

1. Question 1

(a)  $\sum_{i=1}^n (2i - 1)$

$$\begin{aligned} n = 1 & \quad \sum_{i=1}^1 (2i - 1) = 1 \\ n = 2 & \quad \sum_{i=1}^2 (2i - 1) = 1 + 3 = 4 \\ n = 3 & \quad \sum_{i=1}^3 (2i - 1) = 4 + 5 = 9 \\ n = 4 & \quad \sum_{i=1}^4 (2i - 1) = 9 + 7 = 16 \end{aligned}$$

A possible formula for this summation would be  $f(n) = n^2, n \geq 1$

(b) Proof by mathematical induction

$$\begin{aligned} n^2 &= \sum_{i=1}^n (2i - 1) \\ \text{Base case: } 1^2 &= \sum_{i=1}^1 (2i - 1) \\ 1 &= 2 - 1 \end{aligned}$$

Assume:

$$\begin{aligned} n^2 &= \sum_{i=1}^n (2i - 1) \\ (n + 1)^2 &= \sum_{i=1}^{n+1} (2i - 1) \\ \sum_{i=1}^{n+1} (2i - 1) &= \sum_{i=1}^n (2i - 1) + 2(n + 1) - 1 \\ (n + 1)^2 &= n^2 + 2n + 1 \\ n^2 + 2n + 1 &= \sum_{i=1}^n (2i - 1) + 2n + 1 \end{aligned}$$

Subtract  $(2n + 1)$  from each side

$$n^2 = \sum_{i=1}^n (2i - 1)$$

It is assumed that  $n^2 = \sum_{i=1}^n (2i - 1)$  from the inductive step. Therefore, the formula is correct.

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2. Question 2

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3. Question 3

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4. Question 4

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5. Question 5

(a)

(b)

(c)