Measurement Invariance

Adam Garber

Norwegian University of Science and Technology - A Course in `MplusAutomation`

June 01, 2021

DATA SOURCE: This lab exercise utilizes the NCES public-use dataset: Education Longitudinal Study of 2002 (Lauff & Ingels, 2014) See website: nces.ed.gov

```
# load packages
library(MplusAutomation)
library(tidyverse)
library(here)
library(corrplot)
library(gt)
library(reshape2)
library(semPlot)
```

Lab 9 - Begin

Read in data

```
lab_data <- read_csv("https://garberadamc.github.io/project-site/data/els_sub5_data.csv")
```

Preparations: subset, reorder, rename, and recode data

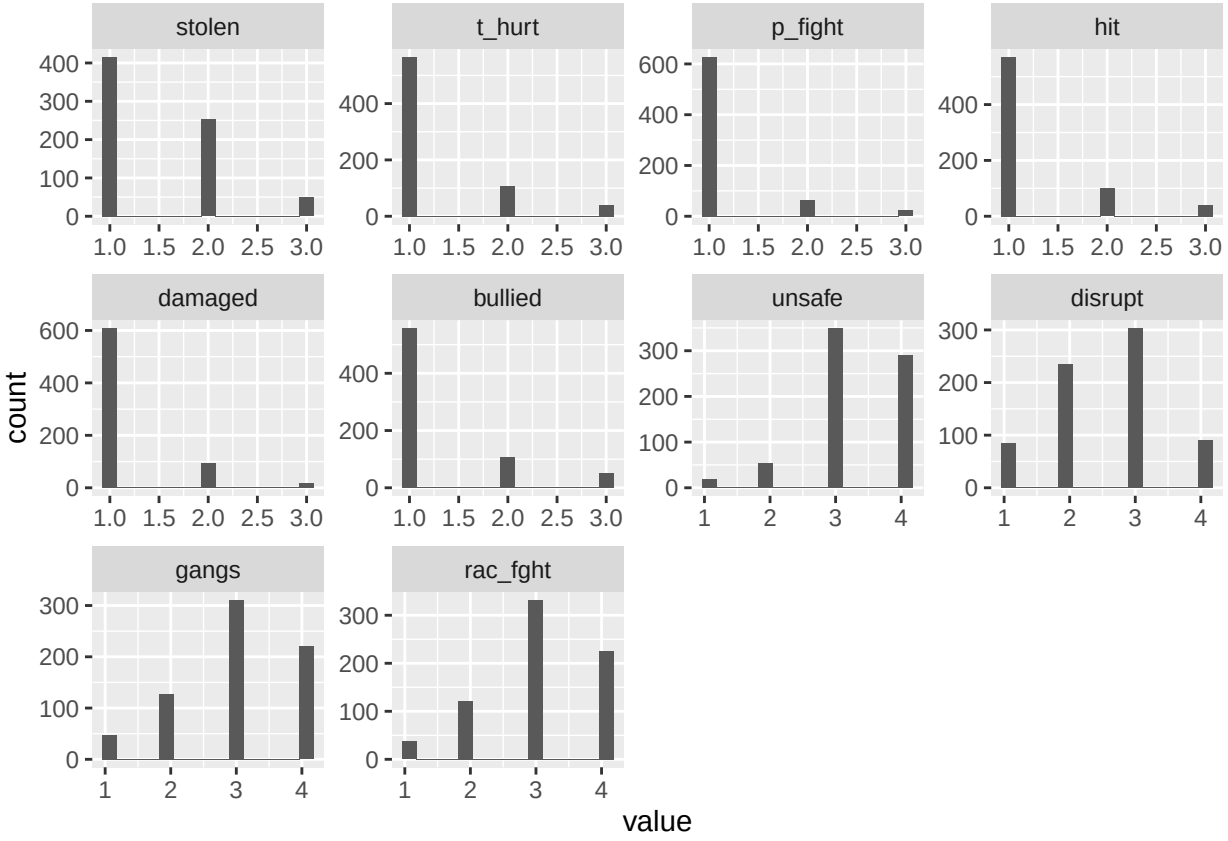
```
invar_data <-  lab_data %>%
  select(bystlang, freelnch, byincome,                    # covariates
         stolen, t_hurt, p_fight, hit, damaged, bullied,  # factor 1 (indicators)
         safe, disrupt, gangs, rac_fght,                  # factor 2 (indicators)
         late, skipped, mth_read, mth_test, rd_test) %>%
  rename("unsafe" = "safe") %>%
  mutate(freelnch = case_when( # Grade 10, percent free lunch - transform to binary
        freelnch <  3 ~ 0,     # school has less than 11%
        freelnch >= 3 ~ 1))    # school has greater than or equal to 11%

table(invar_data$freelnch) # reasonably balanced groups
```

```
##
##   0   1
## 338 388
```

Take a quick look at variable distributions

```
melt(invar_data[,4:13]) %>%
  ggplot(., aes(x=value, label=variable)) +
  geom_histogram(bins = 15) +
  facet_wrap(~variable, scales = "free")
```



Reverse code factor for ease of interpretation

```
cols = c("unsafe", "disrupt", "gangs", "rac_fght")

invar_data[ ,cols] <-  5 - invar_data[ ,cols]
```

Factor names and interpretation:

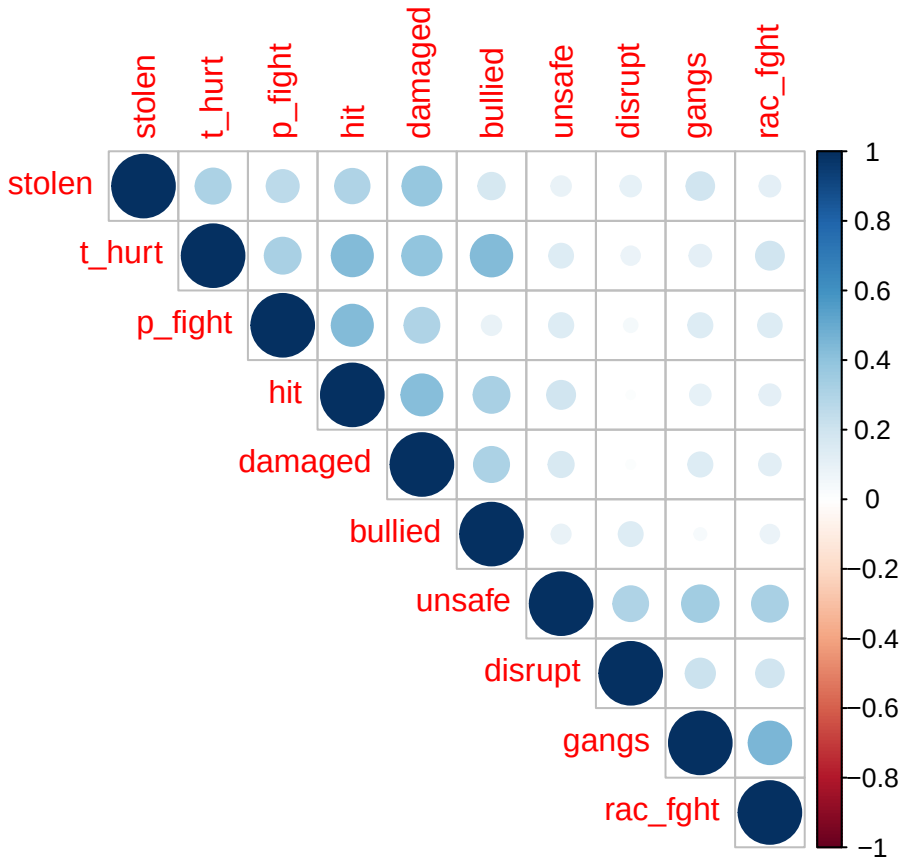
- VICTIM: student reports being a victim of injury to self or property
  - scale range: Never, Once or twice, More than twice
  - higher values indicate greater frequency of victimization events
- NEG_CLIM: Student reports on negative school climate attributes
  - scale range: Strongly Disagree - Strongly Agree

– higher values indicate a more negative climate

Check correct coding, explore correlations

```
cor_matrix <- cor(invar_data[4:13], use = "pairwise.complete.obs")

corrplot(cor_matrix,
         method = "circle",
         type = "upper")
```



Estimate the Unconditional Confirmatory Factor Analysis (CFA) model

---

Number of parameters = 31

- 10 item loadings
- 10 intercepts
- 10 residual variances
- 01 factor co-variances

```
cfa_m0  <- mplusObject(
  TITLE = "model0 - unconditional CFA model",
  VARIABLE =
    "usevar = stolen-rac_fght;",

  ANALYSIS =
    "estimator = mlr;",

  MODEL =
    "VICTIM by stolen* t_hurt p_fight hit damaged bullied;
     VICTIM@1; ! UVI identification

     NEG_CLIM by unsafe* disrupt gangs rac_fght;
     NEG_CLIM@1; ",

  PLOT = "type = plot3;",
  OUTPUT = "sampstat standardized residual modindices (3.84);",

  usevariables = colnames(invar_data),
  rdata = invar_data)

cfa_m0_fit <- mplusModeler(cfa_m0,
              dataout=here("09-invariance", "invar_mplus", "invar_data.dat"),
              modelout=here("09-invariance", "invar_mplus", "m0_cfa_fullsample.inp"),
              check=TRUE, run = TRUE, hashfilename = FALSE)
```

Run separate CFA models for each sub-sample

Group `freelnch = 0` (low) CFA

```
cfa_m1  <- mplusObject(
  TITLE = "CFA model1 - group is 0 for freelnch",
  VARIABLE =
    "usevar = stolen-rac_fght;

     !freelnch (0 = school proportion is less than 11 percent)
     USEOBS = freelnch == 0; ",

  ANALYSIS =
    "estimator = mlr;",

  MODEL =
    "VICTIM by stolen* t_hurt p_fight hit damaged bullied;
     VICTIM@1; ! UVI identification

     NEG_CLIM by unsafe* disrupt gangs rac_fght;
```

```
    NEG_CLIM@1; ",

  PLOT = "type = plot3;",
  OUTPUT = "sampstat standardized residual modindices (3.84);",

  usevariables = colnames(invar_data),
  rdata = invar_data)

cfa_m1_fit <- mplusModeler(cfa_m1,
              dataout=here("09-invariance", "invar_mplus", "invar_data.dat"),
              modelout=here("09-invariance", "invar_mplus", "m1_cfa_freelnch_0.inp"),
              check=TRUE, run = TRUE, hashfilename = FALSE)
```

Group `freelnch = 1` (moderate to high) CFA

```
cfa_m2  <- mplusObject(

  TITLE = "CFA model2 - group is 1 for freelnch",

  VARIABLE =
    "usevar = stolen-rac_fght;

     !freelnch (1 = school proportion is greater than or equal to 11 percent)
     USEOBS = freelnch == 1; ",

  ANALYSIS =
    "estimator = mlr;",

  MODEL =
    "VICTIM by stolen* t_hurt p_fight hit damaged bullied;
     VICTIM@1; ! UVI identification

     NEG_CLIM by unsafe* disrupt gangs rac_fght;
     NEG_CLIM@1; ",

  PLOT = "type = plot3;",

  OUTPUT = "sampstat standardized residual modindices (3.84);",

  usevariables = colnames(invar_data),
  rdata = invar_data)

cfa_m2_fit <- mplusModeler(cfa_m2,
              dataout=here("09-invariance", "invar_mplus", "invar_data.dat"),
              modelout=here("09-invariance", "invar_mplus", "m2_cfa_freelnch_1.inp"),
              check=TRUE, run = TRUE, hashfilename = FALSE)
```

Multi-Group Invariance Models

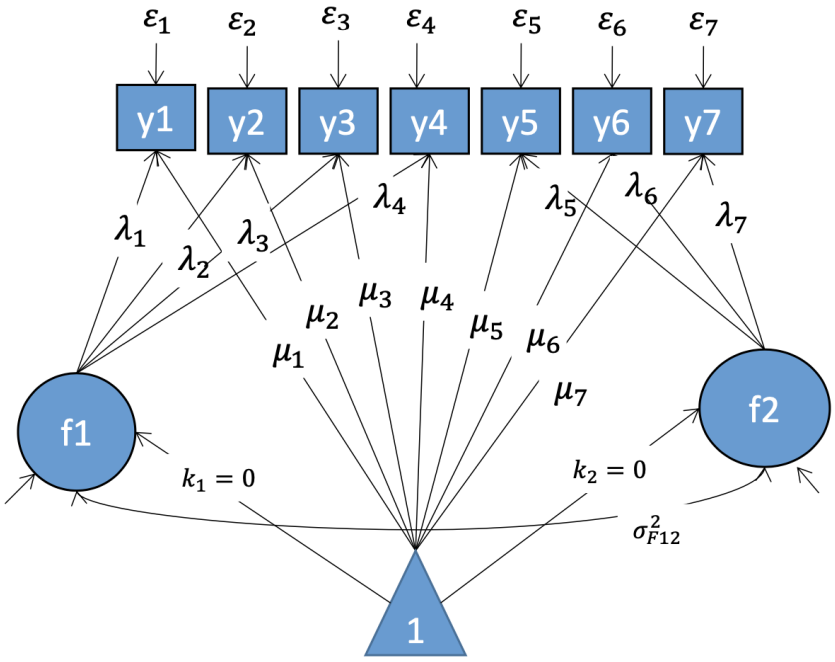


Figure: Picture depicting mean structure from slide by Dr. Karen Nylund-Gibson

---

Configural invariance

- free item loadings, intercepts, and residuals
- factor means fixed to zero
- factor variances fixed to 1

Number of parameters = 62

- 20 item loadings (10items*2groups)
- 20 intercepts
- 20 residual variances
- 02 factor co-variances (1 for each group)

```
cfa_m3  <- mplusObject(

  TITLE = "CFA model3 - configural invariance",

  VARIABLE =
```

```
    "usevar = stolen-rac_fght;

     grouping = freelnch (0=freelnch_0 1=freelnch_1); ",

  ANALYSIS =
    "estimator = mlr;",

  MODEL =
    "VICTIM by stolen* t_hurt p_fight hit damaged bullied;
     VICTIM@1; ! UVI identification

     NEG_CLIM by unsafe* disrupt gangs rac_fght;
     NEG_CLIM@1;

     [VICTIM-NEG_CLIM@0]; !factor means set to zero

     MODEL freelnch_1:

     VICTIM by stolen* t_hurt p_fight hit damaged bullied;
     VICTIM@1;

     [stolen t_hurt p_fight hit damaged bullied]; !free intercepts

     NEG_CLIM by unsafe* disrupt gangs rac_fght;
     NEG_CLIM@1;

     [unsafe disrupt gangs rac_fght]; !free intercepts

     [VICTIM-NEG_CLIM@0]; ",

  PLOT = "type = plot3;",
  OUTPUT = "sampstat standardized residual modindices (3.84);",

  usevariables = colnames(invar_data),
  rdata = invar_data)

cfa_m3_fit <- mplusModeler(cfa_m3,
              dataout=here("09-invariance", "invar_mplus", "invar_data.dat"),
              modelout=here("09-invariance", "invar_mplus", "m3_configural.inp"),
              check=TRUE, run = TRUE, hashfilename = FALSE)
```

Metric invariance

- item loadings (set to equal)
- free intercepts and residuals
- factor means fixed to zero
- free factor variances in group 2

Number of parameters = 54

- 10 item loadings (set to equal)
- 20 intercepts
- 20 residual variances
- 02 factor variances
- 02 factor co-variances

```
cfa_m4  <- mplusObject(
  TITLE = "CFA model4 - metric invariance",
  VARIABLE =
    "usevar = stolen-rac_fght;

     grouping = freelnch (0=freelnch_0 1=freelnch_1); ",

  ANALYSIS =
    "estimator = mlr;",

  MODEL =
    "VICTIM by stolen* t_hurt p_fight hit damaged bullied;
     VICTIM@1; ! UVI identification

     NEG_CLIM by unsafe* disrupt gangs rac_fght;
     NEG_CLIM@1;

     [VICTIM-NEG_CLIM@0];

     MODEL freelnch_1:

     VICTIM; ! free factor variances for group 2

     [stolen t_hurt p_fight hit damaged bullied];

     NEG_CLIM;

     [unsafe disrupt gangs rac_fght];

     [VICTIM-NEG_CLIM@0]; ",

  PLOT = "type = plot3;",
  OUTPUT = "sampstat standardized residual modindices (3.84);",

  usevariables = colnames(invar_data),
  rdata = invar_data)

cfa_m4_fit <- mplusModeler(cfa_m4,
              dataout=here("09-invariance", "invar_mplus", "invar_data.dat"),
              modelout=here("09-invariance", "invar_mplus", "m4_metric.inp"),
              check=TRUE, run = TRUE, hashfilename = FALSE)
```

Scalar invariance

- item loadings (set to equal)
- intercepts (set to equal)
- free residuals
- free factor variances and means in group 2

Number of parameters = 46

- 10 item loadings (set to equal)
- 10 intercepts (set to equal)
- 20 residual variances
- 02 factor variances
- 02 factor co-variances
- 02 factor means

```
cfa_m5  <- mplusObject(
  TITLE = "model5 - scalar invariance",
  VARIABLE =
    "usevar = stolen-rac_fght;

     grouping = freelnch (0=freelnch_0 1=freelnch_1); ",

  ANALYSIS =
    "estimator = mlr;",

  MODEL =
    "VICTIM by stolen* t_hurt p_fight hit damaged bullied;
     VICTIM@1;

     NEG_CLIM by unsafe* disrupt gangs rac_fght;
     NEG_CLIM@1;

     [VICTIM-NEG_CLIM@0];

     MODEL freelnch_1:

     VICTIM; ! free factor variances for group 2

     NEG_CLIM;

     [VICTIM-NEG_CLIM]; ! free factor means",

  PLOT = "type = plot3;",
  OUTPUT = "sampstat standardized residual modindices (3.84);",

  usevariables = colnames(invar_data),
  rdata = invar_data)
```

```
cfa_m5_fit <- mplusModeler(cfa_m5,
              dataout=here("09-invariance", "invar_mplus", "invar_data.dat"),
              modelout=here("09-invariance", "invar_mplus", "m5_scalar.inp"),
              check=TRUE, run = TRUE, hashfilename = FALSE)
```

Strict invariance

- item loadings (set to equal)
- intercepts (set to equal)
- residuals (set to equal)
- free factor variances and means in group 2

Number of parameters = 36

- 10 item loadings (set to equal)
- 10 intercepts (set to equal)
- 10 residual variances
- 02 factor variances
- 02 factor co-variances
- 02 factor means

```
cfa_m6  <- mplusObject(
  TITLE = "model6 - strict invariance",
  VARIABLE =
    "usevar = stolen-rac_fght;

     grouping = freelnch (0=freelnch_0 1=freelnch_1); ",

  ANALYSIS =
    "estimator = mlr;",

  MODEL =
    "VICTIM by stolen* t_hurt p_fight hit damaged bullied;
     VICTIM@1;

     NEG_CLIM by unsafe* disrupt gangs rac_fght;
     NEG_CLIM@1;

     [VICTIM-NEG_CLIM@0];

     stolen-rac_fght(1-10); ! set residuals to be equal across groups

     MODEL freelnch_1:

     VICTIM; ! free factor variances for group 2
```

```
    NEG_CLIM;

    [VICTIM-NEG_CLIM]; ! free factor means

    stolen-rac_fght(1-10); ",

  PLOT = "type = plot3;",
  OUTPUT = "sampstat standardized residual modindices (3.84);",

  usevariables = colnames(invar_data),
  rdata = invar_data)

cfa_m6_fit <- mplusModeler(cfa_m6,
              dataout=here("09-invariance", "invar_mplus", "invar_data.dat"),
              modelout=here("09-invariance", "invar_mplus", "m6_strict.inp"),
              check=TRUE, run = TRUE, hashfilename = FALSE)
```

Structural invariance A (fixed factor variances)

Demonstration of structural invariance using the **Scalar model**

- item loadings (set to equal)
- intercepts (set to equal)
- free residuals (Scalar)
- factor means free in group 2
- factor variances (set to 1)
- free factor covariances

Number of parameters = 44

- 10 item loadings (set to equal)
- 10 intercepts (set to equal)
- 20 residual variances
- 00 factor variances
- 02 factor co-variances
- 02 factor means

```
# fixed factor variances

cfa_m7  <- mplusObject(

  TITLE = "model7 - structural invariance A" ,

  VARIABLE =
```

```
    "usevar = stolen-rac_fght;

     grouping = freelnch (0=freelnch_0 1=freelnch_1); ",

  ANALYSIS =
    "estimator = mlr;",

  MODEL =
    "VICTIM by stolen* t_hurt p_fight hit damaged bullied;
     VICTIM@1;

     NEG_CLIM by unsafe* disrupt gangs rac_fght;
     NEG_CLIM@1;

     [VICTIM-NEG_CLIM@0];

     MODEL freelnch_1:

     [VICTIM-NEG_CLIM]; ! free factor means

     VICTIM@1; NEG_CLIM@1; ! fix factor variance to 1",

  PLOT = "type = plot3;",
  OUTPUT = "sampstat standardized residual modindices (3.84);",

  usevariables = colnames(invar_data),
  rdata = invar_data)

cfa_m7_fit <- mplusModeler(cfa_m7,
              dataout=here("09-invariance", "invar_mplus", "invar_data.dat"),
              modelout=here("09-invariance", "invar_mplus", "m7_structuralA.inp"),
              check=TRUE, run = TRUE, hashfilename = FALSE)
```

Structural invariance B (fixed factor variances and equal covariances)

Demonstration of structural invariance using the **Scalar model**

- item loadings (set to equal)
- intercepts (set to equal)
- free residuals (Scalar)
- factor means free in group 2
- factor variances (set to equal)
- factor covariances (set to equal)

Number of parameters = 43

- 10 item loadings (set to equal)

- 10 intercepts (set to equal)
- 20 residual variances
- 00 factor variances
- 01 factor co-variances
- 02 factor means

```
# equal factor variances and covariances

cfa_m8  <- mplusObject(
  TITLE = "model8 - structural invariance B" ,
  VARIABLE =
    "usevar = stolen-rac_fght;

     grouping = freelnch (0=freelnch_0 1=freelnch_1); ",

  ANALYSIS =
    "estimator = mlr;",

  MODEL =
    "VICTIM by stolen* t_hurt p_fight hit damaged bullied;
     VICTIM@1;

     NEG_CLIM by unsafe* disrupt gangs rac_fght;
     NEG_CLIM@1;

     [VICTIM-NEG_CLIM@0];

     VICTIM with NEG_CLIM (11) ! set covariances to equal;

     MODEL freelnch_1:

     [VICTIM-NEG_CLIM]; ! free factor means

     VICTIM@1; NEG_CLIM@1; ! fix factor variance to 1

     VICTIM with NEG_CLIM (11); ! set covariances to equal",

  PLOT = "type = plot3;",
  OUTPUT = "sampstat standardized residual modindices (3.84);",

  usevariables = colnames(invar_data),
  rdata = invar_data)

cfa_m8_fit <- mplusModeler(cfa_m8,
              dataout=here("09-invariance", "invar_mplus", "invar_data.dat"),
              modelout=here("09-invariance", "invar_mplus", "m8_structuralB.inp"),
              check=TRUE, run = TRUE, hashfilename = FALSE)
```

Latent Factor Means differences:

(model: Step_07_STRUCTURAL)

Mean differences: Students in sub-sample `freelnch_1` have...

```
VICTIM     -0.026      0.091     -0.291      0.771  (not significant)
NEG_CLIM    0.632      0.104      6.104      0.000  (higher scores for "NEG_CLIM")
```

Comparing Fit Across Models

Guidlines: for loadings & fit indices

- **Simple structure:** "0.4 - 0.3 - 0.2" rule Howard (2016) (primary loadings > 0.4 / cross-loadings < 0.3 / minimum difference $= 0.2$)
- **RMSEA:** $< .05$ indicates "good" fit Brown (2015)
- **CFI:** $> .95$ indicates "good" fit Brown (2015)
- **SRMR:** $< .08$ indicates "good" fit Hu and Bentler (1999)
- **Invariance:** Changes in **CFI** less than or equal to **-0.01** are acceptable

---

Read into R summary of all models

```
all_models <- readModels(here("09-invariance", "invar_mplus"), quiet = TRUE)
```

Create table, extract fit statistics, sort by `Filename`

```
invar_summary <- LatexSummaryTable(all_models,
                 keepCols=c("Filename", "Parameters","ChiSqM_Value", "CFI","TLI",
                            "SRMR", "RMSEA_Estimate", "RMSEA_90CI_LB", "RMSEA_90CI_UB"),
                 sortBy = "Filename")

invar_summary %>%
  mutate(Filename = str_remove(Filename, ".out")) %>%
  gt() %>%
  cols_label(
    Filename = "Model",
    Parameters = "Par",
    ChiSqM_Value = "ChiSq",
    CFI = "CFI", TLI = "TLI", SRMR = "SRMR",
    RMSEA_Estimate = "RMSEA",
    RMSEA_90CI_LB = "Lower CI",
    RMSEA_90CI_UB = "Upper CI")
```

Model	Par	ChiSq	CFI	TLI	SRMR	RMSEA	Lower CI	Upper CI
m0_cfa_fullsample	31	121.460	0.898	0.865	0.043	0.060	0.048	0.071
m1_cfa_freelnch_0	31	67.305	0.907	0.877	0.053	0.057	0.036	0.076
m2_cfa_freelnch_1	31	82.253	0.893	0.858	0.049	0.063	0.046	0.081
m3_configural	62	149.315	0.899	0.867	0.051	0.060	0.047	0.073
m4_metric	54	163.312	0.892	0.872	0.061	0.059	0.047	0.072
m5_scalar	46	179.176	0.882	0.874	0.062	0.059	0.047	0.071
m6_strict	36	180.933	0.892	0.897	0.067	0.053	0.041	0.065
m7_structuralA	44	177.512	0.887	0.881	0.063	0.057	0.045	0.069
m8_structuralB	43	177.144	0.888	0.885	0.063	0.056	0.044	0.068

Calculate Satora-Bentler scaled Chi-square difference test (use with MLR estimator)

See website: stats.idre.ucla.edu

- SB0 = null model Chi-square value
- SB1 = alternate model Chi-square value
- c0 = null model scaling correction factor
- c1 = alternate model scaling correction factor
- d0 = null model degrees of freedom
- d1 = alternate model degrees of freedom
- df = Chi-square test degrees of freedom

---

compare configural to metric

```
SB0 <- all_models[["M4_metric.out"]][["summaries"]][["ChiSqM_Value"]]
SB1 <- all_models[["M3_configural.out"]][["summaries"]][["ChiSqM_Value"]]
c0  <- all_models[["M4_metric.out"]][["summaries"]][["ChiSqM_ScalingCorrection"]]
c1  <- all_models[["M3_configural.out"]][["summaries"]][["ChiSqM_ScalingCorrection"]]
d0  <- all_models[["M4_metric.out"]][["summaries"]][["ChiSqM_DF"]]
d1  <- all_models[["M3_configural.out"]][["summaries"]][["ChiSqM_DF"]]
df  <- abs(d0-d1)

# Satora-Bentler scaled Difference test equations
cd <- (((d0*c0)-(d1*c1))/(d0-d1))
t  <- (((SB0*c0)-(SB1*c1))/(cd))

# Chi-square and degrees of freedom
t
```

```
## numeric(0)
```

```
df
```

```
## integer(0)
```

```
# Significance test
pchisq(t, df, lower.tail=FALSE)
```

```
## numeric(0)
```

---

compare metric to scalar

```
SB0 <- all_models[["M5_scalar.out"]][["summaries"]][["ChiSqM_Value"]]
SB1 <- all_models[["M4_metric.out"]][["summaries"]][["ChiSqM_Value"]]
c0  <- all_models[["M5_scalar.out"]][["summaries"]][["ChiSqM_ScalingCorrection"]]
c1  <- all_models[["M4_metric.out"]][["summaries"]][["ChiSqM_ScalingCorrection"]]
d0  <- all_models[["M5_scalar.out"]][["summaries"]][["ChiSqM_DF"]]
d1  <- all_models[["M4_metric.out"]][["summaries"]][["ChiSqM_DF"]]
df  <- abs(d0-d1)

# Satora-Bentler scaled Difference test equations
cd <- (((d0*c0)-(d1*c1))/(d0-d1))
t  <- (((SB0*c0)-(SB1*c1))/(cd))

# Chi-square and degrees of freedom
t
```

```
## numeric(0)
```

```
df
```

```
## integer(0)
```

```
# Significance test
pchisq(t, df, lower.tail=FALSE)
```

```
## numeric(0)
```

Invariance short-cut

```
mx  <- mplusObject(
  TITLE = "INVARIANCE SHORT_CUT - DEMO",
  VARIABLE =
    "usevar = stolen-rac_fght;

     grouping = freelnch (0=freelnch_0 1=freelnch_1); ",

  ANALYSIS =
  "Estimator = MLR;
```

```
   MODEL = CONFIG METRIC SCALAR;",

     MODEL =
    "VICTIM by stolen* t_hurt p_fight hit damaged bullied;
     VICTIM@1;

     NEG_CLIM by unsafe* disrupt gangs rac_fght;
     NEG_CLIM@1;" ,

  PLOT = "",
  OUTPUT = "sampstat residual;",

  usevariables = colnames(invar_data),
  rdata = invar_data)

mx_fit <- mplusModeler(mx,
          dataout=here("09-invariance", "invar_short", "invar_short_cut.dat"),
          modelout=here("09-invariance", "invar_short", "invar_short_cut.inp"),
          check=TRUE, run = TRUE, hashfilename = FALSE)
```

Invariance Testing (Chi-square values - Chi-Square difference p-values are biased)

Model	Number of Parameters	Chi-Square	Degrees of Freedom	P-Value
Configural	62	149.315	68	0.0000
Metric	54	163.312	76	0.0000
Scalar	46	179.176	84	0.0000

Models Compared	Chi-Square	Degrees of Freedom	P-Value
Metric against Configural	14.759	8	0.0640
Scalar against Configural	30.022	16	0.0179
Scalar against Metric	15.444	8	0.0511

End

References

Hallquist, M. N., & Wiley, J. F. (2018). MplusAutomation: An R Package for Facilitating Large-Scale Latent Variable Analyses in Mplus. Structural equation modeling: a multidisciplinary journal, 25(4), 621-638.

Horst, A. (2020). Course & Workshop Materials. GitHub Repositories, https://https://allisonhorst.github.io/

Lauff, E., & Ingels, S. J. (2014). Education Longitudinal Study of 2002 (ELS: 2002): A First Look at 2002 High School Sophomores 10 Years Later. First Look. NCES 2014-363. National Center for Education Statistics.

Muthén, L.K. and Muthén, B.O. (1998-2017). Mplus User's Guide. Eighth Edition. Los Angeles, CA: Muthén & Muthén

R Core Team (2017). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. URL http://www.R-project.org/

Wickham et al., (2019). Welcome to the tidyverse. Journal of Open Source Software, 4(43), 1686, https://doi.org/10.21105/joss.01686