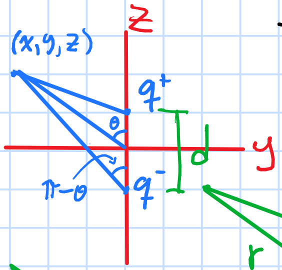
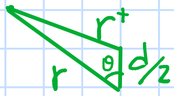


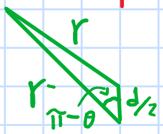
$$\vec{P} = P_0\hat{r} = q\vec{d}$$

$$\varphi_T = \varphi^+ + \varphi^- \qquad \varphi^+ = \frac{1}{4\pi\epsilon_0}\cdot\frac{+q}{r^+} \qquad \varphi^- = \frac{1}{4\pi\epsilon_0}\cdot\frac{-q}{r^-}$$



$$r^+ = \sqrt{\cancel{d^2/4}_{\to 0} + r^2 - dr\cos\theta}$$

$$r^- = \sqrt{d^2/4 + r^2 - dr\cos(\pi-\theta)} = \sqrt{\cancel{d^2/4}_{\to 0} + r^2 + dr\cos\theta}$$



$$r >> d$$

$$r^+ = r\sqrt{1 - \frac{d}{r}\cos\theta} \qquad r^- = r\sqrt{1 + \frac{d}{r}\cos\theta}$$

$$\varphi_T = \frac{q}{4\pi\epsilon_0 r}\left(\overset{(1)}{\frac{1}{\sqrt{1-\frac{d}{r}\cos\theta}}} - \overset{(2)}{\frac{1}{\sqrt{1+\frac{d}{r}\cos\theta}}}\right) \qquad (1+x)^n \approx 1+nx$$

$$\varphi_T = \frac{q}{4\pi\epsilon_0 r}\left(1 + \frac{d}{2r}\cos\theta - 1 + \frac{d}{2r}\cos\theta\right) = \frac{q}{4\pi\epsilon_0 r^2}\cdot d\cos\theta = \frac{P_0\cos\theta}{4\pi\epsilon_0 r^2} \text{ V}$$

$$\vec{E} = -\nabla\varphi = -\frac{\partial\varphi}{\partial r}\hat{r} - \frac{1}{r}\frac{\partial\varphi}{\partial\theta}\hat{\theta} = \frac{P_0}{4\pi\epsilon_0}\left(\frac{2\cos\theta}{r^3}\hat{r} + \frac{\text{sen}\,\theta}{r^3}\hat{\theta}\right) \text{ V}$$

$$= \frac{P_0}{4\pi\epsilon_0}\left(\frac{2\cos\theta}{r^3}(\text{sen}\,\theta\cos\varphi\,\hat{\imath} + \text{sen}\,\theta\,\text{sen}\,\varphi\,\hat{\jmath} + \cos\theta\,\hat{k}) + \frac{\text{sen}\,\theta}{r^3}(\cos\theta\cos\varphi\,\hat{\imath} + \cos\theta\,\text{sen}\,\varphi\,\hat{\jmath} - \text{sen}\,\theta\,\hat{k})\right)$$

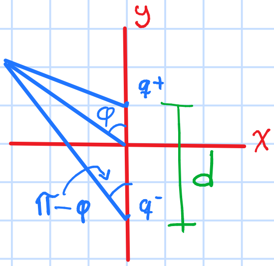
$$= \frac{P_0 r}{4\pi\epsilon_0 r^4}\left[(2\overset{z}{\cos\theta}\,\overset{x}{\text{sen}\,\theta\cos\varphi} + \overset{x}{\text{sen}\,\theta\,\cos\varphi}\,\overset{z}{\cos\theta})\hat{\imath} + (2\overset{z}{\cos\theta}\,\overset{y}{\text{sen}\,\theta\,\text{sen}\,\varphi} + \overset{y}{\text{sen}\,\theta\,\text{sen}\,\varphi}\,\overset{z}{\cos\theta})\hat{\jmath} + \dots\right.$$

$$\left.\dots + (2\cos^2\theta - \text{sen}^2\theta)\hat{k}\right]$$

$$\frac{r^2}{x^2+y^2+z^2}$$

$$= \frac{P_0}{4\pi\epsilon_0 r^4}\left[(3xz)\hat{\imath} + (3zy)\hat{\jmath} + (2\cos^2\theta - \text{sen}^2\theta)\hat{k}\right]$$

$$\frac{2z}{x^2+y^2+z^2} - \frac{x^2+y^2}{x^2+y^2+z^2}$$



$$\varphi^{+} = \frac{1}{4\pi\epsilon_0}\cdot\frac{q^{+}}{r^{+}} \qquad \varphi^{-} = \frac{1}{4\pi\epsilon_0}\cdot\frac{{}^{-}q}{r^{-}}$$

$$r^{+} = \sqrt{\cancel{d^2/4} + r^2 - dr\cos\varphi}$$

$$r^{-} = \sqrt{\cancel{d^2/4} + r^2 - dr\cos\varphi} = \sqrt{r^2 + dr\cos\varphi}$$

$r >> d$

$$\phi_{+} = \frac{q}{4\pi\epsilon_0 r^2}\cdot d\cos\varphi = \frac{P_0\cos\varphi}{4\pi\epsilon_0 r^2} \checkmark$$

$$\vec{E} = -\nabla\phi = -\frac{\partial\phi}{\partial r}\hat{r} - \frac{1}{r\,\mathrm{sen}\theta}\frac{\partial\phi}{\partial\varphi}\hat{\varphi} = \frac{P_0}{4\pi\epsilon_0}\left(\frac{2\cos\varphi}{r^3}\hat{r} + \frac{\mathrm{sen}\varphi}{r^3\mathrm{sen}\theta}\hat{\varphi}\right)$$

$$= \frac{P_0}{4\pi\epsilon_0}\left(\frac{2\cos\varphi}{r^3}(\mathrm{sen}\theta\cos\varphi\,\hat{\imath} + \mathrm{sen}\theta\,\mathrm{sen}\varphi\,\hat{\jmath} + \cos\theta\,\hat{k}) + \frac{\mathrm{sen}\varphi}{r^3\mathrm{sen}\theta}(-\mathrm{sen}\varphi\,\hat{\imath} + \cos\varphi\,\hat{\jmath})\right)$$

$$= \frac{P_0}{4\pi\epsilon_0}\left(\left(\frac{r\,2\cos^2\varphi\,\mathrm{sen}\theta}{r^{\cancel{3}4}} - \frac{\mathrm{sen}^2\varphi}{r^3\mathrm{sen}\theta}\right)\hat{\imath} + \left(\frac{r\,2\cos\varphi\,\mathrm{sen}\theta\,\mathrm{sen}\varphi}{r^{\cancel{3}4}} + \frac{\mathrm{sen}\varphi\cos\varphi}{r^3\mathrm{sen}\theta}\right)\hat{\jmath} + \frac{r\,2\cos\theta\cos\varphi}{r^{\cancel{3}4}}\hat{k}\right)$$

$x \quad x \quad z$

$$= \frac{P_0}{4\pi\epsilon_0}\left(\left(\frac{2\cos\varphi}{r^4\;r\,\mathrm{sen}\theta}x - \frac{\mathrm{sen}\varphi\,\mathrm{sen}\varphi}{r^3\mathrm{sen}\theta\;r\,\mathrm{sen}\theta}\right)\hat{\imath} + \left(\frac{2\,\mathrm{sen}\varphi}{r^4\;r\,\mathrm{sen}\theta}x + \frac{\mathrm{sen}\varphi\cos\varphi}{r^3\mathrm{sen}\theta\;r\,\mathrm{sen}\theta}\right)\hat{\jmath} + \frac{2\cos\varphi}{r^4\;r\,\mathrm{sen}\theta}\hat{k}\right)$$

$$= \frac{P_0}{4\pi\epsilon_0}\left(\left(\frac{2x^2}{r^4\sqrt{x^2+y^2}} - \frac{y\,\mathrm{sen}\varphi}{x^2+y^2}\right)\hat{\imath} + \left(\frac{2yx}{r^4\sqrt{x^2+y^2}} + \frac{y\cos\varphi}{r^2(x^2+y^2)}\right)\hat{\jmath} + \frac{2x}{r^4\sqrt{x^2+y^2}}\hat{k}\right)$$

$$= \frac{P_0}{4\pi\epsilon_0}\left(\left(\frac{2x^2}{r^4\rho} - \frac{y^2}{\rho\cdot\rho^2}\right)\hat{\imath} + \left(\frac{2yx}{r^4\rho} + \frac{yx}{r^2\rho^2\rho}\right)\hat{\jmath} + \frac{2x}{r^4\rho}\hat{k}\right)$$

$$r = \sqrt{x^2+y^2+z^2} \qquad \rho = \sqrt{x^2+y^2}$$

$\vec{P} = K e^{-s^2/\alpha_0}$ $\qquad \vec{D} = \epsilon_0 \vec{E} + \vec{P} \qquad \vec{D} = 0$

$\oint_s \vec{D} \cdot d\vec{s} = q_f = 0$ $\qquad \vec{E} = \dfrac{-\vec{P}}{\epsilon_0} = \dfrac{-K e^{-s^2/\alpha_0}}{\epsilon_0} \hat{\rho}$ $\quad a \leq s < b$

No hay cargas libres

Solo hay campo entre a y b porque solo ahí hay material dieléctrico

$$\vec{E} = \frac{-K e^{-s^2/\alpha_0}}{\epsilon_0} \hat{\rho} = \frac{-K e^{-s^2/\alpha_0}}{\epsilon_0} (\cos\varphi \hat{\imath} + \sin\varphi \hat{\jmath}) * \frac{\rho}{\rho} = \frac{-K e^{-s^2/\alpha_0}}{\epsilon_0} \left(\frac{x}{\rho} \hat{\imath} + \frac{y}{\rho} \hat{\jmath} \right)$$

$$= \frac{-K e^{-s^2/\alpha_0}}{\epsilon_0 \sqrt{x^2+y^2}} (x \hat{\imath} + y \hat{\jmath}) \qquad s = \sqrt{x^2+y^2} \qquad \vec{E} = \frac{-K e^{-\frac{x^2+y^2}{\alpha_0}}}{\epsilon_0 \sqrt{x^2+y^2}} (x \hat{\imath} + y \hat{\jmath})$$

$$\rho_p = -\nabla \cdot \vec{P} = \frac{-1}{s} \frac{\partial (s \vec{P}_s)}{\partial s} = \frac{-1}{s} \frac{\partial}{\partial s} \left(s K e^{-s^2/\alpha_0} \right) = \frac{-1}{s} \left(K e^{-s^2/\alpha_0} - \frac{2 e^{-s^2/\alpha_0} s^2}{\alpha_0} \right)$$

$$q = \int_v \rho \, dv = -2\pi L \int_a^b \left(\frac{K e^{-s^2/\alpha_0}}{s} - \frac{2 e^{-s^2/\alpha_0} s}{\alpha_0} \right) ds$$

$$\left[\frac{1}{2} Ei\left(\frac{-s^2}{\alpha_0} \right) - e^{-s^2/\alpha_0} \right]_a^b$$

$$= 2\pi L \left[\frac{1}{2} Ei\left(\frac{-b^2}{\alpha_0} \right) - \frac{1}{2} Ei\left(\frac{-a^2}{\alpha_0} \right) - e^{-b^2/\alpha_0} + e^{-a^2/\alpha_0} \right] C$$