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Assignment-4

1. Data: $[(0,0),(1,1),(2,3)]$

   (a) A full degree polynomial interpolation using the Lagrange basis is by definition

$$y = \sum_{i=1}^{n} y_i L_i(x_i)$$

where

$$L_i = \prod_{k=1 \;\; k\neq i}^{n} \frac{x - x_k}{x_i - x_k}$$

In this case, $n = 3$ and
$x_1 = 0 \quad x_2 = 1 \quad x_3 = 2$
$y_1 = 0 \quad y_2 = 1 \quad y_3 = 3$

$y = 0L_1(x_1) + 1L_2(x_2) + 3L_3(x_3)$

$$L_2 = \frac{x-0}{1-0} * \frac{x-2}{1-2} = -x^2 + 2x$$

$$L_3 = \frac{x-0}{2-0} * \frac{x-1}{2-1} = \frac{x^2 - x}{2}$$

$$y = -x^2 + 2x + \frac{3}{2}(x^2 - x) = \frac{1}{2}(x^2 + x)$$

Just to ensure the interpolant agrees with the data, a small MATLAB script was used to plot y with the points overlaid.

`holmes5_1.m` script

```
% check for Holmes 5.1

% define calculated interpolant
y = (1/2)*(x.^2 + x);
x = linspace(0,3);

% display results, including original data points
plot(x,y,0,0,'*',1,1,'*',2,3,'*');
legend('Interpolant','(x_1,y_1)','(x_2,y_2)','(x_3,y_3)','Location','Northwest');
```

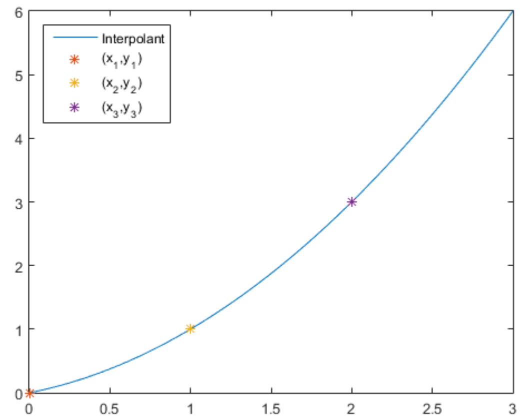

Figure 1: Graph of y and the data points

(b)