

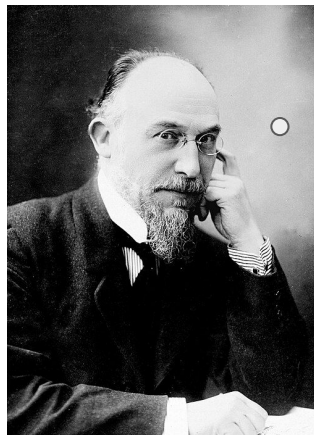


SMT: Satisfiable Music Theory

Karen Haining, Joshua Chen



Constraint-based musical
verification and synthesis

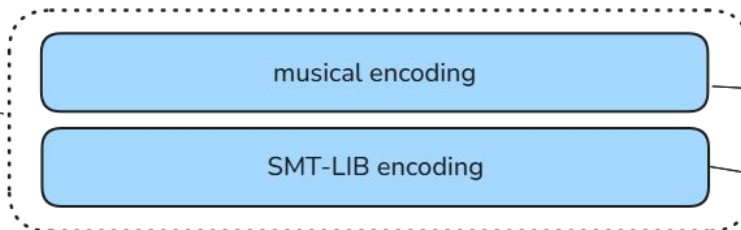


Erik Satie,
French composer and pianist

Satie

Layers of Abstraction

Meant to be able to encode "all" music. In practice, we support a twelve-tone system. Currently supports one note per voice at a time.

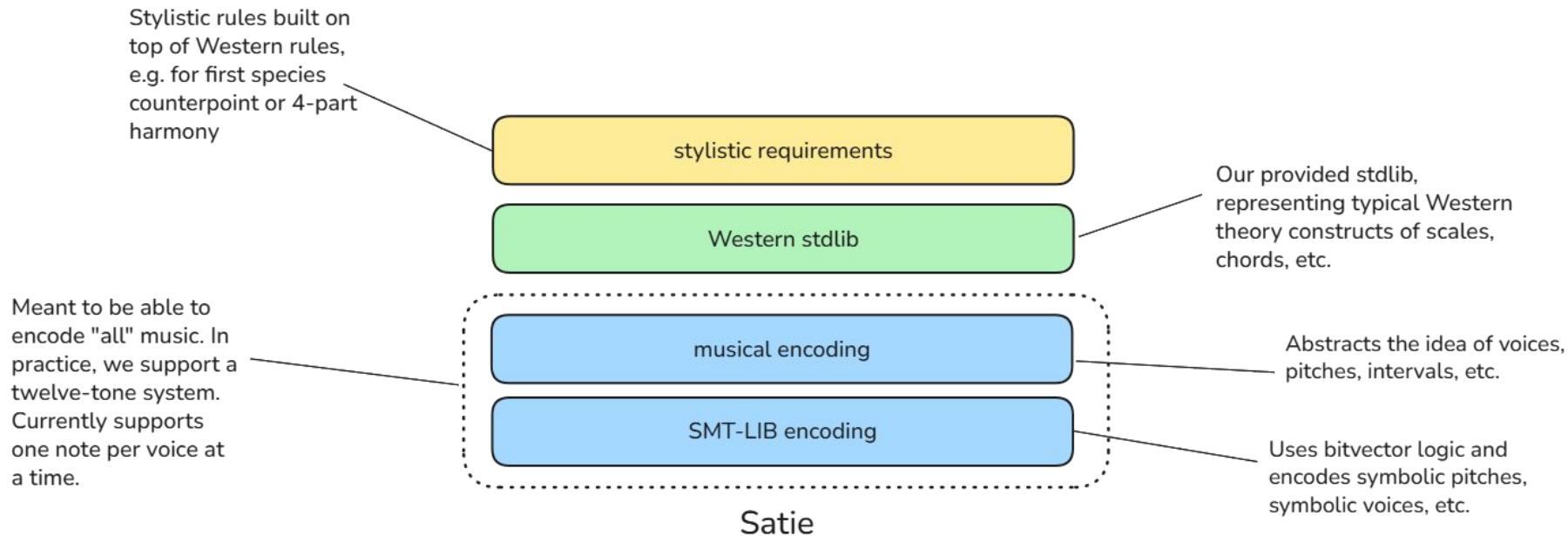


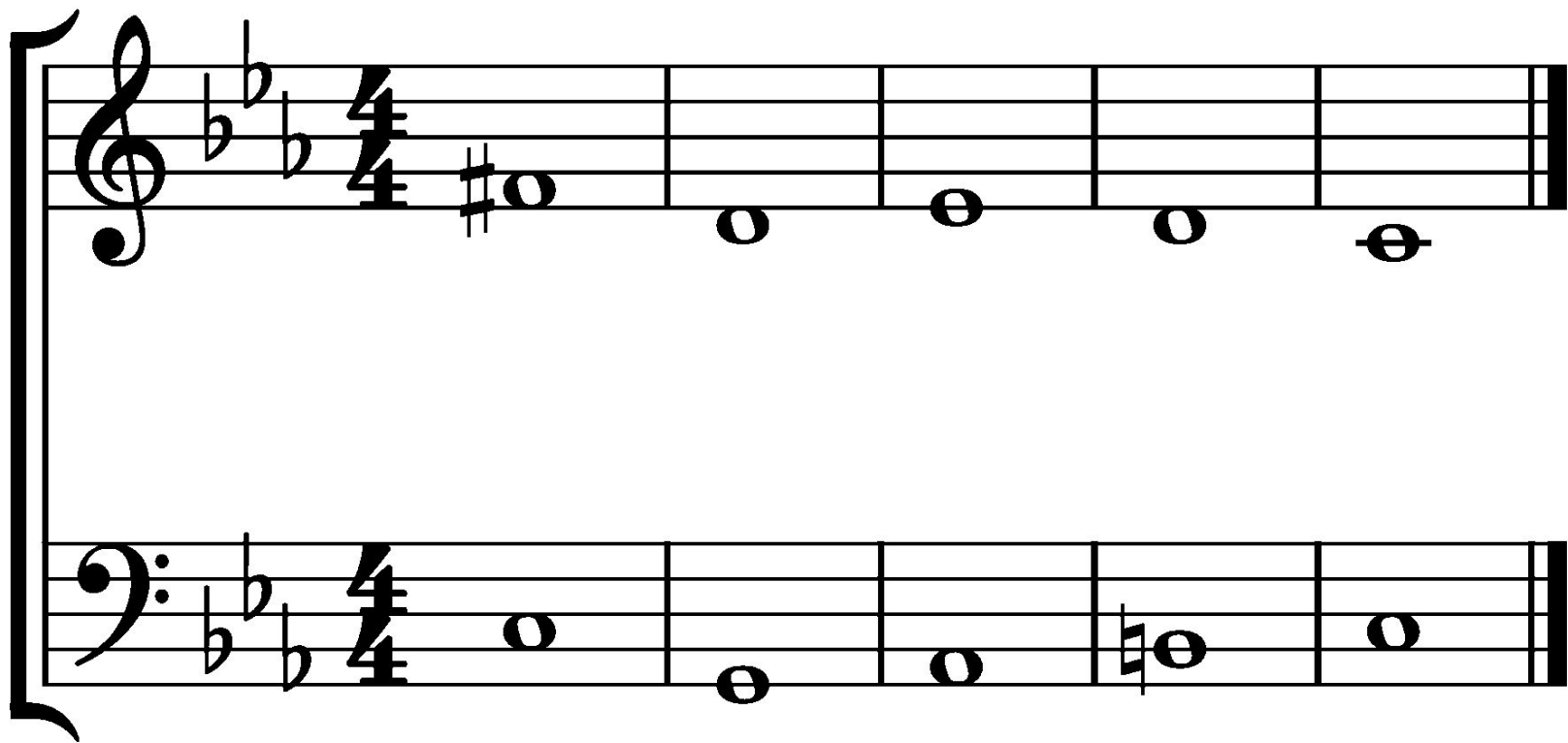
Satie

Abstracts the idea of voices, pitches, intervals, etc.

Uses bitvector logic and encodes symbolic pitches, symbolic voices, etc.

Layers of Abstraction





A musical score for two staves, Treble and Bass, in 4/4 time. The key signature has three flats (B-flat, E-flat, A-flat). The score consists of five measures. The Treble staff contains notes labeled $v_0(0)$ through $v_0(4)$, and the Bass staff contains notes labeled $v_1(0)$ through $v_1(4)$. The notes are as follows:

Measure	Treble Staff Note	Bass Staff Note
1	$v_0(0)$ (F#4)	$v_1(0)$ (F3)
2	$v_0(1)$ (F4)	$v_1(1)$ (E3)
3	$v_0(2)$ (G4)	$v_1(2)$ (F3)
4	$v_0(3)$ (F4)	$v_1(3)$ (E3)
5	$v_0(4)$ (E4)	$v_1(4)$ (F3)

$v_0(0)$

$v_0(1)$

$v_0(2)$

$v_0(3)$

$v_0(4)$

$v_1(0)$

$v_1(1)$

$v_1(2)$

$v_1(3)$

$v_1(4)$

Pitches

In Satie: pitches-of <voice> [at <time>]

- Represented in .midi files as integers ranging from 0 - 127

66p	62p	63p	62p	60p
$v_0(0)$	$v_0(1)$	$v_0(2)$	$v_0(3)$	$v_0(4)$

- Represented in the "Bachend encoding" as symbolic constants of type BitVec 8 (extra bit to allow for negatives when computing intervals)

(declare-const v0t0 (_ BitVec 8)	48p	43p	45p	47p	48p
(declare-const v0t1 (_ BitVec 8)					
(declare-const v0t2 (_ BitVec 8)	$v_1(0)$	$v_1(1)$	$v_1(2)$	$v_1(3)$	$v_1(4)$
...					

Intervals

- Represent the distance between two pitches
- Can have specified direction (ascending or descending), or be unspecified
- Encoded as the difference between two symbolic pitch constants

$v_0(0)$

$v_0(1)$

$v_0(2)$

$v_0(3)$

$v_0(4)$

$v_1(0)$

$v_1(1)$

$v_1(2)$

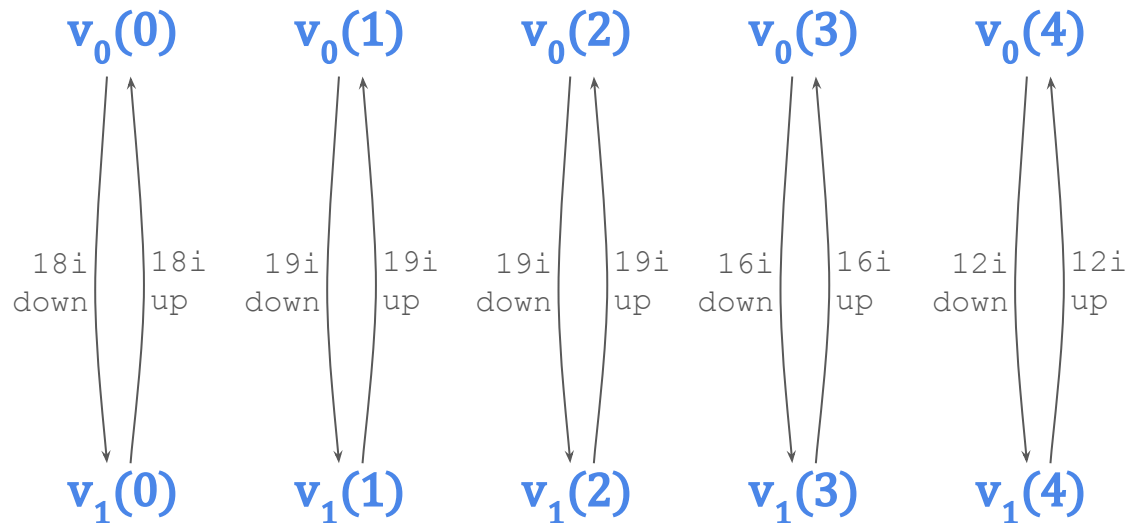
$v_1(3)$

$v_1(4)$

Diads

In Satie: diads-of(<voice1>, <voice2>) [at <time>]

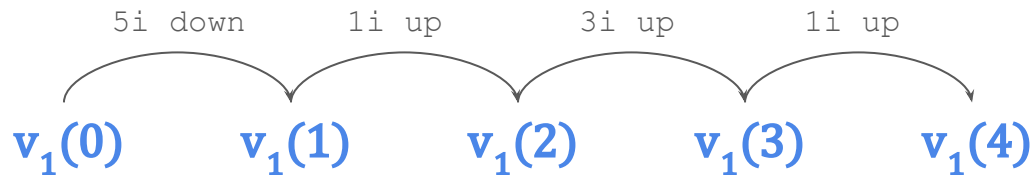
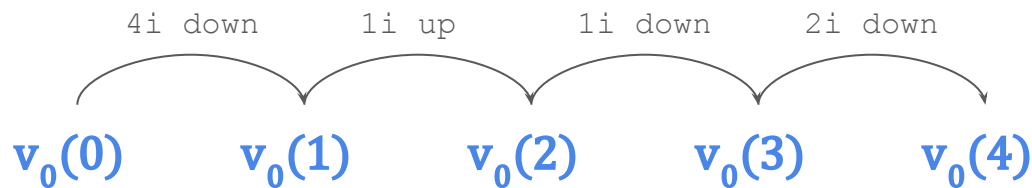
- The "vertical" intervals between two voices



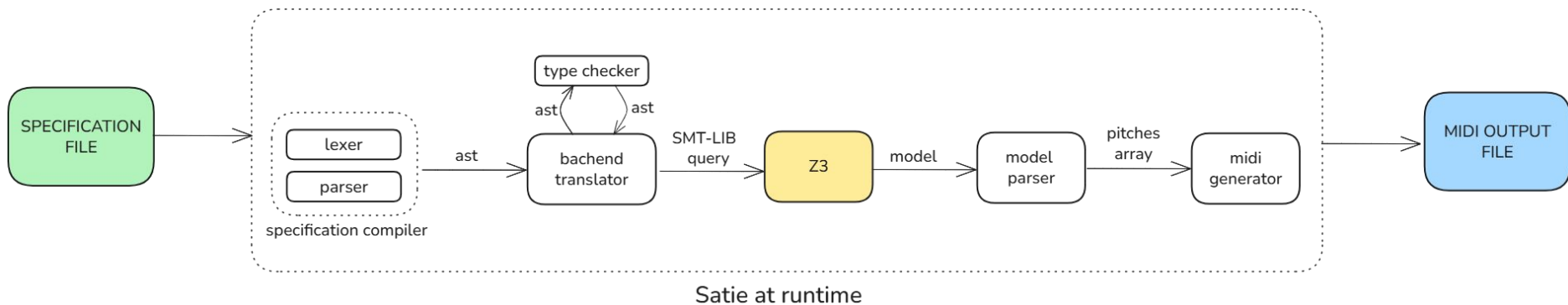
Contours

In Satie: contour-of <voice> [at <time>]

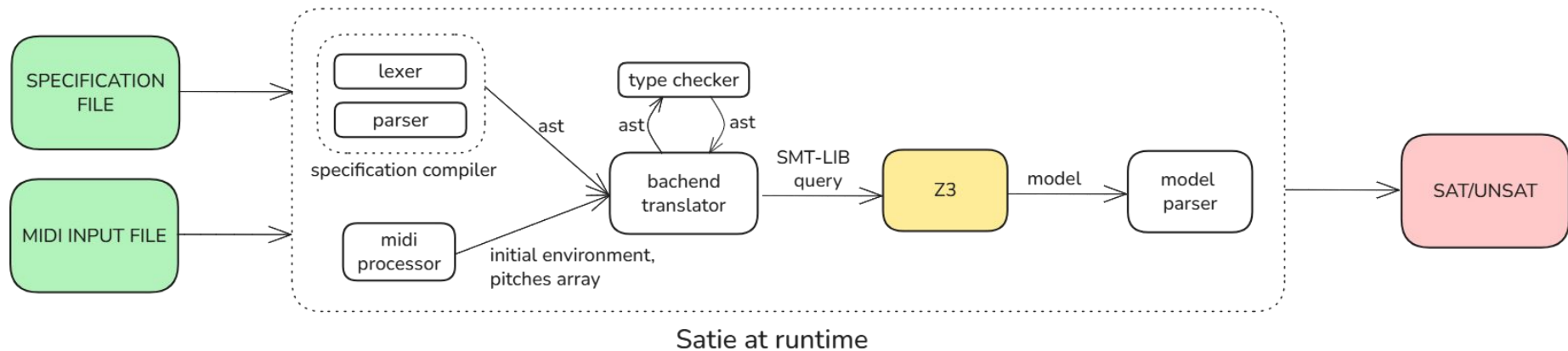
- The "horizontal shape" of a particular voice



Synthesis Pipeline



Verification Pipeline



Demo!