1 Questions

1. What is the probability of getting exactly 3 heads in 10 coin flips?
2. You have a biased coin with probability p of heads. What is the expected number of flips until you get heads?
3. Alice and Bob take turns rolling a fair 6-sided die. The first to roll a 6 wins. What is the probability that Alice wins if she goes first?
4. You randomly pick 3 cards from a standard 52-card deck. What is the probability of getting 3 of a kind?
5. A jar contains 10 red marbles, 20 blue marbles, and 30 green marbles. You draw marbles one at a time without replacement until you get a red one. What is the expected number of draws?
6. There are 100 closed lockers in a hallway. A person walks through and opens every locker. A second person then closes every 2nd locker. A third person toggles every 3rd locker (closes if open, opens if closed). This continues for 100 people. Which lockers are open at the end?
7. A stick is broken randomly into 3 pieces. What is the probability the pieces can form a triangle?
8. You flip a fair coin repeatedly until you get two heads in a row. What is the expected number of flips?
9. Two trains 200km apart are moving toward each other at 70km/h each. A bird flies back and forth between them at 120km/h until they collide. How far does the bird fly?
10. 100 passengers board a plane with assigned seats, but the first person sits in a random seat. Each subsequent passenger sits in their assigned seat if available, otherwise a random open seat. What is the probability the last passenger gets their assigned seat?
11. You have 12 balls, 11 weigh the same and 1 is slightly heavier. Using a balance scale, what is the minimum number of weighings needed to identify the heavy ball?
12. A clock loses 2 minutes every hour. What time will it show 6 hours after it is set to 12:00?
13. What is the probability of rolling at least one 6 with 6 dice?
14. You have two eggs and are in a 100-story building. You want to find the highest floor from which an egg can be dropped without breaking. What is the minimum number of drops needed to determine this in the worst case?
15. A group of 23 people is in a room. What is the probability that at least two people share the same birthday? (Assume 365 days in a year and ignore leap years)
16. Alice flips a fair coin 100 times. Bob flips a fair coin 200 times. What is the probability Alice gets more heads than Bob?
17. You throw a dart randomly at a square dartboard. Given that the dart hits the board, what is the probability it lands closer to the center than to any edge?
18. A stick of length 1 is broken into three pieces at random. What is the probability that the length of the longest piece is less than 1/2?

19. You have 3 boxes: one contains 2 gold coins, one contains 2 silver coins, and one contains 1 gold and 1 silver coin. The boxes are labeled, but all labels are incorrect. You can draw one coin from one box. What is the minimum number of draws needed to determine the contents of all boxes?

20. What is the covariance between X and Y if $Var(X) = 4$, $Var(Y) = 9$, and $Var(X+Y) = 25$?

21. If X and Y are independent standard normal random variables, what is the probability that $X^2 + Y^2 < 1$?

22. A linear regression is performed on a dataset and the coefficient of determination (R^2) is found to be 0.64. What does this mean?

23. In a simple linear regression, what is the relationship between the correlation coefficient r and the slope b of the regression line?

24. What is the difference between the sample covariance and population covariance formulas? Why is there a difference?

25. Explain the concept of heteroscedasticity in linear regression. How can it be detected and addressed?

26. What is multicollinearity in multiple linear regression? How can it be detected and what are its consequences?

27. Explain the difference between Type I and Type II errors in hypothesis testing.

28. What is the central limit theorem and why is it important in statistics?

29. Explain the concept of maximum likelihood estimation. How is it used to estimate parameters?

30. What is the difference between a parametric and non-parametric statistical test? Give an example of each.

31. Explain the concept of p-value in hypothesis testing. What are some common misinterpretations of p-values?

32. What is the difference between a confidence interval and a prediction interval in linear regression?

33. Explain the concept of regularization in linear regression (e.g. Ridge, Lasso). Why is it used?

34. In a simple linear regression, derive the least squares estimators for the slope and intercept.

35. Prove that in a simple linear regression, the least squares line always passes through the point $(\bar{x}, \bar{y})$.

36. In a simple linear regression, show that the residuals are uncorrelated with the predicted values.

37. Derive the formula for R^2 in terms of SST, SSR, and SSE in a simple linear regression.

38. Prove that in a multiple linear regression, the sum of the residuals is always zero when an intercept term is included.

39. In a multiple linear regression, show that the residuals are orthogonal to each predictor variable.

40. Derive the normal equations for multiple linear regression using matrix notation.

41. Prove that the least squares estimator $\hat{\beta} = (X'X)^{-1}X'y$ is unbiased in multiple linear regression.

42. Derive the variance-covariance matrix of the least squares estimator in multiple linear regression. I apologize for the oversight. Here are the 10 challenging probability questions in LaTeX

43. format:

44. Consider a stick of length 1. It is broken at a random point, creating two pieces. The longer piece is then broken again at a random point. What is the expected length of the shortest of the three resulting pieces?

45. In a room of n people, each person randomly chooses another person to shake hands with. What is the expected number of people who will not shake hands?

46. You have an urn with w white balls and b black balls. You draw balls without replacement until you draw a white ball. What is the expected number of black balls drawn?

47. A points to B with probability p and to C with probability 1-p. B points to A with probability q and to C with probability 1-q. C always points to A. If you start at a random point and follow the pointing, what is the probability you end up at A?

48. You roll a fair 6-sided die repeatedly. What is the expected number of rolls until you see a 6 followed by a number less than 6?

49. Consider an infinite sequence of independent coin flips, each with probability p of heads. What is the expected number of flips until you see a run of k consecutive heads?

50. In a group of n people, everyone writes their name on a slip of paper and puts it in a hat. The slips are then randomly redistributed to the group. What is the probability that no one receives their own name?

51. You have n fair coins. You flip them all at once. You remove all coins that came up heads and flip the remaining coins. You repeat this process until no coins are left. What is the expected number of rounds needed?

52. A deck of 52 cards is shuffled. You start turning over cards from the top. What is the expected number of cards you need to turn over to see the first Ace?

53. Two players take turns removing 1, 2, or 3 matches from a pile that starts with n matches. The player who takes the last match loses. For what values of n does the first player have a winning strategy?

54. You play a game where you draw cards from a standard 52-card deck without replacement until you draw an ace. You win 10 for each card drawn, including the ace. What is the expected value of this game?

2 Solutions

1. The probability of getting exactly 3 heads in 10 coin flips can be calculated using the binomial probability formula: $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$ Where n = 10 (total number of flips), k = 3 (number of heads), p = 0.5 (probability of heads on a single flip) $P(X = 3) = \binom{10}{3}(0.5)^3(0.5)^7 = 120 \times 0.125 \times 0.0078125 = 0.1171875$ So the probability is approximately 0.1172 or 11.72

2. Let X be the number of flips until the first head. X follows a geometric distribution with parameter p. The expected value of a geometric distribution is given by: $E[X] = \frac{1}{p}$ So the expected number of flips until you get heads is $\frac{1}{p}$.

3. Let p be the probability that Alice wins. We can set up the following equation: $p = \frac{1}{6} + \frac{5}{6}(1-p)$ The first term represents Alice winning on her first roll. The second term represents Alice not winning on her first roll (probability 5/6) and then Bob not winning on his roll (probability 5/6) and the game returning to Alice's turn. Solving for p: $p = \frac{1}{6} + \frac{5}{6} - \frac{5p}{6}$ $\frac{11p}{6} = \frac{11}{6}$ $p = \frac{6}{11} \approx 0.5455$ So Alice has about a 54.55

4. To get 3 of a kind, we need to choose 1 of the 13 ranks, then choose 3 of the 4 cards of that rank, and then choose 0 of the remaining 48 cards. The probability is: $\frac{\binom{13}{1}\binom{4}{3}\binom{48}{0}}{\binom{52}{3}}$ $= \frac{13 \times 4 \times 1}{22100} = \frac{52}{22100} = \frac{1}{425} \approx 0.00235$ So the probability is about 0.235

5. Let's approach this step-by-step:

   First, calculate the probability of drawing a red marble on each draw:

   1st draw: 10/60 2nd draw: 10/59 3rd draw: 10/58 ...and so on.

   The probability of NOT drawing a red marble is the complement of these probabilities. The expected number of draws is: $E[X] = 1 \cdot P(X = 1) + 2 \cdot P(X = 2) + 3 \cdot P(X = 3) + ...$ $= \frac{10}{60} + 2 \cdot \frac{50}{60} \cdot \frac{10}{59} + 3 \cdot \frac{50}{60} \cdot \frac{49}{59} \cdot \frac{10}{58} + ...$ This series can be simplified to: $E[X] = \frac{60}{10} = 6$

   Therefore, the expected number of draws is 6.

6. A locker will be open at the end if and only if it has an odd number of factors. Perfect squares are the only numbers with an odd number of factors. This is because factors come in pairs (if a is a factor of n, then n/a is also a factor of n), except for the square root of a perfect square, which pairs with itself. The perfect squares less than or equal to 100 are: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100. Therefore, lockers 1, 4, 9, 16, 25, 36, 49, 64, 81, and 100 will be open at the end.

7. This problem can be solved using geometry. Let the lengths of the three pieces be x, y, and z. For these to form a triangle, the sum of any two sides must be greater than the third side. This gives us three conditions: