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1. Question 1

   (a) Let $\mathbb{S}$ be the set of odd multiples of 3
   Basis step: $3 \in \mathbb{S}$
   Recursive step: If $x \in \mathbb{S} \wedge y \in \mathbb{S}$
   $x + 2y \in \mathbb{S}$
   $x - 2y \in \mathbb{S}$
   Because $x, y \in \mathbb{S}$, both are known to be odd. It is also known that $even * odd = even$, and that $even + odd = odd$. Each of the terms is also a multiple of 3, and multiplying that value by an integer does not change that fact. Therefore, an odd multiple of 3 $\pm$ an even multiple of 3 generates a new odd multiple of 3.

   (b) Let $\mathbb{S}$ be the set of bit strings with an even number of zeros
   Note: This question is done assuming that a string of zero length has an even number of zeros.
   Basis Step: $\emptyset \in \mathbb{S}$
   Recursive Step: $x \in \mathbb{S}$
   $1x \in \mathbb{S}$
   $x1 \in \mathbb{S}$
   $00x, 0x0, x00 \in \mathbb{S}$
   Adding a 1 to the string will not affect the number of zeros, so the string is still valid. Adding two zeros in any location will then maintain an even number of zeros.

   (c) Let $\mathbb{S}$ be the set of strings of even length from the alphabet $\Sigma = \{a, b\}$
   Basis Step: $\emptyset \in \mathbb{S}$
   Recursive Step: $X \in \mathbb{S}$
   $Xaa, Xab, Xba, Xbb \in \mathbb{S}$

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2. Question 2

   (a) The maximum number of members is the product of the possible choices for each character in the membership number.

   Total capital letters: 26

   Total number of length 5 bit strings: $2^5 = 32$

   Total number of 2 digit numers where the second digit is less than the first:

   $(1+2+3+4+5+6+7+8+9) = 45$

   $26 * 32 * 45 = 37440$ possible unique member numbers

   (b) In this case, the lowest possible number of shared first letters would be $\frac{1000}{26}$. This is because all numbers must start with a capital letter. Any one letter not being shared means the rest must be shared by more people. So, the lowest number for any given letter would be in an even distribution, which is represented by $\frac{1000}{26}$. This means that there are at least 38 people with the same first letter in a group of 1000.

   (c) The total number of possible numbers that don't start with "J" is

   $25 * 32 * 45 = 36000$

   This answer means that there are more than enough possible numbers for 2000 members that do not begin with "J". So, in a group of 2000, there are at least 0 membership numbers that begin wtih "J".

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3. Question 3

   Take an arbitrary vertex V_1

   Because the graph is complete, V_1 shares an edge (i.e. has some relationship with) all of the eight other vertices.

   It is given that in any complete, two color, 9 vertex graph, there is at least one node incident to 6 red edges, or 4 blue edges. We will then assume that this is the case for V_1

   We then consider the set of 6 friends of V_1, from the previous statement. This "sub graph" can be treated as its own 6 vertex party, which had been covered previously.

   For a 6 vertex graph, it is known that there exists a 3-clique of either friends or enemies.

   If there is a 3 clique of friends, and all members of the clique are also friends with V_1, then we have found a 4-clique of friends (red edges)

   If there is a 3 clique of enemies, then we have found the 3-clique of enemies we were looking for in the beginning.

   $\therefore$ Any complete 2-colored graph with 9 vertices contains either a red 4-clique or a blue 3-clique

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4. Question 4

   (a)

   (b)

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5. Question 5

    (a)

    (b)

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6. Question 6

   (a)

   (b)

   (c)