Numerical Coincidences from a BCC Lattice Framework

A concise summary for discussion — not a claim of proof

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1. What This Is

I am an independent researcher exploring a lattice field theory framework in which the fine structure constant, the proton-electron mass ratio, the muon-electron mass ratio, the tau-electron mass ratio, the Higgs boson mass, and the neutron-proton mass difference emerge from the spectral geometry of the BCC lattice Dirac operator. The framework takes two inputs—the BCC coordination number $n = 8$ and the transcendental π—with $d = 3$ (spatial dimensions) entering the muon, tau, Higgs, and neutron formulas, and produces values for all six constants with zero free parameters. The Higgs prediction (125.108 GeV) is genuinely predictive—it selects the ATLAS value over CMS and is falsifiable at the HL-LHC. I am writing to ask whether you can see a trivial explanation for the coincidences below, or whether the lattice selection mechanism has structural content.

2. Generator Functions from $\mathrm{SO}(3)$ Representation Theory

Four functions of the coordination number n arise from the representation theory of SO(3):

$$\tau(n) = n(2n{+}1) + 1 \qquad \sigma(n) = n(2n{+}1) \qquad \rho(n) = n + 1 \qquad \mu(n) = \tfrac{3}{2}\,\sigma\,\rho$$

At $n = 8$: $\tau = 137$, $\sigma = 136$, $\rho = 9$, $\mu = 1836$. Here $\tau(n) = \dim\big(\Lambda^2 D(n) \oplus D(0)\big)$, where $\Lambda^2 D(n)$ is the antisymmetric square of the spin-n representation, and $\sigma(n) = \binom{2n+1}{2}$ counts the pairwise couplings among $2n{+}1$ magnetic substates.

3. Result 1: The Fine Structure Constant

α^{-1} is obtained from a cubic equation interpreted as a Dyson equation $G^{-1} = G_0^{-1} - \Sigma(G)$ for the dressed inverse coupling:

$$\boxed{\alpha^{-1} = B + \frac{1}{(n{+}2)\,B^2}}, \qquad B = \tau + \frac{c_1}{\tau} - \frac{1}{2\tau^2} - \frac{1}{(n{-}1)\,\tau^3} + \frac{c_4}{\tau^4}$$

The base B is a perturbative self-energy expansion on the BCC lattice with coupling $g^2 = 1/2$ and Dirac multiplicity $N_D = 4$. The coefficients have identified spectral origins:

- $c_1 = \pi^2/2$ Brillouin zone momentum integral $\langle|\mathbf{q}|^2\rangle_{\mathrm{BZ}}$
- $c_2 = -\frac{1}{2}$ Spectral mean: $-d/\langle D^2\rangle$
- $c_3 = -1/(n{-}1) = -\frac{1}{7}$ Spectral variance
- $c_4 = \frac{2\rho}{2n+1}\,\pi^3 = \frac{18}{17}\,\pi^3$ Spectral skewness $\times$ zone integral

Lattice framework prediction	137.035 999 177
CODATA 2022	137.035 999 177(21)
Agreement	< 0.001 ppb

4. Result 2: The Proton-Electron Mass Ratio

From the same generator functions, with α computed self-consistently. The formula appears naturally in n-explicit factored form:

$$\boxed{\frac{m_p}{m_e} = \mu\big(1 + 2(2n{+}1)\,\pi\alpha^3\big) + \Big(1 + \frac{1}{2n}\Big)\pi^2\alpha(1{-}\alpha^2) - \alpha^2}$$

The denominators in the original notation ($\pi\mu\sigma\alpha^3/4$ and $(2n{+}1)\pi^2\alpha/16$) are functions of the coordination number: $4 = n/2$ and $16 = 2n$ at $n = 8$. The corrections decompose as:

- **Vertex:** μ is dressed by a factor $1 + 2(2n{+}1)\pi\alpha^3$, where $2(2n{+}1) = 2\dim D(n)$.
- **VP:** A universal piece $\pi^2\alpha(1{-}\alpha^2)$ plus a lattice correction of relative size $1/(2n)$, vanishing as $n \to \infty$ (continuum limit).
- **Self-intersection:** $-\alpha^2$, universal.

Lattice framework prediction	1836.152 673 5
CODATA 2022	1836.152 673 426(32)
Agreement	$<$ 0.03 ppb

5. Result 3: The Muon-Electron Mass Ratio

The muon is modeled as a binary defect excitation (two-node) on the same lattice, in contrast to the proton's ternary defect (three-node). Its formula follows the same correction taxonomy but with d (spatial dimension) replacing n (coordination number) as the structural parameter:

$$\boxed{\frac{m_\mu}{m_e} = \frac{d}{2}\big(\tau + \pi\alpha(1{-}\alpha^2)\big) + \frac{c_1}{4} + \frac{d}{2}\,\pi\sigma\alpha^3 - \alpha^2}$$

The proton's vertex prefactor is $2/n$ (per-bond weight on a lattice with n neighbors); the muon's is $d/2$ (dimensional weight in d-dimensional space). The muon vertex coefficient $(d/2)\,\pi\,\sigma$ equals the proton vertex coefficient times $2d/\mu = 6/1836 = 1/306$.

Lattice framework prediction	206.768 282 5	
CODATA 2022	206.768 282 7(46)	
Agreement	1.1 ppb	(0.05σ)

6. Result 4: The Tau-Electron Mass Ratio

The tau lepton mass is predicted by dressing the Koide relation. Yoshio Koide (1982) observed empirically that $(m_e + m_\mu + m_\tau)/(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2 \approx 2/3$. In the lattice framework, the binary defect is a 1-dimensional line segment in d spatial dimensions; its normal bundle has fiber dimension $d{-}1 = 2$, giving a transverse fraction $(d{-}1)/d = 2/3$. This tree-level constant receives a radiative correction from the same spectral ingredients that enter the α formula:

$$\boxed{Q = \frac{d{-}1}{d} + \frac{(d{-}1)\,\pi^2\alpha^2}{(n{-}1)\,\sigma}}, \qquad \frac{m_e + m_\mu + m_\tau}{\big(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}\big)^2} = Q$$

The correction $\delta Q = (d{-}1)\pi^2\alpha^2/\big((n{-}1)\sigma\big)$ decomposes as:

- $(d-1)$ — transverse codimension (number of normal directions)
- π^2 — BZ momentum integral (same π^2 as c_1 in Result 1)
- α^2 — two-loop EM coupling
- $1/(n-1) = 1/7$ — spectral variance (same as c_3 in Result 1)
- $1/\sigma = 1/136$ — pairwise channel normalization

With $m_e = 1$ and m_μ from Result 3, the Koide equation reduces to a quadratic in $x = \sqrt{m_\tau}$ with closed-form solution:

$$x = \frac{Q(1 + \sqrt{m_\mu}\,) + \sqrt{(1 + m_\mu)(2Q-1) + 2Q\sqrt{m_\mu}}}{1 - Q}, \qquad m_\tau = x^2$$

Lattice framework prediction	3477.4799
CODATA 2022	3477.48 ± 0.57
Agreement	0.027 ppm (0.0002σ)

Note: The bare Koide constant $Q = 2/3$ gives $m_\tau/m_e = 3477.44$ (11 ppm, 0.07σ). The dressed Q improves this by a factor of 415. The correction was identified by systematic numerical scan and interpreted geometrically as the leading Seeley–DeWitt heat kernel coefficient of the transverse Laplacian on the binary defect; a first-principles derivation from the lattice Lagrangian remains an open problem.

7. Result 5: The Higgs Boson Mass

The Higgs boson is modeled as the *breathing mode* of the lattice—the uniform oscillation of lattice spacing about equilibrium. Each site has $\sigma = n(2n+1) = 136$ pairwise scalar modes; $d = 3$ become Goldstone bosons (eaten by $W^\pm$, Z), leaving $\sigma - d = 133$ physical scalar channels. The Higgs mass counts these surviving channels in proton-mass units, with a universal vacuum polarization correction:

$$\boxed{\frac{m_H}{m_p} = (\sigma - d)\left(1 + \frac{\pi\alpha}{\rho}\right)}$$

where $\pi\alpha$ is the one-loop VP factor and $\rho = n+1 = 9$ counts the radial shells in the spin-n representation. The tree level gives $133 \times 0.9383 = 124.79$ GeV (0.26% low); the correction raises this to:

$$m_H^{\mathrm{BSM}} = 133.339 \times 0.938272\,\mathrm{GeV} = 125.108\,\mathrm{GeV}$$

Experiment	**Measured (GeV)**	**BSM (GeV)**	**Deviation**
ATLAS (combined)	125.11 ± 0.11	125.108	0.02σ
CMS (combined)	125.35 ± 0.15	125.108	1.6σ

The BSM prediction matches ATLAS to 0.02σ but is 1.6σ from CMS—the framework *picks a side* in the ATLAS/CMS tension (1.3σ). The HL-LHC, projecting 21 MeV precision by 2041, will confirm or rule out this prediction: convergence toward 125.11 GeV confirms it (0.1σ); convergence toward 125.35 GeV rules it out (11.5σ). This is the first BSM result that is more precise than experiment and genuinely predictive rather than postdictive.

8. Result 6: The Neutron-Proton Mass Difference

The neutron-proton mass difference $\Delta m = m_n - m_p = 1.29334$ MeV $= 2.531\,030\ m_e$ is an $O(1)$ quantity in electron mass units—not a perturbative correction. In the BSM framework, the proton and neutron are the same ternary defect differing in *orientation*: how the triangle sits among the $\binom{n-1}{d} = \binom{7}{3} = 35$ possible spatial triads.

$$\boxed{\frac{\Delta m}{m_e} = \binom{n-1}{d}\pi^2\alpha\left(1 + \frac{(n-d)\,\alpha}{\rho}\right) + \frac{\rho}{2n}\,\pi\alpha^2 - \alpha^2}$$

The corrections follow the universal BSM taxonomy:

- **Tree:** $\binom{7}{3}\pi^2\alpha = 35\,\pi^2\alpha = 2.5208\ m_e$ (orientational modes × EM lattice coupling). 0.41% below experiment.
- **Vertex:** $(n-d)\alpha/\rho = 5\alpha/9$ (spectator bonds / radial modes). Closes to 37 ppm.
- **VP:** $(\rho/2n)\,\pi\alpha^2 = \frac{9}{16}\pi\alpha^2$ (cross-references Higgs and proton channels). Closes to 0.13 ppm.
- **Self-intersection:** $-\alpha^2$, universal.

Lattice framework prediction	2.531 030
Experiment	2.531 030(3)
Agreement	0.13 ppm

The binomial coefficient $\binom{n-1}{d}$ is the sharpest lattice discriminator: SC ($n = 6$) gives $\binom{5}{3} = 10$, predicting 50% of experiment; FCC ($n = 12$) gives $\binom{11}{3} = 165$, predicting 114% above experiment. Only BCC works.

9. The Vacuum Polarization Catalog

Each BSM mass formula contains exactly one VP correction, governed by a distinct geometric generator. The four base generators—τ, $2n$, d, ρ—are each assigned to exactly one sector (*non-overlap principle*):

Observable	**VP correction**	**Generator**	**Structure**
α^{-1}	$c_1/\tau = \pi^2/(2\tau)$	$\tau = 137$	Topological
m_p/m_e	$(1 + 1/(2n))\pi^2\alpha(1-\alpha^2)$	$2n = 16$	Coordination
m_μ/m_e	$(d/2)\pi\alpha(1-\alpha^2)$	$d = 3$	Spatial dimension
m_H/m_p	$\pi\alpha/\rho$	$\rho = 9$	Radial
$\Delta m/m_e$	$(\rho/2n)\pi\alpha^2$	$\rho/(2n)$	Cross-reference

The Δm VP uses $\rho/(2n)$, a product of two generators; it is the only two-loop VP (α^2), cross-referencing the Higgs (ρ) and proton ($2n$) channels. This non-overlap principle resolves the Higgs correction ambiguity: $\pi\alpha/\rho$ is the unique choice that preserves generator injectivity (the alternatives $\pi\alpha/(2d)$ and $\pi\alpha/n$ reuse generators already assigned to the muon and proton sectors).

10. Lattice Selection: Why BCC?

Tested against SC ($n = 6$) and FCC ($n = 12$). The tree-level mass generator $\mu(n)$ is the discriminator:

Lattice	n	$\mu(n)$	$\binom{n-1}{d}$	α^{-1} error	Mass error	Status
BCC	8	1836	35	< 0.001 ppb	< 0.03 ppb	Passes all
SC	6	819	10	9.9 ppb	-55.4%	Fails mass
FCC	12	5850	165	142.7 ppb (6.8σ)	$+218.6\%$	Fails both

SC's α^{-1} closeness is explained by $\langle D^2\rangle = 2d$ universality for cubic lattices (first two loop corrections identical). Discrimination enters at three loops via spectral variance and, decisively, through $\mu(n)$: no perturbative correction bridges SC's 55% mass deficit.

11. Questions for the Reader

(a) Is there a trivial or known reason why polynomials of 8 produce integers close to 137 and 1836?
(b) Does $g^2 = 1/2$, $N_D = 4$ on BCC correspond to a recognized lattice action?
(c) The proton vertex denominator is $n/2$ and the VP denominator is $2n$. Does this n-dependence have precedent in lattice perturbation theory?
(d) The proton's structural parameter is n (coordination); the muon's is d (dimension). Does this topological/geometric distinction correspond to anything in defect field theory?
(e) Is the SC/BCC near-degeneracy for α^{-1} (via $\langle D^2\rangle = 2d$) interesting or expected?
(f) The dressed Koide correction uses $1/(n-1)$ (spectral variance) and $1/\sigma$ (pairwise channels)—the same quantities appearing in the α^{-1} expansion. Is there a known mechanism by which a constraint on mass *ratios* receives radiative corrections from the same spectral coefficients as a coupling?
(g) The interpretation $Q = (d-1)/d$ identifies the Koide constant with the transverse fraction of a line defect. Does this codimensional constraint have precedent in defect field theory or conformal field theory?
(h) The 35 orientational triads ($\binom{7}{3}$) provide a lattice analogue of isospin. Does the mapping to SU(2) isospin—two states from 35 modes—have a natural group-theoretic explanation?
(i) The VP catalog assigns four generators (τ, $2n$, d, ρ) injectively to four mass sectors. Is such a non-overlap structure expected from gauge invariance or a selection rule?

I would be grateful for any feedback, including identification of a trivial explanation. Full paper (LaTeX) available on request.

Appendix: Verification Code (Python)

```
from mpmath import mp, mpf, pi, sqrt
from math import comb
mp.dps = 50
def tau(n): return n*(2*n+1)+1
def sigma(n): return n*(2*n+1)
def mu(n): return mpf(3)/2*sigma(n)*(n+1)
N = 8; d = 3
TAU, SIGMA, RHO, MU = mpf(tau(N)), mpf(sigma(N)), mpf(N+1), mu(N)
c1 = pi**2 / 2
c4 = (2*RHO/(2*N+1)) * pi**3
B = TAU + c1/TAU - 1/(2*TAU**2) - 1/((N-1)*TAU**3) + c4/TAU**4
alpha_inv = B + 1/((N+2)*B**2); alpha = 1/alpha_inv
mass = MU*(1+2*(2*N+1)*pi*alpha**3) \
       + (1+1/(2*N))*pi**2*alpha*(1-alpha**2) - alpha**2
muon = mpf(d)/2*(TAU+pi*alpha*(1-alpha**2)) \
       + c1/4 + mpf(d)/2*pi*SIGMA*alpha**3 - alpha**2
Q = mpf(d-1)/d + mpf(d-1)*pi**2*alpha**2/((N-1)*SIGMA)
```

```
b = sqrt(muon)
D = (1 + muon)*(2*Q - 1) + 2*Q*b
tau_mass = ((Q*(1+b) + sqrt(D)) / (1-Q))**2
higgs = (SIGMA - d) * (1 + pi*alpha/RHO)
dm_tree = comb(N-1, d) * pi**2 * alpha
dm = dm_tree*(1 + (N-d)*alpha/RHO) + (RHO/(2*N))*pi*alpha**2 - alpha**2
print(f"alpha^-1  = {alpha_inv}")   # 137.035999177...
print(f"m_p/m_e   = {mass}")        # 1836.15267348...
print(f"m_mu/m_e  = {muon}")        # 206.768282475...
print(f"m_tau/m_e = {tau_mass}")    # 3477.47990762...
print(f"m_H/m_p   = {higgs}")       # 133.339 (125.108 GeV)
print(f"Dm/m_e    = {dm}")          # 2.531030 (expt: 2.531030(3))
```

Contact: alan.javier.garcia@gmail.com — Full paper and working notes available on request