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A Survey of DSP Methods for Rotating Machinery Analysis, What is Needed, What is Available

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Abstract

This paper will present a summary of currently available DSP methods used for rotating machinery analysis with brief summaries of each method that emphasize the strengths and weaknesses of each method. This summary will be followed by a list of desired types of analyses and an analysis of current methods and their limitations with respect to these desired types of analyses. With the increased desire by industry to more fully understand issues such as sound quality and time domain transfer path analysis there is an ever increasing need for more computationally efficient and powerful filtering and synthesis methods as well as several other needs. In discussing these desired types of analysis, proposed directions of research and solutions to these problems will be introduced to encourage thoughts on how currently available methods may be adapted to meet these needs or on what requirements and major obstacles new methods will have to possess to meet these needs.
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1. Introduction

The analysis of non-stationary noise and vibration signals on rotating machinery is commonly performed through the use of specialized digital signal processing (DSP) techniques. DSP methods developed primarily for rotating machinery analysis are called order tracking techniques. Order tracking is the analysis of non-stationary frequency components whose frequencies are related to the rotational frequency of the non-stationary operating machine. The analysis of non-stationary conditions requires additional information, as compared to steady state conditions, for accurate results to be obtained. This additional information is usually presented in the form of a tachometer signal measured on a reference shaft of the machine.

An order is a time varying phasor that rotates with an instantaneous frequency related to the rotational frequency of the reference shaft, as shown in Equation 1. It can be seen that the rotating phasor will contain a frequency that varies as the period of rotation, or rpm, varies.

$$X(t) = A(k,t)\sin(2\pi i(k/p)t + \phi_k) \quad (1)$$

where: A(k,t) is the amplitude of order k as a function of time.
ϕ_k is the phase angle of order k.
p is the period of primary order in seconds.
t is time.
k is the order being tracked.
k = 0 DC offset.
k < 0 Negative frequencies.

Multiple orders are normally present in a dataset acquired from an operating machine. These orders may be described mathematically by a summation of time varying phasors.

Order functions can be generated by any rotating input on an operating machine and may vary in amplitude and/or frequency as a function of time. This amplitude varying property causes errors in any type of order tracking analysis. All order tracking techniques consider the amplitude of an order to be semi-constant over the analysis period used to estimate the amplitude and phase of the order. This assumption can cause considerable errors in the analysis.

2. Order Tracking Theory

2.1 Fourier Transform Based Order Analysis.

Fast Fourier transform, FFT, based order tracking is both the simplest to implement and the most commonly used. This method is available in virtually all of the commercial order tracking software.

2.1.1 Fourier Transform Based Order Analysis Theory

The sampling theorem that the FFT is based on is the well understood and documented Shannon's sampling theorem. This sampling theorem is based on acquiring data samples with a uniform time spacing, Δt. Since this sampling theorem is for the general use FFT it is not related in any way to the behavior of the rotating machine. This implies that the result from the transform is based on frequencies and not orders. This property is further shown by the transform itself that is presented in Equation 2.

$$\begin{aligned} a_m &= \frac{1}{N}\sum_{n=1}^{N} x(n\Delta t)\cos(2\pi f_m n\Delta t) \\ b_m &= \frac{1}{N}\sum_{n=1}^{N} x(n\Delta t)\sin(2\pi f_m n\Delta t) \end{aligned} \quad (2)$$

where: $x(n\Delta t)$ is the nth discrete data sample.
f_m is the frequency of the sine/cosine terms, $m\Delta f$.
a_m, b_m are the estimated Fourier coefficients.

The properties of the sampling, kernels, and the type of data that the FFT is well suited to analyze are shown graphically in Figure 1. The kernels of the FFT are constant frequency, constant amplitude sine and cosine functions for each Δf. This figure also shows that the kernel has the same shape as the constant frequency data that it was developed to analyze. Mathematically, a transform works well if the shape of the kernel matches the characteristic of the data of interest. Clearly, the shape of the FFT kernel does not match that of data with frequency content that varies as a function of time.

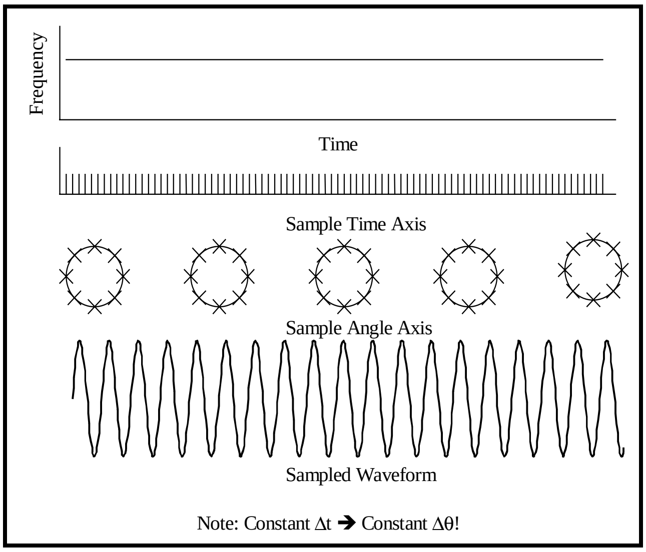

Figure 1: Graphical representation of Fourier transform, its properties, and sampled constant frequency data.

FFT based order tracking performs sliding FFTs on time domain data. The average rpm over which the transform is performed is also calculated. This average rpm is then used to estimate the frequency, and therefore the frequency bin, of the orders of interest for each estimated spectra. Once the frequency bin of an order is known, the amplitude and phase of the order can be extracted from the FFT spectra. Since the order is not in any way locked to the Fourier transform, the order will not necessarily fall on one spectral line. For this reason, oftentimes multiple spectral lines are summed around the center bin determined from the average rpm. Since multiple spectral lines are being summed to arrive at an amplitude/phase estimate, the correction factor for the window applied to the data, usually a Hanning window, is typically the energy correction factor.

2.1.2 Fourier Transform Based Order Tracking Limitations and Errors

The FFT based order tracking technique has limitations based on the constant time over which the transform is performed, regardless of the rpm of the machine. The constant frequency bandwidth of the FFT results in a varying number of orders present in each FFT spectra if the speed of the machine is changing. There will be a higher number of orders present in the spectra at low rpm values than at high rpm values. This implies that the maximum frequency of the highest order of interest must be calculated at the highest rpm of a sweep and that bandwidth used for the entire sweep.

One limitation that all Fourier transform based techniques possess is based on the assumption that the sinusoidal functions of interest are constant amplitude over the transform time. The assumption of constant amplitude leads to underestimates of the amplitudes of the order if the order's amplitude varies over the integration time. The transform will estimate the average amplitude over the integration time. This can clearly be seen from the kernel presented in

Equation 3. Underestimation of the amplitude can be severe if a lightly damped resonance is excited by the order. The resonance may first increase in amplitude, peak, and then decay in amplitude, during the integration time. An example of the differences in the estimated amplitude of a varying amplitude sine wave with different analysis blocksizes is shown in Table 1 for the sine wave presented in Figure 2.

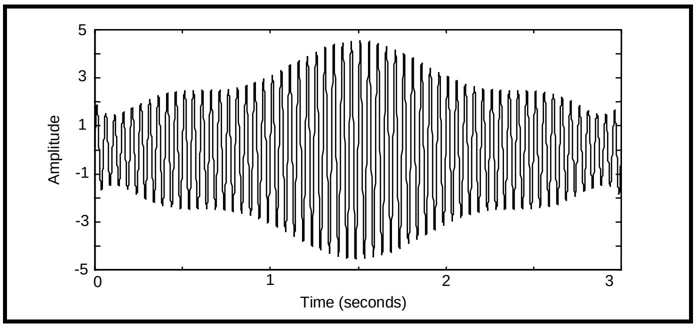

Figure 2: Varying amplitude sine wave.

Table 1 was generated by using 4 different blocksizes, all centered around the center of the block of data. It can be seen that the difference in the amplitude estimates varies from 3.8099 to 4.5476. All estimates were obtained using a Hanning window to minimize leakage.

BLOCKSIZE	AMPLITUDE ESTIMATE
512	4.5476
1024	4.4850
2048	4.2923
4096	3.8099

Table 1: Amplitude estimate vs. Blocksize for time function shown in Figure 2.

A leakage error is also present in all orders that are estimated with an FFT based technique. Since the orders can vary in frequency during the integration time, the orders will not be stationary on one spectral line of the transform. There is also no provision in the FFT techniques to analyze an integer number of revolutions to minimize the leakage error. A Hanning window is normally applied to reduce leakage. The sampled waveform that results when a constant Δt sample rate is used and the frequency is allowed to vary is shown in Figure 3. Obviously, this sampled waveform does not resemble the shape of the kernel of the Fourier transform. This leads to both the leakage and smearing problems that hinder the use of the FFT as an order tracking method.

The varying frequency of the order relative to the constant frequency kernels of the Fourier transform can limit the sweep rate. The sweep rate and the order being analyzed control the rate at which the frequency of the order varies. If the order is to be analyzed accurately, this variation in frequency must fall within the bandwidth which is extracted from the FFT spectra. The necessary equations to estimate the actual bandwidth characteristics are given in Reference 1.

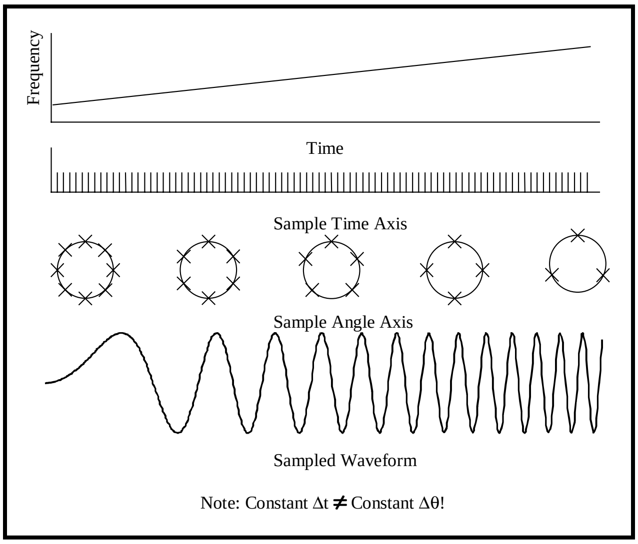

Figure 3: Varying frequency sine wave sampled with a constant Δt.

After determining the sweep rates and order bandwidths that can be analyzed with the Fourier transform, it becomes obvious that the Fourier transform is not ideally suited to order tracking. At low rpm values, the orders will be close together in frequency. The Fourier transform does not account for this and therefore close orders may not be easily separated. At the high rpm values, the orders are very well separated in frequency. This implies that a shorter integration time, T, may be analyzed and the orders still separated. Analyzing the data with a shorter integration time would result in a better estimate of the peak amplitude that an order contains because the averaging time of the FFT would be shorter.

2.2 Angle Domain Sampling Based Order Tracking

The second most common order tracking methods in use in commercial software and dynamic signal analyzers are the digital resampling based order tracking methods. These methods are the digital equivalent of the analog order tracking methods that use a tracking anti-alias filter and a frequency ratio synthesizer as an external sample clock. The digital resampling based methods are implemented in a completely digital manner with only a single frequency analog anti-alias filter, and are frequently referred to as computed order tracking methods.

The first published material on this type of order tracking method was by Potter, et al from Hewlett Packard in 1989[ref. 2-8]. Hewlett Packard considers the exact implementation of the technique to be proprietary and as such has not published many of the details. Recently many other dynamic signal analyzer manufacturers have begun to offer a type of resampling based order tracking. Again, these manufacturers consider their exact implementation to be proprietary and have not published their methods of implementation.

2.2.1 Digital Resampling Based Order Tracking Theory.

The resampling based order tracking methods acquire data with a uniform Δt with standard data acquisition equipment. The uniformly sampled time data is then digitally resampled to the angle domain through the use of an adaptive resampling algorithm.

The uniformly spaced angle data is then processed using the Fourier transform to obtain amplitude and phase estimates of the orders of interest. The significance of angle domain sampling is that this data has the same properties as a stationary frequency sine wave sampled with uniform time intervals. The adaptive resampling from the time domain to the angle domain transforms non-stationary time domain data into stationary angle domain data that can then be analyzed with standard digital signal processing methods. The result of resampling a varying frequency sine wave is shown in Figure 4. This figure shows that the chirp function, which is shown in Figure 3 sampled with a uniform Δt, appears to be a sine wave after angle domain resampling. This function now matches the shape of the Fourier transform kernel, which implies a more accurate analysis of the signal is possible.

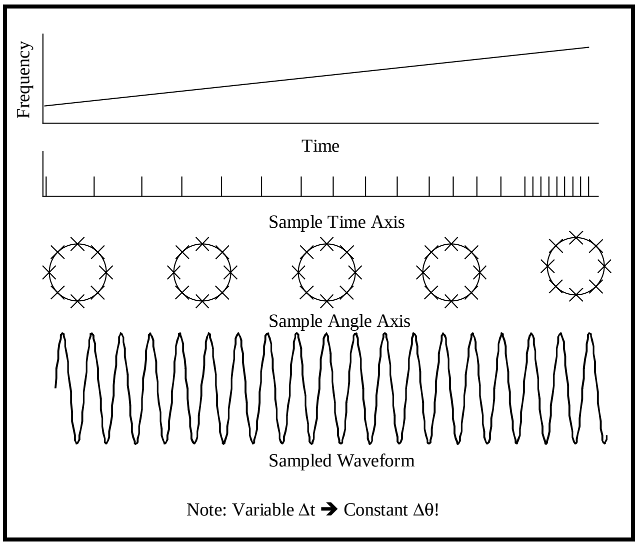

Figure 4: Chirp function resampled to angle domain.

To perform a Fourier transform on the angle domain data the Fourier transform sampling equations must be reformulated in terms of angle and order. The reformulated sampling equations are given in Equation 3.

$$\begin{aligned} \Delta o &= \frac{1}{R} = \frac{1}{N * \Delta\theta} \\ R &= N * \Delta\theta \\ O_{nyquist} &= O_{\max} = \frac{O_{sample}}{2} \\ O_{sample} &= \frac{1}{\Delta\theta} \end{aligned} \qquad (3)$$

where: Δo is the order spacing of the resulting order spectrum.
R is the total number of revolutions that are analyzed.
N is the total number of time points over which the transform is performed.
$\Delta\theta$ is the angular spacing of the resampled samples.
O_{sample} is the angular sample rate at which the data is sampled.
$O_{nyquist}$ is the Nyquist order.
O_{max} is the maximum order that can be analyzed.

As shown in Equation 3, there are analogous quantities for each of the time domain sampling parameters in the angle domain. The order resolution is related to the number of revolutions that the machine turns through over the transform period. It should also be noted that the minimum sampling rate needed to avoid aliasing is two samples per cycle of the highest order of interest, the same as that required with time domain data.

The kernels of the Fourier transform are also reformulated in terms of the uniform angular intervals. These kernels are presented in Equation 4.

$$\begin{aligned} a_m &= \frac{1}{N}\sum_{n=1}^{N} x(n\Delta\theta)\cos(2\pi o_m n\Delta\theta) \\ b_m &= \frac{1}{N}\sum_{n=1}^{N} x(n\Delta\theta)\sin(2\pi o_m n\Delta\theta) \end{aligned} \tag{4}$$

where: o_m is the order which is being analyzed, mΔo.
a_m is the Fourier coefficient of the cosine term for o_m.
b_m is the Fourier coefficient of the sine term for o_m.

The result from this angle domain Fourier transform is that the orders fall on spectral lines, regardless of the speed variations over which the transform is applied. Typically a DFT is performed instead of an FFT so that the transform can be applied over the number of points which corresponds to an integer number of revolutions of the machine’s rotation. This leads to a leakage free order estimate for orders that fall on spectral lines. Since the data is sampled in the angle domain there is a constant number of orders present in the data regardless of the rotational speed of the machine. This also results in a constant order resolution, this is beneficial because the integration time is shorter at higher rpm values. This property is desired since orders will change frequency more rapidly at higher rpm values and therefore are more likely to change amplitude more rapidly as well. Constant order resolution also allows closer orders to be analyzed at low rpm values where they are close together in frequency.

2.2.2 Digital Resampling Based Order Tracking Limitations and Errors.

Resampling based order tracking, while being more accurate than the FFT based order tracking, still has limitations and errors associated with it.

An obvious limitation with any FFT type analysis is the finite order resolution. This presents a problem if there are orders present that do not fall on spectral lines. These orders are difficult to analyze with an FFT approach.

The limitation of the FFT that was discussed above in the analysis of non-stationary amplitude data is still present with the angle domain Fourier transform. The resampling process does nothing to overcome this limitation other than allow the transform to be applied over a shorter period of time at the higher rpm values. The transform is applied over a shorter time at higher rpm values because a shaft requires less time to rotate through the desired number of revolutions.

Another restriction with resampling based order tracking is that orders may only be tracked relative to one rotating shaft. This restriction also implies that orders that cross one another can not be analyzed accurately. Orders which cross one another may be generated by two different rotating components which are not rigidly coupled to one another. This may occur across a torque converter in an automatic transmission or in a continuously variable transmission. To separate the crossing orders or orders that are not relative to the same shaft requires the use of multiple tachometer signals, one for each independent input to the system. Resampling based order tracking cannot use multiple tachometer signals to estimate the response of the system due to the multiple inputs present.

2.3 Kalman Filter Based Order Tracking.

An order tracking method that overcomes many of the limitations of order resolution is the Kalman filter based order tracking. The Kalman filter methods allow the extraction of the time history of the order as well as the estimate of the amplitude and phase of an order. The Kalman filter was first adapted to order tracking by Vold and Leuridan [ref. 9,10]. Since this original implementation, Vold has continued to develop more advanced filters with additional capabilities [ref. 11-13].

2.3.1 Original Kalman Order Tracking Filter Theory.

The Kalman filter approach to estimation requires that apriori information of some type be known [ref. 14,15]. To use the Kalman filter to extract order information from data requires information about the order to be extracted. In this case, the frequency is known from processing of the tachometer signal.

This apriori information is used to formulate the structural equation of the Kalman filter. The structural equation is an equation that describes the mathematical characteristics of the order to be extracted. The structure equation that can be used to mathematically describe a sampled sine wave, and used in the original second order Kalman order tracking filter, is given in Equation 5.

$$x(n\Delta t) - 2\cos(\omega\Delta t)x((n-1)\Delta t) + x((n-2)\Delta t) = 0 \qquad (5)$$

where: $x(n\Delta t)$ is the n^{th} discrete time sample.
ω is the instantaneous frequency of the sine wave.

Equation 5 describes a sine wave whose frequency and amplitude is constant over three consecutive time points. The frequency of an order is allowed to vary with time, which implies that the frequency of the sine wave is not constant. The structure equation is then re-written to account for this, shown in Equation 6.

$$x(n\Delta t) - 2\cos(\omega\Delta t)x((n-1)\Delta t) + x((n-2)\Delta t) = \varepsilon(n) \qquad (6)$$

where: $\varepsilon(n)$ is the nonhomogeneity term.

The nonhomogeneity term is used to describe the amplitude and frequency variations from a perfect sine wave. Mathematically, if a sine wave is amplitude modulated there must be other frequency components present in the data. These additional frequencies are sidebands which allow the amplitude of the sine wave to change with time. If the amplitude is to change quickly, then more frequency information must be allowed to pass through the filter and the nonhomogeneity term must be larger.

The second equation that the Kalman filter is based on is the data equation. The data equation describes the relationship between the order, x(n), and the measured data, y(n). The measured data contains not only the order of interest but all orders generated by the machine and other random noise present in the data. This equation is written in Equation 7.

$$y(n) = x(n) + \eta(n) \tag{7}$$

where: $\eta(n)$ is the nuisance component.

The nuisance component, $\eta(n)$, is the portion of the signal containing the non-tracked orders and random noise. If the nuisance term is large it indicates that a significant portion of the measured signal, y(n), is attributable to non-tracked orders and random noise.

The structure and data equations are combined into a set of linear equations to solve for the amplitudes of the order of interest. Normally, the least squares formulation is formulated in a matrix form to solve for all points of a time history simultaneously [ref. 9,21,22]. This formulation is technically a Kalman smoothing algorithm, as opposed to a filtering algorithm, because it can use time points both before and after the desired time point to obtain the order estimate [ref 14,15].

A weighted solution to this problem is formed by ratioing the standard deviations of the structure and data equations. This ratio is what is referred to as the *Harmonic Confidence Factor*, HCF, in commercial implementations of this filter. The value of this parameter is what determines the tracking characteristics of the filter.

Choosing a relatively high value for the HCF weights the structure equation more heavily in the solution process and results in a filter shape that is very narrow. This allows very little sideband information to pass through the filter and hence only allows the amplitude to change slowly. A relatively high HCF then gives a very sharp filter with very good frequency discrimination.

Choosing a relatively low value for the HCF has the effect of weighting the data equation more heavily in the solution process and results in a filter that does not possess as sharp a rolloff and therefore is not as frequency discriminating as the high HCF filter. Using a low HCF allows the amplitude of the filtered order to vary much more quickly than the high HCF filter. This behavior may be necessary around lightly damped resonances or in fast speed sweeps where the frequency and amplitude of the order must be allowed to vary quickly. The various filter characteristics and how their tracking characteristics vary relative to the HCF are well documented [ref. 9,21,22].

2.3.2 Vold-Kalman Order Tracking Filter.

Vold both simplified and extended the original Kalman order tracking filter into the Vold-Kalman order tracking filter [ref. 11]. This extended filter can be formulated with different numbers of poles to alter its bandpass characteristics. The filter may also be applied in either an iterative or direct solution to separate the contributions of very close or crossing orders.

The Vold-Kalman form of the structure equation has several advantages over the original second order formulation [ref. 11,21,22]. One key advantage of this form of the structure equation is that the frequency term can be eliminated and therefore it becomes obvious that there is absolutely no frequency or slew rate limitations. It is also possible to reformulate the structure equation to gain computational efficiency in its solution [ref. 16,17].

An additional capability of the Vold-Kalman filter is the ability to reformulate the filter as a higher order filter in order to have a broader passband region while improving the sideband rejection. This filter has a flatter top relative to the single point peak which the first order filter possesses. The penalty paid for these improved filter characteristics is computational complexity. A higher order formulation requires the solution of a more heavily banded matrix as opposed to the first order's tri-diagonal matrix. For example, a 2^{nd} order formulation has a banded matrix with 5 fully populated diagonals. The actual formulation of the higher order filters is considered to be proprietary by Vold.

While the higher order filter formulations can possess much sharper passbands, they may not be the filter of choice for very close orders because of their broader top. They are more forgiving if an imperfect tachometer signal was measured. The higher order filters should be used if there are high amplitude orders that are close to the tracked order but outside of its passband. The sharper filter skirts will allow these other orders to be more effectively eliminated from the tracked order.

Probably, the most important characteristic which has been added to the order tracking analysis capability by the Vold-Kalman filter is the ability to separate either very close or crossing orders. The ability to separate interacting orders may be implemented through either an iterative solution or a direct solution. The formulation and solution of these filters are considered to be proprietary by Vold. This ability is commercially available in multiple software packages.

2.3.3 Kalman/Vold-Kalman Applications and Realizations.

The formulations and discussions of both the original and the Vold-Kalman filters are centered about a weighting factor that is constant as a function of time. This is not a requirement of either filter's formulation. In fact, all commercial implementations of either filter allow the weighting factor to vary as a function of time or rpm. If the weighting factor is varied as a function of rpm, a pseudo-constant order bandwidth filter may be obtained. Another strategy to vary the weighting factor of the filter is based on instants of known transient activity in the data. Examples of this transient activity are gear shifts or clutch engagements. In these areas of transient activity, it is assumed that the amplitude of a tracked order may change very quickly, thus the weighting factor is reduced in these regions to allow more sideband energy to pass through the filter. The additional sideband energy allows the amplitude of the order to change very quickly in these regions, allowing a realistic amplitude profile to be estimated.

The best frequency discrimination that can be expected from the Kalman filters is the same as that of the Fourier transform. Frequencies closer together than the inverse of the total length of time of the data cannot be effectively separated, as defined by Rayleigh's criteria.

While the Kalman order tracking methods have many advantages over the traditional order tracking methods, including better dynamic range and the time domain order extraction, they do have some disadvantages. Computational complexity is one disadvantage of the adaptive filters. The largest disadvantage, however, is the experience required to get valid accurate results for each order extraction. The experience is required in choosing the appropriate weighting factor to extract the order with a minimal bandwidth while tracking the amplitude profile accurately. The weighting factor that is necessary for an accurate extraction is a function of the other orders present in the data, the sweep rate, and the properties of excited resonances. All of these items, which can vary from channel to channel, change the rate at which the amplitude of an order changes and therefore affect the width of the filter necessary for an accurate extraction.

2.3.4 Iterative Time Varying HCF Vold-Kalman Order Tracking Filter.

Realizing the limitations of the commercially available Kalman/Vold-Kalman order tracking filter implementations, a new filter implementation was explored [ref. 23]. The two largest shortfalls in the current Kalman implementations are the determination of the HCF and how to determine if an order has been accurately filtered.

Two new approaches were developed to overcome these limitations, one method more automated than the other. Both of these methods made an attempt to vary the HCF based on an initial guess at an HCF value and the results of applying that HCF to the data. After applying the filter with this constant HCF value the amplitude profile of the filtered order was used to reshape the HCF and the data re-filtered. Where large amplitude changes occurred quickly the HCF was reduced, where there were essentially no amplitude changes over a period of time the HCF was increased. This process was repeated until an amplitude envelope for the order of interest was converged upon. The difference between the two methods developed were that in one case the results were iterated upon over the entire time history while in the other case the time history was divided into pieces and the iterations performed over the pieces.

Both of these iterative filter implementations showed that this is a feasible way to apply the Vold-Kalman filter. More development and evaluation of these methods is necessary as they are still in the research stages of implementation. Further improvements in computational efficiency are also required as in most cases it requires 4-6 iterations for the filter's response to stabilize.

2.4 Time Variant Discrete Fourier Transform Order Tracking.

2.4.1 Time Variant Discrete Fourier Transform Order Tracking Theory.

The time variant discrete Fourier transform (TVDFT) method of order tracking is a special case of the chirp-z transform. The chirp-z transform is defined as a type of Fourier transform with a kernel whose frequency and damping vary as a function of time [ref. 18]. The TVDFT is defined as a discrete Fourier transform whose kernel varies as a function of time defined by the rpm of the machine, but the damping does not vary as a function of time. The TVDFT has many of the advantages of the resampling based order tracking methods, while reducing the computational load of the calculations considerably [ref. 19,20].

The TVDFT method is based on constant delta-t sampled data. Whether it is desired to analyze data with a constant frequency or constant order bandwidth determines whether the sampling theorem used is based on constant Δt data or constant $\Delta\theta$ data.

The TVDFT is based on the transform shown in Equation 8. It should be noted that the kernel of this transform appears as a portion of the structure equation used in the order tracking Kalman filter [ref. 11]. This kernel is a cosine or sine function of unity amplitude with an instantaneous frequency matching that of the tracked order at each instant in time. This kernel may also be formulated in a complex exponential format similar to the corresponding Fourier transform.

$$\begin{aligned} a_n &= \frac{1}{N}\sum_{n=1}^{N} x(n\Delta t)\cos\left(2\pi\int_0^{n\Delta t}(o_n * \Delta t * rpm/60)dt\right) \\ b_n &= \frac{1}{N}\sum_{n=1}^{N} x(n\Delta t)\sin\left(2\pi\int_0^{n\Delta t}(o_n * \Delta t * rpm/60)dt\right) \end{aligned} \tag{8}$$

where: o_n is the order which is being analyzed.

a_n is the Fourier coefficient of the cosine term for o_n.

b_n is the Fourier coefficient of the sine term for o_n.

rpm is the instantaneous rpm of the machine.

This transform is best suited to estimate an order with a constant order bandwidth. A constant order bandwidth estimate may be obtained by performing the transform over the number of time points required to achieve the desired order resolution, as defined by Equation 3. This implies that as the rpm increases, the transform will be applied over a shorter time, giving a wider Δf equivalent to a constant order bandwidth. This behavior was also exhibited by the resampling methods and was determined to be advantageous for order tracking. This transform is normally only performed for the orders that are desired and not for a full spectrum as was done in the previously described order tracking methods.

Since the frequency of the kernel of this transform matches the frequency of the order of interest at each instant in time, there is no leakage due to the order not falling on a spectral line. There will, however, be leakage effects from other orders that are present in the data. These orders can “leak” into the frequency band of analysis around the order. Typically used windows for conventional FFT analysis are also used with this transform. Since all windows have a frequency resolution/amplitude estimate tradeoff, the window chosen can have a significant effect on the results. Which window to use depends on the order content of the data and the aspects of the order estimate the user feels are most important.

The TVDFT order tracking method presented here is a very practical order tracking method which can be implemented in a very efficient manner on a computer. This method contains many of the advantages of the resampling based algorithms without much of the computational load and complexity. Computational efficiency is gained for large numbers of channels by computing the transform kernel once, storing it, then applying it to each channel. Any window used in the analysis should be applied to this pre-computed kernel, since the window only has to be applied once if it is applied to the kernel instead of once for each channel. This method provides better order estimates than the FFT based methods.

2.4.2 Orthogonality Compensation Matrix Theory.

To enhance the capabilities of the TVDFT for tracking orders and to reduce the errors due to non-orthogonality of the kernels, an orthogonality compensation matrix (OCM) may be applied. The application of the OCM allows faster sweep rates to be analyzed, as well as closely spaced and crossing orders to be analyzed more accurately. This OCM is applied as a post-processing of the order estimates from the TVDFT analysis.

To apply the OCM, all orders of interest are first tracked using the TVDFT with either a constant frequency or constant order bandwidth. This tracking should be done intelligently, as the quality of the compensation is related to the quality of the original order estimates. This implies that the user may want to apply a Hanning window to increase out of band rejection. The bandwidth used may be somewhat wider than is minimally necessary to separate closely spaced orders. The amount of relaxation of the minimum bandwidth depends on the window used in the analysis. This relaxation of the bandwidth allows fewer revolutions to be analyzed at a time if desired, which allows faster sweep rates to be analyzed.

The application of the OCM is a linear equations formulation that is shown in Equation 9.

$$\begin{bmatrix} e_{11} & e_{12} & e_{13} & \cdots & e_{1m} \\ e_{21} & e_{22} & e_{23} & & \\ e_{31} & e_{32} & e_{33} & & \vdots \\ \vdots & & & \ddots & \\ e_{m1} & & \cdots & & e_{mm} \end{bmatrix} \begin{Bmatrix} o_1 \\ o_2 \\ o_3 \\ \vdots \\ o_m \end{Bmatrix} = \begin{Bmatrix} \tilde{o}_1 \\ \tilde{o}_2 \\ \tilde{o}_3 \\ \vdots \\ \tilde{o}_m \end{Bmatrix} \tag{9}$$

where: e_{ij} is the cross orthogonality contribution of order *i* in the estimate of order *j*.
o_i is the compensated value of order *i*.
$\tilde{o}_i$ is the estimated value of order *i* obtained using the TVDFT.

The cross orthogonality terms, e_{ij}, are calculated by applying the kernel of order *i* to the kernel of order *j*, as shown in Equation 10.

$$\begin{aligned} e_{ij} = \frac{1}{N}\sum_{n=1}^{N}\left\{\exp\left(2\pi\int_{0}^{n\Delta t}\left(o_i * \Delta t * rpm / 60\right)dt\right)\times Window\right\}\times \\ \exp\left(2\pi\int_{0}^{n\Delta t}\left(o_j * \Delta t * rpm / 60\right)dt\right)^{*} \end{aligned} \tag{10}$$

The window used in the original order estimate is applied to order *i* to compensate for any correction factor that may need to be applied to scale the data correctly. It also includes the effects of the shape of the window in the compensation. Each term in the matrix represents the amount that the orders' kernels interact with one another in the transform estimation. If the orders included in the calculation of the OCM are orthogonal, the off diagonal terms of this matrix will be zero, as is the case for the standard Fourier transform kernels. Since the effects of any orders not included in this calculation are not compensated, it is recommended that all significant orders be included in the compensation calculation.

Very closely coupled orders are normally very difficult to separate using standard FFT or resampling techniques, as well as the TVDFT without compensation, because the orders may beat with one another. However, with compensation the TVDFT can separate the contributions of the orders effectively. Initially, the orders should be tracked with a bandwidth that is at its largest approximately equal to the spacing of the closely coupled orders. If the orders are tracked with this bandwidth using a Hanning window, the order estimates will contain beating of the two orders. This beating effect can be removed by applying OCM.

Crossing orders pose a similar problem to that of closely spaced orders. Oftentimes, if two orders cross one another, the order estimates are incorrect at the crossing rpm due to the interaction of the orders. Tracking the orders and then applying the OCM allows the separation of the contributions from each order.

2.5 Other Order Tracking Methods.

Several other order tracking methods have been developed that are not currently available in widespread commercial application packages. These methods include methods based on the Prony Residue Estimation process, methods based on the Maximum Likelihood process, as well as other methods based on conventional digital filtering methods.

The Prony Residue Estimation process, more commonly known as a Complex Exponential Method from Modal Analysis parameter estimation, has several disadvantages over the methods discussed above. The largest two disadvantages that the method possesses are its computational complexity and the requirement that a model order of the data be known. The determination of the model order, or number of orders and resonances in the data, is not trivial because it is typically a function of rpm and which resonances are excited at each instant in time.

The other methods based on conventional digital filtering methods all appear inferior to the Kalman/Vold-Kalman filtering methods because of their passband shapes. These filtering methods also tend to be computationally demanding, though not as demanding as the Kalman/Vold-Kalman implementations.

3. Order Tracking Method Summary.

To summarize the various problems of the order tracking methods presented it is required that the two primary difficulties in tracking orders be first stated. As discussed above, the first difficulty that is encountered when attempting to track orders from rotating machinery is that the frequency components of interest, or orders, change as a function of time. Assuming that an accurate tachometer signal has been measured this limitation can be overcome by most of the order tracking methods as they explicitly use rpm in their theoretical formulations.

The more difficult problem to overcome is the fact that the amplitude of each order varies as a function of time/rpm based on both the forcing function supplying energy to the system of interest changing its amplitude, as in an unbalance, and due to resonances that get excited. The error associated with this problem is two-fold. One problem being that the peak amplitude that an order may have will always be underestimated by the order tracking methods. This is because all of the methods assume the amplitude of the order to be constant, or very slowly varying, over at least a short period of time. Each of the different order tracking methods responds to this error differently. No order tracking method has been developed to this date that can effectively handle this problem in an automated fashion.

The other facet of this amplitude variation is the fact that for an amplitude to change there must be other frequency content present. No current order tracking method has the ability to predict when or to what level this frequency content is present. For instance, when an order excites a resonance the other frequency content present will be the frequency of the resonance. This energy will be present until the order's frequency has moved far enough away from the resonance so as not to excite it anymore and the energy associated with the impulse response of the resonance has decayed down to a level in the noise floor of the data.

4. Future of Order Tracking Analysis.

Current and future demands of order tracking analysis will require DSP methods to continue to become more sophisticated and simultaneously more computationally efficient. For example, while frequency domain transfer path analysis, or conventional TPA, has been in use for several years, it has required that the latest computers be employed to perform the necessary calculations in a minimum amount of time. The major computations this analysis requires are a matrix inverse at each frequency line for each operating condition.

Recently a major software developer has introduced a type of analysis that is referred to as time domain transfer path analysis. This time domain approach requires the same matrix inverses required for conventional TPA to be computed as well as all major orders filtered from the operating data and subsequently a two-dimensional surface to be fit to the background noise. This procedure of filtering and surface fitting is required for each channel in the analysis. This tool is used more as a sound quality application than a force estimation tool. New time histories are synthesized for each channel after the operating forces are estimated and altered. These time histories can then be played back by the user to assess the effectiveness of changing hardware on the sound quality of a machine. The requirement that new order tracking analysis methods have in this type of analysis is that they must be computationally efficient filtering algorithms that completely remove the orders of interest. If there exists close or crossing orders in the data these must also be handled correctly.

As the computational complexity of the NVH analysis methods and particularly order tracking and adaptive filtering continue to increase it would be very beneficial to take advantage of the multiple processors available in computers. Taking advantage of multiple processors requires that either new methods be developed that can explicitly be solved in parallel processes or that current methods must be adapted and re-written.

As more companies are forced to work on reducing the noise and/or vibrations of their particular products there will be personnel involved who do not have a formal training in NVH. Software packages must be designed so that it is not overly difficult for non-NVH personnel to learn to use them and to perform many of the more basic types of analysis. For instance the current Kalman/Vold-Kalman implementations are not appropriate for non-experienced engineers. Research must be performed to try to ease the learning curve on these implementations. Several major software developers are beginning to develop and market products to fill this void in the marketplace.

Conclusions

Theory and implementation of commonly available order tracking methods was presented. The two major sources of error in these analysis methods were discussed. These errors are due to the frequency changing as a function of time and the amplitude varying with respect to time. From the discussion above it is evident that a new order tracking method that was both easy to use and could accurately handle these two errors would be welcome in the NVH analysis community.

Demands placed on current order tracking analysis methods include ease of use and computational efficiency. With the common availability of multi-processor computers and the computational complexity of current order tracking methods it becomes obvious that an order tracking method should be developed or implemented in a manner that it takes advantage of the specific processing advantages of parallel processors.

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