A library of antiassociative magmas of small order

0.2.10

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Chapter 1

smallantimagmas automatic generated documentation

1.1 smallantimagmas automatic generated documentation of properties

1.1.1 IsAntiassociative (for IsMagma)

```
gap> IsAntiassociative(OneSmallGroup(16));
false
gap> IsAntiassociative(OneSmallAntimagma(2));
true
gap> IsAntiassociative(OneSmallAntimagma(3));
true
```

1.1.2 IsLeftCyclic (for IsMagma)

1.1.3 IsRightCyclic (for IsMagma)

1.1.4 IsLeftCancellative (for IsMagma)

```
gap> M := SmallAntimagma(2, 1);
<magma with 2 generators>
gap> Display( MultiplicationTable(M) );
[ [ 2, 1 ],
      [ 2, 1 ] ]
gap> IsRightCancellative(M);
false
gap> IsLeftCancellative(M);
true
gap> List(AllSmallAntimagmas(2), M -> IsLeftCancellative(M));
[ true, false ]
```

1.1.5 IsRightCancellative (for IsMagma)

1.1.6 IsCancellative (for IsMagma)

1.2 smallantimagmas automatic generated documentation of attributes

1.2.1 AssociativityIndex (for IsMagma)

```
\triangleright AssociativityIndex(M) (attribute)
```

identifies associativity index of M.

```
gap> OneSmallAntimagma(2);
<magma with 2 generators>
gap> AssociativityIndex(OneSmallAntimagma(2));
0
gap> OneSmallGroup(4);
<pc group of size 4 with 2 generators>
gap> AssociativityIndex(OneSmallGroup(4));
64
gap> AssociativityIndex(OneSmallGroup(4)) = 4 ^ 3;
true
```

1.2.2 DiagonalOfMultiplicationTable (for IsMagma)

▷ DiagonalOfMultiplicationTable(M)

(attribute)

computes diaognal of multiplication table of M.

```
Example

gap> List(AllSmallAntimagmas(3), M -> DiagonalOfMultiplicationTable((M)));

[ [ 2, 1, 1 ], [ 2, 1, 1 ],
        [ 2, 3, 2 ], [ 2, 1, 1 ],
        [ 2, 1, 1 ], [ 2, 1, 2 ],
        [ 2, 3, 2 ], [ 2, 1, 2 ],
        [ 2, 3, 1 ], [ 2, 3, 1 ]

]
```

1.2.3 CommutativityIndex (for IsMagma)

▷ CommutativityIndex(M)

(attribute)

identifies commutativity index of M.

Example _____

1.2.4 AnticommutativityIndex (for IsMagma)

▷ AnticommutativityIndex(M)

(attribute)

calculates anticommutativity index of M.

_____ Example __

1.2.5 SquaresIndex (for IsMagma)

▷ SquaresIndex(M)

(attribute)

computes squares index of M so the order of $\{m^2 | m \in M\}$.

```
Example

gap> List(AllSmallAntimagmas(2), M -> List(M, m -> m * m));

[ [ m2, m1 ], [ m2, m1 ] ]

gap> List(AllSmallAntimagmas(2), M -> SquaresIndex(M));

[ 2, 2 ]

gap> List(AllSmallAntimagmas(3), M -> SquaresIndex(M));

[ 2, 2, 2, 2, 2, 2, 2, 3, 3 ]
```

1.2.6 IdSmallAntimagma (for IsMagma)

▷ IdSmallAntimagma(M)

(attribute)

identifies class of antiassociative magma M.

```
gap> IsAntiassociative(OneSmallGroup(16));
false
gap> IsAntiassociative(OneSmallAntimagma(2));
true
gap> IsAntiassociative(OneSmallAntimagma(3));
true
```

1.2.7 LeftOrder (for IsExtLElement)

```
    LeftOrder([m])
    (attribute)
```

returns a left order of element m.

1.2.8 RightOrder (for IsExtRElement)

```
▷ RightOrder([m])

(attribute)
```

returns a right order of element m.

1.2.9 LeftOrdersOfElements (for IsMagma)

```
    LeftOrdersOfElements([m])
    (attribute)
```

returns a left order of element m.

1.2.10 RightOrdersOfElements (for IsMagma)

```
    RightOrdersOfElements([m])
    (attribute)
```

returns a left order of element m.

1.3 smallantimagmas automatic generated documentation of global functions

1.3.1 AllSubmagmas

```
▶ AllSubmagmas (M) (function)
```

builds a collection of non-isomorphic submagmas of M.

```
gap> AllSmallAntimagmas(2);

[ <magma with 2 generators>, <magma with 2 generators> ]

gap> List(AllSmallAntimagmas(2), M -> AllSubmagmas(M));

[ [ <magma with 1 generator> ], [ <magma with 1 generator> ] ]
```

(function)

1.3.2 MagmaIsomorphismInvariantsMatch

▶ MagmaIsomorphismInvariantsMatch(M) (function)
computes isomorphism invariants of M.

1.3.3 IsMagmaIsomorphic

▷ IsMagmaIsomorphic(M, N) (function)

identifies whether magmas M, N are isomorphic.

```
gap> M := SmallAntimagma(2, 1);
<magma with 2 generators>
gap> N := SmallAntimagma(2, 2);
<magma with 2 generators>
gap> T := MagmaByMultiplicationTable([[2, 1], [2, 1]]);
<magma with 2 generators>
gap> IsMagmaIsomorphic(M, M);
true
gap> IsMagmaIsomorphic(M, T);
true
gap> IsMagmaIsomorphic(M, N);
false
```

1.3.4 IsMagmaAntiisomorphic

▷ IsMagmaAntiisomorphic([M, N])

identifies whether magmas M, N are antiisomorphic.

```
gap> N := SmallAntimagma(2, 1);
<magma with 2 generators>
gap> M := SmallAntimagma(2, 1);
<magma with 2 generators>
gap> N := SmallAntimagma(2, 2);
<magma with 2 generators>
gap> IsMagmaAntiisomorphic(M, M);
false
gap> IsMagmaAntiisomorphic(M, N);
true
gap> IsMagmaAntiisomorphic(M, TransposedMagma(M));
true
```

1.3.5 TransposedMagma

□ TransposedMagma([M]) (function)
 generates transposed magma M.

```
gap> M := SmallAntimagma(2, 1);
<magma with 2 generators>
gap> IsMagmaAntiisomorphic(M, TransposedMagma(M));
true
gap> IsMagmaIsomorphic(M, TransposedMagma(TransposedMagma(M)));
true
gap> M := SmallAntimagma(2, 1);
<magma with 2 generators>
gap> Display(MultiplicationTable(M));
[[ 2,  1 ],
  [ 2,  1 ]]
gap> Display(MultiplicationTable(TransposedMagma(M)));
[[ 2,  2 ],
  [ 1,  1 ]]
```

1.3.6 LeftPower

```
    LeftPower([m, k])

returns a left k-power of element m.

(function)
```

1.3.7 RightPower

```
⊳ RightPower([m, k]) (function)
returns a right k-power of element m.
```

1.3.8 AllSmallAntimagmas

```
▷ AllSmallAntimagmas(n) (function)
```

returns all antiassociative magmas of specified size n (a number)

1.3.9 NrSmallAntimagmas

```
▷ NrSmallAntimagmas(n) (function)
```

counts number of antiassociative magmas of specified size n (a number).

```
gap> NrSmallAntimagmas(2);
2
gap> NrSmallAntimagmas(3);
10
gap> NrSmallAntimagmas(4);
17780
```

1.3.10 SmallAntimagma

```
\triangleright SmallAntimagma(n, i)
```

returns antiassociative magma of id [n, i].

```
gap> SmallAntimagma(2, 1);

<magma with 2 generators>
gap> SmallAntimagma(4, 5);

<magma with 4 generators>
```

1.3.11 OneSmallAntimagma

▷ OneSmallAntimagma(n)

(function)

(function)

returns a random antiassociative magma of size n.

```
gap> OneSmallAntimagma(2);
<magma with 2 generators>

gap> OneSmallAntimagma(3);
<magma with 3 generators>
```

1.3.12 ReallyAllSmallAntimagmas

▷ ReallyAllSmallAntimagmas(n)

(function)

returns really-all antiassociative magmas, isomorphic, of specified size n (a number)

```
gap> ReallyAllSmallAntimagmas(2);
[ <magma with 2 generators>, <magma with 2 generators>]
```

1.3.13 ReallyNrSmallAntimagmas

(function)

counts number of antiassociative magmas of specified size n (a number)

```
gap> ReallyNrSmallAntimagmas(3);
52
```

1.3.14 AntimagmaGeneratorPossibleDiagonals

▷ AntimagmaGeneratorPossibleDiagonals(n)

(function)

returns all possible diagonals of multiplication table for [n] -antimagma.

```
Example

gap> AntimagmaGeneratorPossibleDiagonals(2);

[ [ 2, 1 ] ]

gap> AntimagmaGeneratorPossibleDiagonals(3);

[
      [ 2, 1, 1 ], [ 2, 1, 2 ], [ 2, 3, 1 ], [ 2, 3, 2 ],

      [ 3, 1, 1 ], [ 3, 1, 2 ], [ 3, 3, 1 ], [ 3, 3, 2 ]

]
```

1.3.15 AntimagmaGeneratorFilterNonIsomorphicMagmas

▷ AntimagmaGeneratorFilterNonIsomorphicMagmas(Ms)

(function)

filters non-isomorphic magmas m.

1.4 smallantimagmas automatic generated documentation of methods

1.4.1 MagmaIsomorphism (for IsMagma, IsMagma)

▷ MagmaIsomorphism(M, N)

(operation)

computes an isomoprhism between magmas M, N.

```
gap> M := SmallAntimagma(2, 1);
<magma with 2 generators>
gap> N := MagmaByMultiplicationTable([ [2, 1], [2, 1] ]);
<magma with 2 generators>
gap> MagmaIsomorphism(M, N);
<mapping: Domain([ m1, m2 ]) -> Domain([ m1, m2 ]) >
```

1.4.2 MagmaAntiisomorphism (for IsMagma, IsMagma)

▷ MagmaAntiisomorphism(M, N)

(operation)

creates an antiisomoprhism between magmas M, N.

```
gap> M := SmallAntimagma(2, 1);
<magma with 2 generators>
gap> N := SmallAntimagma(2, 2);
<magma with 2 generators>
gap> MagmaAntiisomorphism(M, N);
<mapping: Domain([ m1, m2 ]) -> Domain([ m1, m2 ]) >
```

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