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1. Question 1

   (a) $f(x) = -3x^2 + 7$
   $f(-1) = 4$
   $f(1) = 4$

   Obviously, $1 \in \mathbb{R} \wedge -1 \in \mathbb{R}$, so both are within the domain of f. This goes against the definition of a bijection, since two different elements in the domain of f have the same image. In order to rectify this, the domain of f should be $\{x \in \mathbb{R} : x \geq 0\}$. The range would also have to be modified to be $\{y \in \mathbb{R} : y \geq 7\}$, since this function will never be less than 7.

   Inverse: $x = -3y^2 + 7$
   $\frac{7-x}{3} = y^2$
   $y = \sqrt{\frac{7-x}{3}}$
   $f^{-1}(x) = \sqrt{\frac{7-x}{3}}$
   This is, of course, given the modified domain and range.

   (b) $f(x) = \frac{x+1}{x+2}$
   $f(-2) = DNE$

   In order for f to be a bijection, every element in its domain must have an image in its range. Since $f(-2)$ is undefined, the conditions are not satisfied. Here, the domain of f could be modified to be $\{x \in \mathbb{R} : x \neq -2\}$. The range could be modified to be $\{y \in \mathbb{R} : y \neq 1\}$, since this function, by definition, can never be equal to 1.

   Inverse: $x = \frac{y+1}{y+2}$
   $x * (y + 2) = y + 1$
   $xy + 2x = y + 1$
   $2x - 1 = y - xy$
   $2x - 1 = y(1 - x)$
   $f^{-1}(x) = \frac{2x-1}{1-x}$

   (c) $f(x) = x^5 + 1$
   This function is a bijection, since every element in the domain has exactly one unique image.
   Inverse: $x = y^5 + 1$
   $x - 1 = y^5$
   $f^{-1}(x) = \sqrt[5]{x-1}$ $\{y \in \mathbb{R}\}$ $\{x \in \mathbb{R}\}$