

1. Question 1

- (a) Let  $\mathbb{S}$  be the set of odd multiples of 3

Basis step:  $3 \in \mathbb{S}$

Recursive step: If  $x \in \mathbb{S} \wedge y \in \mathbb{S}$

$$x + 2y \in \mathbb{S}$$

$$x - 2y \in \mathbb{S}$$

Because  $x, y \in \mathbb{S}$ , both are known to be odd. It is also known that  $even * odd = even$ , and that  $even + odd = odd$ . Each of the terms is also a multiple of 3, and multiplying that value by an integer does not change that fact. Therefore, an odd multiple of 3  $\pm$  an even multiple of 3 generates a new odd multiple of 3.

- (b) Let  $\mathbb{S}$  be the set of bit strings with an even number of zeros

Note: This question is done assuming that a string of zero length has an even number of zeros.

Basis Step:  $\emptyset \in \mathbb{S}$

Recursive Step:  $x \in \mathbb{S}$

$$1x \in \mathbb{S}$$

$$x1 \in \mathbb{S}$$

$$00x, 0x0, x00 \in \mathbb{S}$$

Adding a 1 to the string will not affect the number of zeros, so the string is still valid. Adding two zeros in any location will then maintain an even number of zeros.

- (c) Let  $\mathbb{S}$  be the set of strings of even length from the alphabet  $\Sigma = \{a, b\}$

Basis Step:  $\emptyset \in \mathbb{S}$

Recursive Step:  $X \in \mathbb{S}$

$$Xaa, Xab, Xba, Xbb \in \mathbb{S}$$

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2. Question 2

- (a) The maximum number of members is the product of the possible choices for each character in the membership number.

Total capital letters: 26

Total number of length 5 bit strings:  $2^5 = 32$

Total number of 2 digit numbers where the second digit is less than the first:

$$(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9) = 45$$

$$26 * 32 * 45 = 37440 \text{ possible unique member numbers}$$

- (b) In this case, the lowest possible number of shared first letters would be  $\frac{1000}{26}$ . This is because all numbers must start with a capital letter. Any one letter not being shared means the rest must be shared by more people. So, the lowest number for any given letter would be in an even distribution, which is represented by  $\frac{1000}{26}$ . This means that there are at least 38 people with the same first letter in a group of 1000.

- (c) The total number of possible numbers that don't start with "J" is

$$25 * 32 * 45 = 36000$$

This answer means that there are more than enough possible numbers for 2000 members that do not begin with "J". So, in a group of 2000, there are at least 0 membership numbers that begin with "J".

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3. Question 3

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4. Question 4

(a)

(b)

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5. Question 5

(a)

(b)

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6. Question 6

(a)

(b)

(c)