Beyond Alternating Projections: Accelerated Alternating Linear Minimizations

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Hiermit erkläre ich, dass ich die vorliegende Arbeit selbstständig und eigenhändig sowie ohne unerlaubte fremde Hilfe und ausschließlich unter Verwendung der aufgeführten Quellen und Hilfsmittel angefertigt habe.

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Abstract

The alternating projections method involves finding a pair of points with minimal distance between two sets. While the resulting sequence ensures its convergence, the computational complexity of projection operators is potentially expensive, particularly in higher-dimensional problems. Addressing these challenges, Braun et al. (2023) introduced alternating linear minimizations (`ALM`), replacing projection oracles with linear minimization oracles (`LMO`). This algorithm inherits properties from `LMO`-based methods, such as sparsity of solutions and lower iteration costs. Despite the extensive research on `LMO`-based algorithms, little is known about `ALM` beyond its canonical convergence rate, which is generally non-improvable. This paper presents acceleration schemes for `ALM` by characterizing the geometry of various feasible regions. We propose that when a pair of points with minimal distance is unique, acceleration is possible regardless of how disjointed the two sets are. The results yield several accelerated convergence rates for `ALM`, enhancing its efficiency in solving feasibility problems.

References

Gábor Braun, Sebastian Pokutta, and Robert Weismantel. Alternating linear minimization: Revisiting von neumann's alternating projections, 2023.