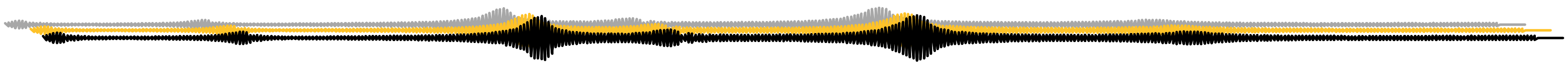

Wavelets:
What are they?
What good are they?

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Overview of Presentation

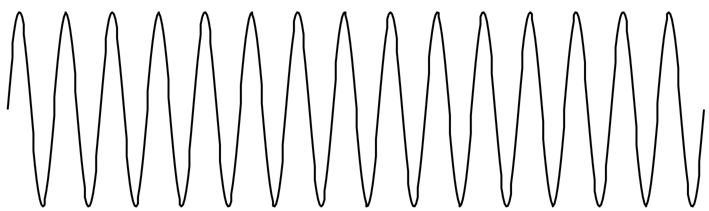
- Overview of FFT.
- Overview of Wavelets.
  - What is a wavelet?
  - Discrete wavelet transform.
  - Continuous wavelet transform.
- Overview of Wigner-Ville transform.

- LMS Wavelet Implementation.
  - Practical application.
- MATLAB Wavelet Toolbox.
  - Practical application.

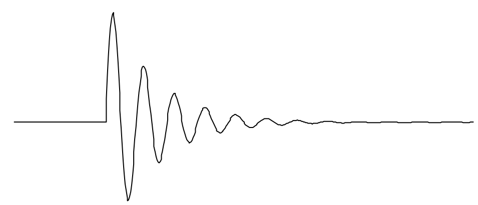
- What Good are Wavelets?

Fast Fourier Transform and Transient Signals.

- FFT is based upon sine/cosine transform.
  - Sine/cosine transform is infinite in length.
  - Would you use this function to analyze a transient signal?

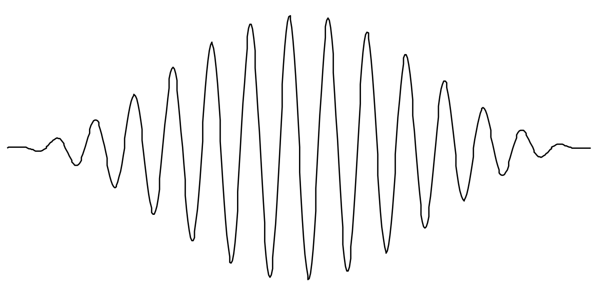


Analyzing signal



Transient Signal

  - This function is more appropriate, isn't it?
    - Use *Window* function to truncate in time.



Analyzing signal

FFT Sampling Properties.

- Windowing the sines/cosines of the FFT gives time localization. Gabor or Short Time Fourier Transform (STFT)
- What penalty do we pay?
  - Shannon’s sampling theorem tells us that we lose frequency resolution when we gain time resolution!

$$\Delta f = \frac{1}{T} = \frac{1}{N * \Delta t}$$

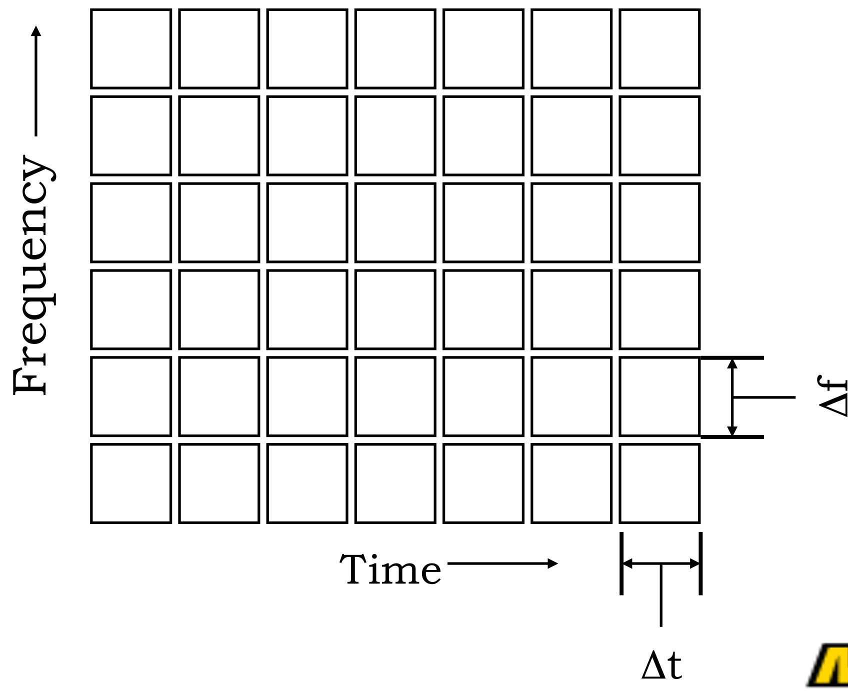
$$T = N * \Delta t$$

$$F_{nyquist} = F_{\max} = \frac{F_{sample}}{2}$$

$$F_{sample} = \frac{1}{\Delta t}$$

STFT Resolution.

- The time/frequency resolution of the STFT can be represented by the grid shown below.
  - STFT contains the same time resolution regardless of the frequency analyzed.



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What is a Wavelet?

- A wavelet is a mathematical entity.
  - Many different wavelets exist.
    - Which wavelet do you use where? … Good question!!
- Most wavelets are orthogonal, not a requirement!
- Most wavelets are invertible, not a requirement!
- Wavelets possess a variable time/frequency resolution.
  - Octave like frequency resolution, $\Delta t * \Delta f = \text{constant}$.
- Signal analysis tool.
  - Remember that sines/cosines also solve differential equations!
- Can think of a wavelet as a special digital filter.

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What is the wavelet transform?

- Continuous Wavelet transform is shown here.
  - Note similarity to Fourier transform.

$$WT(a,\tau) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} f(t)\psi\left(\frac{t-\tau}{a}\right)dt$$

$$f = \frac{f_0}{a}$$

What does a wavelet kernel look like?

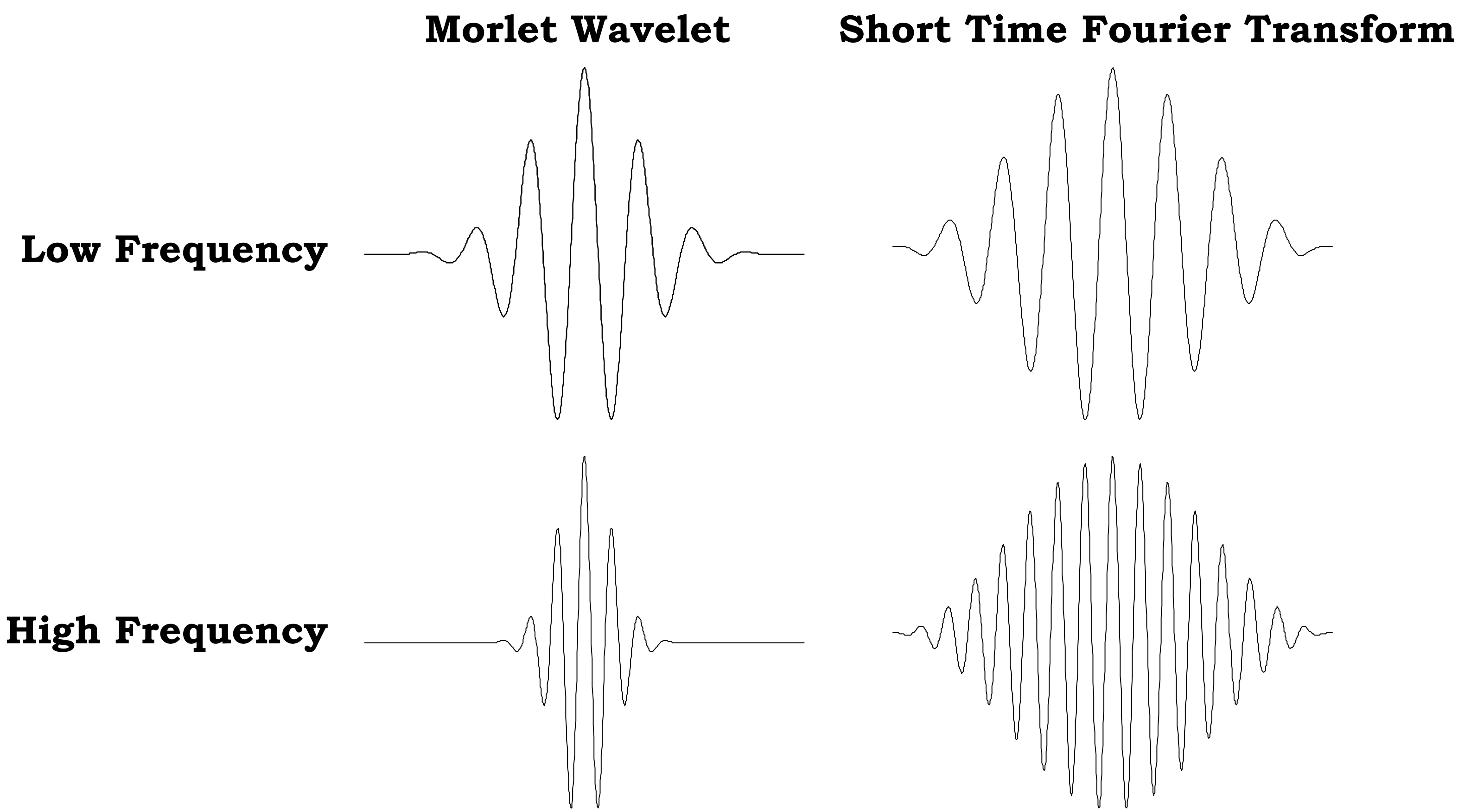
- Morlet wavelet is one wavelet used for NVH.

$$\psi(t) = e^{-j\pi n\frac{t}{T}} e^{\frac{-t^2}{2T^2}}$$

Exponential kernel

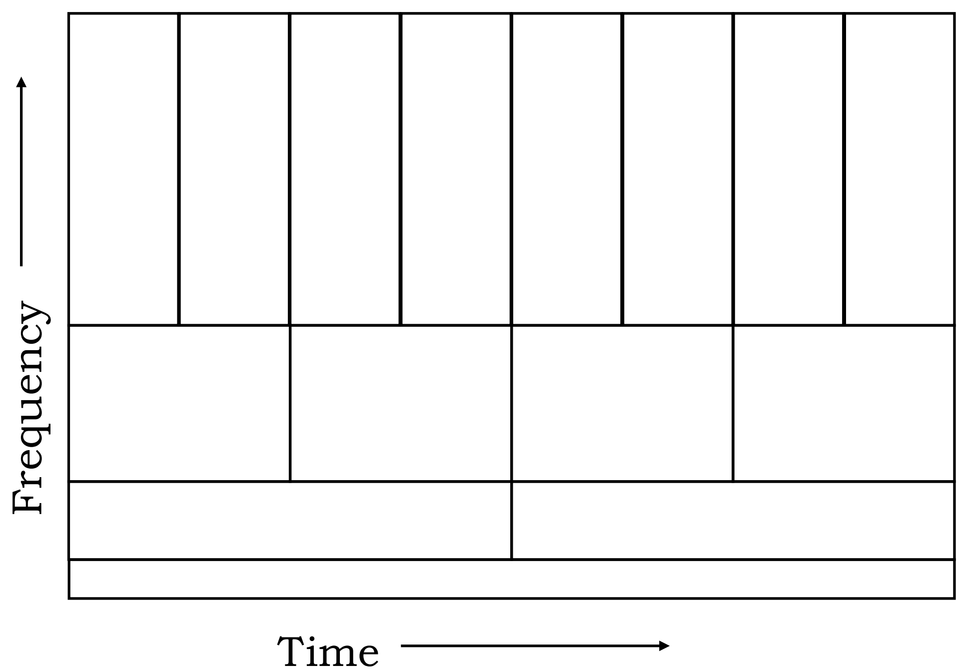
Gaussian window

What does a wavelet look like?



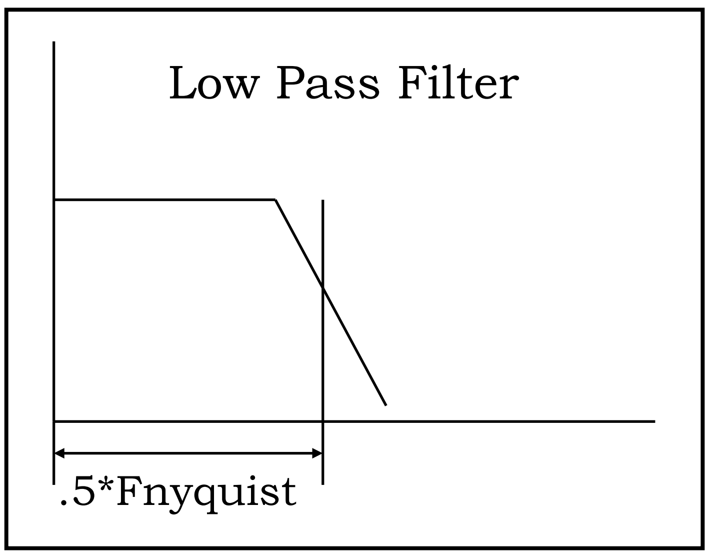
Discrete Wavelet Transform.

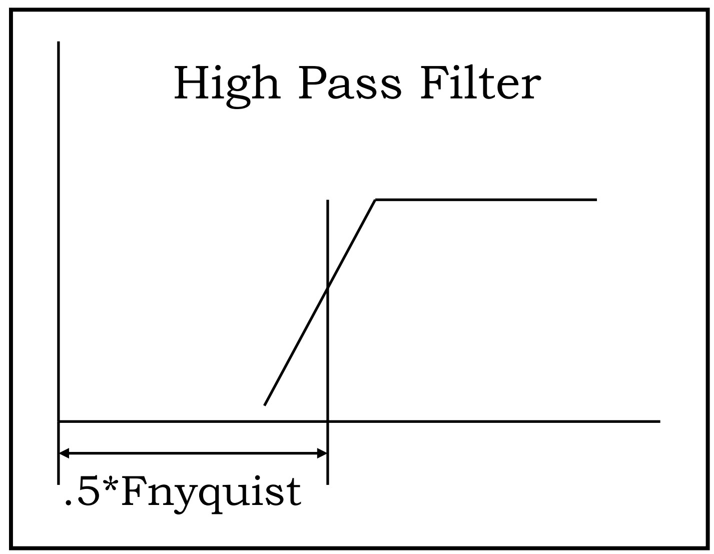
- Discrete wavelet transform is one common implementation of wavelet analysis.
  - Result is discrete time/scale map.



Continuous Wavelet Transform.

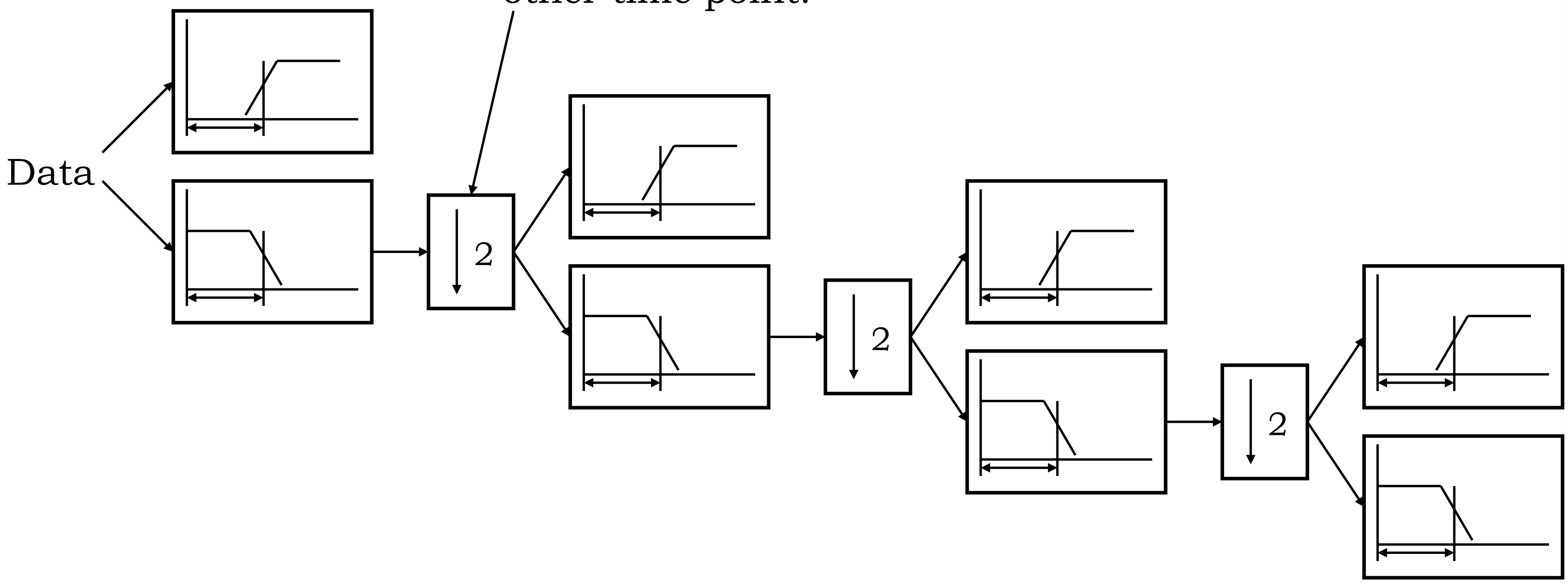
- Continuous wavelet transform can result in time history outputs.
  - One time history for each scale which is analyzed.
  - Typically reversible type transform.
  - Compute with pair of Quadrature Mirror Filters (QMF).
    - What is a QMF?





How is a Continuous Wavelet Transform Calculated?

Decimate - Discard every other time point.



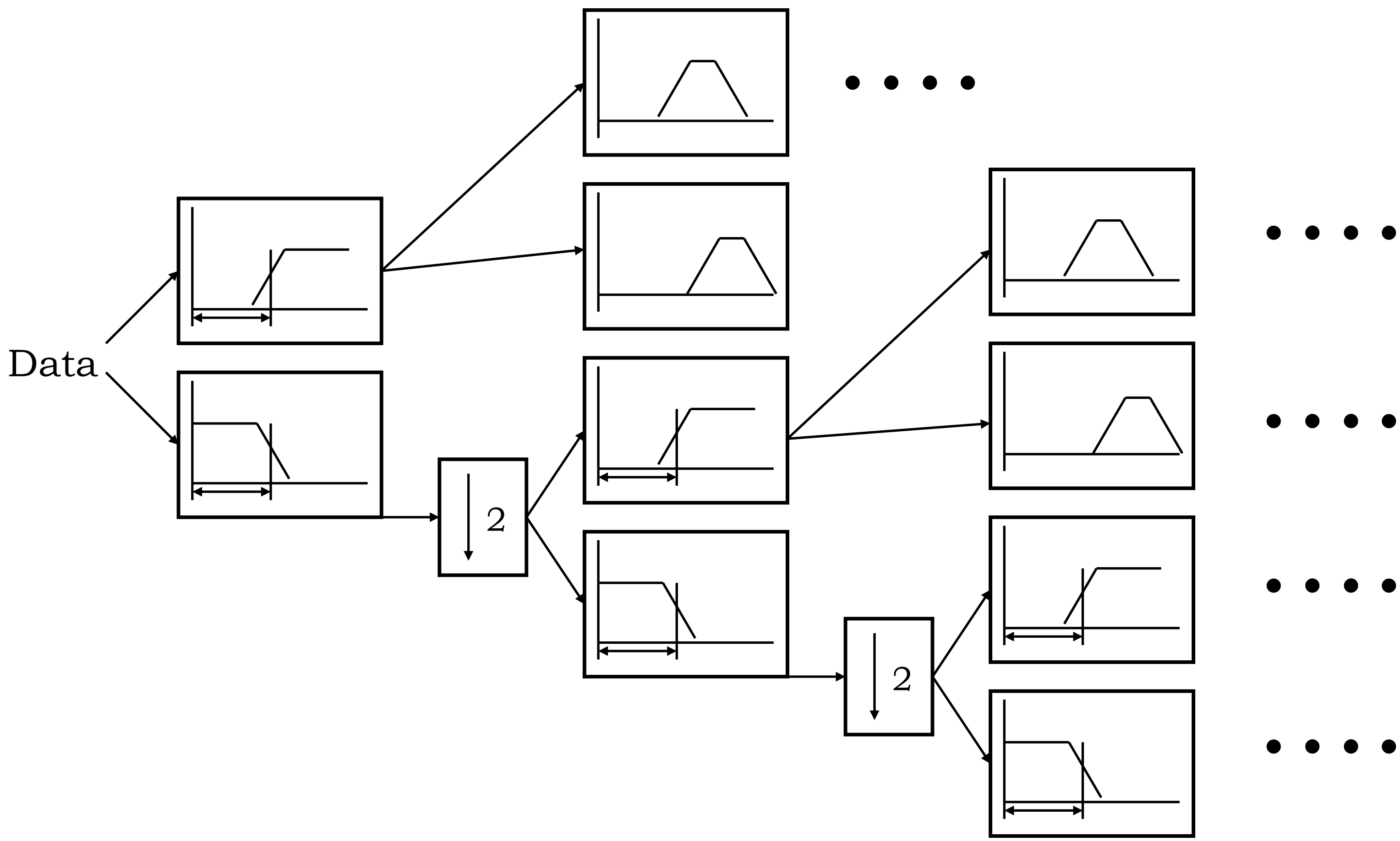
Detail - output of each high pass filter.

Approximation - output of each low pass filter.

What is Wavelet Packet Analysis?

- Wavelet packet analysis is an extension of the *Continuous Wavelet Transform.*
  - More flexible frequency resolution.
  - Filter the details, high pass filter output, into separate frequency bands.
    - Oftentimes not every detail is filtered.
      - Filter details with lots of energy.
      - Filter all details same number of times.

How is Wavelet Packet Analysis Calculated?



What is the Wigner-Ville Transform?

- Wigner-Ville transform is another transform used for time/frequency analysis.
  - Several enhanced versions have been developed:
    - Choi-Williams, Pseudo-WVT, Smoothed-WVT, ...etc.

$$W_s(\tau, f) = \int_{-\infty}^{+\infty} s\left(t + \frac{\tau}{2}\right) s^*\left(t - \frac{\tau}{2}\right) e^{-j2\pi f t} dt$$

Auto-correlation function

Exponential Kernel

Properties of Wigner-Ville Transform.

- Disadvantages:
  - Artifacts can appear along the frequency axis if harmonics or multiple frequencies are present.
    - Can result in negative energy in time/frequency distribution.
  - Artifacts can appear along the time axis if multiple events are present in signal along time axis.
  - Requires twice the normal sample rate to avoid aliasing.
    - Signal can be digitally upsampled to obtain this.
      - Results in twice as much time data.

- Advantages:
  - Time resolution is that of original sample rate.
  - Frequency resolution is dependent on blocksize analyzed.

Overcoming Limitations of Wigner-Ville Transform.

- Digitally upsample data to avoid aliasing.
- Smoothing - Usually Gaussian shaped window.
  - Apply window in frequency domain to reduce frequency domain artifacts.
  - Apply window in time domain to reduce time axis artifacts.

- Avoid time domain artifacts by processing one event at a time.
  - ie. Separate front and rear wheel bump impacts into two separate time histories.
    - Disadvantage can be loss of frequency resolution due to short time block.

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LMS Wavelet Implementation.

- LMS utilizes the Morlet wavelet in a Discrete Wavelet Transform.
- Result is time/frequency colormap.
- Frequency resolution is defined by choosing number of voices/octave … 1/12 octave, 1/6 octave …etc.
- Frequency range is defined by number of scales or levels.
  - More scales results in lower frequencies being broken into smaller bands.

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How do we use LMS Wavelets.

- Use Wavelet, STFT, and Wigner-Ville transforms on same data to generate colormaps.
  - By observing all three colormaps we get the best of all worlds.
    - Low frequency: STFT good time resolution, Wavelet good frequency resolution, WVT best compromise if no artifacts.
    - High frequency: STFT good frequency resolution, Wavelet good time resolution, WVT best compromise if no artifacts.
  - Wavelets should be beneficial for determining *when transient impact occurs.*
    - Clutch engagements, impact bumps, gear shifts…etc.
      - These events all possess high frequencies and impart discontinuities into otherwise periodic data…Wavelets work well for finding when a discontinuity occurs.

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MATLAB Wavelet Toolbox.

- MATLAB Wavelet toolbox is a general purpose wavelet toolbox.
  - Includes both Discrete and Continuous Wavelet Transforms.
    - Includes two different implementations of Wavelet Packet Analysis.
    - Includes option of Morlet as well as many other wavelets.
    - Includes 2-D Wavelet utilities for image processing and compression.
  - Use Discrete Wavelet Transform to obtain results similar to LMS.
  - Use Continuous or Packet Analysis to filter signal to remove frequency ranges from data or to observe the filtered time history from a particular frequency range.
    - Essentially special set of digital filters.

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How do we use MATLAB Wavelets?

- MATLAB has all capabilities of LMS *except Wigner-Ville transform.*
  - Easy to implement Wigner-Ville transform in MATLAB.
- User is responsible for proper scaling in many cases to generate accurate amplitudes. (FFT scale for # of points)
- MATLAB has additional capabilities of filtering with Wavelets.
- MATLAB has Wavelet packet analysis for better frequency resolution at higher frequencies!

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What Good are Wavelets?

- Wavelets are a special set of computationally efficient digital filters.
  - Wavelet filter bank is pre-defined and easy to apply.
    - Must choose an appropriate Wavelet to use!!…Morlet for NVH.
      - More research is appropriate here for other kinds of Wavelets.

- Wavelets are currently a signal analysis and decomposition tool.
  - Research is required to determine if Wavelets have further application in determining system properties.
    - Reduce Leakage? …etc.
  - Frequency resolution decreases at high frequencies, this is contrary to structure behavior where natural frequencies get closer together as frequency increases.

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