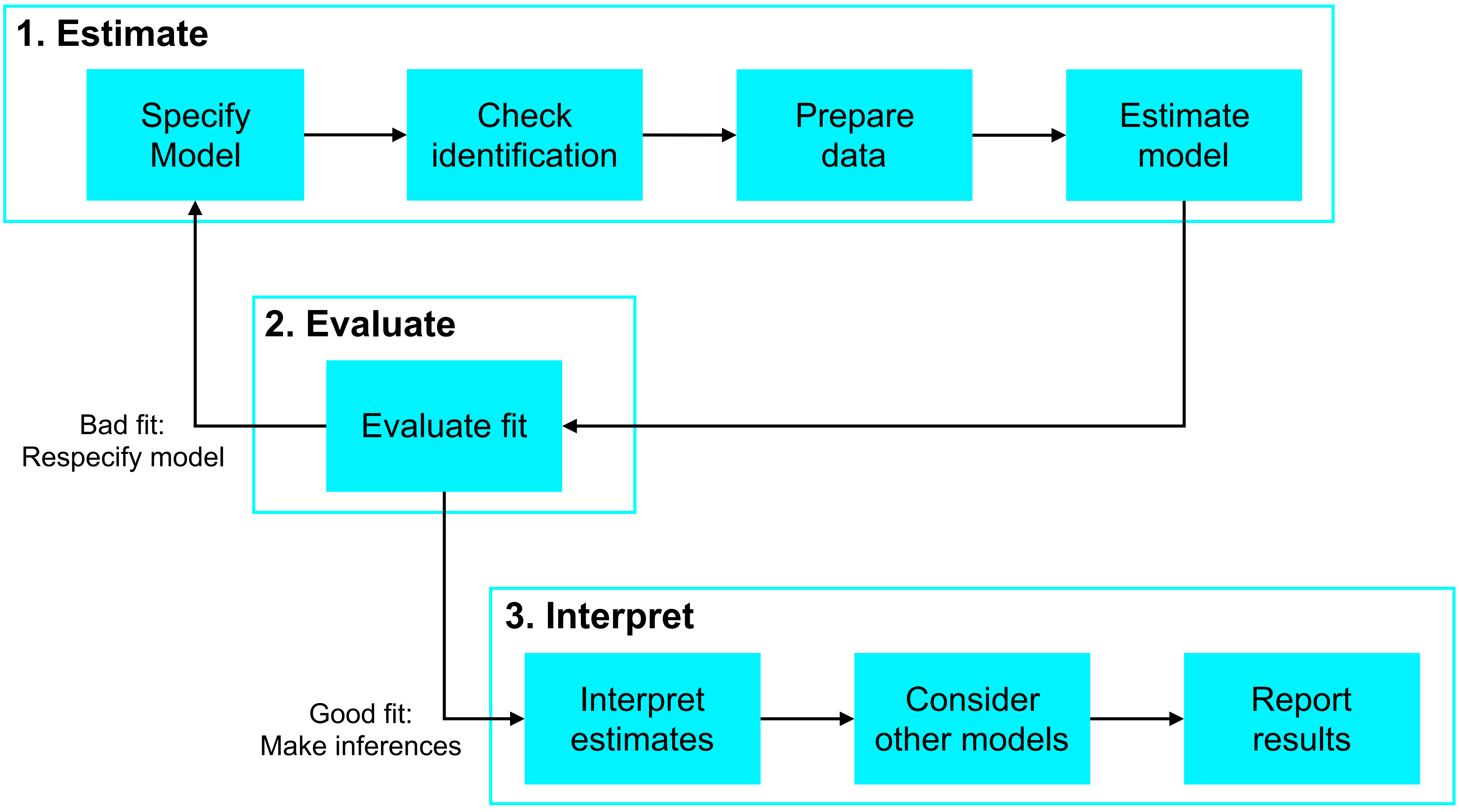
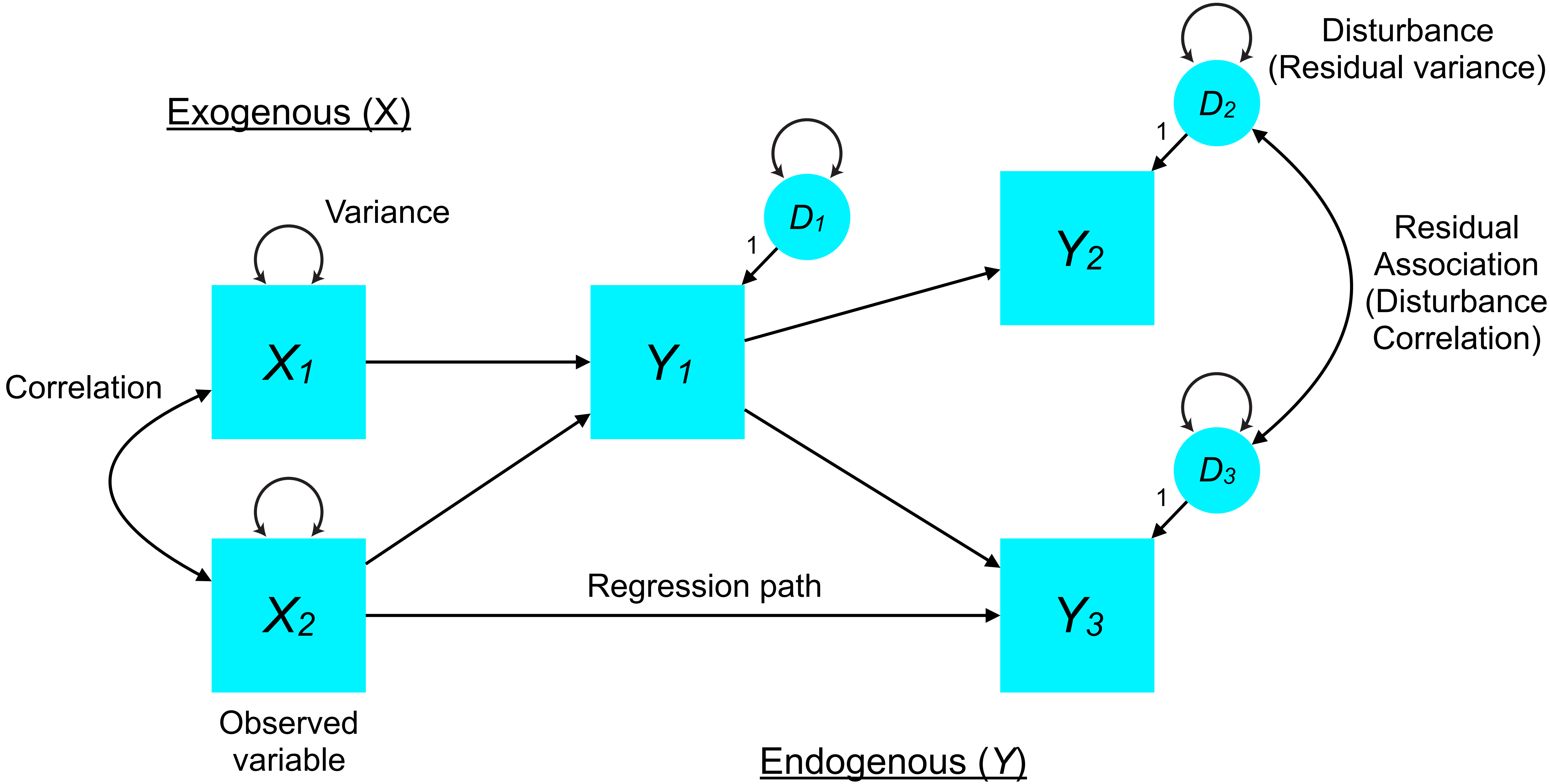
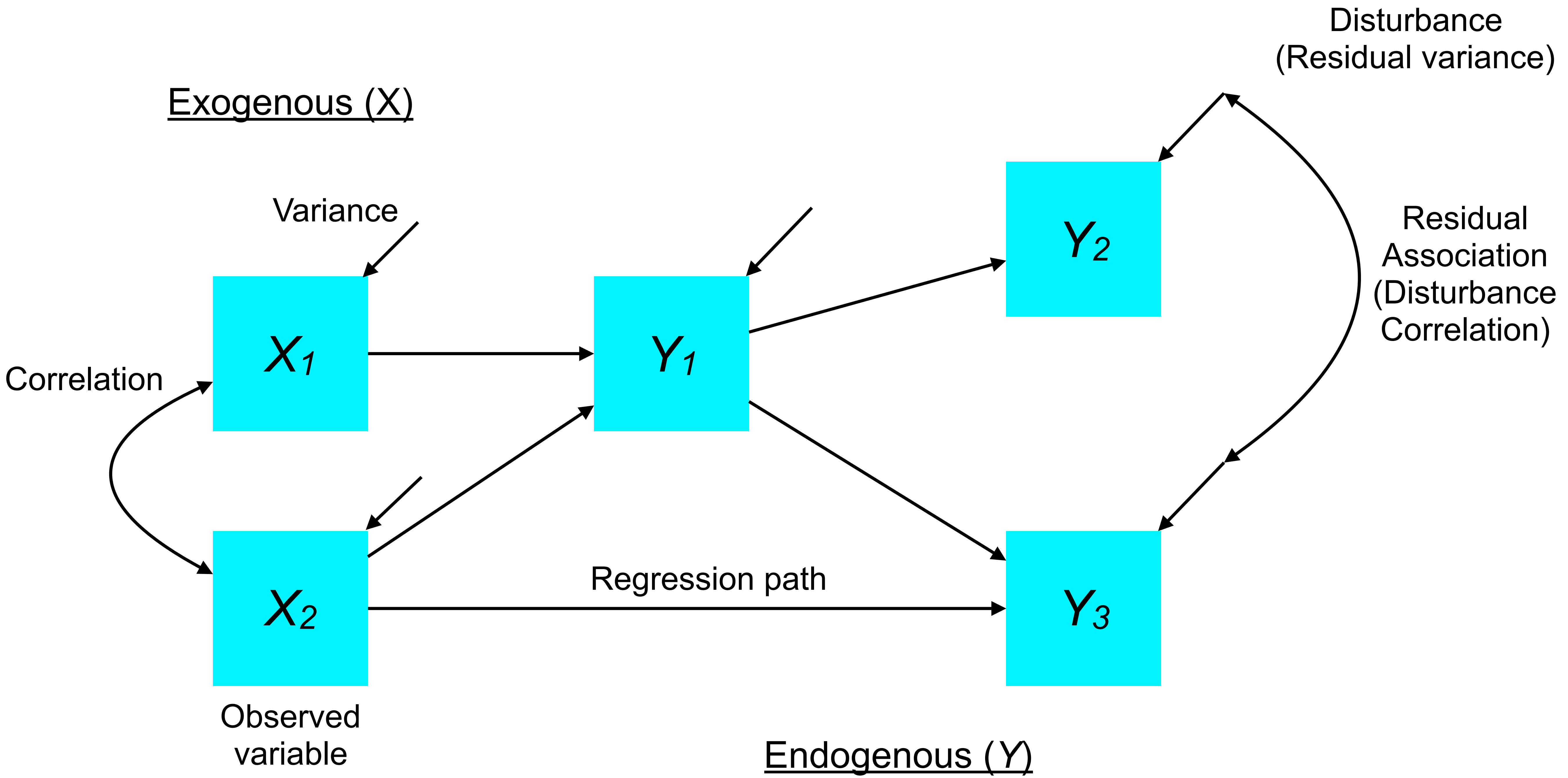
The process of fitting structural equation models



SEM notation redux (RAM notation)

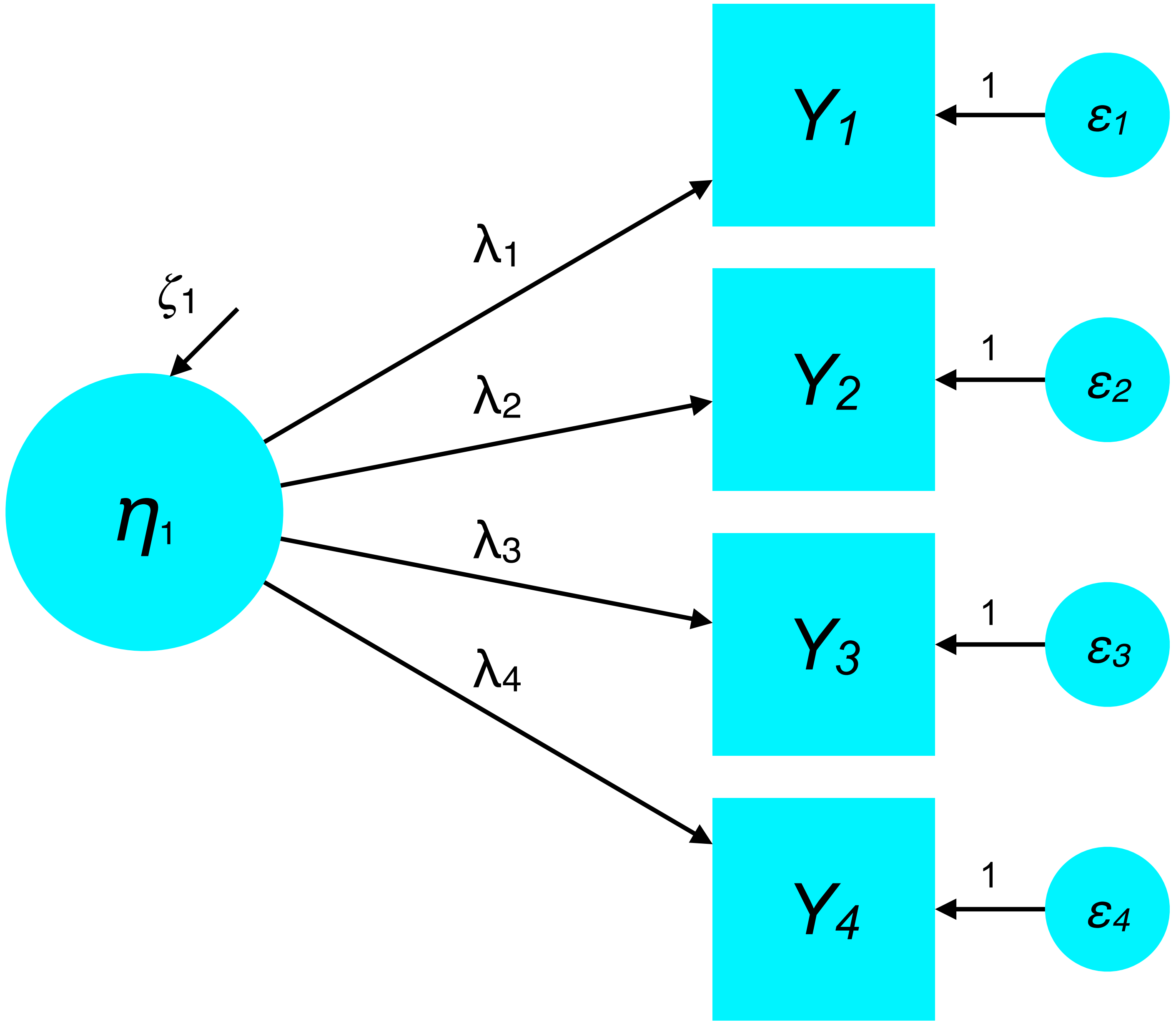


SEM notation redux (simplified RAM notation)

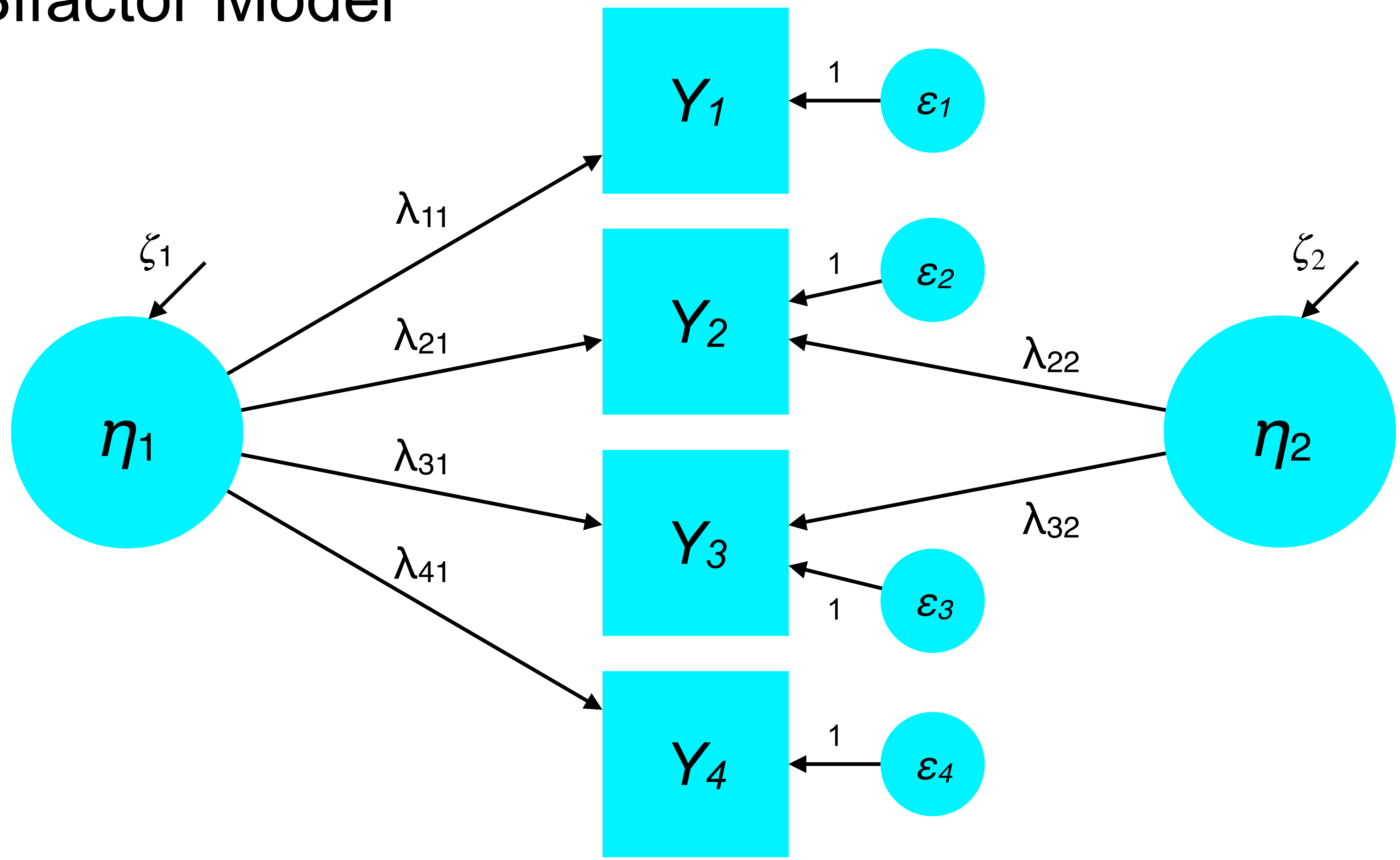


Common Factor Model

$$y_1 = \alpha_1 + \lambda_1 \eta_1 + \varepsilon_1$$
$$y_2 = \alpha_2 + \lambda_2 \eta_1 + \varepsilon_2$$
$$y_3 = \alpha_3 + \lambda_3 \eta_1 + \varepsilon_3$$
$$y_4 = \alpha_4 + \lambda_4 \eta_1 + \varepsilon_4$$



Bifactor Model



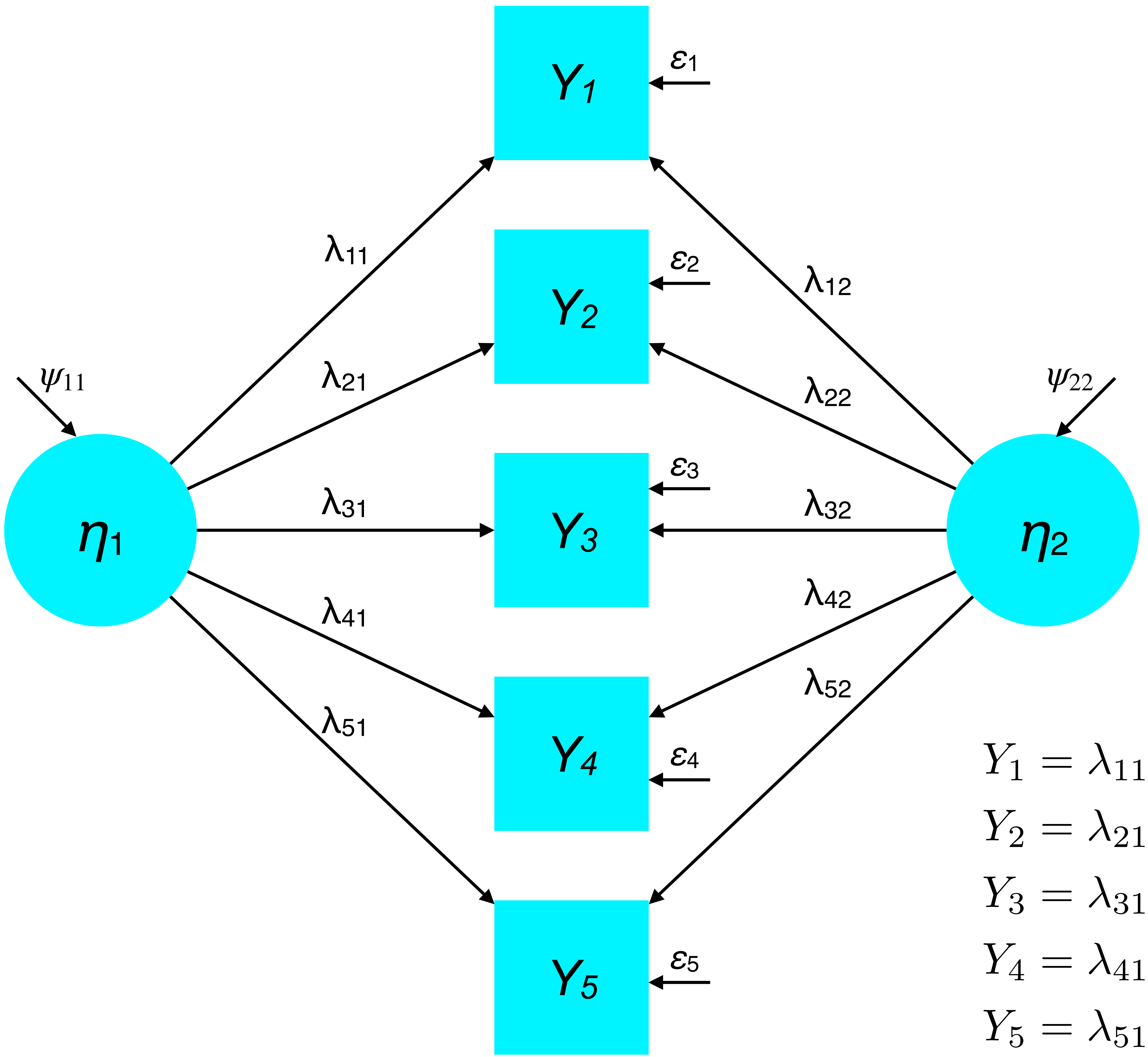
$$y_1 = \alpha_1 + \lambda_{11}\eta_1 + \varepsilon_1$$

$$y_2 = \alpha_2 + \lambda_{21}\eta_1 + \lambda_{22}\eta_2 + \varepsilon_2$$

$$y_3 = \alpha_3 + \lambda_{31}\eta_1 + \lambda_{32}\eta_2 + \varepsilon_3$$

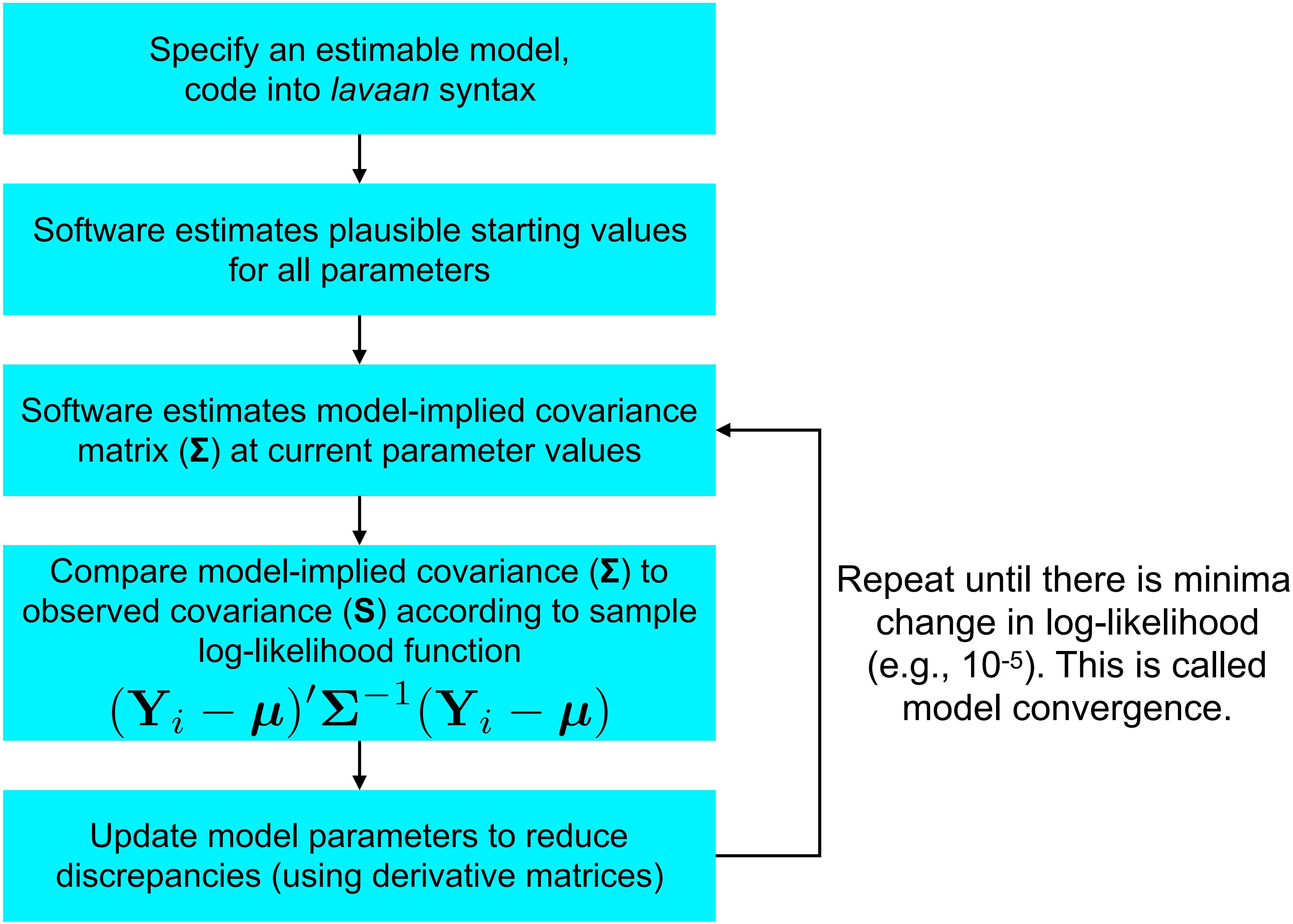
$$y_4 = \alpha_4 + \lambda_{41}\eta_1 + \varepsilon_4$$

Two-factor model as typically instantiated in EFA



$$Y_1 = \lambda_{11}\boldsymbol{\eta}_1 + \lambda_{12}\boldsymbol{\eta}_2 + \varepsilon_1$$
$$Y_2 = \lambda_{21}\boldsymbol{\eta}_1 + \lambda_{22}\boldsymbol{\eta}_2 + \varepsilon_2$$
$$Y_3 = \lambda_{31}\boldsymbol{\eta}_1 + \lambda_{32}\boldsymbol{\eta}_2 + \varepsilon_3$$
$$Y_4 = \lambda_{41}\boldsymbol{\eta}_1 + \lambda_{42}\boldsymbol{\eta}_2 + \varepsilon_4$$
$$Y_5 = \lambda_{51}\boldsymbol{\eta}_1 + \lambda_{52}\boldsymbol{\eta}_2 + \varepsilon_5$$

Estimating SEMs



A book worth reading…

Latent Curve Models

A Structural Equation Perspective

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Department of Sociology
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PATRICK J. CURRAN

University of North Carolina
Department of Psychology
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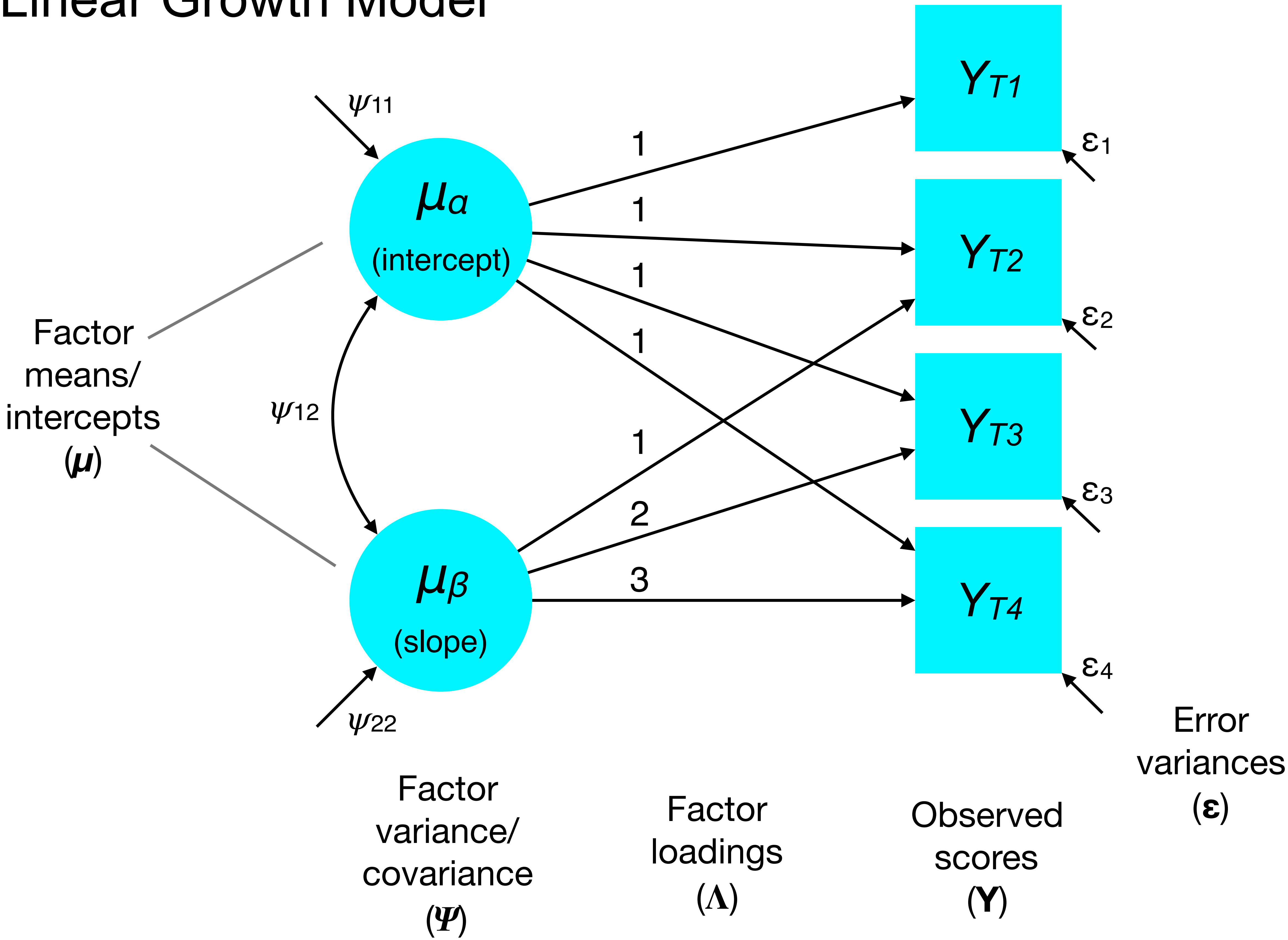
- Forms of change
- Mean-level
- rank-order
- ipsative

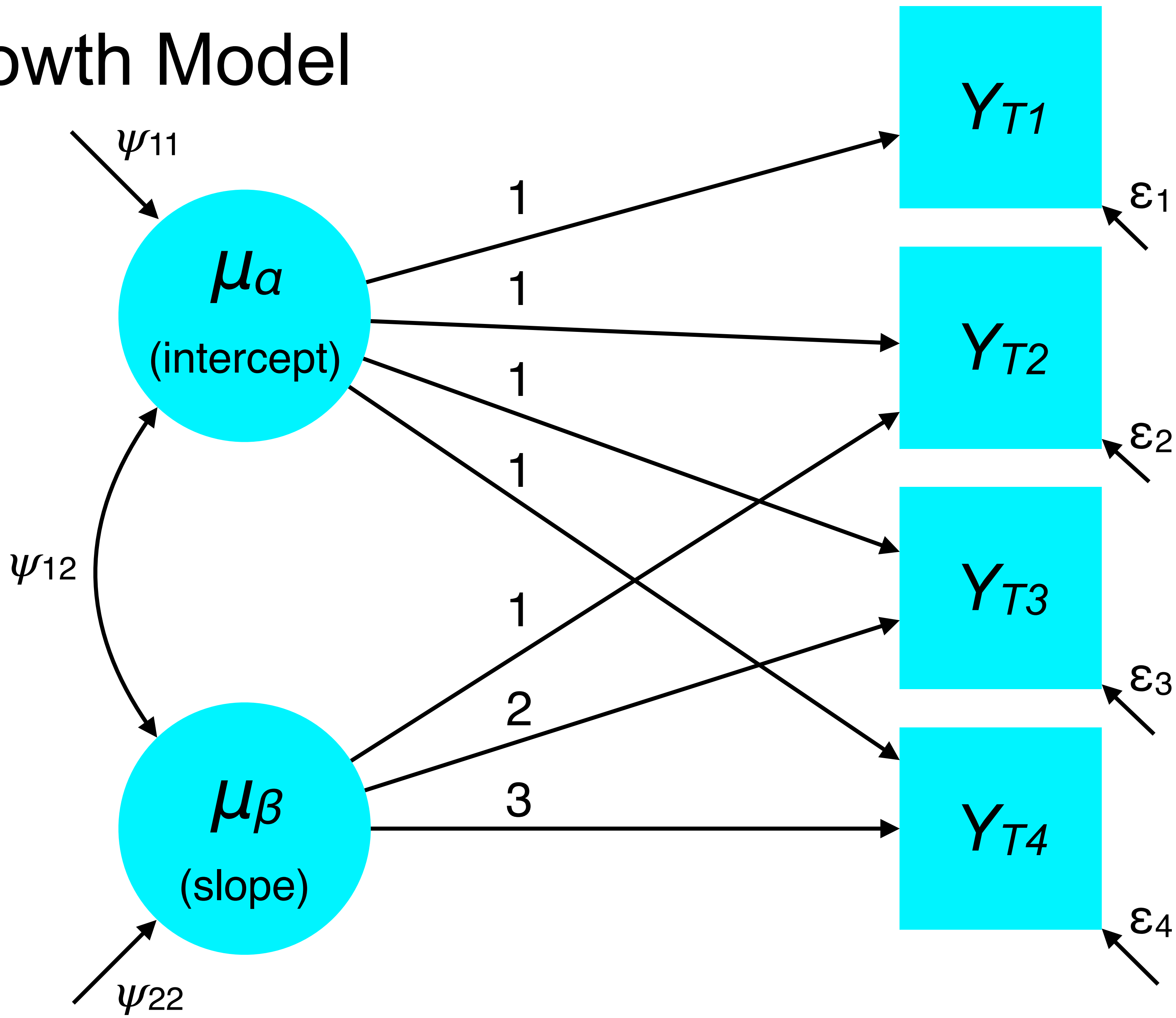
Ergodicity

- Adapt Molenaar here: process of change within-individual may differ from mean-level change *between* individuals

Linear Growth Model



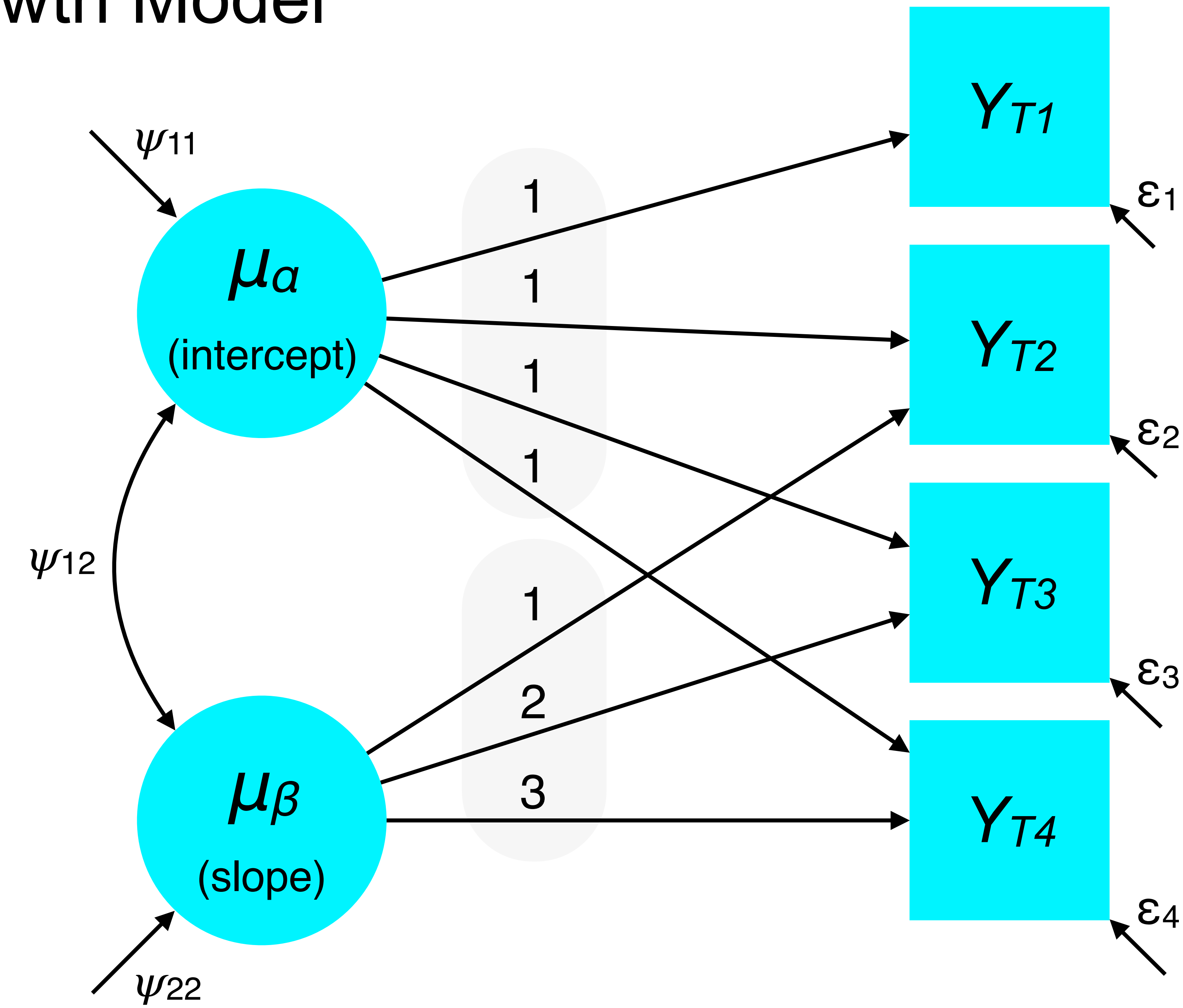
Linear Growth Model



$$y_{i,T1} = \alpha_i + \beta_i \lambda_{T1} + \varepsilon_{i,T1}$$
$$y_{i,T2} = \alpha_i + \beta_i \lambda_{T2} + \varepsilon_{i,T2}$$
$$y_{i,T3} = \alpha_i + \beta_i \lambda_{T3} + \varepsilon_{i,T3}$$
$$y_{i,T4} = \alpha_i + \beta_i \lambda_{T4} + \varepsilon_{i,T4}$$

$$\alpha_i = \mu_\alpha + \zeta_{\alpha_i}$$
$$\beta_i = \mu_\beta + \zeta_{\beta_i}$$
$$Var(\zeta_{\boldsymbol{\alpha}}) = \psi_{11}$$
$$Var(\zeta_{\boldsymbol{\beta}}) = \psi_{22}$$

Linear Growth Model



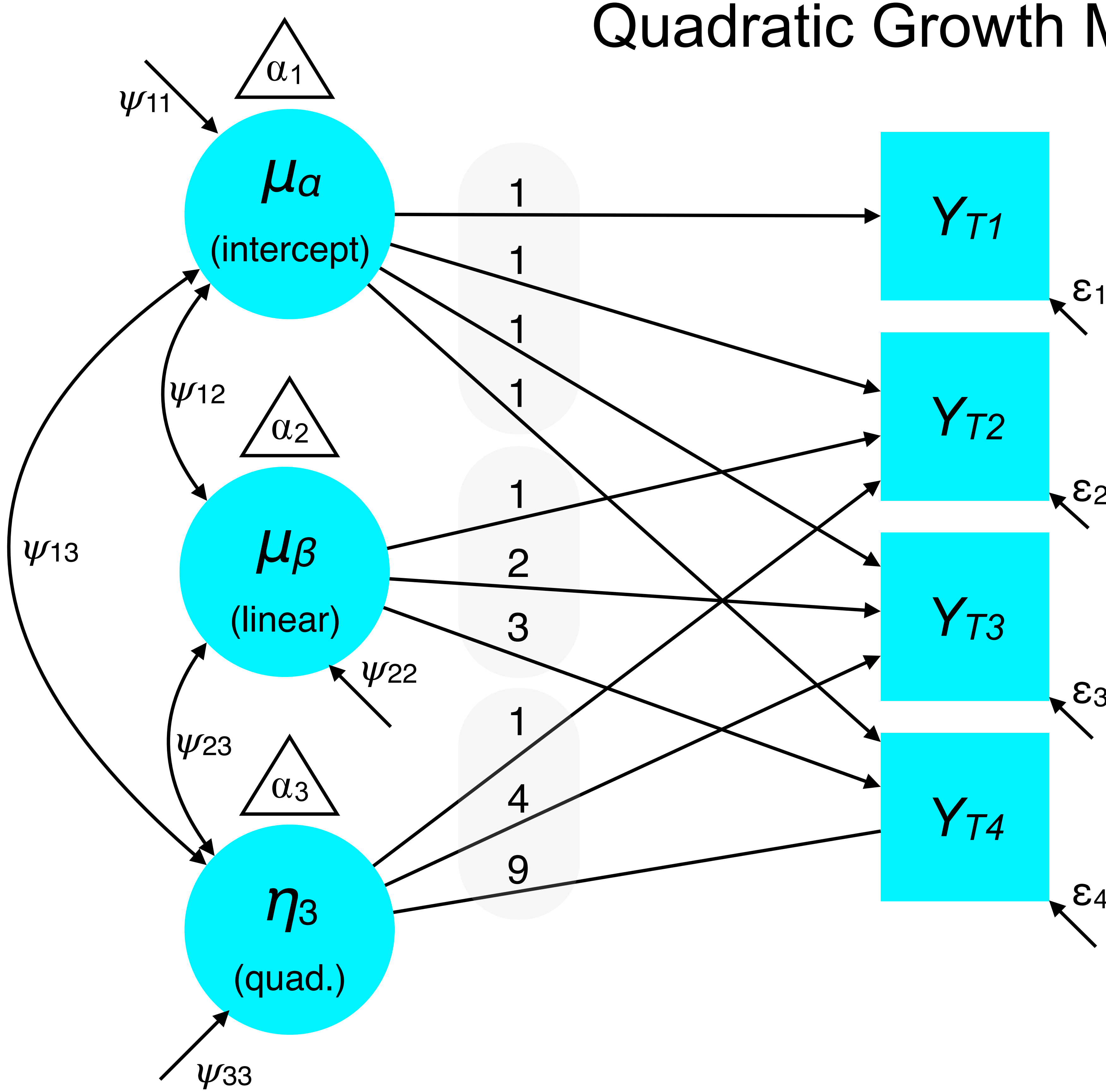
Fixed loadings denote:

a) functional form of change

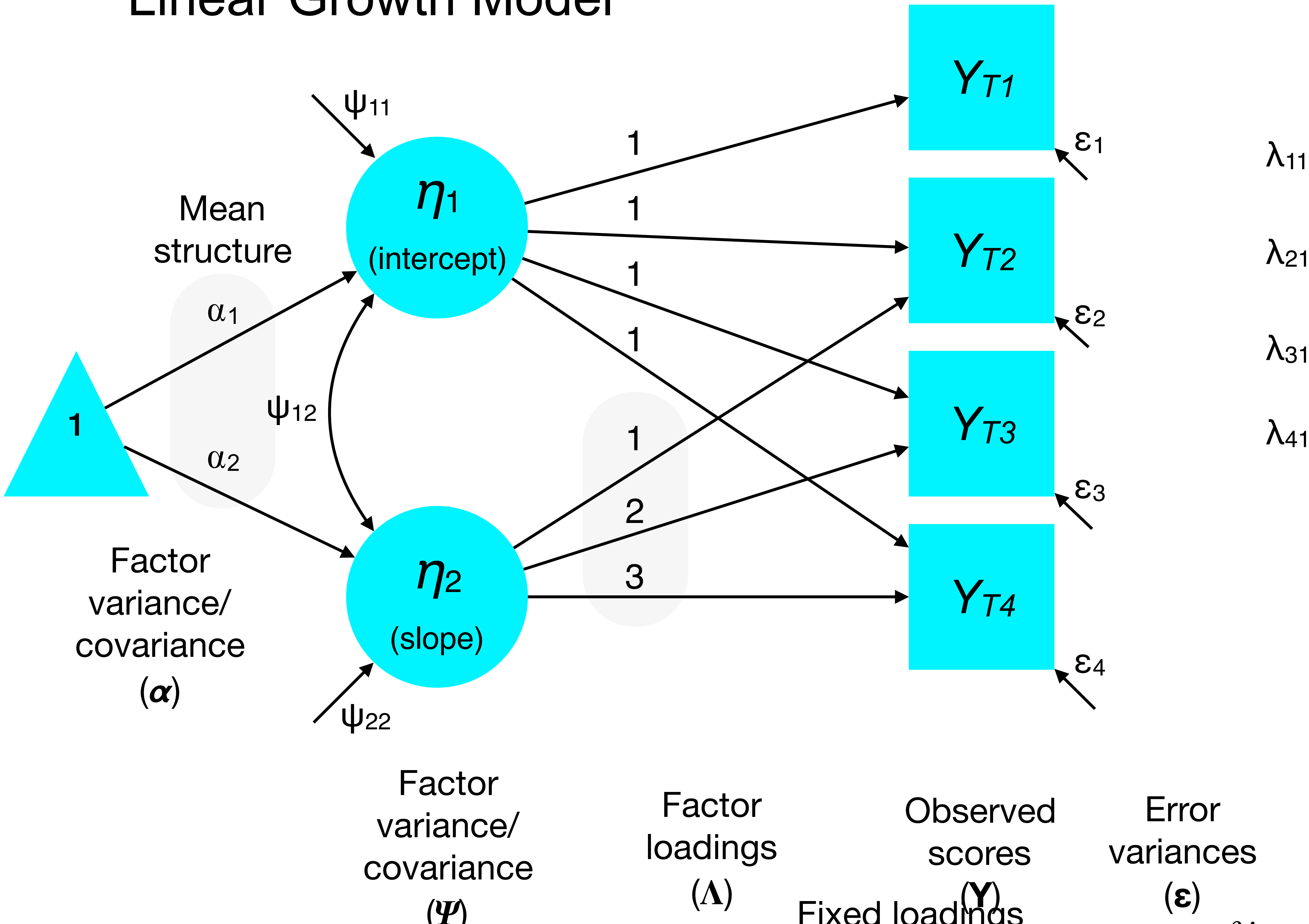
and

b) the spacing between observations

Quadratic Growth Model



Linear Growth Model



Fixed loadings
denote functional
form of change

Fixed loadings
denote functional

$$y_1 = \alpha_1$$

$$y_2 = \alpha_2$$

Organize wrt age, not wave

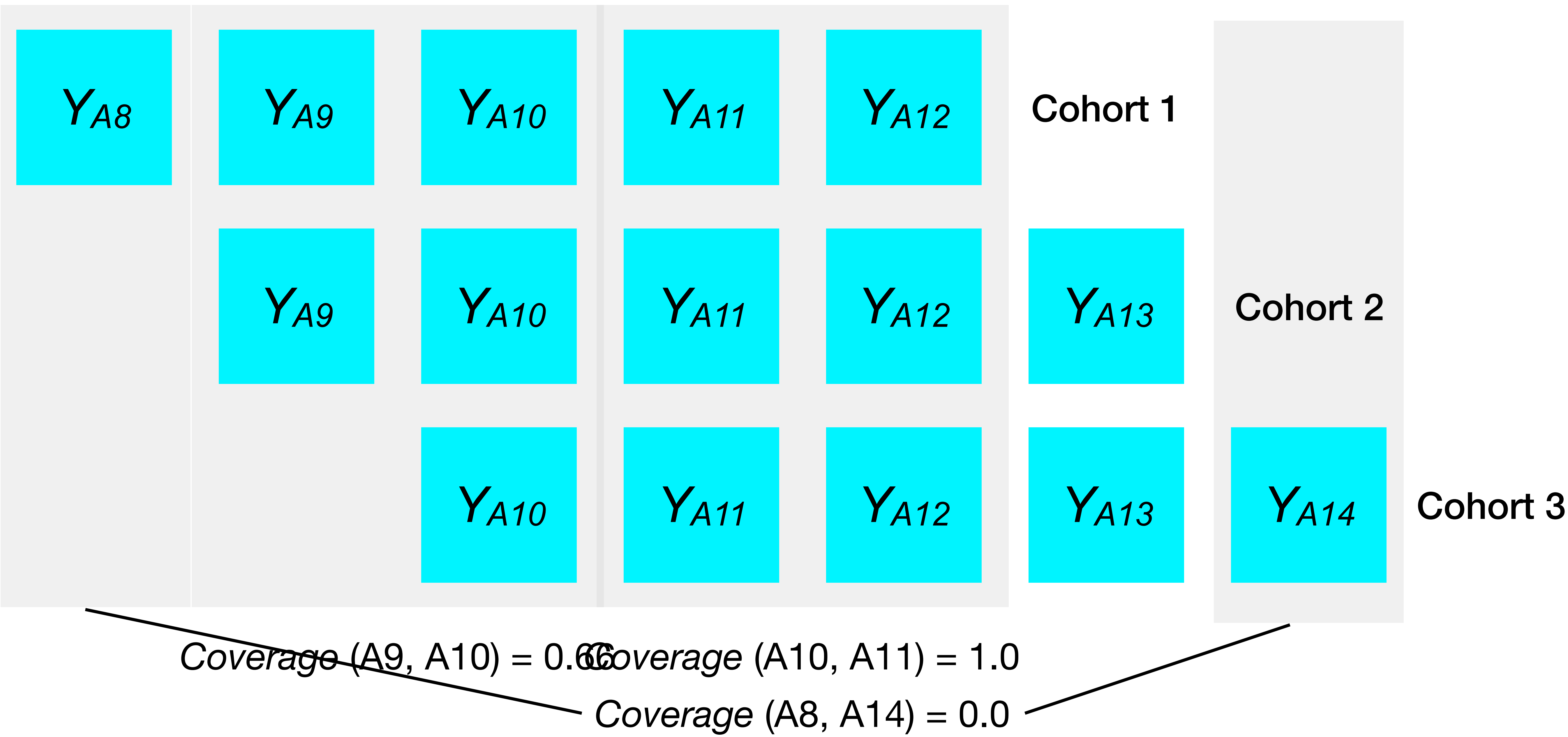
Points around missingness, covariance coverage
Accelerated longitudinal design, but only works if there are many more waves than there are age *cohorts.*

Ad hoc accelerated longitudinal will not have covariance coverage! So if you are accelerating, strongly recommend cohorts and not spacing the out too much

Organizing wrt wave means we are pooling estimates (e.g., means or variances) that reflect different moments in developmental time (e.g., age)

Accelerated Longitudinal Cohort Design

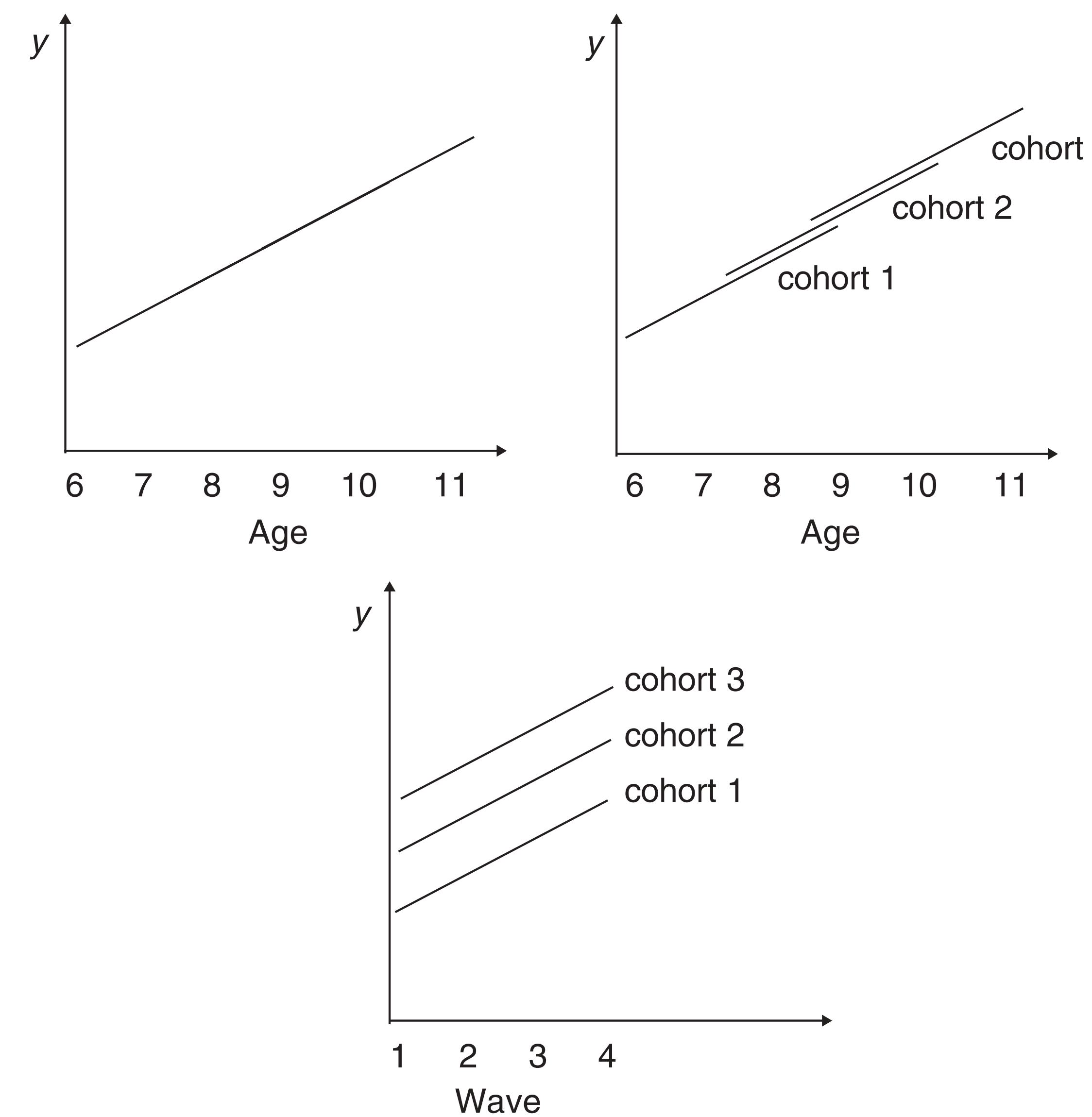
aka Cohort-Sequential Design



(Assuming Equal n per cohort)

Estimation in cohort designs

- Risks of organizing data with respect to cohorts if there are meaningful developmental differences among them (esp. age)



- Missing by design (in sequential-cohort or ad hoc accelerated design) is MCAR by definition.

- Thus, FIML is an acceptable estimation method

- Based on the fact that the residual covariance between indicators is *not* a feature of the model — i.e., we assume conditional independence.

- If we thought there were some meaningful residual covariation (e.g., age 8 with age 14), we should worry about covariance coverage because the structural equations for **y** now depend on the theta $\boldsymbol{\Theta}$ matrix (residual covariance structure). (i.e., a diagonal matrix)

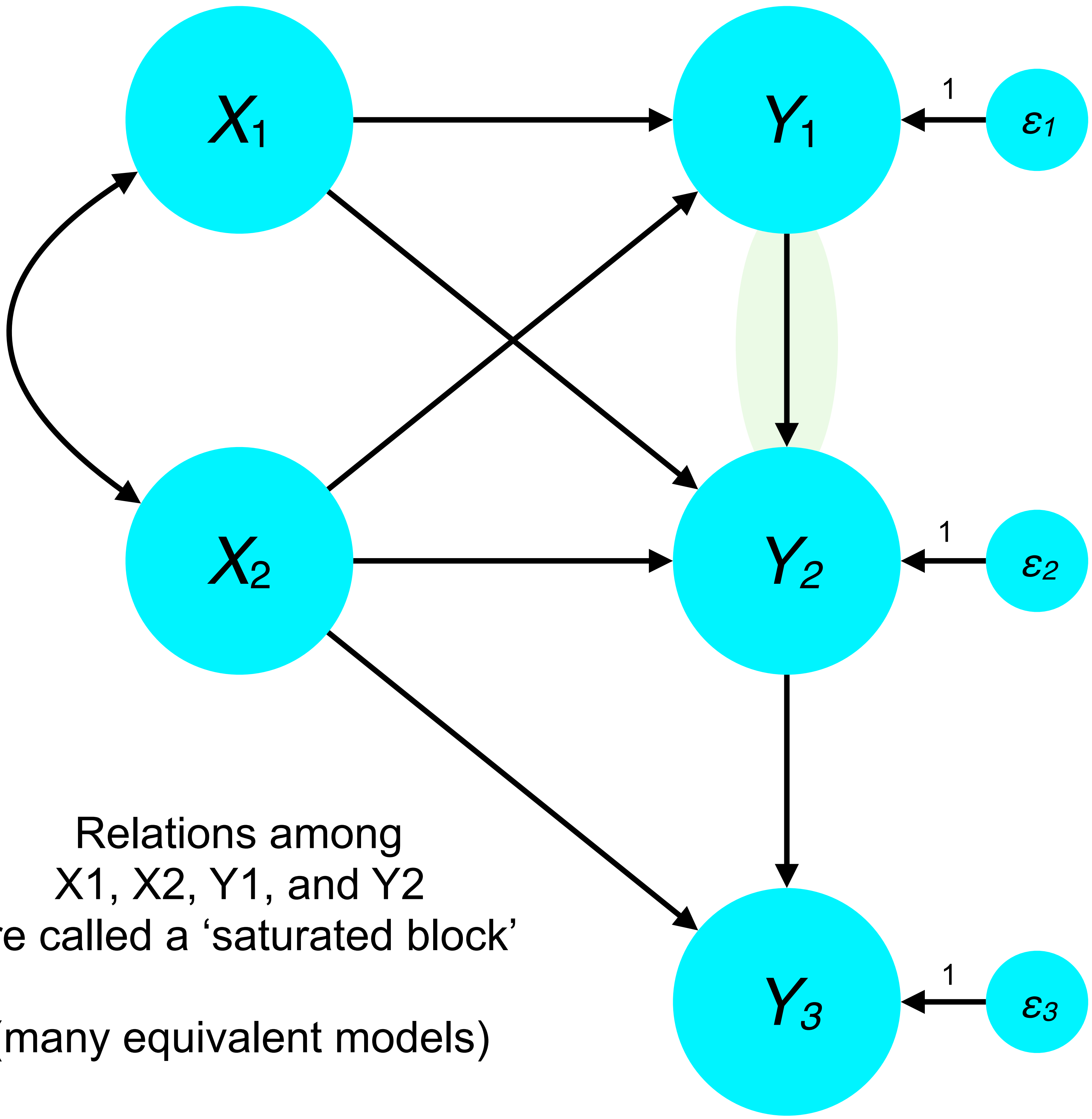
$$\boldsymbol{\theta} = \begin{bmatrix} \varepsilon_1 & \\ 0 & \varepsilon \\ 0 & \\ 0 & \end{bmatrix}$$

- d+2 rule

- Mean structure captures average mean level (when slope loading is 0) and average slope

- Variances capture individual differences in level and rate of change in growth process

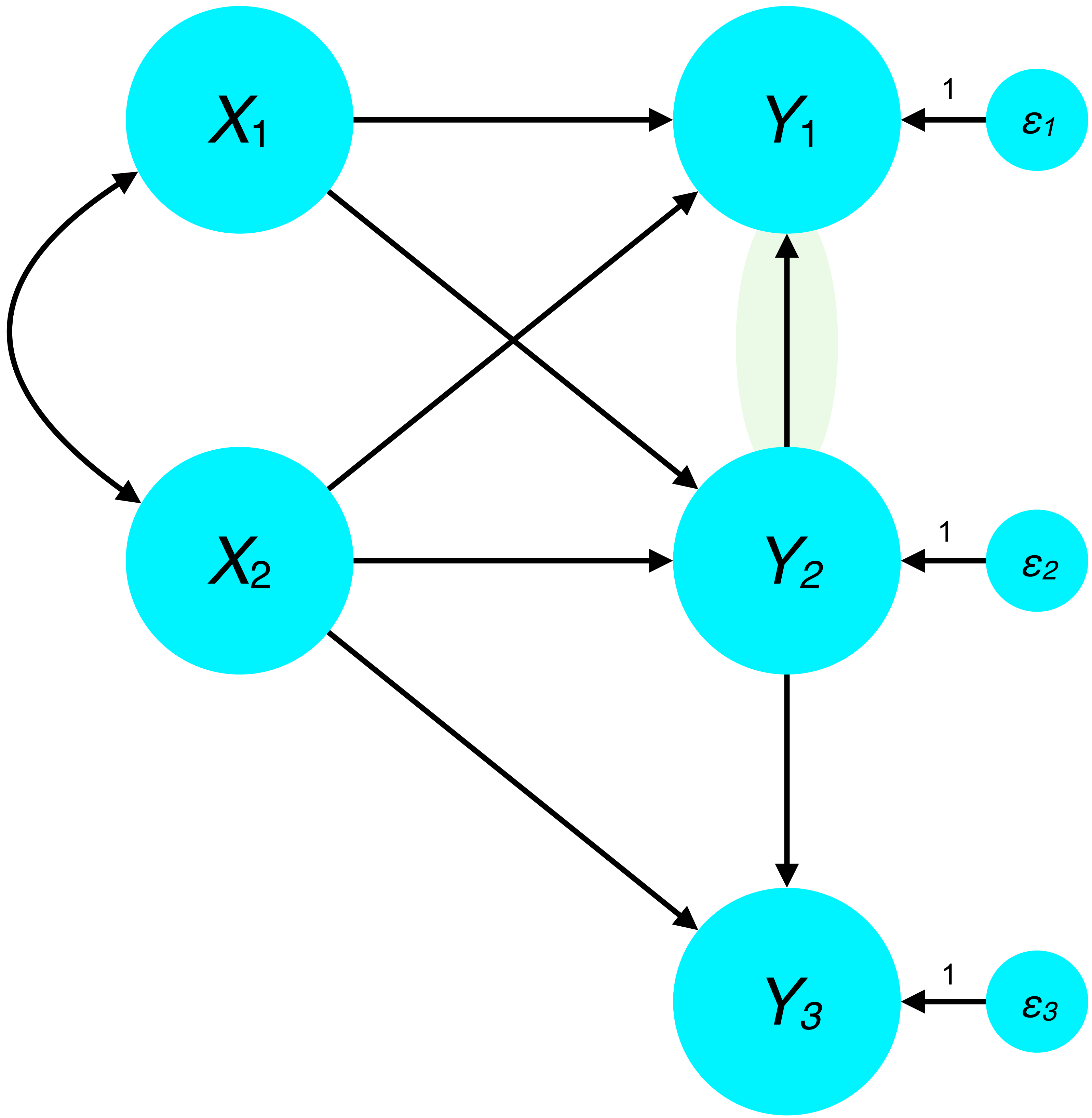
Equivalent Models: A



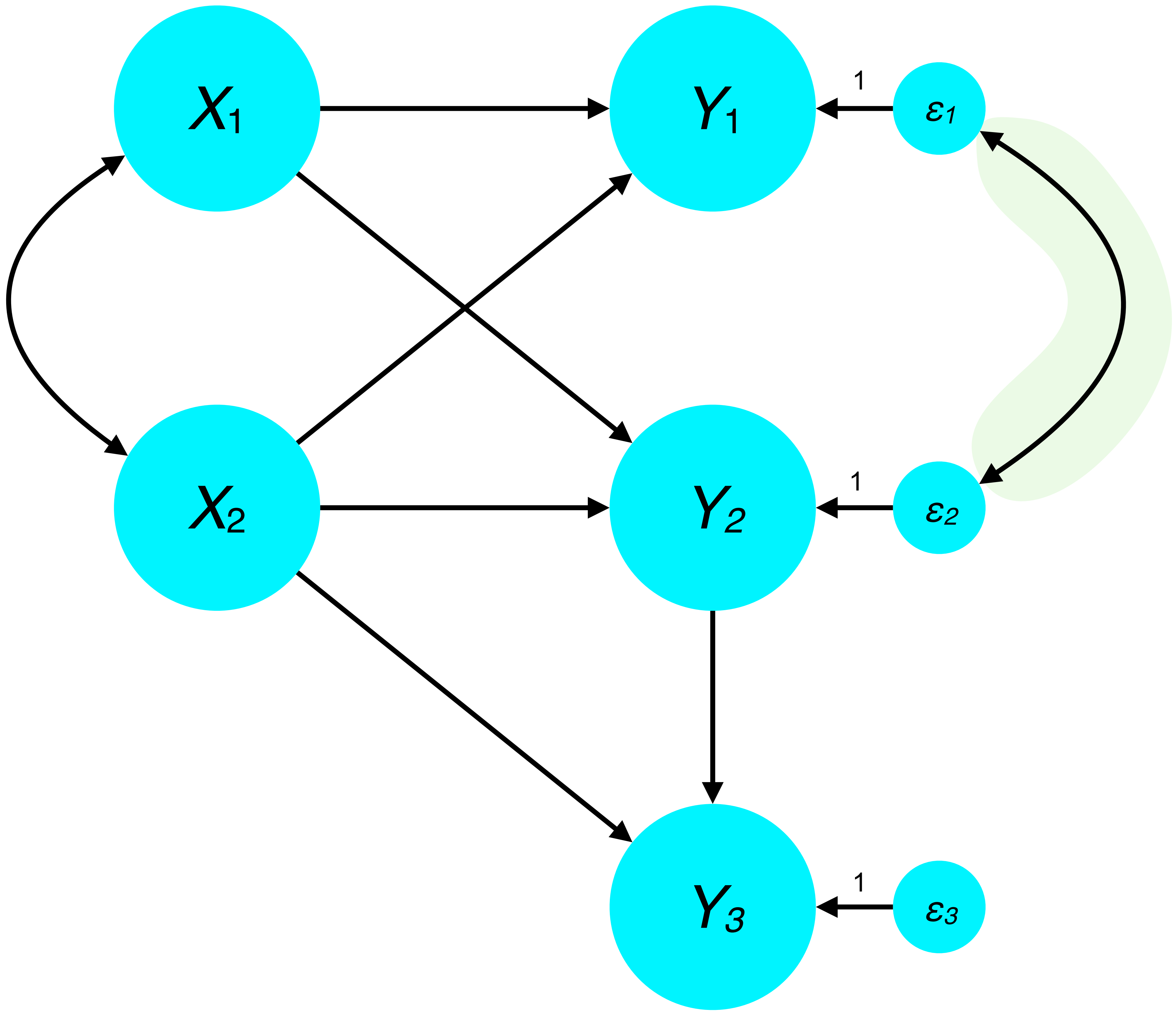
Relations among
X1, X2, Y1, and Y2
are called a ‘saturated block’

(many equivalent models)

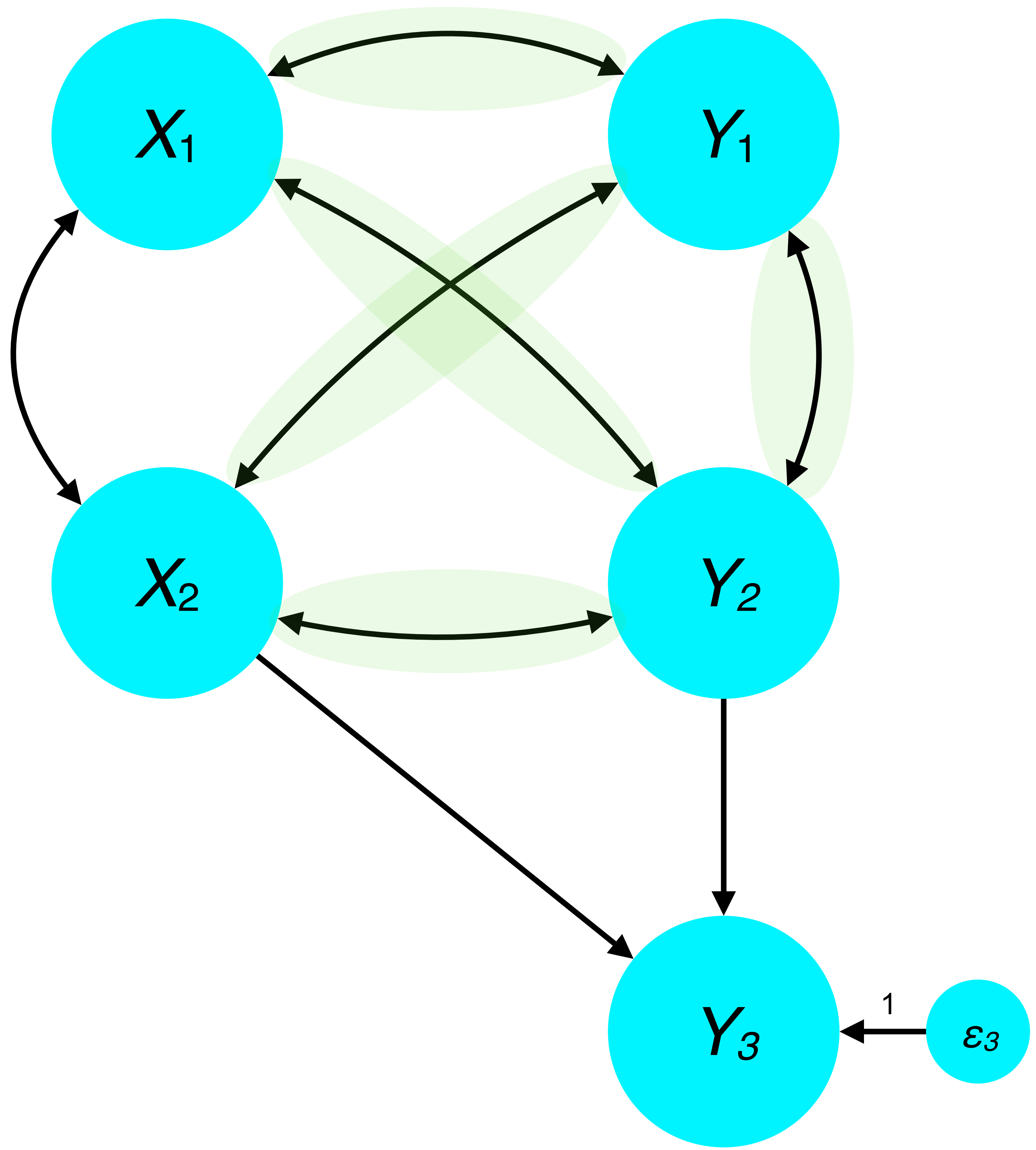
Equivalent Models: B



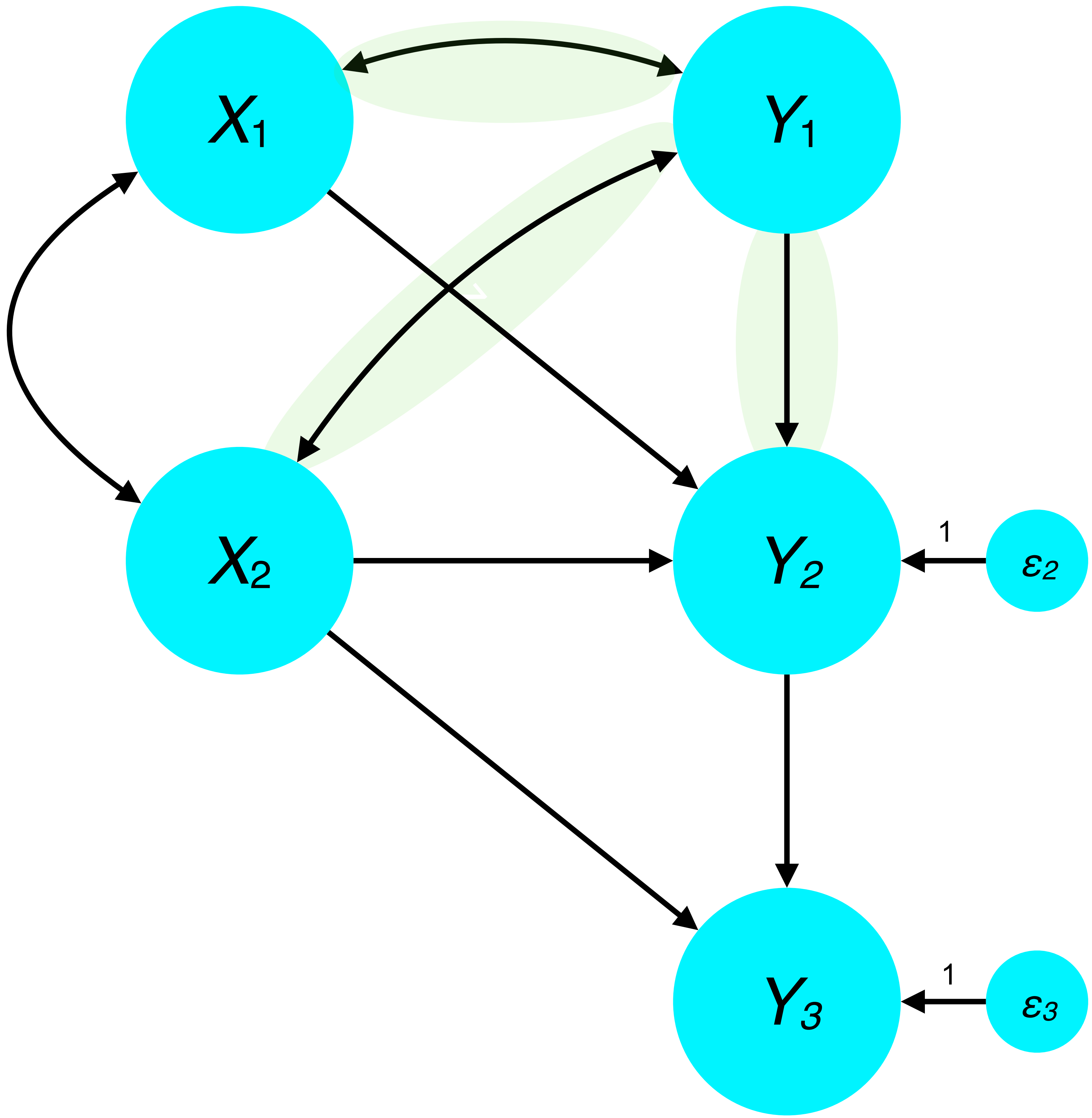
Equivalent Models: C



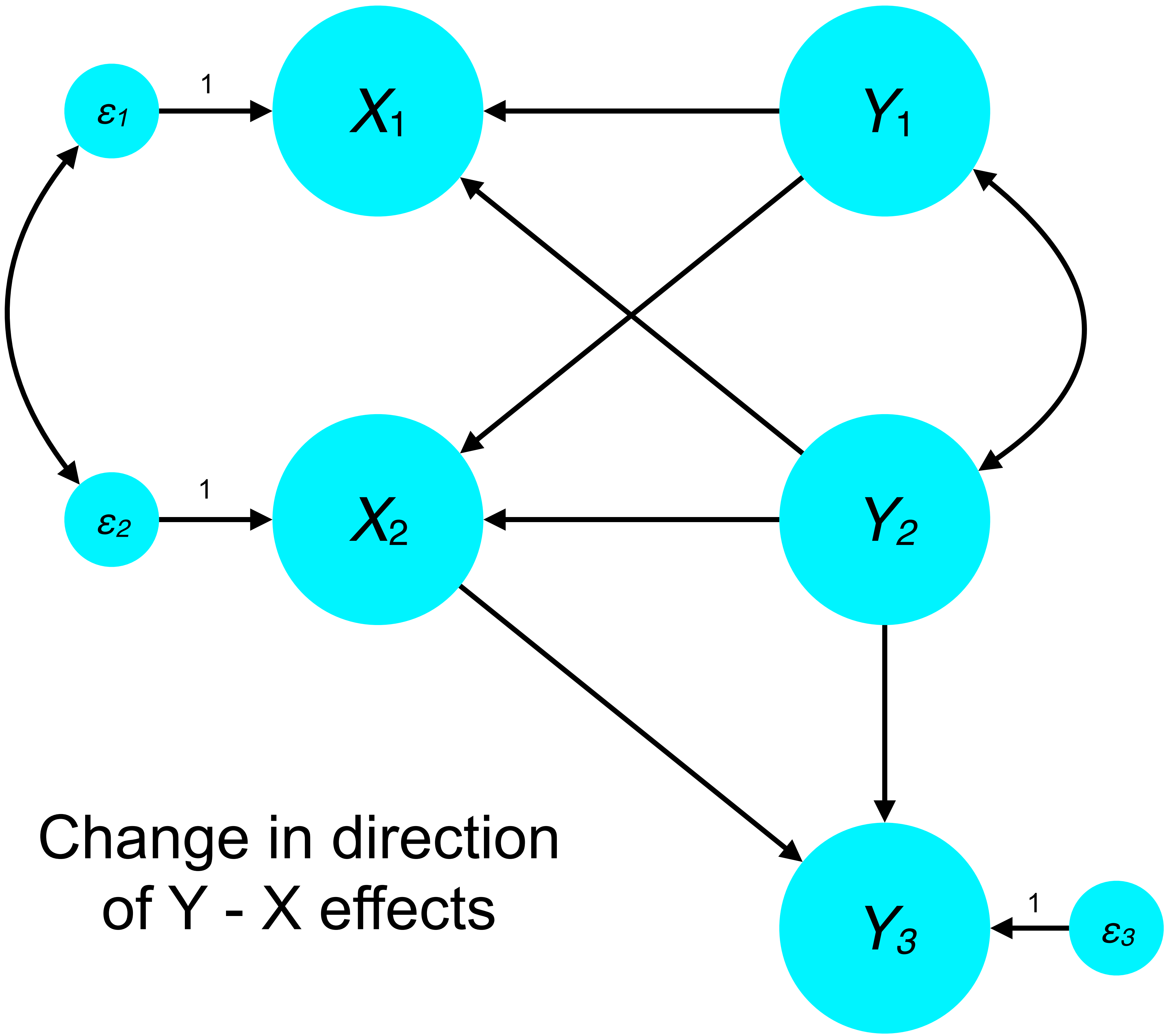
Equivalent Models: D



Equivalent Models: E



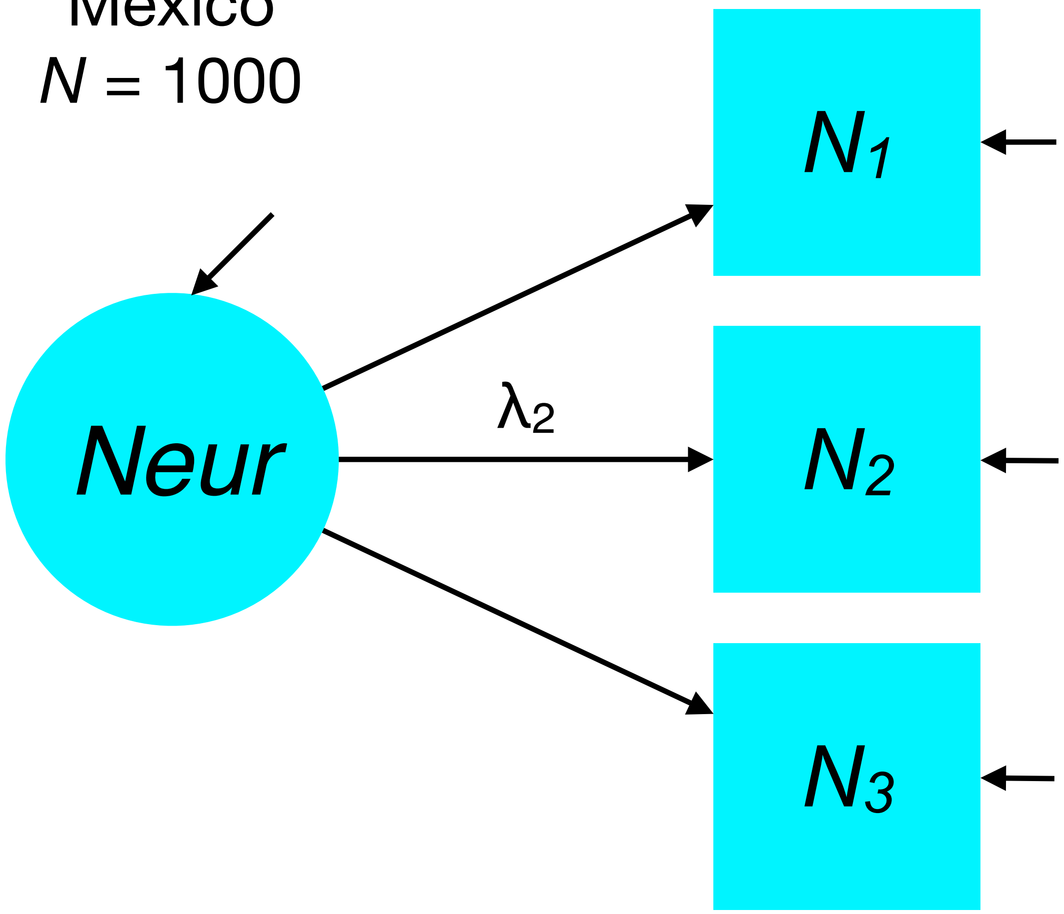
Equivalent Models: F



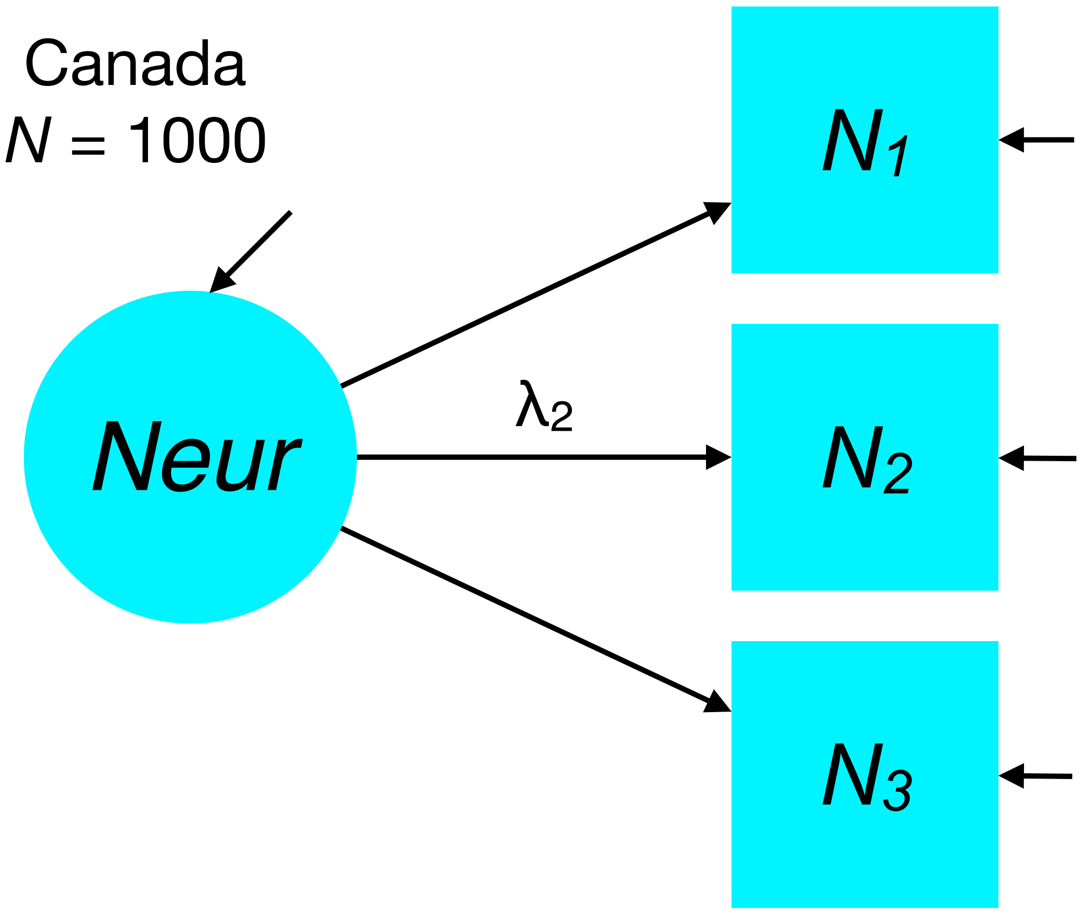
Change in direction
of Y - X effects

Constraints in Factor Models

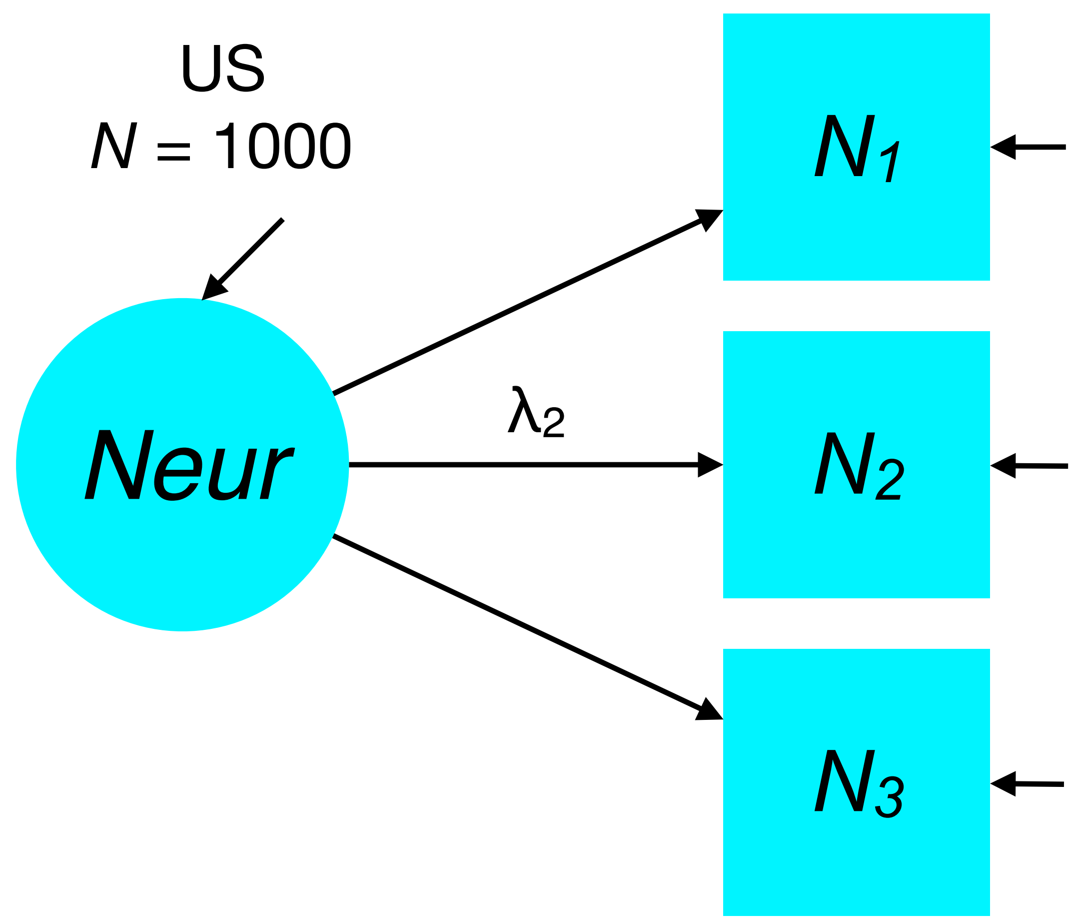
Mexico
N = 1000



Canada
N = 1000

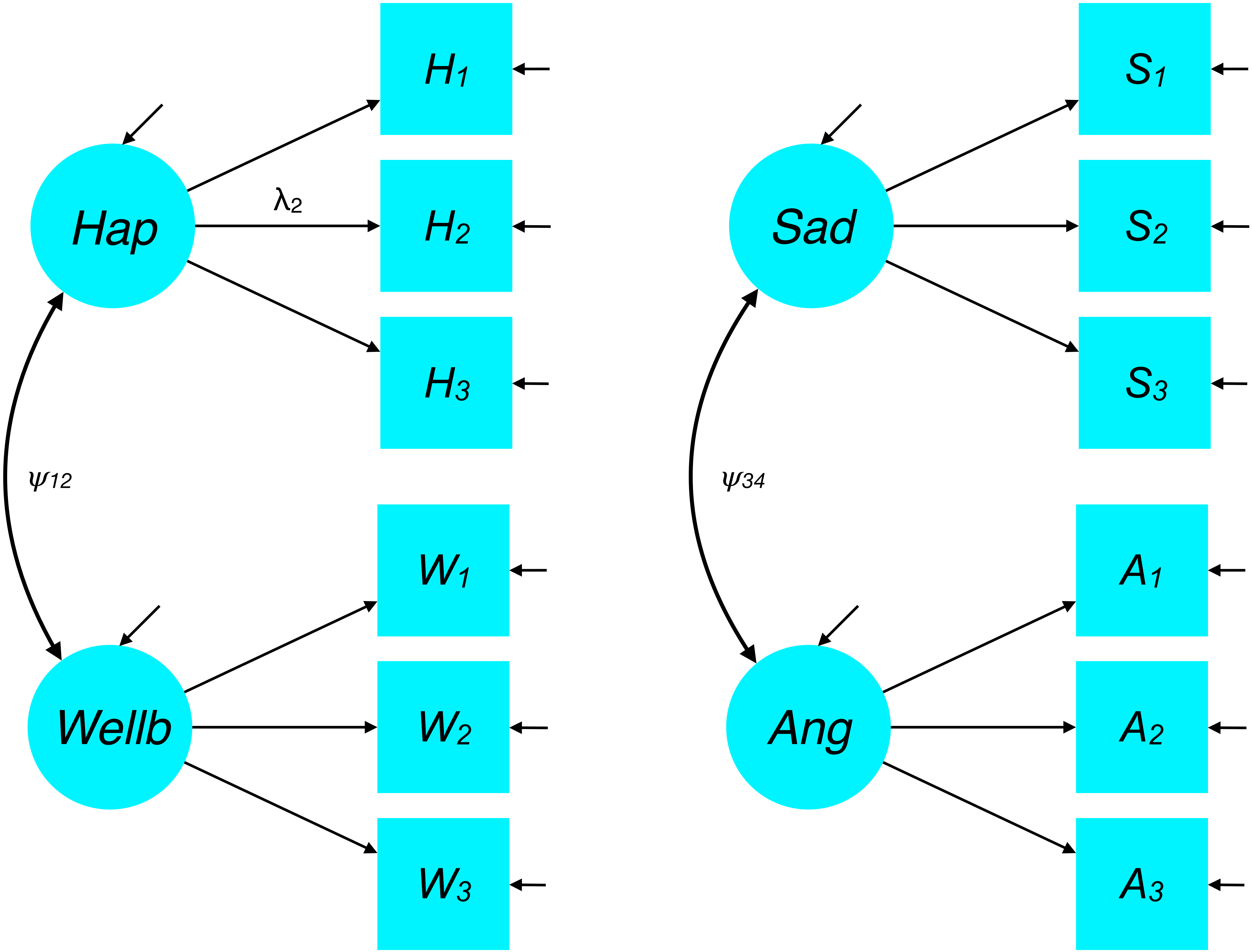


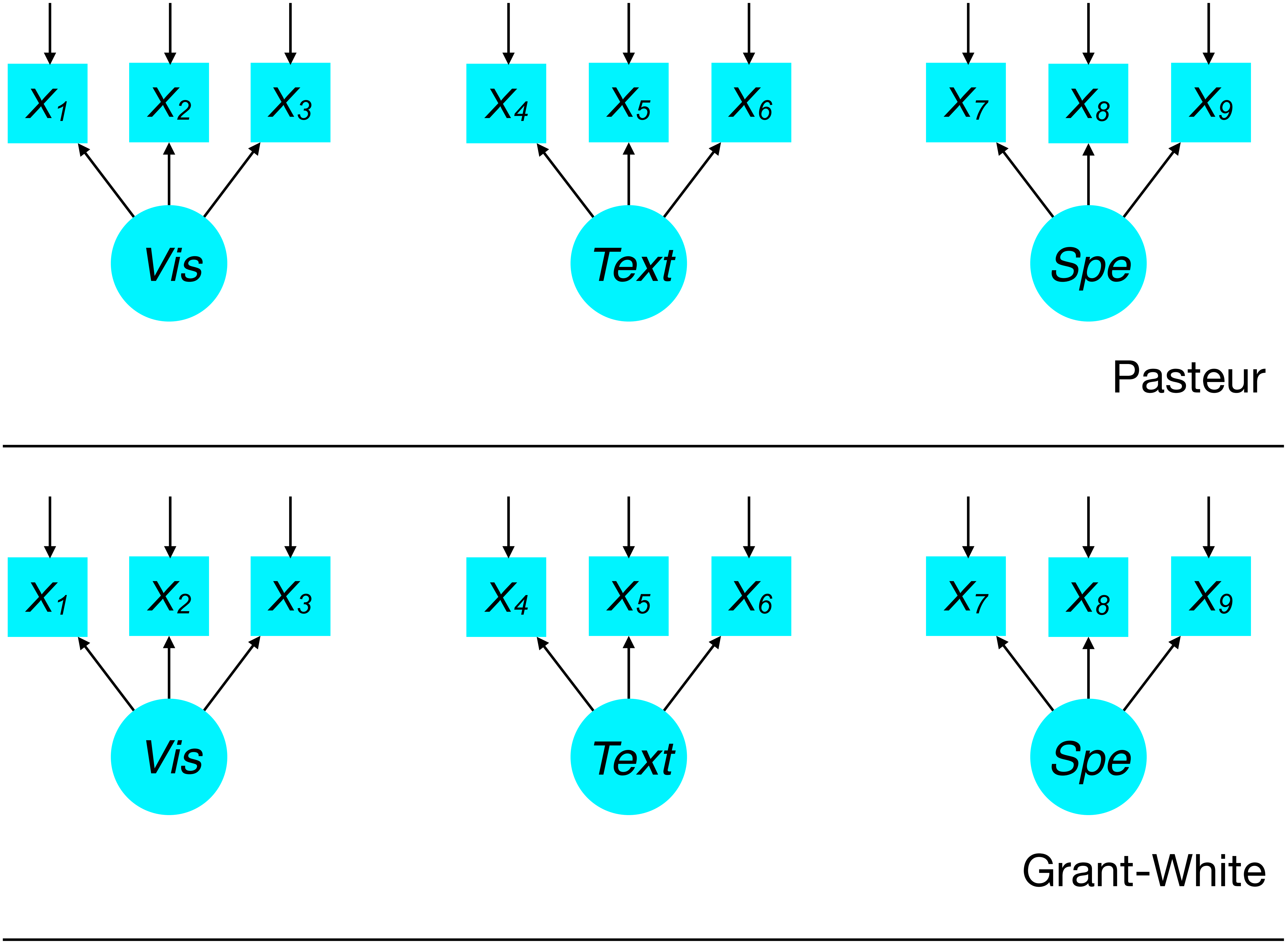
US
N = 1000



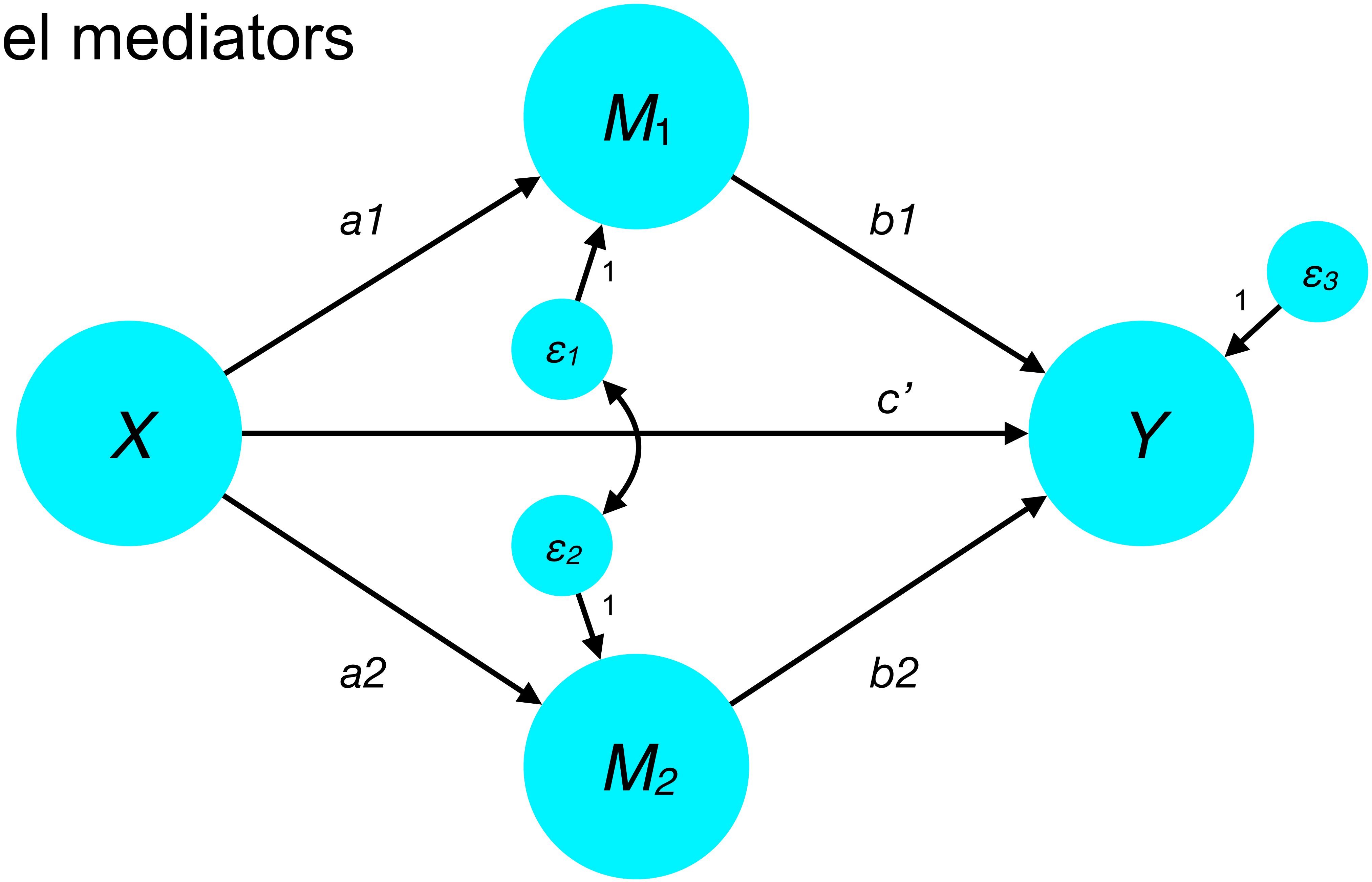
N_2: "I worry a lot" (1-5)

Is the loading of N_2 on the Neuroticism factor similar across countries?


Hap
Wellb
Sad
Ang
H_1
H_2
H_3
W_1
W_2
W_3
S_1
S_2
S_3
A_1
A_2
A_3
λ_2
ψ_{12}
ψ_{34}


X1
X2
X3
X4
X5
X6
X7
X8
X9
Vis
Text
Spe
Pasteur
X1
X2
X3
X4
X5
X6
X7
X8
X9
Vis
Text
Spe
Grant-White

Parallel mediators

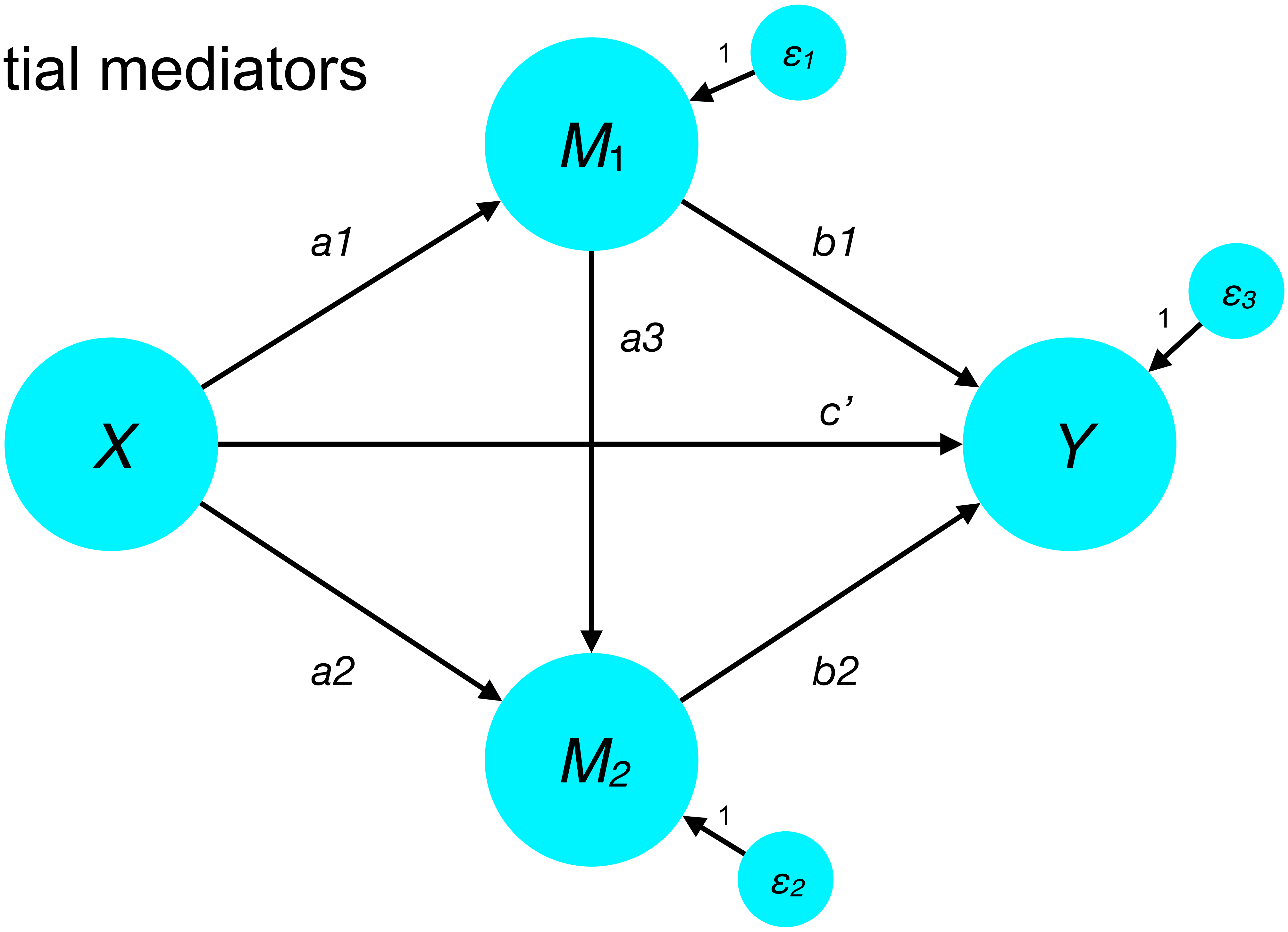


$$\text{Total} = a1b1 + a2b2 + c' = c$$

$$\text{Total indirect} = a1b1 + a1b2$$

$$\text{Direct} = c'$$

Sequential mediators

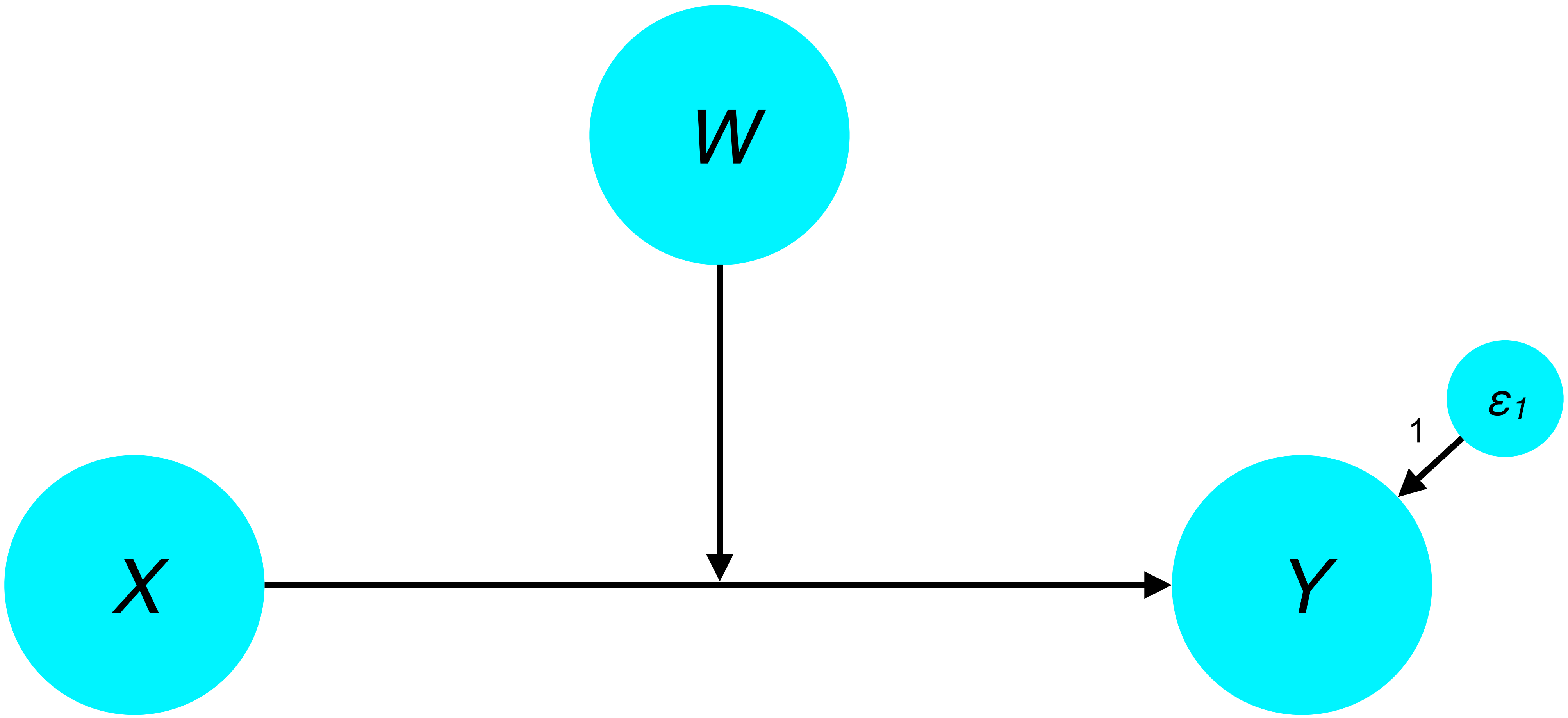


$$\text{Total} = a1b1 + a2b2 + a1a3b2 + c' = c$$

$$\text{Total indirect} = a1b1 + a1b2 + a1a3b2$$

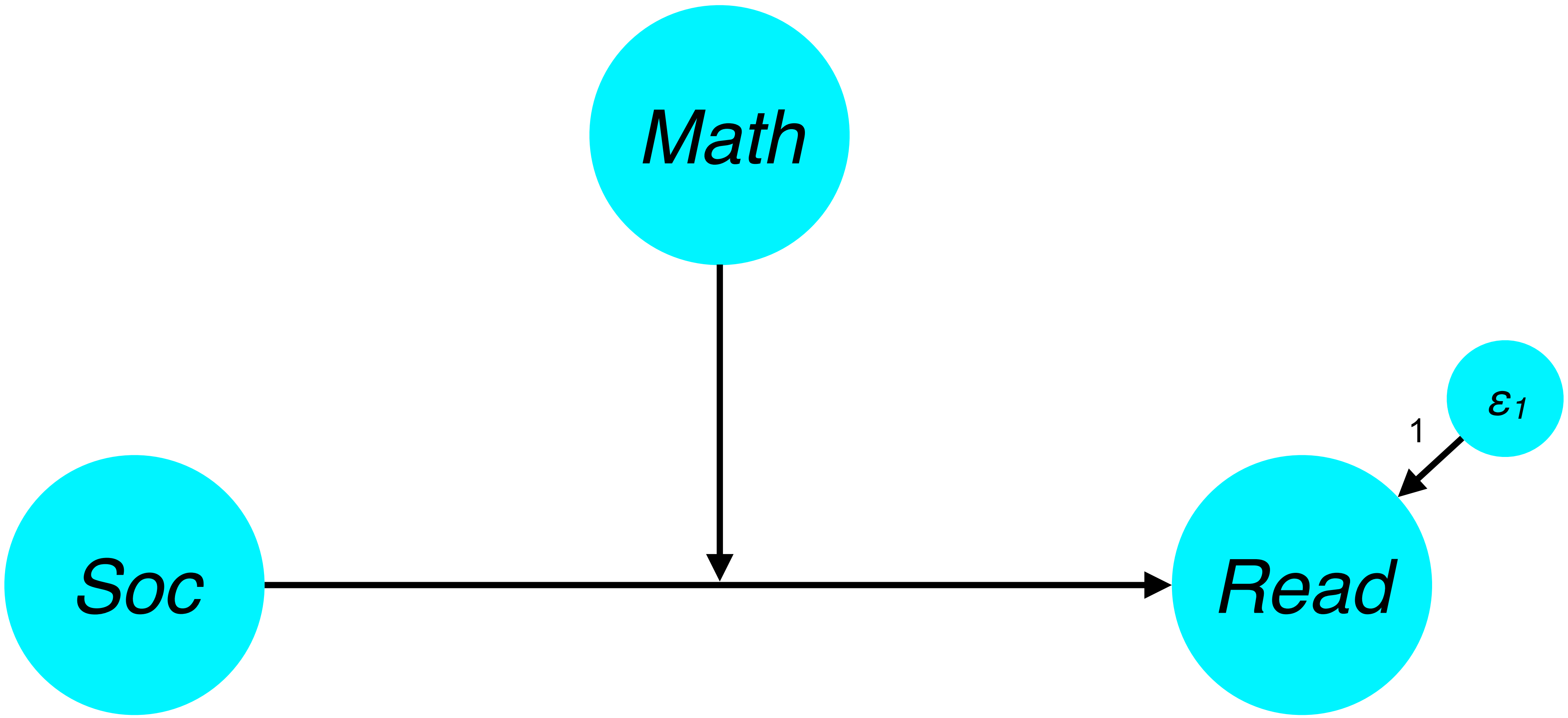
$$\text{Direct} = c'$$

Moderation (conceptual)



The relationship between X and Y depends on W

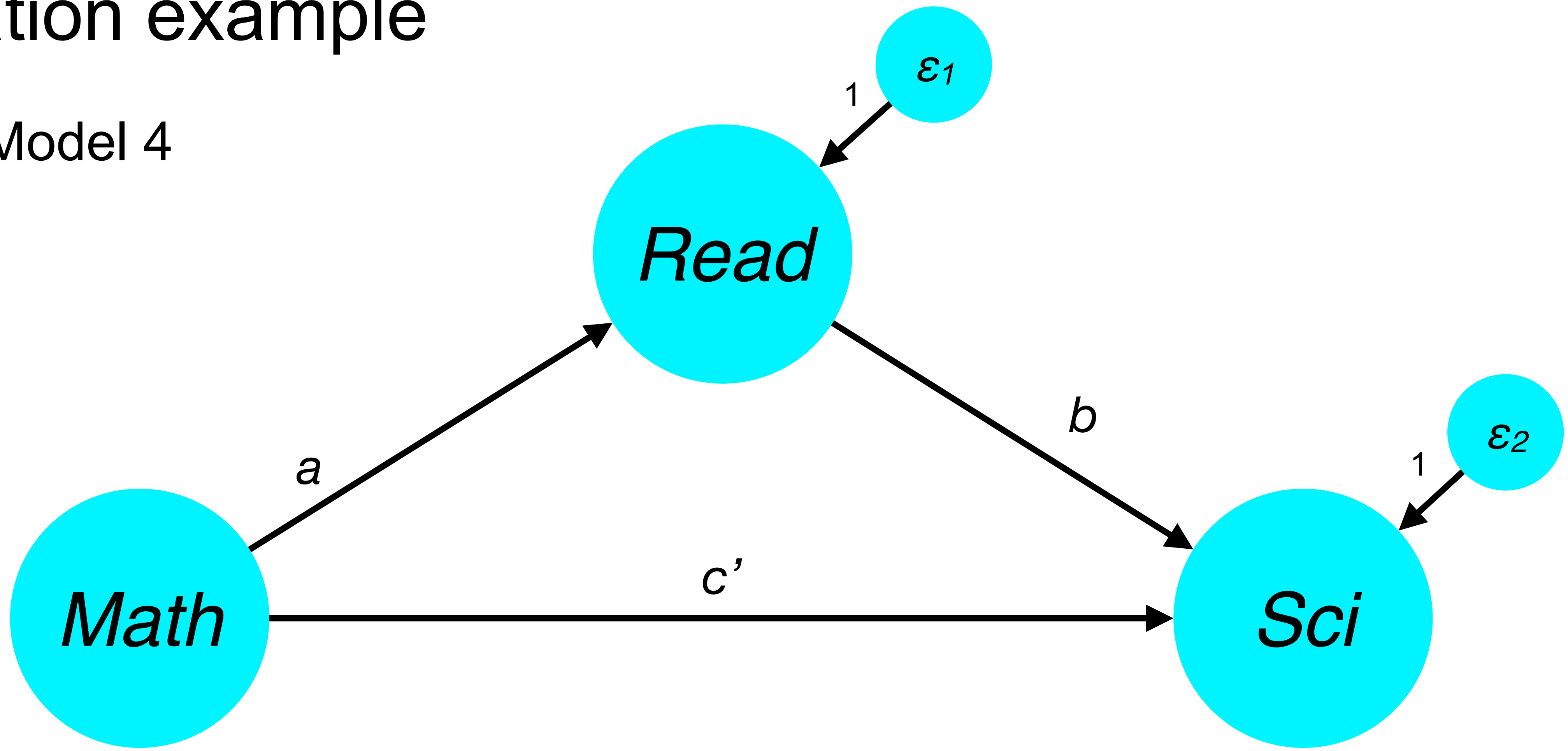
Moderation example



The relationship between social studies (*X*) and reading (*Y*) depends on math (*W*)

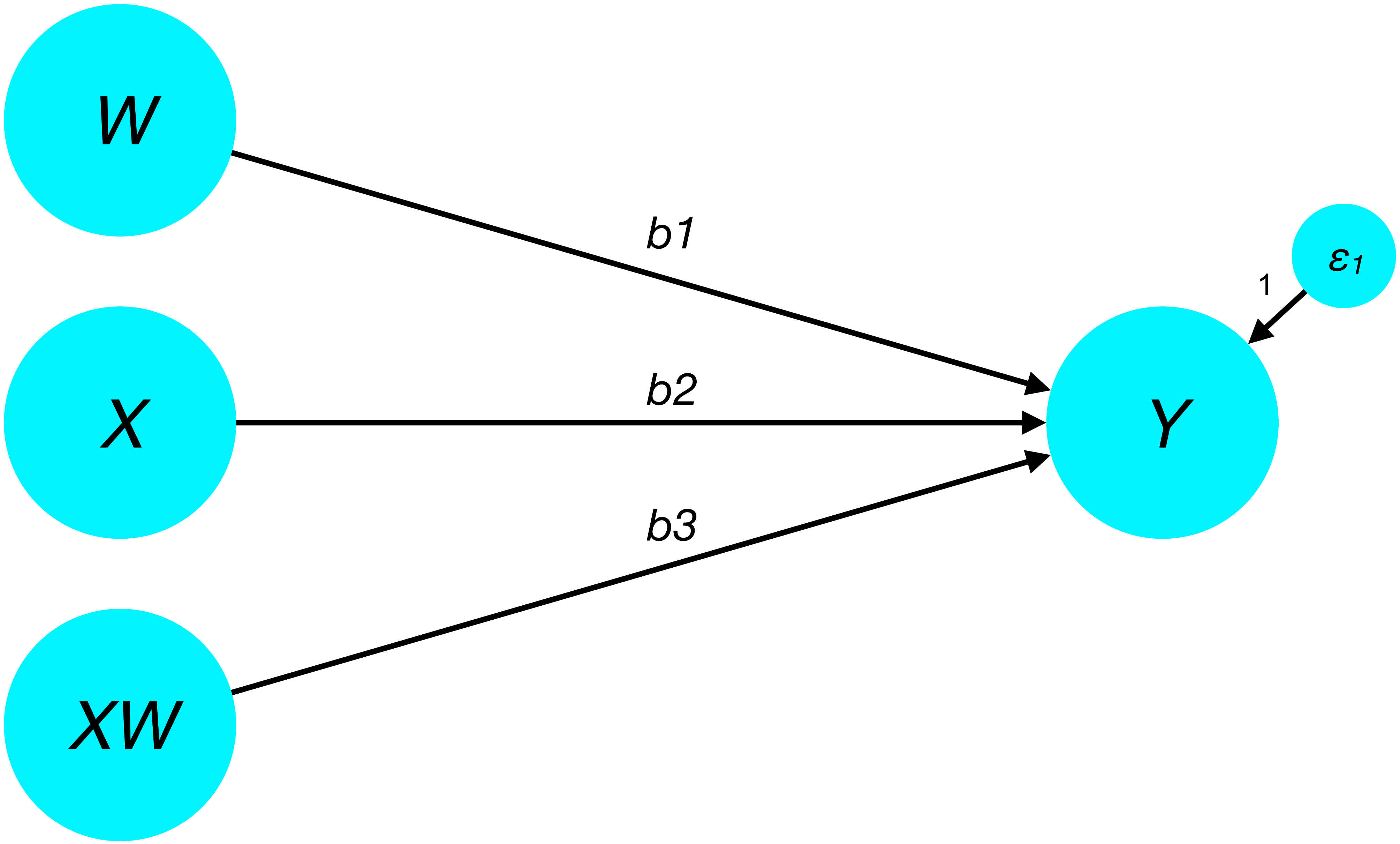
Mediation example

Hayes Model 4



Moderation (statistical)

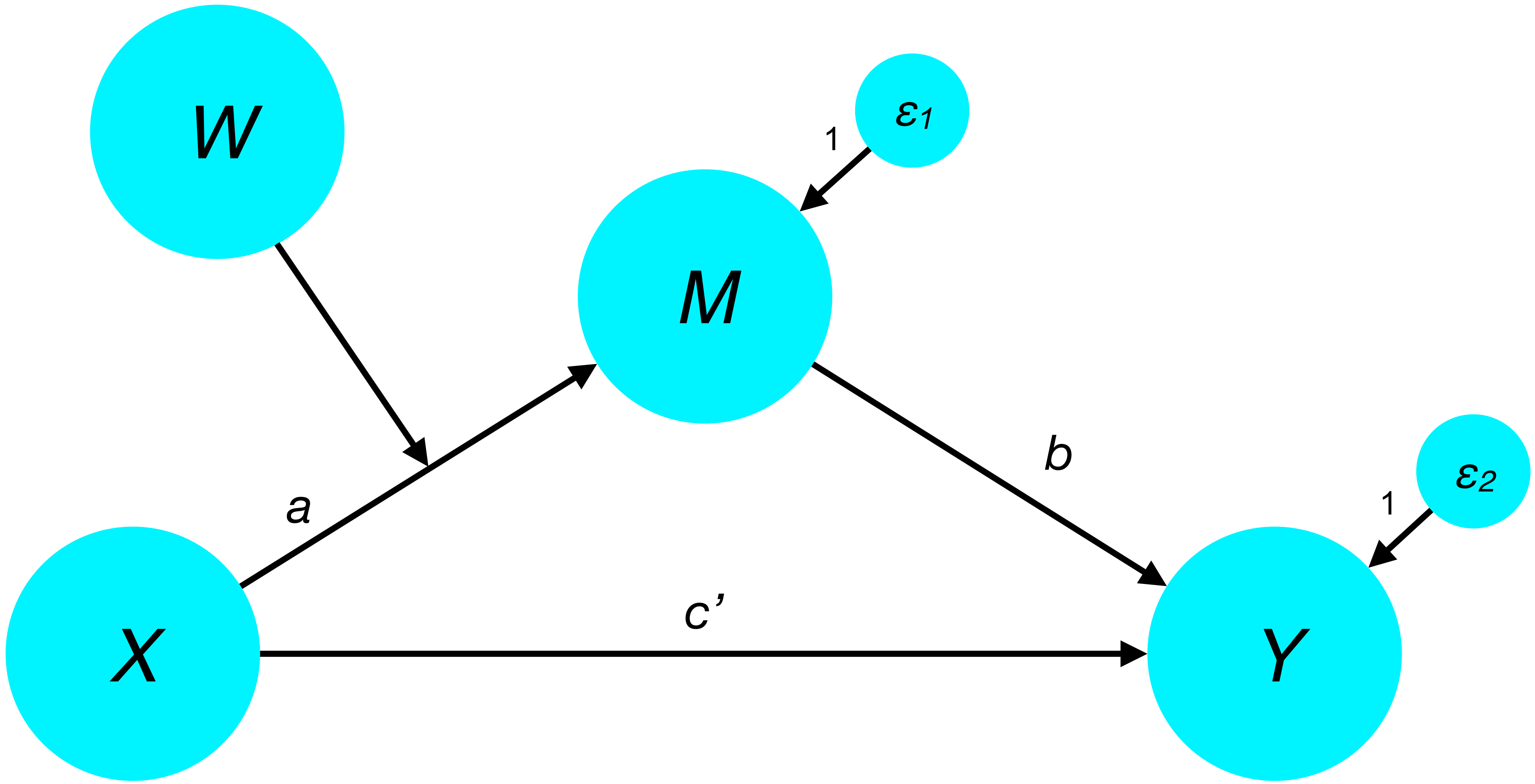
Hayes Model 1



Conditional effect of *X* on *Y* = *b*1 + *b*3 *W*

First-stage moderated mediation (conceptual)

Hayes Model 7

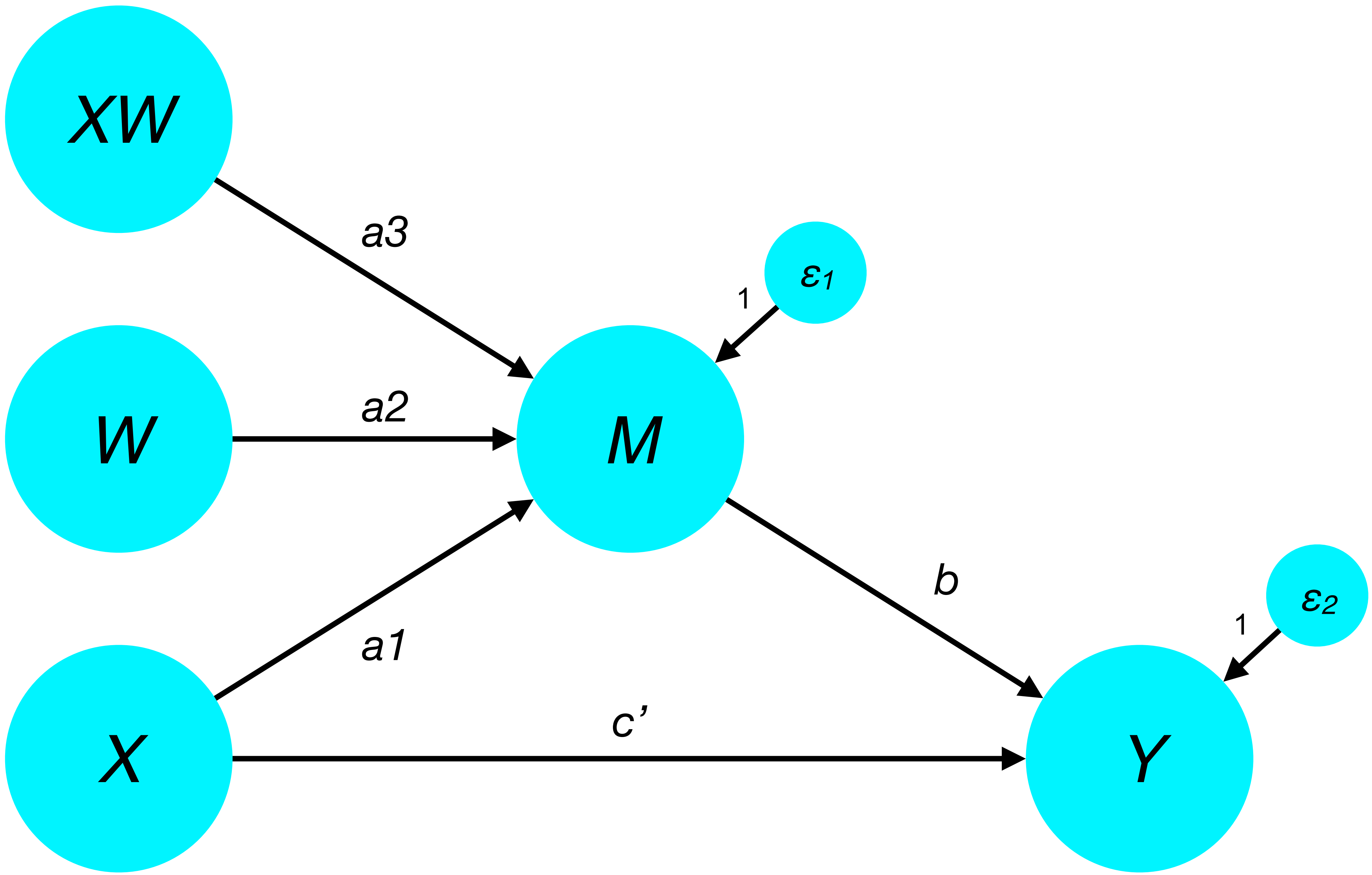


The indirect effect of *X* on *Y* via *M* depends on the value of *W.*

The relationship between *X* and *M* only exists at certain levels of *W*.

First-stage moderated mediation (statistical)

Hayes Model 7



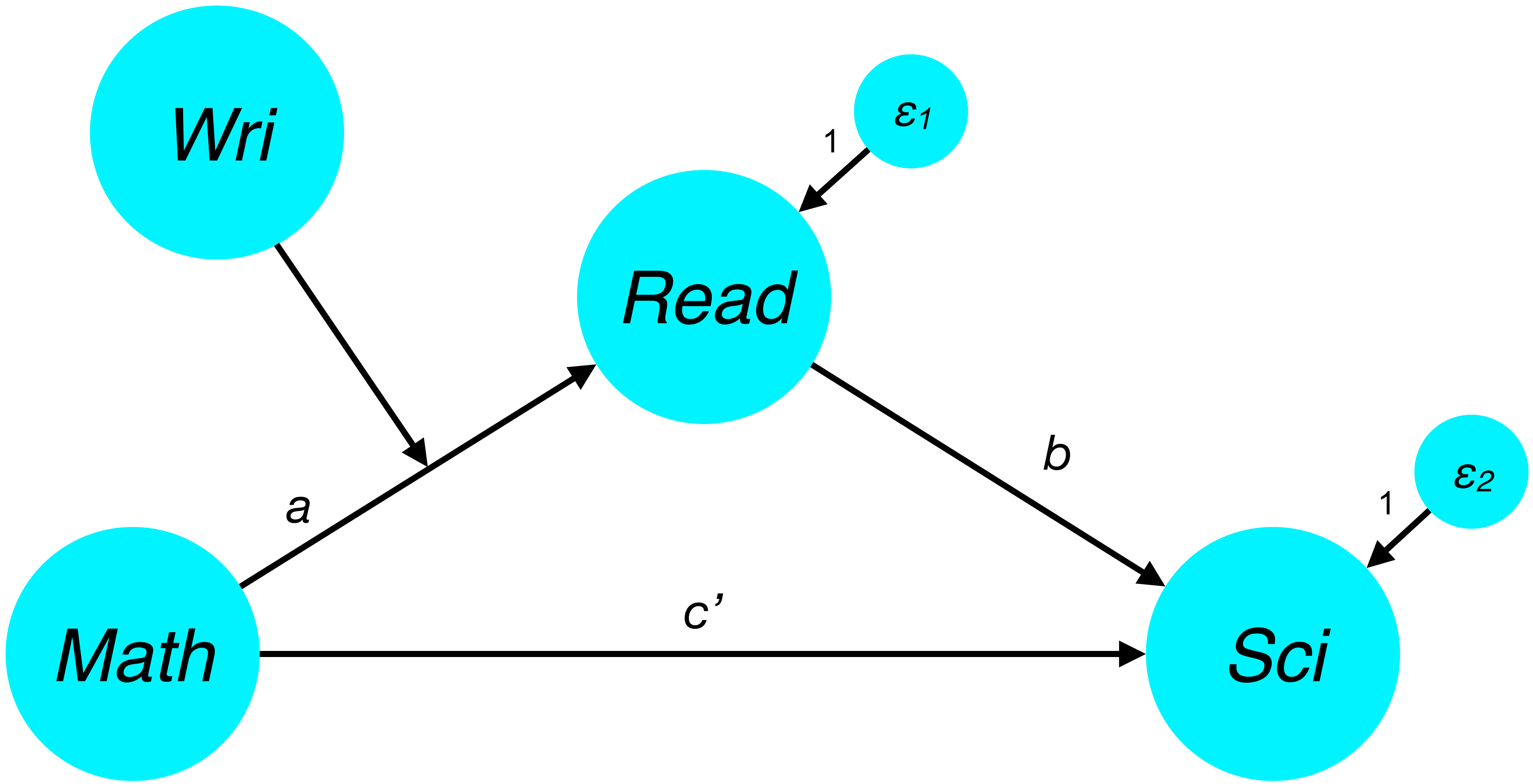
$$M = i_M + a_1 X + a_2 W + a_3 XW + e_M$$

$$Y = i_Y + c' X + bM + e_Y$$

$$\text{imm} = a3b$$

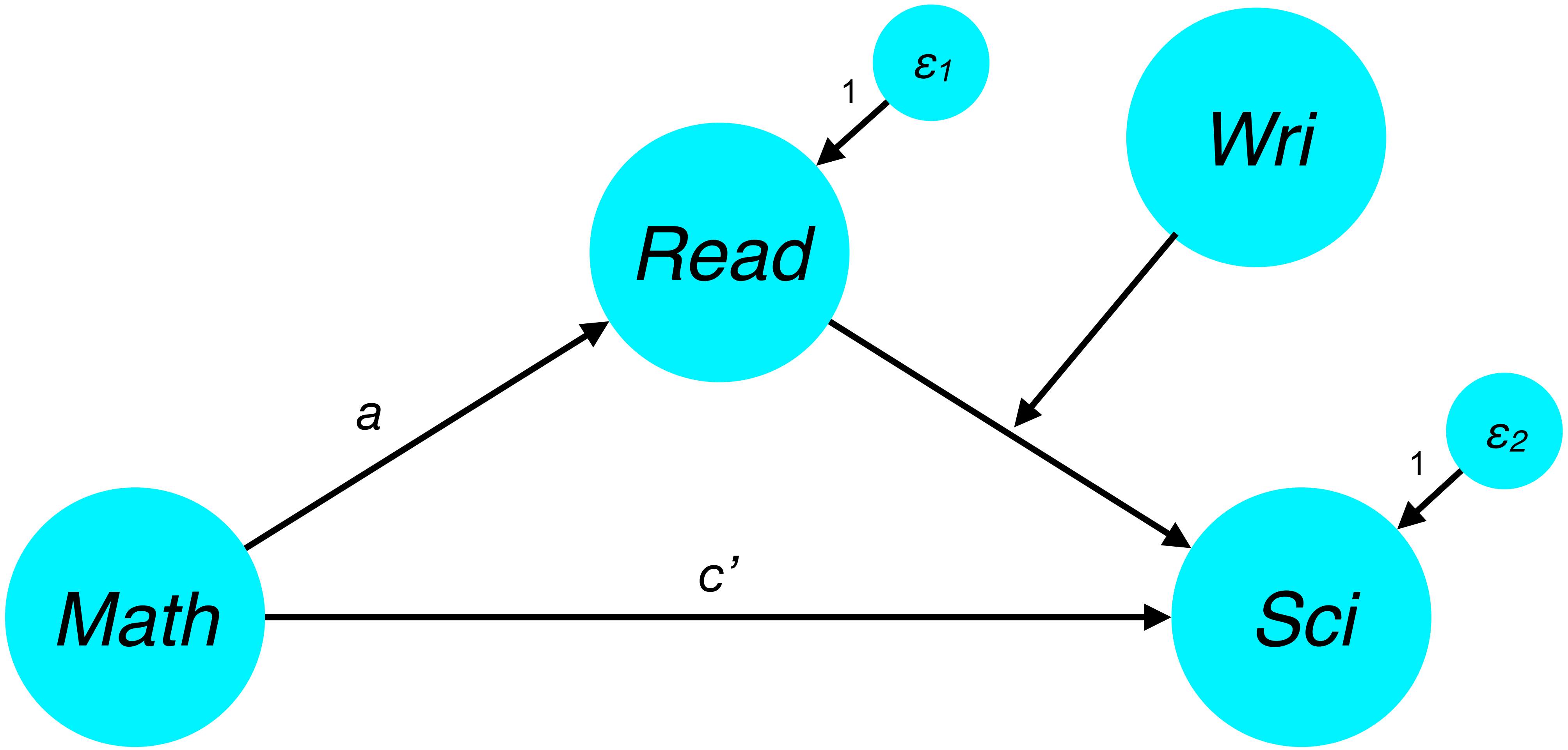
First-stage moderated mediation (example)

Hayes Model 7



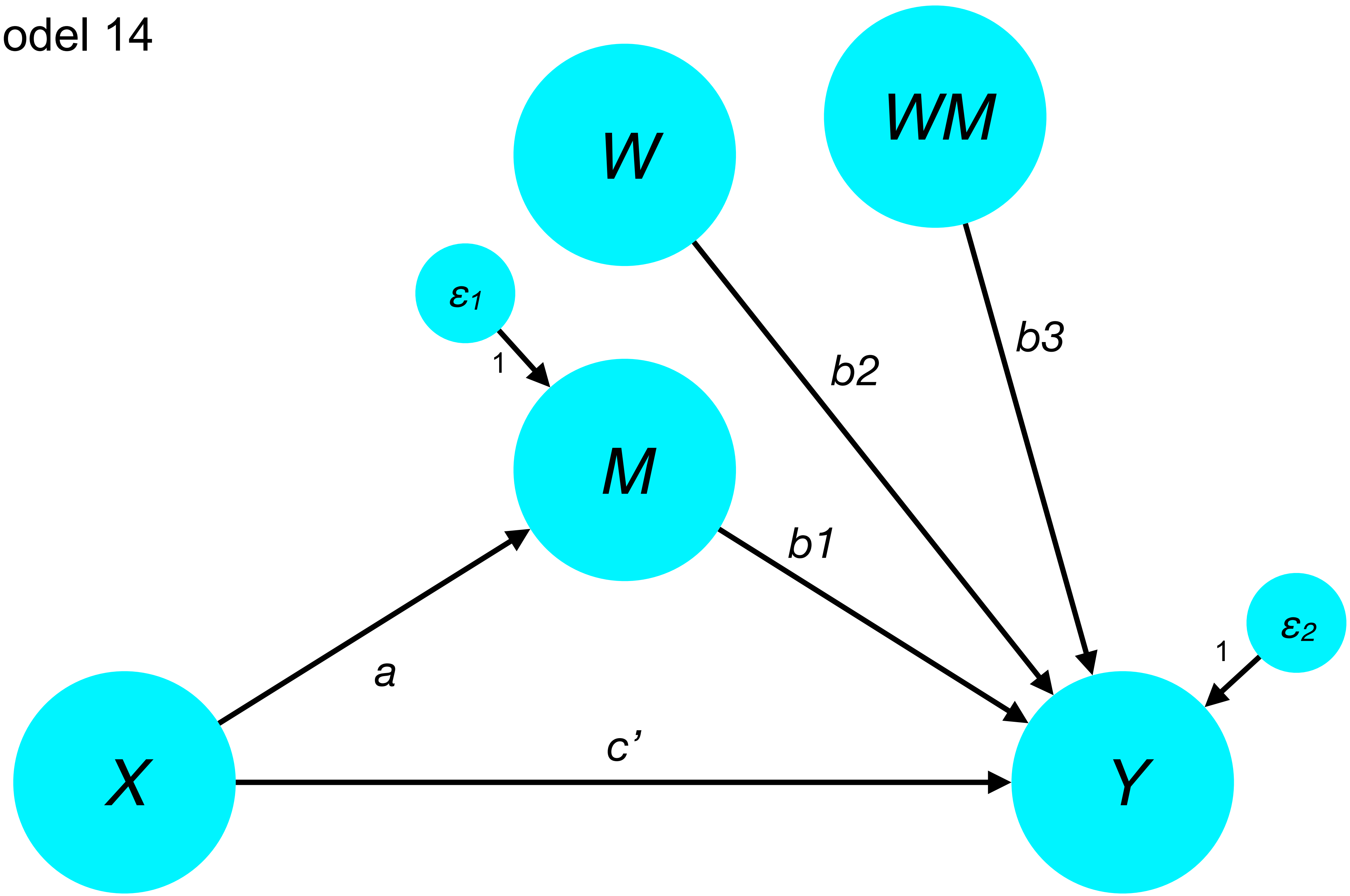
Second-stage moderated mediation (example)

Hayes Model 14



Second-stage moderated mediation (statistical)

Hayes Model 14



$$M = i_M + aX + e_M$$

$$Y = i_Y + c'X + b1M + b2W + b3WM + e_Y$$

$$\text{imm} = ab3$$