WQU

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Table 1: Group members

630GWP3

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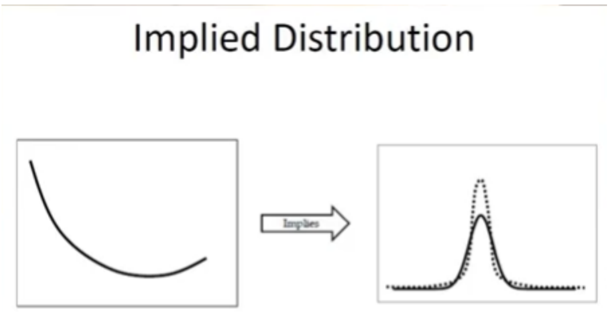
1 Introduction

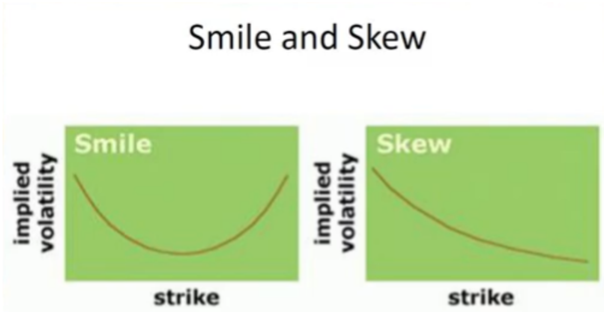
This paper compares similarities and contrasts the differences among five major topics: 1) Vanilla Options versus Barrier Options focusing on payoffs, 2) Out-of-the-Money Call Options versus At-the-Money Call Options focusing on the volatility smile, 3) Default-Free Option Pricing versus Credit Valuation Adjustment Option Pricing, 4) Analytical Pricing Methods versus Monte Carlo Simulation Methods, and 5) Black-Scholes versus Heston versus Local volatility methods.

1.1 Options and Payoffs: Vanilla vs. Barrier

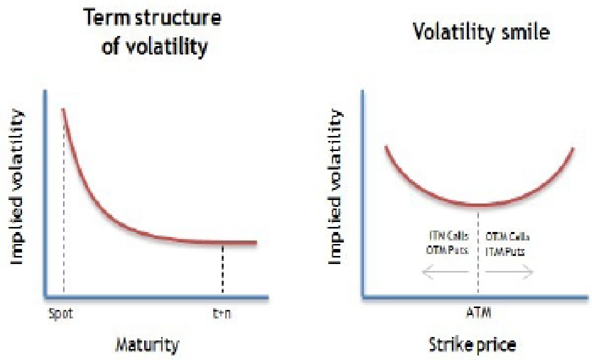
1.2 Call Option Pricing: Out-of-the-Money vs. At-the-Money

A Volatility Smile appeared after the 1987 crash and is a graph of plots of various strike prices against each their implied volatility for a set of options all under the same underlying and with the same expiration date. It is used to imply a distribution that is different to the standard normal distribution found in the Black-Scholes model. The graph looks either mostly flat similar to a human 'smirk' or skew from strikes with higher mean reversions descriptive of the phenomenon of at-the-money options (strike price near the market price); or very curved upward similar to a human 'smile' from strikes with lower volatility clustering and lower mean reversion descriptive of the phenomenon of out-of-the-money options (strike price farther-from the market price). [5]



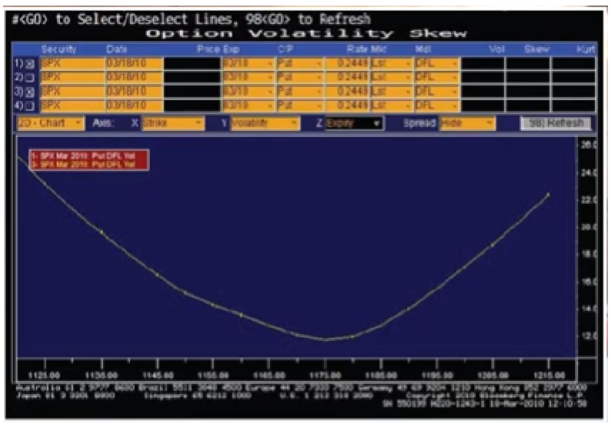


When selecting a strike price careful consideration should be practiced to avoid too large or too small values; e.g. If the strike is too high an option would be too deep in-the-money and require an early exercise, however if the strike is too low an option would be deep out-of-the-money having nearly zero value. [2]



Observing the change in the smile or skew over time can also be used as a indicator or measure of market fear since in a panicked market option writers charge more for their options because of the fat-tailed nature of the distribution and the expectation of it having higher standard deviations.

Real Smile/Skew Example



1.3 Option Pricing: Default-Free vs. CVA

Credit Valuation Adjustment (CVA) is the market value of the counter-party credit risk, the value difference between two differently valued portfolios: a portfolio not accounting for this type of default risk (modeled using Black-Scholes) and a portfolio that does account for default risk (modeled using Black-Scholes-Merton treating firm value as an asset). The CVA calculation is dependent on three types of exposure: current, expected, and potential, each at a specified current, future, or future time, respectively, and quantified with two parameters: a recovery rate and a stopping time. The counter-party can either never default (CVA = zero) or default before the position is closed (CVA below).

If the counter-party never defaults $\tau = \infty$.

$$CVA = E^Q[e^{-rT}X_0] \tag{1}$$

Using the risk-free Black-Scholes modeling, if counter-party does default,

$$CVA = E^Q[e^{-rT}(1-\delta)V(\tau)I_{\{\tau<=T\}}] \tag{2}$$

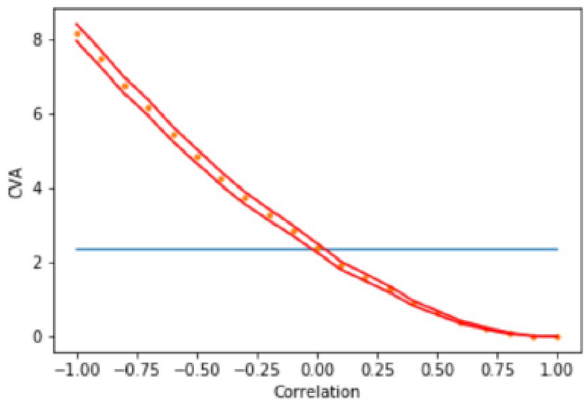
where T is the time at close position, r is risk-free rate, τ is stopping time, δ is recovery rate, $V(\tau)$ is total portfolio value at time of default, $1-\delta$ is the fraction of the portfolio lost, e^{-rt} is the current discounted portfolio value, E^Q is the risk-neutral calculation, and $I_{\{\tau<=T\}}$

Using the risk-free Black-Scholes-Merton modeling (and firm value) if counter-party does default,

$$CVA = E^Q[e^{-rT}(1-\delta)X_T I_{\{\tau=T\}}] \tag{3}$$

where T is the time at close position, r is risk-free rate, τ is stopping time, δ is recovery rate, X_T is portfolio value at time of default, $1-\delta$ is the fraction of the portfolio lost, e^{-rt} is the current discounted portfolio value, E^Q is the risk-neutral calculation, and $I_{\{\tau<=T\}}$

A Monte Carlo CVA graph can be used to visualize the various levels of portfolio values and firm values; the correlation between the stock on which an option is written and the value of the counter-party. Note the three standard deviation error bounds and the horizontal flat line for the zero correlation CVA. This reveals the concepts of wrong and right way risk observed with decreasing or increasing, respectively, portfolio value to risk of counter-party default.



1.4 Simulation Methods: Analytical-Pricing vs. Monte Carlo

Black Scholes is an analytic pricing market model that uses a filtered probability space and the four assumptions of 1) no default risk (credit risk), 2) constant risk-free rate, 3) unlimited short-selling freedom, and 4) no arbitrage opportunities, i.e., parameters of a standard Brownian motion, stock price at a specified time, a constant mean return, and a constant volatility to price a financial derivative, i.e., vanilla European option.

Monte Carlo Simulation (non-analytic) is a technique that uses random sampling, the Law of Large Numbers, and the Central Limit Theorem to numerically evaluate (estimate) a complex integral with no known solution by re-expressing this integral as an expectation of a function of a random variable over a given set to price a financial derivative i.e., look-back and asian option.

1.5 Volatility: Black-Scholes vs. Heston vs. Local

The Black-Scholes model assumes constant volatility at all strikes and/or maturities for an option and by extension, constant volatility for its underlying, but in the real-world constant volatility does not exist. The discovery of an implied volatility surface in 1978 by Breeden and Litzenberger solved this problem by considering affects of varying strike values and/or maturity times on an option's volatility using non-constant volatility models to more closer model the real world markets.

The Heston model, published by Steven L. Heston in 1993, can be used to analytically price both vanilla call and put options using stochastic volatility due to a discovery of a closed-form pricing solution using characteristic functions and Fourier techniques.

[4] [3]

2 Conclusion

This paper illustrated and explained the differences among 1) vanilla options and barrier options, 2) OTM and ATM call options, 3) default-free and CVA option pricing, 4) analtyical-pricing and Monte Carlo simulation methods, 5) Black-Scholes, Heston, and Local volatility methods.

References

[1] MScFE 630 Computational Finance - Notes

[2] Ye, Ziqun, The Black-Scholes and Heston Models for Option Pricing, University of Waterloo, 2013

[3] https://en.wikipedia.org/wiki/Heston$_{m}odel$

[4] Homescu, Cristian, July 9, 2011, Implied Volatility Surface: Construction Methodologies and Characteristics

[5] Patrick Boyle, The Volatility Smile - Options Trading Lessons