Alexander Garcia
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1. Question 1

   (a) When rolling the Zocchihedron the first time, you are guaranteed to get a number. The probability of rolling the same number in any of the next four rolls is $\frac{1}{100}$. The sum of these probabilities is $\frac{4}{100} = \frac{1}{25} = 0.04$

   (b) A fair die will have exactly $\frac{1}{100}$ chance of rolling a given number. Using the same logic as in part (a), we must solve for the number of rolls, rather than the probability.
   $\frac{n}{100} = 0.3$
   $n = 30$ rolls

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2. Question 2

   (a) Probability Distribution: $\sum_{s \in S} p(s) = 1$
   $p(100) = 5p(n), n \in \{1, 2, \cdots, 99\}$
   $p(1) = p(2) = \cdots = p(99)$
   $99 * p(1) + 5 * p(1) = 1$
   $104 * p(1) = 1$

   $p(i) = \frac{1}{104}, i \in \{1, 2, \cdots, 99\}$
   $p(100) = \frac{5}{104}$

   (b) For any one roll, the $p(100) = \frac{5}{104}$. Thus, the probability of rolling a 100 in any n rolls is $\frac{5n}{104}$, since the probability each time compounds, adding another $\frac{5}{104}$

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3. Question 3

   (a) In order for a bit string of length n to contain an equal number of 1's and 0's, it must have $\frac{n}{2}$ of each (1, 0 are treated as heads and tails over n flips). For a bit string of length n there are $C(n, \frac{n}{2})$ strings with equal 1's and 0's.

   There are 2^n total possible outcomes of flipping the coin n times, and $C(n, \frac{n}{2})$ of these will have an even number of heads and tails.

   The probability is then $\frac{\frac{n!}{(n/2)!(n/2)!}}{2^n}$.

   (b) It will not affect the original probability.

   Limiting the first flip to heads limits the number of total possible outcomes to 2^{n-1}.

   Of the remaining flips, there must be exactly one more tail result than heads. This number of strings is $C(n-1, \lceil \frac{n}{2} - 1 \rceil)$, where $\lceil \frac{n}{2} - 1 \rceil = \frac{n-1}{2}$, since n must be odd.

   Probability $= \frac{\frac{(n-1)!}{(\frac{n-1}{2})!(\frac{n-1}{2})!}}{2^{n-1}} * \frac{\frac{n}{n/2}}{2} = \frac{\frac{n!}{(n/2)!(n/2)!}}{2^n}$

   Since the ratios are the same, the probabilities are the same.

   (c) $E(X) = \sum_{t \in S} p(t)X(t)$

   $X(t) = \sum_{i=1}^{n} X_i(t)$

   $E(X) = \sum_{i=1}^{n} (\sum_{j=1}^{i} 0.5 * 2 + \sum_{j=1}^{i} 0.5 * -1)$

   $= \sum_{i=1}^{n} i - \frac{i}{2} = \sum_{i=1}^{n} \frac{i}{2}$

   The expected value is the sum of all the possibilities that the random variable $X(T)$ can take, weighted by their probabilities. There is a 0.5 probability that a flip will come up heads, and the same for tails. Here, n would be the maximum number of flips, and i is the number of flips for the current trial.

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4. Question 4

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5. Question 5

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6. Question 6

   (a)

   (b)