ST 551 Homework 3

Student's Name Goes Here

Fall 2022

Instructions

This assignment is due by 11:59 PM, October 26th on Canvas via Gradescope. **You should submit your assignment as a typed PDF which you can compile using the provide .Rmd (R Markdown) template.** Include your code in your solutions and indicate where the solutions for individual problems are located when uploading into Gradescope. You should also use complete, grammatically correct sentences for your solutions.

Question 1 (18 Points)

The function `binom.test()` in R performs the *exact* binomial test discussed in class. This function returns an object that includes the statistic (observed value of X), the parameter n, the p-value (two-sided by default), an estimate (the sample proportion, X/n), and a confidence interval (95% by default). See the help document `help(binom.test)` for guidance on how to use the function.

a. (3 points) Use `binom.test()` function to perform a two-sided test of the null hypothesis that $p = 0.4$ when you observe $X = 9$ successes in $n = 15$ trials. What is the reported p-value?

```
# Write the code needed to conduct the test here
```

Confirm that this p-value agrees with the method discussed in class. That is, use `dbinom()` to find the probabilities of each outcome $X = 0, 1, 2, \ldots, 15$, and add up the probabilities of outcomes as likely as, or less likely, than $X = 9$ when $p = 0.4$.

```
# Write the code to accomplish this task here
```

b. (3 points) Based on the p-value from part (a), use a sentence which states whether you would reject the null hypothesis $H_0 : p = 0.4$ vs. $H_A : p \neq 0.4$ at level $\alpha = 0.05$?

c. (3 points) What is the reported confidence interval for p?

d. (3 points) Now use the function `binom.test()` to perform a two-sided test of the null hypothesis $H_0 : p = 0.33$. What is the reported p-value? Based on this p-value, would you reject the null hypothesis $H_0 : p = 0.33$ vs. $H_A : p \neq 0.33$ at level $\alpha = 0.05$?

e. (3 points) Compare your results from part (d) above to the confidence interval in parts (c) and (d). Do these results agree? Should we expect them to agree? Discuss (i.e. Write a few sentences explaining the results).

f. (3 points) How could you construct a confidence interval that would agree with the p-value from the exact binomial test? Describe the process, and implement it (using R) to find a confidence interval for p that agrees with the exact binomial two-sided p-value in this setting where you observed $X = 9$ successes in $n = 15$ trials.

Question 2 (10 points)

The usual z-test (i.e. the *score test*) for a binomial proportion to test $H_0 : p = p_0$ uses the test statistic

$$z(p_0) = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}}$$

where $\hat{p} = X/n$ is the sample proportion. For a two-sided level α test, the null hypothesis is rejected when

$$|z(p_0)| > z_{1-\alpha/2}.$$

Compare the center and length of the resulting interval to the more commonly used *Wald interval*:

$$\hat{p} \pm z_{1-\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}.$$

That is, **derive** how these two values are mathematically different in terms of (1) where they are centered and (2) how wide the intervals are. Discuss the details and compare the two centers/lengths.

> Hint (1): We know that we would not reject the null hypothesis whenever $|z(p_0)| < z_{1-\alpha/2}$. Use this fact to first solve for the values of p_0 that would not lead to rejecting in a two-sided α level test.
>
> Hint (2): At some point, you'll want to consider applying our old friend and ally, the quadratic formula.

Question 3 (10 points)

Prove that the one-sided upper z-test for a binomial proportion is a consistent test: If the true value of $p = p_A > p_0$, what is $P_{p_A}(z(p_0) > z_{1-\alpha})$ as $n \to \infty$?