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1. Question 1

   (a) Let $\mathbb{S}$ be the set of odd multiples of 3
   Basis step: $3 \in \mathbb{S}$
   Recursive step: If $x \in \mathbb{S} \land y \in \mathbb{S}$
   $x + 2y \in \mathbb{S}$
   $x - 2y \in \mathbb{S}$
   Because $x, y \in \mathbb{S}$, both are known to be odd. It is also known that $even * odd = even$, and that $even + odd = odd$. Each of the terms is also a multiple of 3, and multiplying that value by an integer does not change that fact. Therefore, an odd multiple of 3 $\pm$ an even multiple of 3 generates a new odd multiple of 3.

   (b) Let $\mathbb{S}$ be the set of bit strings with an even number of zeros
   Note: This question is done assuming that a string of zero length has an even number of zeros.
   Basis Step: $\emptyset \in \mathbb{S}$
   Recursive Step: $x \in \mathbb{S}$
   $1x \in \mathbb{S}$
   $x1 \in \mathbb{S}$
   $00x, 0x0, x00 \in \mathbb{S}$
   Adding a 1 to the string will not affect the number of zeros, so the string is still valid. Adding two zeros in any location will then maintain an even number of zeros.

   (c) Let $\mathbb{S}$ be the set of strings of even length from the alphabet $\Sigma = \{a, b\}$
   Basis Step: $\emptyset \in \mathbb{S}$
   Recursive Step: $X \in \mathbb{S}$
   $Xaa, Xab, Xba, Xbb \in \mathbb{S}$

2. Question 2

   (a)

   (b)

   (c)

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3. Question 3

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4. Question 4

   (a)

   (b)

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5. Question 5

   (a)

   (b)

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6. Question 6

   (a)

   (b)

   (c)