

1. Question 1

(a) $\sum_{i=1}^n (2i - 1)$

$$n = 1 \quad \sum_{i=1}^1 (2i - 1) = 1$$

$$n = 2 \quad \sum_{i=1}^2 (2i - 1) = 1 + 3 = 4$$

$$n = 3 \quad \sum_{i=1}^3 (2i - 1) = 4 + 5 = 9$$

$$n = 4 \quad \sum_{i=1}^4 (2i - 1) = 9 + 7 = 16$$

A possible formula for this summation would be $f(n) = n^2, n \geq 1$

(b) Proof by mathematical induction

$$n^2 = \sum_{i=1}^n (2i - 1)$$

Base case: $1^2 = \sum_{i=1}^1 (2i - 1)$

$$1 = 2 - 1$$

Assume:

$$n^2 = \sum_{i=1}^n (2i - 1)$$

$$(n + 1)^2 = \sum_{i=1}^{n+1} (2i - 1)$$

$$\sum_{i=1}^{n+1} (2i - 1) = \sum_{i=1}^n (2i - 1) + 2(n + 1) - 1$$

$$(n + 1)^2 = n^2 + 2n + 1$$

$$n^2 + 2n + 1 = \sum_{i=1}^n (2i - 1) + 2n + 1$$

Subtract $(2n + 1)$ from each side

$$n^2 = \sum_{i=1}^n (2i - 1)$$

It is assumed that $n^2 = \sum_{i=1}^n (2i - 1)$ from the inductive step. Therefore, the formula is correct.

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2. Question 2