphericalPresentation(F,R,n)` Inputs a free group F, a set R of words in F which constitute an asph
`ResolutionBieberbachGroup( G )`
`ResolutionBieberbachGroup( G, v )` Inputs a Bieberbach group G (represented using AffineCrystGroupOnRight
`ResolutionDirectProduct(R,S)` `ResolutionDirectProduct(R,S,"internal")` Inputs a ZG-resolution R and
`ResolutionExtension(g,R,S)` `ResolutionExtension(g,R, S,"TestFiniteness")` `ResolutionExtensio`
`ResolutionFiniteDirectProduct(R,S)` `ResolutionFiniteDirectProduct(R,S, "internal")` Inputs a ZG
`ResolutionFiniteExtension(gensE,gensG,R,n)` `ResolutionFiniteExtension(gensE,gensG,R,n,true)`
`ResolutionFiniteGroup(gens,n)` `ResolutionFiniteGroup(gens,n,true)` `ResolutionFiniteGroup(gens`
`ResolutionFiniteSubgroup(R,K)` `ResolutionFiniteSubgroup(R,gensG,gensK)` Inputs a ZG-resolution for a
`ResolutionGraphOfGroups(D,n)` `ResolutionGraphOfGroups(D,n,L)` Inputs a graph of groups D and a positi
`ResolutionNilpotentGroup(G,n)` `ResolutionNilpotentGroup(G,n,"TestFiniteness")` Inputs a nilpotent
`ResolutionNormalSeries(L,n)` `ResolutionNormalSeries(L,n,true)` `ResolutionNormalSeries(L,n,fa`
`ResolutionPrimePowerGroup(P,n)` `ResolutionPrimePowerGroup(G,n,p)` Inputs a p-group P and integer $n>($
`ResolutionSmallFpGroup(G,n)` `ResolutionSmallFpGroup(G,n,p)` Inputs a small finitely presented group G
`ResolutionSubgroup(R,K)` Inputs a ZG-resolution for an (infinite) group G and a subgroup K of finite index $|G:K|$.
`ResolutionSubnormalSeries(L,n)` Inputs a positive integer n and a list $L = [L_1, \ldots, L_k]$ of subgroups L_i of a finite
`TwistedTensorProduct(R,S,EhomG,GmapE,NhomE,NEhomN,EltsE,Mult,InvE)` Inputs a ZG-resolution R, a ZN-res

Chapter 2

Resolutions of modules

`ResolutionFpGModule(M,n)` Inputs an FpG-module M and a positive integer n. It returns n terms of a minimal free F

Chapter 3

Induced equivariant chain maps

`EquivariantChainMap(R,S,f)` Inputs a ZG-resolution R, a ZG'-resolution S, and a group homomorphism $f : G \longrightarrow$

Chapter 4

Functors

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`HomToIntegers(X)` Inputs either a ZG-resolution $X = R$, or an equivariant chain map $X = (F : R \longrightarrow S)$. It returns th

`HomToIntegersModP(R)` Inputs a ZG-resolution R and returns the cochain complex obtained by applying $HomZG(,Z$

`HomToIntegralModule(R,f)` Inputs a ZG-resolution R and a group homomorphism $f : G \longrightarrow GL_n(Z)$ to the group c

`LowerCentralSeriesLieAlgebra(G)` `LowerCentralSeriesLieAlgebra(f)` Inputs a pcp group G. If each quo

`TensorWithIntegers(X)` Inputs either a ZG-resolution $X = R$, or an equivariant chain map $X = (F : R \longrightarrow S)$. It retu

`TensorWithIntegersModP(X,p)` Inputs either a ZG-resolution $X = R$, or an equivariant chain map $X = (F : R \longrightarrow S$

`TensorWithRationals(R)` Inputs a ZG-resolution R and returns the chain complex obtained by tensoring with the tri

Chapter 5

Chain complexes

`ChevalleyEilenbergComplex(X,n)` Inputs either a Lie algebra $X = A$ (over the ring of integers Z or over a field K)
`LeibnizComplex(X,n)` Inputs either a Lie or Leibniz algebra $X = A$ (over the ring of integers Z or over a field K) or

Chapter 6

Homology and cohomology groups

`Cohomology(X)` Inputs either a cochain complex $X = C$ or a cochain map $X = (C \longrightarrow D)$ over the integers Z.If $X = C$
`GroupCohomology(X,n)` `GroupCohomology(X,n,p)` Inputs a positive integer n and either a finite group $X = G$ or a
`GroupHomology(X,n)`
`GroupHomology(X,n,p)` Inputs a positive integer n and either a finite group $X = G$ or a Coxeter diagram $X = D$ represe
`Homology(X,n)` Inputs either a chain complex $X = C$ or a chain map $X = (C \longrightarrow D)$.If $X = C$ then the torsion coefficien
`HomologyPb(C,n)` This is a back-up function which might work in some instances where $Homology(C,n)$ fails. It is mo
`LeibnizAlgebraHomology(A,n)` Inputs a Lie or Leibniz algebra $X = A$ (over the ring of integers Z or over a field K), t
`LieAlgebraHomology(A,n)` Inputs a Lie algebra A (over the integers or a field) and a positive integer n. It returns the h
`PrimePartDerivedFunctor(G,R,F,n)` Inputs a finite group G, a positive integer n, at least $n+1$ terms of a ZP-resolut
`RankHomologyPGroup(G,n)` `RankHomologyPGroup(R,n)` `RankHomologyPGroup(G,n,"empirical")` Inputs a (sma
`RankPrimeHomology(G,n)` Inputs a (smallish) p-group G together with a positive integer n. It returns a function $dim(k)$

Chapter 7

Poincare series

•

`EfficientNormalSubgroups(G)`
`EfficientNormalSubgroups(G,k)` Inputs a prime-power group G and, optionally, a positive integer k. The default is k
`ExpansionOfRationalFunction(f,n)` Inputs a positive integer n and a rational function $f(x) = p(x)/q(x)$ where the d
`PoincareSeries(G,n)` `PoincareSeries(R,n)`
`PoincareSeries(L,n)`
`PoincareSeries(G)` Inputs a finite p-group G and a positive integer n. It returns a quotient of polynomials $f(x) = P$
`PoincareSeriesPrimePart(G,p,n)` Inputs a finite group G, a prime p, and a positive integer n. It returns a quotient o
`Prank(G)` Inputs a p-group G and returns the rank of the largest elementary abelian subgroup.

Chapter 8

Cohomology ring structure

`IntegralCupProduct(R,u,v,p,q)`

`IntegralCupProduct(R,u,v,p,q,P,Q,N)` (Various functions used to construct the cup product are also `CR`$_f$*unctio*

`IntegralRingGenerators(R,n)` Inputs at least $n+1$ terms of a ZG-resolution and integer $n>0$. It returns a minima

`ModPCohomologyRing(G,n)`

`ModPCohomologyRing(R)` Inputs either a p-group G and positive integer n, or else n terms of a minimal Z_pG-resoluti

`ModPRingGenerators(A)` Inputs a mod p cohomology ring A (created using the preceeding function). It returns a ger

Chapter 9

Commutator and nonabelian tensor computations

•

`BaerInvariant(G,c)` Inputs a nilpotent group G and integer c>0. It returns the Baer invariant $M^(c)(G)$ defined as fo
`Coclass(G)` Inputs a group G of prime-power order p^n and nilpotency class c say. It returns the integer $r = n - c$.
`EpiCentre(G,N)`
`EpiCentre(G)` Inputs a finite group G and normal subgroup N and returns a subgroup $Z^*(G,N)$ of the centre of N. The
`NonabelianExteriorProduct(G,N)` Inputs a finite group G and normal subgroup N. It returns a record E with the fo
`NonabelianTensorProduct(G,N)` Inputs a finite group G and normal subgroup N. It returns a record E with the foll
`NonabelianTensorSquare(G)`
`NonabelianTensorSquare(G,m)` Inputs a finite or nilpotent infinite group G and returns a record T with the followin
`RelativeSchurMultiplier(G,N)` Inputs a finite group G and normal subgroup N. It returns the homology group H_2
`TensorCentre(G)` Inputs a group G and returns the largest central subgroup N such that the induced homomorphism
`ThirdHomotopyGroupOfSuspensionB(G)`
`ThirdHomotopyGroupOfSuspensionB(G,m)` Inputs a finite or nilpotent infinite group G and returns the abelian invar
`UpperEpicentralSeries(G,c)` Inputs a nilpotent group G and an integer c. It returns the c-th term of the upper epic

Chapter 10

Generators and relators of groups

•

`CayleyGraphDisplay(G,X)`
`CayleyGraphDisplay(G,X,"mozilla")` Inputs a finite group G together with a subset X of G. It displays the corres
`IsAspherical(F,R)` Inputs a free group F and a set R of words in F. It performs a test on the 2-dimensional CW-spa
`PresentationOfResolution(R)` Inputs at least two terms of a reduced ZG-resolution R and returns a record P with
`TorsionGeneratorsAbelianGroup(G)` Inputs an abelian group G and returns a generating set $[x_1,\ldots,x_n]$ where no p

Chapter 11

Orbit polytopes and fundamental domains

•

`FundamentalDomainAffineCrystGroupOnRight(v,G)` Inputs a crystallographic group G (represented using AffineC
`OrbitPolytope(G,v,L)` Inputs a permutation group or matrix group G of degree n and a rational vector v of length n

The function uses Polymake software.

`PolytopalComplex(G,v)`
`PolytopalComplex(G,v,n)`

Inputs a permutation group or matrix group G of degree n and a rational vector v of length n. In both cases there is a natural action of G on v. Let $P(G,v)$ be the convex polytope arising as the convex hull of the Euclidean points in the orbit of v under the action of G. The cellular chain complex $C_* = C_*(P(G,v))$ is an exact sequence of (not necessarily free) ZG-modules. The function returns a component object R with components:

- $R!.dimension(k)$ is a function which returns the number of G-orbits of the k-dimensional faces in $P(G,v)$. If each k-face has trivial stabilizer subgroup in G then C_k is a free ZG-module of rank $R.dimension(k)$.
- $R!.stabilizer(k,n)$ is a function which returns the stabilizer subgroup for a face in the n-th orbit of k-faces.
- If all faces of dimension $<k+1$ have trivial stabilizer group then the first k terms of C_* constitute part of a free ZG-resolution. The boundary map is described by the function $boundary(k,n)$. (If some faces have non-trivial stabilizer group then C_* is not free and no attempt is made to determine signs for the boundary map.)
- $R!.elements$, $R!.group$, $R!.properties$ are as in a ZG-resolution.

If an optional third input variable n is used, then only the first n terms of the resolution C_* will be computed.

The function uses Polymake software.

`PolytopalGenerators(G,v)`

Inputs a permutation group or matrix group G of degree n and a rational vector v of length n. In both cases there is a natural action of G on v, and the vector v must be chosen so that it has trivial stabilizer subgroup in G. Let $P(G,v)$ be the convex polytope arising as the convex hull of the Euclidean points in the orbit of v under the action of G. The function returns a record P with components:

- *P.generators* is a list of all those elements g in G such that $g \cdot v$ has an edge in common with v. The list is a generating set for G.
- *P.vector* is the vector v.
- *P.hasseDiagram* is the Hasse diagram of the cone at v.

The function uses Polymake software. The function is joint work with Seamus Kelly.

`VectorStabilizer(G,v)`

Inputs a permutation group or matrix group G of degree n and a rational vector of degree n. In both cases there is a natural action of G on v and the function returns the group of elements in G that fix v.

Chapter 12

Cocycles

`CocycleCondition(R,n)` Inputs a resolution R and an integer $n>0$. It returns an integer matrix M with the following

`StandardCocycle(R,f,n)`

`StandardCocycle(R,f,n,q)` Inputs a ZG-resolution R (with contracting homotopy), a positive integer n and an integ

`Syzygy(R,g)` Inputs a ZG-resolution R (with contracting homotopy) and a list $g=[g[1],\ldots,g[n]]$ of elements in G. It r

Chapter 13

Words in free ZG-modules

`AddFreeWords(v,w)` Inputs two words v, w in a free ZG-module and returns their sum $v+w$. If the characteristic of Z

`AddFreeWordsModP(v,w,p)` Inputs two words v, w in a free ZG-module and the characteristic p of Z. It returns the su

`AlgebraicReduction(w)`

`AlgebraicReduction(w,p)` Inputs a word w in a free ZG-module and returns a reduced version of the word in which

`Multiply Word(n,w)` Inputs a word w and integer n. It returns the scalar multiple $n \cdot w$.

`Negate([i,j])` Inputs a pair $[i, j]$ of integers and returns $[-i, j]$.

`NegateWord(w)` Inputs a word w in a free ZG-module and returns the negated word $-w$.

`PrintZGword(w,elts)` Inputs a word w in a free ZG-module and a (possibly partial but sufficient) listing elts of the e

`TietzeReduction(S,w)` Inputs a set S of words in a free ZG-module, and a word w in the module. The function retu

Chapter 14

FpG-modules

`DirectSumOfFpGModules(M,N)`
`DirectSumOfFpGModules([ M[1], M[2], ..., M[k] ]))` Inputs two FpG-modules M and N with common grou
`FpGModule(A,P)`
`FpGModule(A,G,p)` Inputs a p-group P and a matrix A whose rows have length a multiple of the order of G. It returns
`FpGModuleDualBasis(M)` Inputs an FpG-module M. It returns a record R with two components:$R.freeModule$ is the
`FpGModuleHomomorphism(M,N,A)`
`FpGModuleHomomorphismNC(M,N,A)` Inputs FpG-modules M and N over a common p-group G. Also inputs a list A
`DesuspensionFpGModule(M,n)`
`DesuspensionFpGModule(R,n)` Inputs a positive integer n and and FpG-module M. It returns an FpG-module D^nM
`RadicalOfFpGModule(M)` Inputs an FpG-module M with G a p-group, and returns the Radical of M as an FpG-mod
`GeneratorsOfFpGModule(M)` Inputs an FpG-module M and returns a matrix whose rows correspond to a minimal ge
`ImageOfFpGModuleHomomorphism(f)` Inputs an FpG-module homomorphism $f : M \longrightarrow N$ and returns its image $f($
`IntersectionOfFpGModules(M,N)` Inputs two FpG-modules M,N arising as submodules in a common free module
`IsFpGModuleHomomorphismData(M,N,A)` Inputs FpG-modules M and N over a common p-group G. Also inputs a l
`MultipleOfFpGModule(w,M)` Inputs an FpG-module M and a list $w := [g_1, ..., g_t]$ of elements in the group $G = M!.g$
`ProjectedFpGModule(M,k)` Inputs an FpG-module M of ambient dimension $n|G|$, and an integer k between 1 and n.
`RandomHomomorphismOfFpGModules(M,N)` Inputs two FpG-modules M and N over a common group G. It returns a
`Rank(f)` Inputs an FpG-module homomorphism $f : M \longrightarrow N$ and returns the dimension of the image of f as a vector
`SumOfFpGModules(M,N)` Inputs two FpG-modules M,N arising as submodules in a common free module $(FG)^n$ whe
`SumOp(f,g)` Inputs two FpG-module homomorphisms $f,g : M \longrightarrow N$ with common sorce and common target. It retu
`VectorsToFpGModuleWords(M,L)` Inputs an FpG-module M and a list $L = [v_1,\dots,v_k]$ of vectors in M. It returns a li

Chapter 15

Meataxe modules

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`DesuspensionMtxModule(M)` Inputs a meataxe module M over the field of p elements and returns an FpG-module DM.
`GeneratorsOfMtxModule(M)` Inputs a meataxe module M acting on, say, the vector space V. The function returns a m

Chapter 16

Coxeter diagrams and graphs of groups

`CoxeterDiagramComponents(D)` Inputs a Coxeter diagram D and returns a list $[D_1, ..., D_d]$ of the maximal connected

`CoxeterDiagramDegree(D,v)` Inputs a Coxeter diagram D and vertex v. It returns the degree of v (i.e. the number of

`CoxeterDiagramDisplay(D)`

`CoxeterDiagramDisplay(D,"web browser")` Inputs a Coxeter diagram D and displays it as a .gif file. It uses the M

`CoxeterDiagramFpArtinGroup(D)` Inputs a Coxeter diagram D and returns the corresponding finitely presented Art

`CoxeterDiagramFpCoxeterGroup(D)` Inputs a Coxeter diagram D and returns the corresponding finitely presented C

`CoxeterDiagramIsSpherical(D)` Inputs a Coxeter diagram D and returns ”true” if the associated Coxeter groups is

`CoxeterDiagramMatrix(D)` Inputs a Coxeter diagram D and returns a matrix representation of it. The matrix is giver

`CoxeterSubDiagram(D,V)` Inputs a Coxeter diagram D and a subset V of its vertices. It returns the full sub-diagram

`CoxeterDiagramVertices(D)` Inputs a Coxeter diagram D and returns its set of vertices.

`EvenSubgroup(G)` Inputs a group G and returns a subgroup G^+. The subgroup is that generated by all products xy wh

`GraphOfGroupsDisplay(D)`

`GraphOfGroupsDisplay(D,"web browser")` Inputs a graph of groups D and displays it as a .gif file. It uses the Mo

`GraphOfGroupsTest(D)` Inputs an object D and itries to test whether it is a Graph of Groups. However, it DOES NO

Chapter 17

Some functions for accessing basic data

`BoundaryMap(C)` Inputs a resolution, chain complex or cochain complex C and returns the function $C!.boundary$.
`BoundaryMatrix(C,n)` Inputs a chain or cochain complex C and integer $n>0$. It returns the n-th boundary map of C
`Dimension(C)`
`Dimension(M)` Inputs a resolution, chain complex or cochain complex C and returns the function $C!.dimension$. Alte
`EvaluateProperty(X,"name")` Inputs a component object X (such as a ZG-resolution or chain map) and a string "n
`GroupOfResolution(R)` Inputs a ZG-resolution R and returns the group G.
`Length(R)` Inputs a resolution R and returns its length (i.e. the number of terms of R that HAP has computed).
`Map(f)` Inputs a chain map, or cochain map or equivariant chain map f and returns the mapping function (as opposed
`Source(f)` Inputs a chain map, or cochain map, or equivariant chain map, or FpG-module homomorphism f and retu
`Target(f)` Inputs a chain map, or cochain map, or equivariant chain map, or FpG-module homomorphism f and retu

Chapter 18

Miscellaneous

`BigStepLCS(G,n)` Inputs a group G and a positive integer n. It returns a subseries $G = L_1 > L_2 > \ldots L_k = 1$ of the lowe

`Compose(f,g)` Inputs two FpG-module homomorphisms $f : M \longrightarrow N$ and $g : L \longrightarrow M$ with $Source(f) = Target(g)$.

`HAPcopyright()` This function provides details of HAP'S GNU public copyright licence.

`IsLieAlgebraHomomorphism(f)` Inputs an object f and returns true if f is a homomorphism $f : A \longrightarrow B$ of Lie alge

`IsSuperperfect(G)` Inputs a group G and returns "true" if both the first and second integral homology of G is trivial

`MakeHAPManual()` This function creates the manual for HAP from an XML file.

`PermToMatrixGroup(G,n)` Inputs a permutation group G and its degree n. Returns a bijective homomorphism $f : G$

`SolutionsMatDestructive(M,B)` Inputs an $m \times n$ matrix M and a $k \times n$ matrix B over a field. It returns a k$\times$*mmatri*

`TestHap()` This runs a representative sample of HAP functions and checks to see that they produce the correct output

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