

1. Question 1

$$\begin{aligned} \text{(a)} \quad f(x) &= -3x^2 + 7 \\ f(-1) &= 4 \\ f(1) &= 4 \end{aligned}$$

Obviously,  $1 \in \mathbb{R} \wedge -1 \in \mathbb{R}$ , so both are within the domain of  $f$ . This goes against the definition of a bijection, since two different elements in the domain of  $f$  have the same image. In order to rectify this, the domain of  $f$  should be  $\{x \in \mathbb{R} : x \geq 0\}$ . The range would also have to be modified to be  $\{y \in \mathbb{R} : y \geq 7\}$ , since this function will never be less than 7.

$$\text{Inverse: } x = -3y^2 + 7$$

$$\frac{7-x}{3} = y^2$$

$$y = \sqrt{\frac{7-x}{3}}$$

$$f^{-1}(x) = \sqrt{\frac{7-x}{3}}$$

This is, of course, given the modified domain and range.

$$\begin{aligned} \text{(b)} \quad f(x) &= \frac{x+2}{x+2} \\ f(-2) &= DNE \end{aligned}$$

In order for  $f$  to be a bijection, every element in its domain must have an image in its range. Since  $f(-2)$  is undefined, the conditions are not satisfied. Here, the domain of  $f$  could be modified to be  $\{x \in \mathbb{R} : x \neq -2\}$ . The range could be modified to be  $\{y \in \mathbb{R} : y \neq 1\}$ , since this function, by definition, can never be equal to 1.

$$\text{Inverse: } x = \frac{y+1}{y+2}$$

$$x * (y + 2) = y + 1$$

$$xy + 2x = y + 1$$

$$2x - 1 = y - xy$$

$$2x - 1 = y(1 - x)$$

$$f^{-1}(x) = \frac{2x-1}{1-x}$$

$$\text{(c)} \quad f(x) = x^5 + 1$$

This function is a bijection, since every element in the domain has exactly one unique image.

$$\text{Inverse: } x = y^5 + 1$$

$$x - 1 = y^5$$

$$f^{-1}(x) = \sqrt[5]{x-1} \quad \{y \in \mathbb{R}\} \quad \{x \in \mathbb{R}\}$$

2. Question 2

$$f(x) = ax + b$$

$$g(x) = cx + d$$

$$\{a, b, c, d \in \mathbb{R}\}$$

$$f \circ g = a(cx + d) + b$$

$$g \circ f = c(ax + b) + d$$

$$a(cx + d) + b = c(ax + b) + d$$

$$acx + ad + b = cax + cb + d$$

$$ad + b = cb + d$$

$$(f \circ g = g \circ f) \leftrightarrow (ad + b = cb + d)$$