



# SMT: Satisfiable Music Theory

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Constraint - based musical  
verification and synthesis

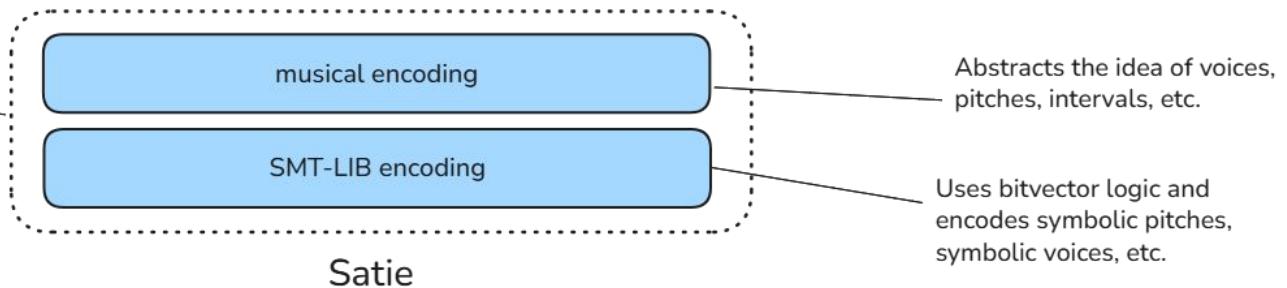


# Satie

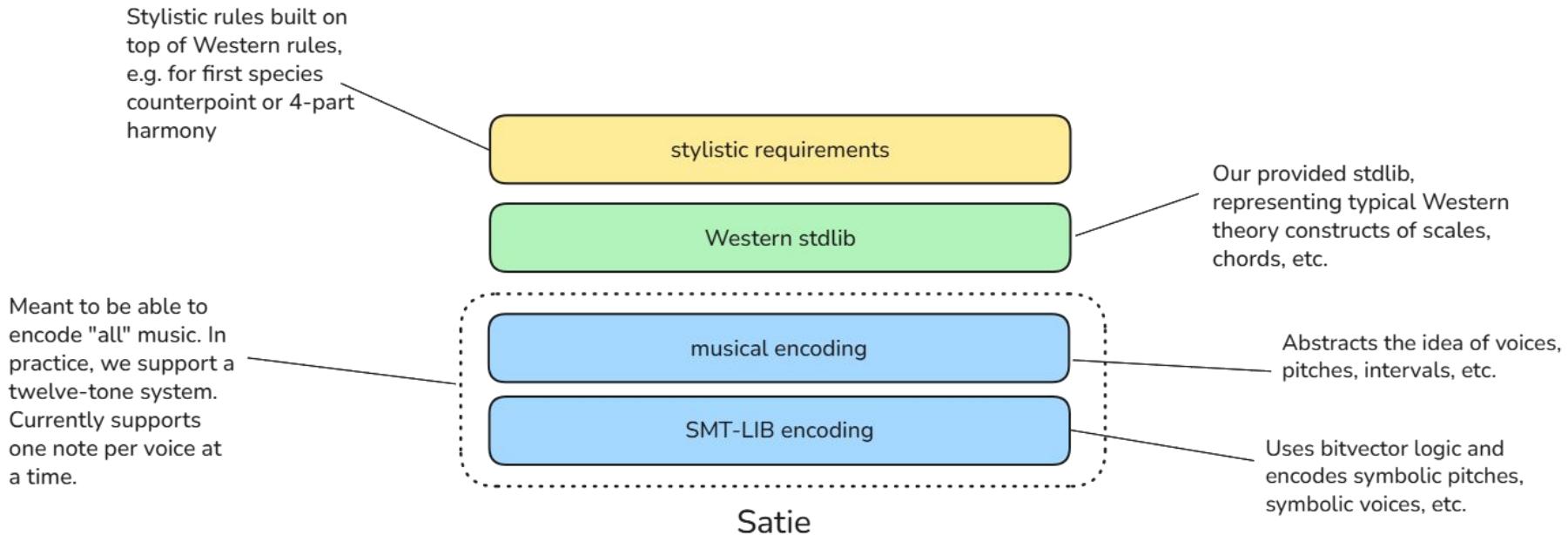
Erik Satie,  
French composer and pianist

# Layers of Abstraction

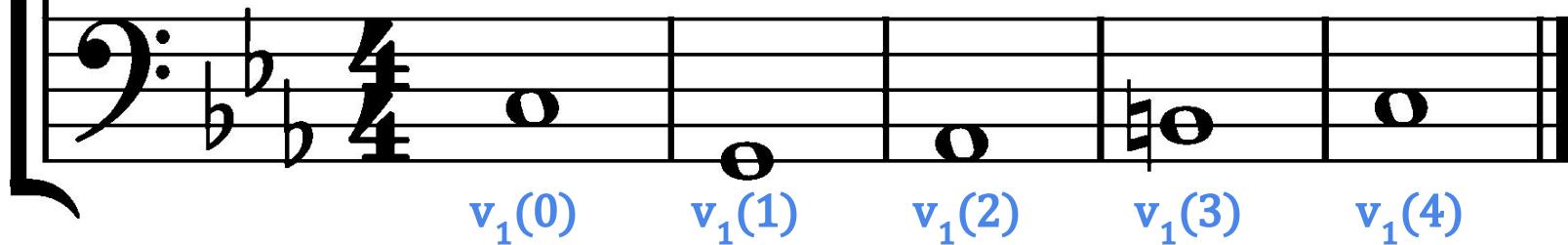
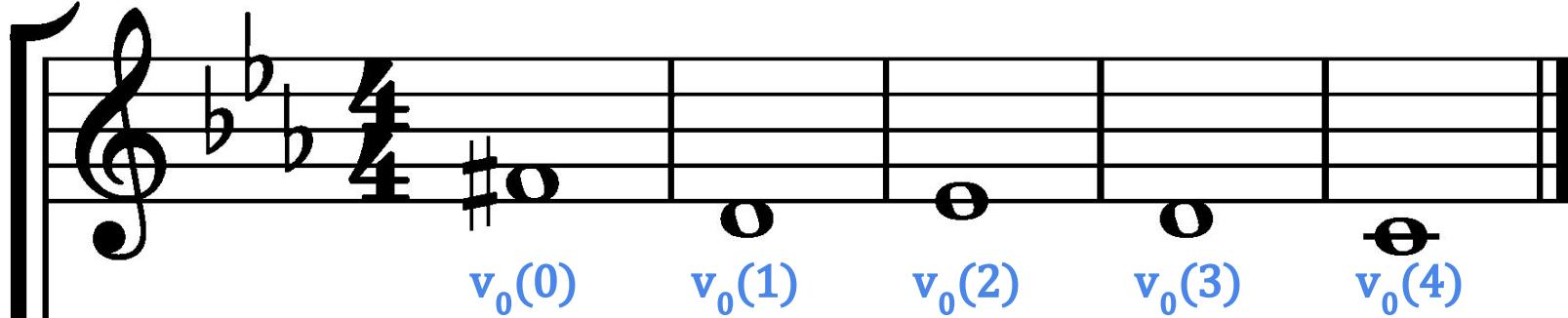
Meant to be able to encode "all" music. In practice, we support a twelve-tone system. Currently supports one note per voice at a time.



# Layers of Abstraction



A musical score consisting of two staves. The top staff is for the soprano voice, indicated by a treble clef, and the bottom staff is for the basso continuo, indicated by a bass clef. Both staves are in common time (indicated by a '4'). The key signature is one flat, represented by a 'b' symbol. The soprano part begins with a whole note (o) followed by a half note (e). The basso continuo part begins with a half note (o), followed by a whole note (e), another whole note (e), and a half note (e).



$v_0(0)$  $v_0(1)$  $v_0(2)$  $v_0(3)$  $v_0(4)$  $v_1(0)$  $v_1(1)$  $v_1(2)$  $v_1(3)$  $v_1(4)$

# Pitches

In Satie: pitches-of <voice> [at <time>]

- Represented in .midi files as integers ranging from 0 - 127

66p	62p	63p	62p	60p
$v_0(0)$	$v_0(1)$	$v_0(2)$	$v_0(3)$	$v_0(4)$

- Represented in the "Bachend encoding" as symbolic constants of type BitVec 8 (extra bit to allow for negatives when computing intervals)

```
(declare-const v0t0 (_ BitVec 8)  
(declare-const v0t1 (_ BitVec 8)  
(declare-const v0t2 (_ BitVec 8)  
....
```

48p	43p	45p	47p	48p
$v_1(0)$	$v_1(1)$	$v_1(2)$	$v_1(3)$	$v_1(4)$

# Intervals

- Represent the distance between two pitches

$v_0(0)$

$v_0(1)$

$v_0(2)$

$v_0(3)$

$v_0(4)$

- Can have specified direction (ascending or descending), or be unspecified

- Encoded as the difference between two symbolic pitch constants

$v_1(0)$

$v_1(1)$

$v_1(2)$

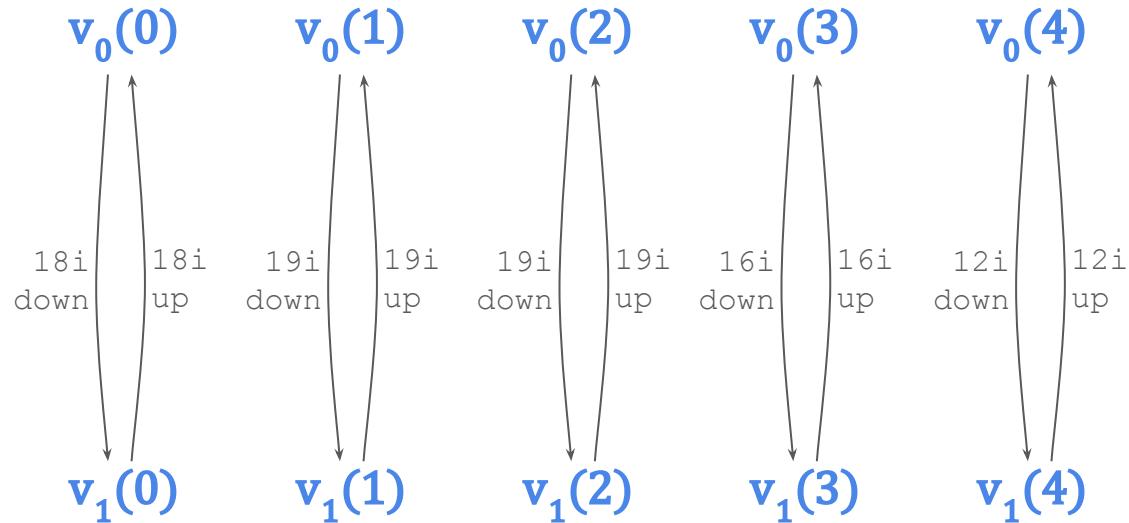
$v_1(3)$

$v_1(4)$

# Diads

In Satie: diads-of(<voice1>, <voice2>) [at <time>]

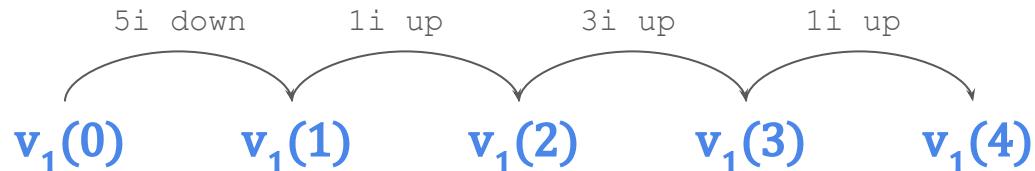
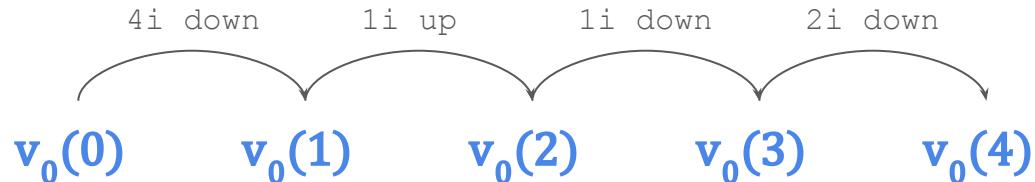
- The "vertical" intervals between two voices



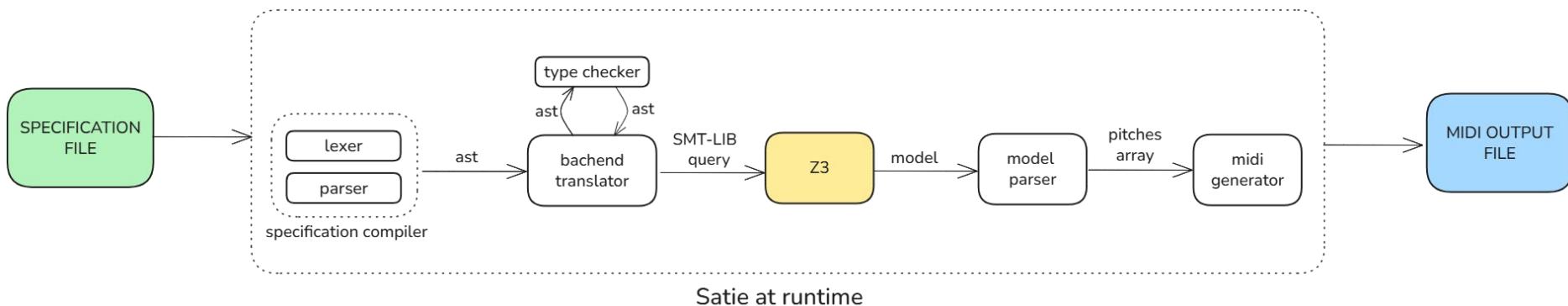
# Contours

In Satie: contour-of <voice> [at <time>]

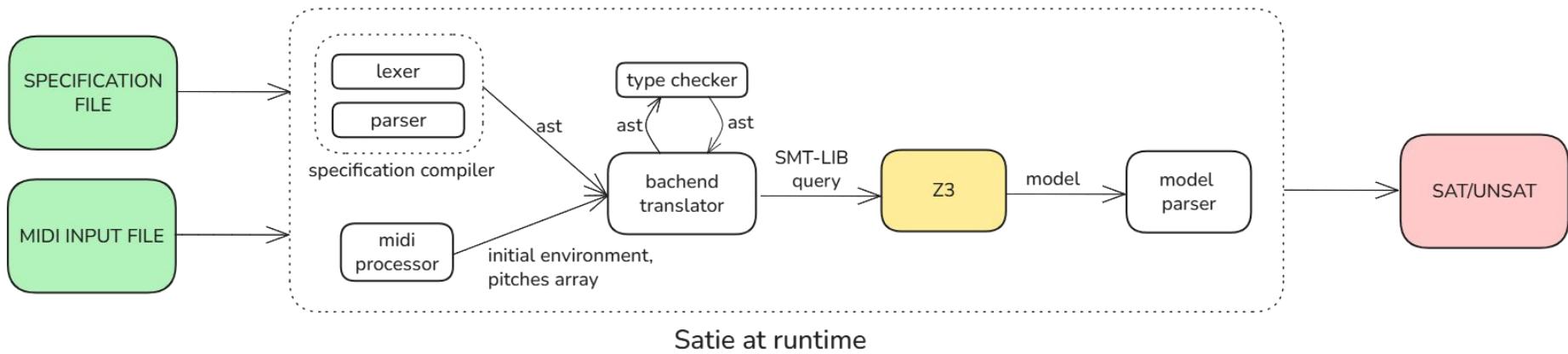
- The "horizontal shape" of a particular voice



# Synthesis Pipeline



# Verification Pipeline



# Demo!