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- (a) When rolling the Zocchihedron the first time, you are guaranteed to get a number. The probability of rolling the same number in any of the next four rolls is $\frac{1}{100}$. The sum of these probabilities is $\frac{4}{100} = \frac{1}{25} = 0.04$
- (b) A fair die will have exactly $\frac{1}{100}$ chance of rolling a given number. Using the same logic as in part (a), we must solve for the number of rolls, rather than the probability.

$$\frac{n}{100} = 0.3$$

$$n = 30 \text{ rolls}$$

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- (a) Probability Distribution: $\sum_{s \in S} p(s) = 1$ $p(100) = 5p(n), n \in \{1, 2, \dots, 99\}$ $p(1) = p(2) = \dots = p(99)$ 99 * p(1) + 5 * p(1) = 1 104 * p(1) = 1 $p(i) = \frac{1}{104}, i \in \{1, 2, \dots, 99\}$ $p(100) = \frac{5}{104}$
- (b) For any one roll, the $p(100) = \frac{5}{104}$. Thus, the probability of rolling a 100 in any n rolls is $\frac{5n}{104}$, since the probability each time compounds, adding another $\frac{5}{104}$

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3. Question 3

(a) In order for a bit string of length n to contain an equal number of 1's and 0's, it must have $\frac{n}{2}$ of each (1, 0 are treated as heads and tails over n flips). For a bit string of length n there are $C(n, \frac{n}{2})$ strings with equal 1's and 0's.

There are 2^n total possible outcomes of flipping the coin n times, and $C(n, \frac{n}{2})$ of these will have an even number of heads and tails.

The probability is then $\frac{n!}{(n/2)!(n/2)!(2^n)}$.

(b) It will not affect the original probability.

Limiting the first flip to heads limits the number of total possible outcomes to 2^{n-1} .

Of the remaining flips, there must be exactly one more tail result than heads. This number of strings is $C(n-1, \lceil \frac{n}{2}-1 \rceil)$, where $\lceil \frac{n}{2}-1 \rceil = \frac{n}{2}$, since n must be odd.

Probability = $\frac{(n-1)!}{(n/2)!(n/2)!(2^{n-1})} = \frac{n}{2} * \frac{n!}{(n/2)!(n/2)!(2^n)}$

Since the ratios are the same, the probabilities are the same.

(c)

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- 6. Question 6
 - (a)
 - (b)