SMT: Satisfiable Music Theory

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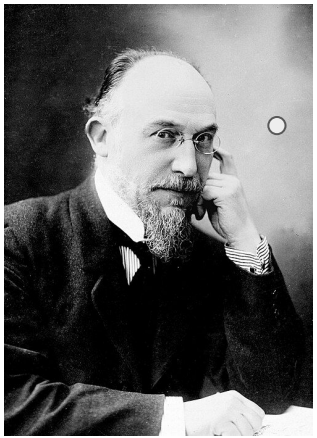
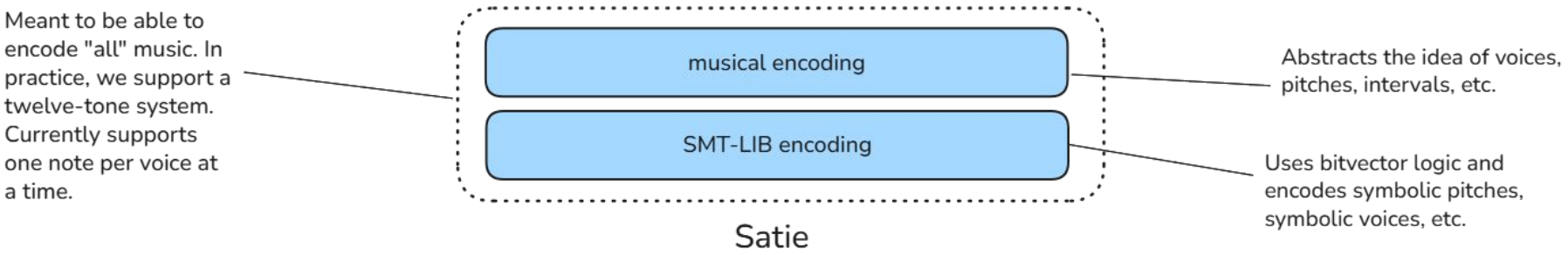
Constraint-based musical verification and synthesis



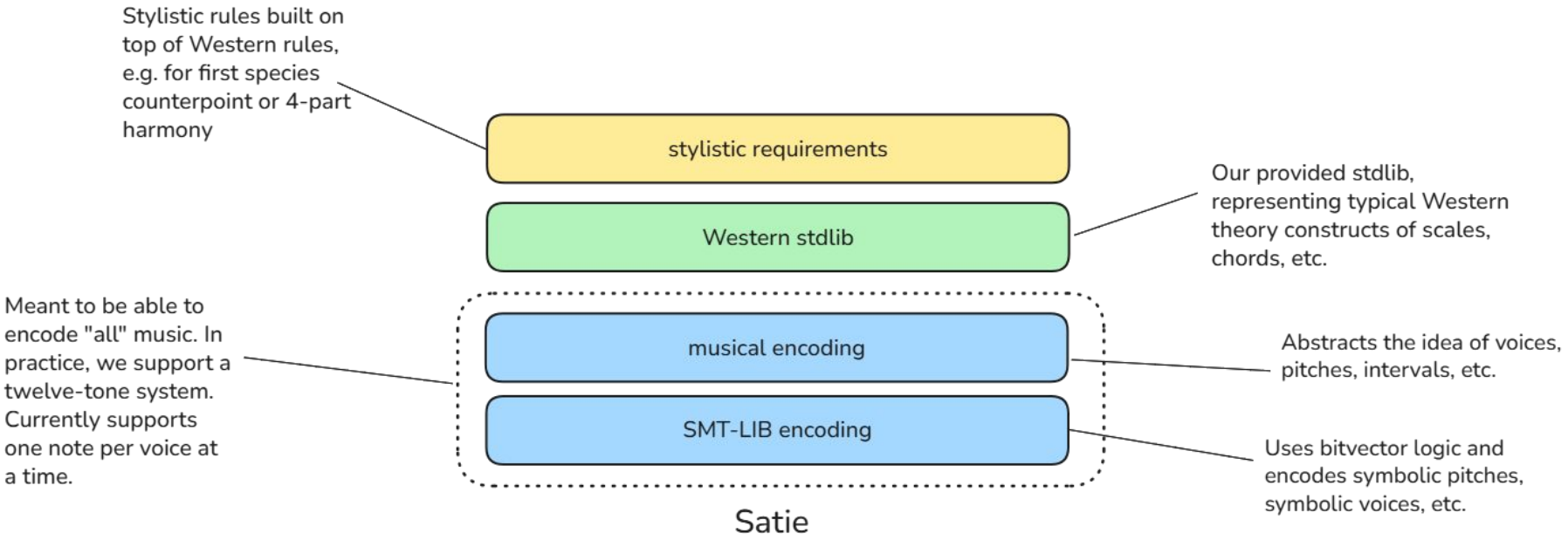
Erik Satie,
French composer and pianist

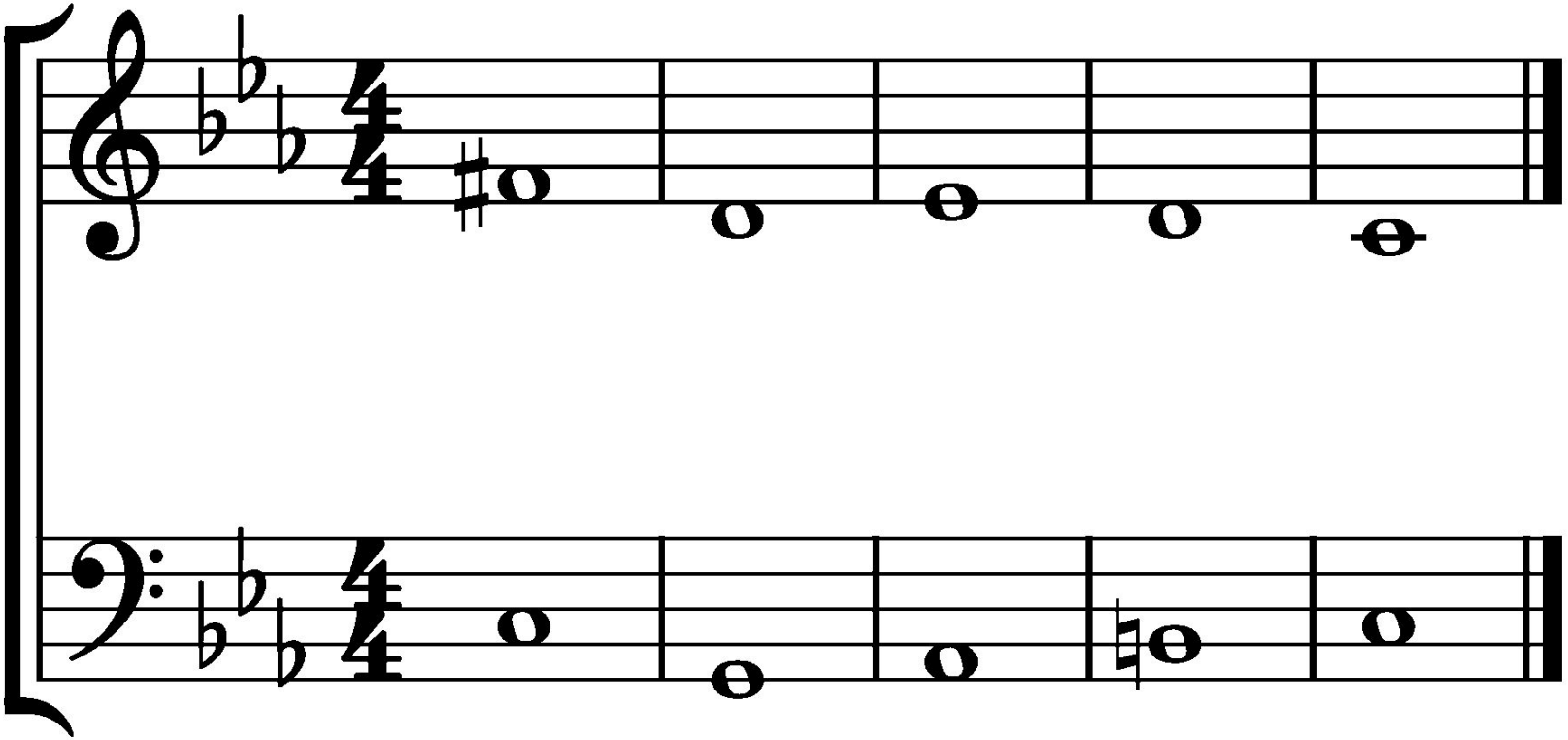
Satie

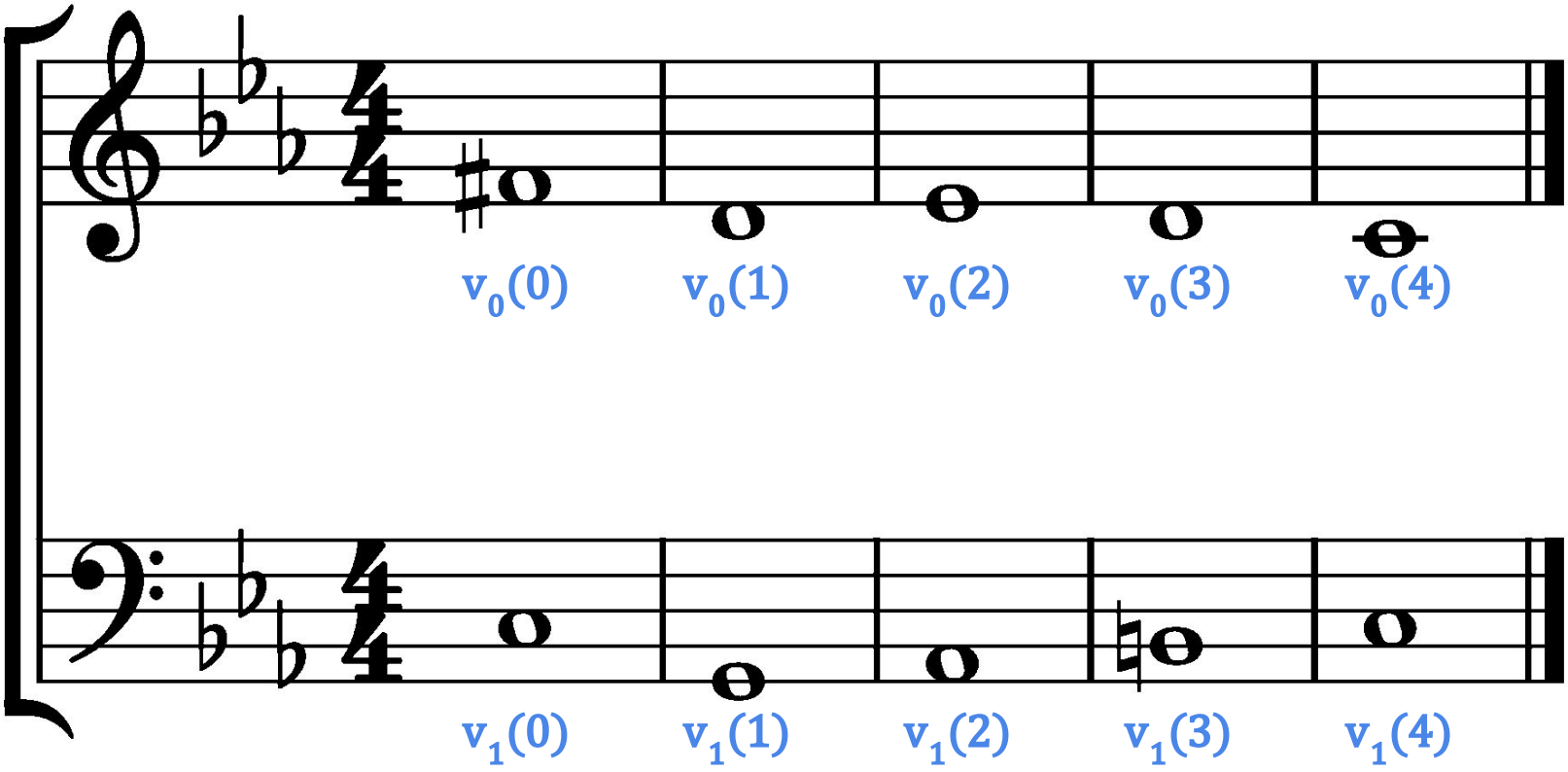
Layers of Abstraction



Layers of Abstraction






$v_0(0)$
$v_0(1)$
$v_0(2)$
$v_0(3)$
$v_0(4)$
$v_1(0)$
$v_1(1)$
$v_1(2)$
$v_1(3)$
$v_1(4)$

$v_0(0)$ $v_0(1)$ $v_0(2)$ $v_0(3)$ $v_0(4)$

$v_1(0)$ $v_1(1)$ $v_1(2)$ $v_1(3)$ $v_1(4)$

Pitches

In Satie: pitches-of <voice> [at <time>]

- Represented in .midi files as integers ranging from 0 - 127

- Represented in the "Bachend encoding" as symbolic constants of type BitVec 8 (extra bit to allow for negatives when computing intervals)

```
(declare-const v0t0 (_ BitVec 8)
(declare-const v0t1 (_ BitVec 8)
(declare-const v0t2 (_ BitVec 8)
        ...
```

66p	62p	63p	62p	60p
$v_0(0)$	$v_0(1)$	$v_0(2)$	$v_0(3)$	$v_0(4)$
48p	43p	45p	47p	48p
$v_1(0)$	$v_1(1)$	$v_1(2)$	$v_1(3)$	$v_1(4)$

Intervals

- Represent the distance between two pitches
- Can have specified direction (ascending or descending), or be unspecified
- Encoded as the difference between two symbolic pitch constants

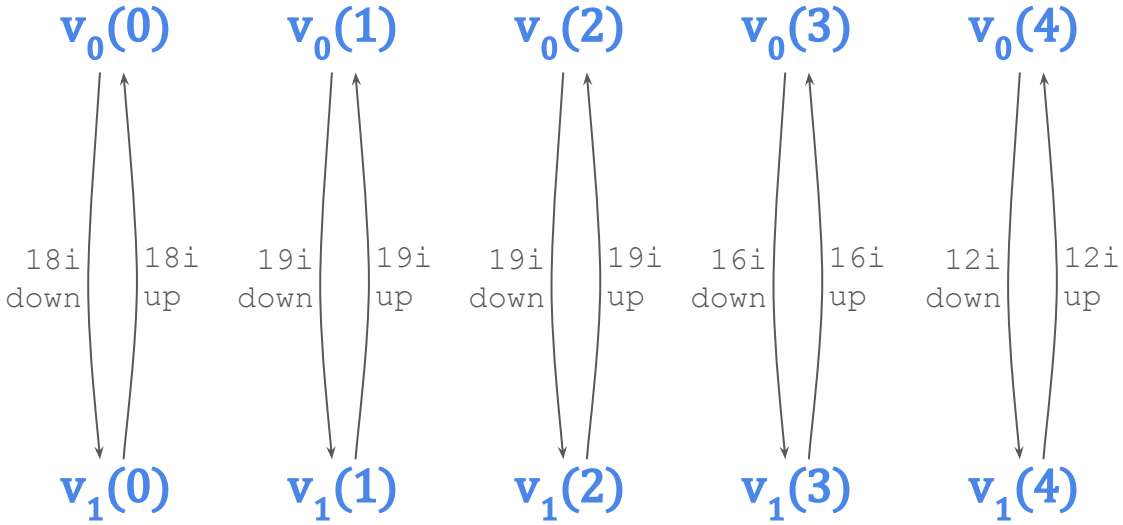
$v_0(0)$ $v_0(1)$ $v_0(2)$ $v_0(3)$ $v_0(4)$

$v_1(0)$ $v_1(1)$ $v_1(2)$ $v_1(3)$ $v_1(4)$

Diads

In Satie: diads-of(<voice1>, <voice2>) [at <time>]

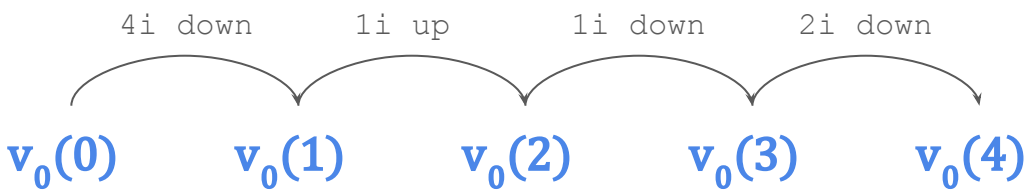
- The "vertical" intervals between two voices

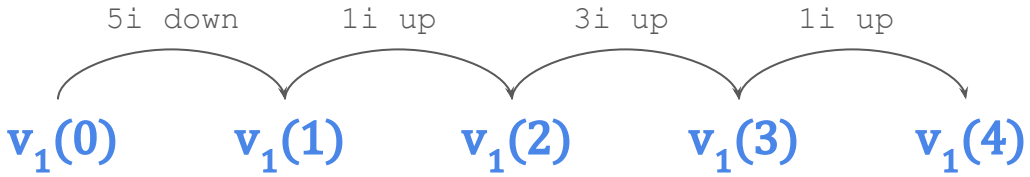


Contours

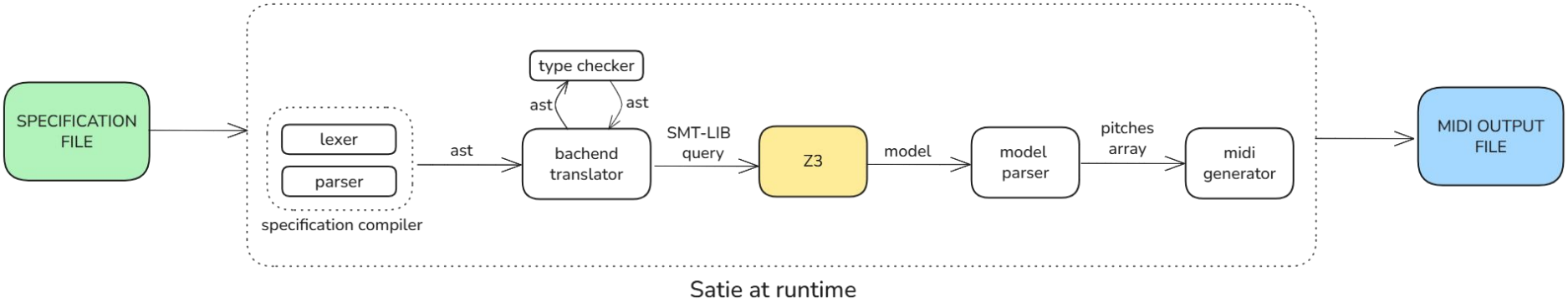
- The "horizontal shape" of a particular voice

In Satie: contour-of <voice> [at <time>]

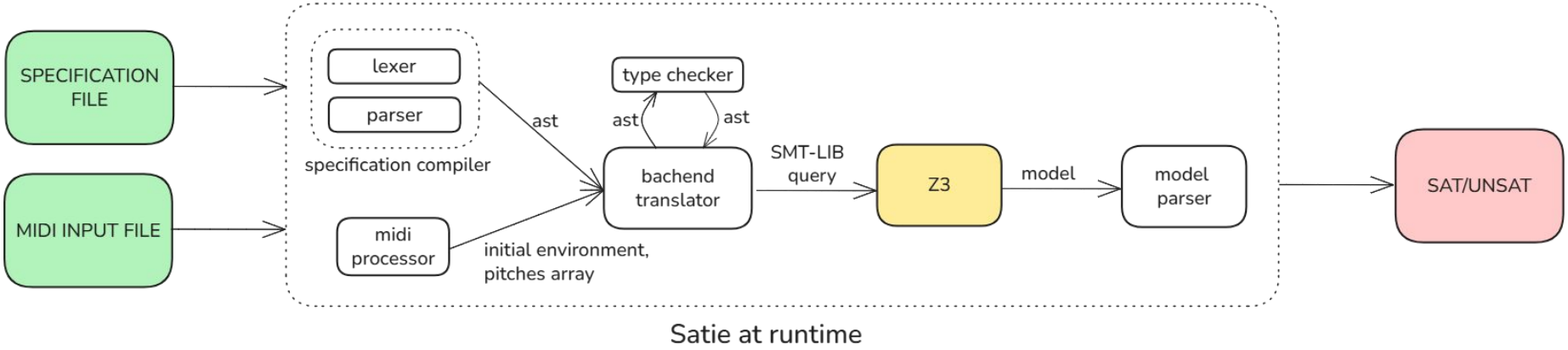




Synthesis Pipeline



Verification Pipeline



Demo!