Alexander Garcia
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1. Question 1

   (a) $S \to 00S1$
   $S \to 1S00$
   The first two are clear rules of the language. When adding a single 1, you must also add two 0's

   $S \to SS$
   Concatenating two existing strings in the grammar will also produce a vaild string. Both have the correct relationship between 1's and 0's, so the relationship does not change when they are combined

   $S \to 0S1S0$
   The forth rule allows the concatenation of two existing strings, and adding two 0's and a 1, allowing for "random" placement of 1's and 0's.

   $S \to \lambda$
   The fifth rule allows for the termination of a derivation

   (b)

$S \to 00S1$	Rule 1
$00S1 \to 00[1S00]1$	Rule 2
$00[1S00]1 \to 00[1[0S1S0]00]1$	Rule 4
$00[1[0S1S0]00]1 \to 001010001$	Rule 5 for both S

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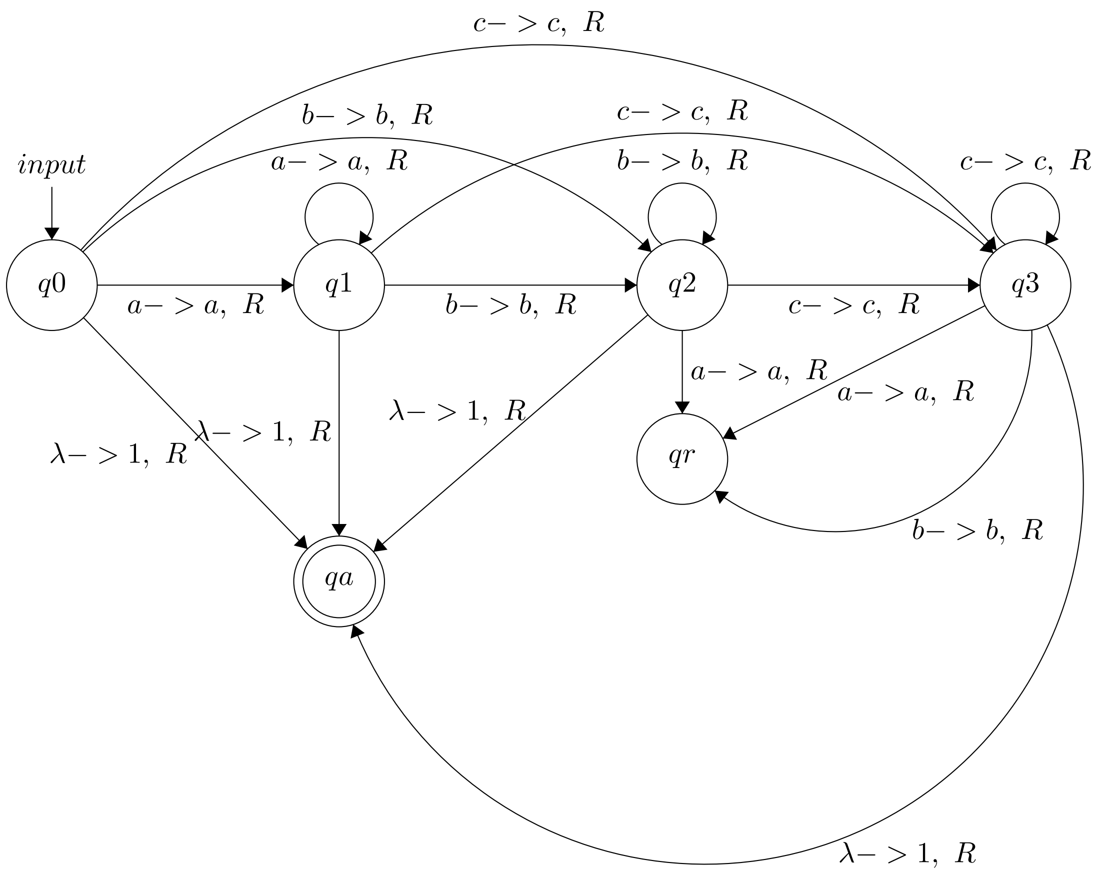
2. Question 2



The diagram of this machine is quite complicated so this is a summary of its functionality.

The string starts accepting input. Any character in the alphabet, including λ, is sent to a specific state. If a letter that comes after the first letter is seen, the machine immediately goes to the reject state. If the same letter is seen, it stays in the previous state. If a "higher" letter is seen, it goes to that letter's state.

As long as the sequence is lexographic, the machine will stay in one of the three letters' states during the whole input, not modifying the input in any way. When a blank space is seen, if the machine is in one of the three states, it will write a 1, and enter the accept state.

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3. Question 3

   (a) The main idea behind this machine is to erase pairs of 1's and 0's.

   Start cycling through the tape. If you see a 1, remove it, then continue cycling until you reach a 0 or the end. If you reach a 0, remove it as well, and start searching for another 1, repeating the process.

   If you reach the end of the input string, start over at the beginning and repeat the process of removing one 1, skipping to the next 0, then removing that 0. If you are able to make it to the end of the tape without removing any digits, you have reached a conclusion. If there are only 0's left, the machine enters the accept state, as there were more 0's than 1's. If there are no digits left, there were exactly as many 0's as there were 1's, and the machine enters the reject state, as it only recognizes words with more 0's than 1's. If there are 1's remaining, it enters the reject state also.

   (b) The runtime of this machine is as follows.
   $r = n + m$
   m = number of 1's, n = number of 0's, r = length of the string
   The worst case runtime would be if there are equal numbers of 1's and 0's, or $m = n = \frac{r}{2}$.
   Because of the way this machine operates, the worst case order of digits would be $\frac{r}{2}$1's followed by $\frac{r}{2}$0's, as the machine would then only remove 2 items per pass.
   So, the machine would read r digits the first pass, $r - 2$ digits the second pass, $r - 4$ the third, and so on.
   This summation is $\sum_{i=1}^{r/2} r - 2j = \frac{1}{4}(r-2) * r$
   In terms of order notation, this makes the algorithm $O(r^2)$

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4. Question 4

   (a) $L = \{w\#w^R\#w | w \in \{a,b\}^+\}$

   Assuming the input string w is on a single tape, and there is one empty string for reading/writing

   (1) Jump across the tape, going from the first position on the input tape to the first position after the 2nd #
   (2) Repeat this process until the left side reaches a break (#)
   (3) If the two sides differ at any point, reject the input, as the string does not appear on either side of two #s
   (4) If each side had the same symbol up to that point, start working backwards from the # to the start of the input tape, writing the character to the head of the second tape
   (5) Iterate through the input tape again, starting this time from the first position after the first #. Compare these inputs to those written to the secondary tape until the next # is reached. If they differ at any point, reject the input. If they are identical, accept the input

   (b) Worst case runtime for this machine

   The machine searches for a pattern of length w. For each string it tests (input up to the first #), it must test another string of the same length, checking the string will in fact repeat. It must then read backwards through w positions while it writes to the second tape. It then compares w characters from the reversed string to the middle segment.

   One cycle = (beginning string + repeated string) + backwards read/write + middle compare

   $O(w) = 4w$

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5. Question 5

   In order for $\mathbb{N} \times \mathbb{N}$ to be countable, every pair of numbers $(\mathbb{N}, \mathbb{N})$ must map to a unique element of $\mathbb{N}$

   $\mathbb{N}$ is countable by definition, since a set is countable if every element in the set can be mapped to $\mathbb{N}$

   Take a number $k = 2^x * 3^y$ where $k, x, y \in \mathbb{N}$

   According to the fundamental theorem of arithmetic, this is the only prime factorization of k, and therefore the only possible values of x and y

   We treat this as the function $f(x, y) = 2^x * 3^y$

   x, y are each in the domain $\mathbb{N}$, giving the function $f(x, y)$ the domain of $\mathbb{N}^2$

   The right side of the function maps each combination of (x, y) to a single value of $\mathbb{N}$

   The function maps $\mathbb{N}^2 \to \mathbb{N}$

   $$\therefore \mathbb{N} \times \mathbb{N} \text{ is countable}$$