aeet
ASOCIACIÓN
ESPAÑOLA DE
ECOLOGIA
TERRESTRE

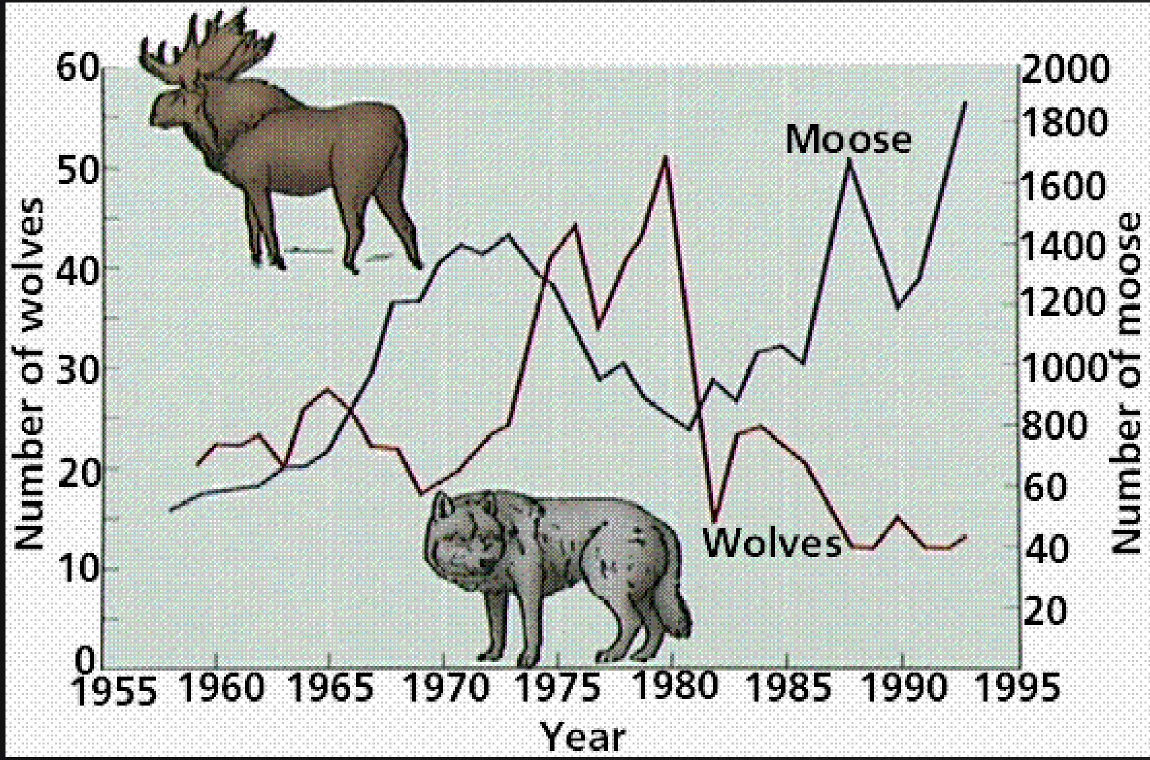
Population models in R

David García-Callejas & Virginia Domínguez-García

¿Qué es Modelar?

¿Qué queremos Modelar?

POBLACIONES



1. UNA ESPECIE AISLADA

Ratio de reproducción constante

$$r = nº$$

$$\frac{dN}{dt} = rN$$

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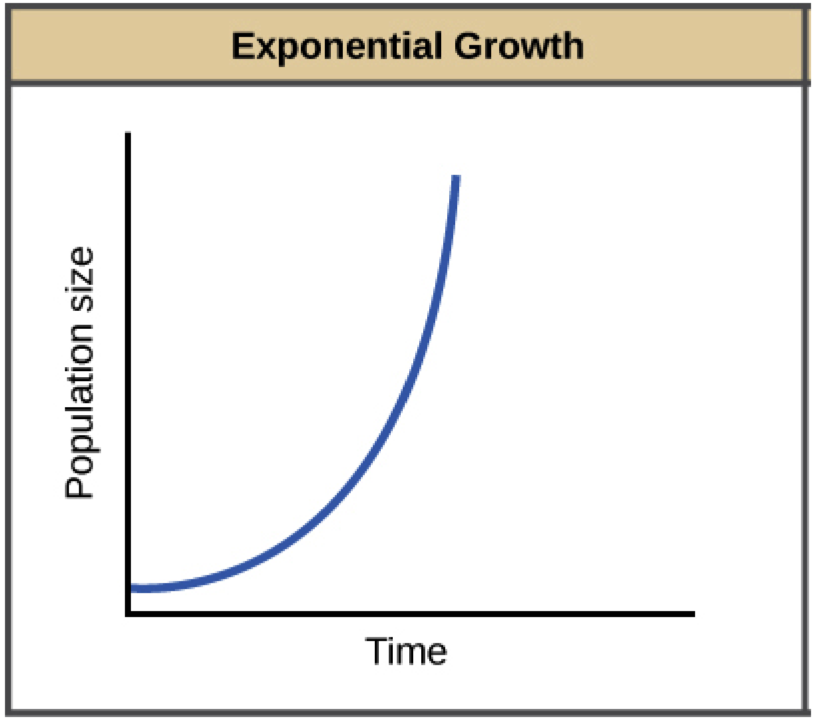

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$$N(t + \Delta t) = N(t) + rN(t)\Delta t$$

Ejemplo

1. UNA ESPECIE AISLADA

$$N(t+\Delta t) = N(t) + r(t)N(t)\Delta t$$

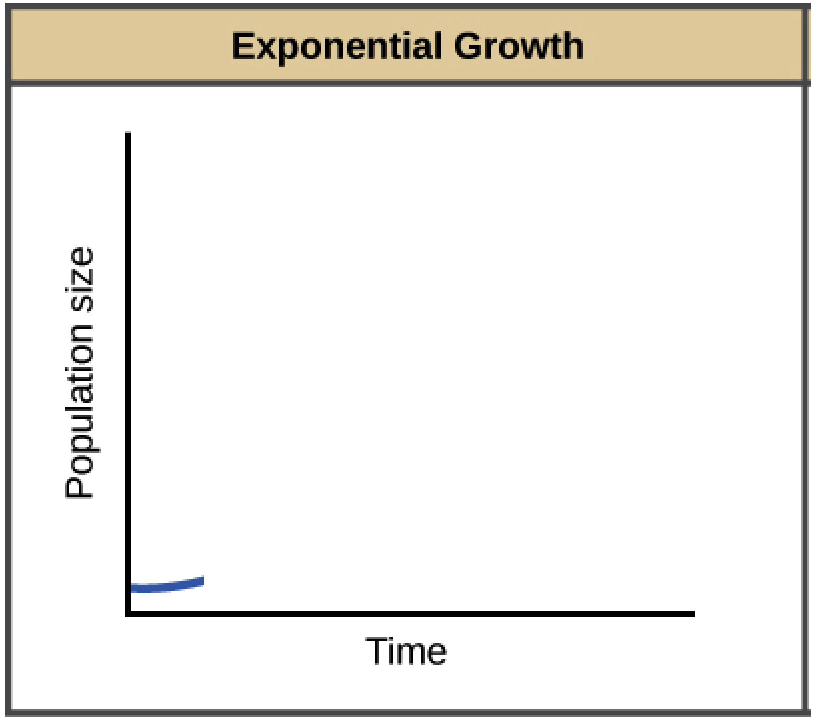
$$N(t+1) = N(t) + dN$$

1. UNA ESPECIE AISLADA

Ratio de reproducción constante

$$r = 2$$

$$\frac{dN}{dt} = rN \qquad N(t) = N_0 e^{rt}$$



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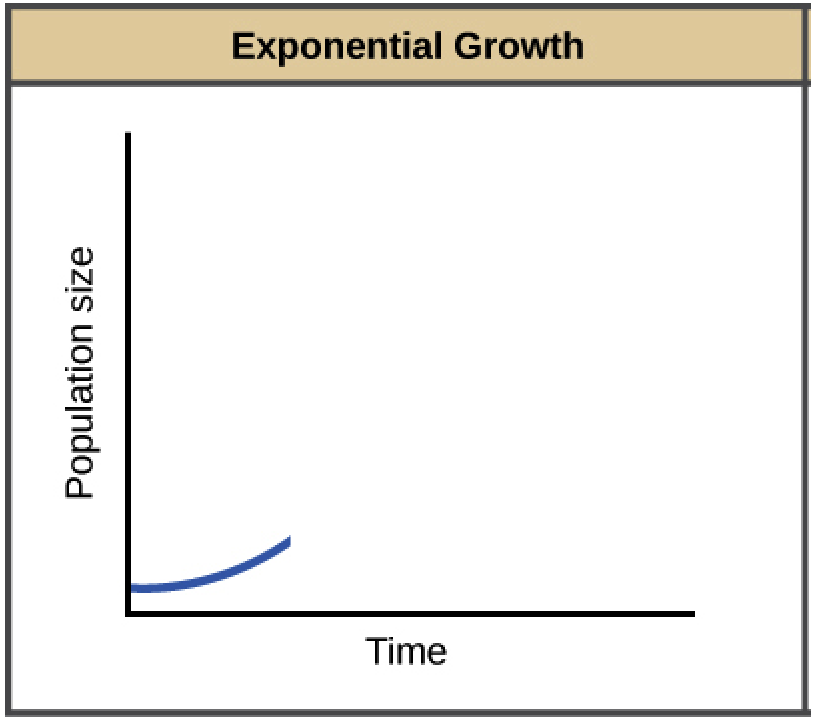

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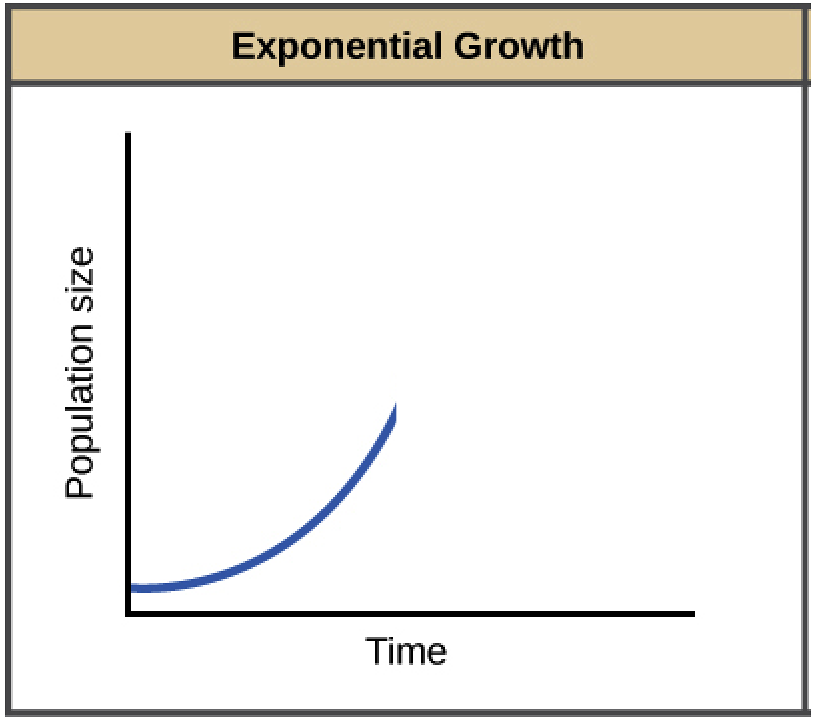

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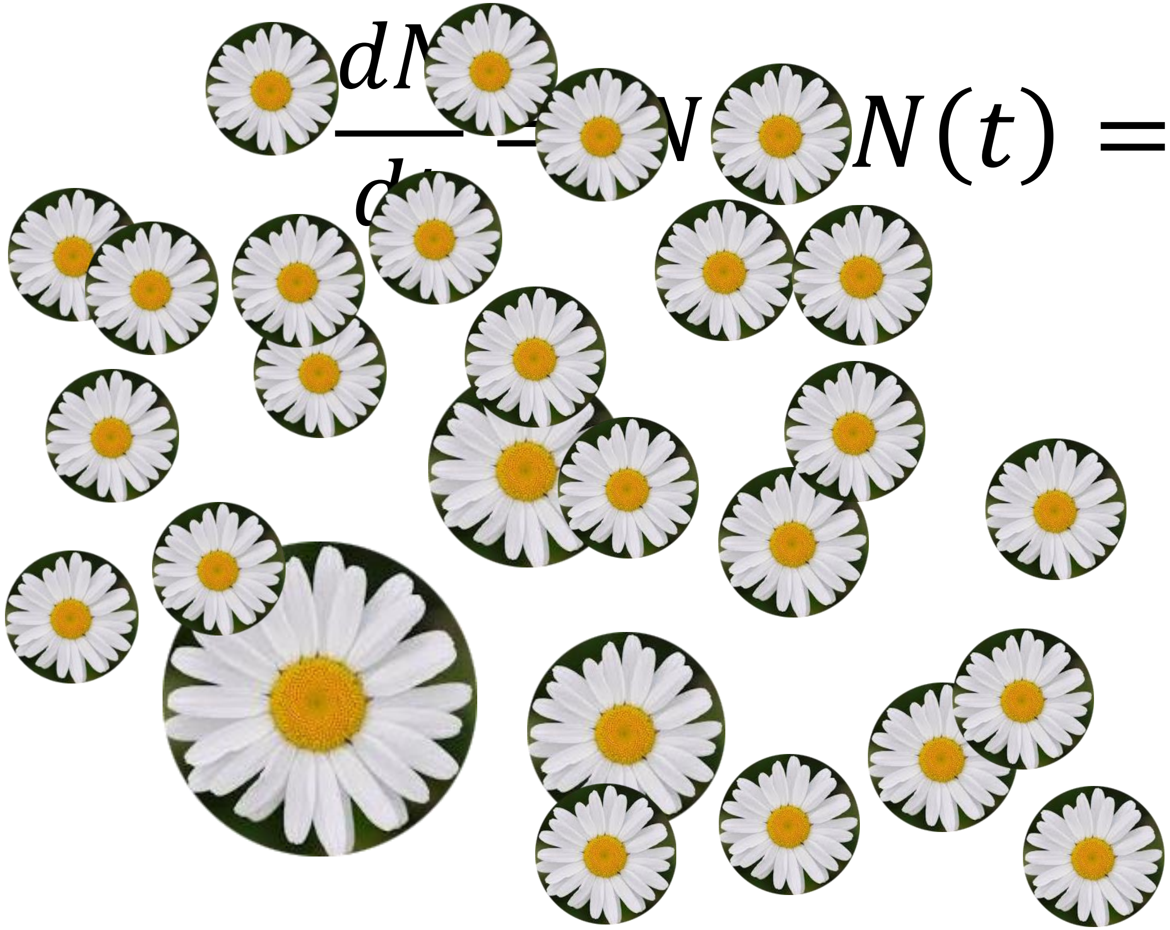

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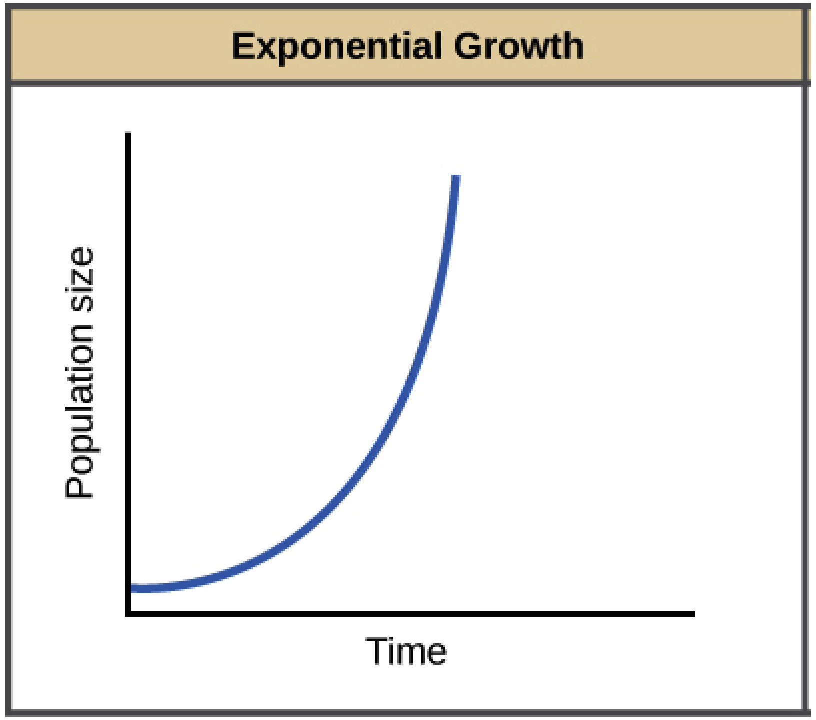

II. UNA ESPECIE AUTO-LIMITADA

¡reproducción variable!

$$\frac{dN}{dt} = \boxed{r\left(\frac{K-N}{K}\right)}N$$

Ratio de reproducción dependiente de la densidad (intraspecific competition)

$$r = f(N, K)$$

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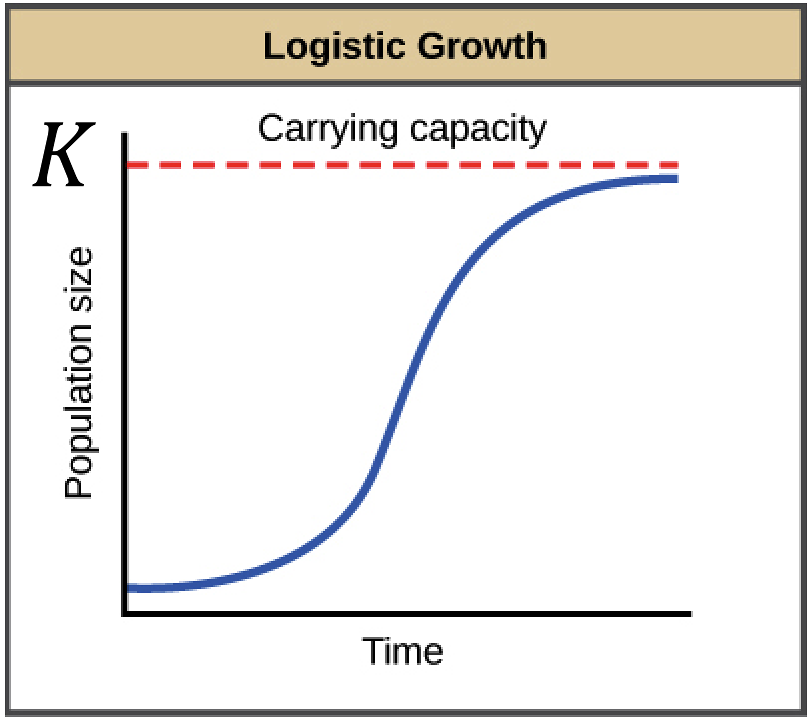
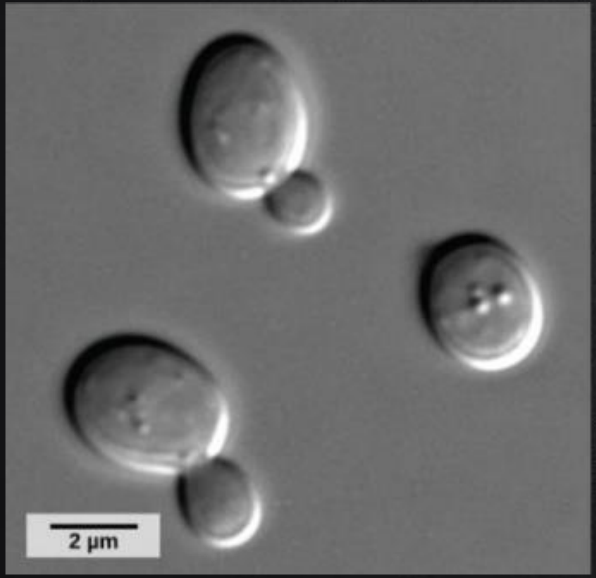
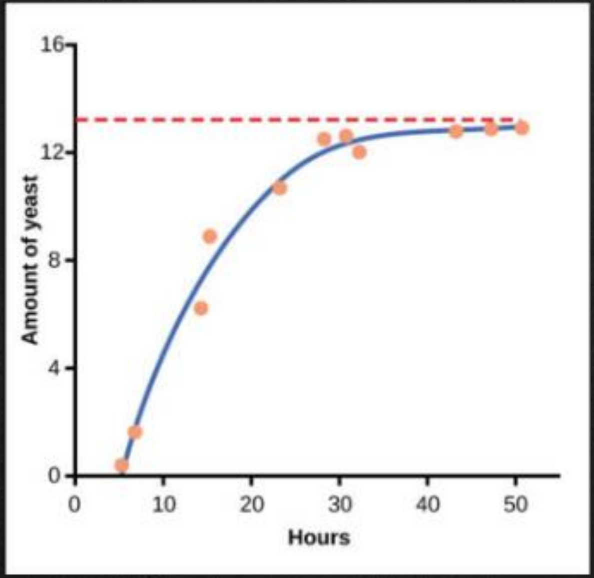
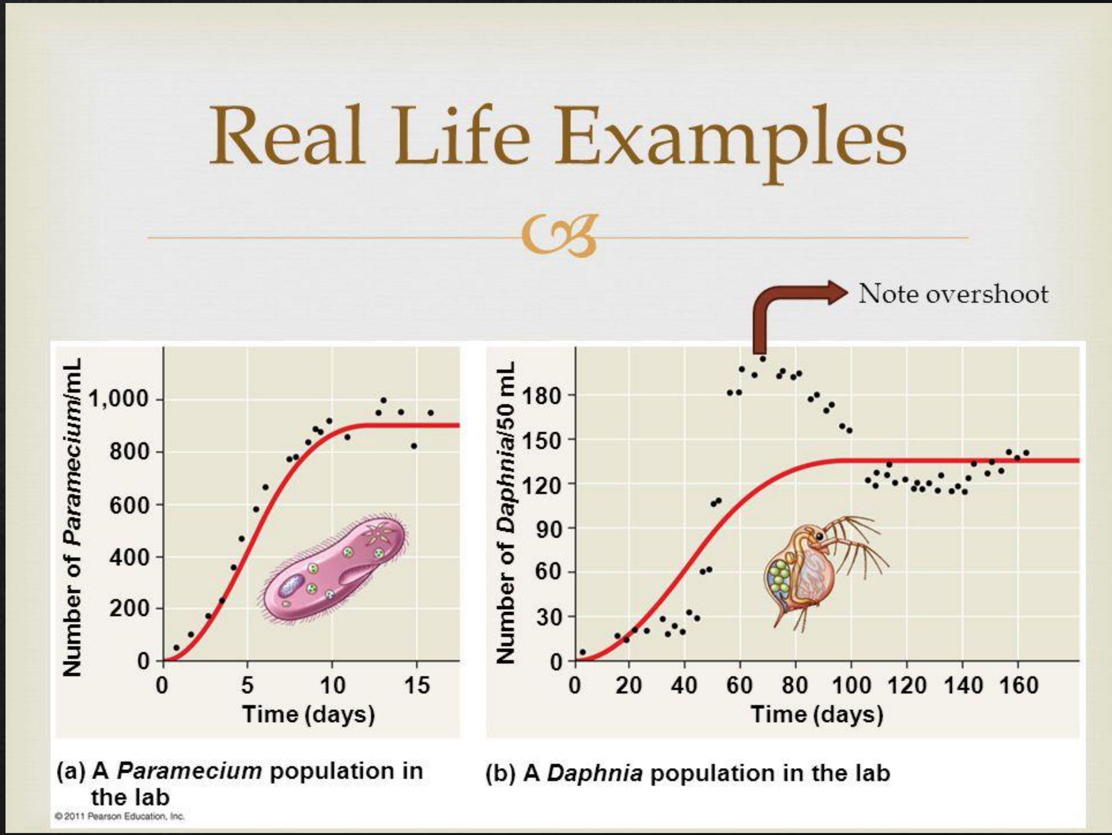


¿Qué queremos Modelar?

POBLACIONES...







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$$N(t + \Delta t) = N(t) + r(t)N(t)\Delta t$$

Ejemplo

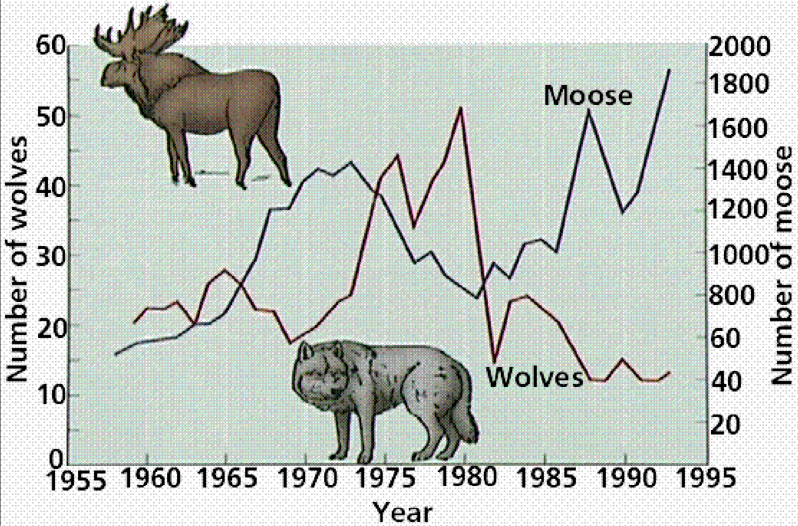
II. UNA ESPECIE AUTO-LIMITADA

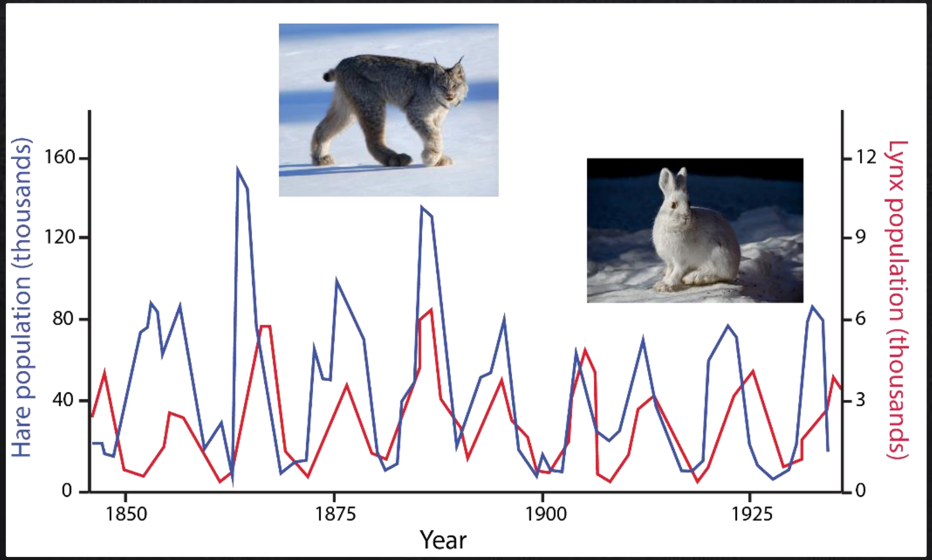
$$N(t+\Delta t) = N(t) + r\,(1 - \frac{N(t)}{K})N(t)\Delta t$$

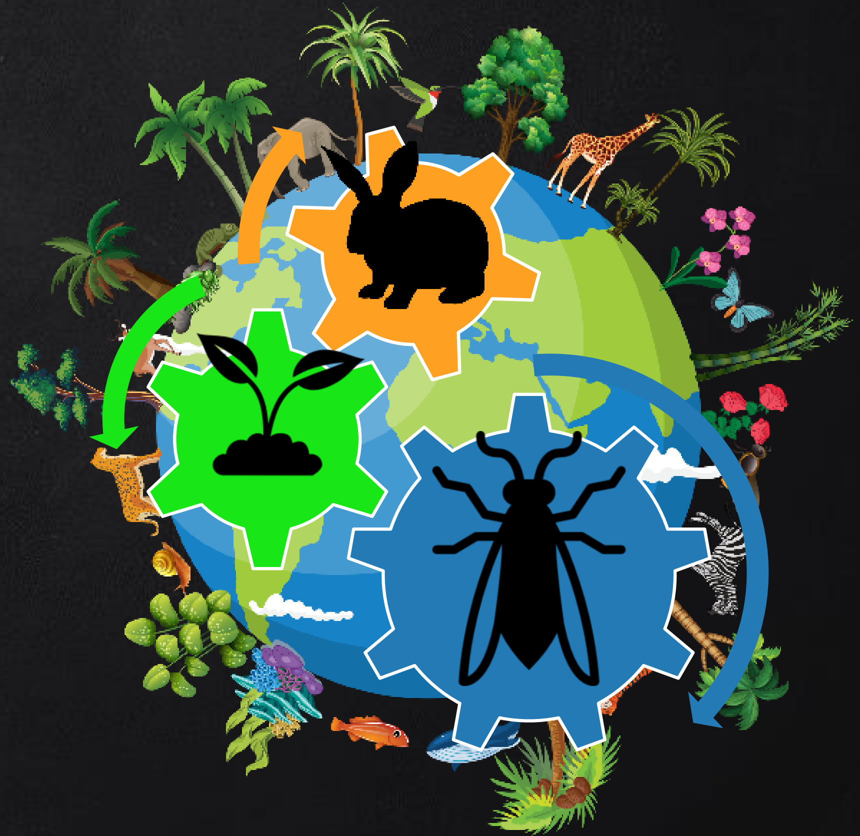
$$N(t+1) = N(t) + dN$$

¿Qué queremos Modelar?

¡Interacciones!



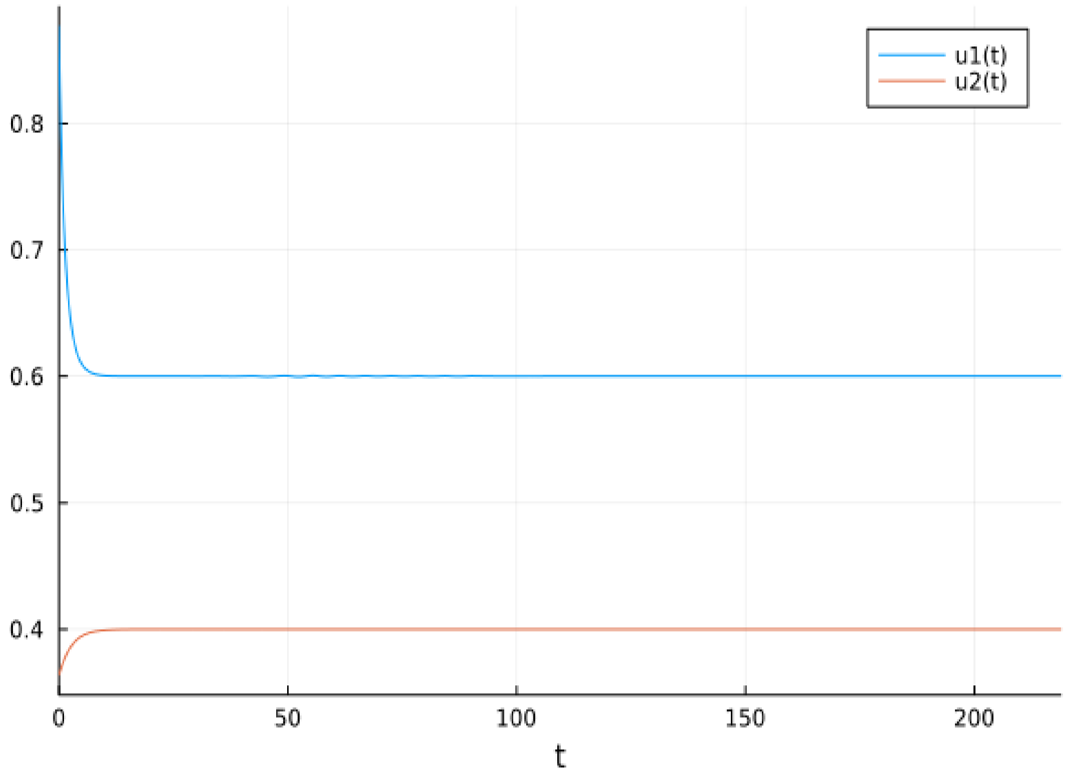




III. DOS ESPECIES

$$\frac{dN_1}{dt} = r_1 N_1 - a_{11} N_1^2$$

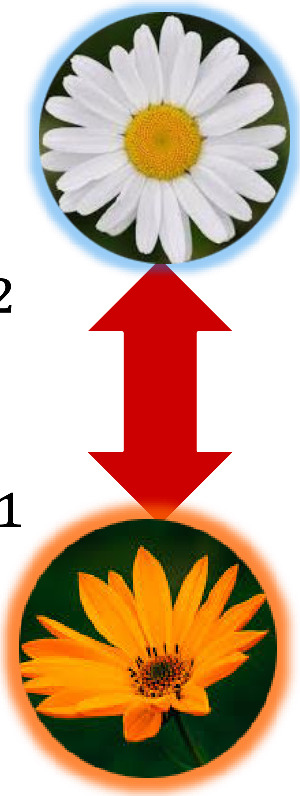
$$\frac{dN_2}{dt} = r_2 N_2 - a_{22} N_2^2$$

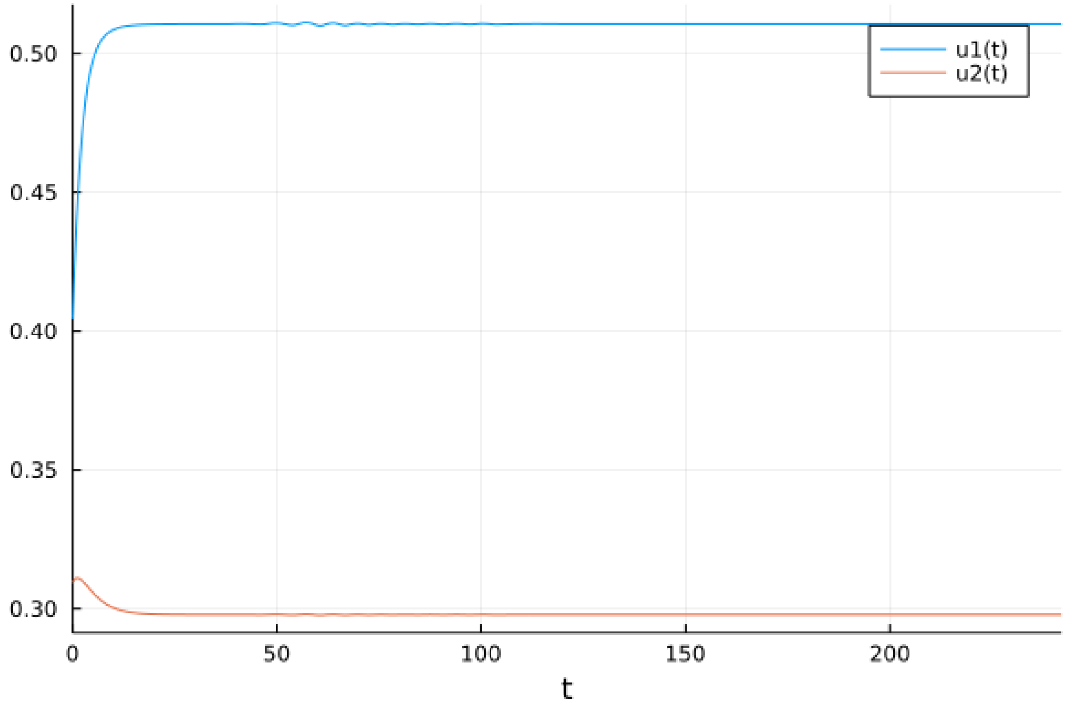


III. DOS ESPECIES COMPITIENDO

$$\frac{dN_1}{dt} = r_1 N_1 - a_{11} N_1^2 - a_{12} N_1 N_2$$

$$\frac{dN_2}{dt} = r_2 N_2 - a_{22} N_2^2 - a_{21} N_2 N_1$$





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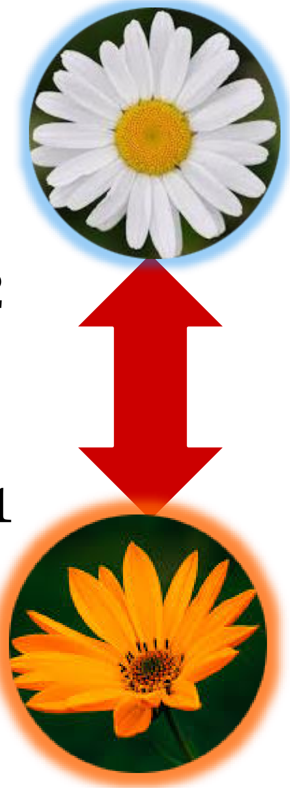


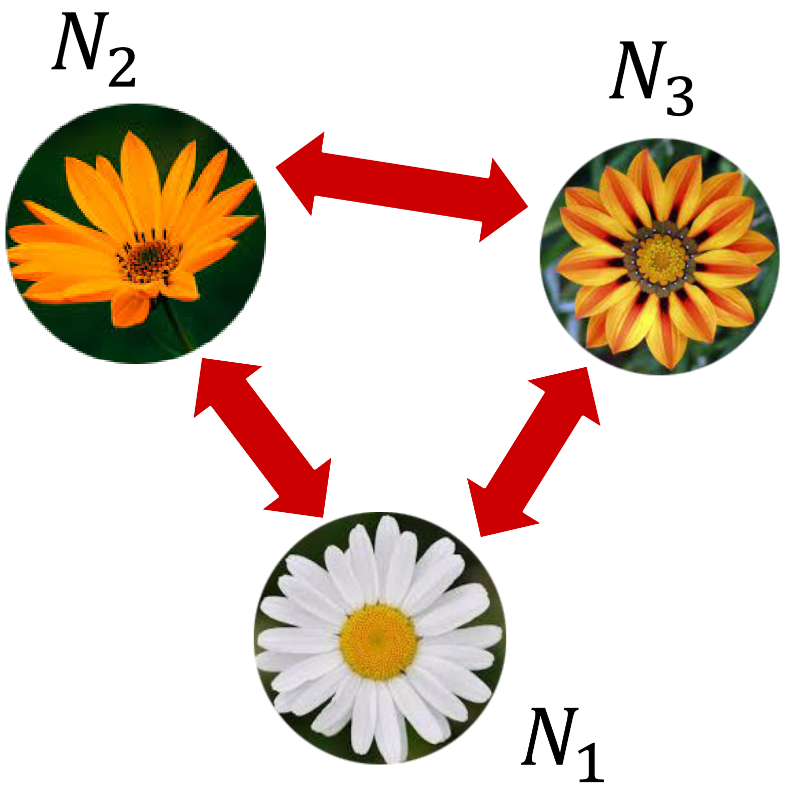
Vector de crecimiento

$\vec{r}$

Matriz de interacción
(Community matrix)

$\hat{A}$

III. TRES ESPECIES COMPITIENDO



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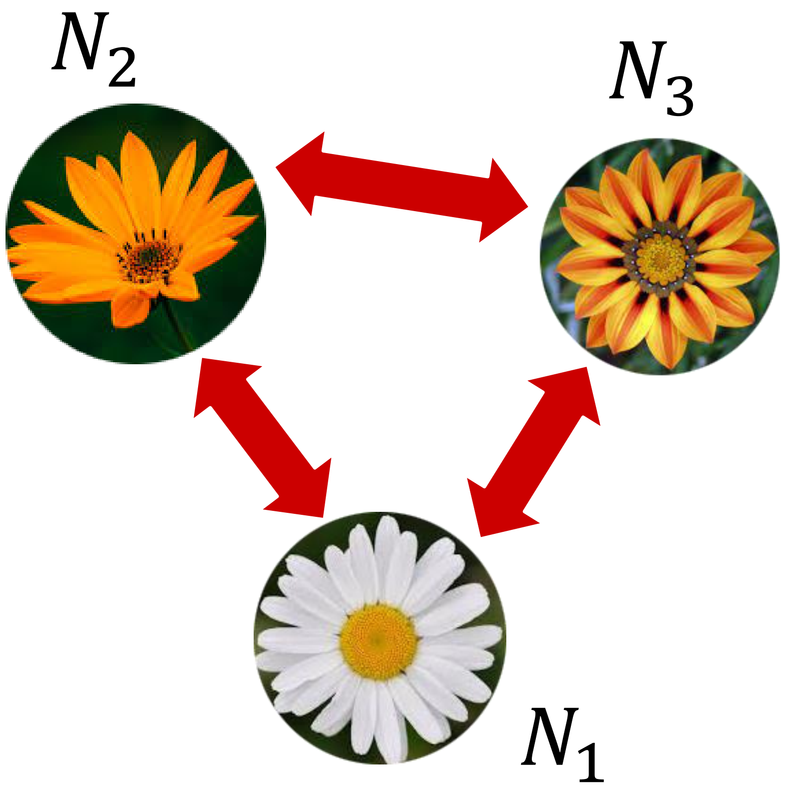

$$\frac{dN_i}{dt} = N_i \left(r_i + \sum_{j=1}^{S} a_{ij} N_j \right)$$

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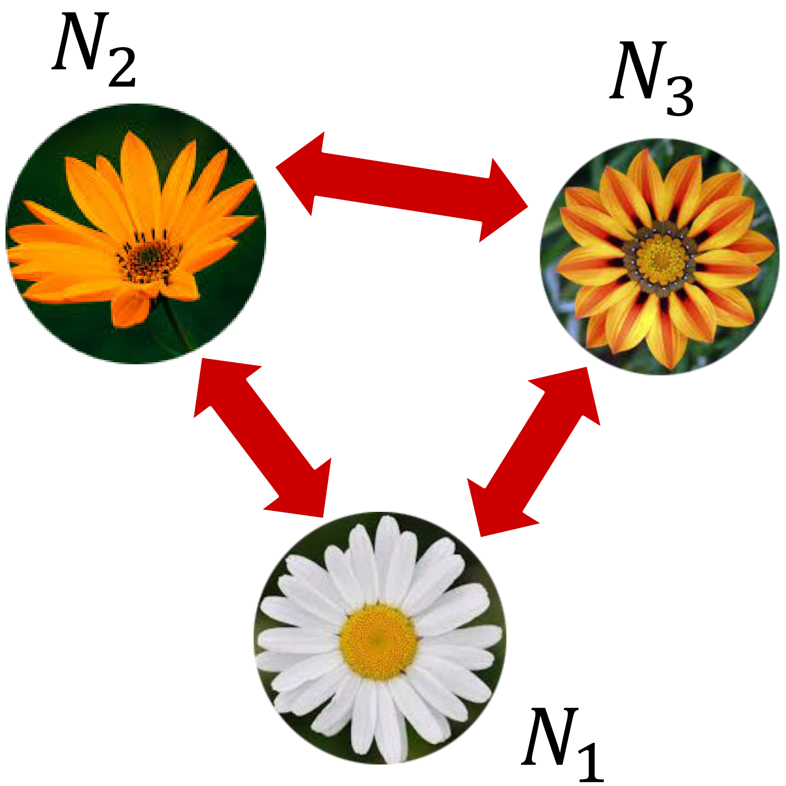


$$\frac{dN_i}{dt} = N_i\left(r_i + \sum_{j=1}^{S} a_{ij}N_j\right)$$

Encontrar equilibrio
Solucionar Sistema de ecuaciones

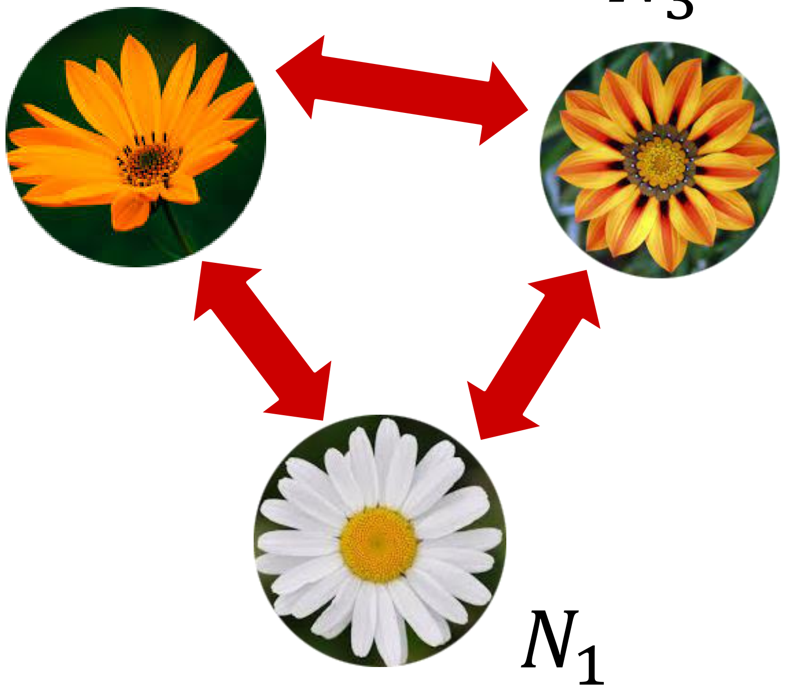
$$N_i^*\left(r_i + \sum_{j=1}^{S} a_{ij}N_j^*\right) = 0.$$

III. TRES ESPECIES COMPITIENDO

$$\frac{dN_i}{dt} = N_i\left(r_i + \sum_{j=1}^{S} a_{ij}N_j\right)$$

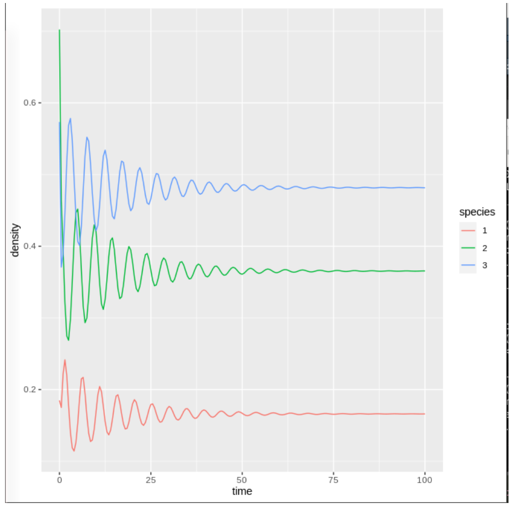
$$r = [1 \quad 1 \quad 1]$$

N_2 N_3



N_1

$$AI = \begin{bmatrix} 10 & 7 & 12 \\ 15 & 10 & 8 \\ 7 & 11 & 10 \end{bmatrix}$$



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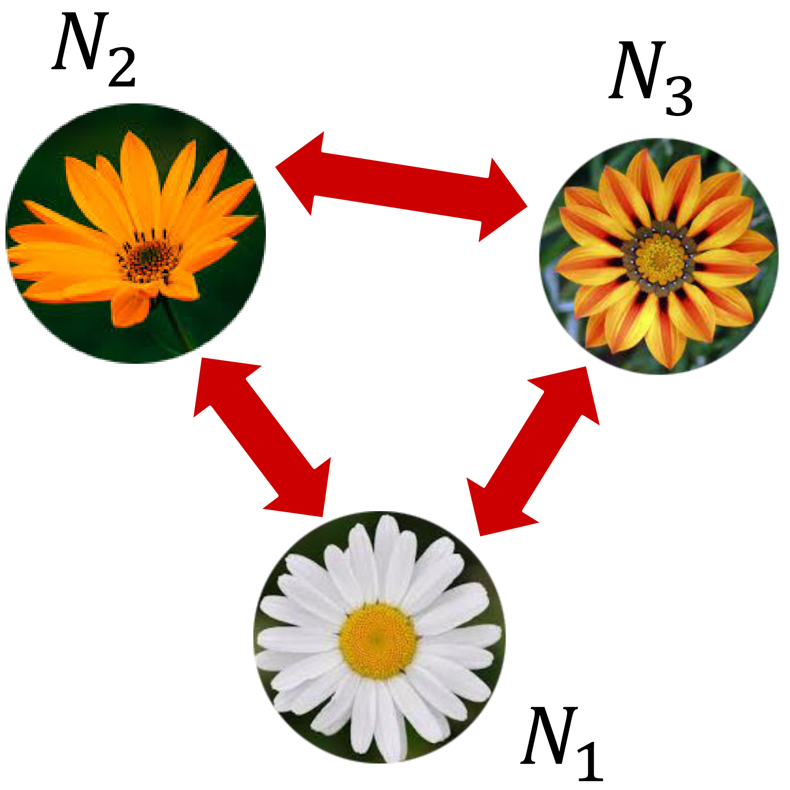
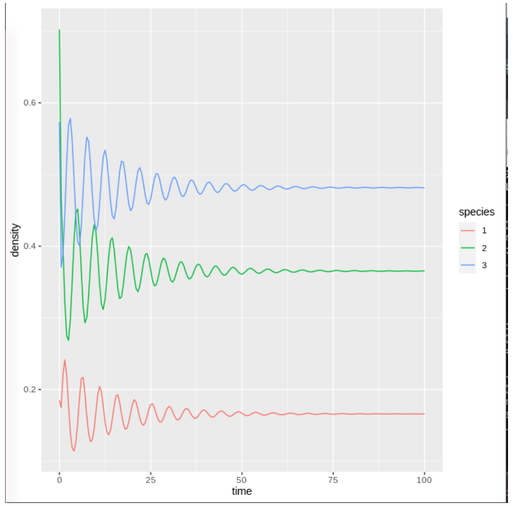
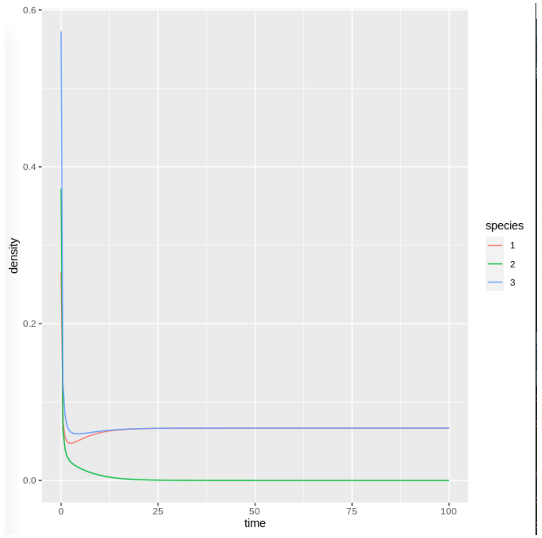


$$AI = \begin{bmatrix} 10 & 7 & 12 \\ 15 & 10 & 8 \\ 7 & 11 & 10 \end{bmatrix}$$

$$A2 = \begin{bmatrix} 10 & 9 & 5 \\ 9 & 10 & 9 \\ 5 & 9 & 10 \end{bmatrix}$$





Vale, tenemos las poblaciones, pero …

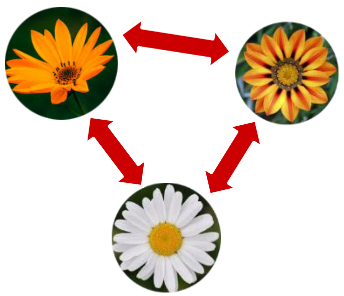
¿Podemos saber algo más de ellas?

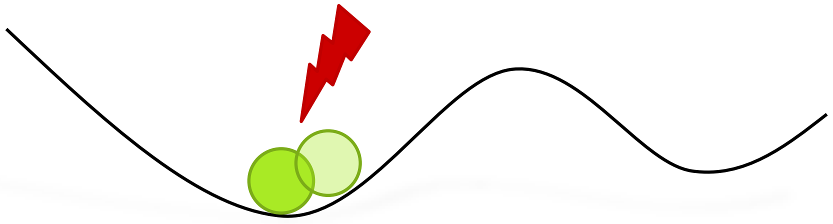
Estabilidad Lineal

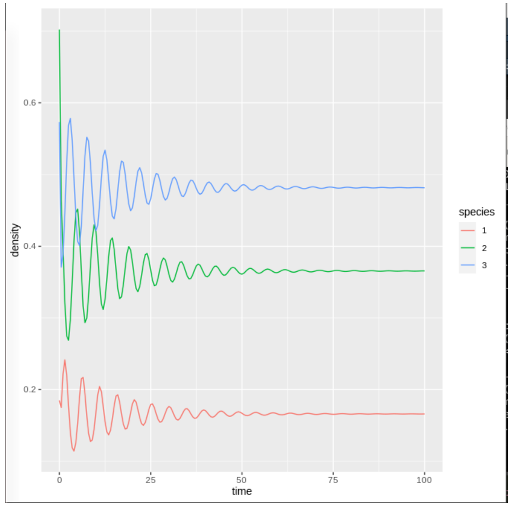
III. TRES ESPECIES COMPITIENDO

Measuring Linear Stability

Instantaneous Change:







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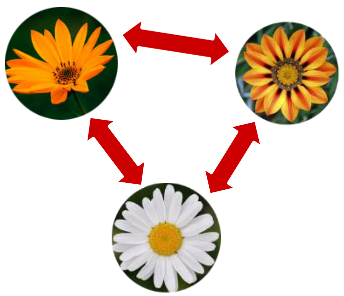

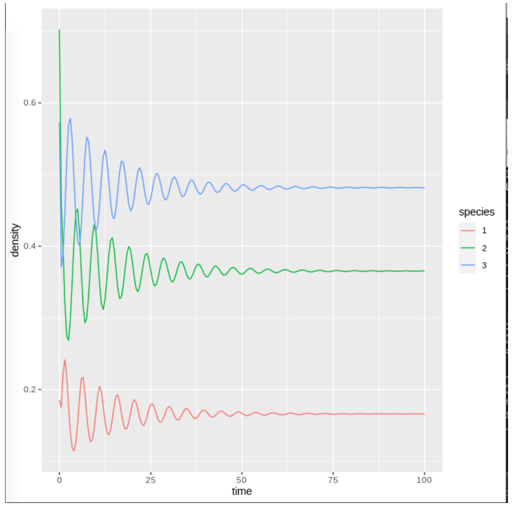

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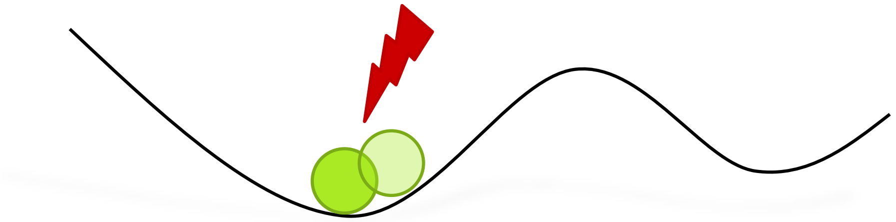

$$J = \frac{\partial F}{\partial x}\Big|^{*}$$

The matrix J has all the information of how the systems **behaves near the steady state**

$$\hat{J} = diag(N^{*})\hat{A}$$

<u>En LV</u>

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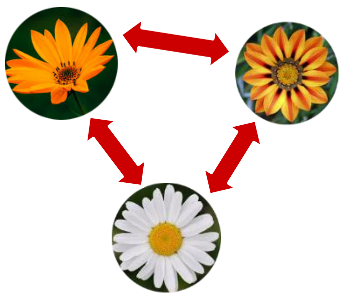

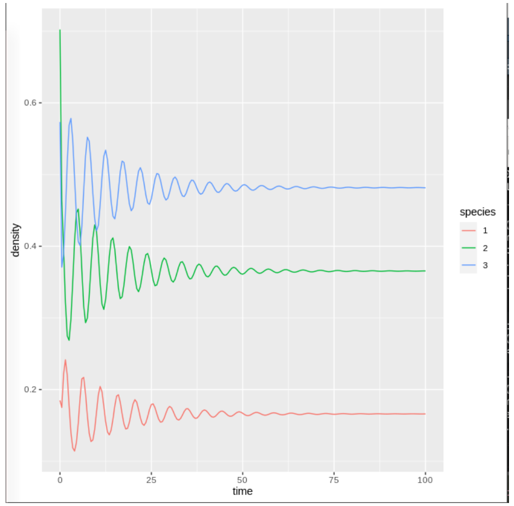

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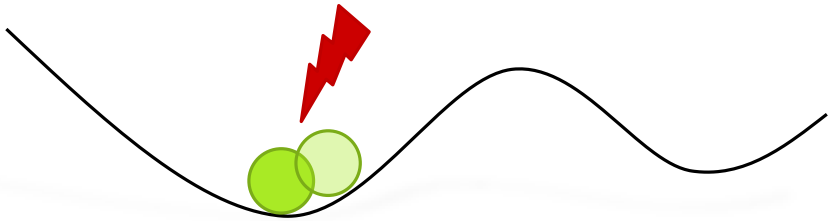

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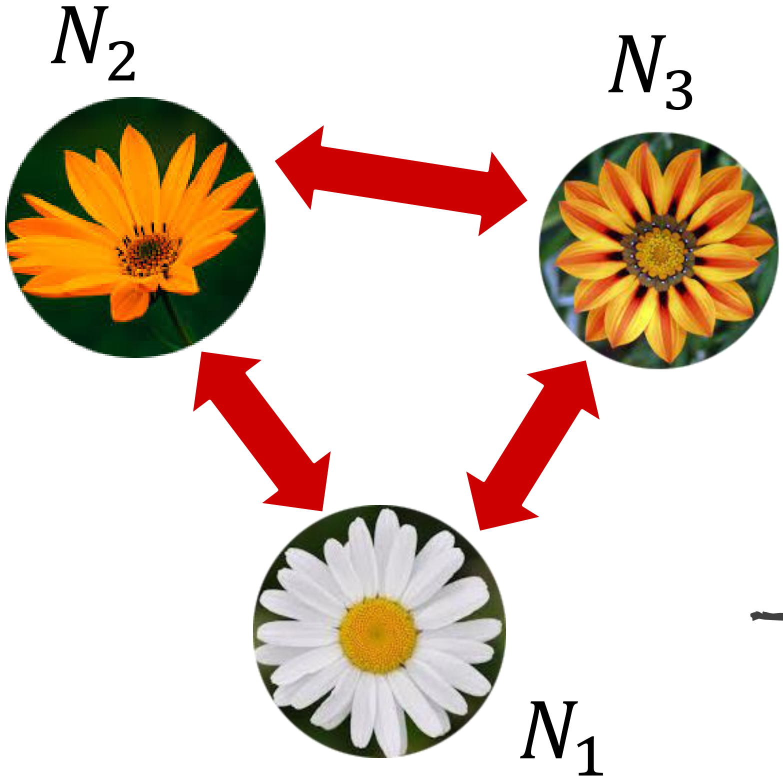
En LV

$$Re(\lambda_{max}) < 0$$

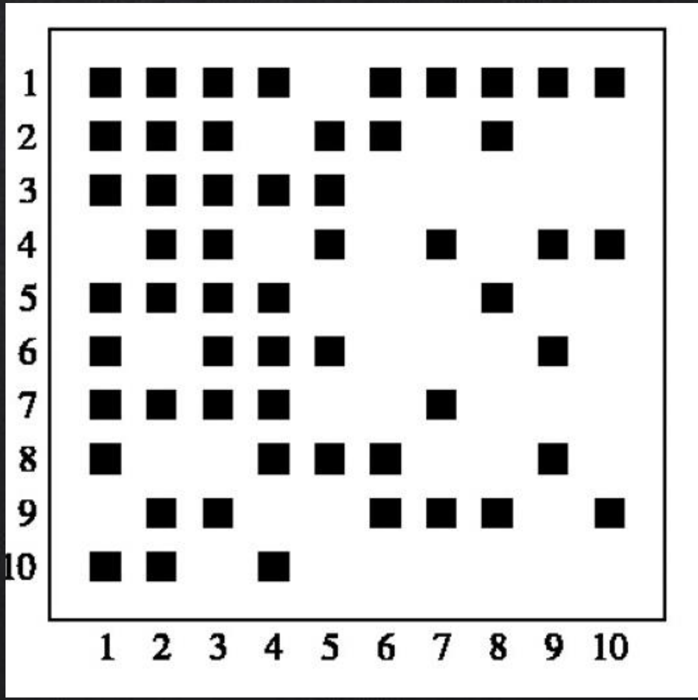
IV. TRES ESPECIES EN INTERACCION



$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

¿Qué queremos Modelar?

POBLACIONES CON MUCHAS INTERACCIONES



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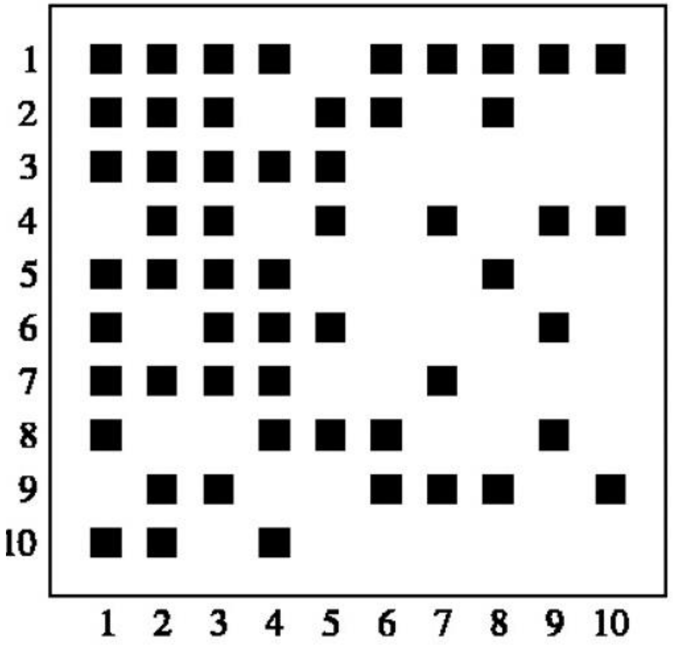

$$\begin{bmatrix} a_{11} & \cdots & a_{1N} \\ \vdots & \ddots & \vdots \\ a_{N1} & \cdots & a_{NN} \end{bmatrix}$$

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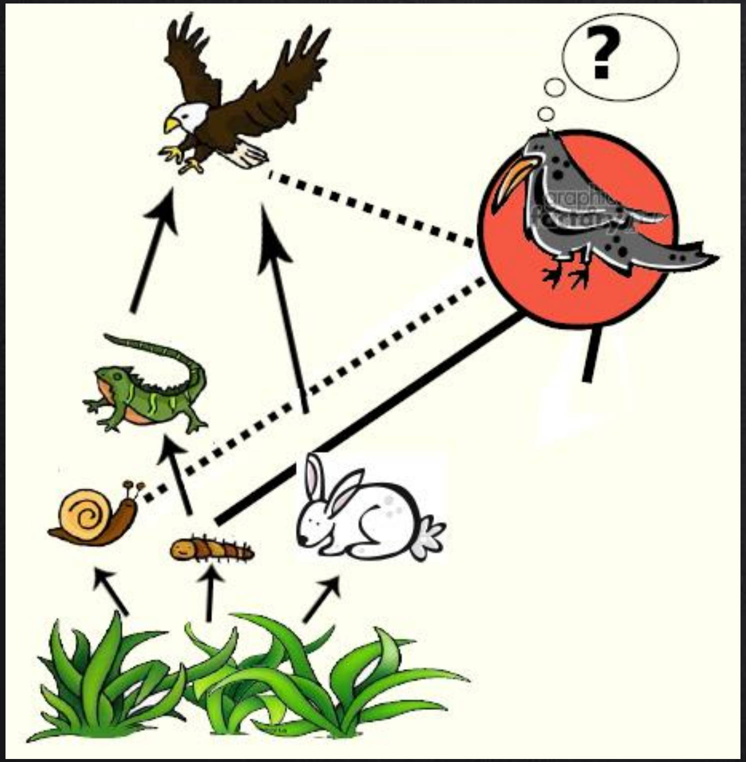

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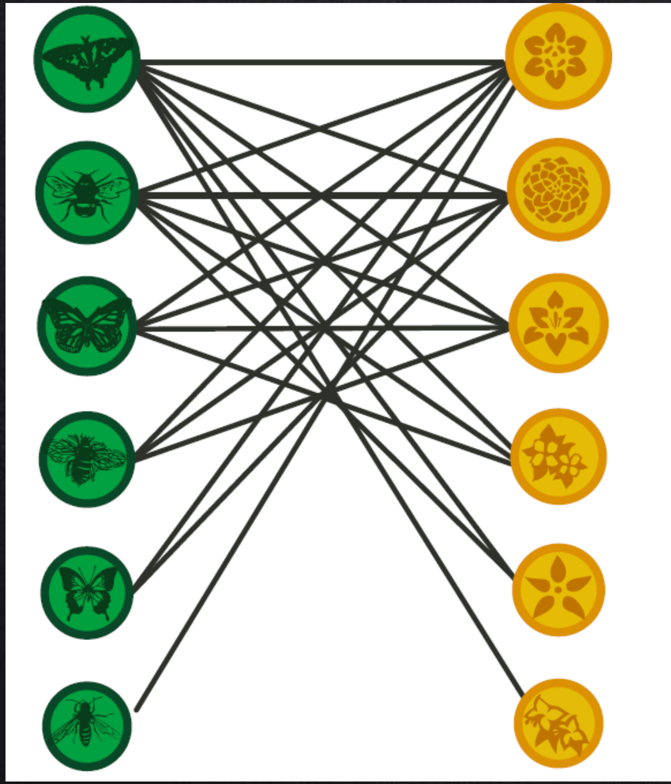

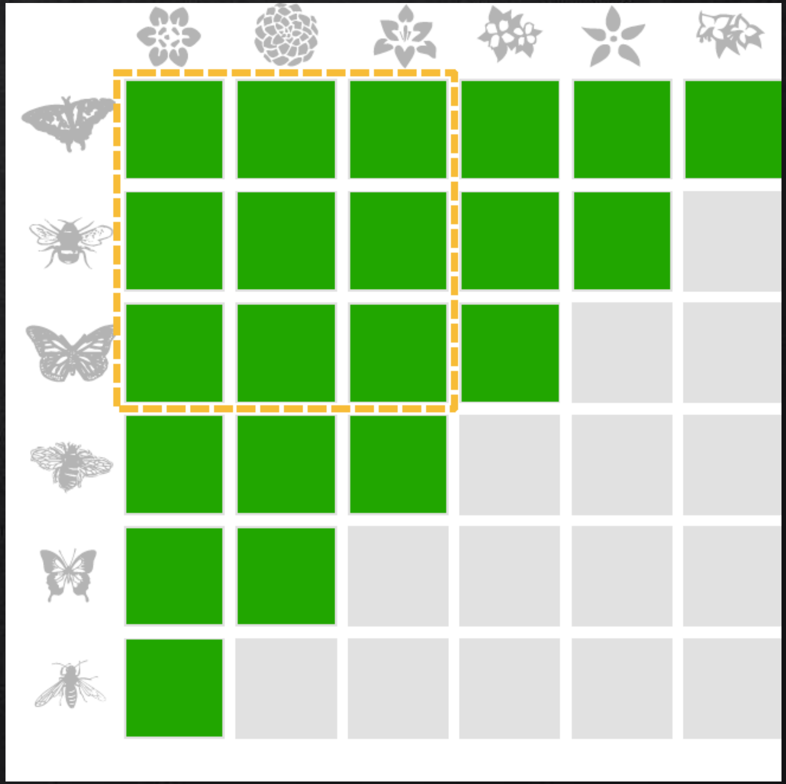

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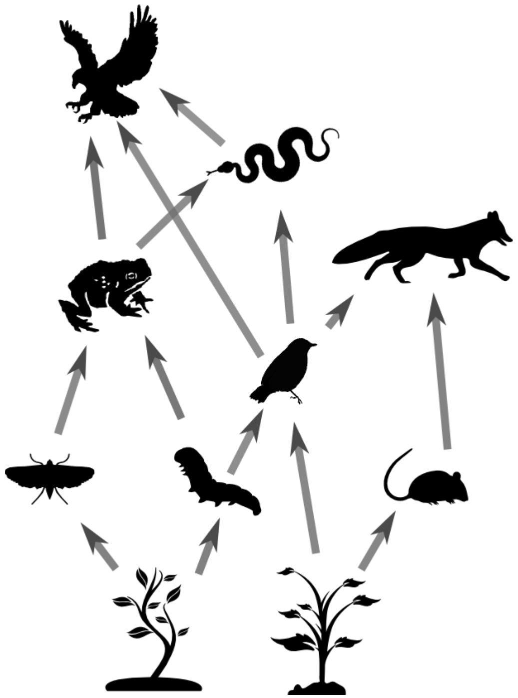

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