# *Rotating Equipment and Order Tracking*

# What is order tracking?
# What is an order?

***Order Tracking*** is the analysis of a time varying frequency signal whose frequency is proportional to the speed of rotation of a machine.

$$X(t) = A(k,t)\sin(2\pi(k/p)t + \phi_k)$$

A(k,t) is the amplitude of order k as a function of time.

$f_k$ is the phase angle of order k.

p is the period of primary order in seconds.

t is time.

k is the order being tracked.

k = 0 DC offset.

k < 0 Negative frequencies

# *How do we talk about orders?*

- *Orders are always defined relative to a reference shaft's rpm.*
  - 1st order - event happens 1 time per revolution of reference shaft
  - 2nd order - event happens 2 times per revolution of reference shaft

  - 4 stroke engine
    - 4 cylinder - 2nd order is considered firing order
      - 2 pistons have combustion event each revolution of crankshaft
  - 2 stroke engine
    - 4 cylinder - 4th order is considered firing order
      - all 4 pistons have combustion event each revolution of crankshaft

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# *What Causes Orders?*

- Number of Pistons in AC Compressor ($N_{pistons}$*rpm/60).
- Number of Poles in Alternator ($N_{poles}$*rpm/60).
- Number of Vanes in Vane Pump ($N_{vanes}$*rpm/60).
- Number of Blades on a Fan ($N_{blades}$*rpm/60).
- Number of Gear Teeth ($N_{teeth}$*rpm/60).
- Shaft Unbalance (1*rpm/60).
- Looseness/Rubbing of Shaft (Harmonics of rpm/60).
- Shaft Misalignment.
  - 2*rpm/60.
  - Large axial vibration levels.

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# *More Orders!*

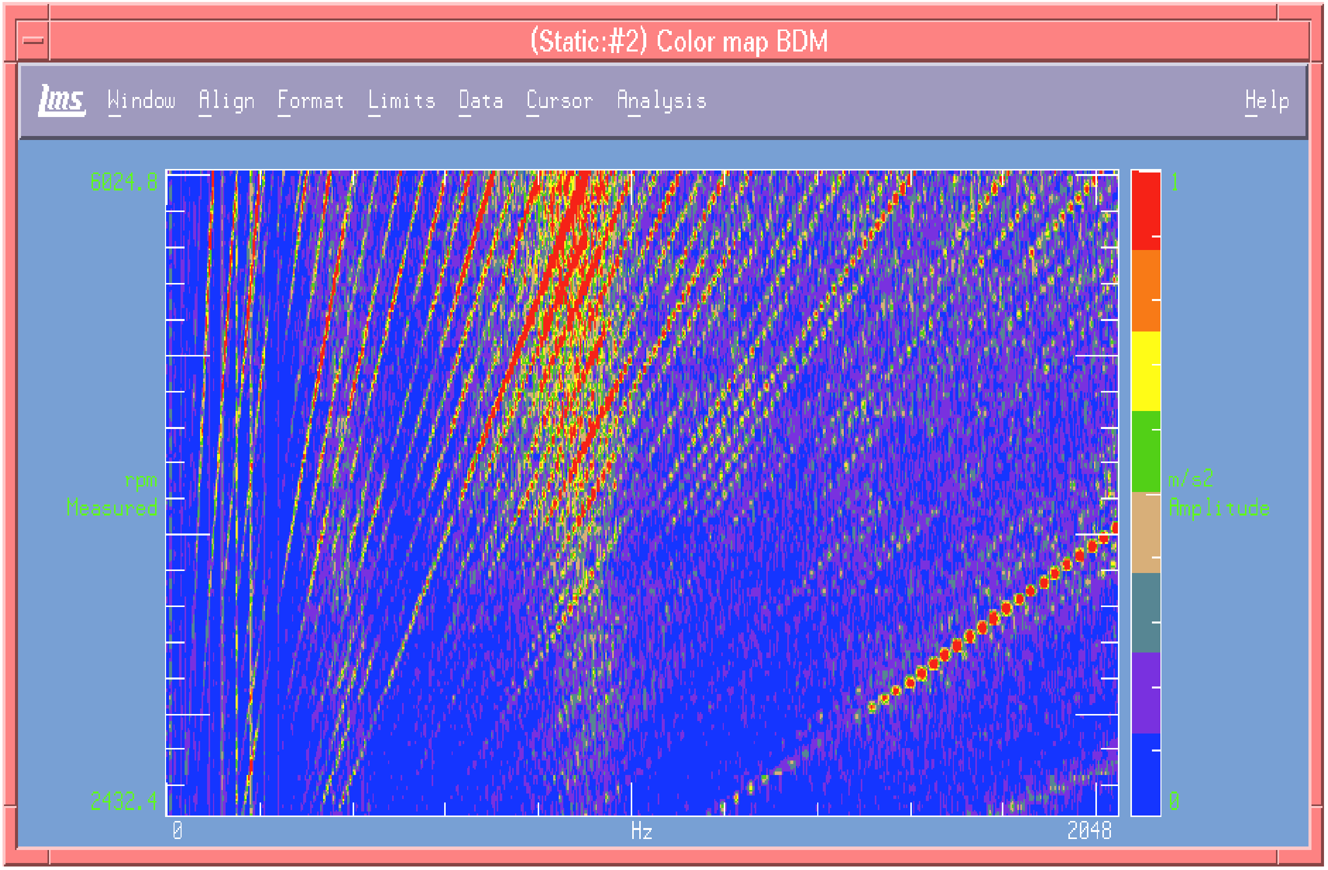
- Universal Joints.
  - 2*rpm/60.
  - Amplitude function of angle of Universal Joint.
- Misfiring Engine Cylinder or Cylinder to Cylinder Combustion Differences.
  - .5*rpm/60 in four cycle engine.
- Sidebands around gear mesh frequency usually appear at rpm/60 of bad gear.
- Gear defects produce large components at gear mesh frequency.
- Fluid Film Bearing faults typically appear at 0.43-0.48*rpm/60.

# *Still More Orders!!*

- Bearings.
  - Inner Race defect approximated at 0.6X # Balls *rpm/60.
  - Outer Race defect approximated at 0.4X # Balls *rpm/60.

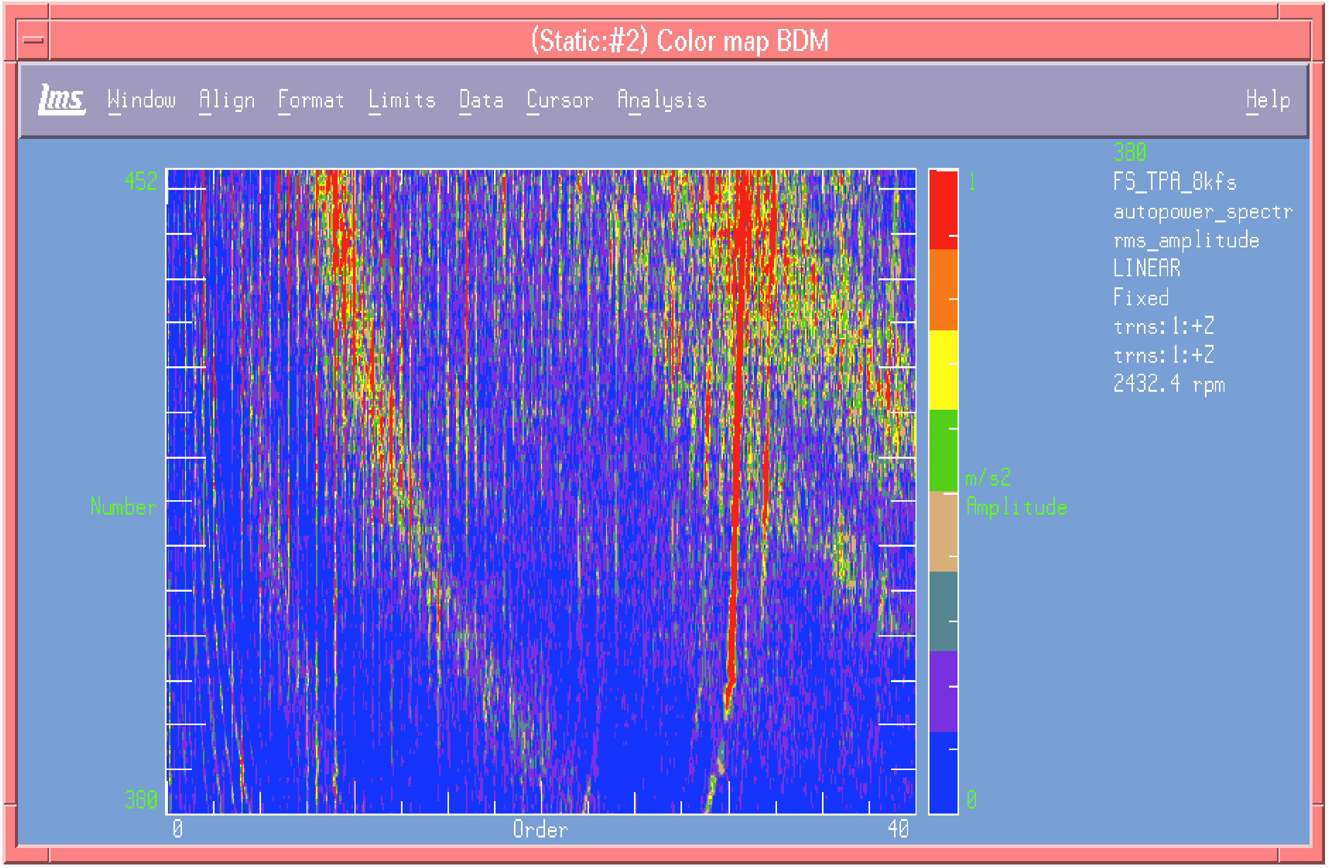
- ***Ghost Noise*** is the phenomena where a gear in the machine which manufactures another gear is worn, this wear causes an order to be generated which does not correlate with any physical component in the machine in which the gear is operating.  Through an analysis of the machinery used to produce the gear it is possible to determine the cause of this order.

# FFT Colormap, Frequency X-Axis.




## Constant Frequency - Vertical Line.

# FFT Colormap, Order X-Axis.




**Constant Order - Vertical Line.**

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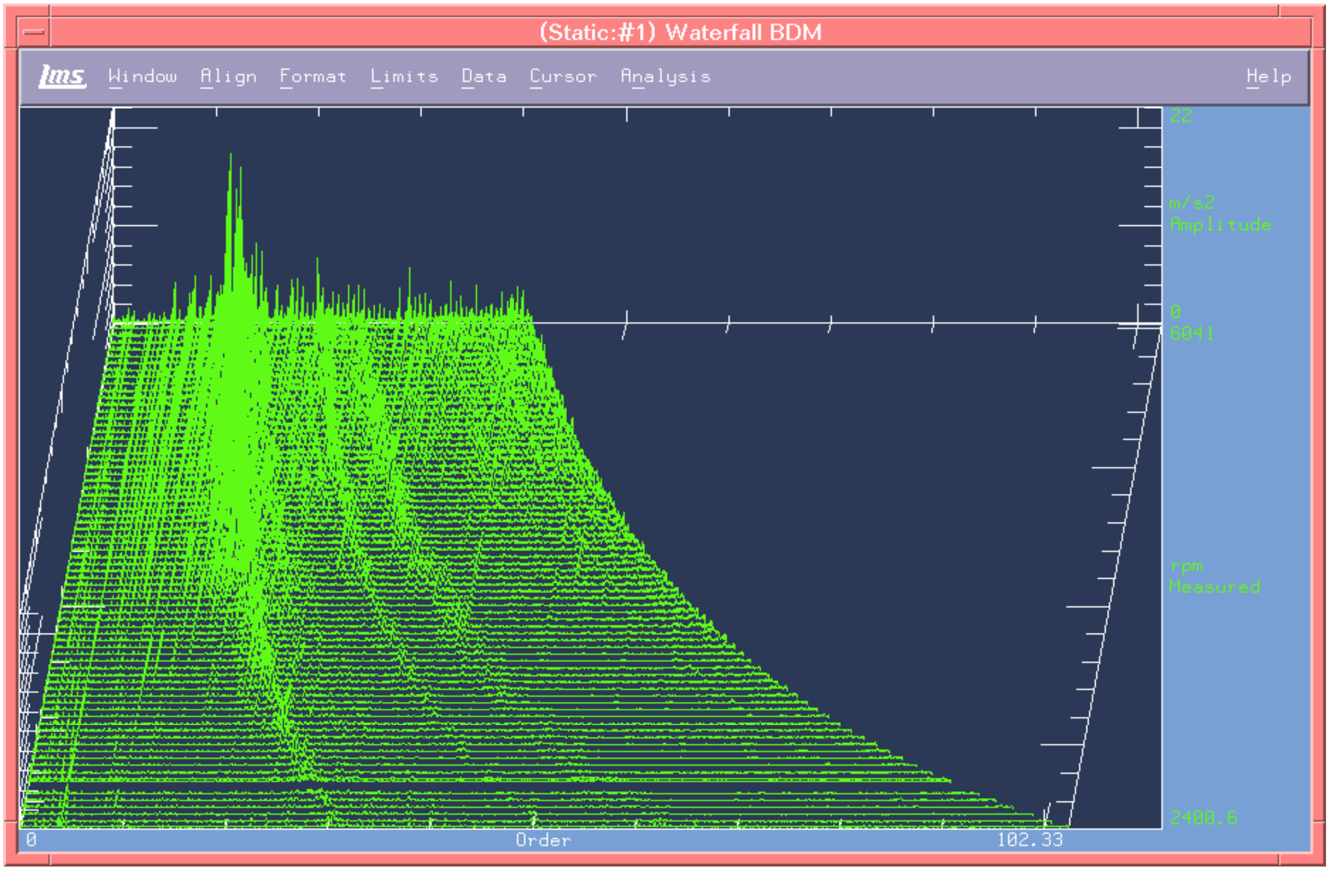
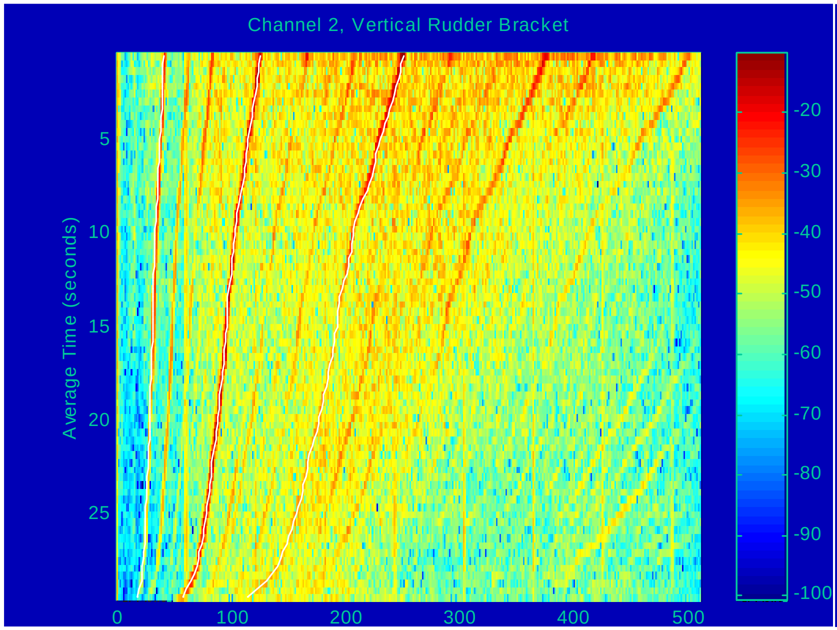
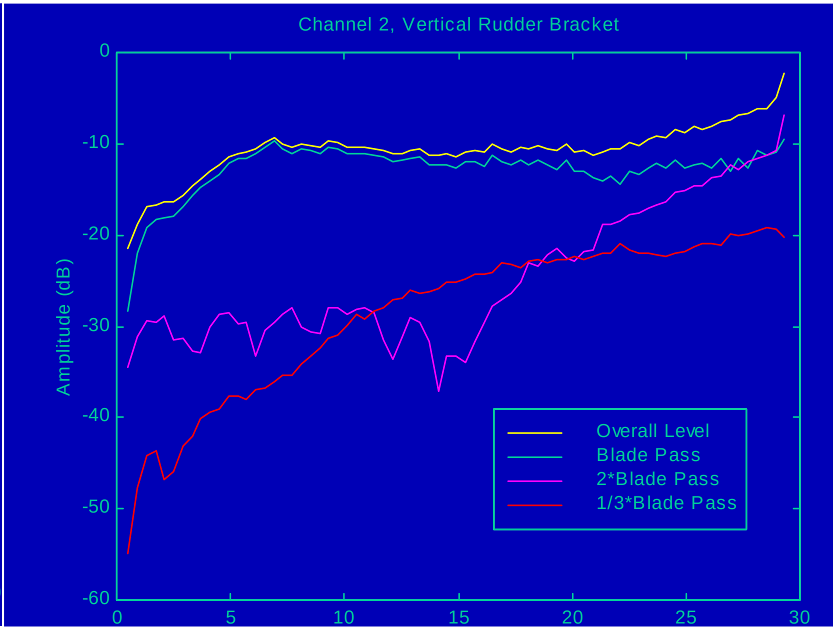
# FFT Waterfall, Order X-Axis.




***Note: Number of orders changes with RPM!***

# *Data Acquired on a Boat.*



# *Dynamometer Test Stand*

# Chassis Dynamometer

# *How do we Order Track?*

- Time Domain Sampling Based Fast Fourier Transform Order Analysis.
  - Based on standard FFT analysis.
- Angle Domain Computed Order Tracking: Time/Angle Domain Resampling.
  - Resample data from time domain to angle domain (either real-time or post-process.)
- Kalman Filter Based Order Tracking.
  - Based on Active Control type filter.
  - Interactive process which requires experience.
- Vold-Kalman Filter Based Order Tracking.
  - Multiple pole formulation.
  - Separation of close/crossing orders.
  - Improved tachometer processing.
- Frequency Domain Order Tracking (FRF Based)
- Miscellaneous methods…

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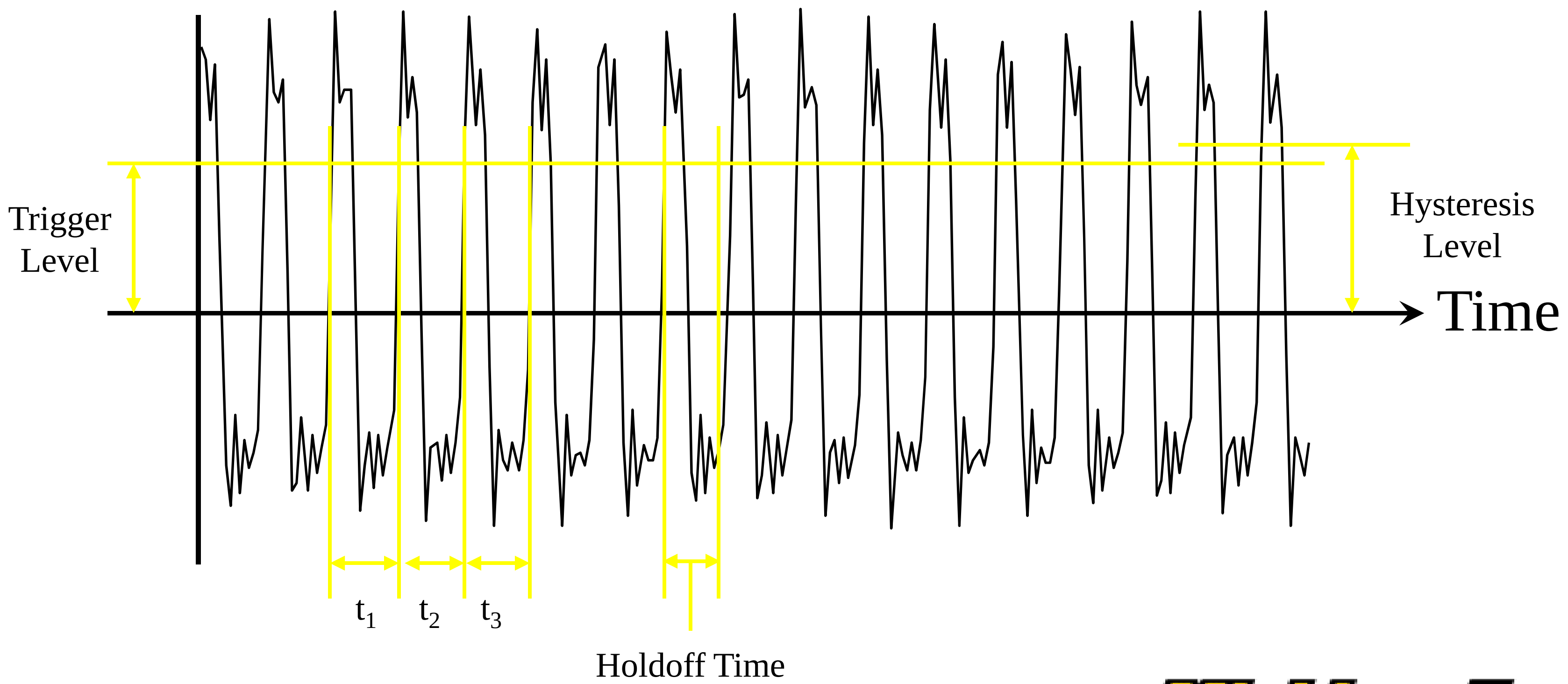
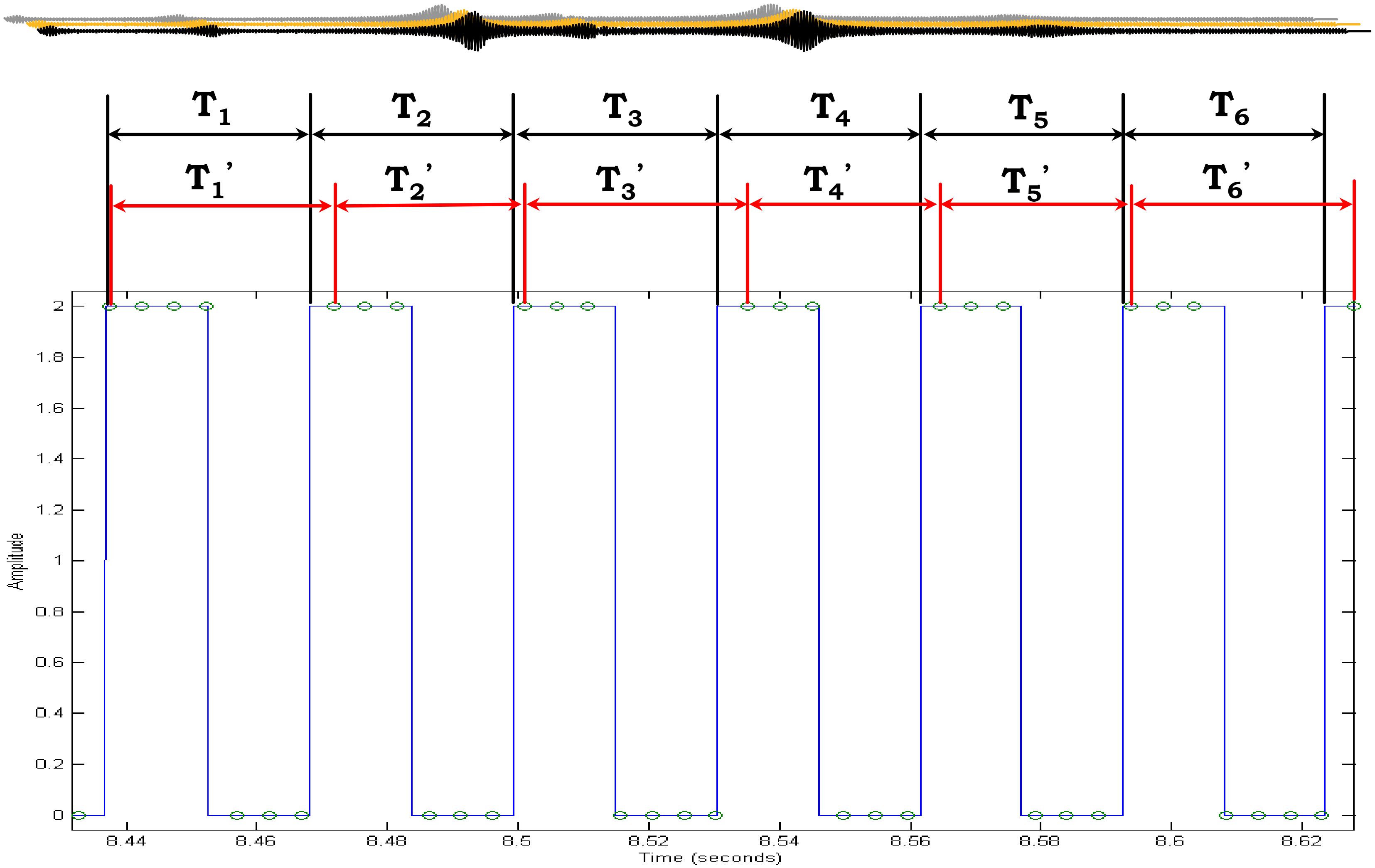
# Order Tracking Analysis (Order Cuts)

# *Processing Tachometer Signals.*

- Tachometer signal processing is very important since the resampling process is driven by this information.
  - Estimate the period between tachometer pulses.

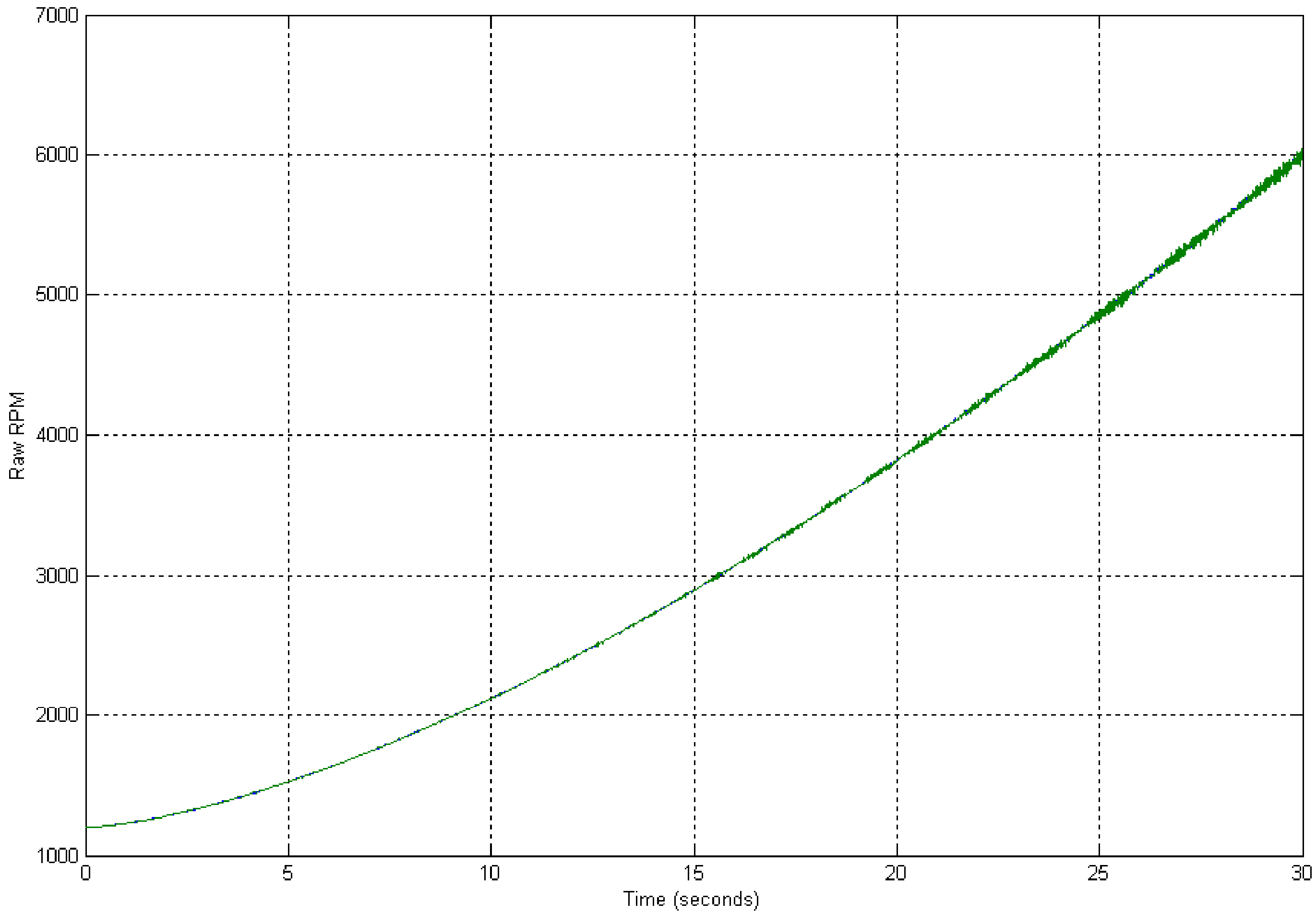


# *Sampled Tachometer Pulsetrain*

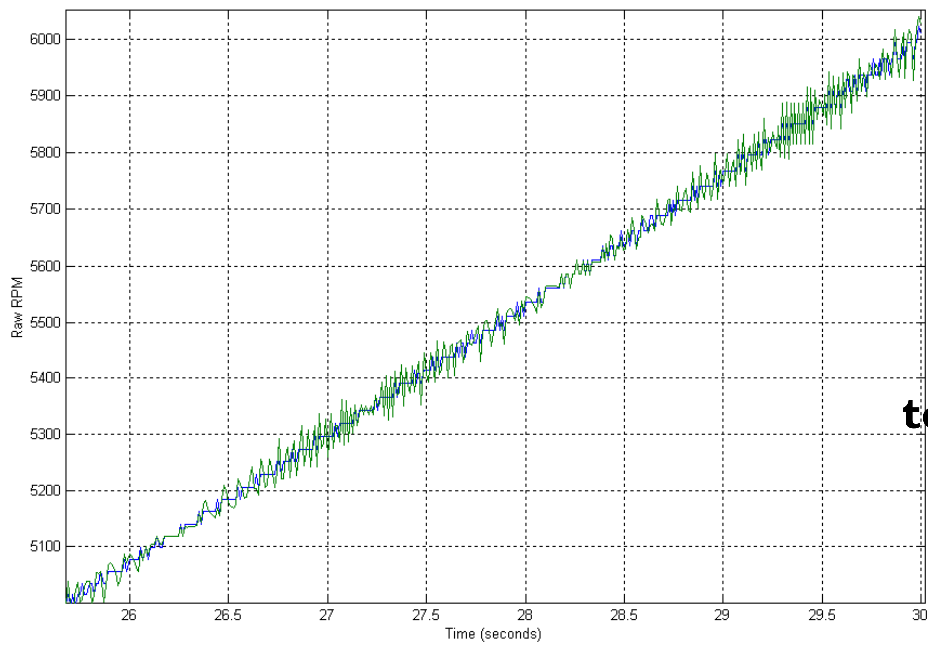



**Once sampled** $T_i \neq T_i'$

# Raw RPM Estimates from Tach Signal

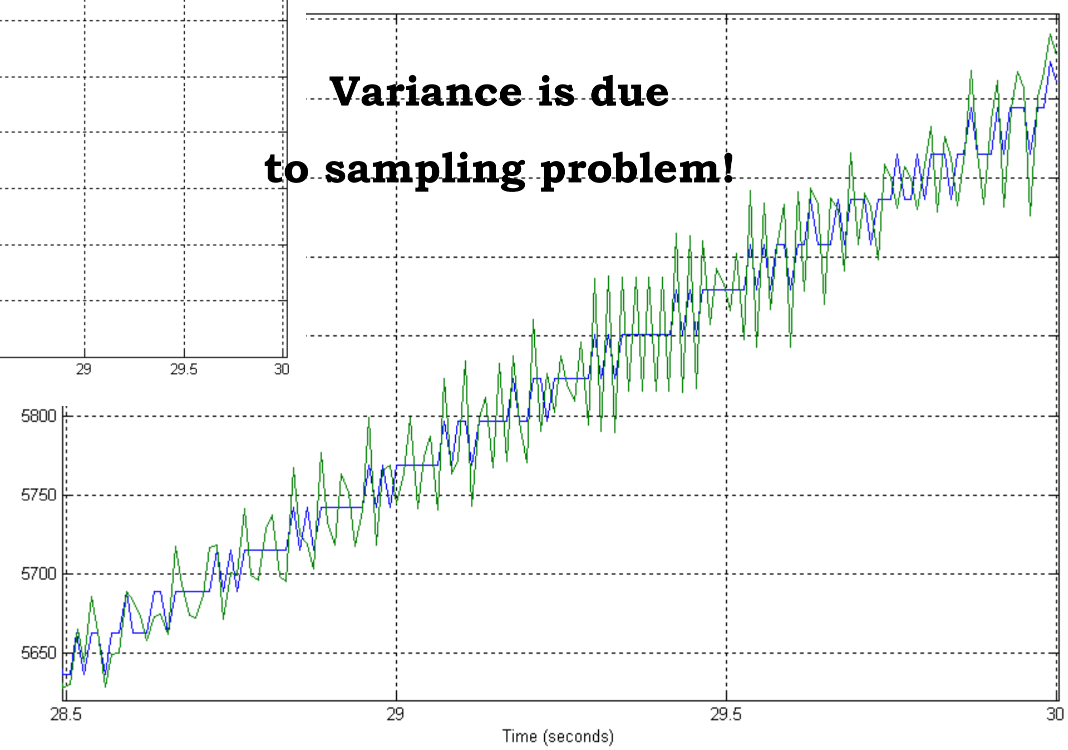


# *Raw RPM Estimates from Tach Signal*

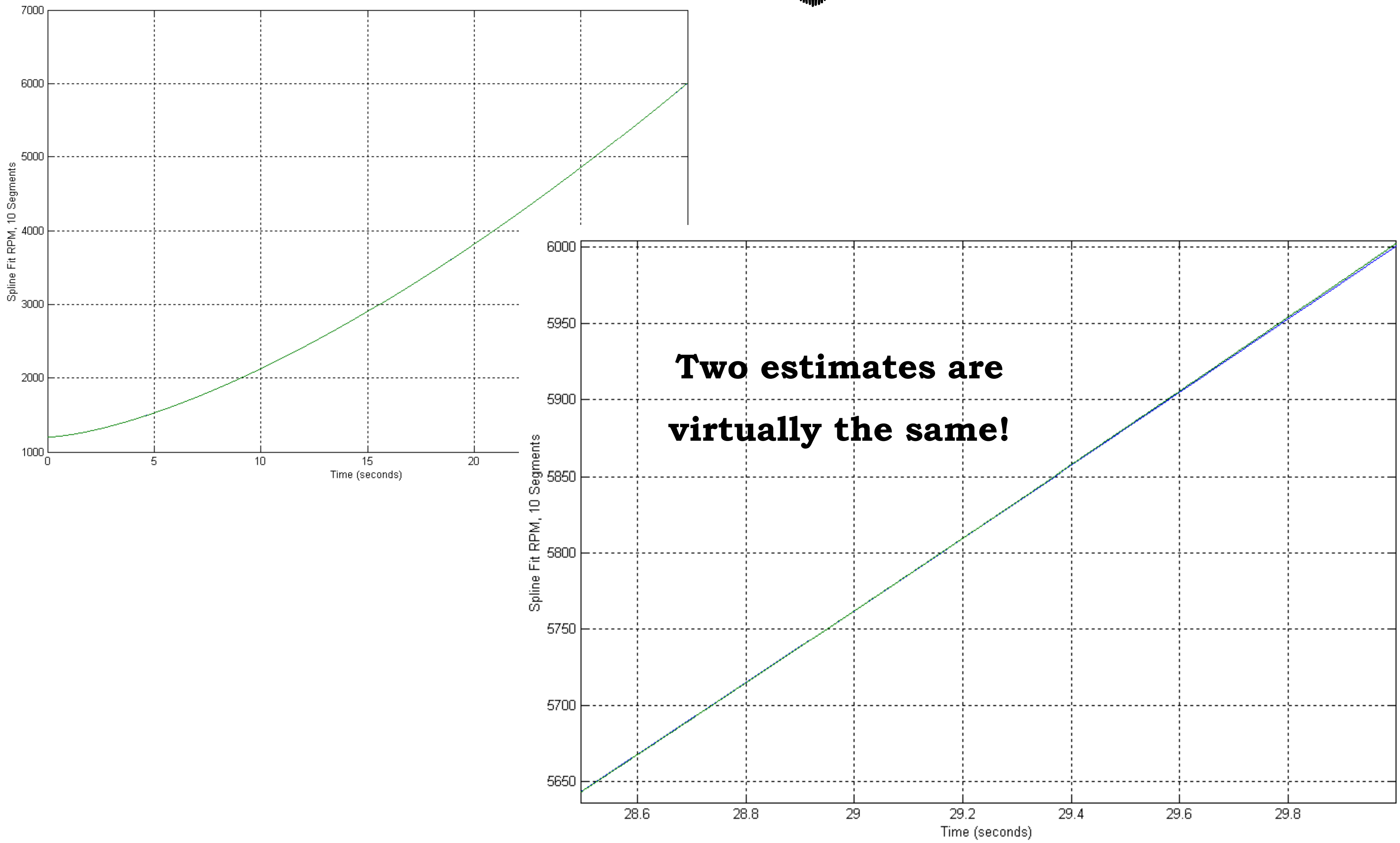



**Variance is due**

**to sampling problem!**



# *Spline Fit RPM Estimates*



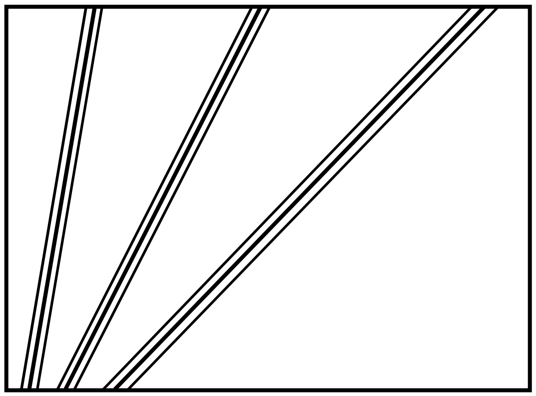

**Two estimates are virtually the same!**

# *FFT Order Tracking.*

- What is done to account for frequency change?
  - Determine frequency of order from tachometer signal.
  - Extract frequency bins which correspond to frequency of order.
    - Integrate over multiple spectral lines to account for frequency variation over transform time, T.
      - Use constant frequency, order, or percentage bandwidths.
      - Apply energy correction for window!

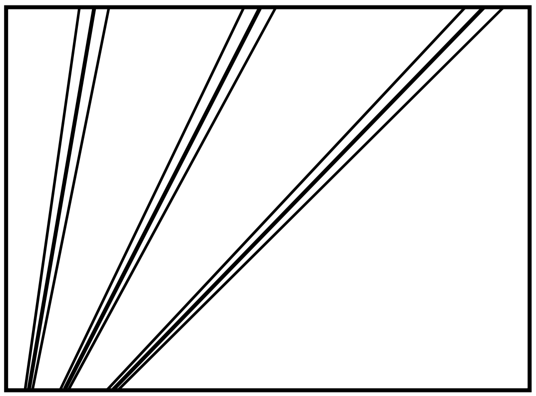
  - These strategies work well for slow sweep rates!
    - Fast sweep rates require more sophisticated methods!

# *FFT Bandwidths.*



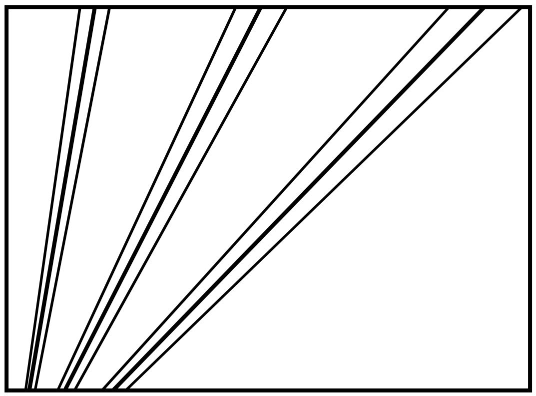
Constant Frequency Bandwidth

- Bandwidth is independent of order/frequency.



Constant Order Bandwidth

- Bandwidth is dependent on frequency of 1st order.



Constant Percentage Bandwidth

- Bandwidth is dependent on frequency.

# *FFT Sampling Relationships.*

- FFT based order tracking is the simplest order tracking method.
  - Based on standard FFT sampling laws.

$$\Delta f = \frac{1}{T} = \frac{1}{N * \Delta t}$$

$$T = N * \Delta t$$

$$F_{nyquist} = F_{\max} = \frac{F_{sample}}{2}$$

$$F_{sample} = \frac{1}{\Delta t}$$

  - Note: Sampling relationships are based on time and frequency!

# FFT Kernel.

- FFT kernel is based on constant frequency sines/cosines.
  - Wait a minute…. Don't the frequencies of orders change with time/rpm??
    - Why use a transform which doesn't account for this?
      - Simple and numerically efficient.

$$a_n = \frac{1}{N}\sum_{n=1}^{N} x(n\Delta t)\cos\left(2\pi f_n n\Delta t\right)$$

$$b_n = \frac{1}{N}\sum_{n=1}^{N} x(n\Delta t)\sin\left(2\pi f_n n\Delta t\right)$$

# *Amplitude Variation.*

- So…frequency variation is accounted for!
  - Wait…Can't an order change amplitude with time/rpm?
    - YES…What can be done about this?
      - NOTHING!!…FFT Kernel is based on constant amplitude sines/cosines!
      - Amplitude which FFT estimates is average amplitude over transform time.
      - Let's look at what effect this has on our amplitude estimates.
        - Depends on blocksize.
        - Depends on damping in resonance.

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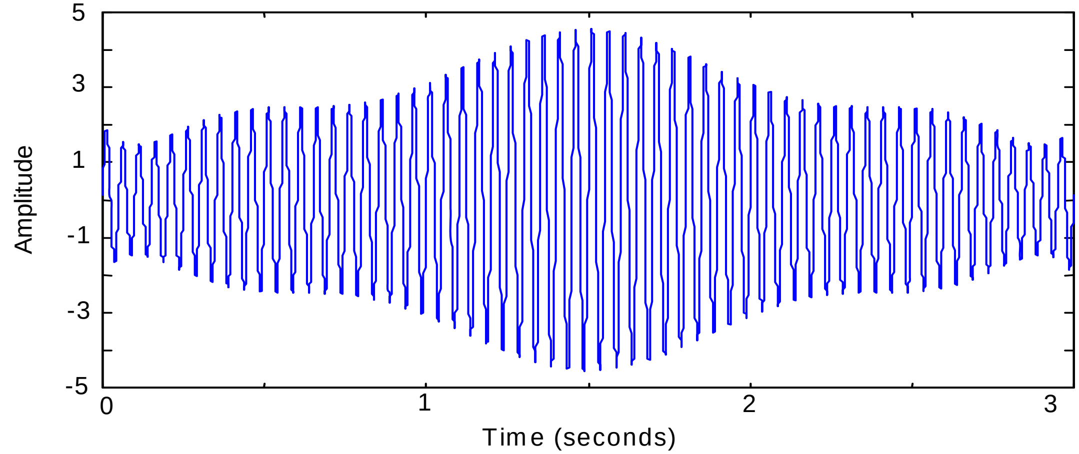
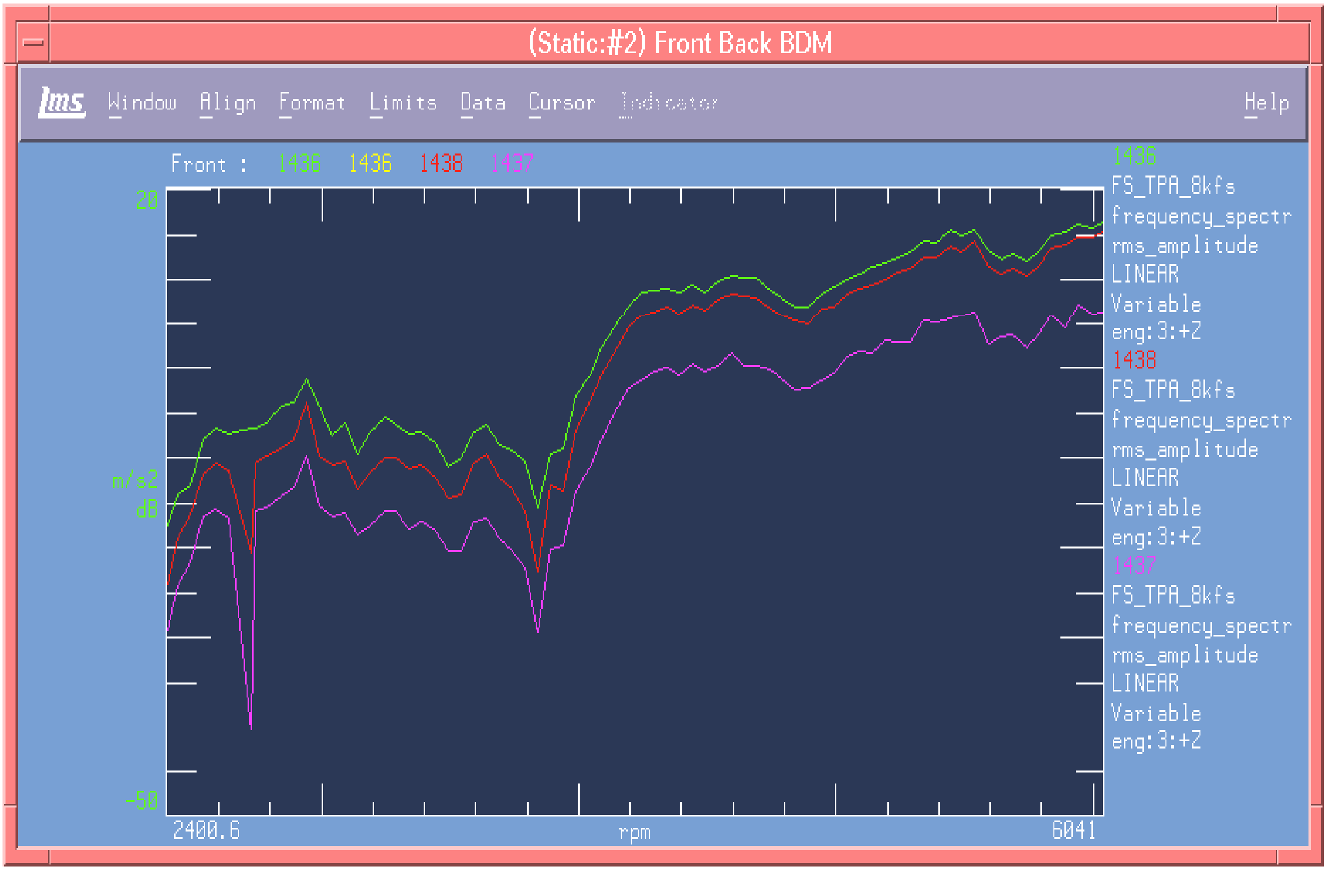
# FFT Based Order Tracking - Errors

- Errors exist because an order is allowed to change amplitude as a function of time!
  - All Fourier transform type of algorithms have this limitation!




| BLOCKSIZE | AMPLITUDE ESTIMATE |
|---|---|
| 512 | 4.5476 |
| 1024 | 4.4850 |
| 2048 | 4.2923 |
| 4096 | 3.8099 |

# Same Order, Different Bandwidths.




***Note: Same order, different amplitudes!***

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# *FFT Order Tracking Summary.*

- Kernel based on time/frequency.
  - Leakage is a problem.
    - Use windows to minimize.
  - Time varying frequency is a problem.
    - Integrate over multiple spectral lines.
  - Time varying amplitude is a problem.
    - Amplitude is average over transform time.
  - Varying number of orders present in data depending on rpm.
    - Sample rate not dependent on rpm.
  - Poor order resolution at low rpm values.
    - Same $\Delta f$ regardless of rpm.

# *Why Resample to Angle Domain?*

- Resampling time data to angle data straightens out orders.
  - Orders now fall on one spectral line regardless of rpm.
  - Provides leakage free estimates of orders.
- Number of orders present in data independent of rpm.
- Order resolution is independent of rpm.
  - Good order resolution at low speeds.

- All standard DSP techniques for constant frequency analysis in the time domain can be performed on orders in the angle domain!
  - ie. FFT, Digital Filtering, Synchronous Averaging, …etc.

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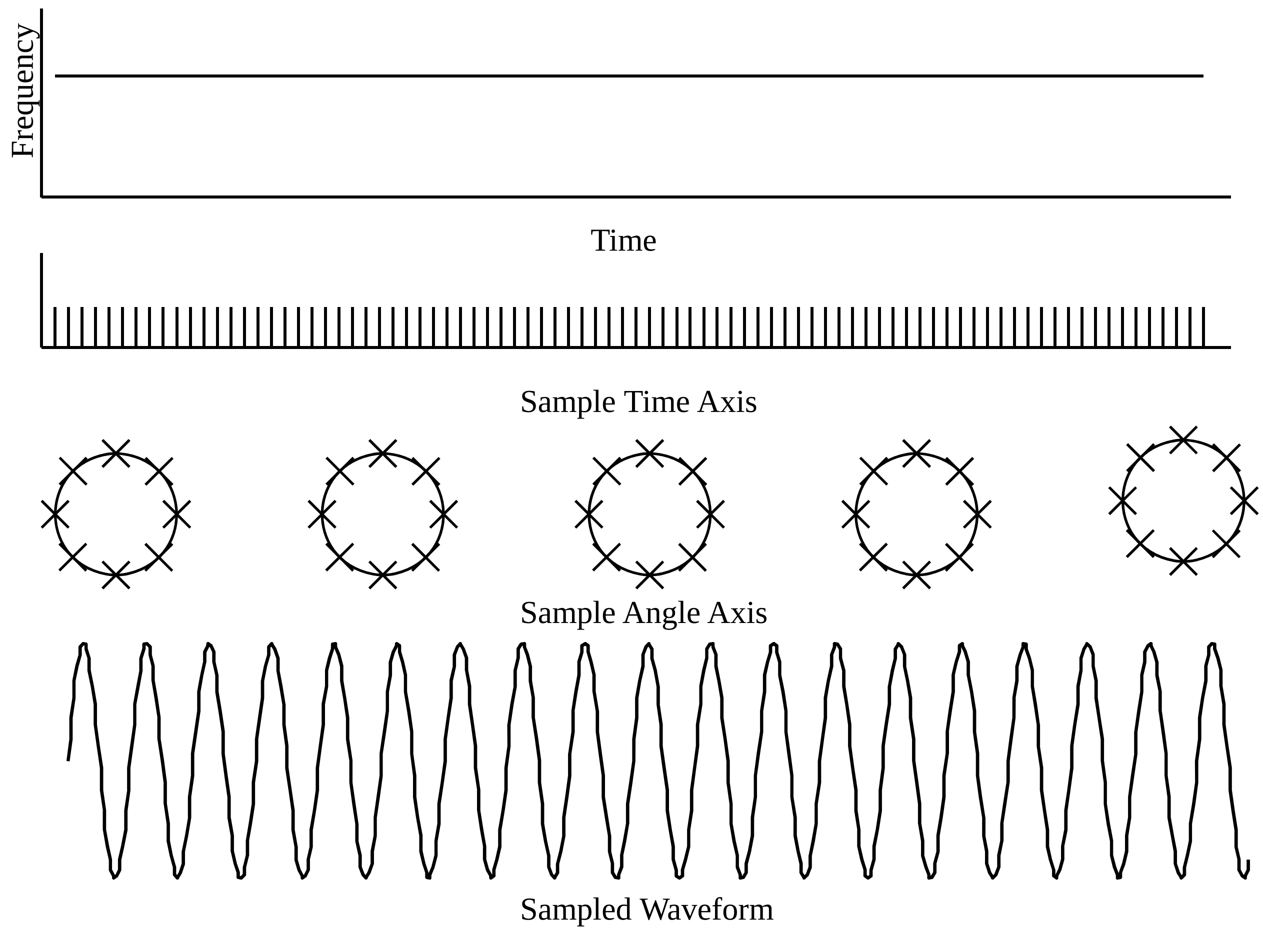
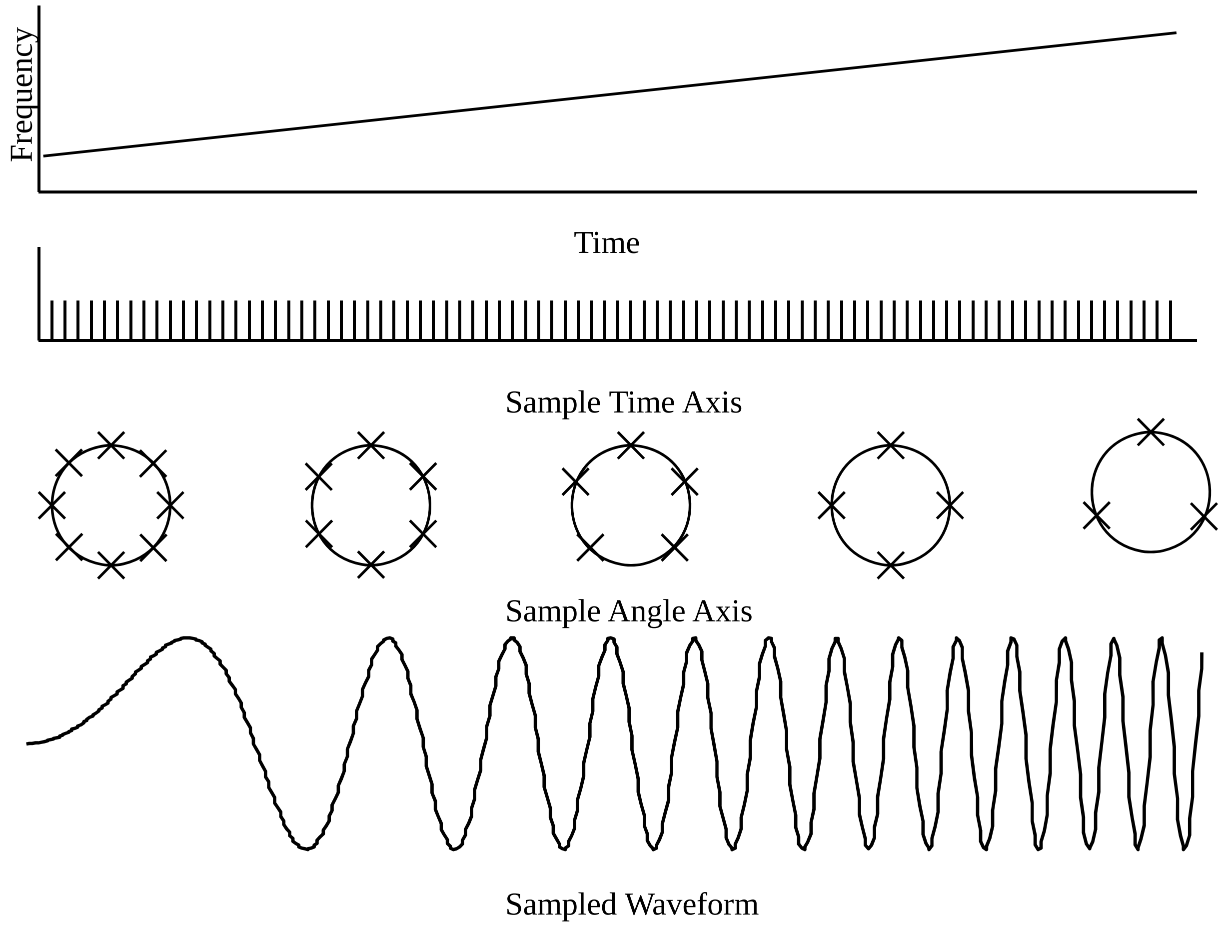
# *Constant Frequency, Constant Δ t*



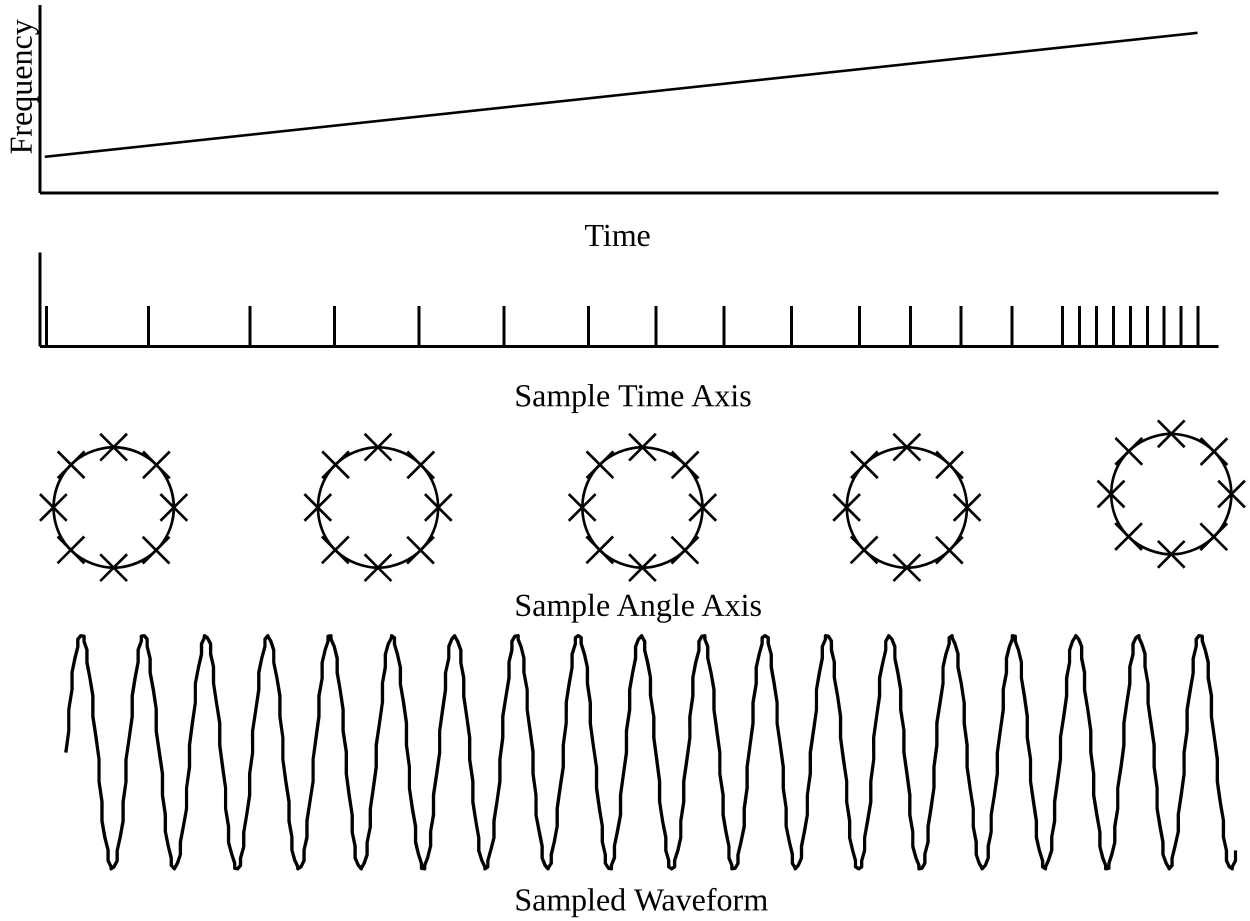

Note: Constant $\Delta t$ ➔ Constant $\Delta\theta$!

# *Variable Frequency, Constant Δ t*




Note: Constant $\Delta t$ $\not\Rightarrow$ Constant $\Delta\theta$!

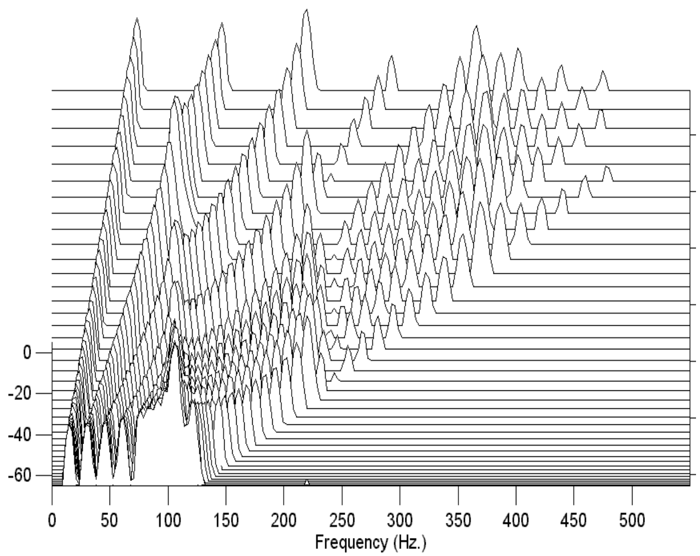
# *Variable Frequency, Constant Δθ*

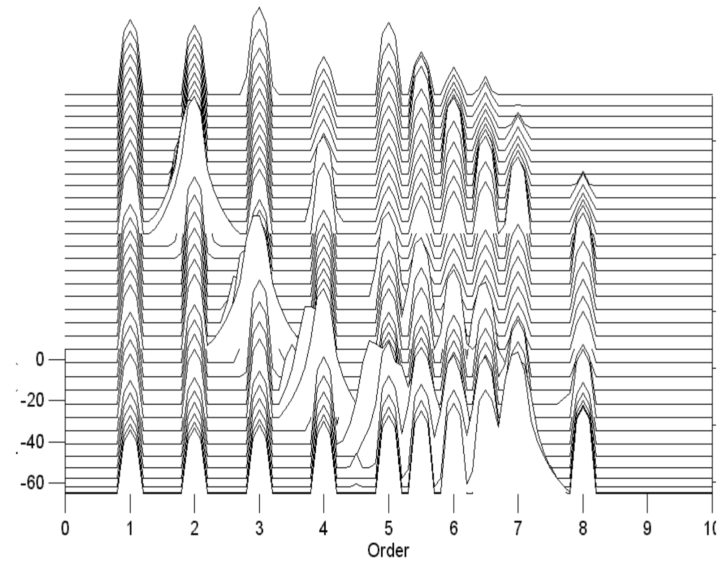



Note: Variable Δt ➔ Constant Δθ!

# What Happens in the Order Domain?

- Let's see what happens in the order domain!
  - Orders which changed frequency now appear as constant frequency!



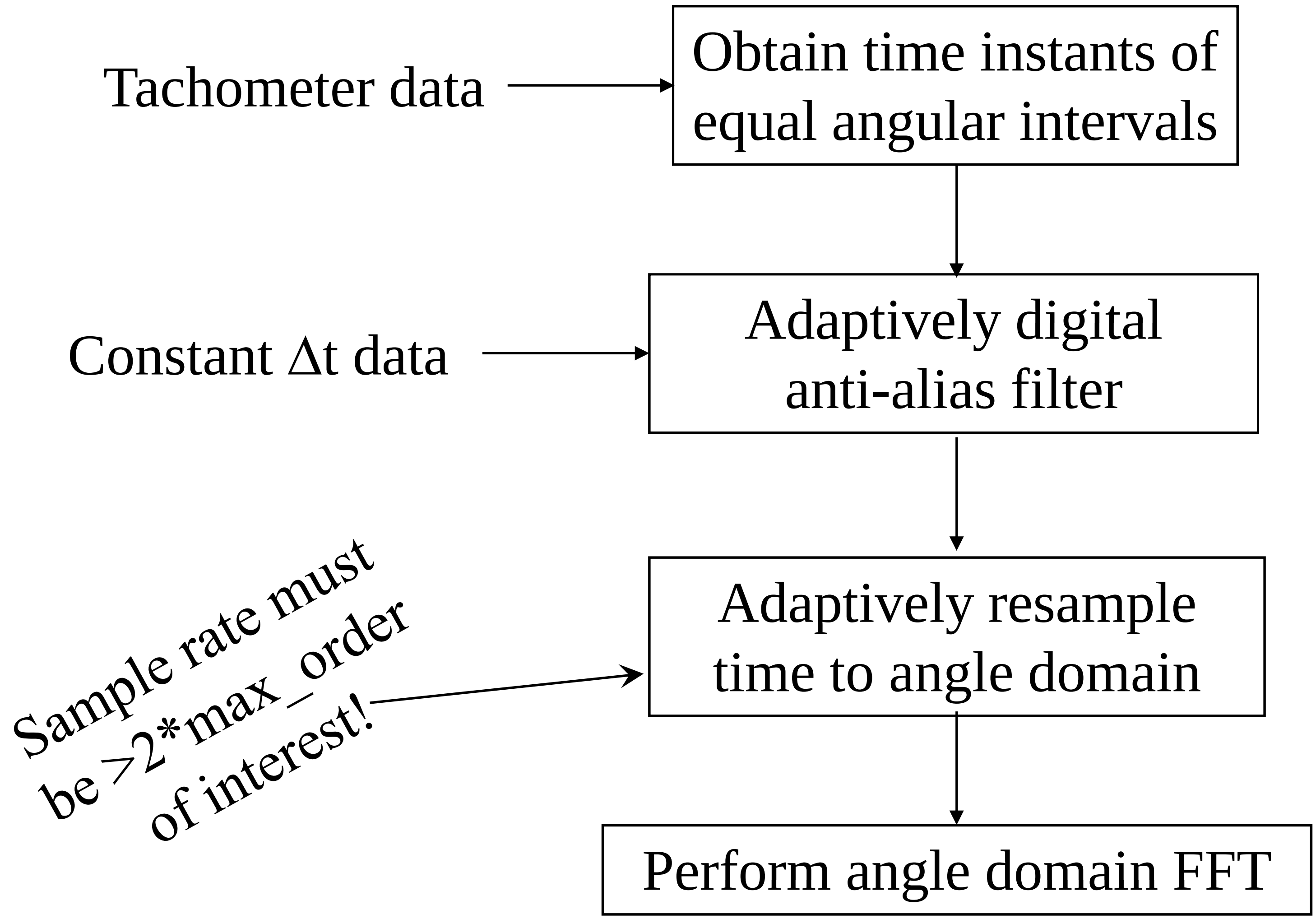



# *How Do We Get There?*

- Now… we know why we go to the angle domain.
- But… How we get there?

- Acquire constant $\Delta t$ data.
  - Adaptively set bandwidth to ensure highest order of interest is acquired.
- Resample $\Delta t$ data to constant $\Delta\theta$ data.
  - Process is driven by accurate tachometer signal processing.
- Estimate orders through use of reformulated FFT.

# *Let's Look at the Resampling Process!*



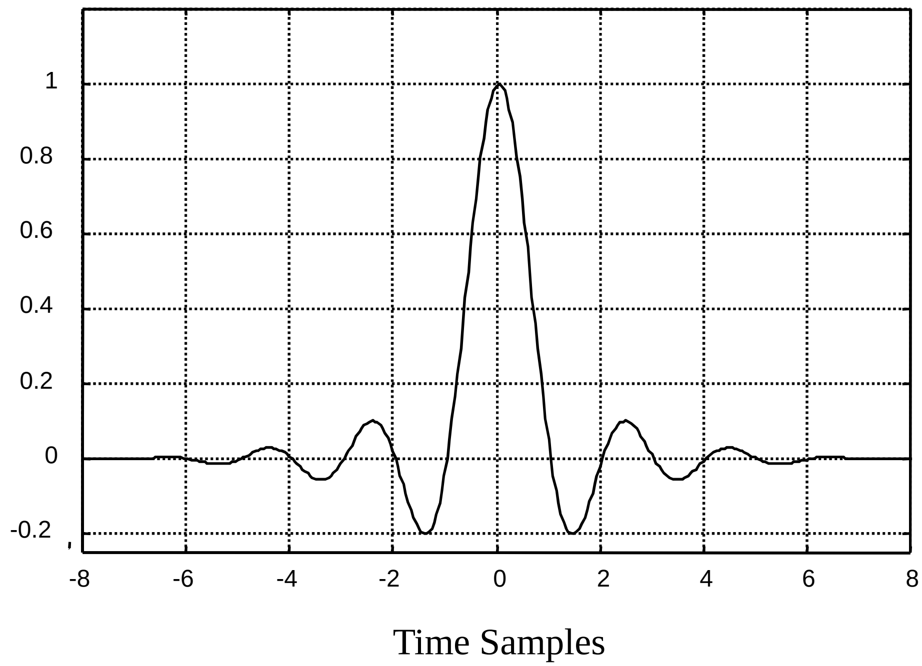
# What do we do with New Time Instants?

- Angle domain sample rate must be fast enough to prevent aliasing in order domain!
  - The $\Delta\theta$ values are evenly spaced to protect against aliasing.

- Use new time instants to resample the data!
  - Resampling is done through the use of an interpolation filter.
    - Interpolation filters preserve the original frequency content of the signal!
    - Adaptive interpolation filter can provide new data estimate at any point in time!

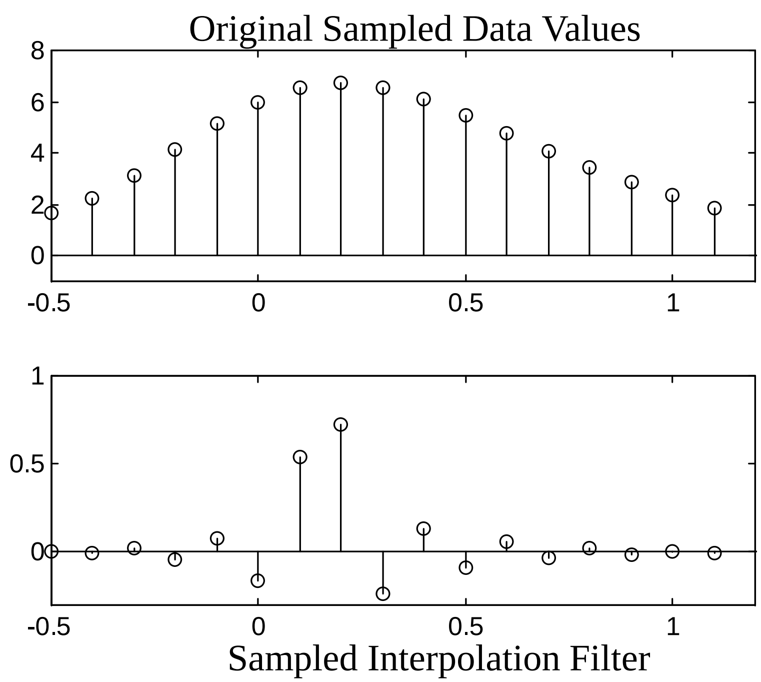
# *What is an Interpolation Filter?*

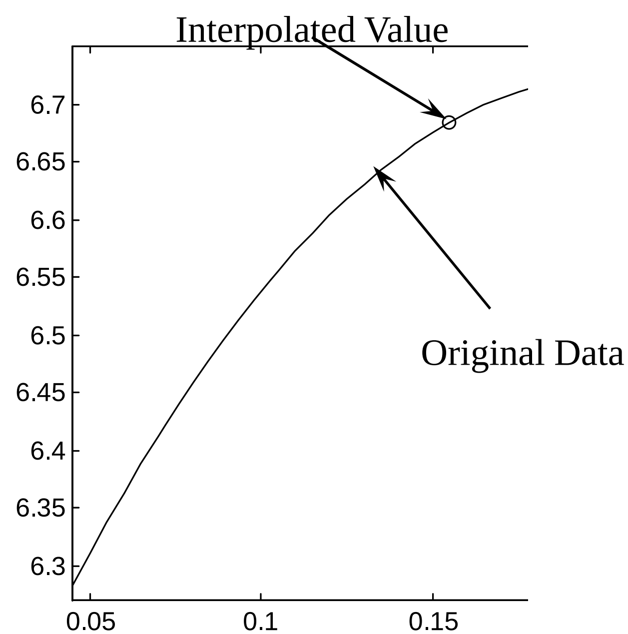
- Digital sampling process can be described by a sinc (sinx/x) function!
  - But… A sinc function is infinite in length so we have to truncate it with a window!



# *Let's See How It Works!*

- Center the interpolation filter at the time at which an interpolated sample is desired.



# *Overcoming FFT Limitations.*

- Computed order tracking overcomes most limitations of FFT!
  - Data is sampled with a constant angular interval.
    - Data resampled from time to angular intervals.
  - FFT sampling relationships are reformulated in terms of angle/order.
  - FFT kernel is reformulated in terms of angle/order.
  - Faster sweep rates are possible.

  - Computed order tracking requires a powerful DSP.
    - Available on much of today's data acquisition equipment.

# *Now What?*

- Now we have resampled angle domain data… what do we do with it?
  - Reformulate FFT sampling relationships.

$$\Delta o = \frac{1}{R} = \frac{1}{N * \Delta\theta}$$

$$R = N * \Delta\theta$$

$$O_{nyquist} = O_{\max} = \frac{O_{sample}}{2}$$

$$O_{sample} = \frac{1}{\Delta\theta}$$

  - Note: No time or frequency in these equations!

# What does the FFT Kernel Look Like?

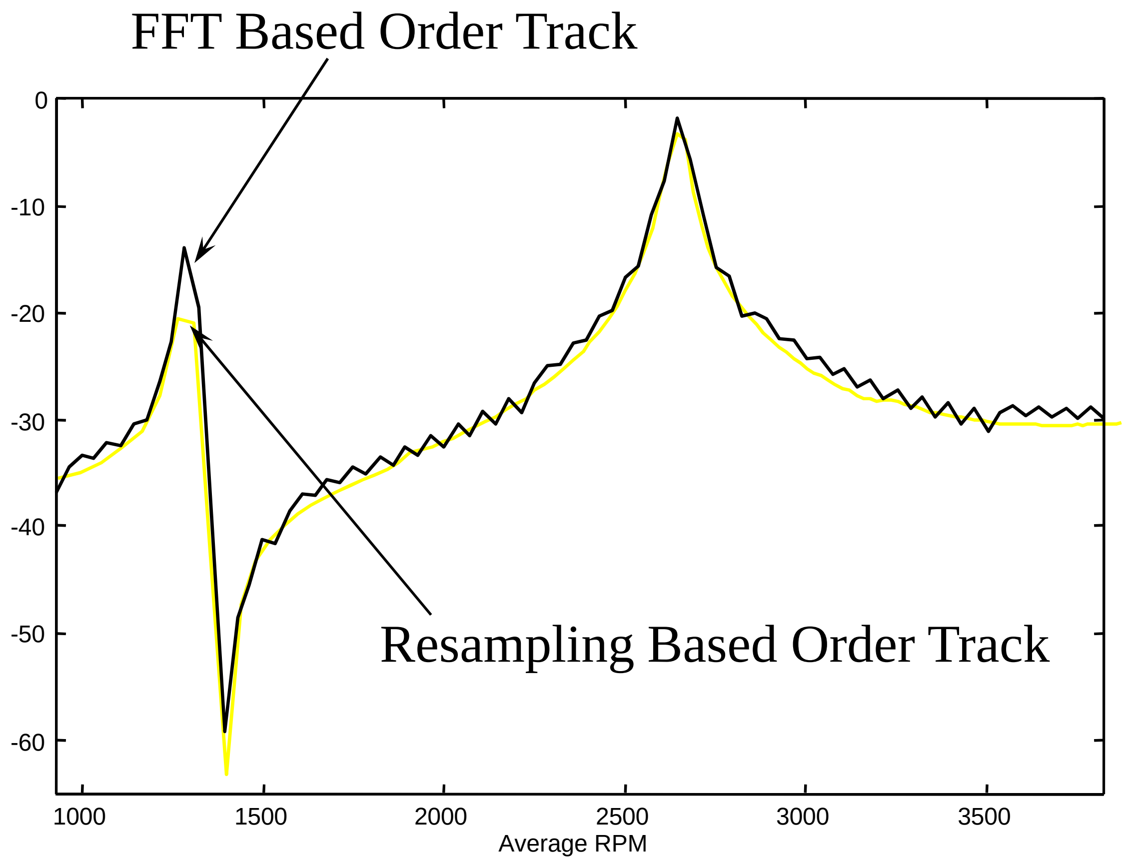
- Reformulate the FFT kernel in terms of angle/order.

$$a_m = \frac{1}{N}\sum_{n=1}^{N} x(n\Delta\theta)\cos(2\pi o_m n\Delta\theta)$$

$$b_m = \frac{1}{N}\sum_{n=1}^{N} x(n\Delta\theta)\sin(2\pi o_m n\Delta\theta)$$

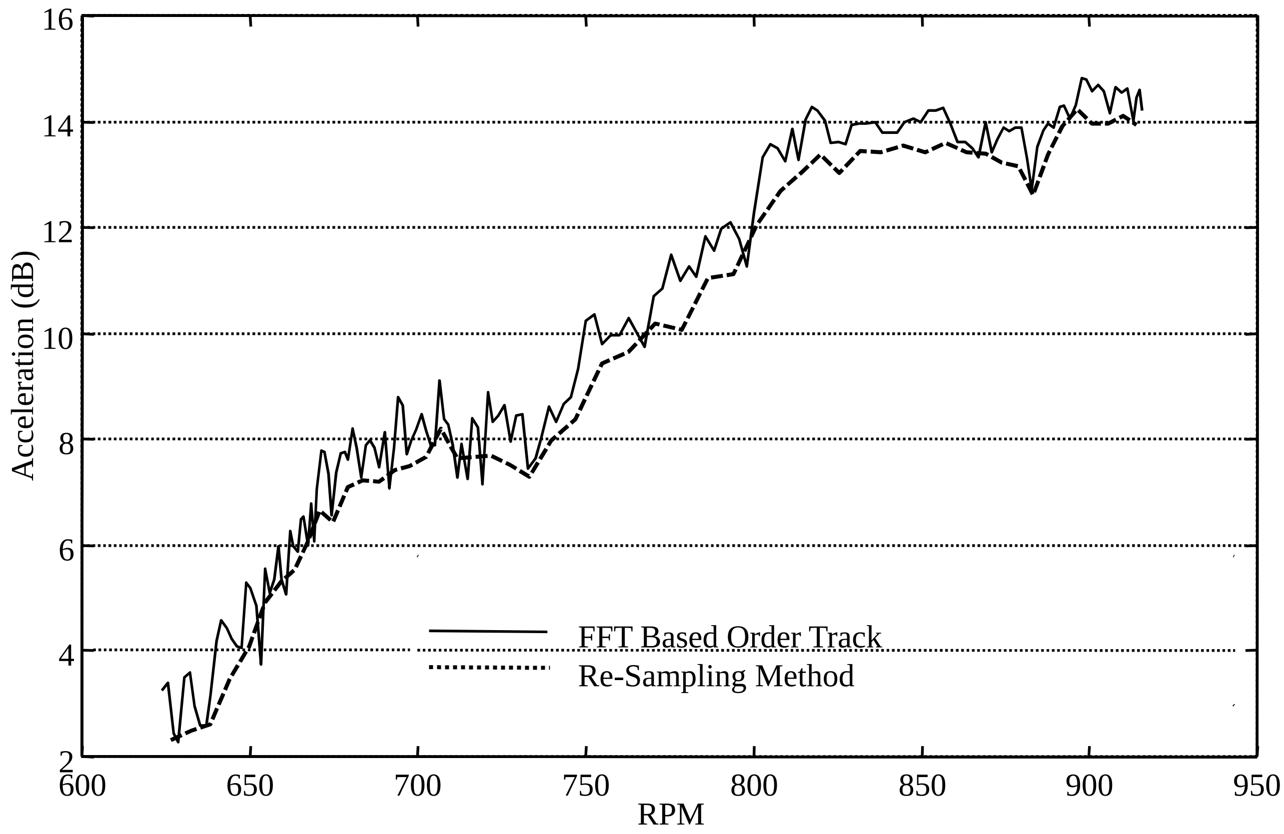
- Note: These kernels operate on orders and produce constant order bandwidth results.
  - Orders which fall on spectral lines are leakage free!!

# *Order Track Results.*




***Note: Amplitude differences due to different integration lengths.***

# Order Track Results.



# *Kalman/Vold-Kalman Tracking Filters*

- Adaptive filters based on two equations
  - Structure equation

**Kalman:** $$x(n\Delta t) - 2\cos(\omega\Delta t)\, x((n-1)\Delta t) + x((n-2)\Delta t) = \varepsilon(n)$$

**1st order Vold-Kalman:** $$x(n+1) - x(n)\exp(i\omega\Delta t) = \varepsilon(n)$$

  - Data equation

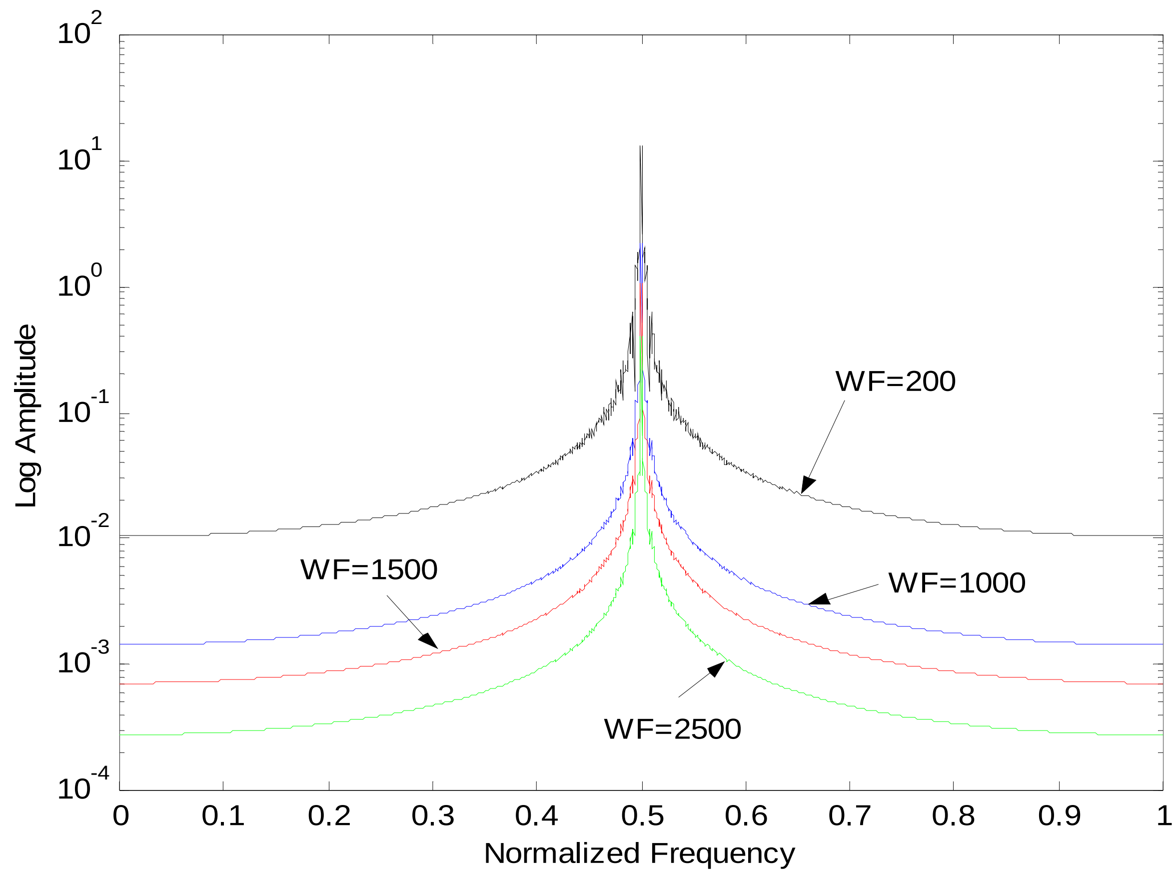
$$y(n) = x(n) + \eta(n)$$

  - Weighted against each other in least squares equation

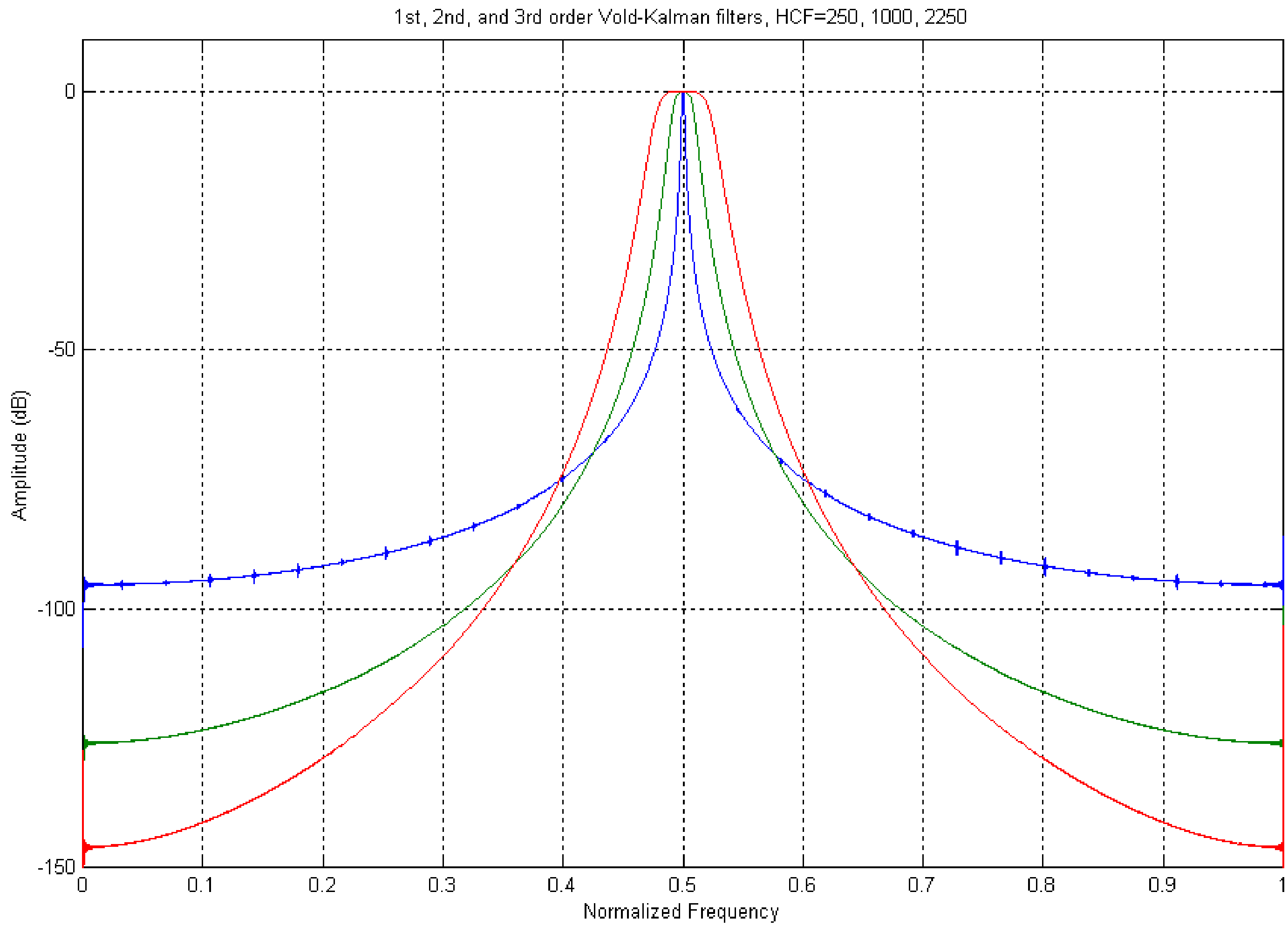
$$r(n) = \frac{s_\varepsilon(n)}{s_\eta(n)}$$

$$\begin{bmatrix} 1 & -2\cos(\omega\Delta t) & 1 \\ 0 & 0 & r(n) \end{bmatrix} \begin{Bmatrix} x(n-2) \\ x(n-1) \\ x(n) \end{Bmatrix} = \begin{Bmatrix} 0 \\ r(n)\, y(n) \end{Bmatrix}$$

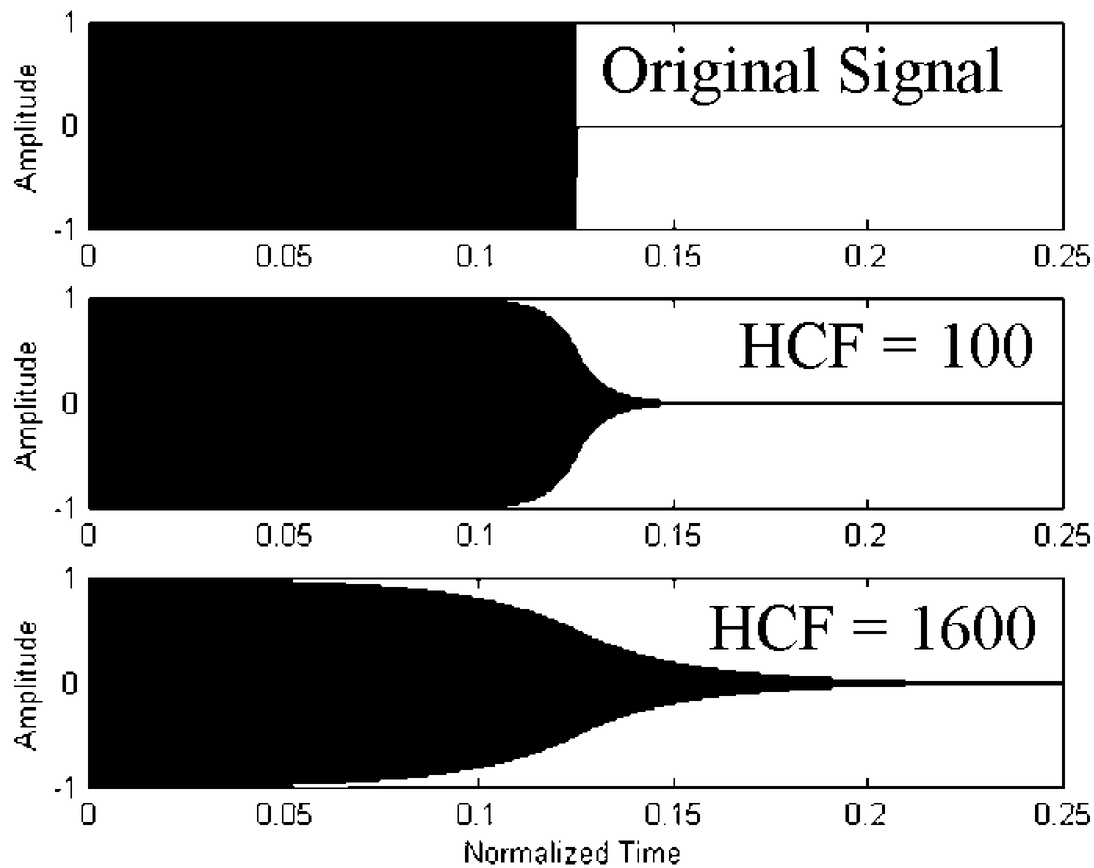
# 1st order Vold-Kalman Filter Shape



# Vold-Kalman Filter Shapes



# *Effect of HCF on Kalman Filter Amplitude Tracking*



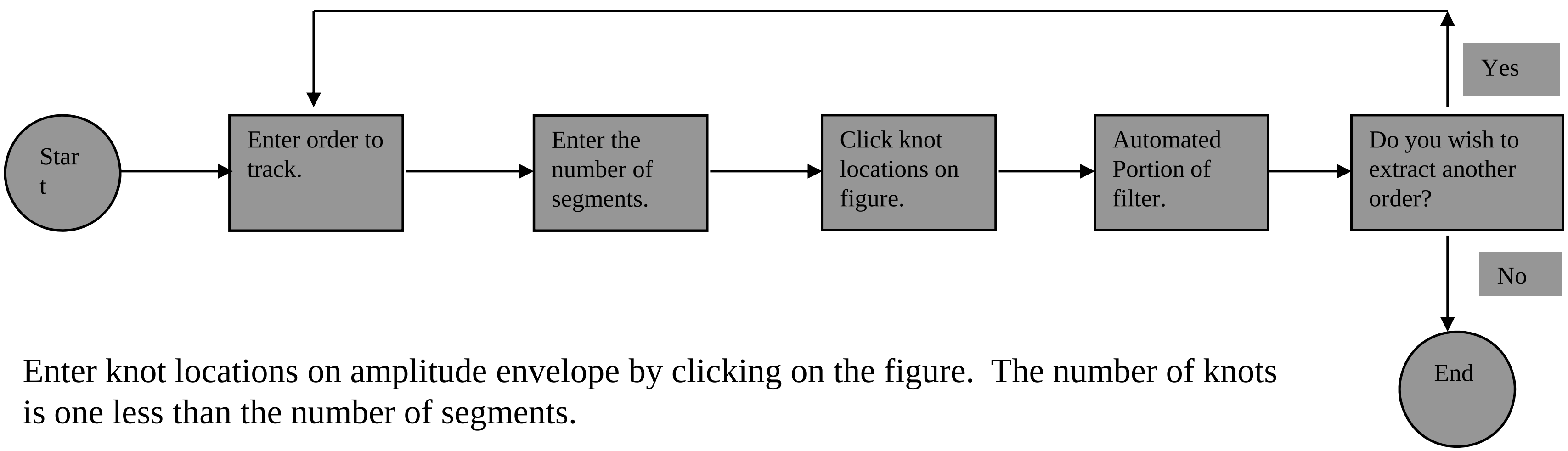
# Kalman/Vold-Kalman Weighting Factor

- Difficulty in using Kalman/Vold-Kalman filters arises in determination of weighting factor.
  - LMS software uses Harmonic Confidence Factor
    - inverse of weighting factor
  - B&K software allows user to input bandwidth in either frequency or orders

- What bandwidth or HCF is necessary to effectively extract all energy relative to an order?
  - Difficult to determine near resonance conditions!

# Iterative Time Varying HCF Vold-Kalman Order Tracking Filter

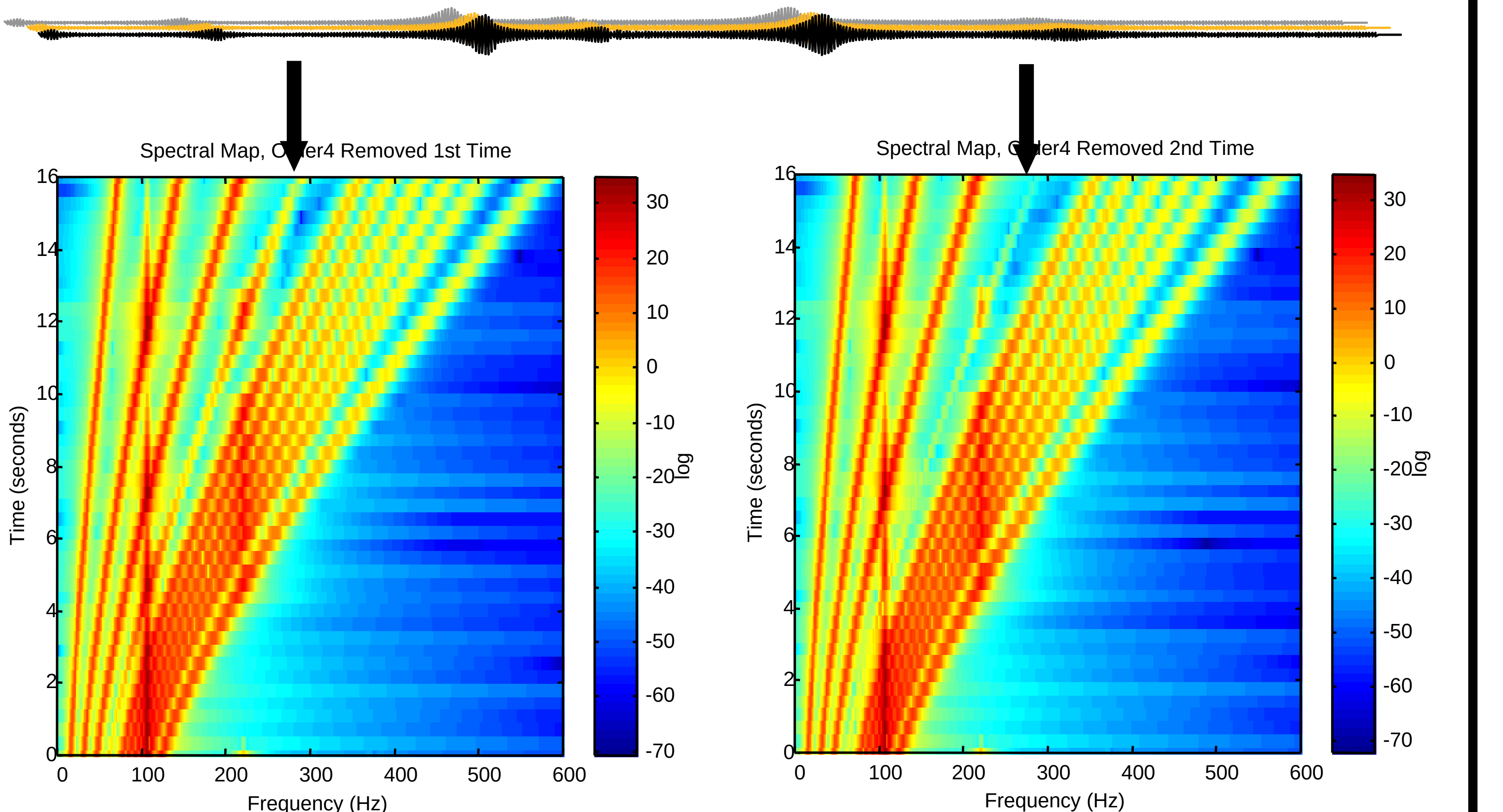
- Procedure developed to automate and optimize the choice of the weighting factor.




Enter knot locations on amplitude envelope by clicking on the figure. The number of knots is one less than the number of segments.

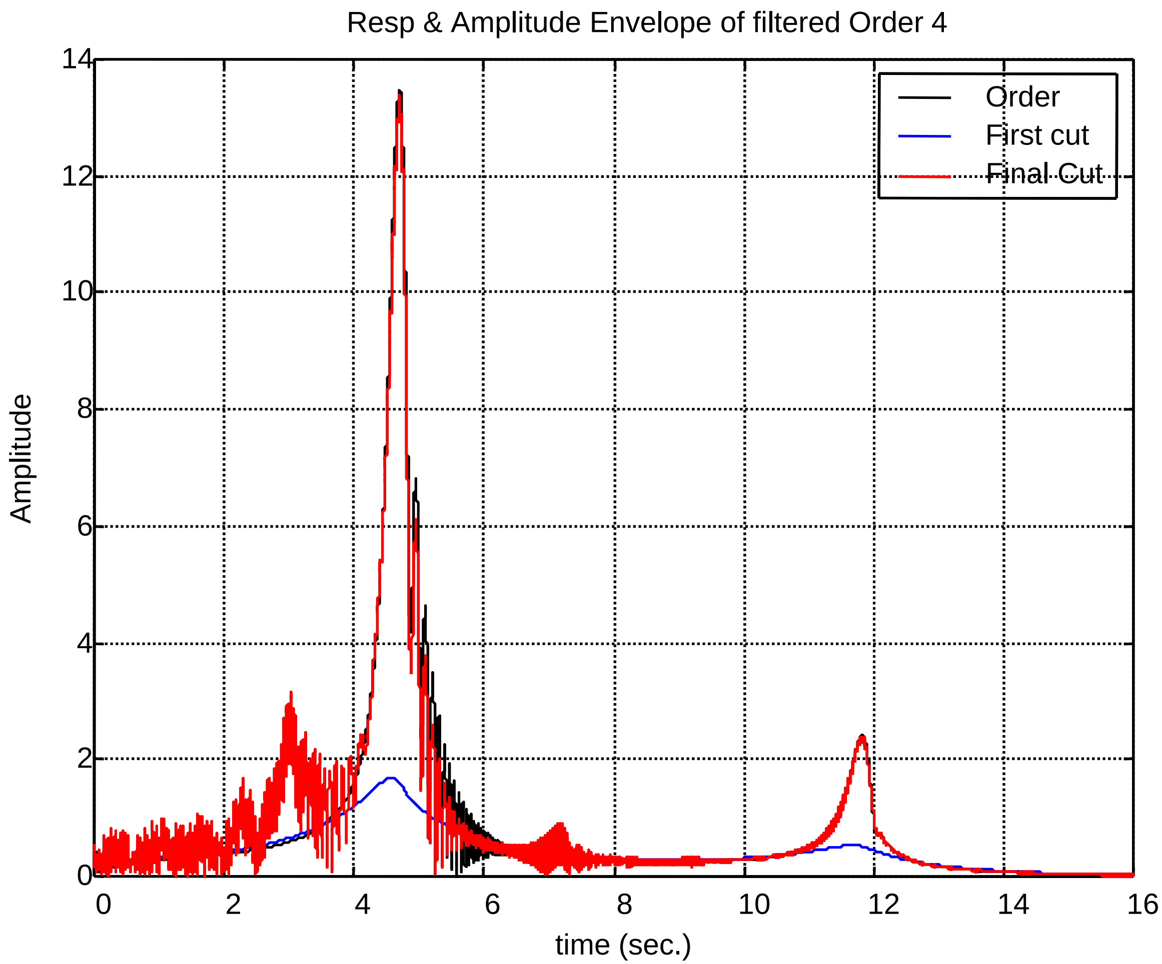
Spline fit amplitude envelope and knots. Inverts and scales spline fit. TVWV extracts order. Compares maximum amplitudes of extracted orders' envelopes, if they are below a specified percentage it lowers the whole TVWV's amplitude a specified amount and continues iterating until they reach the specified percentage. Once the specified percentage is reached, the iterating portion of the program stops and displays a spectral map with the desired order removed and the time history of the removed order.

# *Iterative Filter Results*




**Constant weighting factor** **Time varying weighting factor**

# Iterative Filter Results



# Time Variant Discrete Fourier Transform

- Instantaneous frequency of kernel matches frequency of order of interest

$$a_m = \frac{1}{N}\sum_{n=1}^{N} x(n\Delta t)\cos\left(2\pi \int_0^{n\Delta t} (o_m * \Delta t * rpm / 60) dt\right)$$

$$b_m = \frac{1}{N}\sum_{n=1}^{N} x(n\Delta t)\sin\left(2\pi \int_0^{n\Delta t} (o_m * \Delta t * rpm / 60) dt\right)$$

- Same amplitude estimation problems as both FFT and Angle Domain order tracking methods!

# TVDFT Orthogonality Compensation

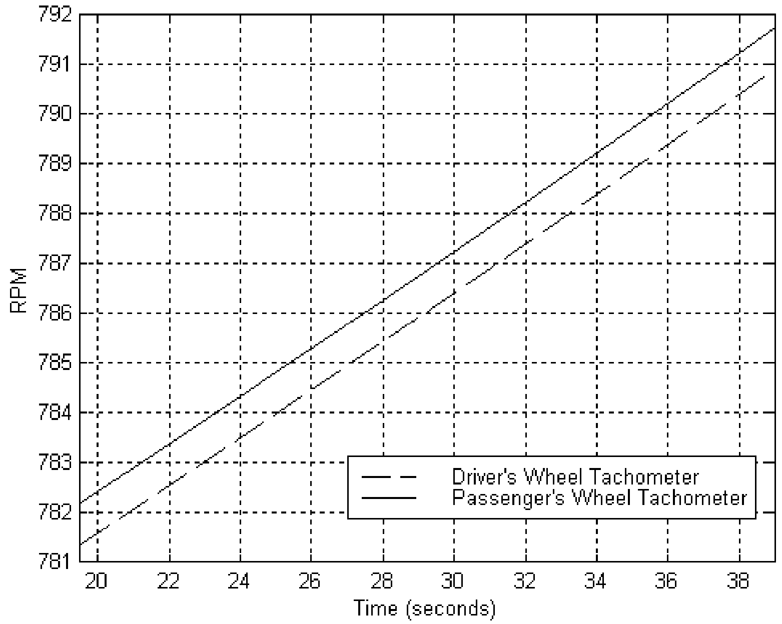
- Post calculation to separate close/crossing orders

$$\begin{bmatrix} e_{11} & e_{12} & e_{13} & \cdots & e_{1m} \\ e_{21} & e_{22} & e_{23} & & \\ e_{31} & e_{32} & e_{33} & & \vdots \\ \vdots & & & \ddots & \\ e_{m1} & & \cdots & & e_{mm} \end{bmatrix} \begin{Bmatrix} o_1 \\ o_2 \\ o_3 \\ \vdots \\ o_m \end{Bmatrix} = \begin{Bmatrix} \tilde{o}_1 \\ \tilde{o}_2 \\ \tilde{o}_3 \\ \vdots \\ \tilde{o}_m \end{Bmatrix}$$

$$e_{ij} = \frac{1}{N} \sum_{n=1}^{N} \left\{ \exp\left( 2\pi \int_0^{n\Delta t} \left( o_i * \Delta t * rpm / 60 \right) dt \right) \times Window \right\} \times \exp\left( 2\pi \int_0^{n\Delta t} \left( o_j * \Delta t * rpm / 60 \right) dt \right)^*$$

# Automotive Example.

- Front wheel drive on 2 wheel chassis dynamometer.
  - 4 psi inflation difference between left and right wheels.
    - Right/Left wheels less than ~0.01 Hz apart.
    - Less than ~0.001 orders apart.
  - Slow speed sweep.
    - ~770-1050 rpm (55-80mph) in 500 seconds.

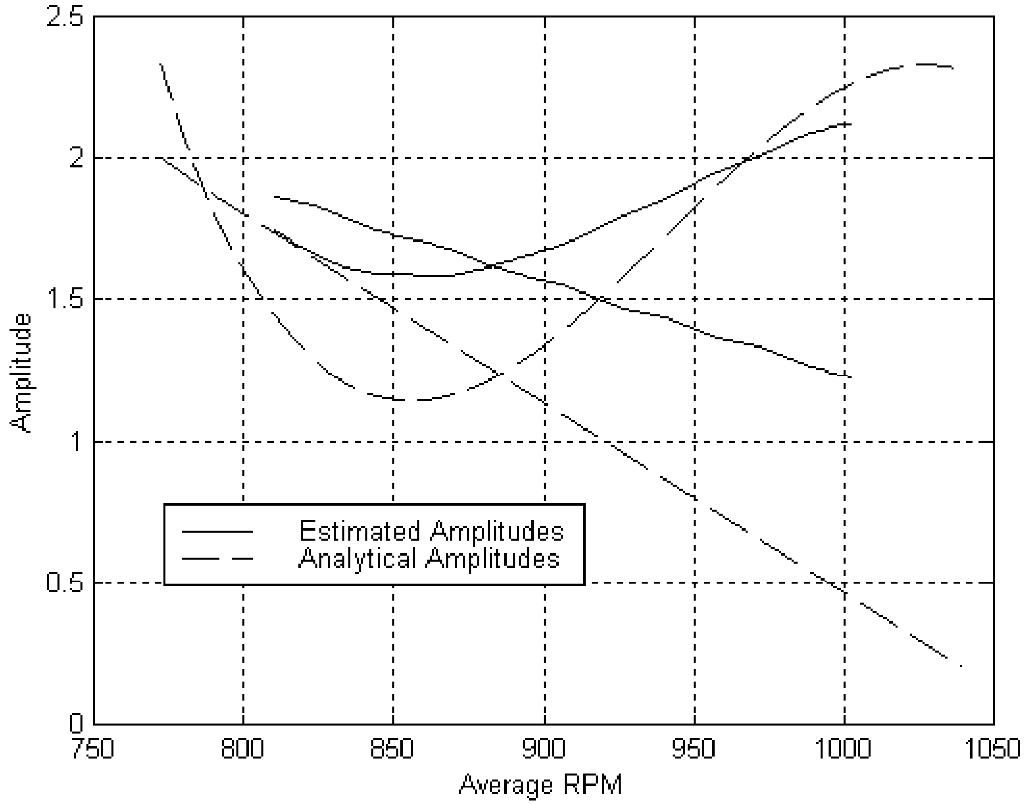


# *Automotive Example - Analytical Verification.*

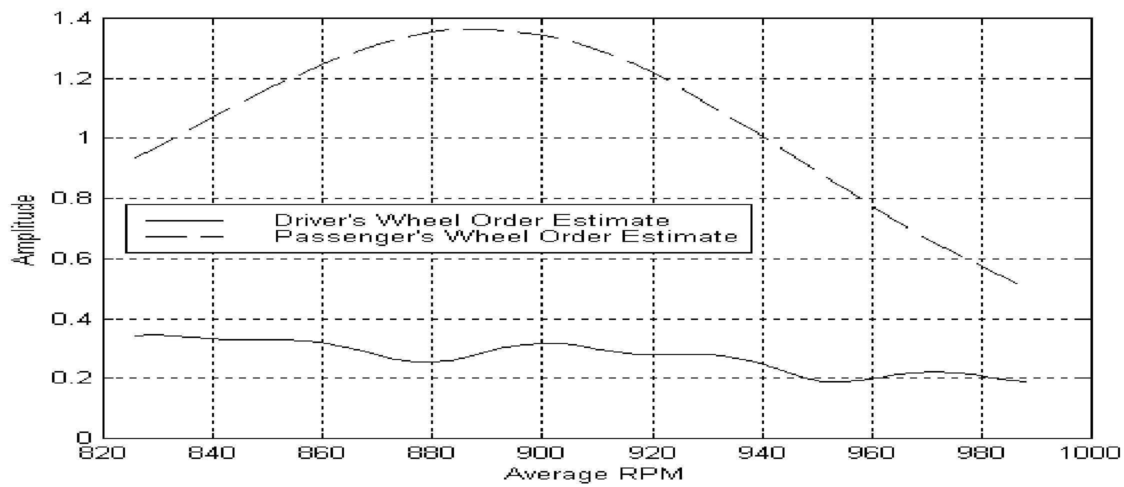
- Analytical data was synthesized to evaluate the performance of the different order tracking methods since the ***right answer*** was not known in the experimental data.
  - Used actual spline fit tach. signals to generate orders.
- Analytical data was evaluated with:
  - 1st order Vold-Kalman filter.
    - Able to separate orders ~0.003 apart but not ~0.001!
  - Tracking filter/Time Domain Residue.
    - Unable to completely eliminate beating.
  - TVDFT w/OCM.
    - Able to separate with Hanning window and bandwidth of 0.0005 orders.

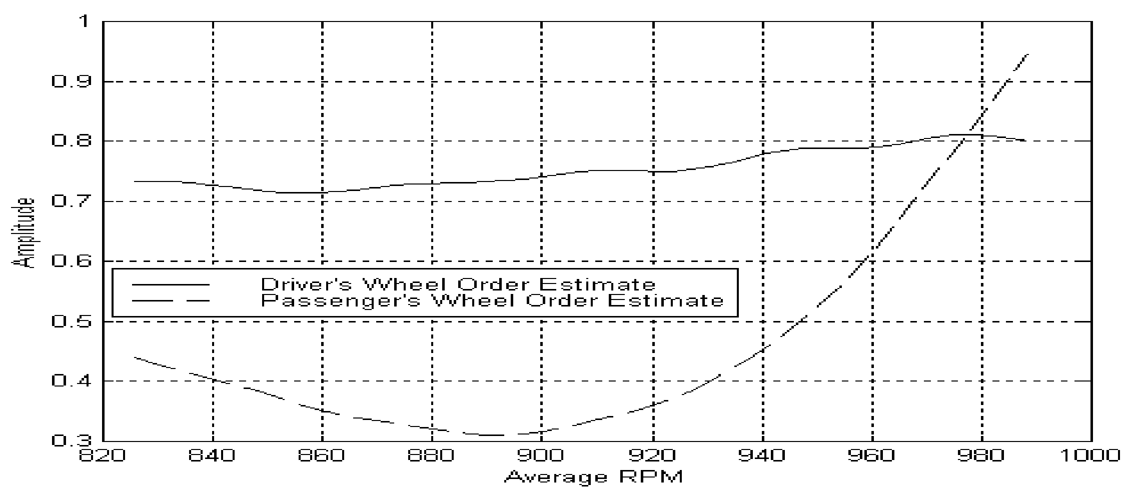
# *TVDFT Analytical Performance.*

- TVDFT w/OCM was able to separate orders but with limited dynamic range.



# TVDFT Automotive Results.

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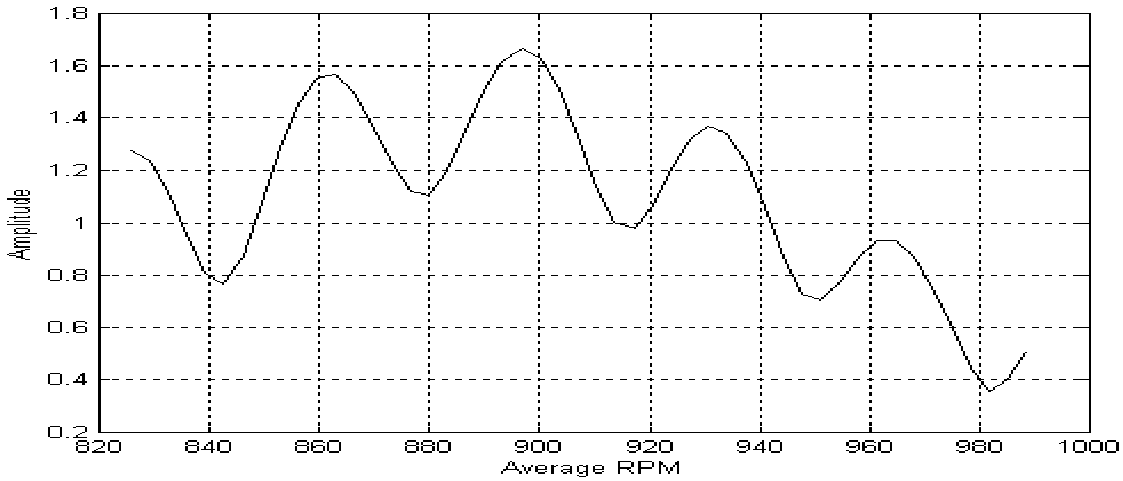
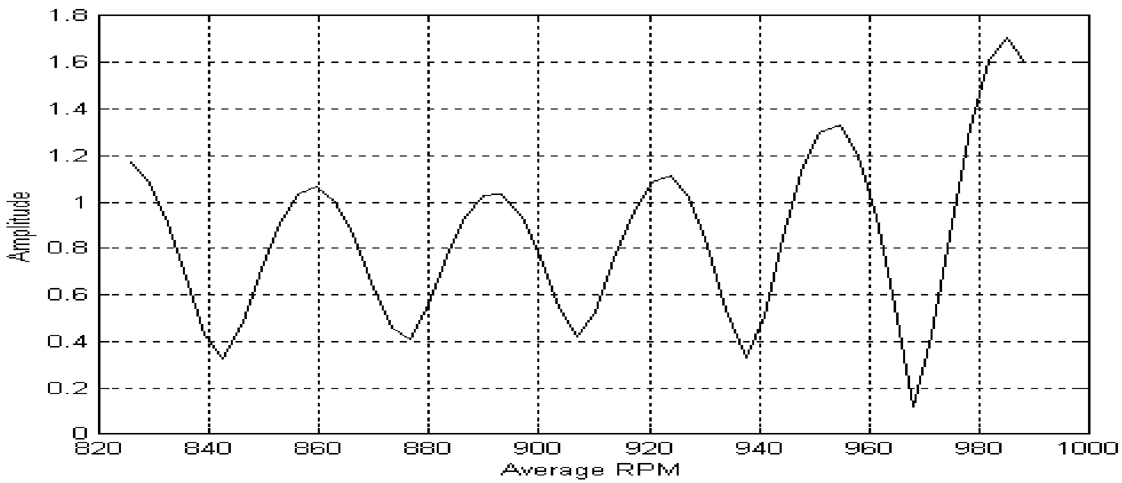
# *Non-Separated Orders.*

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# *TVDFT Conclusions*

- TVDFT estimates orders which closely match those of the resampling based method with much less computational load.
- TVDFT is essentially resampling the kernel of the Fourier transform instead of re-sampling the data.
- TVDFT and resampling methods may be used on data with a Doppler shift.
- FFT windows should be used with all methods to improve sideband energy rejection.
  - Keep Amplitude/Sidelobe width in mind when choosing bandwidth.
- No order tracking method gives correct amplitude at all rpms if lightly damped resonances are present due to time/frequency tradeoff.

# *Summary of Methods*

- FFT based order tracking
  - Limited sweep rate
  - Leakage can be severe

- Computed order tracking
  - Leakage error minimal
  - Track orders relative to one rotating shaft
  - Transform applied over RPM dependent time

- Kalman/Vold-Kalman filters
  - Extract time history of orders
  - Difficult to choose correct weighting factor
  - Computationally demanding
  - Able to separate close/crossing orders

# Summary of Methods (continued)

- Time Variant Discrete Fourier Transform
  - Computationally efficient
  - Amplitude estimation limitation
  - Able to separate close/crossing orders