Alexander Garcia
21 April 2017

1. Question 1

    (a) When rolling the Zocchihedron the first time, you are guaranteed to get a number. The probability of rolling the same number in any of the next four rolls is $\frac{1}{100}$. The sum of these probabilities is $\frac{4}{100} = \frac{1}{25} = 0.04$

    (b) A fair die will have exactly $\frac{1}{100}$ chance of rolling a given number. Using the same logic as in part (a), we must solve for the number of rolls, rather than the probability.
    $\frac{n}{100} = 0.3$
    $n = 30$ rolls

Alexander Garcia

21 April 2017

2. Question 2

    (a) Probability Distribution: $\sum_{s \in S} p(s) = 1$
    $p(100) = 5p(n), n \in \{1, 2, \cdots, 99\}$
    $p(1) = p(2) = \cdots = p(99)$
    $99 * p(1) + 5 * p(1) = 1$
    $104 * p(1) = 1$

    $p(i) = \frac{1}{104}, i \in \{1, 2, \cdots, 99\}$
    $p(100) = \frac{5}{104}$

    (b) For any one roll, the $p(100) = \frac{5}{104}$. Thus, the probability of rolling a 100 in any n rolls is $\frac{5n}{104}$, since the probability each time compounds, adding another $\frac{5}{104}$

Alexander Garcia

21 April 2017

3. Question 3

   (a) In order for a bit string of length n to contain an equal number of 1's and 0's, it must have $\frac{n}{2}$ of each (1, 0 are treated as heads and tails over n flips). For a bit string of length n there are $C(n, \frac{n}{2})$ strings with equal 1's and 0's.

   There are 2^n total possible outcomes of flipping the coin n times, and $C(n, \frac{n}{2})$ of these will have an even number of heads and tails.

   The probability is then $\frac{\frac{n!}{(n/2)!(n/2)!}}{2^n}$.

   (b) It will not affect the original probability.

   Limiting the first flip to heads limits the number of total possible outcomes to 2^{n-1}.

   Of the remaining flips, there must be exactly one more tail result than heads. This number of strings is $C(n-1, \lceil \frac{n}{2} - 1 \rceil)$, where $\lceil \frac{n}{2} - 1 \rceil = \frac{n}{2}$, since n must be odd.

   Probability $= \frac{\frac{(n-1)!}{(\frac{n-1}{2})!(\frac{n-1}{2})!}}{2^{n-1}} = \frac{\frac{n}{n/2}}{2} * \frac{\frac{n!}{(n/2)!(n/2)!}}{2^n}$

   Since the ratios are the same, the probabilities are the same.

   (c)

Alexander Garcia

21 April 2017

4. Question 4

Alexander Garcia

21 April 2017

5. Question 5

Alexander Garcia

21 April 2017

6. Question 6

    (a)

    (b)