A library of antiassociative magmas of small order

0.2.11

28 August 2024

Kamil Zabielski

Kamil Zabielski

Address: Department of Theoretical Computer Science Białystok University of Technology Wiejska 45A 15-325 Białystok Poland

Copyright

© 2024 by Kamil Zabielski

smallantimagmas package is free software; you can redistribute it and/or modify it under the terms of the GNU General Public License as published by the Free Software Foundation; either version 2 of the License, or (at your option) any later version.

Acknowledgements

We appreciate very much all past and future comments, suggestions and contributions to this package and its documentation provided by GAP users and developers.

Contents

1	smallantimagmas automatic generated documentation		4
	1.1	smallantimagmas automatic generated documentation of properties	4
	1.2	smallantimagmas automatic generated documentation of attributes	5
	1.3	smallantimagmas automatic generated documentation of global functions	7
	1.4	smallantimagmas automatic generated documentation of methods	11
Ind	dex		12

Chapter 1

smallantimagmas automatic generated documentation

1.1 smallantimagmas automatic generated documentation of properties

1.1.1 IsAntiassociative (for IsMagma)

```
gap> IsAntiassociative(OneSmallGroup(16));
false
gap> IsAntiassociative(OneSmallAntimagma(2));
true
gap> IsAntiassociative(OneSmallAntimagma(3));
true
```

1.1.2 IsLeftCyclic (for IsMagma)

1.1.3 IsRightCyclic (for IsMagma)

1.1.4 IsLeftCancellative (for IsMagma)

```
gap> M := SmallAntimagma(2, 1);
<magma with 2 generators>
gap> Display( MultiplicationTable(M) );
[ [ 2, 1 ],
      [ 2, 1 ] ]
gap> IsRightCancellative(M);
false
gap> IsLeftCancellative(M);
true
gap> List(AllSmallAntimagmas(2), M -> IsLeftCancellative(M));
[ true, false ]
```

1.1.5 IsRightCancellative (for IsMagma)

1.1.6 IsCancellative (for IsMagma)

1.2 smallantimagmas automatic generated documentation of attributes

1.2.1 AssociativityIndex (for IsMagma)

```
\triangleright AssociativityIndex(M) (attribute)
```

identifies associativity index of M.

```
gap> OneSmallAntimagma(2);
<magma with 2 generators>
gap> AssociativityIndex(OneSmallAntimagma(2));
0
gap> OneSmallGroup(4);
<pc group of size 4 with 2 generators>
gap> AssociativityIndex(OneSmallGroup(4));
64
gap> AssociativityIndex(OneSmallGroup(4)) = 4 ^ 3;
true
```

1.2.2 DiagonalOfMultiplicationTable (for IsMagma)

▷ DiagonalOfMultiplicationTable(M)

(attribute)

computes diaognal of multiplication table of M.

```
Example

gap> List(AllSmallAntimagmas(3), M -> DiagonalOfMultiplicationTable((M)));

[ [ 2, 1, 1 ], [ 2, 1, 1 ],
        [ 2, 3, 2 ], [ 2, 1, 1 ],
        [ 2, 1, 1 ], [ 2, 1, 2 ],
        [ 2, 3, 2 ], [ 2, 1, 2 ],
        [ 2, 3, 1 ], [ 2, 3, 1 ]

]
```

1.2.3 CommutativityIndex (for IsMagma)

▷ CommutativityIndex(M)

(attribute)

identifies commutativity index of M.

Example _____

1.2.4 AnticommutativityIndex (for IsMagma)

▷ AnticommutativityIndex(M)

(attribute)

calculates anticommutativity index of M.

_____ Example __

1.2.5 SquaresIndex (for IsMagma)

▷ SquaresIndex(M)

(attribute)

computes squares index of M so the order of $\{m^2 | m \in M\}$.

```
Example

gap> List(AllSmallAntimagmas(2), M -> List(M, m -> m * m));

[ [ m2, m1 ], [ m2, m1 ] ]

gap> List(AllSmallAntimagmas(2), M -> SquaresIndex(M));

[ 2, 2 ]

gap> List(AllSmallAntimagmas(3), M -> SquaresIndex(M));

[ 2, 2, 2, 2, 2, 2, 2, 3, 3 ]
```

1.2.6 IdSmallAntimagma (for IsMagma)

▷ IdSmallAntimagma(M)

(attribute)

identifies class of antiassociative magma M.

```
gap> IsAntiassociative(OneSmallGroup(16));
false
gap> IsAntiassociative(OneSmallAntimagma(2));
true
gap> IsAntiassociative(OneSmallAntimagma(3));
true
```

1.2.7 LeftOrder (for IsExtLElement)

```
    LeftOrder([m])
    (attribute)
```

returns a left order of element m.

1.2.8 RightOrder (for IsExtRElement)

```
▷ RightOrder([m])

(attribute)
```

returns a right order of element m.

1.2.9 LeftOrdersOfElements (for IsMagma)

```
    LeftOrdersOfElements([m])
    (attribute)
```

returns a left order of element m.

1.2.10 RightOrdersOfElements (for IsMagma)

```
    RightOrdersOfElements([m])
    (attribute)
```

returns a left order of element m.

1.3 smallantimagmas automatic generated documentation of global functions

1.3.1 AllSubmagmas

```
▶ AllSubmagmas (M) (function)
```

builds a collection of non-isomorphic submagmas of M.

```
gap> AllSmallAntimagmas(2);

[ <magma with 2 generators>, <magma with 2 generators> ]

gap> List(AllSmallAntimagmas(2), M -> AllSubmagmas(M));

[ [ <magma with 1 generator> ], [ <magma with 1 generator> ] ]
```

(function)

1.3.2 MagmaIsomorphismInvariantsMatch

▶ MagmaIsomorphismInvariantsMatch(M) (function)
computes isomorphism invariants of M.

1.3.3 IsMagmaIsomorphic

▷ IsMagmaIsomorphic(M, N) (function)

identifies whether magmas M, N are isomorphic.

```
gap> M := SmallAntimagma(2, 1);
<magma with 2 generators>
gap> N := SmallAntimagma(2, 2);
<magma with 2 generators>
gap> T := MagmaByMultiplicationTable([[2, 1], [2, 1]]);
<magma with 2 generators>
gap> IsMagmaIsomorphic(M, M);
true
gap> IsMagmaIsomorphic(M, T);
true
gap> IsMagmaIsomorphic(M, N);
false
```

1.3.4 IsMagmaAntiisomorphic

▷ IsMagmaAntiisomorphic([M, N])

identifies whether magmas M, N are antiisomorphic.

```
gap> N := SmallAntimagma(2, 1);
<magma with 2 generators>
gap> M := SmallAntimagma(2, 1);
<magma with 2 generators>
gap> N := SmallAntimagma(2, 2);
<magma with 2 generators>
gap> IsMagmaAntiisomorphic(M, M);
false
gap> IsMagmaAntiisomorphic(M, N);
true
gap> IsMagmaAntiisomorphic(M, TransposedMagma(M));
true
```

1.3.5 TransposedMagma

□ TransposedMagma([M]) (function)
 generates transposed magma M.

```
gap> M := SmallAntimagma(2, 1);
<magma with 2 generators>
gap> IsMagmaAntiisomorphic(M, TransposedMagma(M));
true
gap> IsMagmaIsomorphic(M, TransposedMagma(TransposedMagma(M)));
true
gap> M := SmallAntimagma(2, 1);
<magma with 2 generators>
gap> Display(MultiplicationTable(M));
[[ 2,  1 ],
  [ 2,  1 ]]
gap> Display(MultiplicationTable(TransposedMagma(M)));
[[ 2,  2 ],
  [ 1,  1 ]]
```

1.3.6 LeftPower

```
    LeftPower([m, k])

returns a left k-power of element m.

(function)
```

1.3.7 RightPower

```
⊳ RightPower([m, k]) (function)
returns a right k-power of element m.
```

1.3.8 AllSmallAntimagmas

```
▷ AllSmallAntimagmas(n) (function)
```

returns all antiassociative magmas of specified size n (a number)

1.3.9 NrSmallAntimagmas

```
▷ NrSmallAntimagmas(n) (function)
```

counts number of antiassociative magmas of specified size n (a number).

```
gap> NrSmallAntimagmas(2);
2
gap> NrSmallAntimagmas(3);
10
gap> NrSmallAntimagmas(4);
17780
```

1.3.10 SmallAntimagma

```
\triangleright SmallAntimagma(n, i)
```

returns antiassociative magma of id [n, i].

```
gap> SmallAntimagma(2, 1);

<magma with 2 generators>
gap> SmallAntimagma(4, 5);

<magma with 4 generators>
```

1.3.11 OneSmallAntimagma

▷ OneSmallAntimagma(n)

(function)

(function)

returns a random antiassociative magma of size n.

```
gap> OneSmallAntimagma(2);
<magma with 2 generators>

gap> OneSmallAntimagma(3);
<magma with 3 generators>
```

1.3.12 ReallyAllSmallAntimagmas

▷ ReallyAllSmallAntimagmas(n)

(function)

returns really-all antiassociative magmas, isomorphic, of specified size n (a number)

```
gap> ReallyAllSmallAntimagmas(2);
[ <magma with 2 generators>, <magma with 2 generators>]
```

1.3.13 ReallyNrSmallAntimagmas

(function)

counts number of antiassociative magmas of specified size n (a number)

```
gap> ReallyNrSmallAntimagmas(3);
52
```

1.3.14 AntimagmaGeneratorPossibleDiagonals

▷ AntimagmaGeneratorPossibleDiagonals(n)

(function)

returns all possible diagonals of multiplication table for [n] -antimagma.

```
Example

gap> AntimagmaGeneratorPossibleDiagonals(2);

[ [ 2, 1 ] ]

gap> AntimagmaGeneratorPossibleDiagonals(3);

[
      [ 2, 1, 1 ], [ 2, 1, 2 ], [ 2, 3, 1 ], [ 2, 3, 2 ],

      [ 3, 1, 1 ], [ 3, 1, 2 ], [ 3, 3, 1 ], [ 3, 3, 2 ]

]
```

1.3.15 AntimagmaGeneratorFilterNonIsomorphicMagmas

▷ AntimagmaGeneratorFilterNonIsomorphicMagmas(Ms)

(function)

filters non-isomorphic magmas m.

1.4 smallantimagmas automatic generated documentation of methods

1.4.1 MagmaIsomorphism (for IsMagma, IsMagma)

▷ MagmaIsomorphism(M, N)

(operation)

computes an isomoprhism between magmas M, N.

```
gap> M := SmallAntimagma(2, 1);
<magma with 2 generators>
gap> N := MagmaByMultiplicationTable([ [2, 1], [2, 1] ]);
<magma with 2 generators>
gap> MagmaIsomorphism(M, N);
<mapping: Domain([ m1, m2 ]) -> Domain([ m1, m2 ]) >
```

1.4.2 MagmaAntiisomorphism (for IsMagma, IsMagma)

▷ MagmaAntiisomorphism(M, N)

(operation)

creates an antiisomoprhism between magmas M, N.

```
gap> M := SmallAntimagma(2, 1);
<magma with 2 generators>
gap> N := SmallAntimagma(2, 2);
<magma with 2 generators>
gap> MagmaAntiisomorphism(M, N);
<mapping: Domain([ m1, m2 ]) -> Domain([ m1, m2 ]) >
```

Index

AllSmallAntimagmas, 9	MagmaAntiisomorphism
AllSubmagmas, 7	for IsMagma, IsMagma, 11
AnticommutativityIndex	MagmaIsomorphism
for IsMagma, 6	for IsMagma, IsMagma, 11
AntimagmaGeneratorFilterNonIsomorphic-	MagmaIsomorphismInvariantsMatch, 8
Magmas, 11	
AntimagmaGeneratorPossibleDiagonals, 11	${ t NrSmallAntimagmas}, 9$
AssociativityIndex	0 0 114 .: 10
for IsMagma, 5	OneSmallAntimagma, 10
· ·	ReallyAllSmallAntimagmas, 10
CommutativityIndex	ReallyNrSmallAntimagmas, 10
for IsMagma, 6	Right Order
Diamonal Officialia attachia mahla	for IsExtRElement, 7
DiagonalOfMultiplicationTable	RightOrdersOfElements
for IsMagma, 6	for IsMagma, 7
IdSmallAntimagma	RightPower, 9
for IsMagma, 6	
IsAntiassociative	SmallAntimagma, 10
for IsMagma, 4	SquaresIndex
IsCancellative	for IsMagma, 6
for IsMagma, 5	
IsLeftCancellative	${\tt TransposedMagma, 8}$
for IsMagma, 4	
IsLeftCyclic	
for IsMagma, 4	
IsMagmaAntiisomorphic, 8	
IsMagmaIsomorphic, 8	
IsRightCancellative	
for IsMagma, 5	
IsRightCyclic	
for IsMagma, 4	
LeftOrder	
for IsExtLElement, 7	
LeftOrdersOfElements	
for IsMagma, 7	
LeftPower, 9	
License, 2	