Numerical Coincidences from a BCC Lattice Framework

A concise summary for discussion — not a claim of proof

Alan Garcia — February 2026 — Independent investigation

1. What This Is

I am an independent researcher exploring a lattice field theory framework in which the fine structure constant, the proton-electron mass ratio, the muon-electron mass ratio, and the tau-electron mass ratio emerge from the spectral geometry of the BCC lattice Dirac operator. The framework takes two inputs—the BCC coordination number $n = 8$ and the transcendental π—with $d = 3$ (spatial dimensions) entering the muon and tau formulas, and produces values for all four constants with zero free parameters. I am writing to ask whether you can see a trivial explanation for the coincidences below, or whether the lattice selection mechanism has structural content.

2. Generator Functions from SO(3) Representation Theory

Four functions of the coordination number n arise from the representation theory of SO(3):

$$\tau(n) = n(2n+1) + 1 \qquad \sigma(n) = n(2n+1) \qquad \rho(n) = n + 1 \qquad \mu(n) = \tfrac{3}{2}\,\sigma\,\rho$$

At $n = 8$: $\tau = 137$, $\sigma = 136$, $\rho = 9$, $\mu = 1836$. Here $\tau(n) = \dim\big(\Lambda^2 D(n) \oplus D(0)\big)$, where $\Lambda^2 D(n)$ is the antisymmetric square of the spin-n representation, and $\sigma(n) = \binom{2n+1}{2}$ counts the pairwise couplings among $2n+1$ magnetic substates.

3. Result 1: The Fine Structure Constant

α^{-1} is obtained from a cubic equation interpreted as a Dyson equation $G^{-1} = G_0^{-1} - \Sigma(G)$ for the dressed inverse coupling:

$$\boxed{\alpha^{-1} = B + \frac{1}{(n+2)\,B^2}}, \qquad B = \tau + \frac{c_1}{\tau} - \frac{1}{2\tau^2} - \frac{1}{(n-1)\,\tau^3} + \frac{c_4}{\tau^4}$$

The base B is a perturbative self-energy expansion on the BCC lattice with coupling $g^2 = 1/2$ and Dirac multiplicity $N_D = 4$. The coefficients have identified spectral origins:

- $c_1 = \pi^2/2$ Brillouin zone momentum integral $\langle|\mathbf{q}|^2\rangle_{\mathrm{BZ}}$
- $c_2 = -\frac{1}{2}$ Spectral mean: $-d/\langle D^2\rangle$
- $c_3 = -1/(n-1) = -\frac{1}{7}$ Spectral variance
- $c_4 = \frac{2\rho}{2n+1}\,\pi^3 = \frac{18}{17}\,\pi^3$ Spectral skewness $\times$ zone integral

Lattice framework prediction	137.035 999 177
CODATA 2022	137.035 999 177(21)
Agreement	< 0.001 ppb

4. Result 2: The Proton-Electron Mass Ratio

From the same generator functions, with α computed self-consistently. The formula appears naturally in n-explicit factored form:

$$\boxed{\frac{m_p}{m_e} = \mu\big(1 + 2(2n{+}1)\,\pi\alpha^3\big) + \Big(1 + \frac{1}{2n}\Big)\pi^2\alpha(1{-}\alpha^2) - \alpha^2}$$

The denominators in the original notation ($\pi\mu\sigma\alpha^3/4$ and $(2n{+}1)\pi^2\alpha/16$) are functions of the coordination number: $4 = n/2$ and $16 = 2n$ at $n = 8$. The corrections decompose as:

- **Vertex:** μ is dressed by a factor $1 + 2(2n{+}1)\pi\alpha^3$, where $2(2n{+}1) = 2\dim D(n)$.
- **VP:** A universal piece $\pi^2\alpha(1{-}\alpha^2)$ plus a lattice correction of relative size $1/(2n)$, vanishing as $n \to \infty$ (continuum limit).
- **Self-intersection:** $-\alpha^2$, universal.

Lattice framework prediction	1836.152 673 5
CODATA 2022	1836.152 673 426(32)
Agreement	< 0.03 ppb

5. Result 3: The Muon-Electron Mass Ratio

The muon is modeled as a binary defect excitation (two-node) on the same lattice, in contrast to the proton's ternary defect (three-node). Its formula follows the same correction taxonomy but with d (spatial dimension) replacing n (coordination number) as the structural parameter:

$$\boxed{\frac{m_\mu}{m_e} = \frac{d}{2}\big(\tau + \pi\alpha(1{-}\alpha^2)\big) + \frac{c_1}{4} + \frac{d}{2}\,\pi\sigma\alpha^3 - \alpha^2}$$

The proton's vertex prefactor is $2/n$ (per-bond weight on a lattice with n neighbors); the muon's is $d/2$ (dimensional weight in d-dimensional space). The muon vertex coefficient $(d/2)\,\pi\,\sigma$ equals the proton vertex coefficient times $2d/\mu = 6/1836 = 1/306$.

Lattice framework prediction	206.768 282 5
CODATA 2022	206.768 282 7(46)
Agreement	1.1 ppb $\quad(0.05\sigma)$

6. Result 4: The Tau-Electron Mass Ratio

The tau lepton mass is predicted by dressing the Koide relation. Yoshio Koide (1982) observed empirically that $(m_e + m_\mu + m_\tau)/(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2 \approx 2/3$. In the lattice framework, the binary defect is a 1-dimensional line segment in d spatial dimensions; its normal bundle has fiber dimension $d{-}1 = 2$, giving a transverse fraction $(d{-}1)/d = 2/3$. This tree-level constant receives a radiative correction from the same spectral ingredients that enter the α formula:

$$\boxed{Q = \frac{d{-}1}{d} + \frac{(d{-}1)\,\pi^2\alpha^2}{(n{-}1)\,\sigma}}, \qquad \frac{m_e + m_\mu + m_\tau}{\big(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}\big)^2} = Q$$

The correction $\delta Q = (d{-}1)\pi^2\alpha^2/\big((n{-}1)\sigma\big)$ decomposes as:

- $(d-1)$ — transverse codimension (number of normal directions)
- π^2 — BZ momentum integral (same π^2 as c_1 in Result 1)
- α^2 — two-loop EM coupling
- $1/(n-1) = 1/7$ — spectral variance (same as c_3 in Result 1)
- $1/\sigma = 1/136$ — pairwise channel normalization

With $m_e = 1$ and m_μ from Result 3, the Koide equation reduces to a quadratic in $x = \sqrt{m_\tau}$ with closed-form solution:

$$x = \frac{Q(1+\sqrt{m_\mu}\,) + \sqrt{(1+m_\mu)(2Q-1) + 2Q\sqrt{m_\mu}}}{1-Q}, \qquad m_\tau = x^2$$

Lattice framework prediction	3477.4799	
CODATA 2022	3477.48 ± 0.57	
Agreement	0.027 ppm	(0.0002σ)

Note: The bare Koide constant $Q = 2/3$ gives $m_\tau/m_e = 3477.44$ (11 ppm, 0.07σ). The dressed Q improves this by a factor of 415. The correction was identified by systematic numerical scan and interpreted geometrically as the leading Seeley–DeWitt heat kernel coefficient of the transverse Laplacian on the binary defect; a first-principles derivation from the lattice Lagrangian remains an open problem.

7. Lattice Selection: Why BCC?

Tested against SC ($n = 6$) and FCC ($n = 12$). The tree-level mass generator $\mu(n)$ is the discriminator:

Lattice	n	$\mu(n)$	α^{-1} **error**	**Mass error**	**Status**
BCC	8	1836	< 0.001 ppb	< 0.03 ppb	Passes all
SC	6	819	9.9 ppb	-55.4%	Fails mass
FCC	12	5850	142.7 ppb (6.8σ)	$+218.6\%$	Fails both

SC's α^{-1} closeness is explained by $\langle D^2 \rangle = 2d$ universality for cubic lattices (first two loop corrections identical). Discrimination enters at three loops via spectral variance and, decisively, through $\mu(n)$: no perturbative correction bridges SC's 55% mass deficit.

8. Questions for the Reader

(a) Is there a trivial or known reason why polynomials of 8 produce integers close to 137 and 1836?

(b) Does $g^2 = 1/2$, $N_D = 4$ on BCC correspond to a recognized lattice action?

(c) The proton vertex denominator is $n/2$ and the VP denominator is $2n$. Does this n-dependence have precedent in lattice perturbation theory?

(d) The proton's structural parameter is n (coordination); the muon's is d (dimension). Does this topological/geometric distinction correspond to anything in defect field theory?

(e) Is the SC/BCC near-degeneracy for α^{-1} (via $\langle D^2 \rangle = 2d$) interesting or expected?

(f) The dressed Koide correction uses $1/(n-1)$ (spectral variance) and $1/\sigma$ (pairwise channels)—the same quantities appearing in the α^{-1} expansion. Is there a known mechanism by which a constraint on mass *ratios* receives radiative corrections from the same spectral coefficients as a coupling?

(g) The interpretation $Q = (d-1)/d$ identifies the Koide constant with the transverse fraction of a line defect. Does this codimensional constraint have precedent in defect field theory or conformal field theory?

I would be grateful for any feedback, including identification of a trivial explanation. Full paper (LaTeX) available on request.

Appendix: Verification Code (Python)

```
from mpmath import mp, mpf, pi, sqrt
mp.dps = 50
def tau(n): return n*(2*n+1)+1
def sigma(n): return n*(2*n+1)
def mu(n): return mpf(3)/2*sigma(n)*(n+1)
N = 8; d = 3
TAU, SIGMA, RHO, MU = mpf(tau(N)), mpf(sigma(N)), mpf(N+1), mu(N)
c1 = pi**2 / 2
c4 = (2*RHO/(2*N+1)) * pi**3
B = TAU + c1/TAU - 1/(2*TAU**2) - 1/((N-1)*TAU**3) + c4/TAU**4
alpha_inv = B + 1/((N+2)*B**2); alpha = 1/alpha_inv
mass = MU*(1+2*(2*N+1)*pi*alpha**3) \
       + (1+1/(2*N))*pi**2*alpha*(1-alpha**2) - alpha**2
muon = mpf(d)/2*(TAU+pi*alpha*(1-alpha**2)) \
       + c1/4 + mpf(d)/2*pi*SIGMA*alpha**3 - alpha**2
Q = mpf(d-1)/d + mpf(d-1)*pi**2*alpha**2/((N-1)*SIGMA)
b = sqrt(muon)
D = (1 + muon)*(2*Q - 1) + 2*Q*b
tau_mass = ((Q*(1+b) + sqrt(D)) / (1-Q))**2
print(f"alpha^-1  = {alpha_inv}")   # 137.035999177...
print(f"m_p/m_e   = {mass}")        # 1836.15267348...
print(f"m_mu/m_e  = {muon}")        # 206.768282475...
print(f"m_tau/m_e = {tau_mass}")    # 3477.47990762...
```

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