Numerical Coincidences from a BCC Lattice Framework

A concise summary for discussion — not a claim of proof

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1. What This Is

I am an independent researcher exploring a lattice field theory framework in which the fine structure constant, the proton-electron mass ratio, and the muon-electron mass ratio emerge from the spectral geometry of the BCC lattice Dirac operator. The framework takes two inputs—the BCC coordination number $n = 8$ and the transcendental π—and produces values for all three constants with zero free parameters. I am writing to ask whether you can see a trivial explanation for the coincidences below, or whether the lattice selection mechanism has structural content.

2. Generator Functions from $\mathrm{SO}(3)$ Representation Theory

Four functions of the coordination number n arise from the representation theory of SO(3):

$$\tau(n) = n(2n+1) + 1 \qquad \sigma(n) = n(2n+1) \qquad \rho(n) = n + 1 \qquad \mu(n) = \tfrac{3}{2}\,\sigma\,\rho$$

At $n = 8$: $\tau = 137$, $\sigma = 136$, $\rho = 9$, $\mu = 1836$. Here $\tau(n) = \dim\big(\Lambda^2 D(n) \oplus D(0)\big)$, where $\Lambda^2 D(n)$ is the antisymmetric square of the spin-n representation, and $\sigma(n) = \binom{2n+1}{2}$ counts the pairwise couplings among $2n+1$ magnetic substates.

3. Result 1: The Fine Structure Constant

α^{-1} is obtained from a cubic equation interpreted as a Dyson equation $G^{-1} = G_0^{-1} - \Sigma(G)$ for the dressed inverse coupling:

$$\boxed{\alpha^{-1} = B + \frac{1}{(n+2)\,B^2}}, \qquad B = \tau + \frac{c_1}{\tau} - \frac{1}{2\tau^2} - \frac{1}{(n-1)\,\tau^3} + \frac{c_4}{\tau^4}$$

The base B is a perturbative self-energy expansion on the BCC lattice with coupling $g^2 = 1/2$ and Dirac multiplicity $N_D = 4$. The coefficients have identified spectral origins:

- $c_1 = \pi^2/2$ Brillouin zone momentum integral $\langle|\mathbf{q}|^2\rangle_{\mathrm{BZ}}$
- $c_2 = -\frac{1}{2}$ Spectral mean: $-d/\langle D^2\rangle$
- $c_3 = -1/(n-1) = -\frac{1}{7}$ Spectral variance
- $c_4 = \frac{2\rho}{2n+1}\,\pi^3 = \frac{18}{17}\,\pi^3$ Spectral skewness × zone integral

Lattice framework prediction	137.035 999 177
CODATA 2022	137.035 999 177(21)
Agreement	< 0.001 ppb

4. Result 2: The Proton-Electron Mass Ratio

From the same generator functions, with α computed self-consistently. The formula appears naturally in n-explicit factored form:

$$\boxed{\frac{m_p}{m_e} = \mu\big(1 + 2(2n{+}1)\,\pi\alpha^3\big) + \Big(1 + \frac{1}{2n}\Big)\pi^2\alpha(1{-}\alpha^2) - \alpha^2}$$

The denominators in the original notation ($\pi\mu\sigma\alpha^3/4$ and $(2n{+}1)\pi^2\alpha/16$) are functions of the coordination number: $4 = n/2$ and $16 = 2n$ at $n = 8$. The corrections decompose as:

- **Vertex:** μ is dressed by a factor $1 + 2(2n{+}1)\pi\alpha^3$, where $2(2n{+}1) = 2\dim D(n)$.
- **VP:** A universal piece $\pi^2\alpha(1{-}\alpha^2)$ plus a lattice correction of relative size $1/(2n)$, vanishing as $n \to \infty$ (continuum limit).
- **Self-intersection:** $-\alpha^2$, universal.

Lattice framework prediction	1836.152 673 5
CODATA 2022	1836.152 673 426(32)
Agreement	< 0.03 ppb

5. Result 3: The Muon-Electron Mass Ratio

The muon is modeled as a binary defect excitation (two-node) on the same lattice, in contrast to the proton's ternary defect (three-node). Its formula follows the same correction taxonomy but with d (spatial dimension) replacing n (coordination number) as the structural parameter:

$$\boxed{\frac{m_\mu}{m_e} = \frac{d}{2}\big(\tau + \pi\alpha(1{-}\alpha^2)\big) + \frac{c_1}{4} + \frac{d}{2}\,\pi\sigma\alpha^3 - \alpha^2}$$

The proton's vertex prefactor is $2/n$ (per-bond weight on a lattice with n neighbors); the muon's is $d/2$ (dimensional weight in d-dimensional space). The muon vertex coefficient $(d/2)\,\pi\,\sigma$ equals the proton vertex coefficient times $2d/\mu = 6/1836 = 1/306$.

Lattice framework prediction	206.768 282 5
CODATA 2022	206.768 282 7(46)
Agreement	1.1 ppb (0.05σ)

6. Lattice Selection: Why BCC?

Tested against SC ($n = 6$) and FCC ($n = 12$). The tree-level mass generator $\mu(n)$ is the discriminator:

Lattice	n	$\mu(n)$	α^{-1} **error**	**Mass error**	**Status**
BCC	8	1836	< 0.001 ppb	< 0.03 ppb	Passes all
SC	6	819	9.9 ppb	-55.4%	Fails mass
FCC	12	5850	142.7 ppb (6.8σ)	$+218.6\%$	Fails both

SC's α^{-1} closeness is explained by $\langle D^2\rangle = 2d$ universality for cubic lattices (first two loop corrections identical). Discrimination enters at three loops via spectral variance and, decisively, through $\mu(n)$: no perturbative correction bridges SC's 55% mass deficit.

7. Questions for the Reader

(a) Is there a trivial or known reason why polynomials of 8 produce integers close to 137 and 1836?
(b) Does $g^2 = 1/2$, $N_D = 4$ on BCC correspond to a recognized lattice action?
(c) The proton vertex denominator is $n/2$ and the VP denominator is $2n$. Does this n-dependence have precedent in lattice perturbation theory?
(d) The proton's structural parameter is n (coordination); the muon's is d (dimension). Does this topological/geometric distinction correspond to anything in defect field theory?
(e) Is the SC/BCC near-degeneracy for α^{-1} (via $\langle D^2 \rangle = 2d$) interesting or expected?

I would be grateful for any feedback, including identification of a trivial explanation. Full paper (LaTeX) available on request.

Appendix: Verification Code (Python)

```
import numpy as np
def tau(n): return n*(2*n+1)+1
def sigma(n): return n*(2*n+1)
def mu(n): return 1.5*sigma(n)*(n+1)
N = 8; d = 3
TAU, SIGMA, RHO, MU = tau(N), sigma(N), N+1, mu(N)
c1 = np.pi**2 / 2
c4 = (2*RHO/(2*N+1)) * np.pi**3
B = TAU + c1/TAU - 1/(2*TAU**2) - 1/((N-1)*TAU**3) + c4/TAU**4
alpha_inv = B + 1/((N+2)*B**2); alpha = 1/alpha_inv
mass = MU*(1+2*(2*N+1)*np.pi*alpha**3) \
       + (1+1/(2*N))*np.pi**2*alpha*(1-alpha**2) - alpha**2
muon = (d/2)*(TAU+np.pi*alpha*(1-alpha**2)) \
       + c1/4 + (d/2)*np.pi*SIGMA*alpha**3 - alpha**2
print(f"alpha^-1  = {alpha_inv:.12f}")  # 137.035999177
print(f"m_p/m_e   = {mass:.9f}")        # 1836.152673485
print(f"m_mu/m_e  = {muon:.10f}")       # 206.7682824754
```

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