

GSERM - Oslo 2019

Panel Data Models for Binary and Count Responses

January 8, 2019 (afternoon session)

Start with:

$$Y_{it}^* = \mathbf{X}_{it}\beta + u_{it}$$
$$Y_{it} = \begin{cases} 0 & \text{if } Y_{it}^* \leq 0 \\ 1 & \text{if } Y_{it}^* > 0 \end{cases}$$

which is generically:

$$Y_{it} = f(\mathbf{X}_{it}\beta + u_{it})$$

What Can Go Wrong?

Suppose:

$$\begin{aligned}X_{it} &= \rho_X \mathbf{X}_{it-1} + \nu_{it} \\ u_{it} &= \rho_u u_{it-1} + \epsilon_{it}\end{aligned}$$

For high values of ρ , logit/probit:

- $\hat{\beta}$ s are consistent, but s.e.s are biased, inefficient (Poirier and Ruud 1988);
- underestimate $\text{Var}(\beta)$ by up to 50 percent (Beck and Katz 1997).

One-way unit effects:

$$Y_{it} = f(\mathbf{X}_{it}\beta + \alpha_i + u_{it})$$

for logit only, so:

$$\Pr(Y_{it} = 1) = \frac{\exp(\mathbf{X}_{it}\beta + \alpha_i)}{1 + \exp(\mathbf{X}_{it}\beta + \alpha_i)} \equiv \Lambda(\mathbf{X}_{it}\beta + \alpha_i)$$

- Nonlinearity \rightarrow inconsistency in both $\hat{\alpha}$ s and $\hat{\beta}$.
- Anderson:

$$L^U = \prod_{i=1}^N \prod_{t=1}^T \Lambda(\mathbf{x}_{it} + \alpha_i)^{Y_{it}} [1 - \Lambda(\mathbf{x}_{it} + \alpha_i)]^{1-Y_{it}}$$

- Chamberlain:

$$L^C = \prod_{i=1}^N \Pr \left(Y_{i1} = y_{i1}, Y_{i2} = y_{i2}, \dots, Y_{iT} = y_{iT} \mid \sum_{t=1}^T Y_{it} \right)$$

Intuition:

- $\Pr(Y_{i1} = 0 \text{ and } Y_{i2} = 0 \mid \sum_T Y_{it} = 0) = 1.0$
- $\Pr(Y_{i1} = 1 \text{ and } Y_{i2} = 1 \mid \sum_T Y_{it} = 2) = 1.0$

Fixed-Effects (continued)

More intuition:

$$\Pr\left(Y_{i1} = 0 \text{ and } Y_{i2} = 1 \mid \sum_T Y_{it} = 1\right) = \frac{\Pr(0, 1)}{\Pr(0, 1) + \Pr(1, 0)}$$

with a similar statement for $\Pr(Y_{i1} = 1 \text{ and } Y_{i2} = 0 \mid \sum_T Y_{it} = 1)$.

Points:

- Fixed effects = no estimates for β_b
- Interpretation: per logit, but $\mid \hat{\alpha}_j$.
- BTSCS in IR: Green et al. (2001) v. B&K (2001).

Model is:

$$\begin{aligned} Y_{it}^* &= \mathbf{X}_{it}\beta + u_{it} \\ Y_{it} &= 0 \text{ if } Y_{it}^* \leq 0 ; \\ &= 1 \text{ if } Y_{it}^* > 0 \end{aligned}$$

with:

$$u_{it} = \alpha_i + \eta_{it}$$

with $\eta_{it} \sim \text{i.i.d. } N(0,1)$, and $\alpha_i \sim N(0, \sigma_\alpha^2)$.

Random Effects (continued)

Implies:

$$\text{Var}(u_{it}) = 1 + \sigma_{\alpha}^2$$

and so:

$$\text{Corr}(u_{it}, u_{is}, t \neq s) \equiv \rho = \frac{\sigma_{\alpha}^2}{1 + \sigma_{\alpha}^2}$$

which means that we can write $\sigma_{\alpha}^2 = \left(\frac{\rho}{1-\rho} \right)$.

Probit:

$$\begin{aligned} L_i &= \text{Prob}(Y_{i1} = y_{i1}, Y_{i2} = y_{i2}, \dots Y_{iT} = y_{iT}) \\ &= \int_{-\infty}^{X_{i1}\beta} \int_{-\infty}^{X_{i2}\beta} \dots \int_{-\infty}^{X_{iT}\beta} \phi(u_{i1}, u_{i2} \dots u_{iT}) du_{iT} \dots du_{i2} du_{i1} \end{aligned}$$

Logit:

$$\begin{aligned} L_i &= \text{Prob}(Y_{i1} = y_{i1}, Y_{i2} = y_{i2}, \dots Y_{iT} = y_{iT}) \\ &= \int_{-\infty}^{X_{i1}\beta} \int_{-\infty}^{X_{i2}\beta} \dots \int_{-\infty}^{X_{iT}\beta} \lambda(u_{i1}, u_{i2} \dots u_{iT}) du_{iT} \dots du_{i2} du_{i1} \end{aligned}$$

Solution?

$$\phi(u_{i1}, u_{i2}, \dots u_{iT}) = \int_{-\infty}^{\infty} \phi(u_{i1}, u_{i2}, \dots u_{iT} \mid \alpha_i) \phi(\alpha_i) d\alpha_i$$

- $\hat{\rho}$ = proportion of the variance due to the α_i s.
- Implementation: Gauss-Hermite quadrature or MCMC.
- Best with N large and T small.
- Critically requires $\text{Cov}(\mathbf{X}, \alpha) = 0$ (see notes re: Chamberlain's CRE Estimator).

R

- `glmmML` (Gauss-Hermite quadrature)
- `pglm` (panel GLMs) (maximum likelihood + quadrature)
- `MCMCpack` (`MCMChlogit`)
- Various user-generated functions (e.g., [here](#)).

Stata

- `xtprobit`, `xtlogit`, `xtcloglog`
- Plus `xttrans` (transition probabilities), `quadchk` (quadrature checking), `xtrho` / `xtrhoi` (estimation of within-unit covariances)

Example: Segal (1986) Search & Seizure Cases

$Y = 1$ (search allowed)

- **warrant**: Whether (=1) or not (=0) a warrant was issued,
- **house**: Whether (=1) or not (=0) the search was of a private home,
- **person**: Whether (=1) or not (=0) the search was of a person,
- **business**: Whether (=1) or not (=0) the search was of a business,
- **car**: Whether (=1) or not (=0) the search was of an automobile,
- **us**: Whether (=1) or not (=0) the U.S. government was the petitioner,
- **except**: The number of “exceptions” outlined by the Court under which the search fell, and
- **justideo**: The justice’s Segal-Cover (1989) ideology score, ranging from zero (most conservative) to 1 (most liberal).

$N = 14$, $\bar{T} = 74.1$.

```
> summary(Segal)
```

justid	caseid	year	vote	warrant
Min. : 1.0	Min. : 1	Min. :63	Min. :0.00	Min. :0.00
1st Qu.: 6.0	1st Qu.: 34	1st Qu.:69	1st Qu.:0.00	1st Qu.:0.00
Median : 8.0	Median : 64	Median :73	Median :1.00	Median :0.00
Mean : 8.1	Mean : 64	Mean :73	Mean :0.53	Mean :0.15
3rd Qu.:11.0	3rd Qu.: 94	3rd Qu.:78	3rd Qu.:1.00	3rd Qu.:0.00
Max. :14.0	Max. :123	Max. :81	Max. :1.00	Max. :1.00

house	person	business	car	us
Min. :0.00	Min. :0.00	Min. :0.00	Min. :0.0	Min. :0.00
1st Qu.:0.00	1st Qu.:0.00	1st Qu.:0.00	1st Qu.:0.0	1st Qu.:0.00
Median :0.00	Median :0.00	Median :0.00	Median :0.0	Median :0.00
Mean :0.23	Mean :0.31	Mean :0.15	Mean :0.2	Mean :0.45
3rd Qu.:0.00	3rd Qu.:1.00	3rd Qu.:0.00	3rd Qu.:0.0	3rd Qu.:1.00
Max. :1.00	Max. :1.00	Max. :1.00	Max. :1.0	Max. :1.00

except	justideo
Min. :0.00	Min. :0.05
1st Qu.:0.00	1st Qu.:0.17
Median :0.00	Median :0.73
Mean :0.35	Mean :0.59
3rd Qu.:1.00	3rd Qu.:0.88
Max. :3.00	Max. :1.00

Plain-Vanilla Logit

```
> SegalLogit<-glm(vote~warrant+house+person+business+car+us+
                  except+justideo,data=Segal,family="binomial")
> summary(SegalLogit)
```

Deviance Residuals:

	Min	1Q	Median	3Q	Max
	-2.3147	-0.9405	0.3898	0.9348	1.9032

Coefficients:

	Estimate	Std. Error	z value	Pr(> z)	
(Intercept)	1.9419	0.2799	6.938	3.97e-12	***
warrant	0.5335	0.2083	2.561	0.010440	*
house	-1.0840	0.2756	-3.934	8.36e-05	***
person	-0.9438	0.2569	-3.674	0.000239	***
business	-1.4722	0.2975	-4.949	7.46e-07	***
car	-1.0066	0.2816	-3.574	0.000351	***
us	0.4824	0.1482	3.254	0.001136	**
except	0.8640	0.1384	6.243	4.29e-10	***
justideo	-2.4026	0.2158	-11.134	< 2e-16	***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 1434.9 on 1036 degrees of freedom
Residual deviance: 1196.7 on 1028 degrees of freedom
AIC: 1214.7

Number of Fisher Scoring iterations: 4

```
> library(glmML)
> SegalFE<-glmboot(vote~warrant+house+person+business+car+us+
  except,data=Segal,family="binomial",
  cluster=justid)
> summary(SegalFE)
```

```
Call: glmboot(formula = vote ~ warrant + house + person + business +
  car + us + except, family = "binomial", data = Segal, cluster = justid)
```

	coef	se(coef)	z	Pr(> z)
warrant	0.599	0.228	2.63	8.7e-03
house	-1.473	0.305	-4.82	1.4e-06
person	-1.124	0.282	-3.99	6.7e-05
business	-1.837	0.326	-5.63	1.8e-08
car	-1.202	0.308	-3.90	9.6e-05
us	0.537	0.162	3.32	9.1e-04
except	1.093	0.155	7.03	2.1e-12

```
Residual deviance: 1050 on 1016 degrees of freedom  AIC: 1090
```

Random Effects

```
> SegalRE<-glmmML(vote~warrant+house+person+business+car+us+
                  except+justideo,data=Segal,family="binomial",
                  cluster=justid)
> summary(SegalRE)
```

```
Call: glmmML(formula = vote ~ warrant + house + person + business +
car + us + except + justideo, family = "binomial", data = Segal, cluster = justid)
```

	coef	se(coef)	z	Pr(> z)
(Intercept)	2.016	0.565	3.57	3.6e-04
warrant	0.594	0.226	2.63	8.5e-03
house	-1.434	0.303	-4.73	2.2e-06
person	-1.104	0.280	-3.95	7.9e-05
business	-1.799	0.324	-5.56	2.7e-08
car	-1.181	0.306	-3.86	1.1e-04
us	0.531	0.160	3.31	9.3e-04
except	1.070	0.154	6.95	3.6e-12
justideo	-2.344	0.737	-3.18	1.5e-03

```
Scale parameter in mixing distribution: 0.926 gaussian
Std. Error: 0.195
```

```
LR p-value for H_0: sigma = 0: 4.63e-24
```

```
Residual deviance: 1100 on 1027 degrees of freedom AIC: 1120
```


Models for Event Counts

Things That Are Not Counts

- Ordinal scales/variables
- Grouped Binary Data
 - $\frac{N \text{ of "successes"}}{N \text{ of "trials"}}$
 - Binomial data
 - = counts only if $\Pr(\text{"success"})$ is small

- Discrete / integer-values
- Non-negative
- “Cumulative”

Count Data: Motivation

$$\text{Arrival Rate} = \lambda$$

$$\Pr(\text{Event})_{t,t+h} = \lambda h$$

$$\Pr(\text{No Event})_{t,t+h} = 1 - \lambda h$$

$$\begin{aligned}\Pr(Y_t = y) &= \frac{\exp(-\lambda h) \lambda h^y}{y!} \\ &= \frac{\exp(-\lambda) \lambda^y}{y!}\end{aligned}$$

- No Simultaneous Events
- Constant Arrival Rate
- Independent Event Arrivals

For M independent Bernoulli trials with (sufficiently small) probability of success π and where $M\pi \equiv \lambda > 0$,

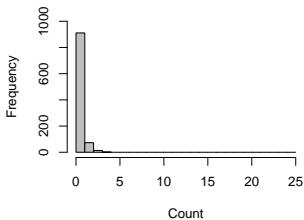
$$\begin{aligned}\Pr(Y_i = y) &= \lim_{M \rightarrow \infty} \left[\binom{M}{y} \left(\frac{\lambda}{M}\right)^y \left(1 - \frac{\lambda}{M}\right)^{M-y} \right] \\ &= \frac{\lambda^y \exp(-\lambda)}{y!}\end{aligned}$$

Poisson: Characteristics

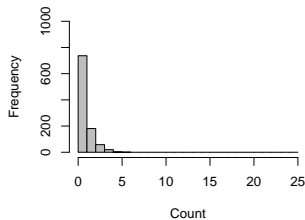
- Discrete
- $E(Y) = \text{Var}(Y) = \lambda$
- Is not preserved under affine transformations...
- For $X \sim \text{Poisson}(\lambda_X)$ and $Y \sim \text{Poisson}(\lambda_Y)$,
 $Z = X + Y \sim \text{Poisson}(\lambda_{X+Y})$ *iff* X and Y are *independent* but
- ...same is not true for differences.
- $\lambda \rightarrow \infty \iff Y \sim N$

Poissons: Examples

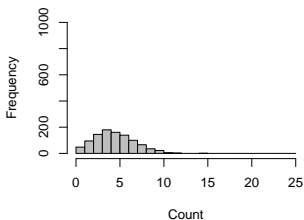
Lambda = 0.5



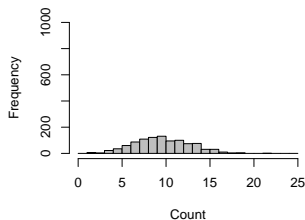
Lambda = 1.0



Lambda = 5



Lambda = 10



Suppose

$$E(Y_i) \equiv \lambda_i = \exp(\mathbf{X}_i\boldsymbol{\beta})$$

then

$$\Pr(Y_i = y | \mathbf{X}_i, \boldsymbol{\beta}) = \frac{\exp[-\exp(\mathbf{X}_i\boldsymbol{\beta})][\exp(\mathbf{X}_i\boldsymbol{\beta})]^y}{y!}$$

$$L = \prod_{i=1}^N \frac{\exp[-\exp(\mathbf{X}_i\boldsymbol{\beta})][\exp(\mathbf{X}_i\boldsymbol{\beta})]^{Y_i}}{Y_i!}$$

$$\ln L = \sum_{i=1}^N [-\exp(\mathbf{X}_i\boldsymbol{\beta}) + Y_i\mathbf{X}_i\boldsymbol{\beta} - \ln(Y_i!)]$$

Event Counts: Unit Effects

$$Y_{it} \sim \text{Poisson}(\mu_{it} = \alpha_i \lambda_{it})$$

with $\lambda_{it} = \exp(\mathbf{X}_{it}\beta)$ implies:

$$\begin{aligned} E(Y_{it} \mid \mathbf{X}_{it}, \alpha_i) &= \mu_{it} \\ &= \alpha_i \exp(\mathbf{X}_{it}\beta) \\ &= \exp(\delta_i + \mathbf{X}_{it}\beta) \end{aligned}$$

where $\delta_i = \ln(\alpha_i)$.

- No “incidental parameters” problem (see e.g. Cameron and Trivedi, pp. 281-2)
- Means “brute force” approach works
- Via `xtpoisson` (and `xtnbreg`) in Stata, `glmmML` in R

$$\begin{aligned}\Pr(Y_{i1} = y_{i1}, \dots, Y_{iT} = y_{iT}) &= \int_0^\infty \Pr(Y_{i1} = y_{i1}, \dots, Y_{iT} = y_{iT}) f(\alpha_i) d\alpha_i \\ &= \int_0^\infty \left[\prod_{t=1}^T \Pr(Y_{it} | \alpha_i) \right] f(\alpha_i) d\alpha_i\end{aligned}$$

- Simplest to assume $\alpha_i \sim \Gamma(\theta)$
- Yields a model with $E(Y_{it}) = \lambda_{it}$ and $\text{Var}(Y_{it}) = \lambda_{it} + \frac{\lambda_{it}^2}{\theta}$
- Via `xtpois`, `re` in Stata and `glmmML` or `glmer` in R
- \exists random effects negative binomial too...

R:

- Tobit = `censReg` (in **`censReg`**)
- Poisson (random effects) = `glmmML` in **`glmmML`** or `glmer` in **`lme4`**
- Poisson (fixed effects) = `glmmML` or “brute force”

Stata:

- Tobit = `xttobit` (re only)
- Poisson / negative binomial = `xtpoisson`, `xtnbreg` (both with `fe`, `re` options)
- See notes for more details / examples

Example: State Failure Task Force

```
> summary(SFTF)
```

countryid	year	sftpPrev	sftpeth	sftpreg
AFG : 9	Min. :1957	Min. :0.0	Min. :0.00	Min. :0.00
ALB : 9	1st Qu.:1967	1st Qu.:0.0	1st Qu.:0.00	1st Qu.:0.00
ARG : 9	Median :1977	Median :0.0	Median :0.00	Median :0.00
AUL : 9	Mean :1979	Mean :0.1	Mean :0.13	Mean :0.12
AUS : 9	3rd Qu.:1992	3rd Qu.:0.0	3rd Qu.:0.00	3rd Qu.:0.00
BEL : 9	Max. :1997	Max. :1.0	Max. :1.00	Max. :1.00

```
(Other):1149
```

sftpGen	poldurab	unuurbpc	ciob	cioc
Min. :0.00	Min. : 0	Min. : 2	Min. : 0	Min. : 0.0
1st Qu.:0.00	1st Qu.: 4	1st Qu.: 23	1st Qu.:14	1st Qu.: 2.0
Median :0.00	Median :12	Median : 41	Median :19	Median : 5.0
Mean :0.08	Mean :21	Mean : 43	Mean :19	Mean : 5.6
3rd Qu.:0.00	3rd Qu.:30	3rd Qu.: 62	3rd Qu.:24	3rd Qu.: 8.0
Max. :1.00	Max. :97	Max. :100	Max. :38	Max. :24.0
	NA's :5	NA's :57		

POLITY	SumEvents
Min. : -10.0	Min. : 0
1st Qu.: -7.0	1st Qu.: 0
Median : -4.0	Median : 0
Mean : -0.7	Mean : 6
3rd Qu.: 8.0	3rd Qu.: 5
Max. : 10.0	Max. :61
NA's :14	NA's :9

```
> pdim(SFTF)
```

```
Unbalanced Panel: n=170, T=1-9, N=1203
```

Panel Tobit: R (see [here](#))

```
> library(plm)
> SFTF.panel<-pdata.frame(SFTF,i="countryid")
> library(censReg)
> Tobit.panel<-censReg(SumEvents~POLITY+unuurbpc+poldurab+year,
+                       data=SFTF.panel,method="BHHH")
> summary(Tobit.panel)
```

Call:

```
censReg(formula = SumEvents ~ POLITY + unuurbpc + poldurab +
        year, data = SFTF.panel, method = "BHHH")
```

Observations:

Total	Left-censored	Uncensored	Right-censored
1132	707	425	0

Coefficients:

	Estimate	Std. error	t value	Pr(> t)
(Intercept)	-1385.21151	60.10481	-23.047	< 2e-16 ***
POLITY	-0.58977	0.09008	-6.547	5.87e-11 ***
unuurbpc	-0.31374	0.03263	-9.616	< 2e-16 ***
poldurab	-0.34628	0.02624	-13.198	< 2e-16 ***
year	0.70470	0.03048	23.121	< 2e-16 ***
logSigmaMu	2.83694	0.05035	56.341	< 2e-16 ***
logSigmaNu	2.58187	0.02160	119.522	< 2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

BHHH maximisation, 40 iterations

Return code 2: successive function values within tolerance limit

Log-likelihood: -2020 on 7 Df

Panel Poisson (Random Effects)

```
> library(lme4)
> Poisson.RE<-glmer(ciob~POLITY+unuurbpc+poldurab+I(year-1900)+(1|countryid),
  data=SFTF,family="poisson")
> summary(Poisson.RE)
Generalized linear mixed model fit by maximum likelihood (Laplace
Approximation) [glmerMod]
Family: poisson ( log )
Formula:
ciob ~ POLITY + unuurbpc + poldurab + I(year - 1900) + (1 | countryid)
Data: SFTF

      AIC      BIC   logLik deviance df.resid
  6811    6841    -3399     6799     1126

Random effects:
Groups   Name      Variance Std.Dev.
countryid (Intercept) 0.159   0.399
Number of obs: 1132, groups:  countryid, 160

Fixed effects:
              Estimate Std. Error z value    Pr(>|z|)
(Intercept)   1.200274   0.063085   19.03 < 2e-16 ***
POLITY        -0.003484   0.001812   -1.92   0.055 .
unuurbpc       0.005996   0.001064    5.64 0.000000017 ***
poldurab       0.001167   0.000672    1.74   0.082 .
I(year - 1900) 0.016385   0.000855   19.16 < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:
              (Intr) POLITY unurbpc poldrb
POLITY        0.354
unuurbpc     -0.075 -0.139
poldurab      0.224  0.348 -0.087
I(yer-1900) -0.628 -0.273 -0.589 -0.313
convergence code: 0
Model failed to converge with max|grad| = 0.00158426 (tol = 0.001, component 1)
Model is nearly unidentifiable: very large eigenvalue
- Rescale variables?
```

Panel Poisson (Random Effects – Alternative)

```
> library(glmML)
> Poisson.RE.alt<-glmML(cio~POLITY+unuurbpc+poldurab+I(year-1900),
                        data=SFTF,cluster=countryid,
                        family="poisson")
> summary(Poisson.RE.alt)
```

```
Call: glmML(formula = cio ~ POLITY + unuurbpc + poldurab + I(year - 1900),
             family = "poisson", data = SFTF, cluster = countryid)
```

	coef	se(coef)	z	Pr(> z)
(Intercept)	1.20027	0.063120	19.02	0.000000000
POLITY	-0.00348	0.001814	-1.92	0.055000000
unuurbpc	0.00600	0.001064	5.63	0.000000018
poldurab	0.00117	0.000672	1.74	0.082000000
I(year - 1900)	0.01639	0.000856	19.15	0.000000000

```
Scale parameter in mixing distribution: 0.399 gaussian
Std. Error: 0.0263
```

```
LR p-value for H_0: sigma = 0: 2.28e-289
```

```
Residual deviance: 1590 on 1126 degrees of freedom AIC: 1600
```

Panel Poisson (Fixed Effects – “brute force”)

```
> Poisson.FE<-glm(cio~POLITY+unuurbpc+poldurab+I(year-1900)+
  as.factor(countryid),data=SFTF,family="poisson")
> summary(Poisson.FE)
```

Call:

```
glm(formula = cio ~ POLITY + unuurbpc + poldurab + I(year -
  1900) + as.factor(countryid), family = "poisson", data = SFTF)
```

Deviance Residuals:

	Min	1Q	Median	3Q	Max
	-4.806	-0.312	0.069	0.364	2.863

Coefficients:

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	1.040769	0.117296	8.87	< 2e-16 ***
POLITY	-0.007437	0.001939	-3.84	0.00013 ***
unuurbpc	0.005011	0.001580	3.17	0.00151 **
poldurab	-0.000477	0.000749	-0.64	0.52386
I(year - 1900)	0.018411	0.001115	16.51	< 2e-16 ***
as.factor(countryid)ALB	-0.376632	0.142587	-2.64	0.00826 **
as.factor(countryid)ALG	0.200591	0.131453	1.53	0.12702
.				
.				
.				
as.factor(countryid)ZAM	0.094994	0.132209	0.72	0.47244
as.factor(countryid)ZIM	-0.053680	0.137511	-0.39	0.69627

Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1

(Dispersion parameter for poisson family taken to be 1)

Null deviance: 4453.85 on 1131 degrees of freedom
Residual deviance: 942.45 on 968 degrees of freedom
(71 observations deleted due to missingness)
AIC: 6483

Number of Fisher Scoring iterations: 5