

GSERM - Oslo 2019

Cox and Discrete-Time Models

January 10, 2019 (afternoon session)

Basic idea:

$$h_i(t) = h_0(t)\exp(\mathbf{X}_i\beta)$$

Note:

- $h_0(t) \equiv h(t|\mathbf{X} = 0)$
- Changes in \mathbf{X} shift $h(t)$ *proportionally*

$$\begin{aligned}\text{HR} &= \frac{h_0(t)\exp(X_1\hat{\beta})}{h_0(t)\exp(X_0\hat{\beta})} \\ &= \exp[(1 - 0)\hat{\beta}] \\ &= \exp(\hat{\beta})\end{aligned}$$

Also, because

$$S(t) = \exp[-H(t)]$$

then

$$\begin{aligned} S(t) &= \exp \left[- \int_0^t h(t) dt \right] \\ &= \exp \left[- \exp(\mathbf{X}_i \beta) \int_0^t h_0(t) dt \right] \\ &= \left[\exp \left(- \int_0^t h_0(t) dt \right) \right]^{\exp(\mathbf{X}_i \beta)} \\ &= [S_0(t)]^{\exp(\mathbf{X}_i \beta)} \end{aligned}$$

Assume N_C distinct event times t_j , with no “ties.”

Then:

$$\begin{aligned}
 & \Pr(\text{Individual } k \text{ experienced the event at } t_j \mid \text{One observation experienced the event at } t_j) \\
 &= \frac{\Pr(\text{At-risk observation } k \text{ experiences the event of interest at } t_j)}{\Pr(\text{One at-risk observation experiences the event of interest at } t_j)} \\
 &= \frac{h_k(t_j)}{\sum_{\ell \in R_j} h_\ell(t_j)}
 \end{aligned}$$

Partial Likelihood (continued)

$$\begin{aligned} L_i &= \frac{h_0(t_j) \exp(\mathbf{X}_i \beta)}{\sum_{\ell \in R_j} h_0(t_j) \exp(\mathbf{X}_\ell \beta)} \\ &= \frac{h_0(t_j) \exp(\mathbf{X}_i \beta)}{h_0(t_j) \sum_{\ell \in R_j} \exp(\mathbf{X}_\ell \beta)} \\ &= \frac{\exp(\mathbf{X}_i \beta)}{\sum_{\ell \in R_j} \exp(\mathbf{X}_\ell \beta)} \end{aligned}$$

$$L = \prod_{i=1}^N \left[\frac{\exp(\mathbf{X}_i \beta)}{\sum_{\ell \in R_j} \exp(\mathbf{X}_\ell \beta)} \right]^{C_i}$$

$$\ln L = \sum_{i=1}^N C_i \left\{ \mathbf{X}_i \beta - \ln \left[\sum_{\ell \in R_j} \exp(\mathbf{X}_\ell \beta) \right] \right\}$$

- PL is
 - Consistent
 - Asymptotically normal
 - Slightly inefficient (but asymptotically efficient)
- Considers order of events, but not actual duration
- Censored events: Modify R_j
- No ties

Example: Interstate War, 1950-1985

- Dyad-years for “politically-relevant” dyads
- $N = 827$, $NT = 20448$.
- Covariates:
 - Whether ($=1$) or not the two countries are *allies*,
 - Whether ($=1$) or not the two countries are *contiguous*,
 - The *capability ratio* of the two countries,
 - The lower of the two countries' (GDP) *growth* (rescaled),
 - The lower of the two countries' *democracy* (POLITY IV) scores (rescaled to $[-1,1]$), and
 - The amount of *trade* between the two countries, as a fraction of joint GDP.

The Data

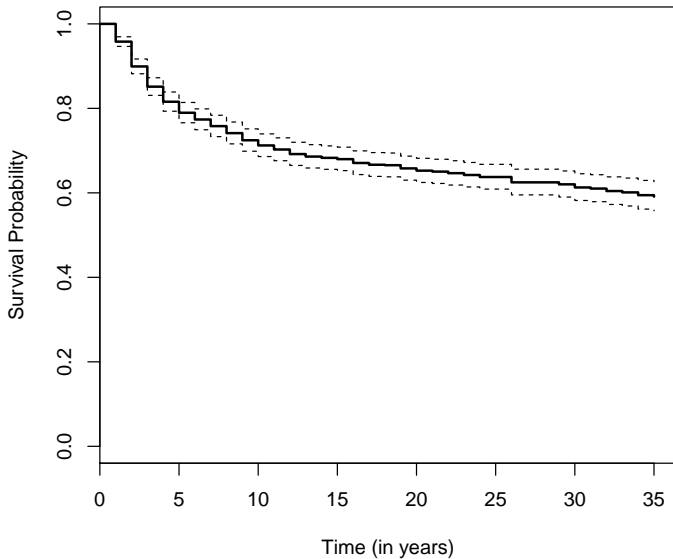
```
> summary(OR)
```

dyadid	year	start	stop	futime
Min. : 2020	Min. :1951	Min. : 0.00	Min. : 1.00	Min. : 5.00
1st Qu.:100365	1st Qu.:1965	1st Qu.: 5.00	1st Qu.: 6.00	1st Qu.:23.00
Median :220235	Median :1972	Median :11.00	Median :12.00	Median :31.00
Mean :253305	Mean :1971	Mean :12.32	Mean :13.32	Mean :28.97
3rd Qu.:365600	3rd Qu.:1979	3rd Qu.:19.00	3rd Qu.:20.00	3rd Qu.:35.00
Max. :900920	Max. :1985	Max. :34.00	Max. :35.00	Max. :35.00

dispute	allies	contig	trade
Min. :0.00000	Min. :0.0000	Min. :0.0000	Min. :0.00000
1st Qu.:0.00000	1st Qu.:0.0000	1st Qu.:0.0000	1st Qu.:0.00000
Median :0.00000	Median :0.0000	Median :0.0000	Median :0.00020
Mean :0.01981	Mean :0.3563	Mean :0.3099	Mean :0.00231
3rd Qu.:0.00000	3rd Qu.:1.0000	3rd Qu.:1.0000	3rd Qu.:0.00120
Max. :1.00000	Max. :1.0000	Max. :1.0000	Max. :0.17680

growth	democracy	capratio
Min. : -0.264900	Min. : -1.0000	Min. : 0.0100
1st Qu.: -0.004800	1st Qu.: -0.8000	1st Qu.: 0.0462
Median : 0.014700	Median : -0.7000	Median : 0.2220
Mean : 0.007823	Mean : -0.3438	Mean : 1.6677
3rd Qu.: 0.027800	3rd Qu.: 0.2000	3rd Qu.: 1.1560
Max. : 0.164700	Max. : 1.0000	Max. :78.9296

The Data (Kaplan-Meier plot)



R:

- `coxph` in `survival` (preferred)
- `cph` in `design`
- Plots: `plot(survfit(PHobject))`

Stata:

- Basic command = `stcox`
- `stset` first
- Options: `robust`, various methods for ties, postestimation commands

Model Fitting

```
> ORCox.br<-coxph(OR.S~allies+contig+capratio+growth+democracy+trade,  
                  data=OR,na.action=na.exclude,method="breslow")
```

```
> summary(ORCox.br)
```

```
n= 20448, number of events= 405
```

	coef	exp(coef)	se(coef)	z	Pr(> z)	
allies	-0.34849	0.70576	0.11096	-3.141	0.001686	**
contig	0.94861	2.58213	0.12173	7.793	6.55e-15	***
capratio	-0.22303	0.80009	0.05164	-4.319	1.57e-05	***
growth	-3.69487	0.02485	1.19950	-3.080	0.002068	**
democracy	-0.38194	0.68254	0.09915	-3.852	0.000117	***
trade	-3.22857	0.03961	9.45588	-0.341	0.732776	

```
---
```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
.  
.   
.
```

Model Fitting (continued)

```
.  
.   
.  
      exp(coef) exp(-coef) lower .95 upper .95  
allies      0.70576      1.4169 5.678e-01 8.772e-01  
contig      2.58213      0.3873 2.034e+00 3.278e+00  
capratio    0.80009      1.2499 7.231e-01 8.853e-01  
growth      0.02485     40.2402 2.368e-03 2.608e-01  
democracy   0.68254      1.4651 5.620e-01 8.289e-01  
trade       0.03961     25.2436 3.540e-10 4.433e+06
```

```
Concordance= 0.714 (se = 0.015 )  
Rsquare= 0.01 (max possible= 0.234 )  
Likelihood ratio test= 210.3 on 6 df, p=0  
Wald test              = 159.8 on 6 df, p=0  
Score (logrank) test = 185.8 on 6 df, p=0
```

Interpretation: Hazard Ratios

$$HR = \exp[(\mathbf{X}_j - \mathbf{X}_k)\hat{\beta}]$$

Means:

- $HR = 1 \Leftrightarrow \hat{\beta} = 0$
- $HR > 1 \Leftrightarrow \hat{\beta} > 0$
- $HR < 1 \Leftrightarrow \hat{\beta} < 0$

$$\text{Percentage difference} = 100 \times \{\exp[(\mathbf{X}_j - \mathbf{X}_k)\hat{\beta}] - 1\}.$$

Example: Hazard Ratios

From above:

	exp(coef)	exp(-coef)	lower .95	upper .95
allies	0.70576	1.4169	5.678e-01	8.772e-01
contig	2.58213	0.3873	2.034e+00	3.278e+00
capratio	0.80009	1.2499	7.231e-01	8.853e-01
growth	0.02485	40.2402	2.368e-03	2.608e-01
democracy	0.68254	1.4651	5.620e-01	8.289e-01
trade	0.03961	25.2436	3.540e-10	4.433e+06

Interpretation:

- Countries which are *allies* have an expected $(0.706 - 1) \times 100 = 29.4$ percent lower hazard of conflict than those that are not.
- *Contiguous* countries have $(2.582 - 1) \times 100 = 158$ percent higher hazards of conflict than non-contiguous ones.
- A one-unit increase in *democracy* corresponds to a $(0.683 - 1) \times 100 = 31.7$ percent decrease in the expected hazard of conflict.

Hazard Ratios: Scaling Covariates

It is good for one-unit changes to be meaningful / realistic...

```
> OR$growthPct<-OR$growth*100  
> summary(coxph(OR.S~allies+contig+capratio+growthPct+democracy+trade,  
               data=OR,na.action=na.exclude, method="breslow"))
```

```
.  
.   
.  
exp(coef) exp(-coef) lower .95 upper .95  
allies    0.70576    1.4169 5.678e-01 8.772e-01  
contig    2.58213    0.3873 2.034e+00 3.278e+00  
capratio  0.80009    1.2499 7.231e-01 8.853e-01  
growthPct 0.96373    1.0376 9.413e-01 9.867e-01  
democracy 0.68254    1.4651 5.620e-01 8.289e-01  
trade     0.03961    25.2436 3.540e-10 4.433e+06
```

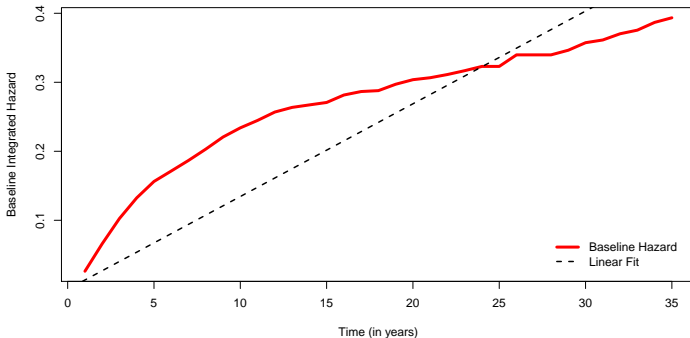
Note:

- Previous HR for growth = 0.02485 \rightarrow 97.5 percent decrease in $\hat{h}(t)$
- HR for growthPct is now 0.964; 1 unit increase \rightarrow 4% decrease in $\hat{h}(t)$
- Same result, proportionally: $0.96373^{100} = 0.02485$

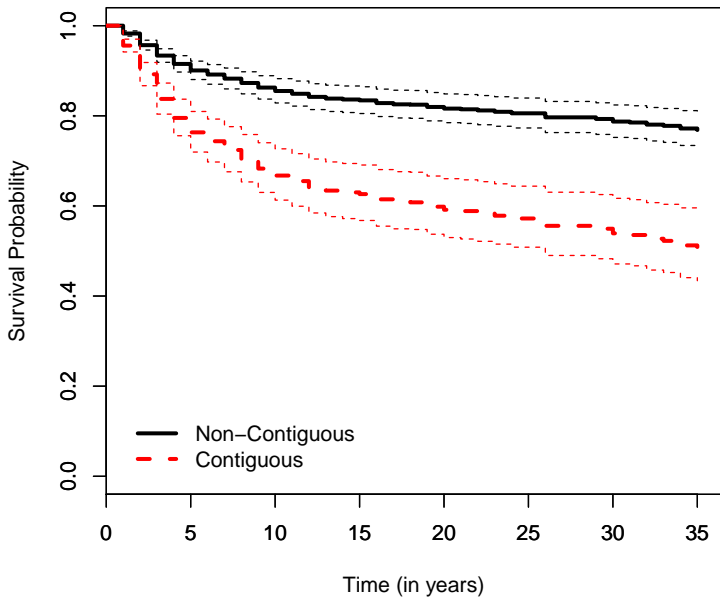
Baseline Hazards

Because the Cox model is semiparametric, it uses a conventional / univariate (Nelson-Aalen) estimate of the “baseline” hazard:

```
OR.BH<-basehaz(ORCox.br,centered=FALSE)
```



Comparing Survival Curves



\exists ties...

Their presence biases Cox $\hat{\beta}$ s toward zero.

- Call $d_j > 0$ the number of events occurring at t_j , and
- D_j the set of d_j observations that have the event at t_j .

Ties (continued)

Means of handling ties:

- Breslow:

$$L_{\text{Breslow}}(\beta) = \prod_{i=1}^N \frac{\exp \left[\left(\sum_{q \in D_j} \mathbf{x}_q \right) \beta \right]}{\left[\sum_{\ell \in R_j} \exp(\mathbf{x}_\ell \beta) \right]^{d_j}}$$

- Efron

$$\ln L_{\text{Efron}}(\beta) = \sum_{j=1}^J \sum_{i \in D_j} \left\{ \mathbf{x}_i \beta - \frac{1}{d_j} \sum_{k=1}^{d_j-1} \ln \left[\sum_{\ell \in R_j} \exp(\mathbf{x}_\ell \beta) \right. \right. \\ \left. \left. - k \left(\frac{1}{d_j} \sum_{\ell \in D_j} \exp(\mathbf{x}_\ell \beta) \right) \right] \right\}$$

Ties (continued)

- “Exact” (partial likelihood)

$$\ln L_{\text{Exact}}(\beta) = \sum_{j=1}^J \left\{ \sum_{i \in R_j} \delta_{ij}(\mathbf{x}_i \beta) - \ln[f(r_j, d_j)] \right\}$$

where

$$f(r, d) = g(r-1, d) + g(r-1, d-1) \exp(\mathbf{x}_k \beta),$$

$k = r$ th observation in R_j ,

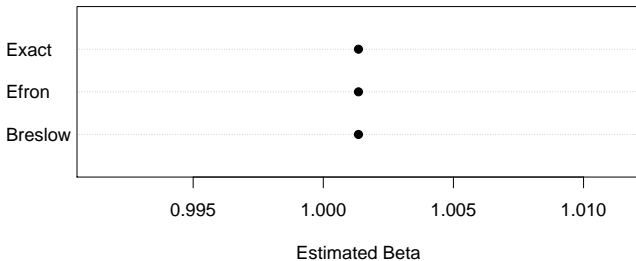
$r_j =$ cardinality of R_j , and

$$g(r, d) = \begin{cases} 0 & \text{if } r < d, \\ 1 & \text{if } d = 0 \end{cases}$$

Ties: Example

```
set.seed(7222009)
Data<-as.data.frame(cbind(c(rep(1,times=400)),
                          c(rep(c(0,1),times=200))))
colnames(Data)<-c("C","X")
Data$T<-rexp(400,exp(0+1*Data$X)) # B = 1.0
Data.S<-Surv(Data$T,Data$C)

D.br<-coxph(Data.S~X,data=Data,method="breslow")
D.ef<-coxph(Data.S~X,data=Data,method="efron")
D.ex<-coxph(Data.S~X,data=Data,method="exact")
```



Ties: Example (continued)

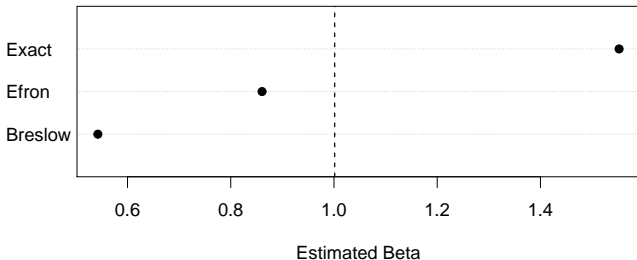
```
Data$Tied<-round(Data$T,0)
```

```
DataT.S<-Surv(Data$Tied,Data$C)
```

```
DT.br<-coxph(DataT.S~X,data=Data,method="breslow")
```

```
DT.ef<-coxph(DataT.S~X,data=Data,method="efron")
```

```
DT.ex<-coxph(DataT.S~X,data=Data,method="exact")
```



- All approx. are identical if \nexists ties
- Few ties = similar results
- When ties are present, Breslow < Efron < “Exact” methods
- If you want to learn more about ties in the Cox model, read my paper....

Cox vs. Parametric Models

Conceptual considerations:

- Theory
- Nature of $h(t)$
- Relative importance: Bias vs. efficiency
- Need / willingness for out-of-sample predictions / forecasting

Cox, On His Model

Reid: “What do you think of the cottage industry that’s grown up around [the Cox model]?”

Cox: “In the light of further results one knows since, I think I would normally want to tackle the problem parametrically... I’m not keen on non-parametric formulations normally.”

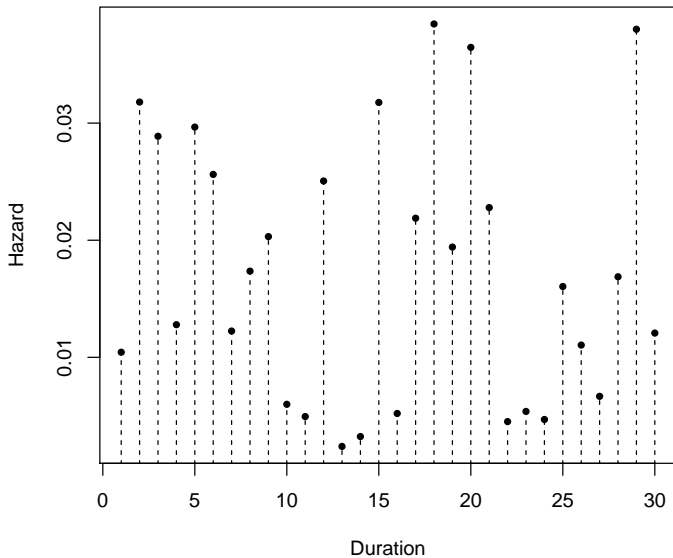
Reid: “So if you had a set of censored survival data today, you might rather fit a parametric model, even though there was a feeling among the medical statisticians that that wasn’t quite right.”

Cox: “That’s right, but since then various people have shown that the answers are very insensitive to the parametric formulation of the underlying distribution. And if you want to do things like predict the outcome for a particular patient, it’s much more convenient to do that parametrically.”

– From Reid (1994).

- Cox Models for *repeated events*
- Models with “frailties”
- Competing risks / “cured” subpopulations
- etc.

The Discrete-Time Idea



A General Discrete-Time Model

Process:

$$t \in \{1, 2, \dots, t_{\max}\}$$

Density:

$$f(t) = \Pr(T = t)$$

CDF:

$$\begin{aligned} F(t) &= \Pr(T \leq t) \\ &= \sum_{j=1}^t f(t_j) \end{aligned}$$

A General Discrete-Time Model

Survival function:

$$\begin{aligned} S(t) &\equiv \Pr(T \geq t) \\ &= 1 - F(t) \\ &= \sum_{j=t}^{t_{\max}} f(t_j) \end{aligned}$$

Hazard function:

$$\begin{aligned} h(t) &\equiv \Pr(T = t | T \geq t) \\ &= \frac{f(t)}{S(t)} \end{aligned}$$

A General Discrete-Time Model

Conditional Pr(Survival):

$$\Pr(T > t | T \geq t) = 1 - h(t)$$

Implies:

$$\begin{aligned} S(t) &= \Pr(T > t | T \geq t) \times \Pr(T > t-1 | T \geq t-1) \times \Pr(T > t-2 | T \geq t-2) \times \dots \\ &\quad \times \Pr(T > 2 | T \geq 2) \times \Pr(T > 1 | T \geq 1) \\ &= [1 - h(t)] \times [1 - h(t-1)] \times [1 - h(t-2)] \times \dots \times [1 - h(2)] \times [1 - h(1)] \\ &= \prod_{j=0}^t [1 - h(t-j)] \end{aligned}$$

A General Discrete-Time Model

which means:

$$\begin{aligned} f(t) &= h(t)S(t) \\ &= h(t) \times [1 - h(t-1)] \times [1 - h(t-2)] \times \dots \\ &\quad \times [1 - h(2)] \times [1 - h(1)] \\ &= h(t) \prod_{j=1}^{t-1} [1 - h(t-j)] \end{aligned}$$

General Discrete-Time Model: Likelihood

$$L = \prod_{i=1}^N \left\{ h(t) \prod_{j=1}^{t-1} [1 - h(t-j)] \right\}^{Y_{it}} \left\{ \prod_{j=0}^t [1 - h(t-j)] \right\}^{1-Y_{it}}$$

Ordered-Categorical Models

For K small:

$$\Pr(T_i \leq k) = \frac{\exp(\tau_k - \mathbf{X}_i\beta)}{1 + \exp(\tau_k - \mathbf{X}_i\beta)}$$

$$\ln \left[\frac{\Pr(T_i \leq \kappa)}{\Pr(T_i > \kappa)} \right] = \tau_\kappa - \mathbf{X}_i\beta$$

Grouped-Data (“BTSCS”) Approaches

$$\Pr(Y_{it} = 1) = f(\mathbf{X}_{it}\beta)$$

- logit
- probit
- c-log-log
- etc.

- Easily estimated, interpreted and understood
- Natural interpretations:
 - $\hat{\beta}_0 \approx$ “baseline hazard”
 - Covariates shift this up or down.
- Can incorporate data in time-varying covariates
- Lots of software

(Potential) Disadvantages

- Requires time-varying data
- *Must deal with time dependence explicitly*

Temporal Issues in Grouped-Data Models

(Implicit) “Baseline” hazard:

$$h_0(t) = \frac{\exp(\beta_0)}{1 + \exp(\beta_0)}$$

→ No temporal dependence / “flat” hazard

Temporal Issues in Grouped-Data Models

Time trend:

$$\Pr(Y_{it} = 1) = f(\mathbf{X}_{it}\beta + \gamma T_{it})$$

- $\hat{\gamma} > 0 \rightarrow$ rising hazard
- $\hat{\gamma} < 0 \rightarrow$ declining hazard
- $\hat{\gamma} = 0 \rightarrow$ “flat” (exponential) hazard

Variants/extensions: Polynomials...

$$\Pr(Y_{it} = 1) = f(\mathbf{X}_{it}\beta + \gamma_1 T_{it} + \gamma_2 T_{it}^2 + \gamma_3 T_{it}^3 + \dots)$$

Temporal Issues in Grouped-Data Models

“Time dummies”:

$$\Pr(Y_{it} = 1) = f[\mathbf{X}_{it}\beta + \alpha_1 I(T_{i1}) + \alpha_2 I(T_{i2}) + \dots + \alpha_{t_{\max}} I(T_{it_{\max}})]$$

→ BKT's cubic splines; might also use:

- Fractional polynomials
- Smoothed duration
- Loess/lowess fits
- Other splines (B-splines, P-splines, natural splines, etc.)

Discrete-Time Model Selection

- Theory
- Formal tests
- Fitted values

Equivalency One: Cox \equiv Conditional Logit

$$\Pr(Y_i = j) = \frac{\exp(\mathbf{X}_{ij}\beta + \mathbf{Z}_j\gamma)}{\sum_{\ell=1}^J \exp(\mathbf{X}_{i\ell}\beta + \mathbf{Z}_\ell\gamma)}$$

$$\Pr(Y_i = j) = \frac{\exp(\mathbf{X}_{ij}\beta)}{\sum_{\ell=1}^J \exp(\mathbf{X}_{i\ell}\beta)}$$

$$L_k = \frac{\exp(\mathbf{X}_k\beta)}{\sum_{\ell \in R_j} \exp(\mathbf{X}_\ell\beta)}.$$

The point: Cox \equiv Conditional logit

Grouped-data duration models and the continuous-time Cox model are equivalent.

Cox-Poisson Equivalence

Cox:

$$S_i(t) = \exp \left[-\exp(\mathbf{X}_i\beta) \int_0^t h_0(t) dt \right]$$

Poisson:

$$\Pr(Y = y) = \frac{\exp(-\lambda)\lambda^y}{y!}$$

$$\begin{aligned} \Pr(Y_{it} = 0) &= \exp(-\lambda) \\ &= \exp[-\exp(\mathbf{X}_i\beta)] \end{aligned}$$

Example: Oneal & Russett (1950-1985)

No time variable / "flat" hazard:

```
> OR.logit<-glm(dispute~allies+contig+capratio+growth+democracy+trade,  
                data=OR,na.action=na.exclude,family="binomial")  
> summary(OR.logit)
```

Coefficients:

	Estimate	Std. Error	z value	Pr(> z)	
(Intercept)	-4.32668	0.11451	-37.785	< 2e-16	***
allies	-0.47969	0.11275	-4.255	2.09e-05	***
contig	1.35358	0.12091	11.195	< 2e-16	***
capratio	-0.19620	0.05011	-3.916	9.01e-05	***
growth	-3.42753	1.25181	-2.738	0.00618	**
democracy	-0.40120	0.10063	-3.987	6.70e-05	***
trade	-21.07611	11.30396	-1.864	0.06225	.

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Example, Continued

Linear trend:

```
> OR$duration<-OR$stop
> OR.trend<-glm(dispute~allies+contig+capratio+growth+democracy+trade
                +duration,data=OR,na.action=na.exclude,family="binomial")
> summary(OR.trend)
```

Coefficients:

	Estimate	Std. Error	z value	Pr(> z)	
(Intercept)	-3.271136	0.134709	-24.283	< 2e-16	***
allies	-0.362966	0.114140	-3.180	0.001473	**
contig	0.996908	0.123978	8.041	8.91e-16	***
capratio	-0.235655	0.052763	-4.466	7.96e-06	***
growth	-3.957428	1.225716	-3.229	0.001244	**
democracy	-0.361150	0.099515	-3.629	0.000284	***
trade	-2.870981	9.861298	-0.291	0.770947	
duration	-0.091189	0.008098	-11.260	< 2e-16	***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Example, Continued

Fourth-Order polynomial trend:

```
OR$d2<-OR$duration^2*0.1
OR$d3<-OR$duration^3*0.01
OR$d4<-OR$duration^4*0.001
```

```
OR.P4<-glm(dispute~allies+contig+capratio+growth+democracy+trade
            +duration+d2+d3+d4,data=OR,na.action=na.exclude,
            family="binomial")
```

```
> summary(OR.P4)
```

Coefficients:

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	-3.401363	0.206815	-16.446	< 2e-16 ***
allies	-0.364127	0.114201	-3.188	0.00143 **
contig	0.995584	0.124074	8.024	1.02e-15 ***
capratio	-0.228355	0.052257	-4.370	1.24e-05 ***
growth	-3.864329	1.245617	-3.102	0.00192 **
democracy	-0.392457	0.100693	-3.898	9.72e-05 ***
trade	-4.032292	9.631171	-0.419	0.67546
duration	0.058036	0.091465	0.635	0.52574
d2	-0.274958	0.128454	-2.141	0.03231 *
d3	0.136086	0.063230	2.152	0.03138 *
d4	-0.018863	0.009914	-1.903	0.05709 .

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Polynomial Improvement?

```
> P4test
Analysis of Deviance Table

Model 1: dispute ~ allies + contig + capratio + growth + democracy + trade
Model 2: dispute ~ allies + contig + capratio + growth + democracy + trade +
  duration + d2 + d3 + d4

   Resid. Df Resid. Dev Df Deviance Pr(>Chi)
1      20441      3693.8
2      20437      3510.0  4    183.76 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```


Example: "Time Dummies"

```
"Time dummies":  
> OR.dummy<-glm(dispute~allies+contig+capratio+growth+democracy+trade  
+as.factor(duration),data=OR,na.action=na.exclude,  
family="binomial")
```

```
> summary(OR.dummy)
```

Coefficients:

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	-3.61115	0.18219	-19.820	< 2e-16 ***
allies	-0.36922	0.11441	-3.227	0.001251 **
contig	0.99389	0.12417	8.005	1.20e-15 ***
capratio	-0.22778	0.05219	-4.364	1.27e-05 ***
growth	-3.97619	1.24940	-3.182	0.001460 **
democracy	-0.39559	0.10077	-3.926	8.65e-05 ***
trade	-3.46727	9.62606	-0.360	0.718700
as.factor(duration)2	0.45489	0.19606	2.320	0.020331 *
as.factor(duration)3	0.36020	0.20632	1.746	0.080843 .
as.factor(duration)4	0.14188	0.22175	0.640	0.522289

<output omitted>

as.factor(duration)33	-1.64467	1.01715	-1.617	0.105891
as.factor(duration)34	-0.86966	0.73158	-1.189	0.234541
as.factor(duration)35	-1.38777	1.01857	-1.362	0.173049

Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1

“Time Dummies,” continued

```
> Test.Dummies<-anova(OR.logit,OR.dummy,test="Chisq")
```

```
> Test.Dummies
```

Analysis of Deviance Table

Model 1: dispute ~ allies + contig + capratio + growth + democracy + trade

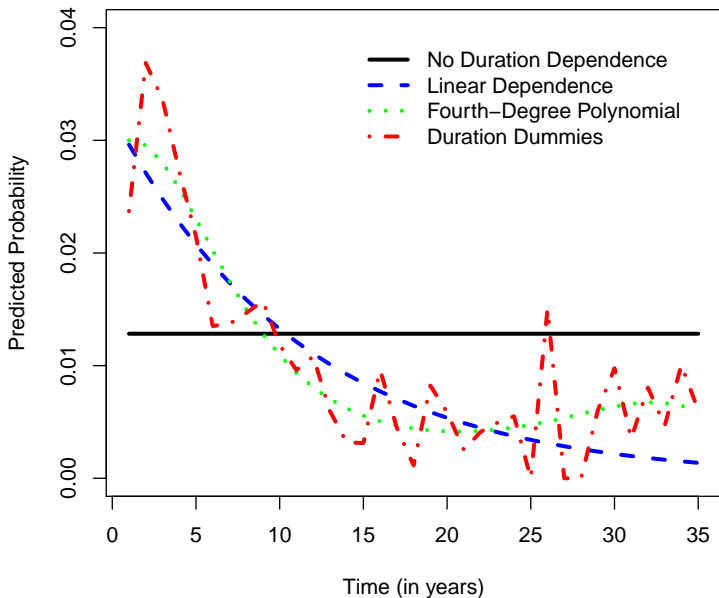
Model 2: dispute ~ allies + contig + capratio + growth + democracy + trade +
as.factor(duration)

	Resid. Df	Resid. Dev	Df	Deviance	Pr(>Chi)
--	-----------	------------	----	----------	----------

1	20441	3693.8			
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2	20407	3464.4	34	229.38	< 2.2e-16 ***
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Predicted “Hazards” (Probabilities)



Cox / Poisson Equivalence

Cox model:

```
OR.Cox<-coxph(Surv(OR$start,OR$stop,OR$dispute)~allies+contig+capratio+  
growth+democracy+trade,data=OR,method="breslow")
```

Poisson:

```
OR.Poisson<-glm(dispute~allies+contig+capratio+growth+democracy+trade  
+as.factor(duration),data=OR,na.action=na.exclude,  
family="poisson")
```

