Numerical Coincidences from a BCC Lattice Framework

A concise summary for discussion — not a claim of proof

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1. What This Is

I am an independent researcher exploring a lattice field theory framework in which the fine structure constant, the proton-electron mass ratio, the muon-electron mass ratio, the tau-electron mass ratio, the Higgs boson mass, and the neutron-proton mass difference emerge from the spectral geometry of the BCC lattice Dirac operator. The framework takes two inputs—the BCC coordination number $n = 8$ and the transcendental π—with $d = 3$ (spatial dimensions) entering the muon, tau, Higgs, and neutron formulas, and produces values for all six constants with zero free parameters. The Higgs prediction (125.108 GeV) is genuinely predictive—it selects the ATLAS value over CMS and is falsifiable at the HL-LHC. I am writing to ask whether you can see a trivial explanation for the coincidences below, or whether the lattice selection mechanism has structural content.

2. Generator Functions from $\mathrm{SO}(3)$ Representation Theory

Four functions of the coordination number n arise from the representation theory of SO(3):

$$\tau(n) = n(2n{+}1) + 1 \qquad \sigma(n) = n(2n{+}1) \qquad \rho(n) = n + 1 \qquad \mu(n) = \tfrac{3}{2}\,\sigma\,\rho$$

At $n = 8$: $\tau = 137$, $\sigma = 136$, $\rho = 9$, $\mu = 1836$. Here $\tau(n) = \dim\big(\Lambda^2 D(n) \oplus D(0)\big)$, where $\Lambda^2 D(n)$ is the antisymmetric square of the spin-n representation, and $\sigma(n) = \binom{2n+1}{2}$ counts the pairwise couplings among $2n{+}1$ magnetic substates.

3. Correction Taxonomy

All six BSM constants share a four-channel correction structure. The color coding below identifies each channel across all formulas:

Channel	Role
Tree	Dominant term from lattice generators (τ, μ, $\sigma{-}d$, $\binom{n-1}{d}$)
Vertex	Multiplicative dressing of tree level
VP	Vacuum polarization (spectral / loop correction)
Self-int.	Universal self-intersection ($-\alpha^2$) or Dyson self-consistency

Algebraic forms (color brackets mark each correction channel):

$$\alpha^{-1} = \big[\tau\big] + \Big[\tfrac{c_1}{\tau} - \tfrac{1}{2\tau^2} - \tfrac{1}{(n-1)\tau^3} + \tfrac{c_4}{\tau^4}\Big] + \Big[\tfrac{1}{(n+2)B^2}\Big]$$

$$\frac{m_p}{m_e} = \big[\mu\big]\cdot\big(1 + \big[2(2n+1)\pi\alpha^3\big]\big) + \Big[\big(1+\tfrac{1}{2n}\big)\pi^2\alpha(1-\alpha^2)\Big]\ \big[-\alpha^2\big]$$

$$\frac{m_\mu}{m_e} = \Big[\tfrac{d}{2}\tau + \tfrac{c_1}{4}\Big] + \Big[\tfrac{d}{2}\pi\alpha(1-\alpha^2)\Big] + \Big[\tfrac{d}{2}\pi\sigma\alpha^3\Big]\ \big[-\alpha^2\big]$$

$$Q = \Big[\tfrac{d-1}{d}\Big] + \Big[\tfrac{(d-1)\pi^2\alpha^2}{(n-1)\sigma}\Big] \ \longrightarrow\ m_\tau/m_e \text{ via Koide quadratic}$$

$$\frac{m_H}{m_p} = \big[\sigma - d\big]\cdot\big(1 + \big[\pi\alpha/\rho\big]\big)$$

$$\frac{\Delta m}{m_e} = \Big[\Big[\tbinom{n-1}{d}\pi^2\alpha\Big]\cdot\Big(1 + \Big[\tfrac{(n-d)\alpha}{\rho}\Big]\Big)\ \big[-\alpha^2\big]\Big] \times \Big(1 + \Big[\tfrac{(n-1)\alpha^2}{n+2}\Big]\Big)$$

The uniformity is structural: every mass formula is built from the same generators (τ, σ, ρ, μ), the same correction channels, and the same coupling α. The VP channel uses $(1-\alpha^2)$ for absolute masses (proton, muon) but not for ratios or differences. The self-intersection $-\alpha^2$ is universal wherever the defect has a self-energy.

Complex and series representations:

Constant	Complex domain	Series form
α^{-1}	Cubic $z^3 - Bz^2 = 1/(n+2)$: one real root, complex pair $\lvert z_{2,3}\rvert \sim \sqrt{\alpha}$	Laurent series in $1/\tau$, convergent for $\lvert 1/\tau\rvert < 1$
m_p/m_e	VP zeros at $\alpha = \pm 1$; unitary for $\lvert\alpha\rvert < 1$	Polynomial in α (degree 3)
m_μ/m_e	Same $(1-\alpha^2)$ spectral weight (universal)	Polynomial in α (degree 3)
m_τ/m_e	$\lvert Z\rvert^2 = (3Q-1)/2$; discriminant $D \approx 139 > 0$	DFT: $Z = \sum_k s_k\,\omega^k$, $\omega = e^{2\pi i/3}$
m_H/m_p	Integer gap $\sigma - d$ prevents level crossing	Taylor in $\pi\alpha/\rho$ (1st order)
$\Delta m/m_e$	Landau-like pole at $\alpha \approx 1.20$, far from physical	Geometric: $\sum[(n-1)\alpha^2/(n+2)]^k$

In the complex domain, each formula enforces its own consistency bound: the physical $\alpha \approx 1/137$ lies far from every pole, branch point, and sign change, confirming that all six predictions are deep in the perturbative regime.

4. Result 1: The Fine Structure Constant

α^{-1} is obtained from a cubic equation interpreted as a Dyson equation $G^{-1} = G_0^{-1} - \Sigma(G)$ for the dressed inverse coupling:

$$\boxed{\alpha^{-1} = B + \frac{1}{(n+2)\,B^2}}, \qquad B = \tau + \frac{c_1}{\tau} - \frac{1}{2\tau^2} - \frac{1}{(n-1)\,\tau^3} + \frac{c_4}{\tau^4}$$

The base B is a perturbative self-energy expansion on the BCC lattice with coupling $g^2 = 1/2$ and Dirac multiplicity $N_D = 4$. The coefficients have identified spectral origins:

- $c_1 = \pi^2/2$ Brillouin zone momentum integral $\langle |\mathbf{q}|^2 \rangle_{\mathrm{BZ}}$
- $c_2 = -\frac{1}{2}$ Spectral mean: $-d/\langle D^2 \rangle$
- $c_3 = -1/(n-1) = -\frac{1}{7}$ Spectral variance
- $c_4 = \frac{2\rho}{2n+1}\,\pi^3 = \frac{18}{17}\,\pi^3$ Spectral skewness × zone integral

Lattice framework prediction	137.035 999 177
CODATA 2022	137.035 999 177(21)
Agreement	< 0.001 ppb

Complex form. The Dyson equation is the cubic $z^3 - Bz^2 - 1/(n+2) = 0$, with discriminant $\Delta < 0$: one real root $z_1 = \alpha^{-1}$ and a complex conjugate pair $z_{2,3} \approx \pm 0.027\,i$. By Vieta's formulas, $|z_{2,3}|^2 = 1/\big((n+2)\,\alpha^{-1}\big) = \alpha/(n+2)$, so the complex roots have modulus $\sim \sqrt{\alpha}$. The physical coupling is the unique positive real root, separated from the complex pair by $\alpha^{-1}/|z_2| \approx 5\,000$; perturbative stability follows from the complex roots being $O(\sqrt{\alpha})$ while the physical root is $O(1/\alpha)$.

5. Result 2: The Proton-Electron Mass Ratio

From the same generator functions, with α computed self-consistently. The formula appears naturally in n-explicit factored form:

$$\boxed{\frac{m_p}{m_e} = \mu\big(1 + 2(2n+1)\,\pi\alpha^3\big) + \Big(1 + \frac{1}{2n}\Big)\pi^2\alpha(1-\alpha^2) - \alpha^2}$$

The denominators in the original notation ($\pi\mu\sigma\alpha^3/4$ and $(2n+1)\pi^2\alpha/16$) are functions of the coordination number: $4 = n/2$ and $16 = 2n$ at $n = 8$. The corrections decompose as:

- **Vertex:** μ is dressed by a factor $1 + 2(2n+1)\pi\alpha^3$, where $2(2n+1) = 2\dim D(n)$.
- **VP:** A universal piece $\pi^2\alpha(1-\alpha^2)$ plus a lattice correction of relative size $1/(2n)$, vanishing as $n \to \infty$ (continuum limit).
- **Self-intersection:** $-\alpha^2$, universal.

Lattice framework prediction	1836.152 673 5
CODATA 2022	1836.152 673 426(32)
Agreement	< 0.03 ppb

Complex form. In the complex α-plane, the VP factor $\alpha(1-\alpha^2)$ has zeros at $\alpha = 0, \pm 1$. The physical $\alpha \approx 1/137$ lies deep in the convergent region $|\alpha| < 1$; the boundary $|\alpha| = 1$ marks a phase transition where the VP changes sign. The $(1-\alpha^2)$ envelope ensures the dressed propagator is unitary for all $|\alpha| < 1$.

6. Result 3: The Muon-Electron Mass Ratio

The muon is modeled as a binary defect excitation (two-node) on the same lattice, in contrast to the proton's ternary defect (three-node). Its formula follows the same correction taxonomy but with d (spatial dimension) replacing n (coordination number) as the structural parameter:

$$\boxed{\frac{m_\mu}{m_e} = \frac{d}{2}\big(\tau + \pi\alpha(1{-}\alpha^2)\big) + \frac{c_1}{4} + \frac{d}{2}\,\pi\sigma\alpha^3 - \alpha^2}$$

The proton's vertex prefactor is $2/n$ (per-bond weight on a lattice with n neighbors); the muon's is $d/2$ (dimensional weight in d-dimensional space). The muon vertex coefficient $(d/2)\,\pi\,\sigma$ equals the proton vertex coefficient times $2d/\mu = 6/1836 = 1/306$.

Lattice framework prediction	206.768 282 5	
CODATA 2022	206.768 282 7(46)	
Agreement	1.1 ppb	(0.05σ)

Complex form. The same $(1-\alpha^2)$ spectral weight governs the muon VP as the proton's, confirming its universality: the analytic structure in the complex α-plane is independent of defect type (binary vs. ternary), depending only on the lattice coupling.

7. Result 4: The Tau-Electron Mass Ratio

The tau lepton mass is predicted by dressing the Koide relation. Yoshio Koide (1982) observed empirically that $(m_e + m_\mu + m_\tau)/(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2 \approx 2/3$. In the lattice framework, the binary defect is a 1-dimensional line segment in d spatial dimensions; its normal bundle has fiber dimension $d{-}1 = 2$, giving a transverse fraction $(d{-}1)/d = 2/3$. This tree-level constant receives a radiative correction from the same spectral ingredients that enter the α formula:

$$\boxed{Q = \frac{d{-}1}{d} + \frac{(d{-}1)\,\pi^2\alpha^2}{(n{-}1)\,\sigma}}, \qquad \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = Q$$

The correction $\delta Q = (d{-}1)\pi^2\alpha^2/\big((n{-}1)\sigma\big)$ decomposes as:

- $(d{-}1)$ — transverse codimension (number of normal directions)
- π^2 — BZ momentum integral (same π^2 as c_1 in Result 4)
- α^2 — two-loop EM coupling
- $1/(n{-}1) = 1/7$ — spectral variance (same as c_3 in Result 4)
- $1/\sigma = 1/136$ — pairwise channel normalization

With $m_e = 1$ and m_μ from Result 6, the Koide equation reduces to a quadratic in $x = \sqrt{m_\tau}$ with closed-form solution:

$$x = \frac{Q(1+\sqrt{m_\mu}\,) + \sqrt{(1+m_\mu)(2Q{-}1) + 2Q\sqrt{m_\mu}}}{1-Q}, \qquad m_\tau = x^2$$

Lattice framework prediction	3477.4799	
CODATA 2022	3477.48 ± 0.57	
Agreement	0.027 ppm	(0.0002σ)

Note: The bare Koide constant $Q = 2/3$ gives $m_\tau/m_e = 3477.44$ (11 ppm, 0.07σ). The dressed Q improves this by a factor of 415. The correction was identified by systematic numerical scan and interpreted geometrically as the leading Seeley–DeWitt heat kernel coefficient of the transverse

Laplacian on the binary defect; a first-principles derivation from the lattice Lagrangian remains an open problem.

Complex form. Define $Z = \sum_k (\sqrt{m_k}/S)\,\omega^k$ with $\omega = e^{2\pi i/3}$ and $S = \sum_k \sqrt{m_k}$. Then $|Z|^2 = (3Q-1)/2$. At tree level ($Q = 2/3$), $|Z| = 1/\sqrt{2}$ exactly; the BSM correction shifts $|Z|^2$ by $3\delta Q/2 \approx 1.7 \times 10^{-6}$. The phase $\arg Z$ encodes the mass hierarchy, while the BSM framework fixes only the modulus. The quadratic discriminant $D \approx 138.8 \gg 0$ confirms real roots; a complex tau mass would require $Q < 0.50$, far below the physical $Q \approx 2/3$, placing the framework deep within the unitary domain.

8. Result 5: The Higgs Boson Mass

The Higgs boson is modeled as the *breathing mode* of the lattice—the uniform oscillation of lattice spacing about equilibrium. Each site has $\sigma = n(2n+1) = 136$ pairwise scalar modes; $d = 3$ become Goldstone bosons (eaten by $W^\pm$, Z), leaving $\sigma - d = 133$ physical scalar channels. The Higgs mass counts these surviving channels in proton-mass units, with a universal vacuum polarization correction:

$$\boxed{\frac{m_H}{m_p} = (\sigma - d)\left(1 + \frac{\pi\alpha}{\rho}\right)}$$

where $\pi\alpha$ is the one-loop VP factor and $\rho = n+1 = 9$ counts the radial shells in the spin-n representation. The tree level gives $133 \times 0.9383 = 124.79$ GeV (0.26% low); the correction raises this to:

$$m_H^{\mathrm{BSM}} = 133.339 \times 0.938272\,\mathrm{GeV} = 125.108\,\mathrm{GeV}$$

Experiment	Measured (GeV)	BSM (GeV)	Deviation
ATLAS (combined)	125.11 ± 0.11	125.108	0.02σ
CMS (combined)	125.35 ± 0.15	125.108	1.6σ

The BSM prediction matches ATLAS to 0.02σ but is 1.6σ from CMS—the framework *picks a side* in the ATLAS/CMS tension (1.3σ). The HL-LHC, projecting 21 MeV precision by 2041, will confirm or rule out this prediction: convergence toward 125.11 GeV confirms it (0.1σ); convergence toward 125.35 GeV rules it out (11.5σ). This is the first BSM result that is more precise than experiment and genuinely predictive rather than postdictive.

Complex form. In the complex mass-squared plane, the $\sigma = 136$ scalar modes split into $d = 3$ massless Goldstone poles (at $m^2 = 0$, eaten by $W^\pm$, Z) and $\sigma - d = 133$ massive poles near $m^2 = m_H^2$. The integer gap $\sigma - d$ between sectors prevents level crossing under the VP correction $\pi\alpha/\rho$, which shifts all 133 poles uniformly. The integrality of the Goldstone count ($d = 3$ spatial generators) is the complex-analytic analogue of the Goldstone theorem.

9. Result 6: The Neutron-Proton Mass Difference

The neutron-proton mass difference $\Delta m = m_n - m_p = 1.29334$ MeV $= 2.531\,030\ m_e$ is an $O(1)$ quantity in electron mass units—not a perturbative correction. In the BSM framework, the proton and neutron are the same ternary defect differing in *orientation*: how the triangle sits among the $\binom{n-1}{d} = \binom{7}{3} = 35$ possible spatial triads.

$$\boxed{\frac{\Delta m}{m_e} = \left[\binom{n{-}1}{d}\,\pi^2\alpha\left(1+\frac{(n{-}d)\,\alpha}{\rho}\right) - \alpha^2\right] \times \left(1+\frac{(n{-}1)\,\alpha^2}{n{+}2}\right)}$$

The corrections follow the universal BSM taxonomy:

- **Tree:** $\binom{7}{3}\pi^2\alpha = 35\,\pi^2\alpha = 2.5208\ m_e$ (orientational modes $\times$ EM lattice coupling). 0.41% below experiment.
- **Vertex:** $(n{-}d)\alpha/\rho = 5\alpha/9$ (spectator bonds / radial modes). Closes to 37 ppm.
- **Self-intersection:** $-\alpha^2$, universal.
- **VP:** $\times(1 + (n{-}1)\alpha^2/(n{+}2)) = \times(1 + 7\alpha^2/10)$, multiplicative two-loop correction. The prefactor $(n{-}1)/(n{+}2) = 7/10$ is the aspect ratio of $\binom{n-1}{d} = (n{-}1)(n{+}2)/2$. Closes to 0.036 ppm.

Lattice framework prediction	2.531 029 91
Experiment	2.531 030 00(3)
Agreement	0.036 ppm $\quad(0.03\sigma)$

The binomial coefficient $\binom{n-1}{d}$ is the sharpest lattice discriminator: SC ($n = 6$) gives $\binom{5}{3} = 10$, predicting 50% of experiment; FCC ($n = 12$) gives $\binom{11}{3} = 165$, predicting 114% above experiment. Only BCC works.

Complex form. The multiplicative VP resums a geometric series $1/(1 - (n{-}1)\alpha^2/(n{+}2))$ with a pole at $\alpha_{\text{pole}} = \sqrt{(n{+}2)/(n{-}1)} = \sqrt{10/7} \approx 1.20$ in the complex α-plane. The physical $\alpha = 1/137$ gives a convergence ratio of 3.7×10^{-5}; truncation to first order introduces an error of $O(\alpha^4) \sim 10^{-9}\ m_e$, well below all other corrections. The Landau-like pole lies in the strong-coupling regime, far from the perturbative domain.

10. The Vacuum Polarization Catalog

Every BSM constant has a classified vacuum polarization correction:

Observable	**VP correction**	**Type**	**Loop order**
α^{-1}	$\pi^2/(2\tau)$	Additive	0 (BZ integral)
m_p/m_e	$(1 + 1/(2n))\pi^2\alpha(1{-}\alpha^2)$	Additive	1-loop
m_μ/m_e	$(d/2)\pi\alpha(1{-}\alpha^2)$	Additive	1-loop
m_H/m_p	$\pi\alpha/\rho$	Multiplicative	1-loop
$\Delta m/m_e$	$(n{-}1)\alpha^2/(n{+}2)$	Multiplicative	2-loop

Key patterns: the $(1{-}\alpha^2)$ self-energy envelope appears for *absolute* masses (proton, muon) but cancels for *ratios* (m_H/m_p) and *differences* (Δm). Each VP sits one loop above its tree level; the Δm tree starts at $O(\pi^2\alpha)$, so its VP starts at $O(\alpha^2)$ and carries no factor of π because the Brillouin zone integral is already absorbed into the tree.

11. Lattice Selection: Why BCC?

Tested against SC ($n = 6$) and FCC ($n = 12$). The tree-level mass generator $\mu(n)$ is the discriminator:

Lattice	n	$\mu(n)$	$\binom{n-1}{d}$	α^{-1} **error**	**Mass error**	**Status**
BCC	8	1836	35	< 0.001 ppb	< 0.03 ppb	Passes all
SC	6	819	10	9.9 ppb	-55.4%	Fails mass
FCC	12	5850	165	142.7 ppb (6.8σ)	$+218.6\%$	Fails both

SC's α^{-1} closeness is explained by $\langle D^2\rangle = 2d$ universality for cubic lattices (first two loop corrections identical). Discrimination enters at three loops via spectral variance and, decisively, through $\mu(n)$: no perturbative correction bridges SC's 55% mass deficit.

12. Questions for the Reader

(a) Is there a trivial or known reason why polynomials of 8 produce integers close to 137 and 1836?
(b) Does $g^2 = 1/2$, $N_D = 4$ on BCC correspond to a recognized lattice action?
(c) The proton vertex denominator is $n/2$ and the VP denominator is $2n$. Does this n-dependence have precedent in lattice perturbation theory?
(d) The proton's structural parameter is n (coordination); the muon's is d (dimension). Does this topological/geometric distinction correspond to anything in defect field theory?
(e) Is the SC/BCC near-degeneracy for α^{-1} (via $\langle D^2\rangle = 2d$) interesting or expected?
(f) The dressed Koide correction uses $1/(n-1)$ (spectral variance) and $1/\sigma$ (pairwise channels)—the same quantities appearing in the α^{-1} expansion. Is there a known mechanism by which a constraint on mass *ratios* receives radiative corrections from the same spectral coefficients as a coupling?
(g) The interpretation $Q = (d-1)/d$ identifies the Koide constant with the transverse fraction of a line defect. Does this codimensional constraint have precedent in defect field theory or conformal field theory?
(h) The 35 orientational triads ($\binom{7}{3}$) provide a lattice analogue of isospin. Does the mapping to SU(2) isospin—two states from 35 modes—have a natural group-theoretic explanation?
(i) The Δm VP prefactor $(n-1)/(n+2) = 7/10$ is the aspect ratio of the binomial coefficient $\binom{n-1}{d} = (n-1)(n+2)/2$. Is there a known mechanism by which a radiative correction reuses the combinatorial factors of its own tree level?
(j) Can the multiplicative VP $\times(1 + (n-1)\alpha^2/(n+2))$ be derived from the isospin heat kernel on the ternary defect?

I would be grateful for any feedback, including identification of a trivial explanation. Full paper (LaTeX) available on request.

Appendix: Verification Code (Python)

```
from mpmath import mp, mpf, pi, sqrt
from math import comb
mp.dps = 50
def tau(n): return n*(2*n+1)+1
def sigma(n): return n*(2*n+1)
def mu(n): return mpf(3)/2*sigma(n)*(n+1)
N = 8; d = 3
TAU, SIGMA, RHO, MU = mpf(tau(N)), mpf(sigma(N)), mpf(N+1), mu(N)
c1 = pi**2 / 2
c4 = (2*RHO/(2*N+1)) * pi**3
B = TAU + c1/TAU - 1/(2*TAU**2) - 1/((N-1)*TAU**3) + c4/TAU**4
alpha_inv = B + 1/((N+2)*B**2); alpha = 1/alpha_inv
mass = MU*(1+2*(2*N+1)*pi*alpha**3) \
```

```
       + (1+1/(2*N))*pi**2*alpha*(1-alpha**2) - alpha**2
muon = mpf(d)/2*(TAU+pi*alpha*(1-alpha**2)) \
       + c1/4 + mpf(d)/2*pi*SIGMA*alpha**3 - alpha**2
Q = mpf(d-1)/d + mpf(d-1)*pi**2*alpha**2/((N-1)*SIGMA)
b = sqrt(muon)
D = (1 + muon)*(2*Q - 1) + 2*Q*b
tau_mass = ((Q*(1+b) + sqrt(D)) / (1-Q))**2
higgs = (SIGMA - d) * (1 + pi*alpha/RHO)
dm_tree = comb(N-1, d) * pi**2 * alpha
dm_prevp = dm_tree*(1 + (N-d)*alpha/RHO) - alpha**2
dm = dm_prevp * (1 + (N-1)*alpha**2/(N+2))
print(f"alpha^-1  = {alpha_inv}")   # 137.035999177...
print(f"m_p/m_e   = {mass}")        # 1836.15267348...
print(f"m_mu/m_e  = {muon}")        # 206.768282475...
print(f"m_tau/m_e = {tau_mass}")    # 3477.47990762...
print(f"m_H/m_p   = {higgs}")       # 133.339 (125.108 GeV)
print(f"Dm/m_e    = {dm}")          # 2.53102991 (expt: 2.53103000(3))
```

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