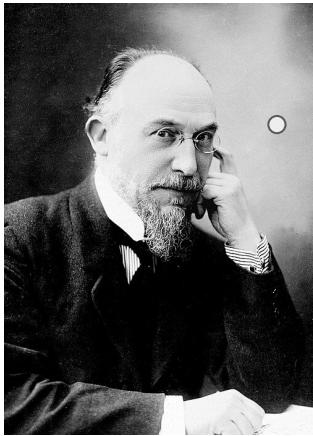




SMT: Satisfiable Music Theory

Karen Haining, Joshua Chen

Constraint - based musical
verification and synthesis

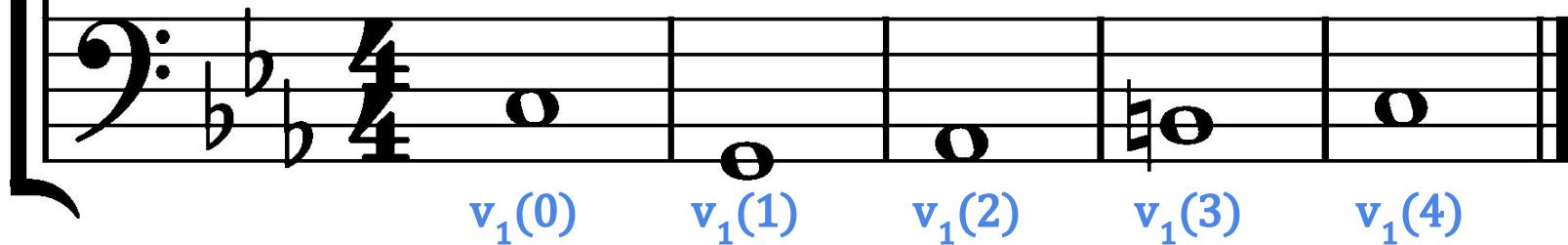
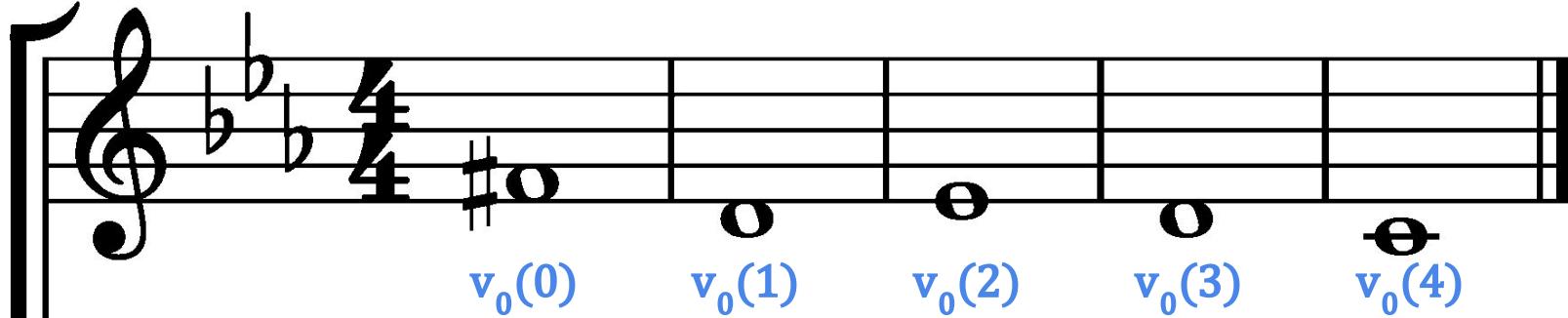


Satie

Erik Satie,
French composer and pianist

The Bachend

A musical score consisting of two staves. The top staff is for the soprano voice, indicated by a treble clef, and the bottom staff is for the basso continuo, indicated by a bass clef. Both staves are in common time (indicated by a '4'). The key signature consists of three flats, which is represented by a '4' with a flat symbol above it and a circle with a sharp symbol below it. The soprano part begins with a note on the first line, followed by a note on the fourth line, then an open circle (rest), then an open circle (rest), then an open circle (rest). The basso continuo part begins with an open circle (rest), followed by an open circle (rest), then an open circle (rest), then an open circle (rest).



$v_0(0)$ $v_0(1)$ $v_0(2)$ $v_0(3)$ $v_0(4)$ $v_1(0)$ $v_1(1)$ $v_1(2)$ $v_1(3)$ $v_1(4)$

Pitches

In Satie: pitches-of <voice> [at <time>]

- Represented in .midi files as integers ranging from 0 - 127

66p	62p	63p	62p	60p
$v_0(0)$	$v_0(1)$	$v_0(2)$	$v_0(3)$	$v_0(4)$

- Represented in the "Bachend encoding" as symbolic constants of type BitVec 8 (extra bit to allow for negatives when computing intervals)

```
(declare-const v0t0 (_ BitVec 8)  
(declare-const v0t1 (_ BitVec 8)  
(declare-const v0t2 (_ BitVec 8)  
....
```

48p	43p	45p	47p	48p
$v_1(0)$	$v_1(1)$	$v_1(2)$	$v_1(3)$	$v_1(4)$

Intervals

- Represent the distance between two pitches

- Can have specified direction (ascending or descending), or be unspecified

- Encoded as the difference between two symbolic pitch constants

$v_0(0)$

$v_0(1)$

$v_0(2)$

$v_0(3)$

$v_0(4)$

$v_1(0)$

$v_1(1)$

$v_1(2)$

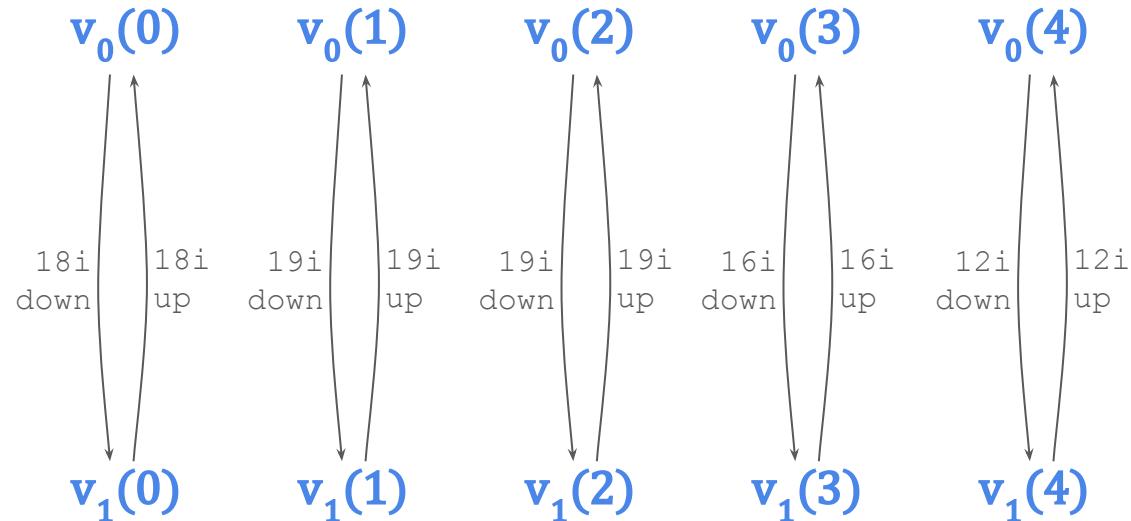
$v_1(3)$

$v_1(4)$

Diads

In Satie: diads-of(<voice1>, <voice2>) [at <time>]

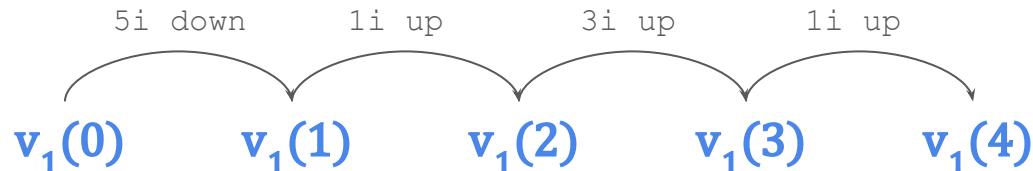
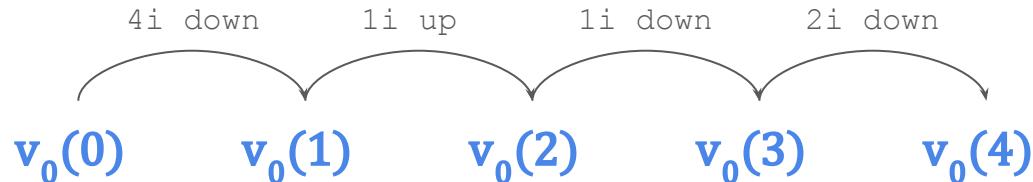
- The "vertical" intervals between two voices



Contours

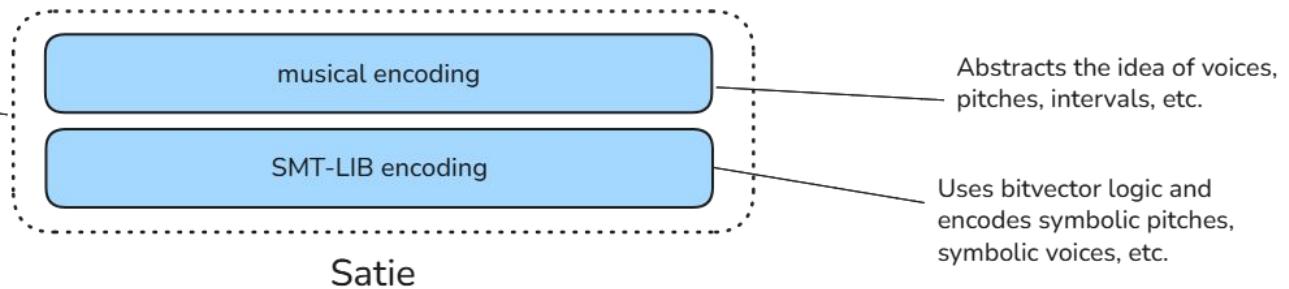
In Satie: contour-of <voice> [at <time>]

- The "horizontal shape" of a particular voice

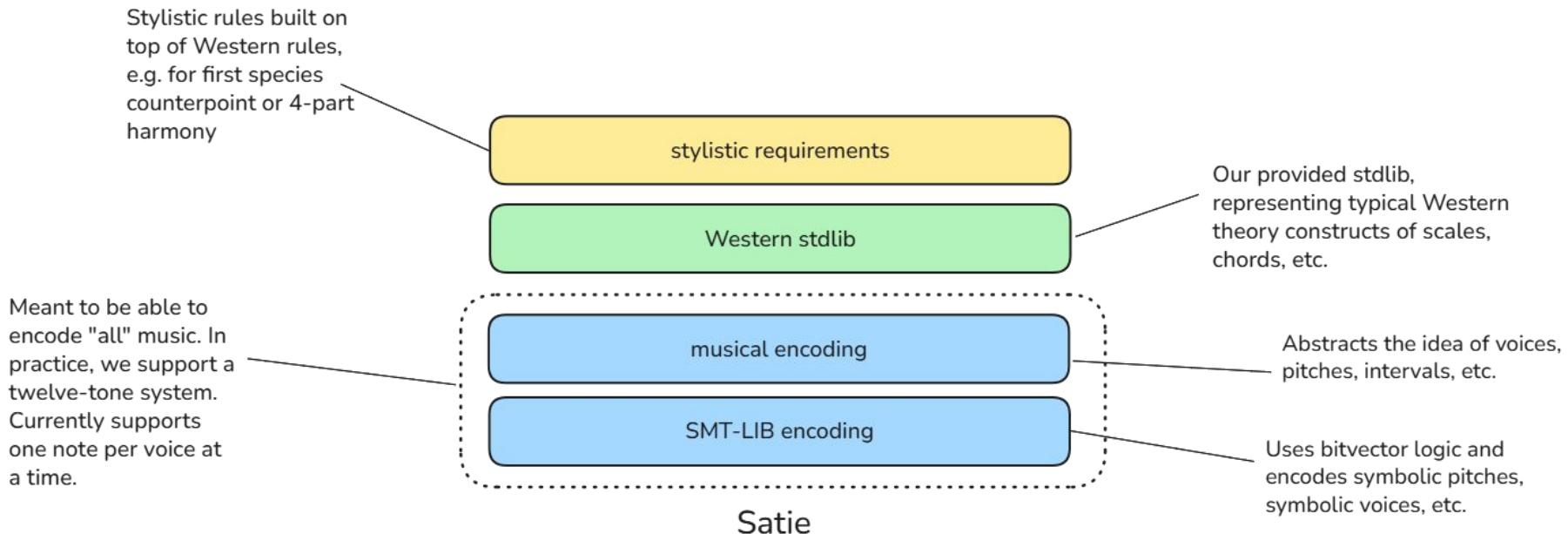


Layers of Abstraction

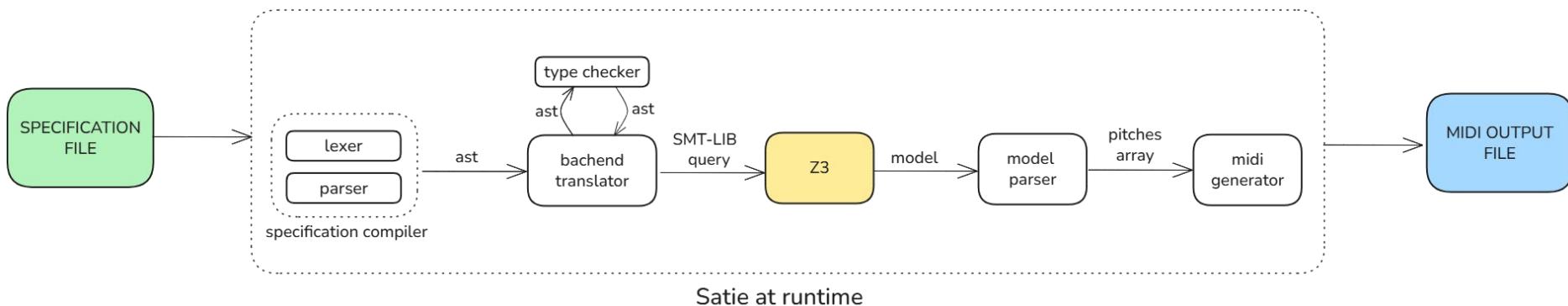
Meant to be able to encode "all" music. In practice, we support a twelve-tone system. Currently supports one note per voice at a time.



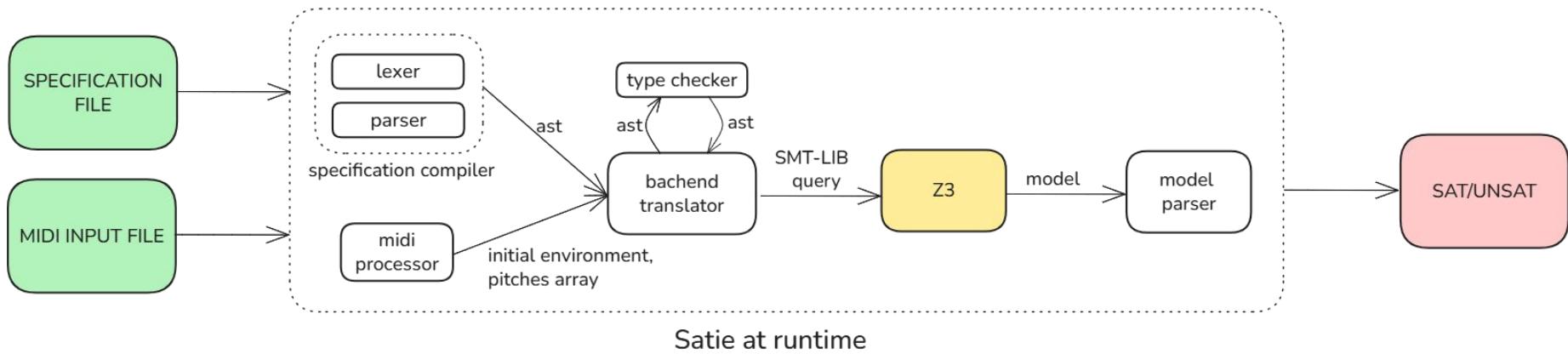
Layers of Abstraction



Synthesis Pipeline



Verification Pipeline



Demo!