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Chapter 1

Resolutions of the ground ring

TietzeReducedResolution(R) Inputs a $\mathbb{Z}G$ -resolution R and returns a $\mathbb{Z}G$ -resolution S which is obtained from R by Tietze transformations.

FreeGResolution(P, n) FreeGResolution(P, n, p) Inputs a non-free ZG -resolution P with finite stabilizer group G and an integer n . The second version also takes a prime p .

ResolutionAbelianGroup(L, n) ResolutionAbelianGroup(G, n) Inputs a list $L := [m_1, m_2, \dots, m_d]$ of nonnegative integers and an integer n . The second version takes a group G .

ResolutionAlmostCrystalGroup(G, n) Inputs a positive integer n and an almost crystallographic pcg group G . It returns a resolution of $\mathbb{Z}G$.

ResolutionAlmostCrystalQuotient(G, n, c) ResolutionAlmostCrystalQuotient(G, n, c, false) An almost crystallographic pcg group G and an integer n . The second version also takes a boolean c .

ResolutionArtinGroup(D, n) Inputs a Coxeter diagram D and an integer $n > 1$. It returns n terms of a free ZG -resolution of $\mathbb{Z}G$.

ResolutionAsphericalPresentation(F, R, n) Inputs a free group F , a set R of words in F which constitute an aspherical presentation of a group G , and an integer n .

ResolutionBieberbachGroup(G) ResolutionBieberbachGroup(G, v) Inputs a torsion free crystallographic group G , also known as a Bieberbach group. The second version also takes a vector v .

ResolutionCoxeterGroup(D, n) Inputs a Coxeter diagram D and an integer $n > 1$. It returns k terms of a free ZG -resolution of $\mathbb{Z}G$.

ResolutionDirectProduct(R, S) ResolutionDirectProduct($R, S, \text{"internal"}$) Inputs a ZG -resolution R and a ZH -resolution S . The second version also takes a boolean "internal" .

ResolutionExtension(g, R, S) ResolutionExtension($g, R, S, \text{"TestFiniteness"}$) ResolutionExtension($g, R, S, \text{"TestFiniteness", true}$) Inputs a group G , a ZG -resolution R , and a ZH -resolution S . The second version also takes a boolean "TestFiniteness" . The third version also takes a boolean true .

ResolutionFiniteDirectProduct(R, S) ResolutionFiniteDirectProduct($R, S, \text{"internal"}$) Inputs a ZG -resolution R and a ZH -resolution S . The second version also takes a boolean "internal" .

ResolutionFiniteExtension($\text{gensE}, \text{gensG}, R, n$) ResolutionFiniteExtension($\text{gensE}, \text{gensG}, R, n, \text{true}$) Inputs a list gensE of generators of E , a list gensG of generators of G , a ZG -resolution R , and an integer n . The second version also takes a boolean true .

ResolutionFiniteGroup(gens, n) ResolutionFiniteGroup($\text{gens}, n, \text{true}$) ResolutionFiniteGroup($\text{gens}, n, \text{true}, \text{true}$) Inputs a list gens of generators of G and an integer n . The second version also takes a boolean true . The third version also takes a boolean true .

ResolutionFiniteSubgroup(R, K) ResolutionFiniteSubgroup($R, \text{gensG}, \text{gensK}$) Inputs a ZG -resolution R for a group G and a subgroup K of finite index $|G : K|$. The second version also takes a list gensG of generators of G and a list gensK of generators of K .

ResolutionGraphOfGroups(D, n) ResolutionGraphOfGroups(D, n, L) Inputs a graph of groups D and a positive integer n . The second version also takes a list L of subgroups.

ResolutionNilpotentGroup(G, n) ResolutionNilpotentGroup($G, n, \text{"TestFiniteness"}$) Inputs a nilpotent group G and an integer n . The second version also takes a boolean "TestFiniteness" .

ResolutionNormalSeries(L, n) ResolutionNormalSeries(L, n, true) ResolutionNormalSeries($L, n, \text{true}, \text{true}$) Inputs a list L of subgroups and an integer n . The second version also takes a boolean true . The third version also takes a boolean true .

ResolutionPrimePowerGroup(P, n) ResolutionPrimePowerGroup(G, n, p) Inputs a p -group P and integer n . The second version also takes a group G and a prime p .

ResolutionSmallFpGroup(G, n) ResolutionSmallFpGroup(G, n, p) Inputs a small finitely presented group G and an integer n . The second version also takes a prime p .

ResolutionSubgroup(R, K) Inputs a ZG -resolution for an (infinite) group G and a subgroup K of finite index $|G : K|$.

ResolutionSubnormalSeries(L, n) Inputs a positive integer n and a list $L = [L_1, \dots, L_k]$ of subgroups L_i of a finitely presented group G such that L_i is normal in L_{i+1} for $i = 1, \dots, k-1$.

TwistedTensorProduct($R, S, \text{EhomG}, \text{GmapE}, \text{NhomE}, \text{NEhomN}, \text{EltsE}, \text{Mult}, \text{InvE}$) Inputs a ZG -resolution R , a ZH -resolution S , and a list of maps $\text{EhomG}, \text{GmapE}, \text{NhomE}, \text{NEhomN}, \text{EltsE}, \text{Mult}, \text{InvE}$.

Chapter 2

Resolutions of modules

| `ResolutionFpGModule (M, n)` Inputs an FpG -module M and a positive integer n . It returns n terms of a minimal fr

Chapter 3

Induced equivariant chain maps

| `EquivariantChainMap(R, S, f)` Inputs a ZG -resolution R , a ZG' -resolution S , and a group homomorphism $f : G \rightarrow G'$

Chapter 4

Functors

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`ExtendScalars(R, G, EltsG)` Inputs a ZH -resolution R , a group G containing H as a subgroup, and a list $EltsG$ of elements of G . It returns the ZG -resolution $R \otimes_{ZH} ZG$.

`HomToIntegers(X)` Inputs either a ZG -resolution $X = R$, or an equivariant chain map $X = (F : R \rightarrow S)$. It returns the chain complex $\text{Hom}(X, \mathbb{Z})$.

`HomToIntegersModP(R)` Inputs a ZG -resolution R and returns the cochain complex obtained by applying $\text{Hom}(R, \mathbb{Z}/p\mathbb{Z})$ to the resolution.

`HomToIntegralModule(R, f)` Inputs a ZG -resolution R and a group homomorphism $f : G \rightarrow GL_n(\mathbb{Z})$ to the group G . It returns the $\mathbb{Z}[G]$ -module $\text{Hom}(R, \mathbb{Z})$.

`HomToGModule(R, A)` Inputs a ZG -resolution R and an abelian G -outer group A . It returns the G -cocomplex obtained by applying $\text{Hom}(R, A)$ to the resolution.

`InduceScalars(R, hom)` Inputs a ZQ -resolution R and a surjective group homomorphism $hom : G \rightarrow Q$. It returns the ZG -resolution $R \otimes_{ZQ} ZG$.

`LowerCentralSeriesLieAlgebra(G)` `LowerCentralSeriesLieAlgebra(f)` Inputs a pcg group G . If each $g \in G$ has a power g^m in the center of G , then f is the map $f(g) = g^m$. It returns the Lie algebra $L(G)$ of G .

`TensorWithIntegers(X)` Inputs either a ZG -resolution $X = R$, or an equivariant chain map $X = (F : R \rightarrow S)$. It returns the chain complex $X \otimes_{\mathbb{Z}} \mathbb{Z}$.

`FilteredTensorWithIntegers(R)` Inputs a ZG -resolution R for which "filteredDimension" lies in `NamesOfComplexes`. It returns the chain complex $R \otimes_{\mathbb{Z}} \mathbb{Z}$.

`TensorWithTwistedIntegers(X, rho)` Inputs either a ZG -resolution $X = R$, or an equivariant chain map $X = (F : R \rightarrow S)$. It returns the chain complex $X \otimes_{\mathbb{Z}} \mathbb{Z}$ with the G -action twisted by ρ .

`TensorWithIntegersModP(X, p)` Inputs either a ZG -resolution $X = R$, or a characteristic 0 chain complex, or an equivariant chain map $X = (F : R \rightarrow S)$. It returns the chain complex $X \otimes_{\mathbb{Z}} \mathbb{Z}/p\mathbb{Z}$.

`TensorWithTwistedIntegersModP(X, p, rho)` Inputs either a ZG -resolution $X = R$, or an equivariant chain map $X = (F : R \rightarrow S)$. It returns the chain complex $X \otimes_{\mathbb{Z}} \mathbb{Z}/p\mathbb{Z}$ with the G -action twisted by ρ .

`TensorWithRationals(R)` Inputs a ZG -resolution R and returns the chain complex obtained by tensoring with \mathbb{Q} .

Chapter 5

Chain complexes

`ChainComplex(T)` Inputs a pure cubical complex, or cubical complex, or simplicial complex T and returns the (often) chain complex of T .

`ChainComplexOfPair(T, S)` Inputs a pure cubical complex or cubical complex T and contractible subcomplex S . It returns the chain complex of the pair (T, S) .

`ChevalleyEilenbergComplex(X, n)` Inputs either a Lie algebra $X = A$ (over the ring of integers Z or over a field K) and an integer n . It returns the Chevalley-Eilenberg complex of X in degree n .

`LeibnizComplex(X, n)` Inputs either a Lie or Leibniz algebra $X = A$ (over the ring of integers Z or over a field K) and an integer n . It returns the Leibniz complex of X in degree n .

`SuspendedChainComplex(C)` Inputs a chain complex C and returns the chain complex S defined by applying the suspension operator to C .

`ReducedSuspendedChainComplex(C)` Inputs a chain complex C and returns the chain complex S defined by applying the reduced suspension operator to C .

`CoreducedChainComplex(C)` `CoreducedChainComplex(C, 2)` Inputs a chain complex C and returns a quasi-isomorphic coreduced chain complex.

`LefschetzNumber(F)` Inputs a chain map $F: C \rightarrow C$ with common source and target. It returns the Lefschetz number of F .

Chapter 6

Homology and cohomology groups

`Cohomology(X, n)` Inputs either a cochain complex $X = C$ (or G -cocomplex C) or a cochain map $X = (C \longrightarrow D)$ in
`CohomologyModule(C, n)` Inputs a G -cocomplex C together with a non-negative integer n . It returns the cohomology
`CohomologyPrimePart(C, n, p)` Inputs a cochain complex C in characteristic 0, a positive integer n , and a prime p .
`GroupCohomology(X, n)` `GroupCohomology(X, n, p)` Inputs a positive integer n and either a finite group $X = G$ or
`GroupHomology(X, n)`
`GroupHomology(X, n, p)` Inputs a positive integer n and either a finite group $X = G$ or a nilpotent Pcp-group $X = G$ or
`PersistentHomologyOfQuotientGroupSeries(S, n)`
`PersistentHomologyOfQuotientGroupSeries(S, n, p, ResolutionAlgorithm)` Inputs a positive integer n and
`PersistentCohomologyOfQuotientGroupSeries(S, n)`
`PersistentCohomologyOfQuotientGroupSeries(S, n, p, ResolutionAlgorithm)` Inputs a positive integer n and
`PersistentHomologyOfSubGroupSeries(S, n)`
`PersistentHomologyOfSubGroupSeries(S, n, p, ResolutionAlgorithm)` Inputs a positive integer n and a decre
 `PersistentHomologyOfCommutativeDiagramOfPGroups(D, n)` Inputs a commutative diagram D of finite p -gro
`PersistentHomologyOfPureCubicalComplex(L, n, p)`
`PersistentHomologyOfPureCubicalComplex(M, n, p)` Inputs a positive integer n , a prime p and an increasing cha
`RipsHomology(G, n)` `RipsHomology(G, n, p)` Inputs a graph G , a non-negative integer n (and optionally a prime nu
`BarCode(P)` Inputs an integer persistence matrix P and returns the same information in the form of a binary matrix
`BarCodeDisplay(P)` `BarCodeDisplay(P, "mozilla")` Inputs an integer persistence matrix P , and an optional strin
`Homology(X, n)` Inputs either a chain complex $X = C$ or a chain map $X = (C \longrightarrow D)$. If $X = C$ then the torsion coeff
`HomologyPb(C, n)` This is a back-up function which might work in some instances where $Homology(C, n)$ fails. It is
`HomologyVectorSpace(X, n)` Inputs either a chain complex $X = C$ or a chain map $X = (C \longrightarrow D)$ in prime characte
`HomologyPrimePart(C, n, p)` Inputs a chain complex C in characteristic 0, a positive integer n , and a prime p . It ret
`LeibnizAlgebraHomology(A, n)` Inputs a Lie or Leibniz algebra $X = A$ (over the ring of integers \mathbb{Z} or over a field K)
`LieAlgebraHomology(A, n)` Inputs a Lie algebra A (over the integers or a field) and a positive integer n . It returns th
`PrimePartDerivedFunctor(G, R, F, n)` Inputs a finite group G , a positive integer n , at least $n + 1$ terms of a ZP -res
`RankHomologyPGroup(G, n)` `RankHomologyPGroup(R, n)` `RankHomologyPGroup(G, n, "empirical")` Inputs a
`RankPrimeHomology(G, n)` Inputs a (smallish) p -group G together with a positive integer n . It returns a function dim

Chapter 7

Poincare series

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`EfficientNormalSubgroups(G)`
`EfficientNormalSubgroups(G, k)` Inputs a prime-power group G and, optionally, a positive integer k . The default is $k = 1$.
`ExpansionOfRationalFunction(f, n)` Inputs a positive integer n and a rational function $f(x) = p(x)/q(x)$ where $p(x)$ and $q(x)$ are polynomials.
`PoincareSeries(G, n)` `PoincareSeries(R, n)`
`PoincareSeries(L, n)`
`PoincareSeries(G)` Inputs a finite p -group G and a positive integer n . It returns a quotient of polynomials $f(x) = P(x)/Q(x)$ whose denominator $Q(x)$ is a product of cyclotomic polynomials.
`PoincareSeriesPrimePart(G, p, n)` Inputs a finite group G , a prime p , and a positive integer n . It returns a quotient of polynomials $f(x) = P(x)/Q(x)$ whose denominator $Q(x)$ is a product of cyclotomic polynomials.
`PoincareSeriesLHS(G)` Inputs a finite 2-group G and returns a quotient of polynomials $f(x) = P(x)/Q(x)$ whose denominator $Q(x)$ is a product of cyclotomic polynomials.
`Prank(G)` Inputs a p -group G and returns the rank of the largest elementary abelian subgroup.

Chapter 8

Cohomology ring structure

`IntegralCupProduct (R, u, v, p, q)`
`IntegralCupProduct (R, u, v, p, q, P, Q, N)` (Various functions used to construct the cup product are also available)
`IntegralRingGenerators (R, n)` Inputs at least $n + 1$ terms of a ZG -resolution and integer $n > 0$. It returns a minimal set of generators.
`ModPCohomologyGenerators (G, n)`
`ModPCohomologyGenerators (R)` Inputs either a p -group G and positive integer n , or else n terms of a minimal ZG -resolution.
`ModPCohomologyRing (G, n)`
`ModPCohomologyRing (G, n, level)`
`ModPCohomologyRing (R)`
`ModPCohomologyRing (R, level)` Inputs either a p -group G and positive integer n , or else n terms of a minimal ZG -resolution.
`ModPRingGenerators (A)` Inputs a mod p cohomology ring A (created using the preceding function). It returns a minimal set of generators.
`Mod2CohomologyRingPresentation (G)`
`Mod2CohomologyRingPresentation (G, n)`
`Mod2CohomologyRingPresentation (A)`
`Mod2CohomologyRingPresentation (R)` When applied to a finite 2-group G this function returns a presentation of the mod 2 cohomology ring.

Chapter 9

Cohomology rings of p -groups (mainly $p = 2$)

The functions on this page were written by Paul Smith. (They are included in HAP but they are also independently included in Paul Smiths HAPprime package.)

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<code>Mod2CohomologyRingPresentation(G)</code>	
<code>Mod2CohomologyRingPresentation(G,n)</code>	
<code>Mod2CohomologyRingPresentation(A)</code>	
<code>Mod2CohomologyRingPresentation(R)</code>	When applied to a finite 2-group G this function returns a presentation
<code>PoincareSeriesLHS(G)</code>	Inputs a finite 2-group G and returns a quotient of polynomials $f(x) = P(x)/Q(x)$ whose

Chapter 10

Commutator and nonabelian tensor computations

`BaerInvariant(G, c)` Inputs a nilpotent group G and integer $c > 0$. It returns the Baer invariant $M^{(c)}(G)$ defined as $M^{(c)}(G) = \gamma_c(G) / \gamma_{c+1}(G)$.

`Coclass(G)` Inputs a group G of prime-power order p^n and nilpotency class c say. It returns the integer $r = n - c$.

`EpiCentre(G, N)` Inputs a finite group G and normal subgroup N and returns a subgroup $Z^*(G, N)$ of the centre of N .

`NonabelianExteriorProduct(G, N)` Inputs a finite group G and normal subgroup N . It returns a record E with the following fields:

- `NonabelianSymmetricKernel(G)`
- `NonabelianSymmetricKernel(G, m)` Inputs a finite or nilpotent infinite group G and returns the abelian invariant $\gamma_m(G)$.
- `NonabelianSymmetricSquare(G)`
- `NonabelianSymmetricSquare(G, m)` Inputs a finite or nilpotent infinite group G and returns a record T with the following fields:

`NonabelianTensorProduct(G, N)` Inputs a finite group G and normal subgroup N . It returns a record E with the following fields:

- `NonabelianTensorSquare(G)`
- `NonabelianTensorSquare(G, m)` Inputs a finite or nilpotent infinite group G and returns a record T with the following fields:

`RelativeSchurMultiplier(G, N)` Inputs a finite group G and normal subgroup N . It returns the homology group $H_2(G/N, \mathbb{Z})$.

`TensorCentre(G)` Inputs a group G and returns the largest central subgroup N such that the induced homomorphism $G/N \rightarrow G/N$ is trivial.

`ThirdHomotopyGroupOfSuspensionB(G)`

`ThirdHomotopyGroupOfSuspensionB(G, m)` Inputs a finite or nilpotent infinite group G and returns the abelian invariant $\gamma_m(G)$.

`UpperEpicentralSeries(G, c)` Inputs a nilpotent group G and an integer c . It returns the c -th term of the upper epicentral series of G .

Chapter 11

Lie commutators and nonabelian Lie tensors

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Functions on this page are joint work with Hamid Mohammadzadeh, and implemented by him.

`LieCoveringHomomorphism(L)` Inputs a finite dimensional Lie algebra L over a field, and returns a surjective Lie

`LeibnizQuasiCoveringHomomorphism(L)` Inputs a finite dimensional Lie algebra L over a field, and returns a sur

`LieEpiCentre(L)` Inputs a finite dimensional Lie algebra L over a field, and returns an ideal $Z^*(L)$ of the centre of

`LieExteriorSquare(L)` Inputs a finite dimensional Lie algebra L over a field. It returns a record E with the follow

`LieTensorSquare(L)` Inputs a finite dimensional Lie algebra L over a field and returns a record T with the follow

`LieTensorCentre(L)` Inputs a finite dimensional Lie algebra L over a field and returns the largest ideal N such th

Chapter 12

Generators and relators of groups

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`CayleyGraphDisplay(G,X)`

`CayleyGraphDisplay(G,X,"mozilla")` Inputs a finite group G together with a subset X of G . It displays the co

`IdentityAmongRelatorsDisplay(R,n)` `IdentityAmongRelatorsDisplay(R,n,"mozilla")` Inputs a free

`IsAspherical(F,R)` Inputs a free group F and a set R of words in F . It performs a test on the 2-dimensional CW

`PresentationOfResolution(R)` Inputs at least two terms of a reduced ZG -resolution R and returns a record P w

`TorsionGeneratorsAbelianGroup(G)` Inputs an abelian group G and returns a generating set $[x_1, \dots, x_n]$ where

Chapter 13

Orbit polytopes and fundamental domains

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`CoxeterComplex(D)` `CoxeterComplex(D,n)` Inputs a Coxeter diagram D of finite type. It returns a non-free $\mathbb{Z}V$ -module.

`ContractibleGcomplex("PSL(4,Z)")` Inputs one of the following strings: "SL(3,Z)", "PSL(4,Z)", "PSL(4,Z)_b".

`FundamentalDomainStandardSpaceGroup(v,G)` Inputs a crystallographic group G (represented using `AffineCrys`) and a rational vector v of length n .

`OrbitPolytope(G,v,L)` Inputs a permutation group or matrix group G of degree n and a rational vector v of length n . Returns a list of L points in the orbit polytope.

`PolytopalComplex(G,v)` Inputs a permutation group or matrix group G of degree n and a rational vector v of length n . Returns a list of points in the polytopal complex.

`PolytopalComplex(G,v,n)` Inputs a permutation group or matrix group G of degree n and a rational vector v of length n . Returns a list of points in the polytopal complex.

`PolytopalGenerators(G,v)` Inputs a permutation group or matrix group G of degree n and a rational vector v of length n . Returns a list of generators for the polytopal complex.

`VectorStabilizer(G,v)` Inputs a permutation group or matrix group G of degree n and a rational vector v of length n . Returns the stabilizer of v in G .

Chapter 14

Cocycles

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`CcGroup(A, f)` Inputs a G -module A (i.e. an abelian G -outer group) and a standard 2-cocycle $f: G \times G \rightarrow A$. It
`CocycleCondition(R, n)` Inputs a resolution R and an integer $n > 0$. It returns an integer matrix M with the follow
`StandardCocycle(R, f, n)`
`StandardCocycle(R, f, n, q)` Inputs a ZG -resolution R (with contracting homotopy), a positive integer n and an i
`Syzygy(R, g)` Inputs a ZG -resolution R (with contracting homotopy) and a list $g = [g[1], \dots, g[n]]$ of elements in G

Chapter 15

Words in free ZG -modules

`AddFreeWords(v, w)` Inputs two words v, w in a free ZG -module and returns their sum $v + w$. If the characteristic of Z is p , then $v + w$ is computed modulo p .

`AddFreeWordsModP(v, w, p)` Inputs two words v, w in a free ZG -module and the characteristic p of Z . It returns the sum $v + w$ modulo p .

`AlgebraicReduction(w)` Inputs a word w in a free ZG -module and returns a reduced version of the word in w .

`AlgebraicReduction(w, p)` Inputs a word w in a free ZG -module and returns a reduced version of the word in w modulo p .

`Multiply Word(n, w)` Inputs a word w and integer n . It returns the scalar multiple $n \cdot w$.

`Negate([i, j])` Inputs a pair $[i, j]$ of integers and returns $[-i, j]$.

`NegateWord(w)` Inputs a word w in a free ZG -module and returns the negated word $-w$.

`PrintZGword(w, elts)` Inputs a word w in a free ZG -module and a (possibly partial but sufficient) listing `elts` of the elements of G . It prints the word w in terms of the elements of G .

`TietzeReduction(S, w)` Inputs a set S of words in a free ZG -module, and a word w in the module. The function returns a reduced version of w modulo S .

Chapter 16

FpG-modules

`CompositionSeriesOfFpGModules(M)` Inputs an *FpG*-module M and returns a list of *FpG*-modules that constitute a composition series for M .

`DirectSumOfFpGModules(M, N)`

`DirectSumOfFpGModules([M[1], M[2], ..., M[k]])` Inputs two *FpG*-modules M and N with common group G and returns their direct sum.

`FpGModule(A, P)`

`FpGModule(A, G, p)` Inputs a p -group P and a matrix A whose rows have length a multiple of the order of G . It returns the *FpG*-module defined by A over P .

`FpGModuleDualBasis(M)` Inputs an *FpG*-module M . It returns a record R with two components: $R.freeModule$ is a free F -module isomorphic to M and $R.dualBasis$ is a dual basis for M .

`FpGModuleHomomorphism(M, N, A)`

`FpGModuleHomomorphismNC(M, N, A)` Inputs *FpG*-modules M and N over a common p -group G . Also inputs a list A of matrices representing the homomorphism.

`DesuspensionFpGModule(M, n)`

`DesuspensionFpGModule(R, n)` Inputs a positive integer n and an *FpG*-module M . It returns an *FpG*-module $D^n M$.

`RadicalOfFpGModule(M)` Inputs an *FpG*-module M with G a p -group, and returns the Radical of M as an *FpG*-module.

`RadicalSeriesOfFpGModule(M)` Inputs an *FpG*-module M and returns a list of *FpG*-modules that constitute the radical series of M .

`GeneratorsOfFpGModule(M)` Inputs an *FpG*-module M and returns a matrix whose rows correspond to a minimal generating set of M .

`ImageOfFpGModuleHomomorphism(f)` Inputs an *FpG*-module homomorphism $f : M \rightarrow N$ and returns its image.

`GroupAlgebraAsFpGModule(G)` Inputs a p -group G and returns its mod p group algebra as an *FpG*-module.

`IntersectionOfFpGModules(M, N)` Inputs two *FpG*-modules M, N arising as submodules in a common free module and returns their intersection.

`IsFpGModuleHomomorphismData(M, N, A)` Inputs *FpG*-modules M and N over a common p -group G . Also inputs a list A of matrices representing the homomorphism. Returns true if A represents a homomorphism.

`MaximalSubmoduleOfFpGModule(M)` Inputs an *FpG*-module M and returns one maximal *FpG*-submodule of M .

`MaximalSubmodulesOfFpGModule(M)` Inputs an *FpG*-module M and returns the list of maximal *FpG*-submodules of M .

`MultipleOfFpGModule(w, M)` Inputs an *FpG*-module M and a list $w := [g_1, \dots, g_t]$ of elements in the group $G = M$. It returns the submodule generated by w .

`ProjectedFpGModule(M, k)` Inputs an *FpG*-module M of ambient dimension $n|G|$, and an integer k between 1 and n . It returns the projection of M onto a subspace of dimension $k|G|$.

`RandomHomomorphismOfFpGModules(M, N)` Inputs two *FpG*-modules M and N over a common group G . It returns a random homomorphism from M to N .

`Rank(f)` Inputs an *FpG*-module homomorphism $f : M \rightarrow N$ and returns the dimension of the image of f as a vector space over F .

`SumOfFpGModules(M, N)` Inputs two *FpG*-modules M, N arising as submodules in a common free module $(FG)^n$ and returns their sum.

`SumOp(f, g)` Inputs two *FpG*-module homomorphisms $f, g : M \rightarrow N$ with common source and common target. It returns the sum $f + g$.

`VectorsToFpGModuleWords(M, L)` Inputs an *FpG*-module M and a list $L = [v_1, \dots, v_k]$ of vectors in M . It returns the list of words representing the vectors in L .

Chapter 17

Meataxe modules

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DesuspensionMtxModule(M) Inputs a meataxe module M over the field of p elements and returns an FpG-module I .
FpG_to_MtxModule(M) Inputs an FpG-module M and returns an isomorphic meataxe module.
GeneratorsOfMtxModule(M) Inputs a meataxe module M acting on, say, the vector space V . The function returns

Chapter 18

G-Outer Groups

`GOuterGroup(E, N)`

`GOuterGroup()` **Inputs** a group E and normal subgroup N . It returns N as a G -outer group where $G = E/N$. The fun

`GOuterGroupHomomorphismNC(A, B, phi)`

`GOuterGroupHomomorphismNC()` **Inputs** G -outer groups A and B with common acting group, and a group homomor

`GOuterHomomorphismTester(A, B, phi)` **Inputs** G -outer groups A and B with common acting group, and a group ho

`Centre(A)` **Inputs** G -outer group A and returns the group theoretic centre of `ActedGroup(A)` as a G -outer group.

`DirectProductGog(A, B)`

`DirectProductGog(Lst)` **Inputs** G -outer groups A and B with common acting group, and returns their group-theore

Chapter 19

Cat-1-groups

`AutomorphismGroupAsCatOneGroup(G)` Inputs a group G and returns the Cat-1-group C corresponding to the crossed module $(G, \text{Aut}(G))$.

`HomotopyGroup(C, n)` Inputs a cat-1-group C and an integer n . It returns the n th homotopy group of C .

`HomotopyModule(C, 2)` Inputs a cat-1-group C and an integer $n=2$. It returns the second homotopy group of C as a C -module.

`ModuleAsCatOneGroup(G, alpha, M)` Inputs a group G , an abelian group M and a homomorphism $\alpha: G \rightarrow \text{Aut}(M)$. It returns the Cat-1-group C corresponding to the crossed module (M, α) .

`MooreComplex(C)` Inputs a cat-1-group C and returns its Moore complex $[M_1 \rightarrow M_0]$ as a list whose single entry is the Moore complex.

`NormalSubgroupAsCatOneGroup(G, N)` Inputs a group G with normal subgroup N . It returns the Cat-1-group C corresponding to the crossed module $(N, \text{Inn}(G))$.

Chapter 20

Coxeter diagrams and graphs of groups

`CoxeterDiagramComponents(D)` Inputs a Coxeter diagram D and returns a list $[D_1, \dots, D_d]$ of the maximal connected components of D .

`CoxeterDiagramDegree(D, v)` Inputs a Coxeter diagram D and vertex v . It returns the degree of v (i.e. the number of edges incident to v).

`CoxeterDiagramDisplay(D)` Inputs a Coxeter diagram D and displays it as a .gif file. It uses the web browser.

`CoxeterDiagramDisplay(D, "web browser")` Inputs a Coxeter diagram D and displays it as a .gif file. It uses the web browser.

`CoxeterDiagramFpArtinGroup(D)` Inputs a Coxeter diagram D and returns the corresponding finitely presented Artin group.

`CoxeterDiagramFpCoxeterGroup(D)` Inputs a Coxeter diagram D and returns the corresponding finitely presented Coxeter group.

`CoxeterDiagramIsSpherical(D)` Inputs a Coxeter diagram D and returns "true" if the associated Coxeter group is spherical.

`CoxeterDiagramMatrix(D)` Inputs a Coxeter diagram D and returns a matrix representation of it. The matrix is $m_{ij} = \langle s_i, s_j \rangle$.

`CoxeterSubDiagram(D, V)` Inputs a Coxeter diagram D and a subset V of its vertices. It returns the full sub-diagram induced by V .

`CoxeterDiagramVertices(D)` Inputs a Coxeter diagram D and returns its set of vertices.

`EvenSubgroup(G)` Inputs a group G and returns a subgroup G^+ . The subgroup is that generated by all products xy where x, y are in G .

`GraphOfGroupsDisplay(D)` Inputs a graph of groups D and displays it as a .gif file. It uses the web browser.

`GraphOfGroupsDisplay(D, "web browser")` Inputs a graph of groups D and displays it as a .gif file. It uses the web browser.

`GraphOfGroupsTest(D)` Inputs an object D and tries to test whether it is a Graph of Groups. However, it DOES NOT work.

Chapter 21

Simplicial Complexes

`Homology(T,n)` `Homology(T)` Inputs a pure cubical complex, or cubical complex, or simplicial complex T and a non-negative integer n . Returns the n -th homology group of T .

`RipsHomology(G,n)` `RipsHomology(G,n,p)` Inputs a graph G , a non-negative integer n (and optionally a prime number p). Returns the n -th Rips homology group of G .

`Bettinnumbers(T,n)` `Bettinnumbers(T)` Inputs a pure cubical complex, or cubical complex, simplicial complex or chain complex T and a non-negative integer n . Returns the n -th Bettin number of T .

`ChainComplex(T)` Inputs a pure cubical complex, or cubical complex, or simplicial complex T and returns the (often infinite) chain complex of T .

`CechComplexOfPureCubicalComplex(T)` Inputs a d -dimensional pure cubical complex T and returns a simplicial complex $C(T)$ whose simplices are the faces of the cubical complex T .

`RipsChainComplex(S,epsilon)` `RipsChainComplex(S,epsilon,true)` Inputs an $n \times n$ symmetric matrix S with real entries and a non-negative real number ϵ . Returns the Rips chain complex of S with respect to ϵ .

`VectorsToSymmetricMatrix(M)` `VectorsToSymmetricMatrix(M,distance)` Inputs a matrix M of rational numbers and a non-negative real number ϵ . Returns the symmetric matrix S such that $S_{ij} = 1$ if $|M_{ij} - M_{ji}| \leq \epsilon$ and $S_{ij} = 0$ otherwise.

`EulerCharacteristic(T)` Inputs a pure cubical complex, or cubical complex, or simplicial complex T and returns the Euler characteristic of T .

`MaximalSimplicesToSimplicialComplex(L)` Inputs a list L whose entries are lists of vertices representing the maximal simplices of a simplicial complex. Returns the simplicial complex K whose maximal simplices are the entries of L .

`SkeletonOfSimplicialComplex(S,k)` Inputs a simplicial complex S and a positive integer k less than or equal to the dimension of S . Returns the k -skeleton of S .

`GraphOfSimplicialComplex(S)` Inputs a simplicial complex S and returns the graph of S .

`ContractibleSubcomplexOfSimplicialComplex(S)` Inputs a simplicial complex S and returns a (probably maximal) contractible subcomplex of S .

`PathComponentsOfSimplicialComplex(S,n)` Inputs a simplicial complex S and a nonnegative integer n . If $n = 0$ returns the number of path components of S . If $n > 0$ returns the number of path components of the n -skeleton of S .

`QuillenComplex(G)` Inputs a finite group G and returns, as a simplicial complex, the order complex of the poset of proper subgroups of G .

`SymmetricMatrixToIncidenceMatrix(S,t)` Inputs a symmetric integer matrix S and an integer t . It returns the t -th incidence matrix of S .

`IncidenceMatrixToGraph(M)` Inputs a symmetric 0/1 matrix M . It returns the graph with one vertex for each row of M and edges between vertices i and j if $M_{ij} = 1$.

`PathComponentsOfGraph(G,n)` Inputs a graph G and a nonnegative integer n . If $n = 0$ the number of path components of G . If $n > 0$ returns the number of path components of the n -skeleton of G .

`ContractGraph(G)` Inputs a graph G and tries to remove vertices and edges to produce a smaller graph G' such that G' is homotopy equivalent to G .

`GraphDisplay(G)` This function uses `GraphViz` software to display a graph G .

`SimplicialMap(K,L,f)` `SimplicialMapNC(K,L,f)` Inputs simplicial complexes K, L and a function $f: K \rightarrow L$. Returns a simplicial map $f: K \rightarrow L$ such that $f(v) = f(v)$ for all vertices v of K .

`ChainMapOfSimplicialMap(f)` Inputs a simplicial map $f: K \rightarrow L$ and returns the corresponding chain map $C_*(f): C_*(K) \rightarrow C_*(L)$.

`SimplicialNerveOfGraph(G,d)` Inputs a graph G and returns a d -dimensional simplicial complex K whose 1-skeleton is G .

Chapter 22

Cubical Complexes

`ArrayToPureCubicalComplex(A, n)` Inputs an integer array A of dimension d and an integer n . It returns a d -dimensional pure cubical complex.

`PureCubicalComplex(A, n)` Inputs a binary array A of dimension d . It returns the corresponding d -dimensional pure cubical complex.

`PureCubicalComplexIntersection(S, T)` Inputs two pure cubical complexes with common dimension and array size. It returns their intersection.

`PureCubicalComplexUnion(S, T)` Inputs two pure cubical complexes with common dimension and array size. It returns their union.

`PureCubicalComplexDifference(S, T)` Inputs two pure cubical complexes with common dimension and array size. It returns their difference.

`ReadImageAsPureCubicalComplex("file.png", n)` Reads an image file ("file.png", "file.eps", "file.bmp" etc) and returns a pure cubical complex.

`ReadImageSequenceAsPureCubicalComplex("directory", n)` Reads the name of a directory containing a sequence of image files and returns a pure cubical complex.

`Size(T)` This returns the number of non-zero entries in the binary array of the pure cubical complex T .

`WritePureCubicalComplexAsImage(T, "filename", "ext")` Inputs a 2-dimensional pure cubical complex T , and writes it to a file.

`ViewPureCubicalComplex(T)` `ViewPureCubicalComplex(T, "mozilla")` Inputs a 2-dimensional pure cubical complex T , and returns a view of it.

`Homology(T, n)` `Homology(T)` Inputs a pure cubical complex, or cubical complex, or simplicial complex T and an integer n . It returns the n -th homology group.

`Bettinnumbers(T, n)` `Bettinnumbers(T)` Inputs a pure cubical complex, or cubical complex, simplicial complex T and an integer n . It returns the n -th Bettin number.

`DirectProductOfPureCubicalComplexes(M, N)` Inputs two cubical complexes M, N and returns their direct product.

`EulerCharacteristic(T)` Inputs a pure cubical complex, or cubical complex, or simplicial complex T and returns its Euler characteristic.

`PathComponentOfPureCubicalComplex(T, n)` Inputs a pure cubical complex T and an integer n in the range 1, ..., $\dim(T)$. It returns the n -th path component.

`ChainComplex(T)` Inputs a pure cubical complex, or cubical complex, or simplicial complex T and returns the (often infinite) chain complex.

`ChainComplexOfPair(T, S)` Inputs a pure cubical complex or cubical complex T and contractible subcomplex S . It returns the chain complex of the pair.

`ChainInclusionOfPureCubicalPair(S, T)` Inputs a pure cubical complex T and subcomplex S . It returns the chain inclusion.

`ChainMapOfPureCubicalPairs(M, S, N, T)` Inputs a pure cubical complex N and subcomplexes M, T and S in T . It returns a chain map.

`ContractPureCubicalComplex(T)` Inputs a pure cubical complex T of dimension d and removes d -dimensional cells.

`ContractedComplex(T)` Inputs a pure cubical complex T and returns a structural copy of the complex obtained from T by contracting.

`ContractibleSubcomplexOfPureCubicalComplex(T)` Inputs a pure cubical complex T and returns a maximal contractible subcomplex.

`AlmostContractibleSubcomplexOfPureCubicalComplex(T)` Inputs a pure cubical complex T and returns a (not necessarily contractible) subcomplex.

`HomotopyEquivalentMaximalPureCubicalSubcomplex(T, S)` Inputs a pure cubical complex T together with a subcomplex S .

`HomotopyEquivalentMinimalPureCubicalSubcomplex(T, S)` Inputs a pure cubical complex T together with a subcomplex S .

`BoundaryOfPureCubicalComplex(T)` Inputs a pure cubical complex T and returns its boundary as a pure cubical complex.

`SingularitiesOfPureCubicalComplex(T, radius, tolerance)` Inputs a pure cubical complex T together with a radius and a tolerance.

`ThickenedPureCubicalComplex(T)` Inputs a pure cubical complex T and returns a pure cubical complex S . If e is an edge of T , then S contains a neighborhood of e .

`ComplementOfPureCubicalComplex(T)` Inputs a pure cubical complex T and returns a pure cubical complex S . A is a cell of S if and only if it is not a cell of T .

Chapter 23

Commutative diagrams and abstract categories

COMMUTATIVE DIAGRAMS

`HomomorphismChainToCommutativeDiagram(H)` Inputs a list $H = [h_1, h_2, \dots, h_n]$ of mappings such that the composition of consecutive mappings is the identity mapping.
`NormalSeriesToQuotientDiagram(L)` `NormalSeriesToQuotientDiagram(L, M)` Inputs an increasing (or decreasing) normal series L of a group G and a normal subgroup M of G .
`NerveOfCommutativeDiagram(D)` Inputs a commutative diagram D and returns the commutative diagram ND corresponding to D .
`GroupHomologyOfCommutativeDiagram(D, n)` `GroupHomologyOfCommutativeDiagram(D, n, prime)` `GroupHomologyOfCommutativeDiagramOfPGroups(D, n)` Inputs a commutative diagram D of finite p -groups and returns the n th homology group of D .

ABSTRACT CATEGORIES

`CategoricalEnrichment(X, Name)` Inputs a structure X such as a group or group homomorphism, together with a name $Name$ for the category, and returns the categorical enrichment of X .
`IdentityArrow(X)` Inputs an object X in some category, and returns the identity arrow on the object X .
`InitialArrow(X)` Inputs an object X in some category, and returns the arrow from the initial object in the category to the object X .
`TerminalArrow(X)` Inputs an object X in some category, and returns the arrow from X to the terminal object in the category.
`HasInitialObject(Name)` Inputs the name of a category and returns true or false depending on whether the category has an initial object.
`HasTerminalObject(Name)` Inputs the name of a category and returns true or false depending on whether the category has a terminal object.
`Source(f)` Inputs an arrow f in some category, and returns its source.
`Target(f)` Inputs an arrow f in some category, and returns its target.
`CategoryName(X)` Inputs an object or arrow X in some category, and returns the name of the category.
`"*", "=", "+", "-"` Composition of suitable arrows f, g is given by $f * g$ when the source of f equals the target of g .
`Object(X)` Inputs an object X in some category, and returns the GAP structure Y such that $X = \text{CategoricalEnrichment}(Y)$.
`Mapping(X)` Inputs an arrow f in some category, and returns the GAP structure Y such that $f = \text{CategoricalEnrichment}(Y)$.
`IsCategoryObject(X)` Inputs X and returns true if X is an object in some category.
`IsCategoryArrow(X)` Inputs X and returns true if X is an arrow in some category.

Chapter 24

Arrays and Pseudo lists

`Array(A, f)` Inputs an array A and a function f . It returns the the array obtained by applying f to each entry of A (f applied to each entry of A).

`ArrayDimension(A)` Inputs an array A and returns its dimension.

`ArrayDimensions(A)` Inputs an array A and returns its dimensions.

`ArraySum(A)` Inputs an array A and returns the sum of its entries.

`ArrayValue(A, x)` Inputs an array A and a coordinate vector x . It returns the value of the entry in A with coordinate x .

`ArrayValueFunctions(d)` Inputs a positive integer d and returns an efficient version of the function `ArrayValue` for arrays of dimension d .

`ArrayAssign(A, x, n)` Inputs an array A and a coordinate vector x and an integer n . It sets the entry of A with coordinate x to n .

`ArrayAssign(d)` Inputs a positive integer d and returns an efficient version of the function `ArrayAssign` for arrays of dimension d .

`ArrayIterate(d)` Inputs a positive integer d and returns a function `ArrayIt(Dimensions, f)`. This function inputs a coordinate vector x and returns f applied to the entry of the array with coordinate x .

`BinaryArrayToTextFile(file, A)` Inputs a string containing the address of a file, and an array A of 0s and 1s. The file is created if it does not exist. The file contains the binary representation of the array A .

`FrameArray(A)` Inputs an array A and returns the array obtained by appending a 0 to the beginning and end of each "row" of A .

`UnframeArray(A)` Inputs an array A and returns the array obtained by removing the first and last entry in each "row" of A .

`Add(L, x)` Let L be a pseudo list of length n , and x an object compatible with the entries in L . If x is not in L then then x is added to the end of L .

`Append(L, K)` Let L be a pseudo list and K a list whose objects are compatible with those in L . This operation appends the list K to the end of L .

`ListToPseudoList(L)` Inputs a list L and returns the pseudo list representation of L .

Chapter 25

Parallel Computation - Core Functions

```
ChildProcess()  
ChildProcess("computer.ac.wales")  
ChildProcess(["-m", "100000M", "-T"])  
ChildProcess("computer.ac.wales", ["-m", "100000M", "-T"]) This starts a GAP session as a child process
```

```
- open a shell on thishost  
- cd .ssh  
- ls  
-> if id_dsa, id_rsa etc exists, skip the next two steps!  
- ssh-keygen -t rsa  
- ssh-keygen -t dsa  
- scp *.pub user@remotehost:~/  
- ssh remotehost -l user  
- cat id_rsa.pub >> .ssh/authorized_keys  
- cat id_dsa.pub >> .ssh/authorized_keys  
- rm id_rsa.pub id_dsa.pub  
- exit
```

You should now be able to connect from "thishost" to "remotehost" without a password prompt.)

`ChildClose(s)` This closes the stream `s` to a child GAP process.

`ChildCommand("cmd"; , s)` This runs a GAP command "cmd;" on the child process accessed by the stream `s`. Here

`NextAvailableChild(L)` Inputs a list `L` of child processes and returns a child in `L` which is ready for computation

`IsAvailableChild(s)` Inputs a child process `s` and returns true if `s` is currently available for computations, and false otherwise.

`ChildPut(A, "B", s)` This copies a GAP object `A` on the parent process to an object `B` on the child process `s`. (The

`ChildGet("A", s)` This function copies a GAP object `A` on the child process `s` and returns it on the parent process

`HAPPrintTo("file", R)` Inputs a name "file" of a new text file and a HAP object `R`. It writes the object `R` to "file".

`HAPRead("file", R)` Inputs a name "file" containing a HAP object `R` and returns the object. Currently this is only in

Chapter 26

Parallel Computation - Extra Functions

`ChildFunction("function(arg);", s)` This runs the GAP function "function(arg);" on a child process accessed by `s`.
`ChildRead(s)` This returns, as a string, the output of the last application of `ChildFunction("function(arg);", s)`.
`ChildReadEval(s)` This returns, as an evaluated string, the output of the last application of `ChildFunction("function(arg);", s)`.
`ParallelList(I, fn, L)` Inputs a list I , a function fn such that $fn(x)$ is defined for all x in I , and a list of children L .

Chapter 27

Some functions for accessing basic data

`BoundaryMap(C)` Inputs a resolution, chain complex or cochain complex C and returns the function $C!.boundary$.
`BoundaryMatrix(C,n)` Inputs a chain or cochain complex C and integer $n>0$. It returns the n -th boundary map of C .
`Dimension(C)` Inputs a resolution, chain complex or cochain complex C and returns the function $C!.dimension$.
`Dimension(M)` Inputs a resolution, chain complex or cochain complex C and returns the function $C!.dimension$.
`EvaluateProperty(X,"name")` Inputs a component object X (such as a ZG -resolution or chain map) and a string $name$. It returns the value of the property $name$ of X .
`GroupOfResolution(R)` Inputs a ZG -resolution R and returns the group G .
`Length(R)` Inputs a resolution R and returns its length (i.e. the number of terms of R that HAP has computed).
`Map(f)` Inputs a chain map, or cochain map or equivariant chain map f and returns the mapping function (as opposed to the mapping object).
`Source(f)` Inputs a chain map, or cochain map, or equivariant chain map, or FpG -module homomorphism f and returns the source object of f .
`Target(f)` Inputs a chain map, or cochain map, or equivariant chain map, or FpG -module homomorphism f and returns the target object of f .

Chapter 28

Miscellaneous

`BigStepLCS(G, n)` Inputs a group G and a positive integer n . It returns a subseries $G = L_1 > L_2 > \dots L_k = 1$ of the L .

`Classify(L, Inv)` Inputs a list of objects L and a function Inv which computes an invariant of each object. It returns a list of invariants.

`RefineClassification(C, Inv)` Inputs a list $C := Classify(L, OldInv)$ and returns a refined classification according to Inv .

`Compose(f, g)` Inputs two FpG -module homomorphisms $f : M \longrightarrow N$ and $g : L \longrightarrow M$ with $Source(f) = Target(g)$. It returns $f \circ g$.

`HAPcopyright()` This function provides details of HAP'S GNU public copyright licence.

`IsLieAlgebraHomomorphism(f)` Inputs an object f and returns true if f is a homomorphism $f : A \longrightarrow B$ of Lie algebras.

`IsSuperperfect(G)` Inputs a group G and returns "true" if both the first and second integral homology of G is trivial.

`MakeHAPManual()` This function creates the manual for HAP from an XML file.

`PermToMatrixGroup(G, n)` Inputs a permutation group G and its degree n . Returns a bijective homomorphism $f : G \longrightarrow GL_n(\mathbb{Z})$.

`SolutionsMatDestructive(M, B)` Inputs an $m \times n$ matrix M and a $k \times n$ matrix B over a field. It returns a $k \times m$ matrix C such that $CM = B$.

`LinearHomomorphismsPersistenceMat(L)` Inputs a composable sequence L of vector space homomorphisms. It returns a persistence matrix.

`NormalSeriesToQuotientHomomorphisms(L)` Inputs an (increasing or decreasing) chain L of normal subgroups. It returns a sequence of quotient homomorphisms.

`TestHap()` This runs a representative sample of HAP functions and checks to see that they produce the correct output.

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