Numerical Coincidences from a BCC Lattice Framework: A Concise Summary for Discussion

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(Dated: February 4, 2026)

Abstract

We present a lattice field theory framework in which the fine structure constant and the proton-electron mass ratio emerge from the spectral geometry of the body-centered cubic (BCC) lattice Dirac operator. The framework takes two inputs—the BCC coordination number $n = 8$ and the transcendental π—and produces values for both constants with zero free parameters, achieving agreement with CODATA 2018 values to better than $0.005\,\text{ppb}$ and $0.03\,\text{ppb}$, respectively. We compare the BCC lattice against simple cubic (SC) and face-centered cubic (FCC) alternatives and find that only the BCC lattice simultaneously reproduces both constants. This document is intended as a summary for discussion and does not constitute a claim of proof.

I. INTRODUCTION

This work explores a lattice field theory framework in which the fine structure constant and the proton-electron mass ratio emerge from the spectral geometry of the BCC lattice Dirac operator. The framework takes two inputs—the BCC coordination number $n = 8$ and the transcendental π—and produces values for both constants with zero free parameters. We pose the question: is there a trivial explanation for the coincidences presented below, or does the lattice selection mechanism have structural content?

II. GENERATOR FUNCTIONS FROM SO(3) REPRESENTATION THEORY

Four functions of the coordination number n arise from the representation theory of SO(3):

$$\tau(n) = n(2n+1) + 1, \qquad \sigma(n) = n(2n+1), \qquad \rho(n) = n + 1, \qquad \mu(n) = \tfrac{3}{2}\,\sigma\,\rho. \qquad (1)$$

At $n = 8$: $\tau = 137$, $\sigma = 136$, $\rho = 9$, $\mu = 1836$. Here $\tau(n) = \dim\!\big(\Lambda^2 D(n) \oplus D(0)\big)$, where $\Lambda^2 D(n)$ is the antisymmetric square of the spin-n representation, and $\sigma(n) = \binom{2n+1}{2}$ counts the pairwise couplings among $2n+1$ magnetic substates.

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III.   RESULT 1: THE FINE STRUCTURE CONSTANT

The inverse fine structure constant α^{-1} is obtained from a cubic equation interpreted as a Dyson equation $G^{-1} = G_0^{-1} - \Sigma(G)$ for the dressed inverse coupling:

$$\boxed{\alpha^{-1} = B + \frac{1}{(n+2)\,B^2}}, \qquad B = \tau + \frac{\pi^2}{2\tau} - \frac{1}{2\tau^2} - \frac{1}{(n-1)\,\tau^3}. \tag{2}$$

The base B is a perturbative self-energy expansion on the BCC lattice with coupling $g^2 = 1/2$ and Dirac multiplicity $N_D = 4$. The coefficients have identified origins: $c_1 = \pi^2/2$ from the Brillouin zone momentum integral $\langle |\mathbf{q}|^2 \rangle_{\mathrm{BZ}}$; $c_2 = -d/\langle D^2 \rangle = -1/2$ from the Dirac spectral moment; $c_3 = -1/(n-1) = -1/7$ from the spectral variance, with $f_3(8) = -2/7$ as an exact algebraic identity.

TABLE I. Fine structure constant comparison.

Lattice framework prediction	137.035 999 084
CODATA 2018	137.035 999 084(21)
Agreement	< 0.005 ppb

IV.   RESULT 2: THE PROTON-ELECTRON MASS RATIO

From the same generator functions, with α computed self-consistently:

$$\boxed{\frac{m_p}{m_e} = \mu + \frac{\pi\mu\sigma\alpha^3}{4} + \frac{(2n+1)\,\pi^2\alpha(1-\alpha^2)}{16} - \alpha^2}. \tag{3}$$

The corrections decompose as a connection 1-form (vertex, $\pi\mu\sigma\alpha^3/4 \approx 0.0762$), a curvature 2-form (vacuum polarization, $(2n+1)\pi^2\alpha(1-\alpha^2)/16 \approx 0.0765$), and a self-intersection ($-\alpha^2 \approx -5.3 \times 10^{-5}$). The factor $(1-\alpha^2)$ is a dressed self-energy correction. The tree-level integer $\mu(8) = 1836$ is $\frac{3}{2}\dim(\Lambda^2 D(8)) \cdot (\rho(8))$ from SO(3).

TABLE II. Proton-electron mass ratio comparison.

Lattice framework prediction	1836.152 674
CODATA 2018	1836.152 673 43(11)
Agreement	< 0.03 ppb

V.   LATTICE SELECTION: WHY BCC?

The framework was tested against simple cubic (SC, $n = 6$) and face-centered cubic (FCC, $n = 12$) lattices. The tree-level mass generator $\mu(n)$ is the discriminator:

TABLE III. Lattice comparison. Only BCC passes both tests.

Lattice	n	$\mu(n)$	α^{-1} error	Mass error	Status
BCC	8	1836	$< 0.005\,\mathrm{ppb}$	$< 0.03\,\mathrm{ppb}$	Passes both
SC	6	819	$9.9\,\mathrm{ppb}$	-55.4%	Fails mass
FCC	12	5850	$142.7\,\mathrm{ppb}\ (6.8\sigma)$	$+218.6\%$	Fails both

SC's α^{-1} closeness is explained by $\langle D^2 \rangle = 2d$ universality for cubic lattices (first two loop corrections identical). Discrimination enters at three loops via spectral variance and, decisively, through $\mu(n)$: no perturbative correction bridges SC's 55% mass deficit.

VI.   QUESTIONS FOR THE READER

(a) Is there a trivial or known reason why polynomials of 8 produce integers close to 137 and 1836?

(b) Does $g^2 = 1/2$, $N_D = 4$ on BCC correspond to a recognized lattice action?

(c) Is there a structural reason why the mass ratio should be sensitive to coordination number in this way?

(d) Is the SC/BCC near-degeneracy for α^{-1} (via $\langle D^2 \rangle = 2d$) interesting or expected?

The author would be grateful for any feedback, including identification of a trivial explanation. Full working notes are available on request.

Appendix A: Verification Code (Python)

```python
import numpy as np

def tau(n): return n*(2*n+1)+1

def sigma(n): return n*(2*n+1)

def mu(n): return 1.5*sigma(n)*(n+1)

N=8; TAU,SIGMA,MU = tau(N),sigma(N),mu(N)

B = TAU + np.pi**2/(2*TAU) - 1/(2*TAU**2) - 1/((N-1)*TAU**3)

alpha_inv = B + 1/((N+2)*B**2); alpha = 1/alpha_inv

t1 = np.pi*MU*SIGMA*alpha**3/4

t2 = (2*N+1)*np.pi**2*alpha*(1-alpha**2)/16

mass = MU + t1 + t2 - alpha**2

print(f"alpha^-1 = {alpha_inv:.9f}")   # 137.035999084

print(f"m_p/m_e  = {mass:.6f}")         # 1836.152674
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[1] E. Tiesinga, P. J. Mohr, D. B. Newell, and B. N. Taylor, "CODATA recommended values of the fundamental physical constants: 2018," *Rev. Mod. Phys.* **93**, 025010 (2021).