General form of 2nd order Taylor Series around $x=a$.

$$\Rightarrow f(a) + f'(a)(x-a) + O(x^2)$$

In our case,

$$\underbrace{w_i(t+1)\Big|_{\varepsilon=0}}_{(i)} + \underbrace{\frac{\partial}{\partial \varepsilon} w_i(t+1)\Big|_{\varepsilon=0}}_{(ii)} \cdot \varepsilon + O(\varepsilon^2)$$

$$(i) \rightarrow \quad w_i(t+1)\Big|_{\varepsilon=0} = \frac{w_i(t)}{\left[\sum_{j=1}^{N}(w_j(t))^2\right]^{1/2}} = w_i(t)$$

$(ii) \rightarrow ①$

$$\frac{\partial}{\partial \varepsilon} w_i(t+1) = \frac{\partial}{\partial \varepsilon} \frac{w_i(t) + \varepsilon y(t) x_i(t)}{\left(\sum_{j=1}^{N}[w_j(t) + \varepsilon y(t) x_j(t)]^2\right)^{1/2}}$$

$$= \frac{y(t) x_i(t)}{\left(\sum_{j=1}^{N}[w_j(t) + \varepsilon y(t) x_j(t)]^2\right)^{1/2}} + \left(w_i(t) + \varepsilon y(t) x_i(t)\right)\left(-\frac{1}{2}\right) \frac{\cancel{2} \cdot \sum_{j=1}^{N}[w_j(t) + \varepsilon y(t) x_j(t)] \cdot (y(t) x_j(t)}{\left(\sum_{j=1}^{N}[w_j(t) + \varepsilon y(t) x_j(t)]^2\right)^{3/2}}$$

(ii) → 2

$$\frac{\partial}{\partial \varepsilon} w_i(t+1)\Big|_{\varepsilon=0}$$

$$= \frac{y(t)\, x_i(t)}{\left(\sum_{j=1}^{N} [w_j(t)]^2\right)^{1/2}} - w_i(t) \cdot \frac{\sum_{j=1}^{N} w_j(t) \cdot y(t)\, x_j(t)}{\left(\sum_{j=1}^{N} [w_j(t)]^2\right)^{3/2}}$$

$$= y(t)\, x_i(t) - y(t)\, w_i(t) \sum_{j=1}^{N} w_j(t)\, x_j(t)$$

$$= y(t)\, x_i(t) - y(t)\, w_i(t)\, y(t)$$

$$= y(t)\,(x_i(t) - y(t)\, w_i(t))$$

$$\therefore\ w_i(t+1) = w_i(t) + \varepsilon\, y(t)\,[x_i(t) - y(t)\, w_i(t)]$$

∎