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9 February 2017
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Homework 2

(1) (a)
$P(x) := x$ drives a Lamborghini
$Q(x) := x$ has received a speeding ticket

1. $P(Jim)$	**Premise**
2. $\forall x(P(x) \to Q(x))$	**Premise**
3. $Q(Jim)$	**Modus Tollens (3) & (2)**
4.$Jim \in D_{Students}$	**Premise**
5. $\therefore \exists x \in D_{Students}(Q(x))$	**Existential Generalization**

(b)
$P(x) := x$ is a thought-provoking movie
$Q(x) := x$ was directed by Clint Eastwood
$R(x) := x$ is a movie about a boxer

1. $\forall x(Q(x) \to P(x))$	**Premise**
2. $\exists x(Q(x) \wedge R(x))$	**Premise**
3. $\exists x R(x)$	**Simplification**
4. $\exists x Q(x)$	**Simplification**
5. $\exists x P(x)$	**Modus Tollens (1) & (4)** (if (1) is $\forall x$ is true, $\exists x$ is true)
6. $\therefore \exists x P(x) \wedge R(x)$	**Conjunction from (3) & (5)**

(c)
$P(x) := x$ is enrolled at RPI
$Q(x) := x$ has lived in in a dormitory

1. $\forall x(P(x) \to Q(x))$	**Premise**
2. $\neg Q(Ryan)$	**Premise**
3. $\forall x(\neg Q(x) \to \neg P(x))$	**Contraposition of (1)**
4. $Ryan \in D_x$	**Premise**
5. $\therefore \neg P(Ryan)$	**Modus Tollens of (2) & (3)**

(d)
$P(x) := x$ is a Kawasaki motorcycle
$Q(x) := x$ is exciting to drive

1. $P(x) \to Q(x)$	**Premise**
2. $\neg P(Isabella's)$	**Premise**
3. $\therefore \neg Q(Isabella's)$	**Is FALSE due to the fallacy of denying hypothesis**

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(2)

2. It is not explicitly stated that $c \in D_x$ (this is probably fine, but just in case)
3. It is not explicitly stated that $\neg Q(c)$, so we cannot be sure that $P(c)$ holds
5. It is not explicitly stated that $\neg P(c)$, so we cannot be sure that $Q(c)$ holds

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(3)

$P(x) :=$ x is a rational number

$\forall x, y \in \mathbb{Q}(P(x) \wedge P(y)) \rightarrow P(x * y)$

$x = \frac{a}{b}; a, b \in \mathbb{Z}$	**Definition of a rational number**
$y = \frac{m}{n}; m, n \in \mathbb{Z}$	**Definition of a rational number**
$xy = \frac{am}{bn}$	**Rules of algebra**
$am \in \mathbb{Z}$	**Under multiplication closure**
$bn \in \mathbb{Z}$	**Under multiplication closure**
$\frac{\mathbb{Z}}{\mathbb{Z}} \in \mathbb{Q}$	**Definition of a rational number**
$\therefore xy \in \mathbb{Q}$	**q.e.d.**

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(4)

$P(x) := x$ is an even number
$\forall m, n \in \mathbb{Z}(P(m * n) \rightarrow (P(m) \vee P(n))),$

$\forall m, n \in \mathbb{Z}((\neg P(m) \wedge \neg P(n)) \rightarrow \neg P(m * n))$	**Contrapositive of initial proposition**
$m = 2k + 1, k \in \mathbb{Z}$	**Definition of an odd number**
$n = 2c + 1, c \in \mathbb{Z}$	**Definition of an odd number**
$mn = 4kc + 2c + 2k + 1$	**Distributive property**
$mn = 2(2kc + c + k) + 1$	**Rule of algebra**
$A = 2kc + c + k$	**Redefinition**
$A \in \mathbb{Z}$	**Under multiplication closure**
$mn = 2A + 1$	**Substitution**
$\therefore \neg P(mn)$	**Defintion of an even number**
q.e.d.	

(5)

$P(x) := x$ is odd

$((P(x) \land \neg P(y)) \lor (\neg P(x) \land P(y))) \to P(5x+5y), x, y \in \mathbb{Z}$	**Premise**
$(P(x) \land \neg P(y)) \to P(5x+5y)$	**Assumption WLOG**
$x = 2k+1, k \in \mathbb{Z}$	**Definition of an odd number**
$y = 2m, m \in \mathbb{Z}$	**Definition of an even number**
$5(2k+1) + 5(2m)$	**Substitution**
$5(2k+1) + 5(2m) = 2c+1, c \in \mathbb{Z}$	**Definition of an odd number**
$10k + 5 + 10m = 2c + 1$	**Distributive property**
$10k + 10m + 4 = 2c$	**Rule of algebra**
$2(5k + 5m + 2) = 2c$	**Factoring**
$A = 5k + 5m + 2, A \in \mathbb{Z}$	**Redefinition**
$2A = 2c; A, c \in \mathbb{Z}$	**Substitution**
$2 = 2$	**Simplification**
$\therefore P(x) \land \neg P(y) \to P(5x+5y)$	**q.e.d.**

The above proof is sufficient for the entire statement. If the roles of x and y are reversed, the proof is still the same. Thus, the first assumption is valid under the idea of "without loss of generality".

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(6) Idea: Only the last 2 digits of n are important to the last 2 digits of n^2

n	n^2 (last 2 digits)
00	00
01	01
02	04
03	09
04	16
05	25
06	36
07	49
08	64
09	81
10	00
11	21
12	44
13	69
14	96
15	25
16	56
17	89
18	24
19	61
20	00
21	41
22	84
23	29
24	76

From this point to 50 ($25^2 \to 49^2$), these values repeat in reverse order. This is because $(50-n)^2 = 2500 - 100n + n^2$. For $25 \leq n < 50$, this means that n^2 and $(50-n)^2$ will have the same final digits, since only the 100s place will be affected by $2500 - 100n$. From $50 \leq n < 100$, the entire pattern from $0^2 \to 49^2$ repeats itself. In this case, we take the relation $(50+n)^2 = 2500 + 100n + n^2$. Again, the only part of the sum that affects the last 2 digits is n^2, making $(50+n)^2 = n^2$. Therefore, the only unique combinations of the final 2 digits are contained within the above table. Repeats were left in the table to maintain continuity.