Fourier Fringe Analysis

Outline

1. Gabor Hologram (on-axis holography)
2. Off-axis Holography
3. Image hologram
4. MATLAB Simulation
5. Difficulties
6. Application to white light reconstruction
7. Conclusion
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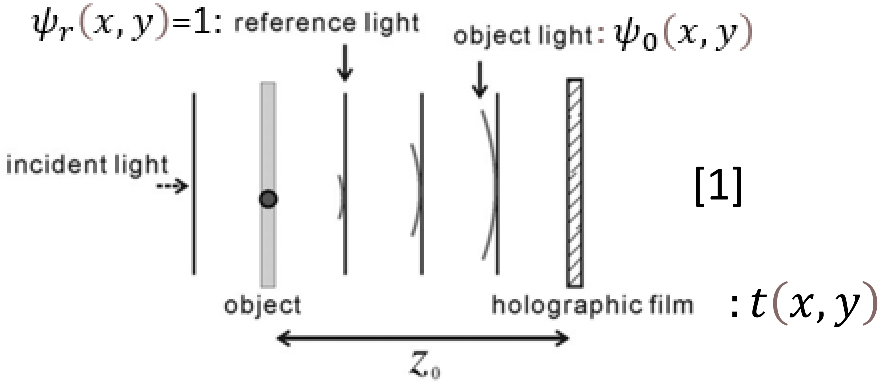
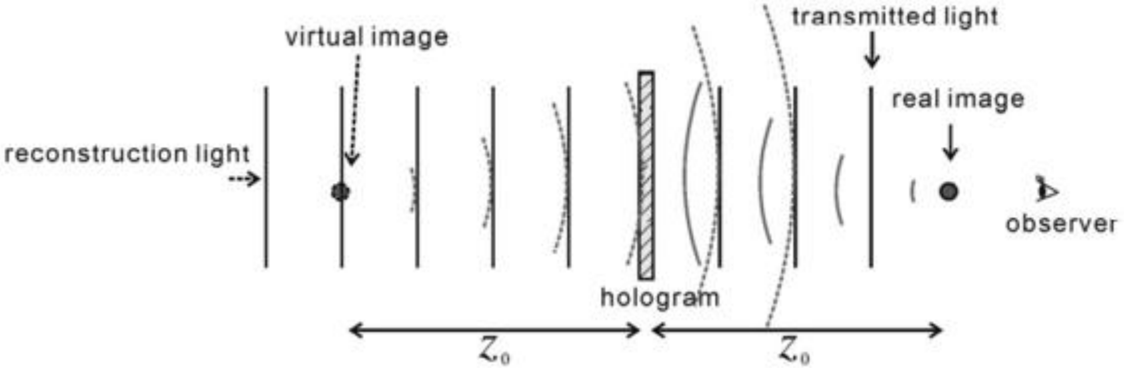
Gabor Hologram (on-axis holography)



[1]

Amplitude transmittance of the hologram: $t(x,y) \propto |\psi_0(x,y)+\psi_r(x,y)|^2$

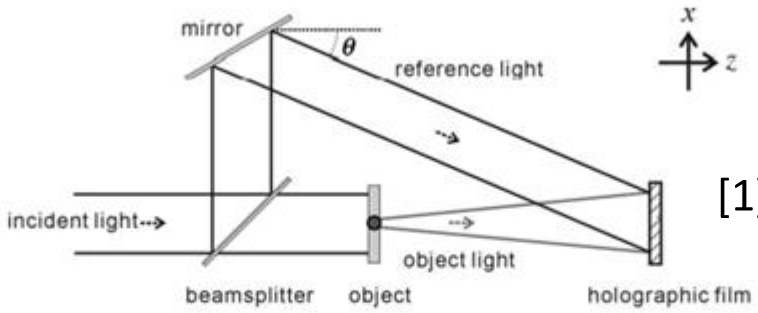
$$\propto |\psi_0(x,y)|^2 + |\psi_r(x,y)|^2 + \psi_0^*(x,y)\,\psi_r(x,y) + \psi_0(x,y)\,\psi_r^*(x,y)$$



[1]

Reconstructed field: $\psi_r(x,y) \times t(x,y) \propto \underbrace{(|\psi_0(x,y)|^2 + |\psi_r(x,y)|^2)\psi_r(x,y)}_{\text{Zeroth-order beam (transmitted light)}} + \underbrace{\psi_0^*(x,y)\,\psi_r^2(x,y)}_{\text{Real image}} + \underbrace{\psi_0(x,y)|\psi_r(x,y)|^2}_{\text{Virtual image}}$

Off-axis Holography-Recording process



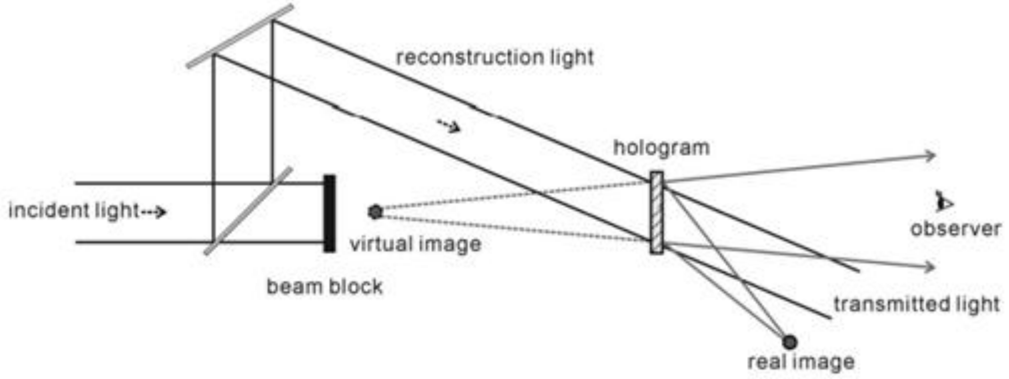
[1]

Amplitude transmittance of the hologram:

$$t(x,y) \propto \left|\, \psi_0(x,y) + \psi_r(x,y)e^{jk_0 sin\theta x} \right|^2$$
$$\propto |\,\psi_0(x,y)|^2 + |\,\psi_r(x,y)|^2 + \psi_0^*(x,y)\,\psi_r(x,y)e^{jk_0 sin\theta x} + \psi_0(x,y)\,\psi_r^*(x,y)e^{-jk_0 sin\theta x}$$
$$\propto |\,\psi_0(x,y)|^2 + |\,\psi_r(x,y)|^2 + 2|\,\psi_0(x,y)||\,\psi_r(x,y)|\cos(2\pi f_x + \phi(x,y))$$

$f_x = \frac{sin\theta}{\lambda}$ Carrier frequency → carrier-frequency Holography.

Off-axis Holography-Reconstructing process



[1]

$$Reconstructed\ field:\ t(x,y)\psi_r(x,y)e^{jk_0 sin\theta x}$$

$$\propto \underbrace{|\psi_0(x,y)|^2\psi_r e^{jk_0 sin\theta x} + |\psi_r(x,y)|^2\psi_r e^{jk_0 sin\theta x}} + \underbrace{\psi_0^*(x,y)\,\psi_r^2 e^{j2k_0 sin\theta x}} + \underbrace{\psi_0(x,y)|\,\psi_r(x,y)|^2}$$

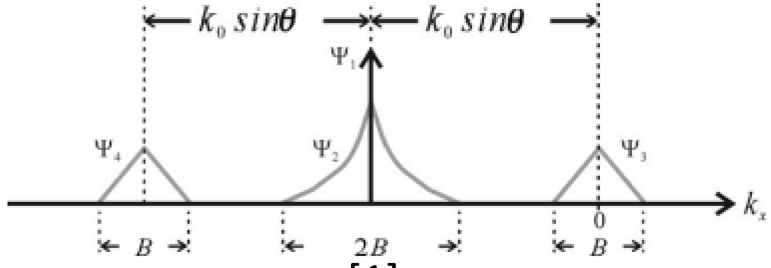
Zeroth-order beam (transmitted light)
Propagating at an angle θ respect to the optical axis

Real image is deflected at an angle $\sin^{-1}(2 * sin\theta)$ respect to the optical axis

Virtual image propagates on the optical axis

Off-axis Holography-Reconstructing process

FFT of the reconstructed field



[1]

B: is the bandwidth of the object spectrum

$$\Psi_1(k_x.k_y) = F\{|\psi_r(x,y)|^2\psi_r e^{jk_0 sin\theta x}\}$$

$$\Psi_2(k_x.k_y) = F\{|\psi_0(x,y)|^2\psi_r e^{jk_0 sin\theta x}\}$$

Zeroth-order beam's spectrum

proportional to the auto-correlation of F$\{\psi_0(x,y)\}$→ its bandwidth is 2B

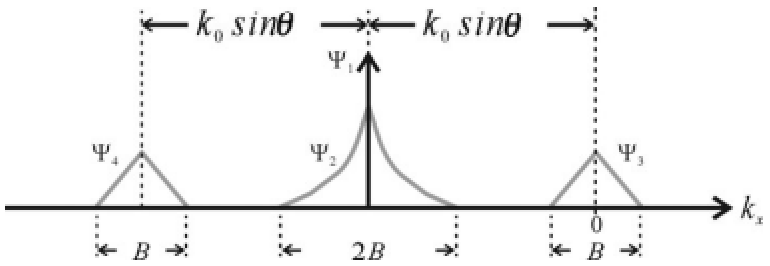
$$\Psi_3(k_x.k_y) = F\{\psi_0(x,y)|\psi_r(x,y)|^2\}$$

Virtual image 's spectrum

$$\Psi_4(k_x.k_y) = F\{\psi_0^*(x,y)\,\psi_r^2 e^{j2k_0 sin\theta x}\}$$

Real image 's spectrum

Off-axis Holography- Minimum offset angle



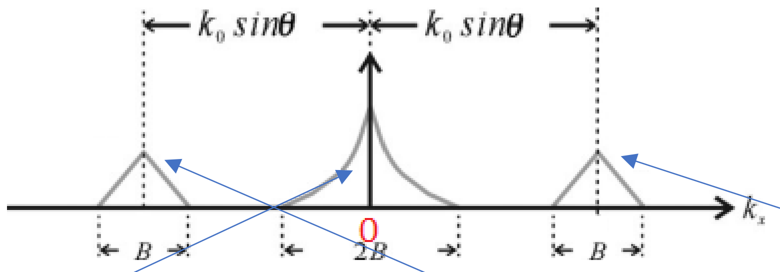
If there is no overlap between the spectrums of the real image and the virtual image and the zeroth-order light [2]
→ They are physically separated from each other due to the different propagation angles or spatial filtering operation can be used to reconstruct/separate the real/virtual image

$$k_0 sin\,\theta \geq \frac{3}{2}B$$

$$\longrightarrow \quad \theta_{min} = \sin^{-1}\left(\frac{3B}{2k_0}\right)$$

Off-axis Holography- recording medium

FFT of the off-axis hologram

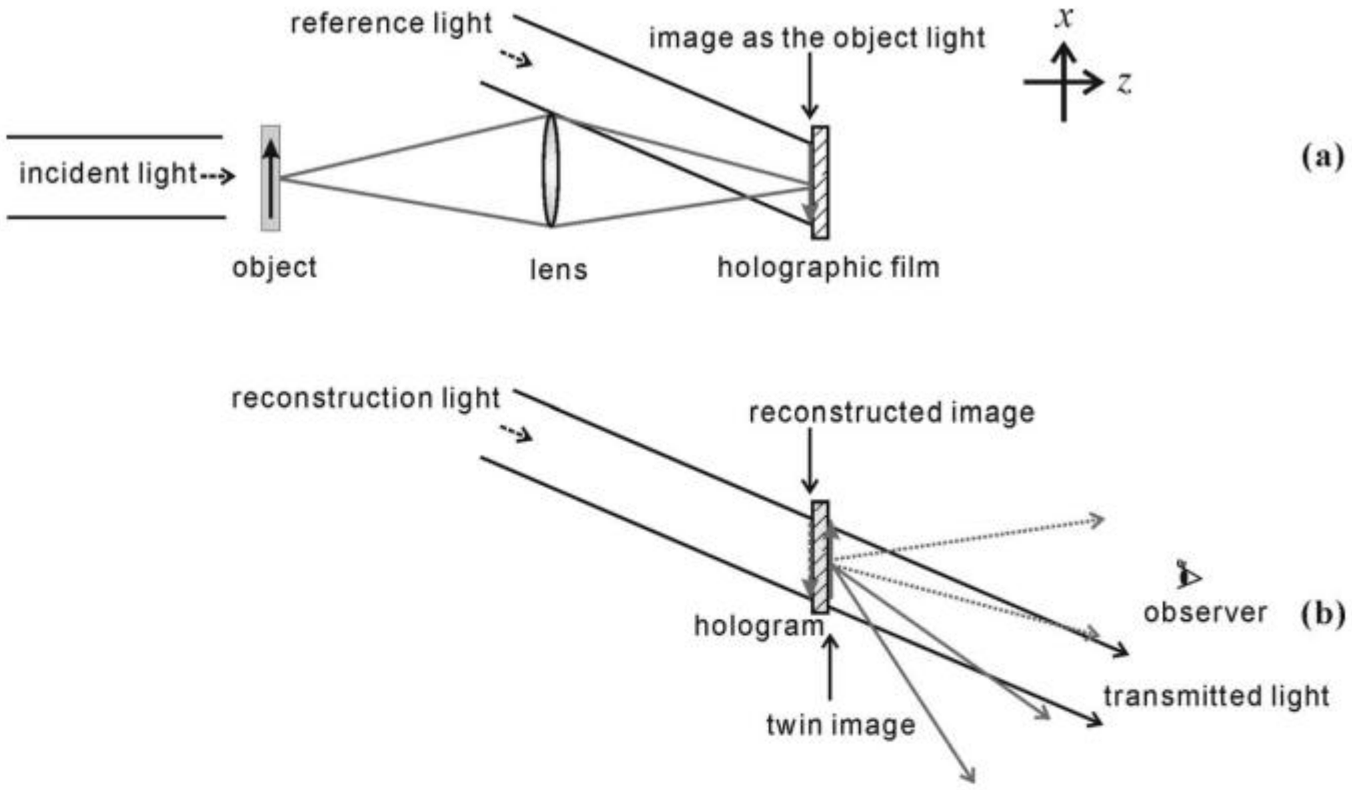


$$t(x,y) \propto |\psi_0(x,y)|^2 + |\psi_r(x,y)|^2 + \psi_0^*(x,y)\,\psi_r(x,y)e^{jk_0 sin\theta x} + \psi_0(x,y)\,\psi_r^*(x,y)e^{-jk_0 sin\theta x}$$

→The recording medium or the CCD must be able to resolve the spatial carrier frequency plus half the bandwidth of the reconstructed image in order to successfully record the image hologram

$$\frac{1}{resolution} = f_{resolvable} = \frac{sin\theta_{min}}{\lambda} + \frac{B}{2}$$

Image hologram



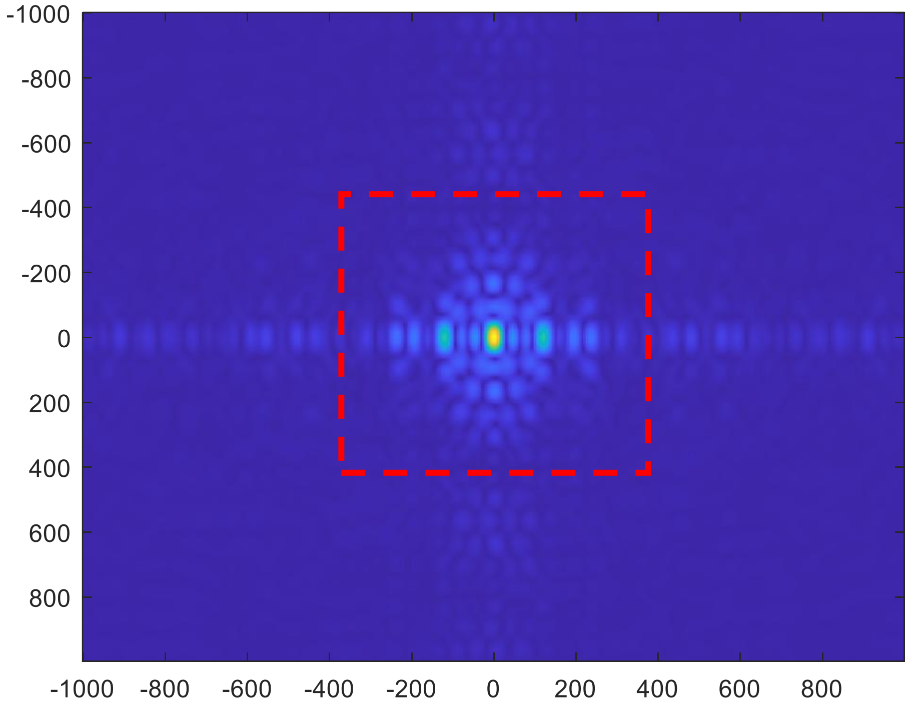
[1]

MATLAB Simulation

Using red laser: $\lambda = 6.5 \times 10^{-5}\ cm$

2D Object pattern without phase

Pixel pitch: 5 µm→ resolution: 10 µm



Object's spectrum . Its bandwidth is 820 lines/cm

→ $\theta_{min} \approx 2.14° \rightarrow The\ chosen\ offset\ angle: 2.15°$

→ $\frac{1}{10^{-3}\ cm} = f_{resolvable} = 1000\ \text{lines/cm} > \frac{sin2.15°}{6.5\times10^{-5}} + \frac{850}{2}$=987 lines/cm

MATLAB Simulation

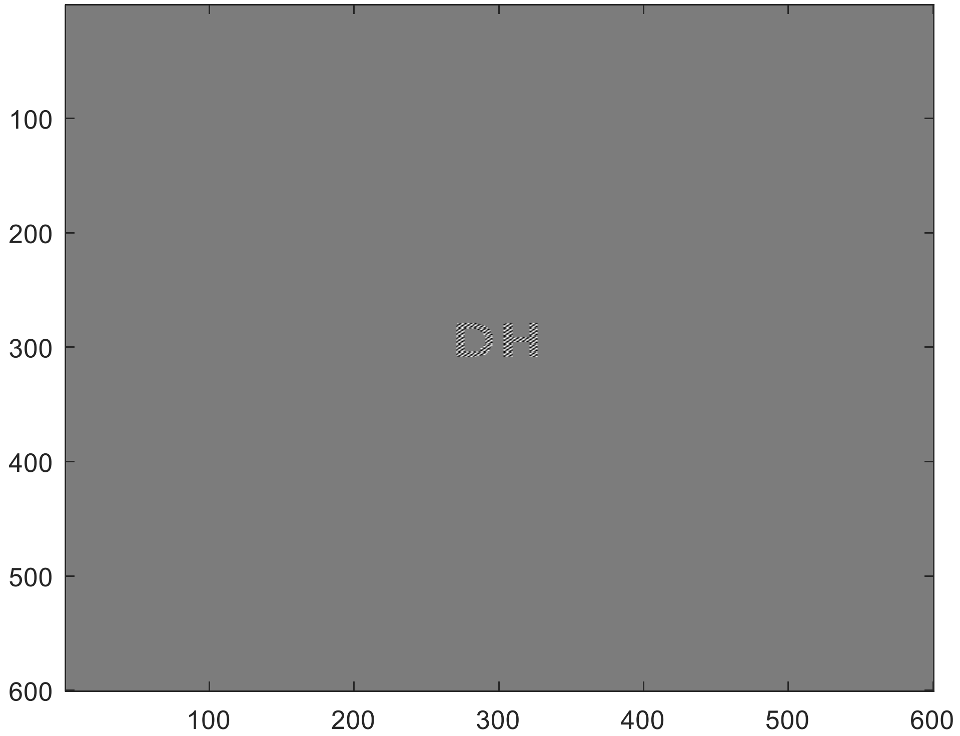


Image hologram

The fringes came from the introduction of the carrier frequency or offset angle

MATLAB Simulation

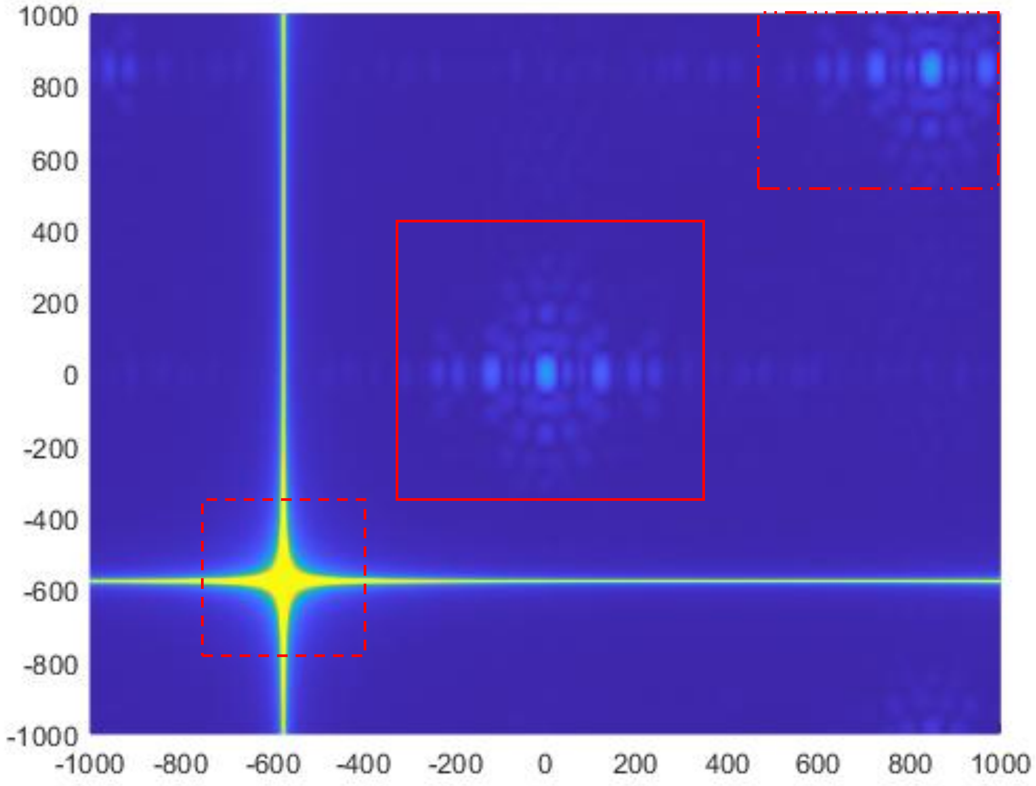
Virtual image spectrum

Real image spectrum

Zeroth-order beam spectrum

Plane wave

$\psi_r e^{jk_0 \sin}$



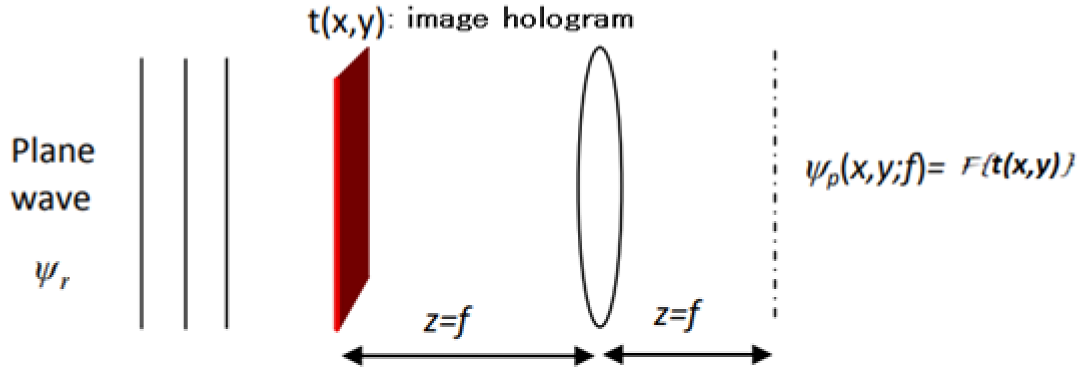
FFT transform of reconstructed field

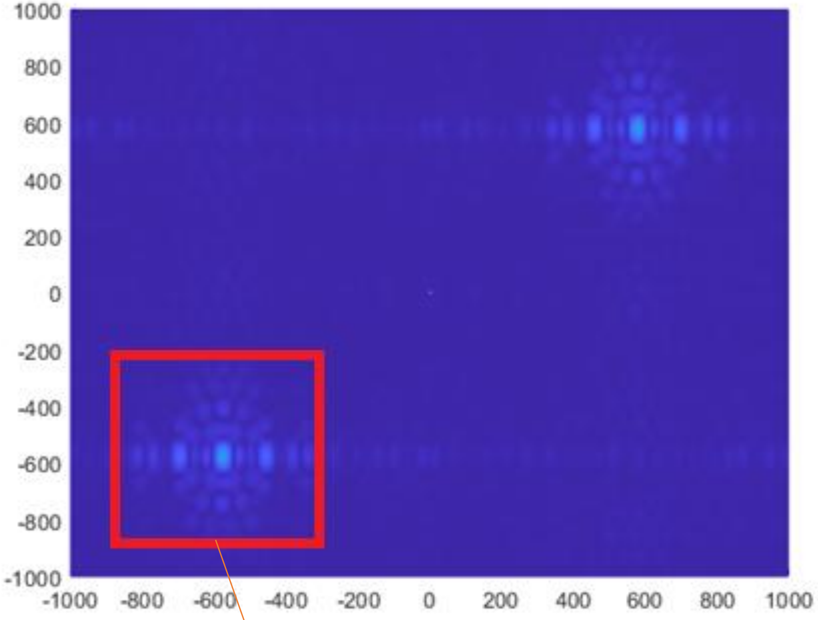
$jk_0 sin\theta x$ }

The real image spectrum should be located at $2f_x = \frac{2sin2.15°}{6.5\times10^{-5}} = 1154\ lines/cm$

→ Out of the frequency range (-1000 lines/cm to 1000 lines/cm)

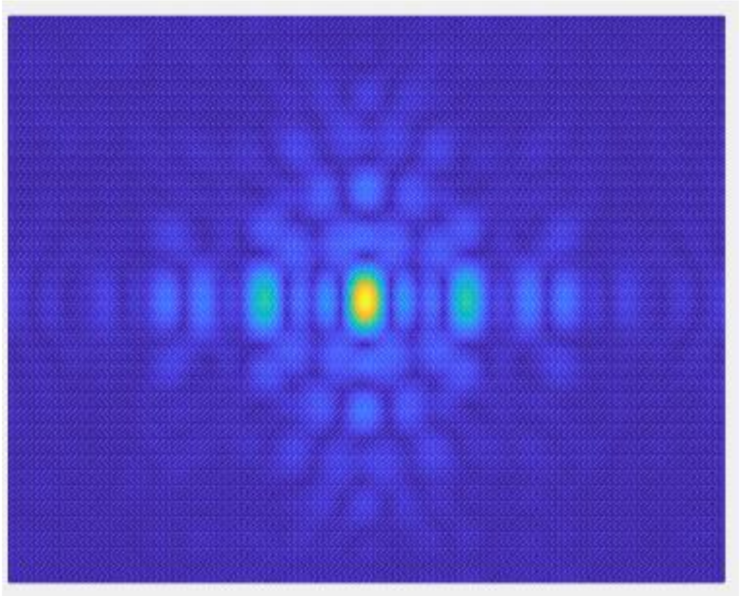
MATLAB Simulation





FFT transform of image hologram

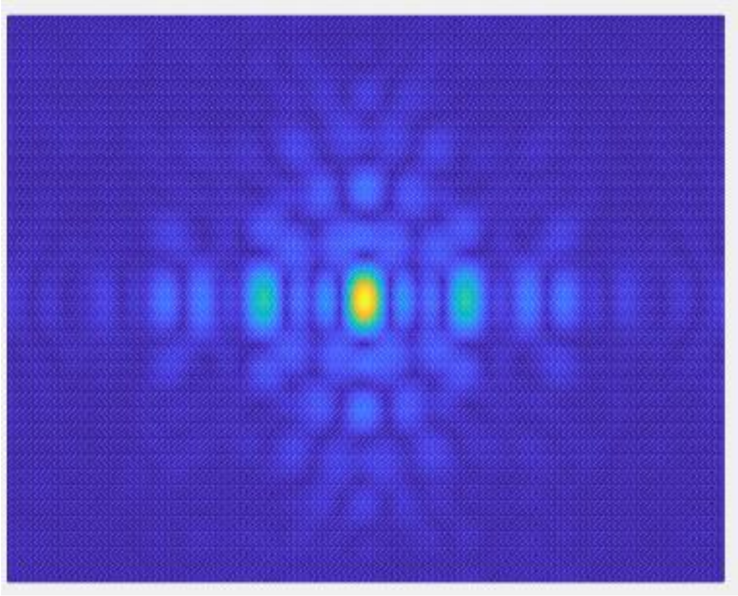
Notch filter



Filtered signal

Real image's spectrum

MATLAB Simulation

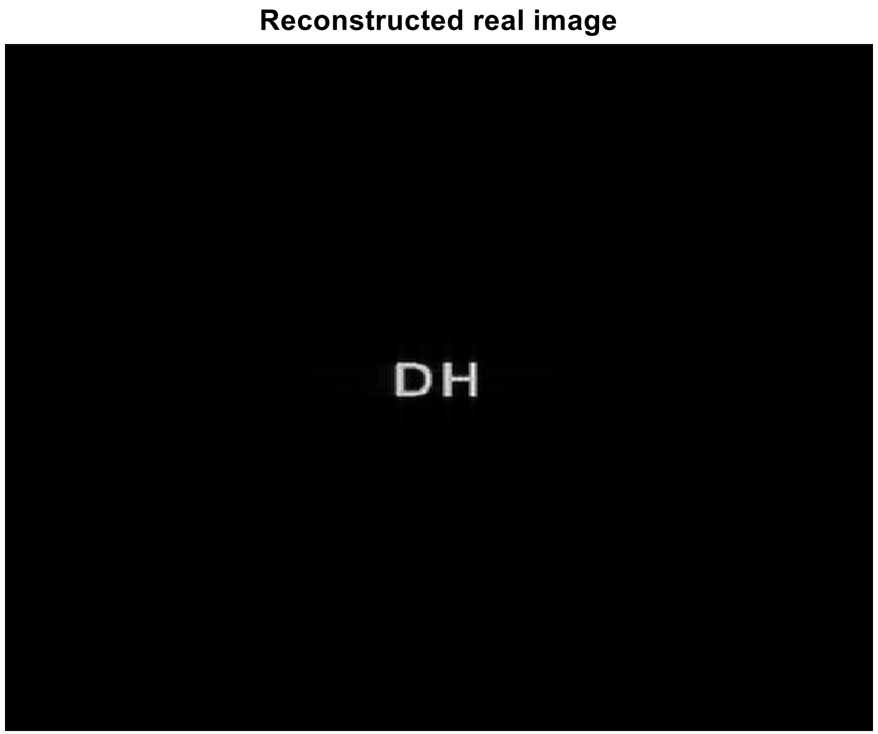


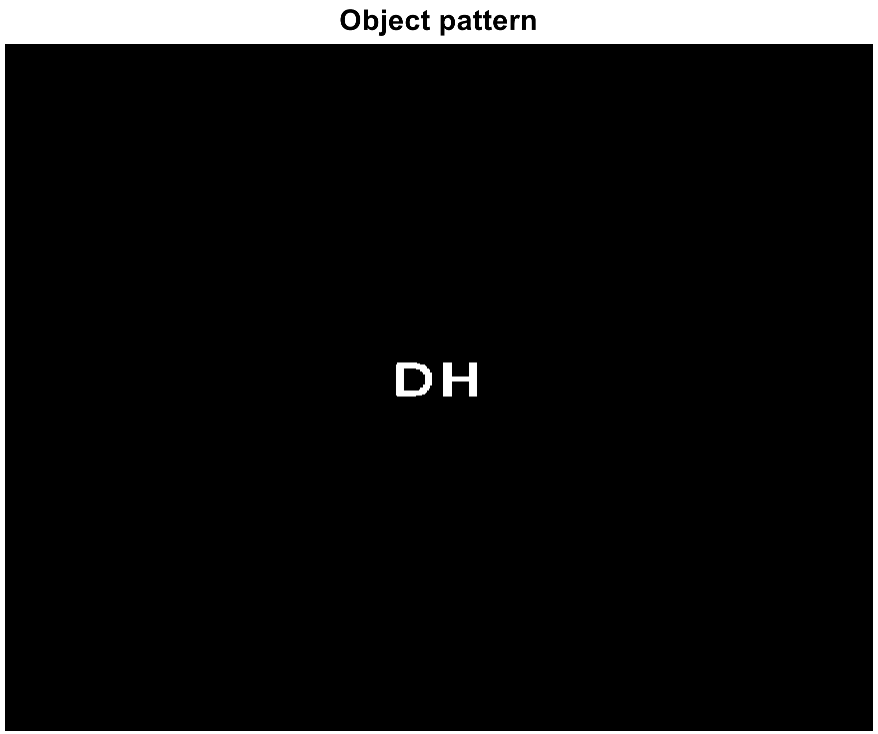
Filtered signal

Zero padding + Inverse FFT

Reconstructed real image

MATLAB Simulation





MSE=0.0054

Difficulties

- The pixel pitch in the simulation is 5 μm. If the size of the pixel of CCD camera is much larger than this value, the image hologram will not be successfully recorded.

Conclusion

- With an appropriate offset angle and recording medium, the problem of the zeroth-order light and the twin image is resolved
- spatial filtering operations using lens system can be also used to reconstruct the virtual or the real image
- Image Hologram is used in white light reconstruction since the reconstructed real and virtual images are at the hologram plane → overcome the chromatic aberration.

References

- [1] T. C. Poon and J. P. Liu, "Introduction to Modern Digital Holography with MATLAB" (2014)
- [2] Joseph W. Goodman, "Introduction to Fourier Optics", 3rd Edition (2005)