Numerical Coincidences from a BCC Lattice Framework

A concise summary for discussion — not a claim of proof

Alan Garcia — February 2026 — Independent investigation

1. What This Is

I am an independent researcher exploring a lattice field theory framework in which the fine structure constant and the proton-electron mass ratio emerge from the spectral geometry of the BCC lattice Dirac operator. The framework takes two inputs—the BCC coordination number $n = 8$ and the transcendental π—and produces values for both constants with zero free parameters. I am writing to ask whether you can see a trivial explanation for the coincidences below, or whether the lattice selection mechanism has structural content.

2. Generator Functions from SO(3) Representation Theory

Four functions of the coordination number n arise from the representation theory of SO(3):

$$\tau(n) = n(2n{+}1) + 1 \qquad \sigma(n) = n(2n{+}1) \qquad \rho(n) = n + 1 \qquad \mu(n) = \tfrac{3}{2}\,\sigma\,\rho$$

At $n = 8$: $\tau = 137$, $\sigma = 136$, $\rho = 9$, $\mu = 1836$. Here $\tau(n) = \dim\big(\Lambda^2 D(n) \oplus D(0)\big)$, where $\Lambda^2 D(n)$ is the antisymmetric square of the spin-n representation, and $\sigma(n) = \binom{2n+1}{2}$ counts the pairwise couplings among $2n{+}1$ magnetic substates.

3. Result 1: The Fine Structure Constant

α^{-1} is obtained from a cubic equation interpreted as a Dyson equation $G^{-1} = G_0^{-1} - \Sigma(G)$ for the dressed inverse coupling:

$$\boxed{\alpha^{-1} = B + \frac{1}{(n{+}2)\,B^2}}, \qquad B = \tau + \frac{\pi^2}{2\tau} - \frac{1}{2\tau^2} - \frac{1}{(n{-}1)\,\tau^3}$$

The base B is a perturbative self-energy expansion on the BCC lattice with coupling $g^2 = 1/2$ and Dirac multiplicity $N_D = 4$. The coefficients have identified origins: $c_1 = \pi^2/2$ from the Brillouin zone momentum integral $\langle|\mathbf{q}|^2\rangle_{\mathrm{BZ}}$; $c_2 = -d/\langle D^2\rangle = -1/2$ from the Dirac spectral moment; $c_3 = -1/(n{-}1) = -1/7$ from the spectral variance, with $f_3(8) = -2/7$ as an exact algebraic identity.

Lattice framework prediction	137.035 999 084
CODATA 2018	137.035 999 084(21)
Agreement	< 0.005 ppb

4. Result 2: The Proton-Electron Mass Ratio

From the same generator functions, with α computed self-consistently:

$$\boxed{\frac{m_p}{m_e} = \mu + \frac{\pi\mu\sigma\alpha^3}{4} + \frac{(2n{+}1)\,\pi^2\alpha(1-\alpha^2)}{16} - \alpha^2}$$

The corrections decompose as a connection 1-form (vertex, $\pi\mu\sigma\alpha^3/4 \approx 0.0762$), a curvature 2-form (vacuum polarization, $(2n+1)\pi^2\alpha(1-\alpha^2)/16 \approx 0.0765$), and a self-intersection ($-\alpha^2 \approx -5.3 \times 10^{-5}$). The factor $(1-\alpha^2)$ is a dressed self-energy correction. The tree-level integer $\mu(8) = 1836$ is $\frac{3}{2}\dim(\Lambda^2 D(8)) \cdot (\rho(8))$ from SO(3).

Lattice framework prediction	1836.152 674
CODATA 2018	1836.152 673 43(11)
Agreement	< 0.03 ppb

5. Lattice Selection: Why BCC?

Tested against SC ($n = 6$) and FCC ($n = 12$). The tree-level mass generator $\mu(n)$ is the discriminator:

Lattice	n	$\mu(n)$	α^{-1} **error**	**Mass error**	**Status**
BCC	8	1836	< 0.005 ppb	< 0.03 ppb	Passes both
SC	6	819	9.9 ppb	-55.4%	Fails mass
FCC	12	5850	142.7 ppb (6.8σ)	$+218.6\%$	Fails both

SC's α^{-1} closeness is explained by $\langle D^2 \rangle = 2d$ universality for cubic lattices (first two loop corrections identical). Discrimination enters at three loops via spectral variance and, decisively, through $\mu(n)$: no perturbative correction bridges SC's 55% mass deficit.

6. Questions for the Reader

(a) Is there a trivial or known reason why polynomials of 8 produce integers close to 137 and 1836?
(b) Does $g^2 = 1/2$, $N_D = 4$ on BCC correspond to a recognized lattice action?
(c) Is there a structural reason why the mass ratio should be sensitive to coordination number in this way?
(d) Is the SC/BCC near-degeneracy for α^{-1} (via $\langle D^2 \rangle = 2d$) interesting or expected?

I would be grateful for any feedback, including identification of a trivial explanation. Full paper (LaTeX) available on request.

Appendix: Verification Code (Python)

```python
import numpy as np
def tau(n): return n*(2*n+1)+1
def sigma(n): return n*(2*n+1)
def mu(n): return 1.5*sigma(n)*(n+1)
N=8; TAU,SIGMA,MU = tau(N),sigma(N),mu(N)
B = TAU + np.pi**2/(2*TAU) - 1/(2*TAU**2) - 1/((N-1)*TAU**3)
alpha_inv = B + 1/((N+2)*B**2); alpha = 1/alpha_inv
t1 = np.pi*MU*SIGMA*alpha**3/4
t2 = (2*N+1)*np.pi**2*alpha*(1-alpha**2)/16
mass = MU + t1 + t2 - alpha**2
print(f"alpha^-1 = {alpha_inv:.9f}")  # 137.035999084
print(f"m_p/m_e  = {mass:.6f}")       # 1836.152674
```

Contact: alan.javier.garcia@gmail.com — Full paper and working notes available on request