Adaptive Estimation of Downhole Pressure for Managed Pressure Drilling Operations

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***Abstract*— The objective of Managed Pressure Drilling (MPD) is to enable precise control of the downhole pressure during drilling and completion operations. An essential part of an automated MPD control system is the downhole pressure estimator, usually referred to as the hydraulic model, which in most cases is the limiting factor for achievable accuracy of the system. In this paper we present a new design of an adaptive observer for online estimation of the downhole pressure in the well. Unlike conventional hydraulic models, which are based on complex partial differential equations, the adaptive observer is based on a simplified lumped model of ordinary differential equations, combined with online adaptation that tunes uncertain model parameters utilizing topside measurements. We demonstrate that the novell adaptive observer design captures the important hydraulics of MPD operations with high accuracy, and fundamentally improves the convergence rate of the parameter estimaties by using delayed observers. The results are demonstrated on field data from an offshore MPD operation.**

I. Introduction

To meet the increasing demand for oil and gas there is a need to find and extract new reserves. Most of the larger fields that are accessible with conventional drilling technology have been drilled. Consequently the remaining fields typically contain less oil and gas, and are harder to drill (located in less accessible formations). It is therefore a strong demand for drilling technologies that can drill where conventional drilling cannot be used, while still being cost and time efficient.

As an introduction to drilling consider the drill rig set-up illustrated in Fig. 1. The figure illustrates a jacket platform performing offshore managed pressure drilling. At the top of the derrick the drill string is attached to the top drive, which is a motor that turns the drill string. The drill string can move up and down inside the derrick as the top drive is attached to a hook that can be lowered or raised. As the drilling progresses the top of the drill string sinks towards the drill floor. After approximately $27m$ a new stand of drill pipe is connected to the top and drilling resumes. This procedure is referred to as a pipe connection. For a typical rate of penetration of $15\frac{m}{hr}$ a pipe connection is performed roughly every two hours.

During drilling, downhole cuttings need to be transported out of the bore hole. This is done by using a drilling fluid

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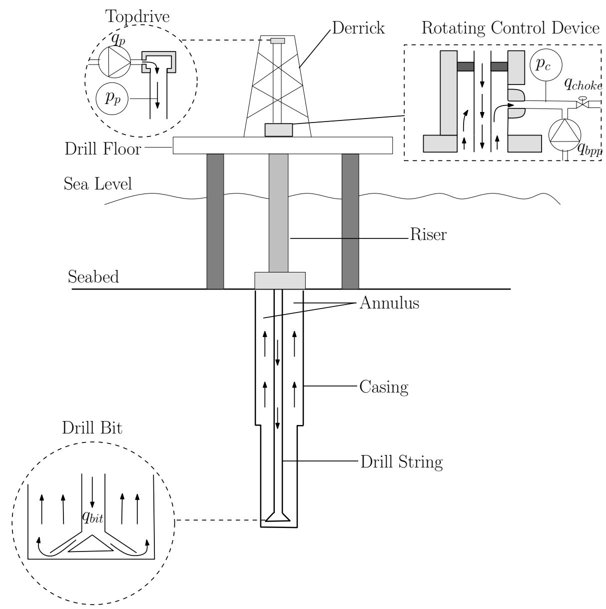


Fig. 1: Offshore drilling from a jacket platform. Drill mud flows from the main pump through the drill string, drill bit and out through the choke. The mud transports cuttings out of the wellbore and helps to maintain the desired pressure in the borehole.

(mud) circulation system. On board the rig, tanks filled with drilling fluid feed the main mud pump, which pumps the drilling fluid through the top drive and into the drill string. The fluid then flows down through the bit and up through the annulus, carrying the cuttings along, before the fluid exits through a choke. After exiting, the fluid is recycled and returned to the mud tanks. The example illustrated in Fig. 1 has a rotating control device which seals off the annulus from the outside, a choke that controls the flow rate of drilling fluid out from the annulus, and an additional back pressure pump that ensures a minimum flow rate through the choke. The rotating control device and the choke thus pressurize the annulus so that pressure can be increased or decreased by manipulating the choke. In conventional drilling there is no rotating control device, choke or back pressure pump, so the means of influencing the pressure in the annulus are limited.

The main reason for pressure control is to maintain the annulus pressure profile within its margins, i.e. above the pore pressure of the reservoir or the collapse pressure of the bore hole, and below the fracturing pressure of the bore hole. If the pressure in the annulus falls below the pore pressure, fluids (e.g. gas) can flow from the formation into the annulus, which is called a kick. If a kick is not detected and dealt with properly it can lead to an uncontrolled surface blowout with large financial losses, environmental damage and possible loss of lives. If the pressure in the annulus falls below the collapse pressure, the well can collapse onto the drill string which in turn gets stuck. In the worst case scenario the pipe must be severed and parts of the well drilled over again. If the pressure in the annulus exceeds the fracturing pressure, drilling fluid can be lost to the formation which can damage the permeability of the reservoir, and if severe losses occur, the annulus pressure might fall below the pore pressure (due to loss of hydrostatic height) inducing a kick.

As a response to the demand for more accurate pressure control a fairly new (for offshore drilling) technology, named Managed Pressure Drilling (MPD), has emerged [1], [2]. Over the last 10 years much research has been devoted to the field offshore MPD and there have been several successful applications, both on the Norwegian continental shelf [3], [4], [5], [6], [7], and in the rest of the world [8], [9], [10]. [1] divides existing MPD technologies into several categories, all of which have their pros and cons. In this paper we consider the so called constant bottomhole pressure variation which uses an additional seal, choke manifold and possibly a back pressure pump to pressurize the annulus, as shown in Fig. 1.

A crucial part of an automated MPD system is the so called hydraulic model or downhole pressure estimator. The estimator provides the choke pressure reference for the choke controller. As choke pressure control can be made very accurate by choosing appropriate hardware and designing high performance control loops, the accuracy of the hydraulic model is in most cases the limiting factor for the achievable accuracy of the entire control system. This has triggered a lot of research into developing advanced hydraulic models in order to capture all aspects of the drilling hydraulics, see e.g. [11], [12], [13], [14]. These models are based on distributed parameter models of multiphase flow and are able to reproduce a wide range of drilling-specific events, such as pipe connections, cementing, gelling and multi-fluid scenarios to an impressingly high degree of detail [15], [16]. Real-time versions of the models have also been developed and used with success [3], [16], [4].

Contrary to using advanced hydraulic models for downhole pressure estimation there have been attempts at using low order models to estimate the pressure. In the work [17] a low order model for underbalanced drilling[1] is derived. The later work [18] and [19] focuses on control and estimation based on the low order model. In this paper we use a simplified lumped model, presented in [20], [21], to derive an adaptive observer that estimates unmeasured states and parameters to improve the downhole pressure estimate. The work presented here contains improvements of earlier work presented in [22], [23]. In particular we remove the need to solve a partial differential equation (PDE) neccessary for the design in [22]. In practice this will allow for more sophisticated and accurate modelling of the frictional pressure drop in the system.

The remainder of the paper is organized as follows. Section II contains the simplified model and the main assumptions that are used as a basis for deriving the adaptive observer. In Section III we highlight the limitations impose by the requirement to solve the PDE in [22] and show that by adding an additional state to the observer we do not need to solve the PDE. We also show that the new observer fits into the framework used in [23], and so the parameter identification properties of the new observer can be improved by using multiple delayed observers, provided that the system is sufficiently excited. Finally, in Section IV we use data from experiments performed in an offshore well to show that the novel observer is able to accurately predict the downhole pressure in the presence of significant parametric uncertainties.

II. Simplified Model for Drilling

We consider the simplified model of a drilling system [20], [21]

$$\frac{V_d}{\beta_d}\dot{p}_p = q_{pump} - q_{bit} \tag{1a}$$

$$\frac{V_a}{\beta_a}\dot{p}_c = q_{bit} + q_{bpp} - q_{choke} \tag{1b}$$

$$M\dot{q}_{bit} = p_p - p_c + F\left(\theta, q_{bit}\right) + G\left(\theta\right) \tag{1c}$$

where p_p is the main pump pressure, q_{bit} is the flow rate through the bit and p_c is the pressure upstream the choke. V_d and V_a are the volumes in the drill string and annulus, β_d and β_a the bulk moduli of the fluid in the drill string and the annulus, and M a lumped density per length paramter. q_p, q_{choke} and q_{bpp} are the main pump, choke and back pressure pump flow rates, which are all measured signals. The pressures p_p and p_c are measured while the bit flow rate is not. As this paper is not concerned with control design we assume that q_p, q_c and q_{bpp} ensure that the solutions $p_p(t)$, $p_c(t)$ and $q_{bit}(t)$ of (1) are bounded. Note that the value of the bound is not used in the implementation of the observers. Furthermore $F(\theta, q_{bit})$ is a sufficiently smooth function describing friction losses in the system and $G(\theta)$ is a function describing the steady-state hydrostatic term affecting the flow. We allow both terms to depend on a vector of uncertain parameters $\theta \in \mathbb{R}^p$. In particular $F(\theta, q_{bit}) = F_a(\theta, q_{bit}) + F_d(\theta, q_{bit})$, where F_a and F_d are functions that describe the uncertain friction losses in the drillstring and annulus respectively. The hydrostatic term can be written as $G(\theta) = G_d(\theta) - G_a(\theta)$, where G_d is the hydrostatic pressure in the drill string and G_a the hydrostatic pressure in the annulus. The pressure at the bit, p_{bit}, is described by

$$p_{bit} = p_c - F_a\left(\theta, q_{bit}\right) + G_a\left(\theta\right). \tag{2}$$

For details on the derivation of the above model confer [20], [21]. The objective is to estimate the unmeasured bit

[1] During underbalanced drilling the well pressure is kept below the pore pressure so that the well is actually producing while drilling.

flow rate, the downhole pressure and the uncertain parameter vector θ.

For the observer design we assume that F and G satisfy the following properties:

P1 Monotonicity: There exists $L_f < \infty$ such that $\left[F(\theta,a) - F(\theta,b)\right](a-b) \le L_f |a-b|^2 \ \forall\ a,b \in \mathbb{R}$

P2 Linear parameterization: $F(\theta,a) - F(\hat{\theta},a) + G(\theta) - G(\hat{\theta}) = (\theta - \hat{\theta})^T \phi(a) \ \ \forall a \in \mathbb{R},\ \theta, \hat{\theta} \in \mathbb{R}^p$ and a continuously differentiable function $\phi : \mathbb{R} \to \mathbb{R}^p$.

P1 imposes a sector condition [24], [25] on the friction loss in the system. Due to the dissipative nature of friction the condition is satsified for drilling systems. **P2** imposes an assumption on linear parameterization that is commonly made in the literature on adaptive systems [26], [27], [28], [29], [30], [31]. Although there have been recent developments of adaptive observervers for systems with nonlinear parameterizations [32], [33] we require the property as it simplifies the adaptive observer design significantly. For future reference, note that by the smoothness of $F(\theta, q_{bit})$ we can always find a smooth function $\rho_F(a,b) : \mathbb{R} \times \mathbb{R} \to \mathbb{R}^+$ so that

$$|F(\theta,a) - F(\theta,b)| \le \rho_F(a,b)\,|a-b| \tag{3}$$

for any $a, b \in \mathbb{R}$, by assuming that θ belongs to a known compact set. Some models of friction loss in drilling fluids contain yield stress terms and consequently do not have a unique derivative at zero flow rates. Although this will violate the assumption on smoothness of $F(\theta, q_{bit})$, it can be ensured by making a smooth approximation of the yield stress terms. Furthermore, one can argue that for very small flow rates it is not that important to estimate the flow rate accurately as one can simply replace (2) with

$$p_{bit} = p_c - F_a(\theta, 0) + G_a(\theta), \tag{4}$$

which would be very accurate.

Adaptive observer design based on the model (1) has been pursued in [21], [22], [23]. In [21], $\phi(q_{bit}) = q_{bit}^2$, which is valid for turbulent flow in a Newtonian fluid [34]. In general, the drilling fluid will be non-Newtonian and both turbulent and laminar flow regimes will exist along the flow path [35], [36], [37]. In [22], [23] this was addressed by using a third order polynomial to more accurately describe the friction loss. In this paper, we improve the results in [22], [23] by allowing any functional description (satisfying **P1** and **P2**) of the friction loss in the system.

III. Adaptive Observer Design

To facilitate the observer design and simplify notation we define

$$y = -(1-\alpha)\frac{V_d}{\beta_d} p_p + \alpha \frac{V_a}{\beta_a} p_c \tag{5}$$

for $\alpha \in [0,1]$. Differentiating y with respect to time using (1a)–(1b) gives

$$\dot{y} = q_{bit} + f_y(t) \tag{6}$$

where $f_y(t) = -(1-\alpha) q_{pump}(t) + \alpha(q_{bpp}(t) - q_{choke}(t))$. y is a measured signal where we can weigh the importance of p_p vs. p_c by the use of the parameter α. On some drilling rigs, q_{choke} is not measured, and consequently one would choose $\alpha = 0$ so that the observer does not rely on this measurement.

Based on the model (1) and (6), we will derive an adaptive observer that generates an estimate $\hat{q}_{bit}$ of q_{bit} and an estimate $\hat{\theta}$ of θ. The downhole pressure estimate, $\hat{p}_{bit}$, will be generated by

$$\hat{p}_{bit} = p_c - F_a(\hat{\theta}, \hat{q}_{bit}) + G_a(\hat{\theta}). \tag{7}$$

Let the estimation errors be denoted as $\tilde{q}_{bit} = q_{bit} - \hat{q}_{bit}$ and $\tilde{\theta} = \theta - \hat{\theta}$. Comparing (7) with (2) we see that if $\lim_{t\to\infty}\left(\tilde{\theta}(t), \tilde{q}_{bit}(t)\right) = 0$, then $\lim_{t\to\infty} \hat{p}_{bit}(t) - p_{bit}(t) = 0$. The objective of the adaptive observer design is thus to ensure that $\lim_{t\to\infty}\left(\tilde{\theta}(t), \tilde{q}_{bit}(t)\right) = 0$.

A. Observer Design from [22]

For completeness and to motivate the work presented in this paper we take a brief look at the observer proposed in [22]. Following the approach in [22] an adaptive observer for the flow rate q_{bit} is

$$\hat{q}_{bit} = \beta_1(y,\xi) = \xi + \frac{l_1}{M} y, \tag{8a}$$

$$M\dot{\xi} = p_p - p_c + F(\hat{\theta}, \hat{q}_{bit}) + G(\hat{\theta}) - l_1(\hat{q}_{bit} + f_y(t)), \tag{8b}$$

$$\hat{\theta} = \sigma + \Gamma \eta(y,\xi), \tag{8c}$$

$$\dot{\sigma} = -\Gamma \frac{\partial \eta}{\partial y}(\hat{q}_{bit} + f_y(t)) - \Gamma \frac{\partial \eta}{\partial \xi}\dot{\xi}, \tag{8d}$$

where $\eta : \mathbb{R} \times \mathbb{R} \to \mathbb{R}^p$ is a function that satisfies

$$\frac{\partial \eta}{\partial y} = \phi(\beta_1(y,\xi)), \tag{9}$$

with $\phi(q_{bit})$ defined in **P2**. From Theorem 2 in [22] we know that the above adaptive observer guarantees that the estimation errors $\tilde{q}_{bit}$ and $\tilde{\theta}$ are globally uniformly bounded and $\lim_{t\to\infty} \tilde{q}_{bit}(t) = 0$. If improved parameter identification is desired one can use multiple delayed observers, as in [23], to guarantee tunable convergence rate of both the state and the *parameter* estimation errors. Equation (9) implies an integrability condition on each element of ϕ. For drilling applications ϕ will be a model of the friction loss in the system plus a hydrostatic term. As most drilling fluids are non-Newtonian [35], [36], [37] the model will, in addition to well geometry, depend on several rheological parameters, making it hard (or at least very inconvenient) to find an η that satisfies (9). In the next section, by adding an additional state to the observer, we remove this obstacle.

B. Improved Observer Design

To eliminate the need to solve the partial differential equation (9) we propose the following observer

$$\hat{q}_{bit} = \beta_1(y,\xi) = \xi_1 + \frac{l_1}{M} y, \tag{10a}$$

$$
\begin{aligned}
M\dot{\xi}_1 &= p_p - p_c + F(\hat{\theta}, \xi_2) + G(\hat{\theta}) \\
&\quad - l_1(\hat{q}_{bit} + f_y(t)), &\text{(10b)} \\
M\dot{\xi}_2 &= p_p - p_c + F(\hat{\theta}, \xi_2) + G(\hat{\theta}) \\
&\quad + l_2(\hat{q}_{bit}, \xi_2)(\hat{q}_{bit} - \xi_2), &\text{(10c)} \\
\hat{\theta} &= \sigma + \Gamma\phi(\xi_2)\, y, &\text{(10d)} \\
\dot{\sigma} &= -\Gamma\phi(\xi_2)(\hat{q}_{bit} + f_y(t)) - \Gamma\frac{\partial\phi(\xi_2)}{\partial\xi_2}\dot{\xi}_2 y. &\text{(10e)}
\end{aligned}
$$

The observer has a similar structure as (8) with a few modifications. We have added an additional estimate ξ_2 which is designed to converge sufficiently fast to $\hat{q}_{bit}$. We now use ξ_2 and not $\hat{q}_{bit}$ in the third term on the right hand side of (10b). This ensures that the regressor ϕ will be a function of ξ_2, which has a *known time-derivative*, which is not the case for $\hat{q}_{bit}$. Consequently, the adaptive law (10d)–(10e) can be stated explicitly without the need to solve the partial differential equation (9).

Before we state the main stability result we derive the relevant error dynamics. To that end, we differentiate (10a) with respect to time and use (6) and (10b) to obtain

$$M\dot{\hat{q}}_{bit} = p_p - p_c + F(\hat{\theta}, \xi_2) + G(\hat{\theta}) + l_1\tilde{q}_{bit}. \tag{11}$$

Subtracting (11) from (1c) using **P2** gives

$$
\begin{aligned}
M\dot{\tilde{q}}_{bit} &= F(\theta, q_{bit}) - F(\theta, \hat{q}_{bit}) + F(\theta, \hat{q}_{bit}) - F(\theta, \xi_2) \\
&\quad + \tilde{\theta}^T\phi(\xi_2) - l_1\tilde{q}_{bit}.
\end{aligned} \tag{12}
$$

Letting $e_q = \hat{q}_{bit} - \xi_2$ and subtracting (10c) from (11) give

$$M\dot{e}_q = l_1\tilde{q}_{bit} - l_2(\hat{q}_{bit}, \xi_2)\, e_q. \tag{13}$$

Finally, for the parameter estimation error, we differentiate (10d) with respect to time and use (6) and (10e) to obtain

$$\dot{\tilde{\theta}} = -\dot{\hat{\theta}} = -\Gamma\phi(\xi_2)\,\tilde{q}_{bit}. \tag{14}$$

The stability result for the adaptive observer is summarized in the following Theorem.

Theorem 1: Let l_1, l_2 and Γ be chosen so that $(l_1 - 3L_f) > 0$ and

$$l_2(\hat{q}_{bit}, \xi_2) = \bar{l}_2 + \frac{l_1}{2} + \frac{\rho_F^2(\hat{q}_{bit}, \xi_2)}{(l_1 - L_f)}, \tag{15}$$

with $\bar{l}_2 > 0$ and $\Gamma = \Gamma^T > 0$. The adaptive observer (10) guarantees that, for any initial conditions, the solutions $\tilde{q}_{bit}(t)$, $e_q(t)$ and $\tilde{\theta}(t)$ to (12), (13) and (14) are globally uniformly bounded and satisfy $\lim_{t\to\infty}(\tilde{q}_{bit}(t), e_q(t)) = 0$.

Proof: Consider the function

$$V\left(\tilde{q}_{bit}, e_q, \tilde{\theta}\right) = \frac{1}{2}M\tilde{q}_{bit}^2 + \frac{1}{2}Me_q^2 + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta}. \tag{16}$$

For notational convenience we have omitted the arguments of V in the following. Differentiating (16) with respect to time using (12), (13) and (14) gives

$$
\begin{aligned}
\dot{V} &= (F(\theta, q_{bit}) - F(\theta, \hat{q}_{bit}))\,\tilde{q}_{bit} + (F(\theta, \hat{q}_{bit}) \\
&\quad - F(\theta, \xi_2))\,\tilde{q}_{bit} - l_1\tilde{q}_{bit}^2 + l_1\tilde{q}_{bit}e_q - l_2(\hat{q}_{bit}, \xi_2)\, e_q^2.
\end{aligned}
$$

Using property **P1** and (3)

$$
\begin{aligned}
\dot{V} &\le L_f\tilde{q}_{bit}^2 + \rho_F(\hat{q}_{bit}, \xi_2)\,|e_q|\,|\tilde{q}_{bit}| - l_1\tilde{q}_{bit}^2 \\
&\quad + l_1\tilde{q}_{bit}e_q - l_2(\hat{q}_{bit}, \xi_2)\, e_q^2.
\end{aligned} \tag{17}
$$

By Young's inequality we have

$$
\begin{aligned}
\rho_F(\hat{q}_{bit}, \xi_2)\,|e_q|\,|\tilde{q}_{bit}| &\le \frac{(l_1 - L_f)}{4}\tilde{q}_{bit}^2 + \frac{\rho_F^2(\hat{q}_{bit}, \xi_2)}{(l_1 - L_f)}e_q^2 \\
|\tilde{q}_{bit}e_q| &\le \frac{1}{2}\tilde{q}_{bit}^2 + \frac{1}{2}e_q^2
\end{aligned}
$$

so that

$$
\begin{aligned}
\dot{V} &\le -\left(l_1 - L_f - \frac{(l_1 - L_f)}{4} - \frac{1}{2}l_1\right)\tilde{q}_{bit}^2 \\
&\quad - \left(l_2(\hat{q}_{bit}, \xi_2) - \frac{1}{2}l_1 - \frac{\rho_F^2(\hat{q}_{bit}, \xi_2)}{(l_1 - L_f)}\right)e_q^2 \\
&= -\frac{1}{4}(l_1 - 3L_f)\,\tilde{q}_{bit}^2 \\
&\quad - \left(l_2(\hat{q}_{bit}, \xi_2) - \frac{l_1}{2} - \frac{\rho_F^2(\hat{q}_{bit}, \xi_2)}{(l_1 - L_f)}\right)e_q^2.
\end{aligned} \tag{18}
$$

Choosing l_1, $l_2(\hat{q}_{bit}, \xi_2)$ and $\bar{l}_2$ as specified in Theorem 1 we get

$$
\begin{aligned}
\dot{V} &\le -\min\left(\bar{l}_2, \frac{(l_1 - 3L_f)}{4}\right)\left(\frac{1}{4}\tilde{q}_{bit}^2 + e_q^2\right) &\text{(19)} \\
&\le -\lambda\left(\tilde{q}_{bit}^2 + e_q^2\right), &\text{(20)}
\end{aligned}
$$

for some $\lambda > 0$. In view of (16) and (20) the result follows by applying the LaSalle-Yoshizawa theorem (see e.g. [27]). ■

C. Improved Parameter Identification Properties

When estimating more than one parameter the adaptive law used in the adaptive observers above has poor parameter identification properties. Since the proposed observer (10) is of the form considered in [23] we can improve the parameter identification properties by the use of multiple delayed observers. The method in [23] estimates the unmeasured state q_{bit} not only at the current time t, but also at $t - \tau^1$, $t - \tau^2, ..., t - \tau^N$, with $\tau^0 = 0$ and $\tau^0 < \tau^1 < \tau^2 ... \tau^{N-1} < \tau^N$. The motivation behind the scheme is to bring more "memory" into the parameter estimation algorithm. For notational convenience we define $\xi = [\xi_1, \xi_2]^T$, $\beta = [\beta_1, \beta_2]^T$ with $\beta_2(y, \xi) = \xi_2$ and

$$
\psi(t, y, \xi) = \frac{1}{M}\times \begin{bmatrix} p_p(t) - p_c(t) - l_1(\beta_1(y, \xi) + f_y(t)) \\ p_p(t) - p_c(t) + l_2(\beta_1(y, \xi), \xi_2)(\beta_1(y, \xi) - \xi_2) \end{bmatrix}, \tag{21}
$$

$$\Phi(q_{bit}) = \begin{bmatrix} \phi^T(q_{bit}) \\ \phi^T(q_{bit}) \end{bmatrix} \tag{22}$$

so that (10b)–(10c) can be written compactly as

$$\dot{\xi} = \psi(t, y, \xi) + \Phi(\xi_2)\,\hat{\theta}, \tag{23}$$

where the gains l_1 and $l_2(\beta_1(y, \xi), \xi_2)$ are chosen as specified in Theorem 1. We also define $K_N = \{0, 1, ..., N\}$, and let $k \in K_N$ and

$$y^k = y^k(t) = y\left(t - \tau^k\right), \tag{24}$$

$$q_{bit}^k = q_{bit}^k(t) = q_{bit}\left(t-\tau^k\right), \quad (25)$$
$$\hat{q}_{bit}^k = \hat{q}_{bit}^k(t) = \beta_1\left(y^k, \xi^k\right), \quad (26)$$
$$\tilde{q}_{bit}^k = q_{bit}^k(t) - \hat{q}_{bit}^k(t), \quad (27)$$
$$e_q^k = q_{bit}^k(t) - \beta_2\left(y^k, \xi^k\right), \quad (28)$$
$$\phi^k(t) = \phi\left(q_{bit}^k(t)\right), \quad (29)$$
$$\hat{\phi}^k(t) = \phi\left(\xi^k(t)\right), \quad (30)$$
$$f_y^k(t) = f_y\left(t-\tau^k\right), \quad (31)$$
$$\tilde{q}_{bit}^T = \left[\tilde{q}_{bit}^0, ..., \left(\tilde{q}_{bit}^N\right)^T\right]^T, \quad (32)$$
$$e_q^T = \left[e_q^0, ..., \left(e_q^N\right)^T\right]^T \quad (33)$$

with $\xi^k = \xi^k(t)$ the solution to

$$\dot{\xi}^k = \psi\left(t-\tau^k, y^k, \xi^k\right) + \left(\hat{\theta}(t)\right)^T \hat{\phi}^k(t). \quad (34)$$

Although (34) looks like a shifted (in time) version of (23), it is not, due to the fact that we use the same parameter estimate $\hat{\theta}(t)$ for all k. The proposed parameter update law is

$$\hat{\theta} = \sigma + \Gamma \sum_{k=0}^{N} \hat{\phi}^k(t)\, y^k, \quad (35a)$$
$$\dot{\sigma} = \Gamma \sum_{k=0}^{N} \left[-\hat{\phi}^k(t)\left(\hat{q}_{bit}^k + f_y^k(t)\right) - \frac{\partial \phi\left(\xi_2^k\right)}{\partial \xi_2^k}\dot{\xi}_2^k y^k\right]. \quad (35b)$$

For $N = 0$ we see that (35) reduces to (10d)–(10e). However, for $N > 0$, it contains more "memory" than (10d)–(10e). In [23], it is proved that this improves the parameter identification properties of the adaptive observer. Before we state the main stability properties for the adaptive observer (26), (34)–(35) we derive the neccessary error dynamics. The dynamics are derived in a similar way to what was done in Section III-B. First, differentiating (26) with respect to time using (6) and (34) gives

$$\begin{aligned} M\dot{\tilde{q}}_{bit}^k &= F\left(\theta, q_{bit}^k\right) - F\left(\theta, \hat{q}_{bit}^k\right) + F\left(\theta, \hat{q}_{bit}^k\right) \\ &\quad - F\left(\theta, \xi_2^k\right) + \tilde{\theta}^T \hat{\phi}^k(t) - l_1 \tilde{q}_{bit}^k \end{aligned} \quad (36)$$

for all $k \in K_N$. Similarly we get

$$M\dot{e}_q^k = l_1 \tilde{q}_{bit}^k - l_2\left(\hat{q}_{bit}^k, \xi_2^k\right) e_q^k, \ \forall\ k \in K_N. \quad (37)$$

Finally, by differentiating (35a) with respect to time using (6) and (35b) gives

$$\dot{\tilde{\theta}} = -\dot{\hat{\theta}} = -\Gamma \sum_{k=0}^{N} \hat{\phi}^k(t)\, \tilde{q}_{bit}^k. \quad (38)$$

The improved parameter identification properties rely on the following assumption of persistency of excitation (PE). Assume that there exists $b_M \geq b_m > 0$ so that

$$b_m I \leq \sum_{k=0}^{N} \left(\phi^k(t)\right)^T \phi^k(t) \leq b_M I. \quad (39)$$

The following result follows from Theorem 8 and Remark 9 in [23].

Corollary 2: The origin of (36), (37) and (38) is uniformly globally stable and $\lim_{t\to\infty}\left(\tilde{q}_{bit}(t), e_q(t)\right) = 0$. Moreover, if Assumption (39) holds, then the origin is uniformly globally asymptotically stable, semi-globally exponentially stable and solutions satisfy

$$\left\| \begin{bmatrix} \tilde{q}_{bit}(t) \\ \tilde{e}_q(t) \\ \tilde{\theta}(t) \end{bmatrix} \right\| \leq \kappa \left\| \begin{bmatrix} \tilde{q}_{bit}(t_0) \\ \tilde{e}_q(t_0) \\ \tilde{\theta}(t_0) \end{bmatrix} \right\| e^{-\alpha(\|w(t_0)\|)(t-t_0)} \quad (40)$$

for some $\kappa > 0$ and any desirable value of $\alpha(\|w(t_0)\|) > 0$ by appropriate choice of l_1, $\bar{l}_2$ and Γ.

IV. Simulation Example

To illustrate the use and performance of the proposed observer, it was tested on a data set obtained from experiments performed in an offshore well. The experiments were performed in a closed well without drilling and the data contains a so-called pipe connection procedure. The pipe connection procedure involves ramping the main pump from full flow rate to zero flow rate and up again, ensuring that the system is sufficiently excited, so that the PE condition (39) is satisfied. To obtain enough data to run the estimation algorithm it was neccessary to duplicate the data. The parameters used in the model (1) are given in Table I. To obtain simple models of the frictional pressure losses two polynomials were fitted to a different data set giving

$$F_a(\theta, q_{bit}) = -\theta_1\left(304.9 q_{bit} + 5188\,|q_{bit}|\, q_{bit}\right), \quad (41)$$
$$F_d(\theta, q_{bit}) = -\theta_1\left(366.6 q_{bit} + 146570\,|q_{bit}|\, q_{bit}\right), \quad (42)$$

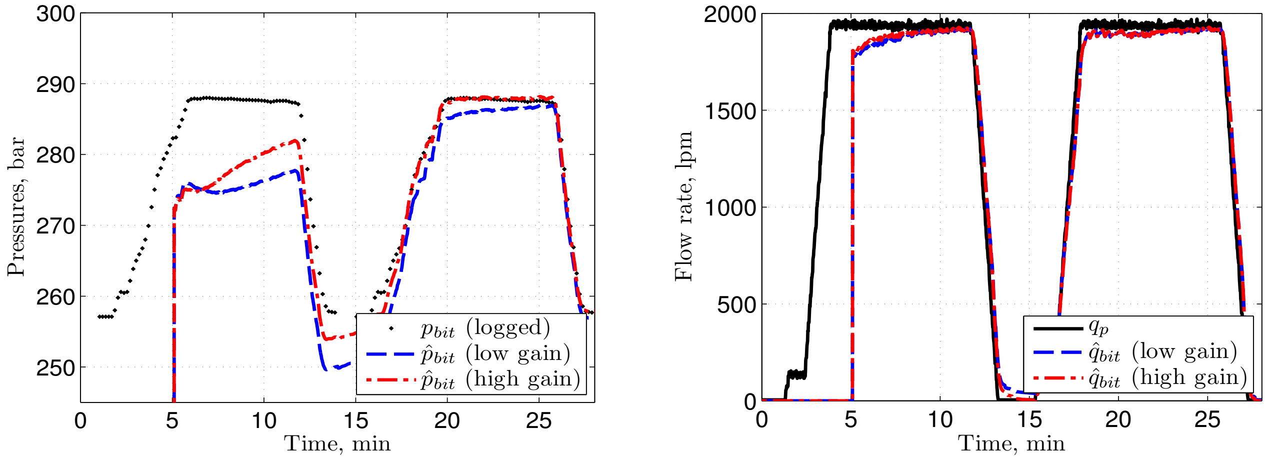
for the friction in the annulus and drill string, respectively. θ_1 is an uncertain correction factor we want to estimate. For the hydrostatic pressure we have

$$G_a(\theta) = \theta_2 \rho g h_{dh}, \quad (43)$$
$$G_d(\theta) = G_d = \rho g h_{dh}, \quad (44)$$

for the annulus and drill string, respectively. θ_2 is a correction factor we want to estimate that allows for variations in the density in the annulus but not the drill string. This is a reasonable assumption since the drilling fluid in the annulus contains an uncertain amount of cuttings due to drilling while the fluid in the drill string is pure drilling mud of known density. For the data set given we know the "true" θ_1 and θ_2 to be close to one, as (41)–(42) is fitted to data with $\theta_1 \equiv 1$ and there is no drilling and thus no cuttings in the annulus. The adaptive observer will provide estimates of q_{bit} and the correction factors θ_1 and θ_2 so that the downhole pressure estimate can be generated according to (7).

To apply the improved observer we need to verify that the properties **P1** and **P2** are satisfied. To that end note that both F_a and F_d are monotonically decreasing in q_{bit}, this implies that **P1** is satisfied with $L_f = 0$. The linear parameterization of (41)–(44) implies that **P2** is satisfied with

$$\phi(q_{bit}) = \begin{bmatrix} 671.5 q_{bit} + 151758\,|q_{bit}|\, q_{bit} \\ -\rho g h_{dh} \end{bmatrix}. \quad (45)$$



(a) Downhole pressure and its estimates.

(b) Main pump flow rate and the estimate of the unmeasured flow rate.

Fig. 2: Adaptive observer used to estimate the unmeasured down hole pressure and the unmeasured bit flow rate.

By assuming that $0.5 \leq \theta_1 \leq 1.5$, ρ_F in (3) can easily be constructed as

$$\rho_F(a,b) = \begin{cases} \frac{F(1.5,a)-F(1.5,b)}{a-b} & a-b > \varepsilon \\ \frac{F(1.5,a+\varepsilon)-F(1.5,a-\varepsilon)}{2\varepsilon} & a-b \leq \varepsilon \end{cases}. \quad (46)$$

for a small number ε, chosen here to be $\varepsilon = \frac{1}{60000}$ corresponding to 1 liter per minute. Note also that $\frac{\partial\phi(\xi_2)}{\partial\xi_2}$ can easily be implemented numerically with sufficient accuracy as

$$\frac{\partial\phi(\xi_2,\rho)}{\partial\xi_2} \approx \frac{\phi(\xi_2+\varepsilon)-\phi(\xi_2-\varepsilon)}{2\varepsilon}. \quad (47)$$

To improve the parameter identification properties of the observer we implemented 20 delayed observers each separated by 45s. That is, we chose $\tau^k = kT$ with $k = \{0, 1, ..., 19\}$ and $T = 45$. The delayed observers thus cover a window of 15 min, corresponding to one pipe connection scenario. The length of this window is an important parameter as it is a trade off between satisfying the PE condition (39) and tracking the uncertain parameters. For the weighting in the measurement equation (5), $\alpha = 0.5$ was chosen, while the injection gains were chosen as $l_1 = 5000$ and $\bar{l}_2 = 200$. To illustrate that the convergence rate of the estimation error can be tuned we chose two sets of adaptation gains, $\Gamma_h = 10^{-3}\times\text{diag}([3, 0.3])$ and $\Gamma_l = 0.5\Gamma_h$, corresponding to a high and low gain, respectively. The observer was started at around $t_0 = 5$min with initial conditions $\xi_1(t_0) = q_p(t_0) - \frac{l_1}{M}y(t_0)$, $\xi_2(t_0) = q_p(t_0)$, $\hat{\theta}_1(t_0) = 1.3$ and $\hat{\theta}_2(t_0) = 0.95$, corresponding to a 30% error in the friction pressure estimate and a 5% error in the density estimate. Fig. 2 shows that the estimates of the downhole pressure and the bit flow rate converge faster when using a high gain compared to a low gain as expected. From Fig. 2a we can see that in particular the high gain downhole pressure estimate is very accurate after initial transients. Looking at the model (1) one would expect $q_p = q_{bit} = q_{choke}$ in steady state ($q_{bpp} = 0$ in the data set), however as Fig. 2b shows, the estimated bit flow rate converges to a value slightly smaller than q_p. The reason for this is that the measured q_{choke} (not

Parameter	Value	Description
h_{dh}	$1632m$	True vertical depth of bit
ρ	$1580\frac{kg}{m^3}$	Density of mud
g	$9.81\frac{m}{s^2}$	Gravitational acceleration
$\beta_d = \beta_a$	$20000bar$	Effective bulk modulus
V_d	$15.5m^3$	Volume of mud in drill string
V_a	$75.4m^3$	Volume of mud in annulus
M	$4100 \times 10^5 \frac{kg}{m^4}$	Integrated density per cross section

TABLE I: Parameters used in the model.

shown) is slightly smaller than q_p which affects the estimate $\hat{q}_{bit}$. Fig. 3 shows that $\hat{\theta}_1$ converges to a value sligtly higher than one. The reason for this is believed to be that the friction models (41)–(42) were fitted to steady state data using q_p. As q_p is slightly larger than $\hat{q}_{bit}$ the frictional pressure loss must be compensated by a slightly higher $\hat{\theta}_1$. Fig. 3 also shows that the density correction factor $\hat{\theta}_2$ converges to one as expected.

V. Conclusion

This paper addresses the topic of adaptive estimation of downhole pressure for managed pressure drilling. The proposed adaptive observer allows for general models of the frictional pressure drop in the drilling fluid and thus extends existing work on adaptive estimation based on low order models. The adaptive observer has been tested on data from an offshore well. It is shown that the observer can estimate both the unmeasured bit flow rate, and two uncertain correction factors, giving a robust estimate of the downhole pressure in the presence of significant parametric uncertainties.

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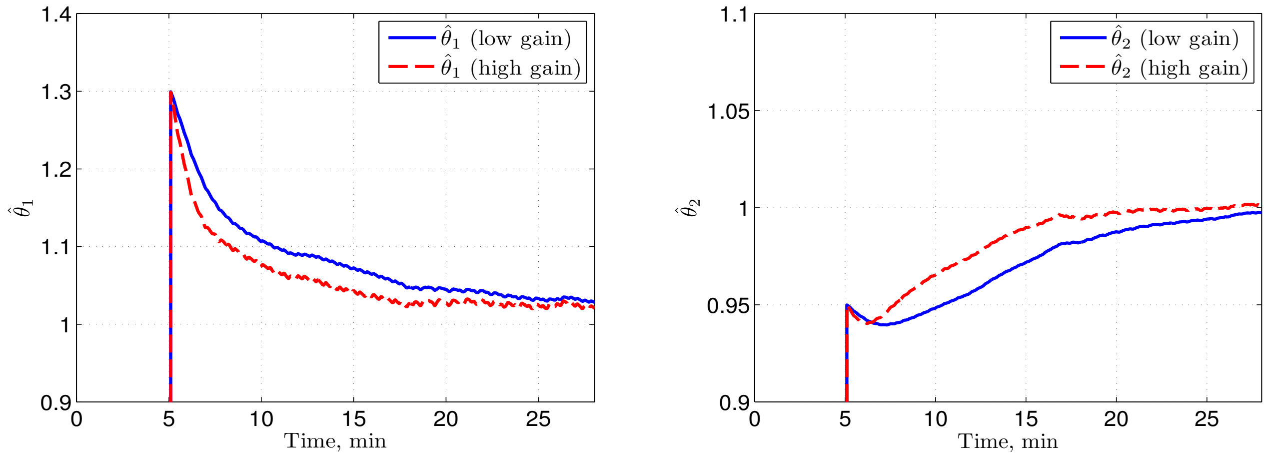

Fig. 3: Parameter estimates.

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