Numerical Coincidences from a BCC Lattice Framework: A Concise Summary for Discussion

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Abstract

We present a lattice field theory framework in which the fine structure constant, the proton-electron mass ratio, and the muon-electron mass ratio emerge from the spectral geometry of the body-centered cubic (BCC) lattice Dirac operator. The framework takes two inputs—the BCC coordination number $n = 8$ and the transcendental π—and produces values for all three constants with zero free parameters, achieving agreement with CODATA 2018 values to better than 0.005 ppb, 0.03 ppb, and 12.6 ppb (0.6σ), respectively. We compare the BCC lattice against simple cubic (SC) and face-centered cubic (FCC) alternatives and find that only the BCC lattice simultaneously reproduces the observed constants. This document is intended as a summary for discussion and does not constitute a claim of proof.

I. INTRODUCTION

This work explores a lattice field theory framework in which the fine structure constant, the proton-electron mass ratio, and the muon-electron mass ratio emerge from the spectral geometry of the BCC lattice Dirac operator. The framework takes two inputs—the BCC coordination number $n = 8$ and the transcendental π—and produces values for all three constants with zero free parameters. We pose the question: is there a trivial explanation for the coincidences presented below, or does the lattice selection mechanism have structural content?

II. GENERATOR FUNCTIONS FROM SO(3) REPRESENTATION THEORY

Four functions of the coordination number n arise from the representation theory of SO(3):

$$\tau(n) = n(2n+1) + 1, \qquad \sigma(n) = n(2n+1), \qquad \rho(n) = n + 1, \qquad \mu(n) = \tfrac{3}{2}\,\sigma\,\rho. \tag{1}$$

At $n = 8$: $\tau = 137$, $\sigma = 136$, $\rho = 9$, $\mu = 1836$. Here $\tau(n) = \dim\big(\Lambda^2 D(n) \oplus D(0)\big)$, where $\Lambda^2 D(n)$ is the antisymmetric square of the spin-n representation, and $\sigma(n) = \binom{2n+1}{2}$ counts the pairwise couplings among $2n+1$ magnetic substates.

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III. RESULT 1: THE FINE STRUCTURE CONSTANT

The inverse fine structure constant α^{-1} is obtained from a cubic equation interpreted as a Dyson equation $G^{-1} = G_0^{-1} - \Sigma(G)$ for the dressed inverse coupling:

$$\boxed{\alpha^{-1} = B + \frac{1}{(n+2)\, B^2}}, \qquad B = \tau + \frac{\pi^2}{2\tau} - \frac{1}{2\tau^2} - \frac{1}{(n-1)\,\tau^3}. \tag{2}$$

The base B is a perturbative self-energy expansion on the BCC lattice with coupling $g^2 = 1/2$ and Dirac multiplicity $N_D = 4$. The coefficients have identified origins: $c_1 = \pi^2/2$ from the Brillouin zone momentum integral $\langle|\mathbf{q}|^2\rangle_{\mathrm{BZ}}$; $c_2 = -d/\langle D^2\rangle = -1/2$ from the Dirac spectral moment; $c_3 = -1/(n-1) = -1/7$ from the spectral variance, with $f_3(8) = -2/7$ as an exact algebraic identity.

TABLE I. Fine structure constant comparison.

Lattice framework prediction	137.035 999 084
CODATA 2018	137.035 999 084(21)
Agreement	< 0.005 ppb

IV. RESULT 2: THE PROTON-ELECTRON MASS RATIO

From the same generator functions, with α computed self-consistently:

$$\boxed{\frac{m_p}{m_e} = \mu + \frac{\pi\mu\sigma\alpha^3}{4} + \frac{(2n+1)\,\pi^2\alpha(1-\alpha^2)}{16} - \alpha^2}. \tag{3}$$

The corrections decompose as a connection 1-form (vertex, $\pi\mu\sigma\alpha^3/4 \approx 0.0762$), a curvature 2-form (vacuum polarization, $(2n+1)\pi^2\alpha(1-\alpha^2)/16 \approx 0.0765$), and a self-intersection ($-\alpha^2 \approx -5.3\times 10^{-5}$). The factor $(1-\alpha^2)$ is a dressed self-energy correction. The tree-level integer $\mu(8) = 1836$ is $\frac{3}{2}\dim(\Lambda^2 D(8)) \cdot (\rho(8))$ from SO(3).

TABLE II. Proton-electron mass ratio comparison.

Lattice framework prediction	1836.152 674
CODATA 2018	1836.152 673 43(11)
Agreement	< 0.03 ppb

V. RESULT 3: THE MUON-ELECTRON MASS RATIO

The framework also predicts the muon-electron mass ratio using the same lattice constants. Where the proton is a bound state whose vacuum polarization term is dressed *subtractively* by $(1-\alpha^2)$, the muon is an excitation whose vacuum correction is dressed *additively* by $(1+d\,\alpha^2)$, where $d=3$ is the spatial dimension:

$$\boxed{\frac{m_\mu}{m_e} = \frac{d}{2}\big(\tau+\pi\alpha\big) + \frac{c_1}{4}\big(1+d\,\alpha^2\big)}, \qquad c_1 = \frac{\pi^2}{2}. \tag{4}$$

The tree-level term $(d/2)\,\tau = 205.5$ sets the scale. The electromagnetic correction $(d/2)\,\pi\alpha$ is a first-order dressing of the topological scale by the coupling. The Brillouin zone integral $c_1/4 = \pi^2/8$ enters as the vacuum correction, and its dressing factor $(1+d\,\alpha^2)$ is the additive counterpart to the proton's $(1-\alpha^2)$: physically, the muon as an excitation *gains* energy from dimensional coupling, whereas the proton as a bound state *loses* energy to self-interaction.

TABLE III. Muon-electron mass ratio comparison.

Lattice framework prediction	206.768 286
Experimental	206.768 283 0(46)
Agreement	12.6 ppb (0.6σ)

VI. LATTICE SELECTION: WHY BCC?

The framework was tested against simple cubic (SC, $n=6$) and face-centered cubic (FCC, $n=12$) lattices. The tree-level mass generator $\mu(n)$ is the discriminator:

TABLE IV. Lattice comparison. Only BCC passes both tests.

Lattice	n	$\mu(n)$	α^{-1} **error**	**Mass error**	**Status**
BCC	8	1836	< 0.005 ppb	< 0.03 ppb	Passes both
SC	6	819	9.9 ppb	-55.4%	Fails mass
FCC	12	5850	142.7 ppb (6.8σ)	$+218.6\%$	Fails both

SC's α^{-1} closeness is explained by $\langle D^2\rangle = 2d$ universality for cubic lattices (first two loop corrections identical). Discrimination enters at three loops via spectral variance and, decisively, through $\mu(n)$: no perturbative correction bridges SC's 55% mass deficit.

VII. QUESTIONS FOR THE READER

(a) Is there a trivial or known reason why polynomials of 8 produce integers close to 137 and 1836?

(b) Does $g^2 = 1/2$, $N_D = 4$ on BCC correspond to a recognized lattice action?

(c) Is there a structural reason why the mass ratio should be sensitive to coordination number in this way?

(d) Is the SC/BCC near-degeneracy for α^{-1} (via $\langle D^2 \rangle = 2d$) interesting or expected?

The author would be grateful for any feedback, including identification of a trivial explanation. Full working notes are available on request.

Appendix A: Verification Code (Python)

```
import numpy as np

def tau(n): return n*(2*n+1)+1

def sigma(n): return n*(2*n+1)

def mu(n): return 1.5*sigma(n)*(n+1)

N=8; d=3; TAU,SIGMA,MU = tau(N),sigma(N),mu(N)

c1 = np.pi**2/2

B = TAU + c1/TAU - 1/(2*TAU**2) - 1/((N-1)*TAU**3)

alpha_inv = B + 1/((N+2)*B**2); alpha = 1/alpha_inv

t1 = np.pi*MU*SIGMA*alpha**3/4

t2 = (2*N+1)*np.pi**2*alpha*(1-alpha**2)/16

mass = MU + t1 + t2 - alpha**2

muon = (d/2)*(TAU + np.pi*alpha) + (c1/4)*(1 + d*alpha**2)

print(f"alpha^-1  = {alpha_inv:.9f}")  # 137.035999084

print(f"m_p/m_e   = {mass:.6f}")       # 1836.152674

print(f"m_mu/m_e  = {muon:.6f}")       # 206.768286
```


[1] E. Tiesinga, P. J. Mohr, D. B. Newell, and B. N. Taylor, "CODATA recommended values of the fundamental physical constants: 2018," *Rev. Mod. Phys.* **93**, 025010 (2021).