Department of Engineering Cybernetics

TTK4215 System Identification and Adaptive Control

Solution 1

Non-parametric Methods

Problem 1 (Impulse-Response Analysis)

a) Since $u(t) = 0$ for $t \neq 0$,the only non-zero term in the sum is for $k = t$. Thus, we have

$$y(t) = \sum_{k=1}^{\infty} g_0(k)u(t-k) + v(t) = g_0(k)\,u(t-k)\big|_{k=t} + v(t) = g_0(t)\,u(0) + v(t) = ag_0(t) + v(t)\,. \tag{1}$$

b) An estimate can be formed by removing the uncertain part $v(t)$. That is

$$\hat{g}_0(t) = \frac{y(t)}{a}. \tag{2}$$

c) The resulting estimation error is

$$g_0(t) - \hat{g}_0(t) = \frac{y(t) - v(t)}{a} - \frac{y(t)}{a} = -\frac{v(t)}{a}. \tag{3}$$

In order to obtain an estimation error that is small, a has to be large compared to $v(t)$. This may be impossible due to saturation in the input (a valve, for instance, restricts $u(t)$ to take values in $[0,1]$). Furthermore, since systems in practice are nonlinear, large perturbations will give nonlinear effects, and possibly as severe as instability.

Problem 2 (Step-Response Analysis)

a) In this case the terms of the sum are non-zero when $k \leq t$, so we have

$$y(t) = \sum_{k=1}^{\infty} g_0(k)u(t-k) + v(t) = a\sum_{k=1}^{t} g_0(k) + v(t)\,. \tag{4}$$

b) From a), we have that

$$\begin{aligned} y(t) - y(t-1) &= a\sum_{k=1}^{t} g_0(k) + v(t) - a\sum_{k=1}^{t-1} g_0(k) - v(t-1) \\ &= ag_0(t) + v(t) - v(t-1)\,. \end{aligned} \tag{5}$$

Removing the uncertain terms $v(t)$ and $v(t-1)$, an estimate can be found using

$$y(t) - y(t-1) = a\hat{g}_0(t)\,,$$

giving

$$\hat{g}_0(t) = \frac{y(t) - y(t-1)}{a}.$$

c) The estimation error is obtained as follows.

$$\begin{aligned} g_0(t) - \hat{g}_0(t) &= \frac{y(t) - y(t-1)}{a} - \frac{v(t) - v(t-1)}{a} - \frac{y(t) - y(t-1)}{a} \\ &= \frac{v(t-1) - v(t)}{a}. \end{aligned} \tag{6}$$

Again, a has to be large compared to the noise, and one will have the same practical problems as for the impuse-response analysis.

d) A first order system with time delay has transfer function

$$h(s) = \frac{k}{Ts+1} e^{-\tau s},$$

where k is the process gain, T is the time constant, and τ is the time delay. These parameters can be read directly from a single step response graph.

Problem 3 (Correlation Analysis)

a) A stationary signal $\{v(t)\}$ has $Ev(t) = c$, for some constant c, so $m_s(t)$ can be taken as c in i). Since $Ev(t)v(t-\tau) = R_v(\tau)$, $\breve{R}_v(t,r)$ in the sum in ii) only depends on $t - r = \tau$ and thus is independent of the summation variable t.

b) We have

$$\bar{E}s(t)s(t-\tau) = \lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} Es(t)s(t-\tau) = \lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} \breve{R}_s(t, t-\tau) = R_s(\tau). \tag{7}$$

c) We have that

$$\begin{aligned} R_{yu}(\tau) &= \bar{E}y(t)u(t-\tau) = \bar{E}[G_0(q)u(t) + v(t)]u(t-\tau) \\ &= \bar{E}G_0(q)u(t)u(t-\tau) + \bar{E}v(t)u(t-\tau). \end{aligned}$$

Since v and u are uncorrelated, the last term is identically zero. Therefore, we can view the system as $y(t) = G_0(q)u(t)$ for the purpose of this computation and exploit the fact that

$$\Phi_{yu}(\omega) = G_0\left(e^{i\omega}\right)\Phi_u(\omega).$$

We have

$$\begin{aligned} R_{yu}(\tau) &= \frac{1}{2\pi}\int_{-\pi}^{\pi} \Phi_{yu}(\omega) e^{i\tau\omega} d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} G_0\left(e^{i\omega}\right)\Phi_u(\omega) e^{i\tau\omega} d\omega \\ &= \frac{1}{2\pi}\int_{-\pi}^{\pi} \sum_{k=1}^{\infty} g_0(k) e^{-ik\omega}\Phi_u(\omega) e^{i\tau\omega} d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} \sum_{k=1}^{\infty} g_0(k)\Phi_u(\omega) e^{i(\tau-k)\omega} d\omega \\ &= \sum_{k=1}^{\infty} g_0(k)\left(\frac{1}{2\pi}\int_{-\pi}^{\pi} \Phi_u(\omega) e^{i(\tau-k)\omega} d\omega\right) = \sum_{k=1}^{\infty} g_0(k) R_u(\tau-k). \end{aligned} \tag{8}$$

d) We can replace $R_u(\tau)$ and $R_{yu}(\tau)$ with their estimates to obtain

$$\hat{R}_{yu}^N(\tau) = \sum_{k=1}^{\infty} g_0(k)\, \hat{R}_u^N(\tau - k). \tag{9}$$

Next, we have to truncate the sum, which we can since the coefficients $g_0(k)$ decay quickly, to obtain

$$\hat{R}_{yu}^N(\tau) = \sum_{k=1}^{M} g_0(k)\, \hat{R}_u^N(\tau - k). \tag{10}$$

Equation (10) can be solved for the coefficients $g_0(k)$.

Problem 4 (Fourier Analysis)

a) Since $\rho_1(N)$ approaches zero for large N, $\hat{G}_N(e^{i\omega})$ is an unbiased estimate of $G_0(e^{i\omega})$. Since $\rho_2(N)$ approaches zero for large N, the variance is given by the noise/signal ratio, which does not decrease with increasing N, and the estimates at different frequencies are uncorrelated (for large N).