smallantimagmas

A library of antiassociative magmas of small order

0.5.1

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smallantimagmas automatic generated documentation

1.1 smallantimagmas automatic generated documentation of properties

1.1.1 IsAntiassociative (for IsMagma)

▷ IsAntiassociative(*M*)   (property)
   Returns: `true` or `false`
   identifies whether magma *M* is antiassociative.

```
━━━━━━━━━━━━━━━━━━━━━━━━━ Example ━━━━━━━━━━━━━━━━━━━━━━━━━
gap> IsAntiassociative(OneSmallGroup(16));
false
gap> IsAntiassociative(OneSmallAntimagma(2));
true
gap> IsAntiassociative(OneSmallAntimagma(3));
true
```

1.1.2 IsLeftCyclic (for IsMagma)

▷ IsLeftCyclic(*M*)   (property)
   Returns: `true` or `false`
   if magma is left cyclic *m*.

1.1.3 IsRightCyclic (for IsMagma)

▷ IsRightCyclic(*M*)   (property)
   Returns: `true` or `false`
   if magma is left cyclic *m*.

1.1.4 IsLeftDistributive (for IsMagma)

▷ IsLeftDistributive(*M*)   (property)
   Returns: `true` or `false`
   if magma is left distributive *m*.

```
 ───────────────── Example ─────────────────
 gap> List(AllSmallAntimagmas(3), M -> IsLeftDistributive(M) );
 [ true, false, false, false, false, false, false, false, false, true ]
```

1.1.5 IsRightDistributive (for IsMagma)

▷ IsRightDistributive(*M*)   (property)

   Returns: true or false

   if magma is right distributive *m*.

```
 ───────────────── Example ─────────────────
 gap> List(AllSmallAntimagmas(3), M -> IsRightDistributive(M) );
 [ false, false, false, false, true, false, false, false, true, false ]
```

1.1.6 IsLeftCancellative (for IsMagma)

▷ IsLeftCancellative(*M*)   (property)

   Returns: true or false

   if magma is left cancellative *m*.

```
 ───────────────── Example ─────────────────
 gap> M := SmallAntimagma(2, 1);
 <magma with 2 generators>
 gap> Display( MultiplicationTable(M) );
 [ [  2,  1 ],
   [  2,  1 ] ]
 gap> IsRightCancellative(M);
 false
 gap> IsLeftCancellative(M);
 true
 gap> List(AllSmallAntimagmas(2), M -> IsLeftCancellative(M));
 [ true, false ]
```

1.1.7 IsRightCancellative (for IsMagma)

▷ IsRightCancellative(*M*)   (property)

   Returns: true or false

   if magma is right cancellative *m*.

```
 ───────────────── Example ─────────────────
 gap> List(AllSmallAntimagmas(2), M -> IsRightCancellative(M));
 [ false, true ]
```

1.1.8 IsCancellative (for IsMagma)

▷ IsCancellative(*M*)   (property)

   Returns: true or false

   if magma is cancellative *m*.

```
 ───────────────── Example ─────────────────
 gap> List(AllSmallAntimagmas(2), M -> IsCancellative(M));
 [ false, false ]
```

1.1.9 IsLeftFPFInducted (for IsMagma)

▷ IsLeftFPFInducted(*M*)   (property)

    Returns: `true` or `false`

    is a left-hand sided fixed-point free inducted `m`.

———————————————— Example ————————————————
```
gap> Display( MultiplicationTable( SmallAntimagma(2, 2) ) );
[ [  2,  2 ],
  [  1,  1 ] ]
gap> IsLeftFPFInducted( SmallAntimagma(2, 2) );
true
```

1.1.10 IsRightFPFInducted (for IsMagma)

▷ IsRightFPFInducted(*M*)   (property)

    Returns: `true` or `false`

    is a right-hand sided fixed-point free inducted `m`.

———————————————— Example ————————————————
```
gap> Display( MultiplicationTable( SmallAntimagma(2, 1) ) );
[ [  2,  1 ],
  [  2,  1 ] ]
gap> IsRightFPFInducted( SmallAntimagma(2, 1) );
true
```

1.1.11 IsLeftDerangementInducted (for IsMagma)

▷ IsLeftDerangementInducted(*M*)   (property)

    Returns: `true` or `false`

    is a left-hand sided derangment inducted `m`.

———————————————— Example ————————————————
```
gap> M := SmallAntimagma(2, 2);
<magma with 2 generators>
gap> IsLeftFPFInducted(M);
true
gap> IsRightFPFInducted(M);
false
gap> IsRightDerangementInducted(M);
false
```

1.1.12 IsRightDerangementInducted (for IsMagma)

▷ IsRightDerangementInducted(*M*)   (property)

    Returns: `true` or `false`

    is a right-hand sided derangment inducted `m`.

———————————————— Example ————————————————
```
gap> M := SmallAntimagma(2, 1);
<magma with 2 generators>
gap> IsLeftFPFInducted(M);
false
```

```
gap> IsRightFPFInducted(M);
true
gap> IsRightDerangementInducted(M);
true
```

1.1.13 IsLeftAlternative (for IsMagma)

▷ IsLeftAlternative(*M*)   (property)
   Returns: `true` or `false`
   is a left-alternatve magma *M*.

———————————————————————— Example ————————————————————————

1.1.14 IsRightAlternative (for IsMagma)

▷ IsRightAlternative(*M*)   (property)
   Returns: `true` or `false`
   is a right-alternatve magma *M*.

———————————————————————— Example ————————————————————————

1.2 smallantimagmas automatic generated documentation of attributes

1.2.1 AssociativityIndex (for IsMagma)

▷ AssociativityIndex(*M*)   (attribute)

   identifies associativity index of *M*.

———————————————————————— Example ————————————————————————
```
gap> OneSmallAntimagma(2);
<magma with 2 generators>
gap> AssociativityIndex(OneSmallAntimagma(2));
0
gap> OneSmallGroup(4);
<pc group of size 4 with 2 generators>
gap> AssociativityIndex(OneSmallGroup(4));
64
gap> AssociativityIndex(OneSmallGroup(4)) = 4 ^ 3;
true
```

1.2.2 DiagonalOfMultiplicationTable (for IsMagma)

▷ DiagonalOfMultiplicationTable(*M*)   (attribute)

   computes diaognal of multiplication table of *M*.

———————————————————————— Example ————————————————————————
```
gap> List(AllSmallAntimagmas(3), M -> DiagonalOfMultiplicationTable((M)));
[ [ 2, 1, 1 ], [ 2, 1, 1 ],
  [ 2, 3, 2 ], [ 2, 1, 1 ],
```

```
    [ 2, 1, 1 ], [ 2, 1, 2 ],
    [ 2, 3, 2 ], [ 2, 1, 2 ],
    [ 2, 3, 1 ], [ 2, 3, 1 ]
  ]
```

1.2.3   CommutativityIndex (for IsMagma)

▷ CommutativityIndex(*M*)   (attribute)

   identifies commutativity index of *M*.

———————————————————————— Example ————————————————————————

1.2.4   AnticommutativityIndex (for IsMagma)

▷ AnticommutativityIndex(*M*)   (attribute)

   calculates anticommutativity index of *M*.

———————————————————————— Example ————————————————————————

1.2.5   SquaresIndex (for IsMagma)

▷ SquaresIndex(*M*)   (attribute)

   computes squares index of *M* so the order of $\left\{m^2 | m \in M\right\}$.

———————————————————————— Example ————————————————————————
```
gap> List(AllSmallAntimagmas(2), M -> List(M, m -> m * m) );
[ [ m2, m1 ], [ m2, m1 ] ]
gap> List(AllSmallAntimagmas(2), M -> SquaresIndex(M ));
[ 2, 2 ]
gap> List(AllSmallAntimagmas(3), M -> SquaresIndex(M ));
[ 2, 2, 2, 2, 2, 2, 2, 2, 3, 3 ]
```

1.2.6   IdSmallAntimagma (for IsMagma)

▷ IdSmallAntimagma(*M*)   (attribute)

   identifies class of antiassociative magma *M*.

———————————————————————— Example ————————————————————————
```
gap> IsAntiassociative(OneSmallGroup(16));
false
gap> IsAntiassociative(OneSmallAntimagma(2));
true
gap> IsAntiassociative(OneSmallAntimagma(3));
true
```

1.2.7 LeftOrder (for IsExtLElement)

▷ LeftOrder(*[m]*) (attribute)

returns a left order of element *m*.

1.2.8 RightOrder (for IsExtRElement)

▷ RightOrder(*[m]*) (attribute)

returns a right order of element *m*.

1.2.9 LeftOrdersOfElements (for IsMagma)

▷ LeftOrdersOfElements(*[m]*) (attribute)

returns a left order of element *m*.

1.2.10 RightOrdersOfElements (for IsMagma)

▷ RightOrdersOfElements(*[m]*) (attribute)

returns a left order of element *m*.

1.3 smallantimagmas automatic generated documentation of global functions

1.3.1 AllSubmagmas

▷ AllSubmagmas(*M*) (function)

builds a collection of non-isomorphic submagmas of *M*.

```
───────────────────── Example ─────────────────────
gap> AllSmallAntimagmas(2);
[ <magma with 2 generators>, <magma with 2 generators> ]
gap> List(AllSmallAntimagmas(2), M -> AllSubmagmas(M));
[ [ <magma with 1 generator> ], [ <magma with 1 generator> ] ]
```

1.3.2 MagmaIsomorphismInvariantsMatch

▷ MagmaIsomorphismInvariantsMatch(*M*) (function)

computes isomorphism invariants of *M*.

1.3.3 IsMagmaIsomorphic

▷ IsMagmaIsomorphic(*M, N*)   (function)

identifies whether magmas *M*, *N* are isomorphic.

```
───────────────────────────── Example ─────────────────────────────
gap> M := SmallAntimagma(2, 1);
<magma with 2 generators>
gap> N := SmallAntimagma(2, 2);
<magma with 2 generators>
gap> T := MagmaByMultiplicationTable([ [2, 1], [2, 1] ]);
<magma with 2 generators>
gap> IsMagmaIsomorphic(M, M);
true
gap> IsMagmaIsomorphic(M, T);
true
gap> IsMagmaIsomorphic(M, N);
false
```

1.3.4 IsMagmaAntiisomorphic

▷ IsMagmaAntiisomorphic(*[M, N]*)   (function)

identifies whether magmas *M*, *N* are antiisomorphic.

```
───────────────────────────── Example ─────────────────────────────
gap> N := SmallAntimagma(2, 1);
<magma with 2 generators>
gap> M := SmallAntimagma(2, 1);
<magma with 2 generators>
gap> N := SmallAntimagma(2, 2);
<magma with 2 generators>
gap> IsMagmaAntiisomorphic(M, M);
false
gap> IsMagmaAntiisomorphic(M, N);
true
gap> IsMagmaAntiisomorphic(M, TransposedMagma(M));
true
```

1.3.5 TransposedMagma

▷ TransposedMagma(*[M]*)   (function)

generates transposed magma *M*.

```
───────────────────────────── Example ─────────────────────────────
gap> M := SmallAntimagma(2, 1);
<magma with 2 generators>
gap> IsMagmaAntiisomorphic(M, TransposedMagma(M));
true
gap> IsMagmaIsomorphic(M, TransposedMagma(TransposedMagma(M)));
true
```

```
gap> M := SmallAntimagma(2, 1);
<magma with 2 generators>
gap> Display(MultiplicationTable(M));
[ [  2,  1 ],
  [  2,  1 ] ]
gap> Display(MultiplicationTable(TransposedMagma(M)));
[ [  2,  2 ],
  [  1,  1 ] ]
```

1.3.6 LeftPower

▷ LeftPower(*[m, k]*)   (function)

  returns a left k-power of element *m*.

1.3.7 RightPower

▷ RightPower(*[m, k]*)   (function)

  returns a right k-power of element *m*.

1.3.8 AllSmallAntimagmas

▷ AllSmallAntimagmas(*n*)   (function)

  returns all antiassociative magmas of specified size *n* (a number)

––––––––––––––––––––––––––––––– Example –––––––––––––––––––––––––––––––
```
gap> AllSmallAntimagmas(2);
[ <magma with 2 generators>, <magma with 2 generators> ]
gap> AllSmallAntimagmas(3);
[
  <magma with 3 generators>, <magma with 3 generators>, <magma with 3 generators>,
  <magma with 3 generators>, <magma with 3 generators>, <magma with 3 generators>,
  <magma with 3 generators>, <magma with 3 generators>,
  <magma with 3 generators>, <magma with 3 generators>
]
```

1.3.9 NrSmallAntimagmas

▷ NrSmallAntimagmas(*n*)   (function)

  counts number of antiassociative magmas of specified size *n* (a number).

––––––––––––––––––––––––––––––– Example –––––––––––––––––––––––––––––––
```
gap> NrSmallAntimagmas(2);
2
gap> NrSmallAntimagmas(3);
10
gap> NrSmallAntimagmas(4);
17780
```

1.3.10 SmallAntimagma

▷ SmallAntimagma(`n, i`)   (function)

    returns antiassociative magma of id `[n, i]`.

────────────────────────── Example ──────────────────────────
```
gap> SmallAntimagma(2, 1);
<magma with 2 generators>
gap> SmallAntimagma(4, 5);
<magma with 4 generators>
```

1.3.11 OneSmallAntimagma

▷ OneSmallAntimagma(`n`)   (function)

    returns a random antiassociative magma of size `n`.

────────────────────────── Example ──────────────────────────
```
gap> OneSmallAntimagma(2);
<magma with 2 generators>

gap> OneSmallAntimagma(3);
<magma with 3 generators>
```

1.3.12 ReallyAllSmallAntimagmas

▷ ReallyAllSmallAntimagmas(`n`)   (function)

    returns really-all antiassociative magmas, isomorphic, of specified size `n` (a number)

────────────────────────── Example ──────────────────────────
```
gap> ReallyAllSmallAntimagmas(2);
[ <magma with 2 generators>, <magma with 2 generators> ]
```

1.3.13 ReallyNrSmallAntimagmas

▷ ReallyNrSmallAntimagmas(`n`)   (function)

    counts number of antiassociative magmas of specified size `n` (a number)

────────────────────────── Example ──────────────────────────
```
gap> ReallyNrSmallAntimagmas(3);
52
```

1.4 smallantimagmas automatic generated documentation of methods

1.4.1 MagmaIsomorphism (for IsMagma, IsMagma)

▷ MagmaIsomorphism(`M, N`)   (operation)

    computes an isomoprhism between magmas `M`, `N`.

```
───────────────────────── Example ─────────────────────────
gap> M := SmallAntimagma(2, 1);
<magma with 2 generators>
gap> N := MagmaByMultiplicationTable([ [2, 1], [2, 1] ]);
<magma with 2 generators>
gap> MagmaIsomorphism(M, N);
<general mapping: Domain([ m1, m2 ]) -> Domain([ m1, m2 ]) >
```

1.4.2 MagmaAntiisomorphism (for IsMagma, IsMagma)

▷ MagmaAntiisomorphism(*M, N*)   (operation)

creates an antiisomoprhism between magmas *M*, *N*.

```
───────────────────────── Example ─────────────────────────
gap> M := SmallAntimagma(2, 1);
<magma with 2 generators>
gap> N := SmallAntimagma(2, 2);
<magma with 2 generators>
gap> MagmaAntiisomorphism(M, N);
<mapping: Domain([ m1, m2 ]) -> Domain([ m1, m2 ]) >
```

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