Local statistics: local indicators of spatial association

GIS 5923 Spatial Statistics

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Local Statistics

- A local statistic is any descriptive statistic associated with a spatial data set whose value varies from place to place.
- In the broadest sense, any spatial data set is a collection of local statistics, since the recorded attribute values are different at each location.
- But, a "local statistic" usually is one that is derived by considering a subset of the data local to or nearby the spatial location where it is being calculated. Example: the localized mean

Two topics in Local Statistics

- We will cover two general topics:

- In the last set of slides, we focused on geographically-weighted regression: regression models in which we allow the coefficients to vary spatially

- In this set of slides, we will cover Local Indicators of Spatial Association (LISAs): used to identify hotspots and outliers

Local statistics

- In principle, almost any spatial statistics can be turned into a "local statistic": instead of summarizing over a whole data set, we summarize over only the data in the locality of each point

- In practice, the term "local statistic" refers most commonly to three statistics: The Getis-Ord G_i and G_i^* statistics, and the local Moran's I statistic

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The weights matrix revisited

$$W = \begin{bmatrix} w_{11} & \cdots & w_{1n} \\ \vdots & \ddots & \vdots \\ w_{n1} & \cdots & w_{nn} \end{bmatrix}$$

- Typical interpretation: the element w_{ij} represents the hypothesized spatial influence of location *j* on location *i*
- Therefore, we can say that a row matrix $W_i = [w_{i1} \quad \cdots \quad w_{in}]$ describes the local neighborhood of each location *i*
- We've seen that there are many different ways to construct the weights matrix: adjacency, distance, k-nearest neighbors, etc.

The Getis-Ord G_i statistic

The Getis-Ord G_i statistic is used to detect local concentrations of high or low values in a variable x. For a location i it is calculated as:

$$G_i = \frac{\sum_j w_{ij} x_j}{\sum_{j=1}^{n} x_j} for\ all\ i \neq j$$

If the weights matrix is based on adjacency, then G_i can be interpreted as the proportion of the sum of all x values in the study area accounted for just by the neighbors of i

The Getis-Ord G_i^* Statistic

- The Getis-Ord G_i^* statistic is calculated by including location *i* itself in the numerator and denominator

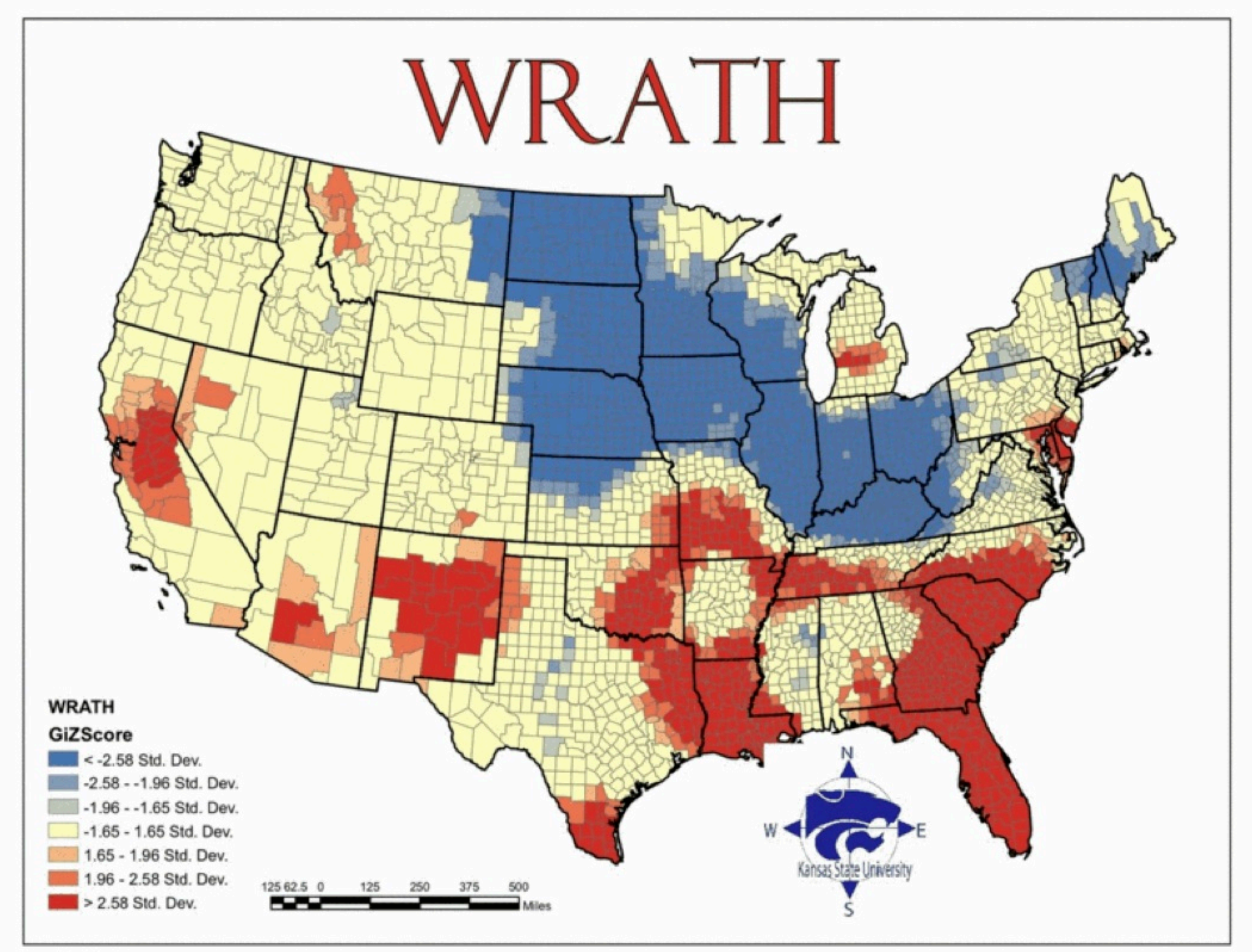
- Important: the attribute under consideration must be a ratio-scale variable with a natural origin
  - For example, you can see that the value of G_i will be different if you add a constant value to every location or if you transform the variable

Statistical significance of G and G^*

- Getis and Ord (1992) give the expected value and variance of G_i and G_i^* under a null hypothesis of CSR.

- As a result, we can assign a z-score to each location's G or G^* value

Wrath: Total number of violent crimes (murder, assault and rape) per capita



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Interpreting z-scores

- The typical approach would be to consider z scores outside the range -1.96 to +1.96 to be significant
- More care is required in making inferences from local statistics.
- Potential pitfalls:
  - 1) When events are rare, the statistic is being calculated on a small number of cases
  - 2) Edge effects
  - 3) Multiple testing problem: if we have 100 areal units in the study area, then on average 5 will be significant just by chance

Multiple test corrections

- The simplest approach to correct for multiple comparisons is to use a Bonferroni correction, and set a' = a / n.
  - Some authors find this too conservative. What is a' if a = 0.05 but n = 100?

- Alternatively, a Monte Carlo approach could be taken by holding the value at location *i* constant and shuffling the other attribute values among locations in the data set

Global Moran's I: an index of autocorrelation

Before discussing local Moran's I, let's review the formula for global Moran's I

Divide by the overall data set variance

Covariance term; subscripts i and j refer to different areal units

$$I = \left[\boxed{\frac{n}{\sum_{i=1}^{n} (y_i - \bar{y})^2}} \right] \times \left[\frac{\boxed{\sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} (y_i - \bar{y})(y_j - \bar{y})}}{\boxed{\sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij}}} \right]$$

Normalize by the total spatial weights in the map

Local Moran's I

- Local Moran's I_i values are onents that are summed to calculate the Global Moran's I

This summation is omitted for local I

Global I

$$I = \left[\frac{n}{\sum_{i=1}^{n} (y_i - \bar{y})^2}\right] \times \left[\frac{\boxed{\sum_{i=1}^{n}}\sum_{j=1}^{n} w_{ij}(y_i - \bar{y})(y_j - \bar{y})}{\boxed{\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}}}\right]$$

Local I is not normalized by the sum of the spatial weights

Local I

$$I_i = \left[\frac{n}{\sum_{i=1}^{n} (y_i - \bar{y})^2}\right] \times \left[\sum_{j=1}^{n} w_{ij}(y_i - \bar{y})(y_j - \bar{y})\right]$$

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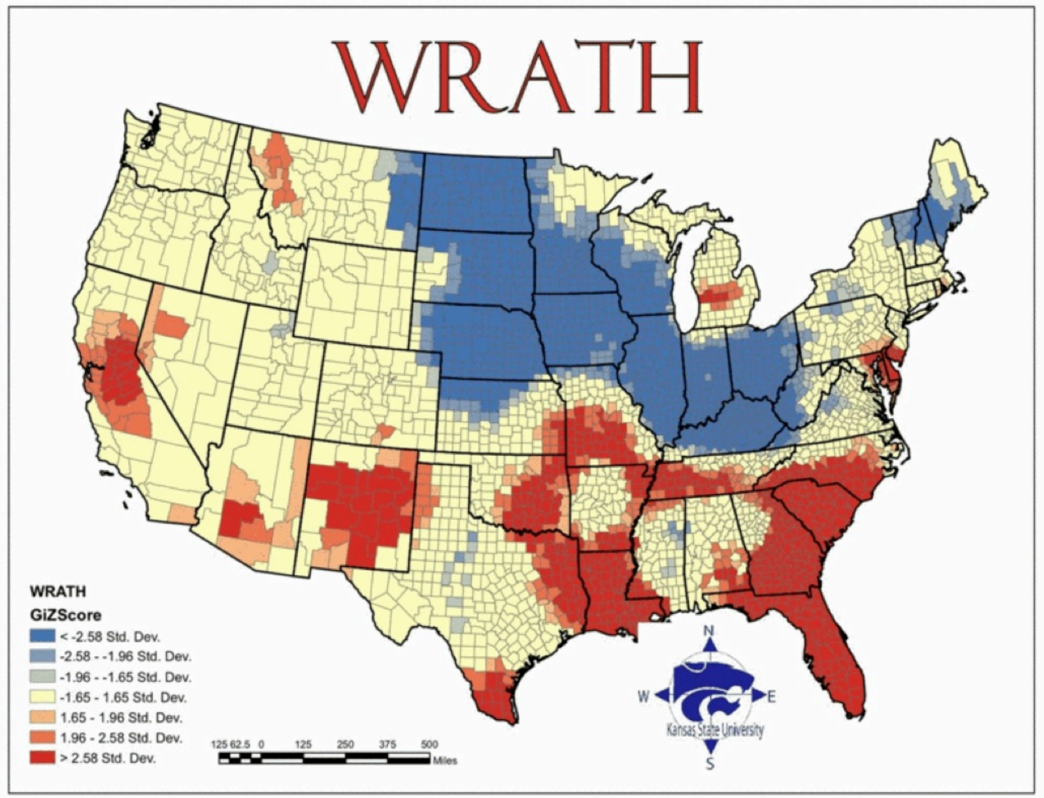
Which statistic to use?

- When exploring spatial data it is suggested that users apply both the G_i and the local Moran's I statistics.
- These statistics explore different but complementary processes underlying the observed spatial distribution of attribute values.



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R

- Let’s look at some examples in R…