

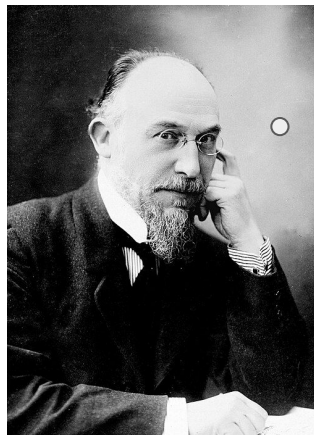


SMT: Satisfiable Music Theory

Karen Haining, Joshua Chen



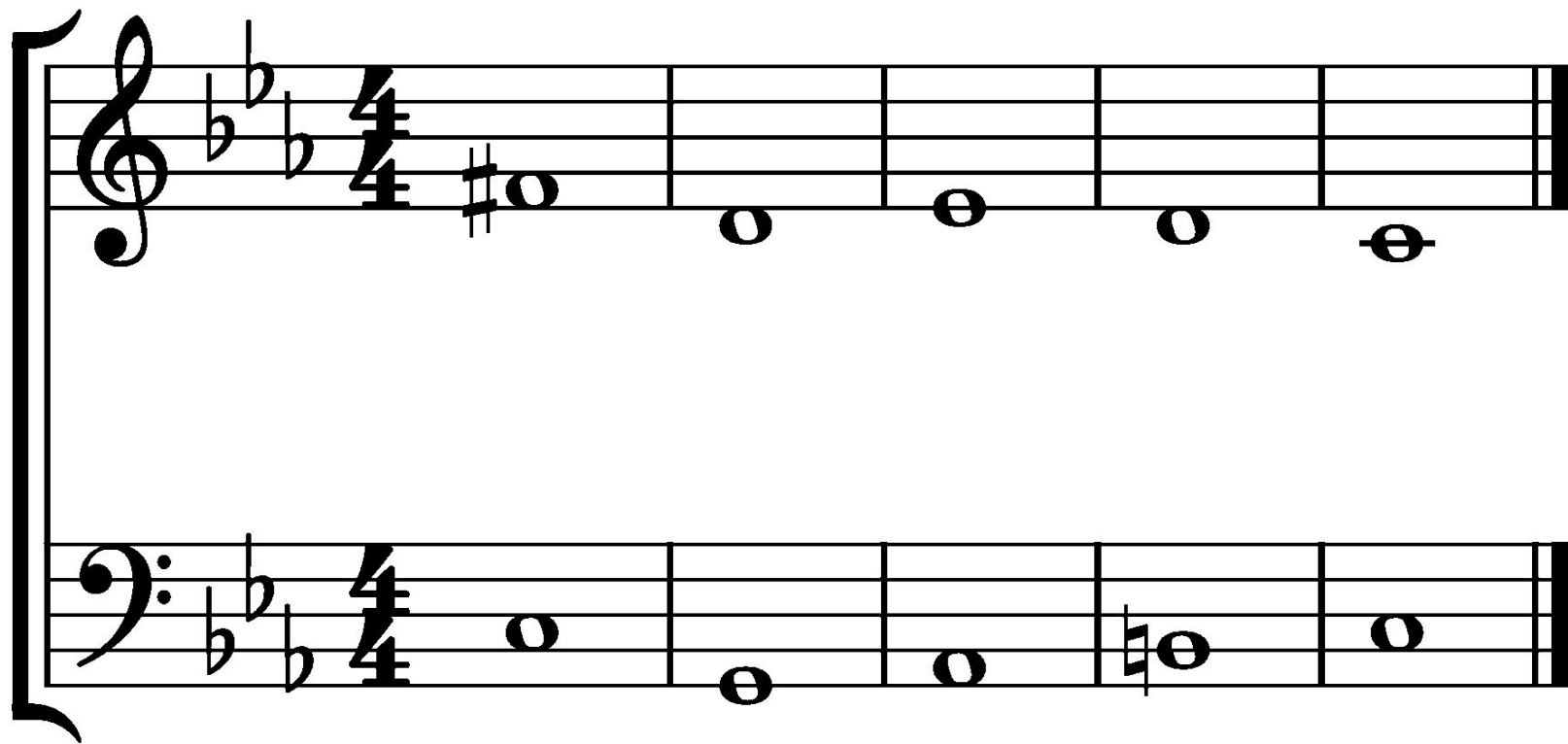
Constraint-based musical
verification and synthesis



Erik Satie,
French composer and pianist

Satie

The Bachend



A musical score consisting of two staves, treble and bass, in 4/4 time. The key signature has three flats (B-flat, E-flat, A-flat). The treble staff contains five notes: a half note G4 with a sharp sign, and four whole notes (F4, E4, D4, C4) with flat signs. The bass staff contains five notes: a half note G3, and four whole notes (F3, E3, D3, C3) with flat signs. The notes are labeled with blue text below the staff.

Measure	Treble Note	Treble Label	Bass Note	Bass Label
1	G4 (sharp)	$v_0(0)$	G3	$v_1(0)$
2	F4 (flat)	$v_0(1)$	F3 (flat)	$v_1(1)$
3	E4 (flat)	$v_0(2)$	E3 (flat)	$v_1(2)$
4	D4 (flat)	$v_0(3)$	D3 (flat)	$v_1(3)$
5	C4 (flat)	$v_0(4)$	C3 (flat)	$v_1(4)$

$v_0(0)$

$v_0(1)$

$v_0(2)$

$v_0(3)$

$v_0(4)$

$v_1(0)$

$v_1(1)$

$v_1(2)$

$v_1(3)$

$v_1(4)$

Pitches

In Satie: pitches-of <voice> [at <time>]

- Represented in .midi files as integers ranging from 0 - 127

66p	62p	63p	62p	60p
$v_0(0)$	$v_0(1)$	$v_0(2)$	$v_0(3)$	$v_0(4)$

- Represented in the "Bachend encoding" as symbolic constants of type BitVec 8 (extra bit to allow for negatives when computing intervals)

(declare-const v0t0 (_ BitVec 8)	48p	43p	45p	47p	48p
(declare-const v0t1 (_ BitVec 8)					
(declare-const v0t2 (_ BitVec 8)	$v_1(0)$	$v_1(1)$	$v_1(2)$	$v_1(3)$	$v_1(4)$
...					

Intervals

- Represent the distance between two pitches
- Can have specified direction (ascending or descending), or be unspecified
- Encoded as the difference between two symbolic pitch constants

$v_0(0)$

$v_0(1)$

$v_0(2)$

$v_0(3)$

$v_0(4)$

$v_1(0)$

$v_1(1)$

$v_1(2)$

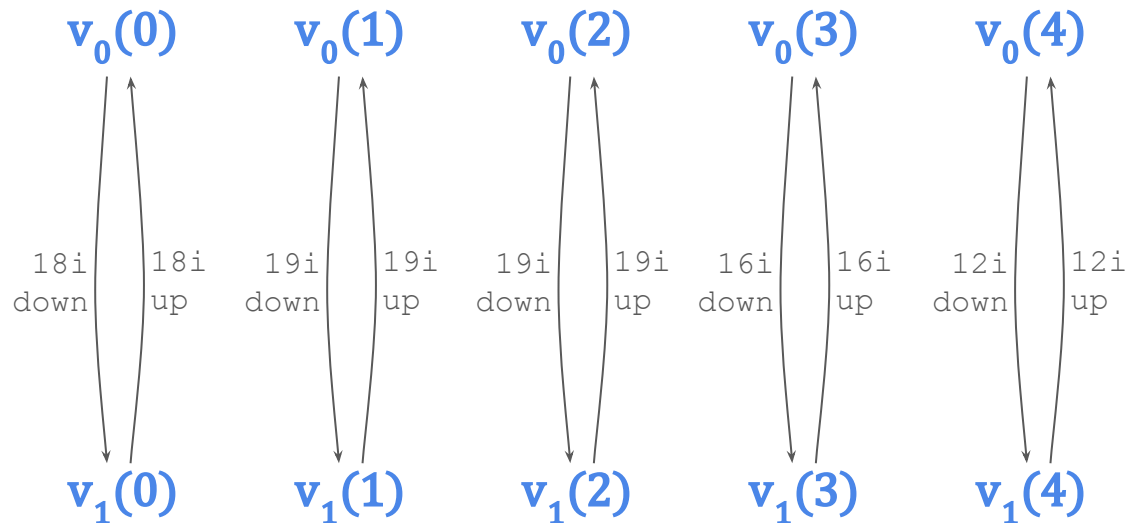
$v_1(3)$

$v_1(4)$

Diads

In Satie: diads-of(<voice1>, <voice2>) [at <time>]

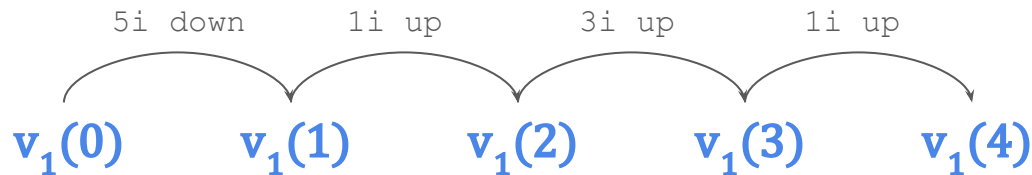
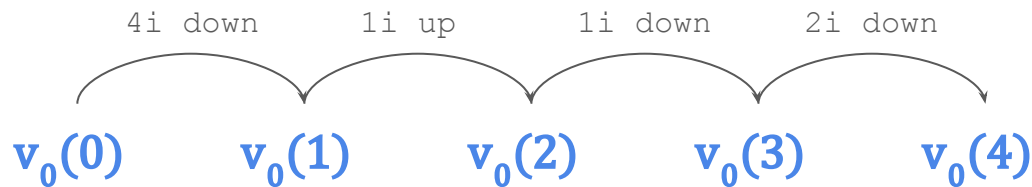
- The "vertical" intervals between two voices



Contours

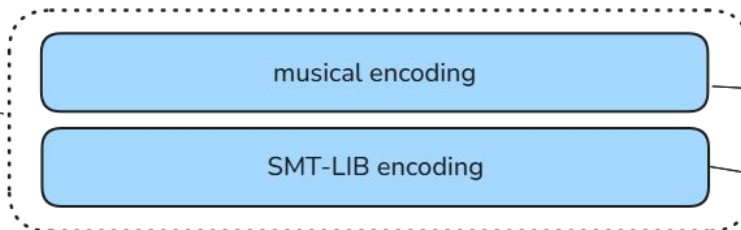
In Satie: contour-of <voice> [at <time>]

- The "horizontal shape" of a particular voice



Layers of Abstraction

Meant to be able to encode "all" music. In practice, we support a twelve-tone system. Currently supports one note per voice at a time.

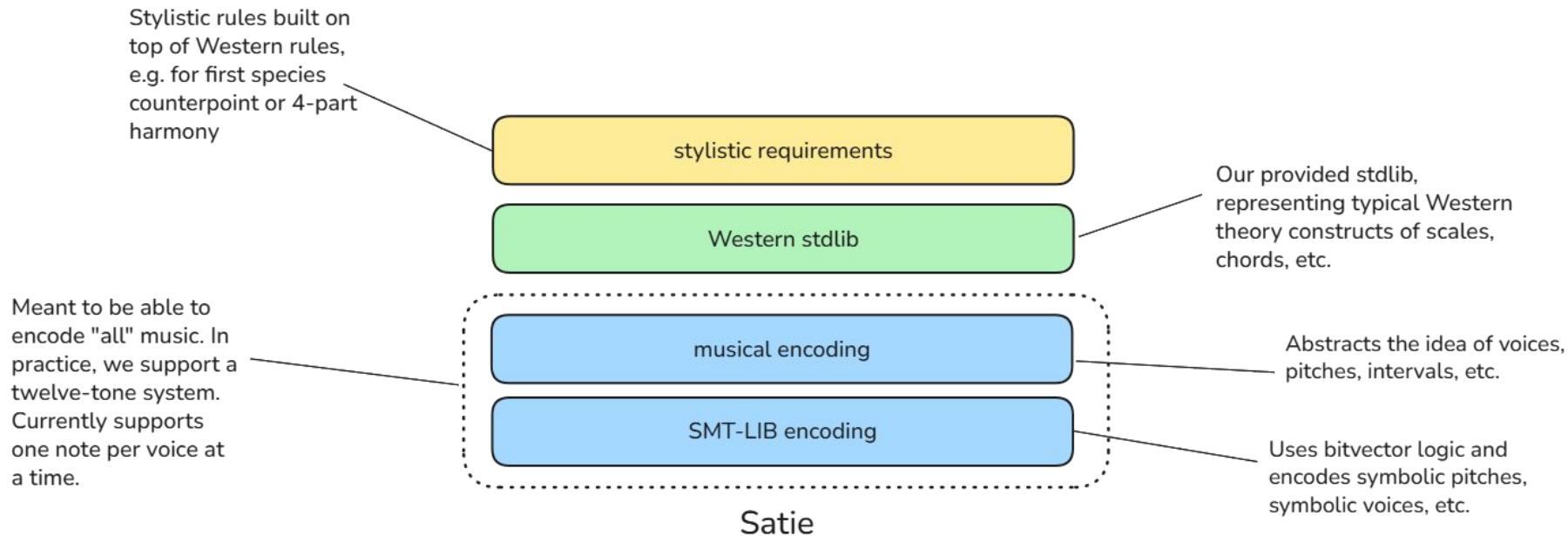


Satie

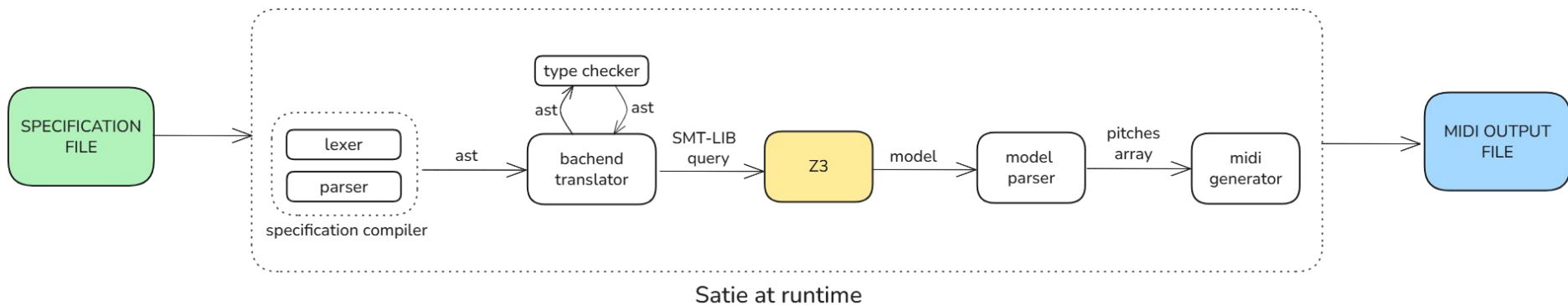
Abstracts the idea of voices, pitches, intervals, etc.

Uses bitvector logic and encodes symbolic pitches, symbolic voices, etc.

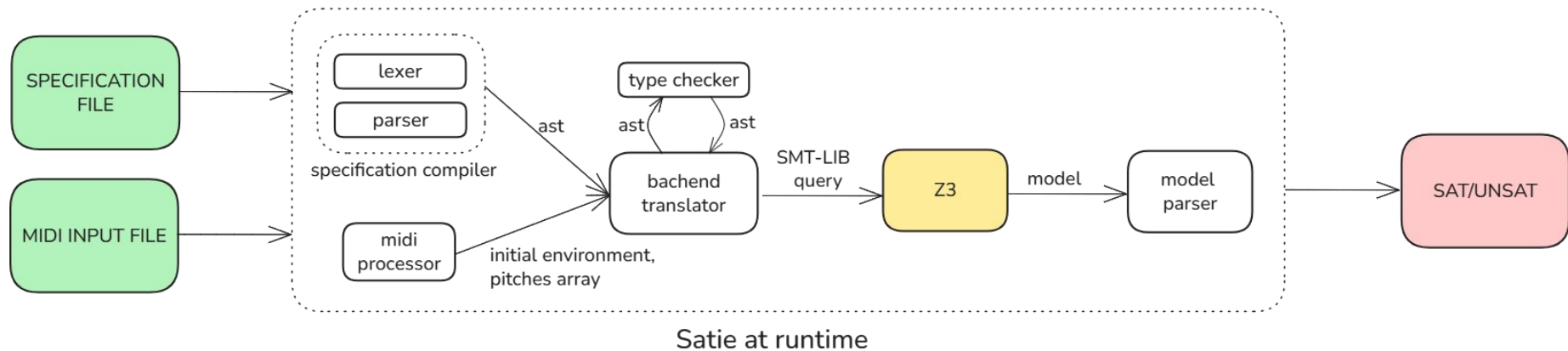
Layers of Abstraction



Synthesis Pipeline



Verification Pipeline



Demo!