Fast Deep Coherence Holography (FDCH) for 3D Object Reconstruction

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Principle of coherence holography

The principle of coherence holography has been described in detail in [4,29]. An off-axis 3D object $g(x, y, z) = |g(x, y, z)| \exp[i\phi(x, y, z)]$ located before the front focal plane of the Fourier Transform (FT) lens generates an object wave at the hologram plane which interferences with the reference wave from a coherent source. The complex amplitude of the object wave at the hologram plane is given by

$$G(\hat{x}, \hat{y}) = |G(\hat{x}, \hat{y})| e^{[i\Phi(\hat{x},\hat{y})]} \propto \iint g_{ASP}(x, y, z = 0) e^{\left[-i\frac{2\pi}{\lambda f}(x\hat{x}+y\hat{y})\right]} dxdy = F\{g_{ASP}(x, y, z = 0)\}|_{k_x=\frac{2\pi}{\lambda f}\hat{x}, k_y=\frac{2\pi}{\lambda f}\hat{y}}, \quad (1)$$

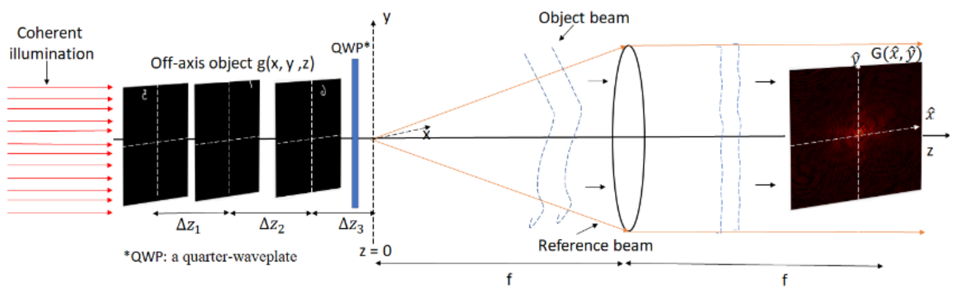

Figure 1. Generation of a Fourier transform coherence hologram.

where λ is the wavelength of light, f is the focal length of the lens, the spatial frequencies are represented by the coordinates $\hat{x}$ and $\hat{y}$. $F\{\cdot\}$ and $F^{-1}\{\cdot\}$ denote forward and inverse 2D Fourier transform operations, respectively, $g_{ASP}(x, y, z = 0) = |g_{ASP}(x, y, z = 0)| \exp(i\phi_{ASP}(x, y, z = 0) = \int \underset{z\to 0}{ASP}\{g(x, y, z)\}dz$ is the field at the front focal plane that results from propagating the object to this plane, $\underset{z\to 0}{ASP}\{\cdot\}= F^{-1}\{F\{g(x, y, z)\} e^{[-ik_z(\hat{x},\hat{y})z]}\}$ denotes the angular spectrum propagation operator. The $\exp[-ik_z(\hat{x}, \hat{y})z]$ term accounts for defocusing, and propagating the angular spectrum of the field by a distance z with $k_z(\hat{x}, \hat{y}) = \frac{2\pi}{\lambda}\sqrt{1 - \left(\frac{\hat{x}}{f}\right)^2 - \left(\frac{\hat{y}}{f}\right)^2}$.

The computer-generated hologram (CGH) of the coherence holography method is slightly different from the one of conventional holography. The term $|G(\hat{x}, \hat{y})|^2$ was removed from the interference fringe intensity to avoid unwanted autocorrelation image, and the intensity (rather than amplitude) transmittance of the hologram is made proportional to the interference fringe pattern [14]. In addition, the term $|G(\hat{x}, \hat{y})|$ is added to make the hologram positive, which allows it to be displayed as an intensity image. The equation of the CGH is then given by

$$H(\hat{x}, \hat{y}) \propto |G(\hat{x}, \hat{y})| + 0.5[G(\hat{x}, \hat{y}) + G^*(\hat{x}, \hat{y})] = |G(\hat{x}, \hat{y})|\{1 + \cos[\Phi(\hat{x}, \hat{y})]\}. \quad (2)$$

The experiment setup for capturing the interferogram used for reconstruction is shown in Figure 2. In this setup, a projector representing for a spatially incoherent light source which is modeled as an optical field with unit amplitude and instantaneous random phase $\Phi_r(\hat{x}, \hat{y})$ in the hologram plane located at the back focal plane of the FT lens L_2 is used to display the CGH. The instantaneous field right at the rear focal plane of the FT lens L2 is

$$u(x,y,z) = \iint \sqrt{H(\hat{x},\hat{y})}\exp\left[i\,\Phi_{\mathrm{r}}(\hat{x},\hat{y})\right]\ \exp[ik_z(\hat{x},\hat{y})z]\exp[i(2\pi/\lambda f)(x\hat{x}+y\hat{y})]\,d\hat{x}d\hat{y}. \quad (3)$$

This field by itself does not reconstruct the object wave since the phase has been scrambled. To eliminate the effect of the incoherence, the mutual intensity between a pair of points or the coherence function, $\Gamma(\Delta x,\Delta y,\Delta z)$, is detected by directing this incoherent illuminated field to the Sagnac radial shearing interferometer. The field intensity at the output of the Sagnac interferometer is

$$I(\Delta x,\Delta y,\Delta z) = 2\Gamma(0,0,0) + 2\mathrm{Re}[\Gamma(\Delta x,\Delta y,\Delta z)], \quad (4)$$

where $\Delta x = x_2 - x_1, \Delta y = y_2 - y_1, \Delta z = z_2 - z_1$ are the difference of the coordinates and

$$\begin{aligned}\Gamma(\Delta x,\Delta y,\Delta z) &= \iint H(\hat{x},\hat{y})\, e^{[ik_z(\hat{x},\hat{y})\Delta z]}\ e^{\left[i\left(\frac{2\pi}{\lambda f}\right)(\hat{x}\Delta x+\hat{y}\Delta y)\right]}d\hat{x}d\hat{y} \\ &\propto \tilde{g}(\Delta x,\Delta y,\Delta z) + \frac{1}{2}\left[g_{ASP}(\Delta x,\Delta y,\Delta z) + g^*_{ASP}(-\Delta x,-\Delta y,-\Delta z)\right],\end{aligned} \quad (5)$$

where $\tilde{g}(\Delta x,\Delta y,\Delta z)$ is an unwanted distribution with peak at the center, $g_{ASP}(\Delta x,\Delta y,\Delta z)$ is the complex object field at Δz. They are given by

$$g_{ASP}(\Delta x,\Delta y,\Delta z) = {}^{ASP}_{0\to\Delta z}\{g_{ASP}(\Delta x,\Delta y,\Delta z=0)\}=F^{-1}\left\{\int F\left\{g_{ASP}(\Delta x,\Delta y,\Delta z=0)\right\}e^{[ik_z(\hat{x},\hat{y})\Delta z]}\right\}. \quad (6)$$

$$\tilde{g}(\Delta x,\Delta y,\Delta z) \propto \iint |G(\hat{x},\hat{y})|\, e^{[ik_z(\hat{x},\hat{y})\Delta z]}\ e^{\left[i\frac{2\pi}{\lambda f}(\hat{x}\Delta x+\hat{y}\Delta y)\right]}\ d\hat{x}d\hat{y}. \quad (7)$$

By substituting Eqs. (5, 6, 7) into Eq. (4), the captured image at Δz is

$$\begin{aligned}I(\Delta x,\Delta y,\Delta z) \propto{} & 2\Gamma(0,0,0)+2Re\{\tilde{g}(\Delta x,\Delta y,\Delta z)\} + |g_{ASP}(\Delta x,\Delta y,\Delta z)|\cos(\phi_{ASP}(\Delta x,\Delta y,\Delta z)) \\ & +|g^*_{ASP}(-\Delta x,-\Delta y,-\Delta z)|\cos(\phi_{ASP}(-\Delta x,-\Delta y,-\Delta z)).\end{aligned} \quad (8)$$

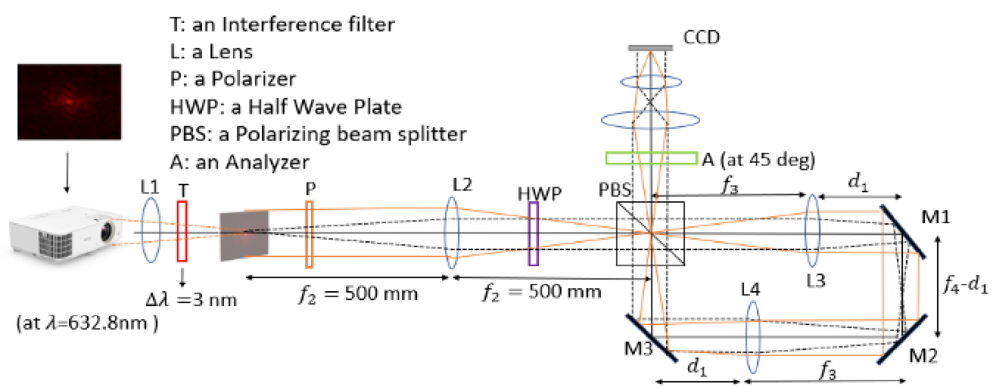

Figure 2. Experimental setup for recording the interferogram used for reconstruction [14]. The CGH is numerically generated. An interference filter T with a bandwidth of $\Delta\lambda = 3nm$ at $\lambda = 632.8$ nm is used to mitigate chromatic aberrations on the recorded interferogram due to the optical elements. The values of the focal lengths f_3, f_4 dictates α and the lateral and axial magnifications for the reconstructed image are (m_x, m_y) and m_z, respectively.

From the properties of the Sagnac interferometer which is described in detail in [14], $\Delta x = -(\alpha - \alpha^{-1})\tilde{x}$, $\Delta y = -(\alpha - \alpha^{-1})\tilde{y}$, and $\Delta z = (\alpha^2 - \alpha^{-2})\tilde{z}$, where $\tilde{x}$, $\tilde{y}$, $\tilde{z}$ are the coordinates of the output plane of the interferometer, and $\alpha = f_3/f_4$. This means that the reconstructed image is magnified in lateral and axial directions by factors $-(\alpha - \alpha^{-1})^{-1}$ and $(\alpha^2 - \alpha^{-2})^{-1}$, respectively. We should note that the lateral magnification must be chosen such that the reconstructed image size fits the CCD aperture.